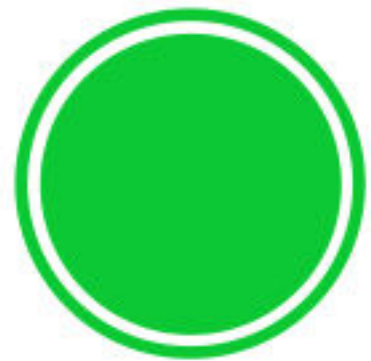


This pdf is collection of **Pathfinder video solutions** (MCQ/BYUs) available on my youtube channel  [Arihant Senior - Ankit Singhvi](#)

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Sept-2023



Revision Book for **Pathfinder XI**

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by Ankit Singhvi

Ankit Singhvi
Dual Degree, IIT Madras

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	1	1a-32	35	1b-30	69	2a-22	103	2b-9	137	2b-38	171	3a-10	205	3b-16	239	4a-18	273	4b-26	307		
	2		36	1b-31	70	2a-23	104	2b-10	138	2b-39	172	3a-11	206	3b-17	240	4a-19	274	4b-27	308		
	3		37	1b-32	71	2a-24	105	2b-11	139	2b-40	173	3a-12	207	3b-18	241	4a-20	275		309		
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1a-3	6	1b-3	40	1b-35	74	2a-27	108	2b-13	142	2b-43	176	3a-15	210	3b-21	244	4a-23	278	4b-29	312		
1a-4	7	1b-4	41	1b-36	75		109	2b-14	143	2b-44	177	3a-16	211	3b-22	245		279	4b-30	313		
1a-5	8	1b-5	42	1b-37	76	2a-28	110	2b-15	144	2b-45	178	3a-17	212	3b-23	246	4a-25	280	4b-31	314		
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1a-7	10	1b-7	44	1b-39	78	2a-30	112		146	2b-46	180	3a-19	214	3b-25	248	4b-1	282	4b-33	316		
1a-8	11		45	1b-40	79	2a-31	113	2b-17	147	2b-47	181		215	3b-26	249	4b-2	283	4b-34	317		
	12	1b-8	46	2a-1	80	2a-32	114	2b-18	148	2b-48	182		216	3b-27	250	4b-3	284	4b-35	318		
1a-9	13	1b-9	47	2a-2	81	2a-33	115	2b-19	149	2b-49	183	3a-20	217	3b-28	251	4b-4	285		319		
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1a-17	22		56	2a-9	90	2a-45	124	2b-27	158	2b-57	192	3b-4	226	4a-6	260	4b-13	294	5a-1	328		
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1a-19	24	1b-19	58	2a-11	92	2a-48	126	2b-30	160		194	3b-6	228	4a-8	262	4b-15	296	5a-3	330		
1a-20	25	1b-20	59	2a-12	93		127	2b-31	161	3a-1	195		229	4a-9	263	4b-16	297	5a-4	331		
1a-21	26	1b-21	60	2a-13	94		128	2b-32	162	3a-2	196	3b-7	230	4a-10	264	4b-17	298	5a-5	332		
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1a-22	28	1b-23	62	2a-15	96	2b-2	130	2b-34	164	3a-4	198	3b-9	232	4a-12	266	4b-19	300	5a-7	334		
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a MCQ b BYU				1 kinematics 2 NLM				3 WPE 4 Impulse - momentum				5 Rotation 6 Gravitation				7 Fluids 8 Thermo		9 Properties of Matter 10 SHM - Waves	
5a-14	341	5b-20	375	6a-3	409	7b-2	443	8a-2	477	8b-4	511		545	10a-5	579	10b-29	613		
5a-15	342	5b-21	376	6a-4	410	7b-3	444	8a-3	478	8b-5	512	8b-34	546	10a-6	580	10b-30	614		
5a-16	343	5b-22	377	6a-5	411	7b-4	445	8a-4	479		513	8b-35	547	10a-7	581	10b-31	615		
5a-17	344	5b-23	378	6a-6	412	7b-5	446	8a-5	480		514	8b-36	548	10a-8	582				
5a-18	345	5b-24	379	6a-7	413	7b-6	447	8a-6	481	8b-6	515	8b-37	549	10a-9	583				
5a-19	346	5b-25	380	6a-8	414	7b-7	448	8a-7	482	8b-7	516	8b-38	550	10a-10	584				
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5a-22	350	5b-28	384	6b-2	418	7b-10	452	8a-11	486	8b-10	520	9a-2	554	10b-4	588				
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5a-23	352	5b-30	386	6b-4	420	7b-12	454	8a-13	488	8b-12	522	9a-4	556	10b-6	590				
5b-1	353	5b-31	387	6b-5	421	7b-13	455	8a-14	489	8b-13	523	9a-5	557	10b-7	591				
5b-2	354	5b-32	388		422	7b-14	456	8a-15	490		524	9a-6	558	10b-8	592				
5b-3	355	5b-33	389	6b-6	423	7b-15	457	8a-16	491	8b-14	525	9b-1	559	10b-9	593				
5b-4	356		390	6b-7	424	7b-16	458	8a-17	492	8b-15	526	9b-2	560	10b-10	594				
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5b-8	361	5b-36	395	7a-2	429	7b-21	463		497	8b-20	531	9b-7	565	10b-15	599				
5b-9	362	5b-37	396	7a-3	430	7b-22	464	8a-22	498	8b-21	532	9b-8	566	10b-16	600				
5b-10	363	5b-38	397		431	7b-23	465	8a-23	499	8b-22	533	9b-9	567	10b-17	601				
5b-11	364	5b-39	398	7a-4	432	7b-24	466	8a-24	500	8b-23	534	9b-10	568	10b-18	602				
	365	5b-40	399	7a-5	433	7b-25	467	8a-25	501	8b-24	535	9b-11	569	10b-19	603				
5b-12	366	5b-41	400	7a-6	434	7b-26	468	8a-26	502	8b-25	536	9b-12	570	10b-20	604				
5b-13	367	5c-10	401	7a-7	435		469	8a-27	503	8b-26	537	9b-13	571	10b-21	605				
5b-14	368	5c-22	402	7a-8	436	7b-27	470	8a-28	504	8b-27	538	9b-15	572		606				
5b-15	369		403	7a-9	437	7b-28	471		505	8b-28	539	9b-16	573	10b-22	607				
5b-16	370	5c-23	404	7a-10	438	7b-29	472	8a-30	506	8b-29	540	9b-17	574	10b-23	608				
5b-17	371	5c-24	405	7a-11	439	7b-30	473		507	8b-30	541	10a-1	575	10b-24	609				
5b-18	372	5c-25	406	7a-12	440	7b-31	474	8b-1	508	8b-31	542	10a-2	576	10b-25	610				
	373	6a-1	407	7a-13	441	7b-32	475	8b-2	509	8b-32	543	10a-3	577	10b-27	611				
5b-19	374	6a-2	408	7b-1	442	8a-1	476	8b-3	510	8b-33	544	10a-4	578	10b-28	612				

1. From a town, cars start at regular intervals of 30 s and run towards another town with a constant speed of 60 km/h. At some point of time, all the cars simultaneously have to reduce their speeds to 40 km/h due to bad weather conditions. What will be the time interval between arrivals of the cars at the second town during the bad weather?

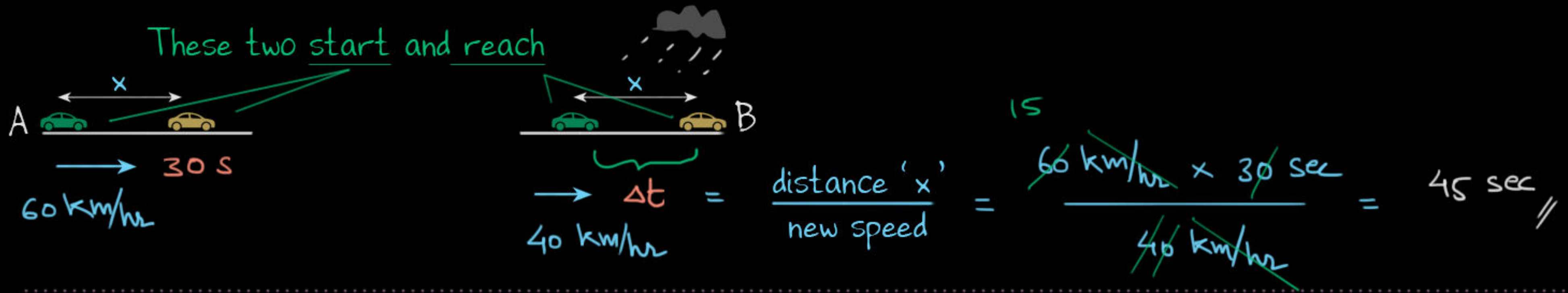
(a) 20 s

(b) 30 s

(c) 40 s

✓(d) 45 s

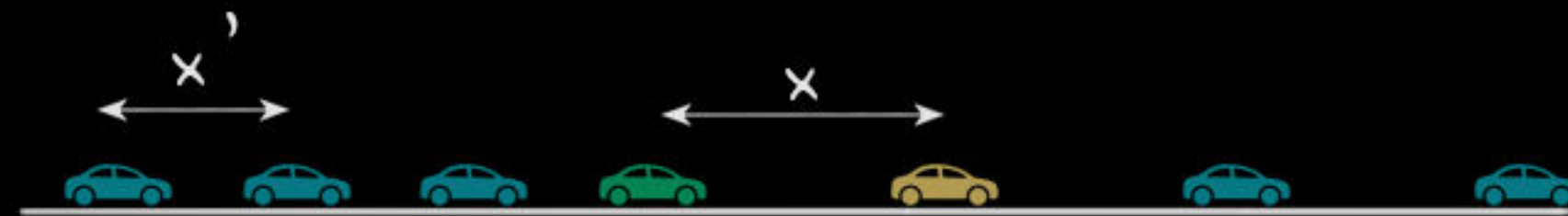
Concept: When bad weather begins, gap 'x' won't change as their relative velocity is still zero.



Note:

Initial gap $x = 60 \text{ km/hr} \times 30 \text{ sec}$

Gap later, $x' = 40 \text{ km/hr} \times 30 \text{ sec}$ and much later cars will resume to arrive in 30 seconds gap again.



2. Sam used to walk to school every morning, and it takes him 20 min. Once on his way, he realized that he had forgotten his homework notebook at home. He knew that if he continued walking to school at the same speed, he would be there 8 min before the bell, so he went back home for the notebook and arrived the school 10 min after the bell. If he had walked all the way with his usual speed, what fraction of the way to school had he covered till the moment he turned back?

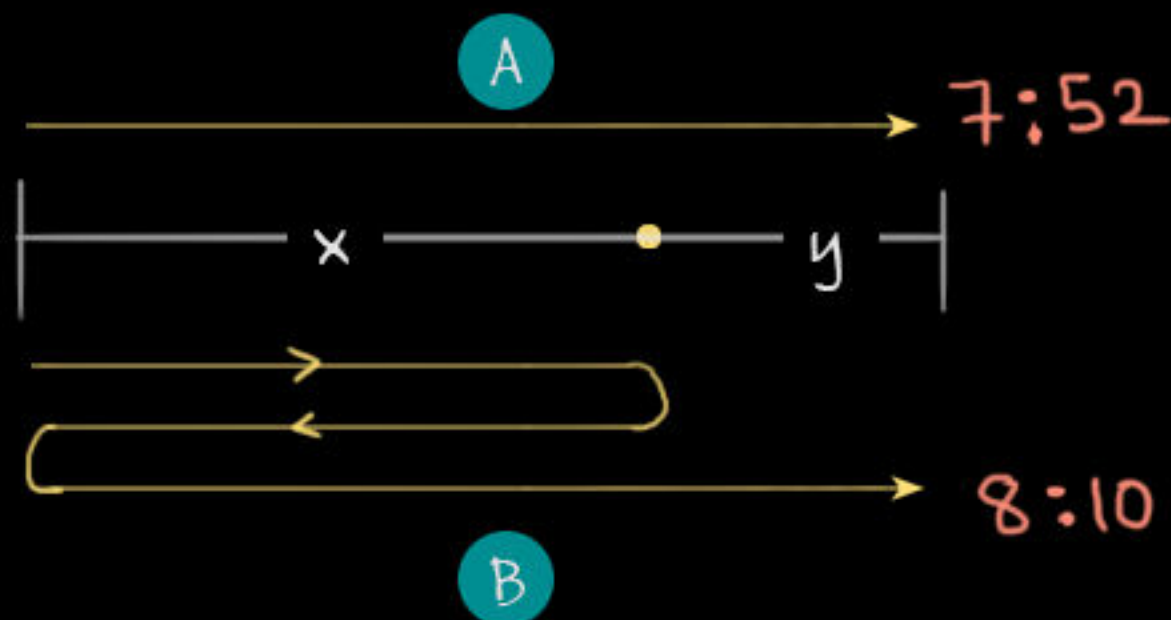
(a) $8/20$

✓ (b) $9/20$

(c) $10/20$

(d) $12/20$

Concept: Simplify. Fix the time, to avoid confusion. Here, let's say the school bell rings at 8:00 AM.



Clearly, path B ($2x+y$) takes 18 mins more than path A ($x+y$)

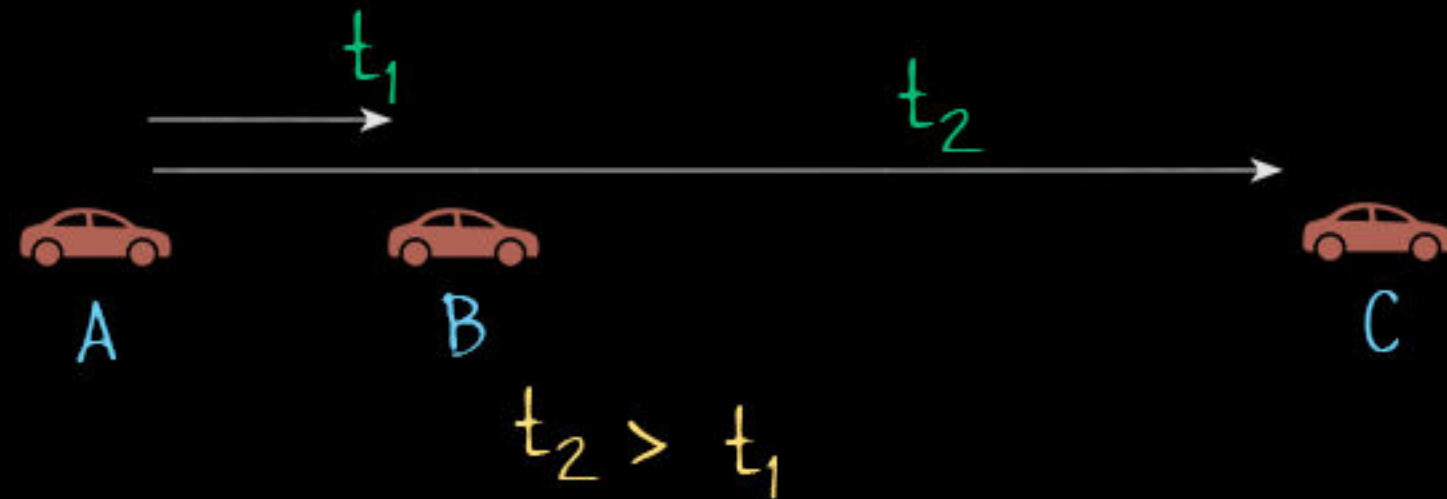
∴ x takes 9 minutes. And,
 $x+y$ takes 20 minutes (given)

Fraction of distance = Fraction of time

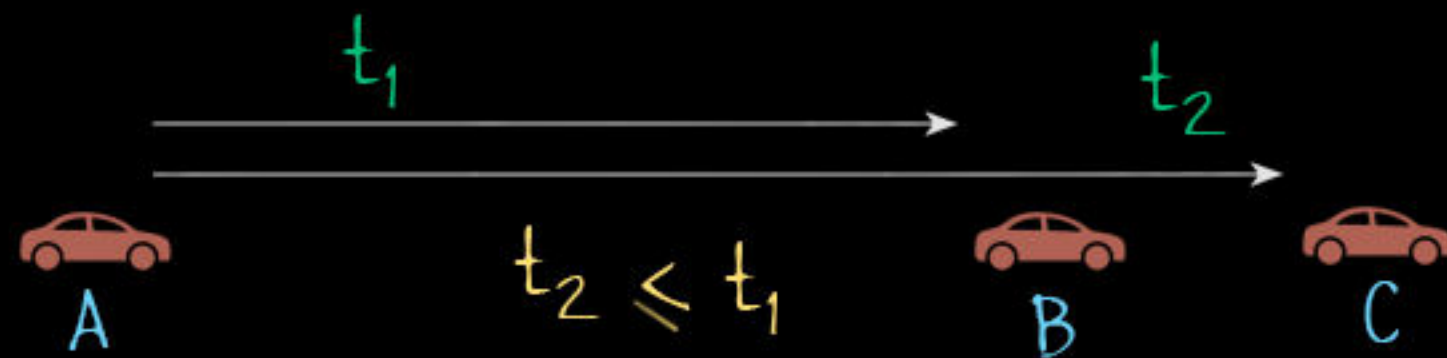
$$\therefore \frac{x}{x+y} = \frac{9}{20} //$$

3. Three particles A, B and C start moving simultaneously with constant velocities from three places. The starting places are collinear and that of B is somewhere in between those of A and C. In the absence of C, particles A and B would have collided t_1 time after they started and in the absence of B, particles A and C would have collided t_2 time after they started. What would have happened, if A were not present?

Simplify as: three cars A, B, C on a road. A overtakes B and C in t_1 and t_2 respectively



⇒ A is going at highest speed and relative low speeds of B and C can be anything.



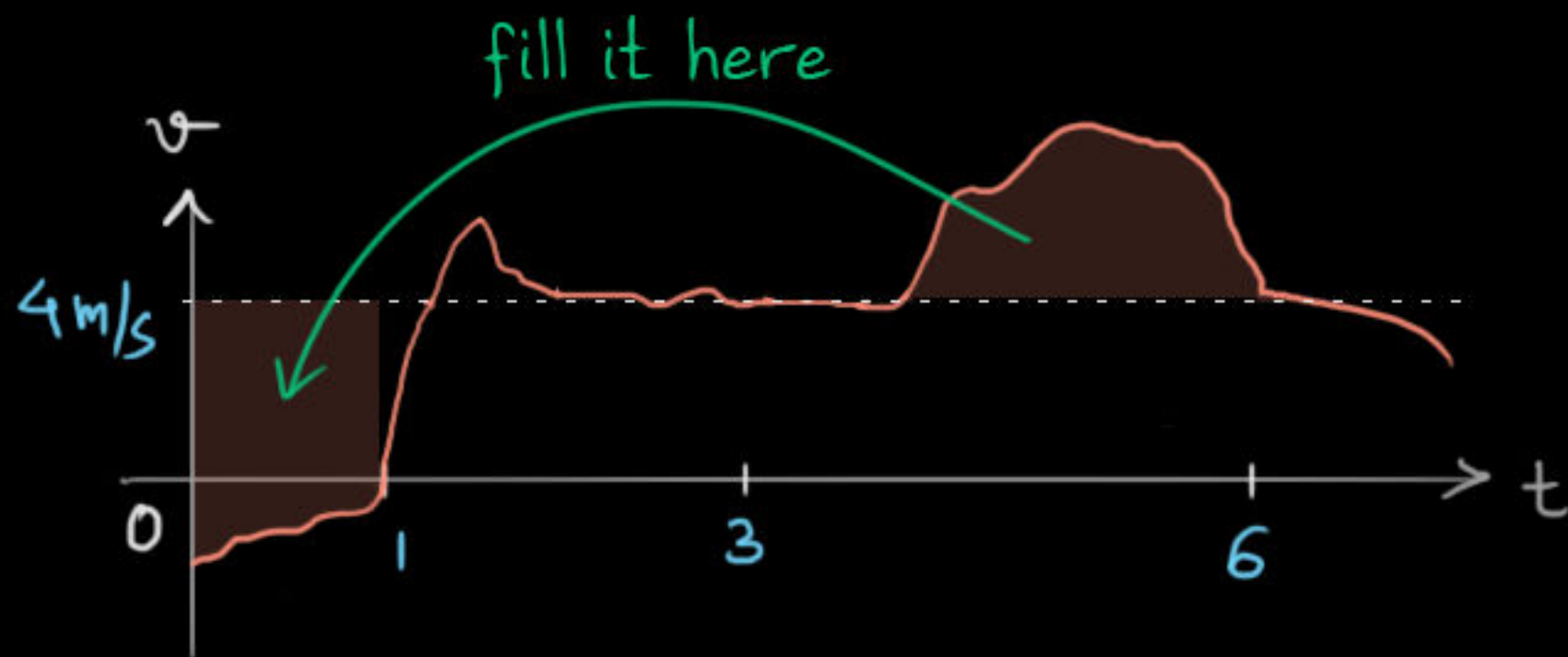
⇒ A will overtake C in t_2 , while B is still ahead.
∴ B will overtake C before A in $t < t_2$.

4. A particle moving continuously in the positive x -direction passes the positions $x = 9 \text{ m}$ and 17 m at the instants $t = 1 \text{ s}$ and 3 s respectively. Its average velocities in the time intervals $[1 \text{ s}, 3 \text{ s}]$ and $[0 \text{ s}, 6 \text{ s}]$ are equal. Which of the following statements is/are correct?

- (a) It was at $x = 5 \text{ m}$ at $t = 0 \text{ s}$.
- (b) It is moving with a uniform speed.
- (c) Average velocity in the interval $[3 \text{ s}, 6 \text{ s}]$ is 4 m/s .
- ✓(d) Information is insufficient to decide any of the above.

Data is rather unusable.

Following graph disproves a, b, c while satisfying what's given in problem.



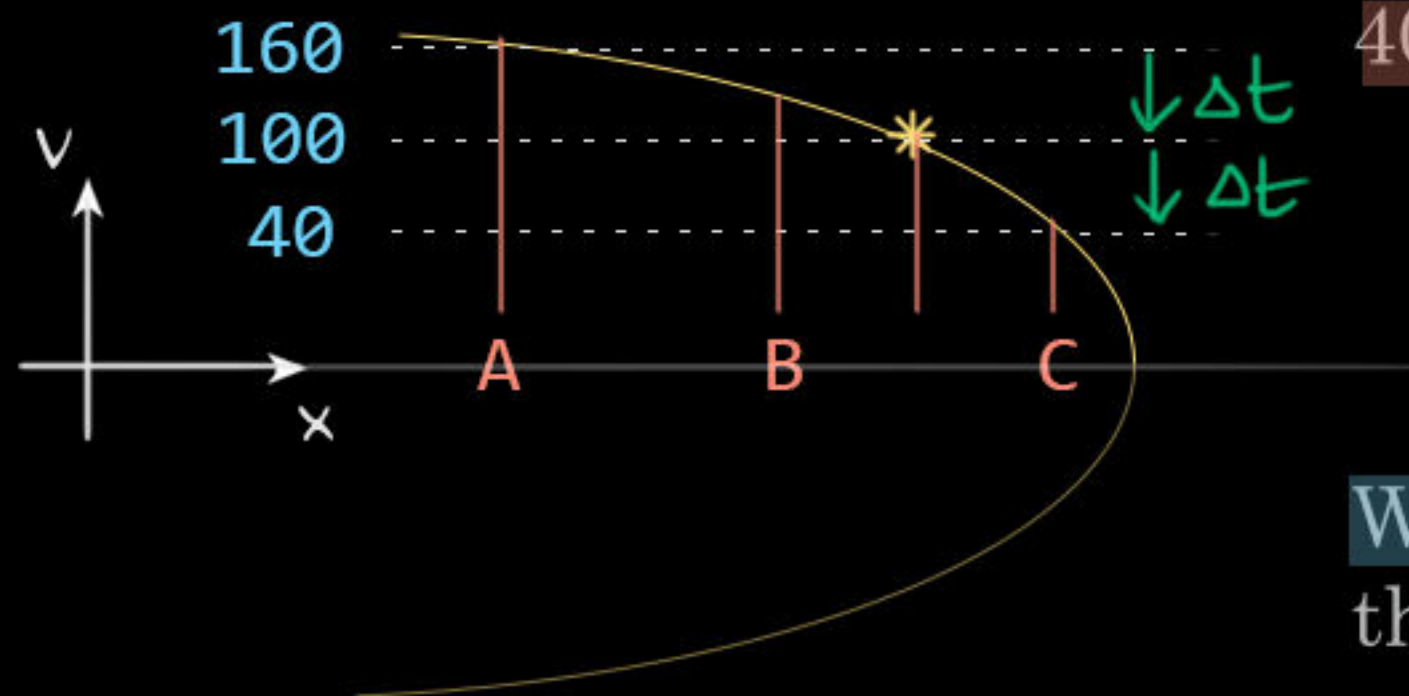
a) $x > 9$ at $t = 0$ ($\because v < 0$ before $t = 1$)

b) clearly non uniform

c) $\langle v \rangle_{t=[3,6]} > 4$

Method 1

$$v^2 = u_0^2 - 2ax$$



5. When a car passes mark-A, driver applies brakes. Thereafter reducing speed uniformly from 160 km/h at A, the car passes mark C with a speed 40 km/h. The marks are at equal distances on the road as shown below.



Mark-A

Mark-B

Mark-C

Where on the road was the car moving with a speed 100 km/h? Neglect the size of the car as compared to the distances involved.

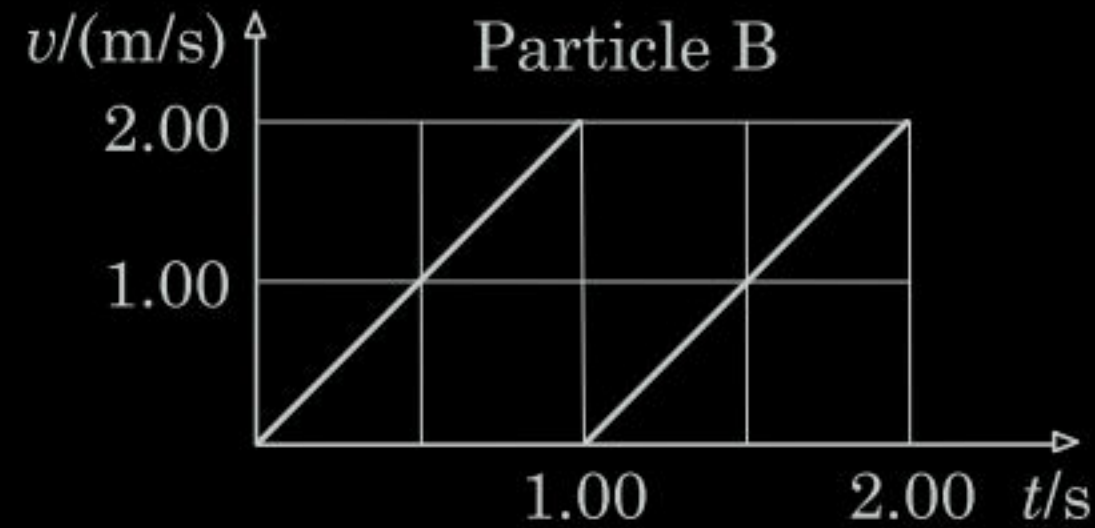
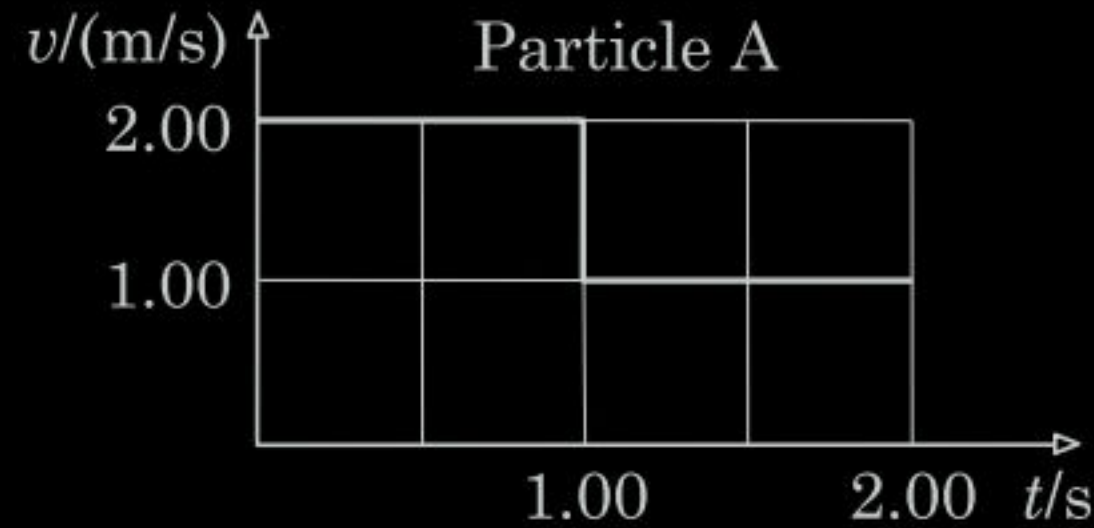
- (a) At mark-B
- (b) Between mark-A and mark-B
- ✓(c) Between mark-B and mark-C
- (d) Information is insufficient to decide.

Method 2

Velocity drop is equal (60 km/h), and acceleration is uniform
 ∴ time t is equal too in both parts.

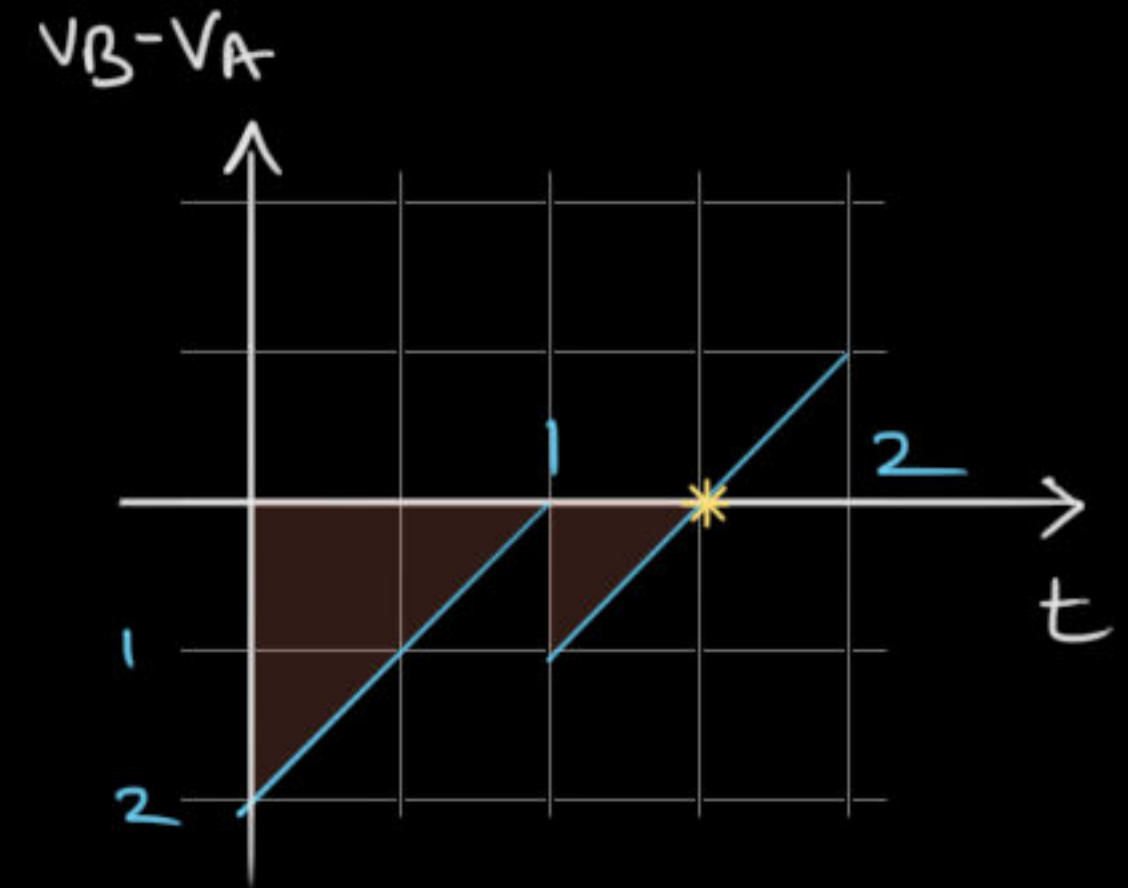
In first part (0- t), average velocity ($\frac{160+140}{2}$) is more than second half ($\frac{100+40}{2}$).
 ∴ It will travel more in first t seconds.

6. Two particles A and B start from the same point and move in the positive x -direction. In a time interval of 2.00 s after they start, their velocities v vary with time t as shown in the following figures. What is the maximum separation between the particles during this time interval?



- (a) 1.00 m
(c) 1.50 m

- ✓ (b) 1.25 m
(d) 2.00 m



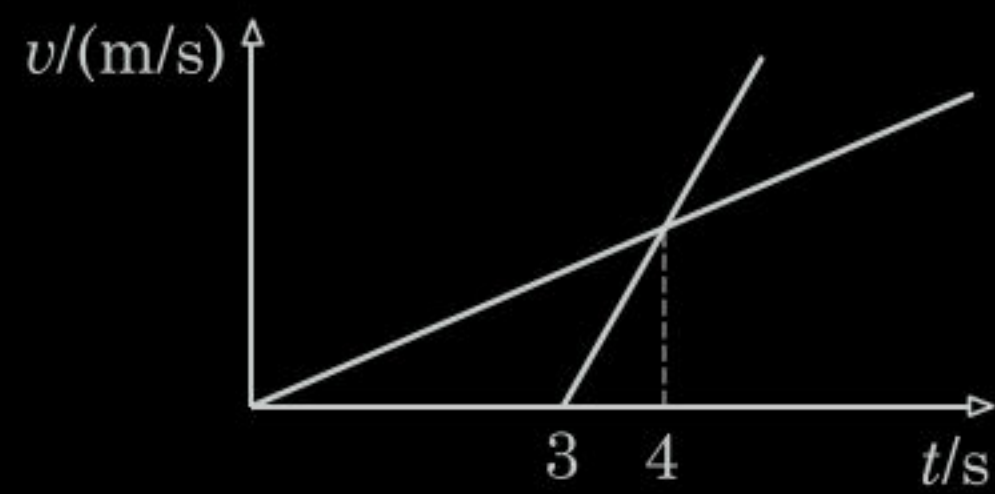
Concept: Separation is area under $v_{rel}-t$ curve.

Concept: Separation need not be minimum or maximum everytime v_{rel} is zero.

For example at $t=1\text{sec}$ here.

$$\therefore \text{Maximum separation} = \frac{1}{2}(1)(2) + \frac{1}{2}(0.5)(1)$$

$$= 1.25 //$$

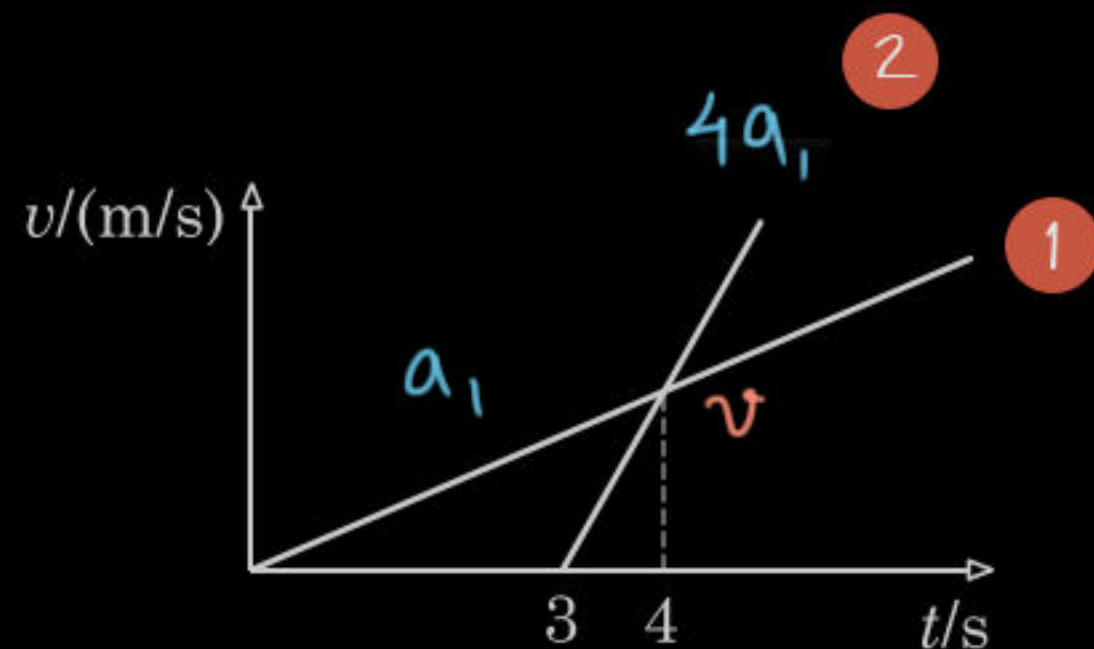


7. A material particle is chasing another one and both of them are moving on the same straight line. After they pass a particular point, their velocities v vary with time t as shown in the figure. When will the chase end?

- (a) 4.0 s
(c) 12 s
✓(b) 6.0 s
(d) Insufficient information.

Concept: Compare the slopes.

$$a_1 = \frac{v}{4}, \quad a_2 = \frac{v}{1} = 4a_1$$



Chase ends, when displacements are same.

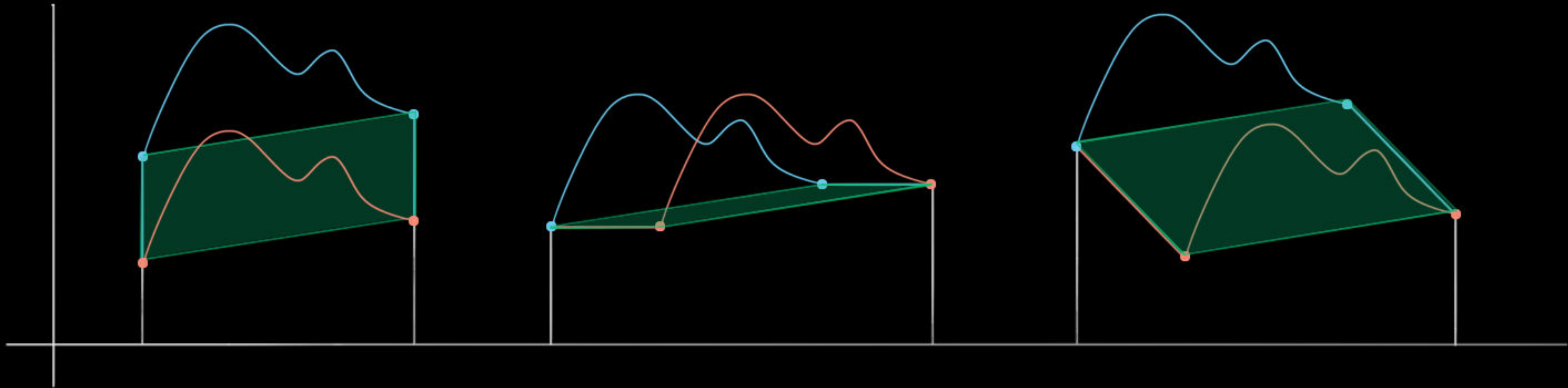
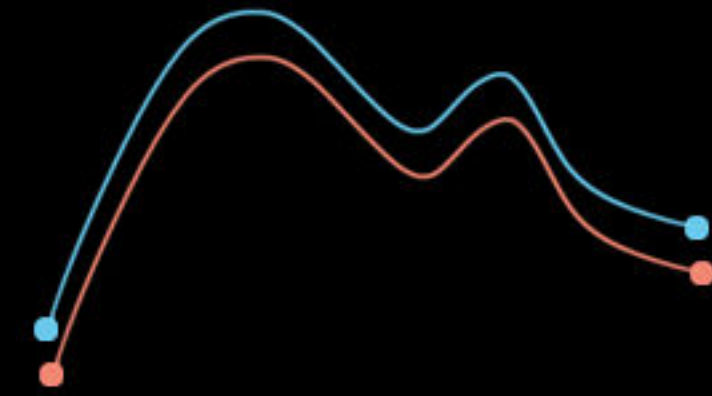
i.e: $S_1 = S_2$

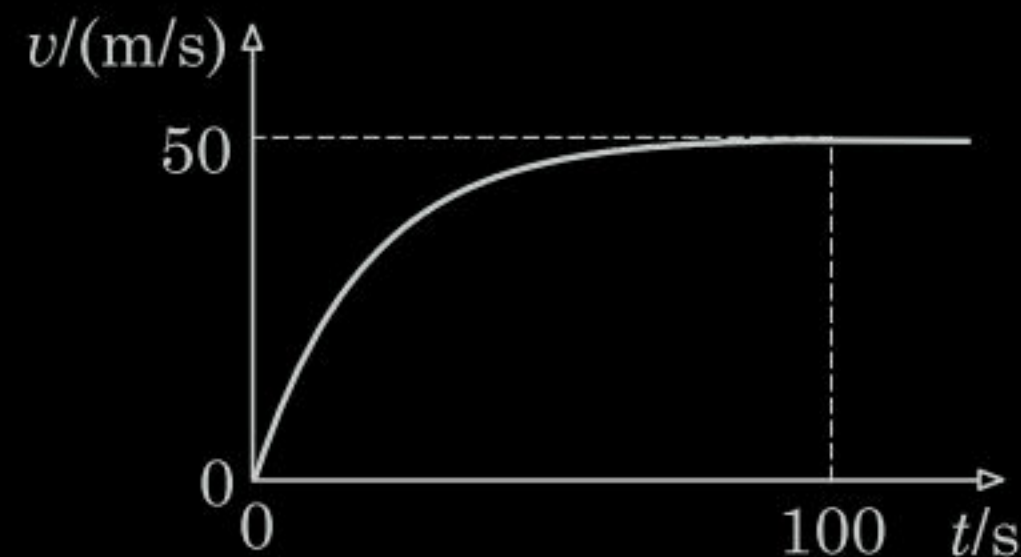
$$\Rightarrow \frac{1}{2} a_1 t^2 = \frac{1}{2} 4a_1 (t-3)^2$$

Reject as 2nd particle doesn't start till 3 seconds.

$$\Rightarrow t = 6, 2$$

A geometrical Concept: Area difference between 'exactly same parallel profile' curves is area of parallelogram between them. (formed by joining the ends of those similar curves)

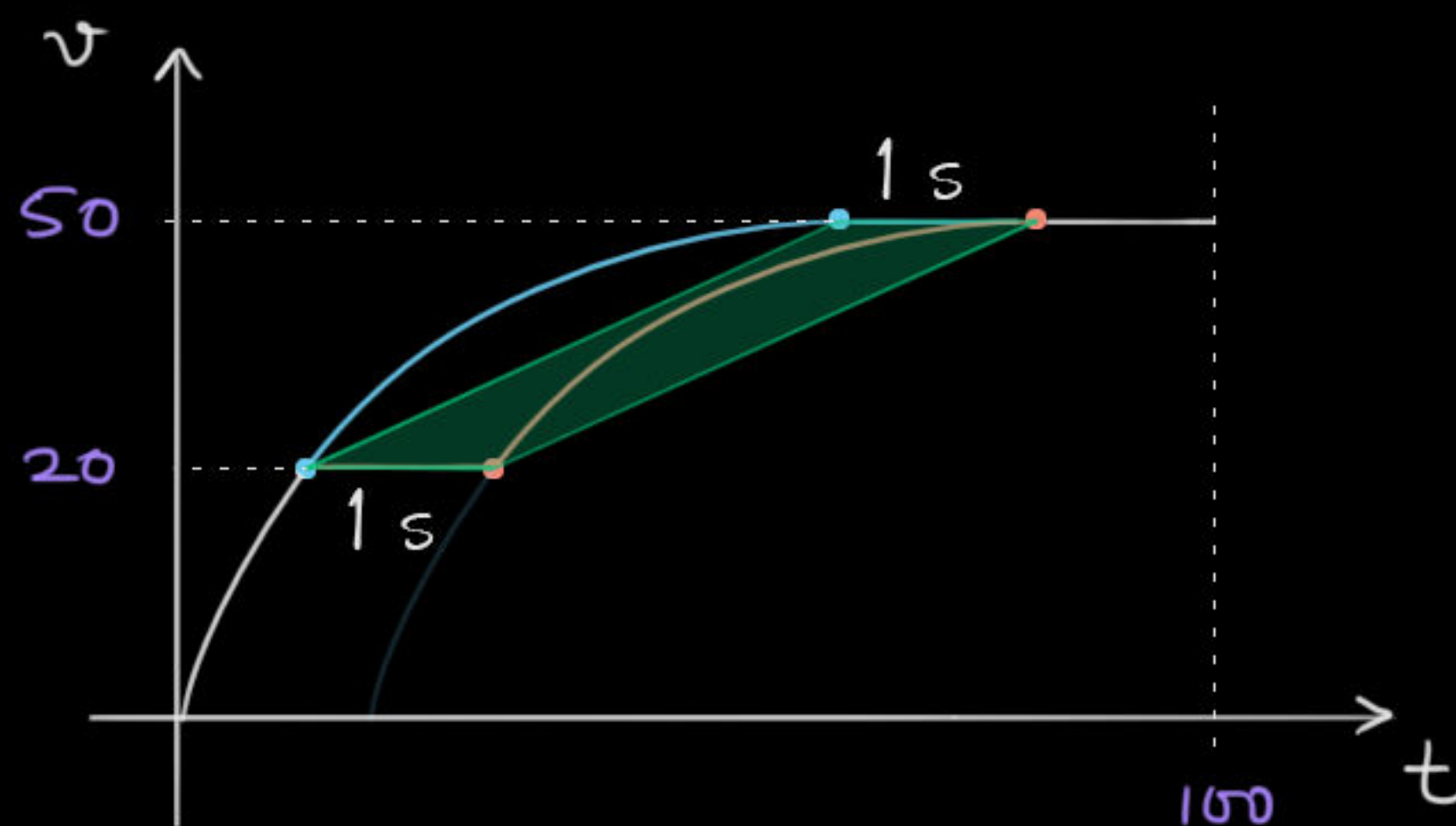




8. Two cars A and B simultaneously start a race. Velocity v of the car A varies with time t according to the graph shown in the figure. It acquires a velocity 50 m/s few seconds before $t = 100$ s and thereafter moves with this speed. Car B runs together with car A till both acquire a velocity 20 m/s; after this, car B moves with zero acceleration for one second and then follows velocity-time profile identical to that of A with a delay of one second. In this way, car B acquires the velocity 50 m/s one second after A acquires it. How much more distance Δs does the car A cover in the first 100 s as compared to the car B?

- ✓(a) $\Delta s = 30$ m
(c) $\Delta s = 20$ m

- (b) $\Delta s < 30$ m
(d) Insufficient information.



$$\Delta S = \Delta \text{Area} = \text{parallelogram} = \text{base} \times \text{height}$$

$$= 1 \times 30$$

$$= 30 \text{ m} //$$

9. A model rocket fired from the ground ascends with a constant upward acceleration. A small bolt is dropped from the rocket 1.0 s after the firing and fuel of the rocket is finished 4.0 s after the bolt is dropped. Air-time of the bolt is 2.0 s. Acceleration of free fall is 10 m/s^2 . Which of the following statements is/are correct?

- ✓(a) Acceleration of the rocket while ascending on its fuel is 8.0 m/s^2 .
- ✓(b) Fuel of the rocket was finished at a height 100 m above the ground.
- ✓(c) Maximum speed of the rocket during its upward flight is 40 m/s .
- ✓(d) Total air-time of the rocket is 15 s.

Let the acceleration be a , and when bolt falls, rocket is at height x and velocity v .

$$x = \frac{1}{2}at^2 = \frac{a}{2} \quad \& \quad v_x = at = a$$

For free fall of bolt: $s = ut + \frac{1}{2}at^2$

$$\therefore -x = (a)(2) - \frac{1}{2}g(2)^2$$

$$\Rightarrow -\frac{a}{2} = 2a - 2g \Rightarrow a = 8 \text{ m/s}^2$$

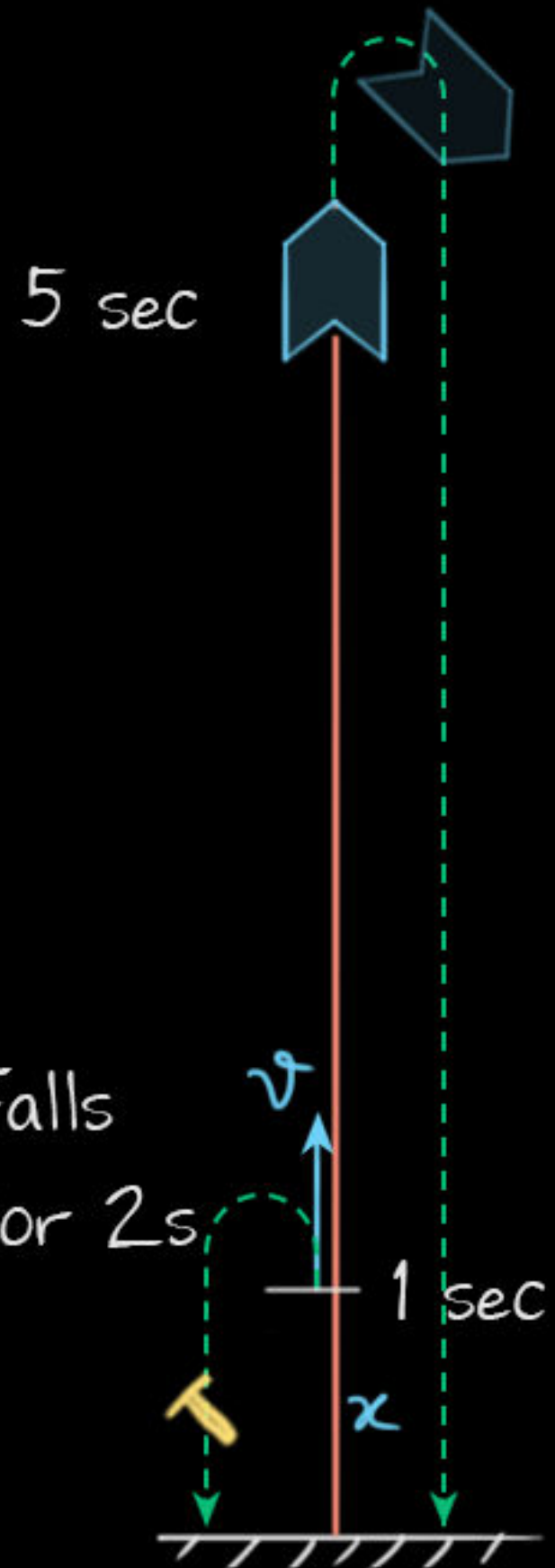
$$h = \frac{1}{2}at^2 = \frac{1}{2}8(25) = 100 \text{ m}$$

$$v_{\max} = at = 8(5) = 40 \text{ m/s}$$

$$T = 5 + t = 5 + 10 = 15 \text{ s}$$

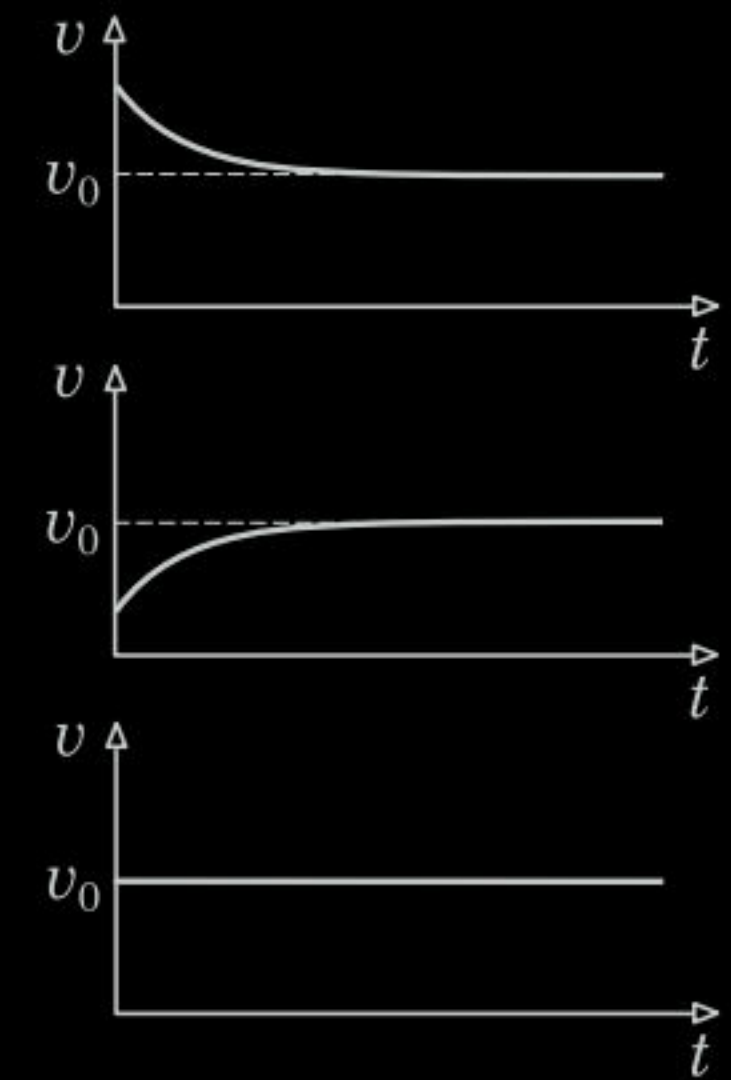
$$(-100 = 40t - \frac{1}{2}gt^2)$$

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10. Drag force of a fluid on a body is proportional to the velocity of the body relative to the fluid. A student drops several small identical stones from different heights over a deep lake and prepares graphs between speed v of every stone in the water and time t . The graphs can be divided into three categories as shown here. Which of the following explanations of these graphs appear reasonable?

- (a) The first graph is for a stone dropped from a small height, and the second is for a stone dropped from a large height.
- ✓(b) The first graph is for a stone dropped from a large height, and the second is for a stone dropped from a small height.
- ✓(c) The third graph is for a stone dropped from a height sufficient to acquire the speed v_0 at the instant it enters the water.
- (d) The third graph is not possible.

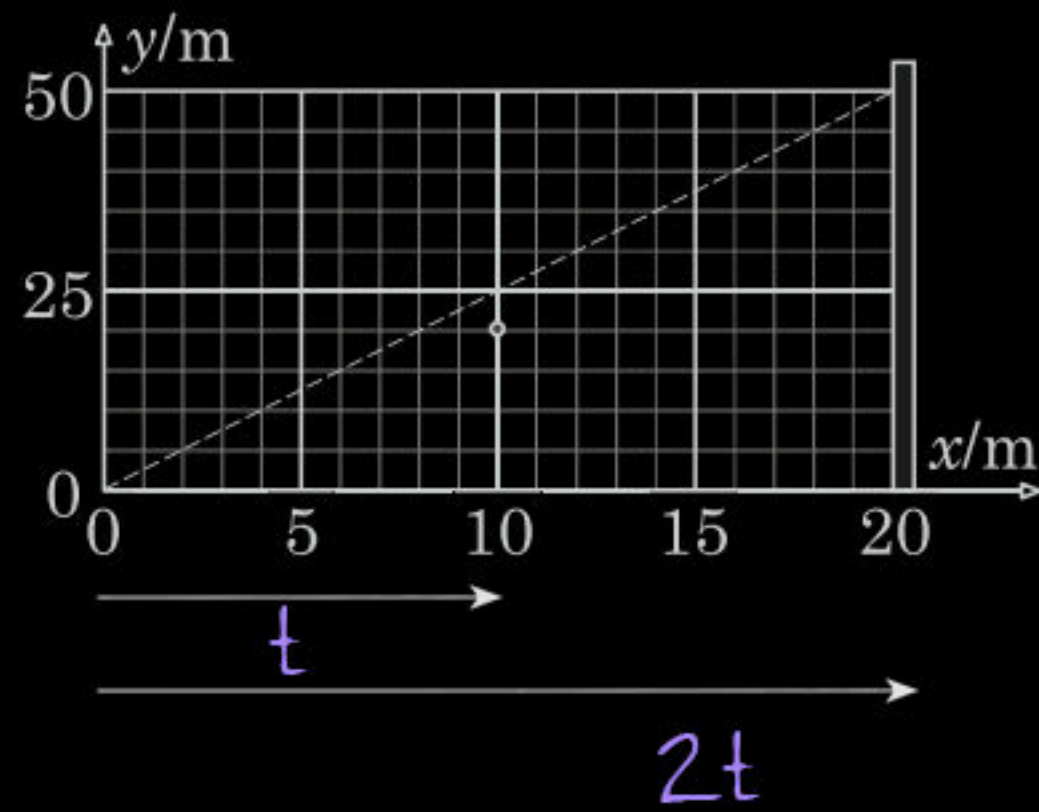


Concept: Eventually all bodies will achieve terminal velocity (v_T).

Graph 1: When dropped from large height, splash velocity $> v_T$, so it will reduce to v_T soon

Graph 2: When dropped from small height, splash velocity $< v_T$, so it will increase to v_T soon

Graph 3: When splash velocity $= v_T$, it will maintain v_T .

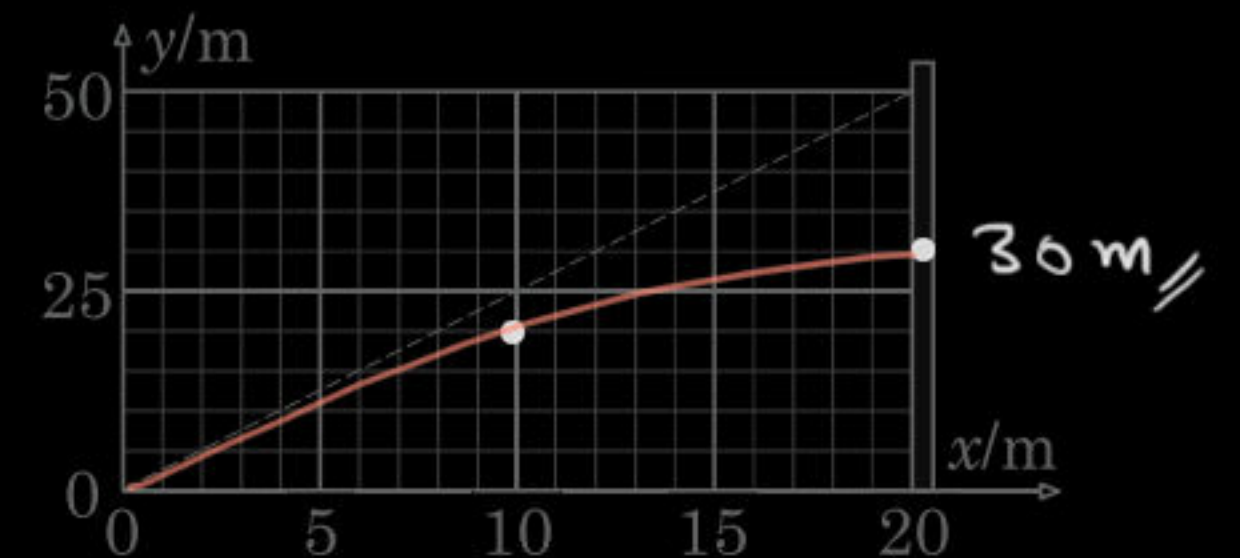
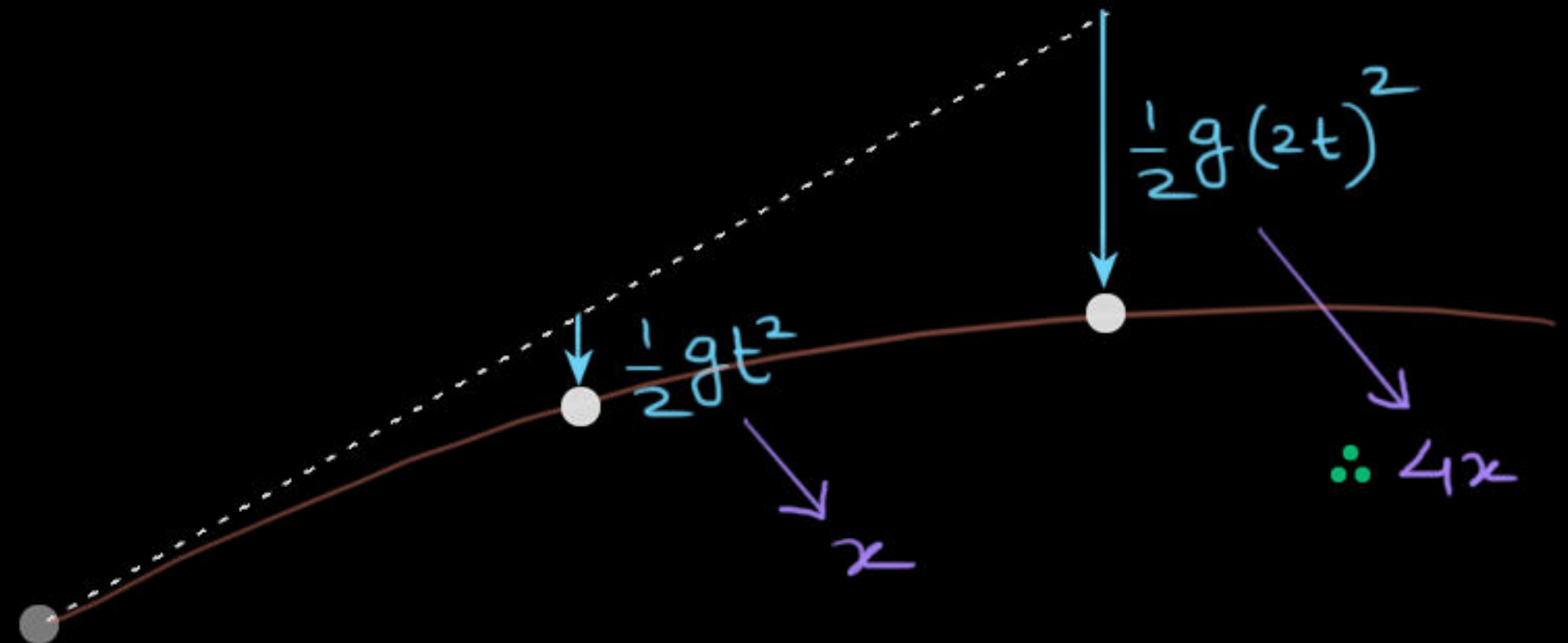
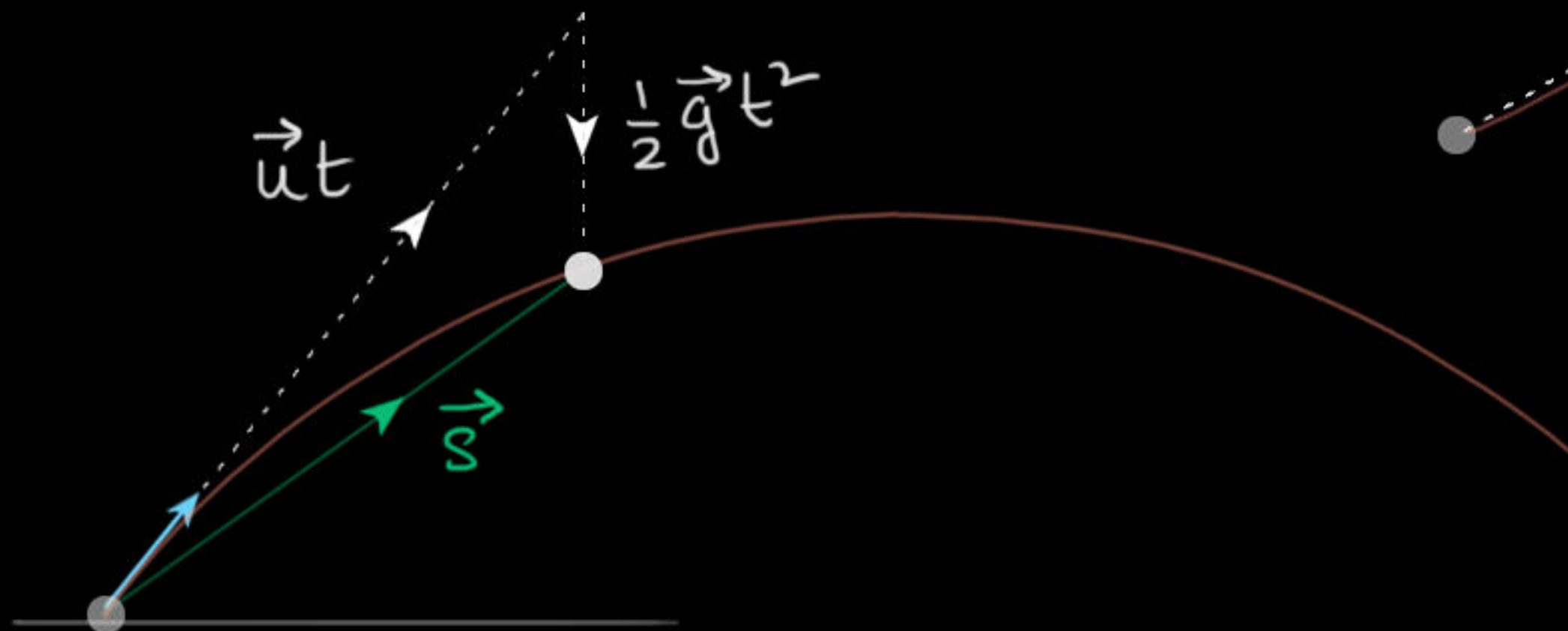


11. From a place on the ground that is 20 m away from a wall, a bullet is fired aiming at a 50 m high mark on the wall. The location of the bullet is shown by a circular dot at some point of time. Where on the wall will the bullet hit?

- (a) 20 m mark
 ✓ (c) 30 m mark

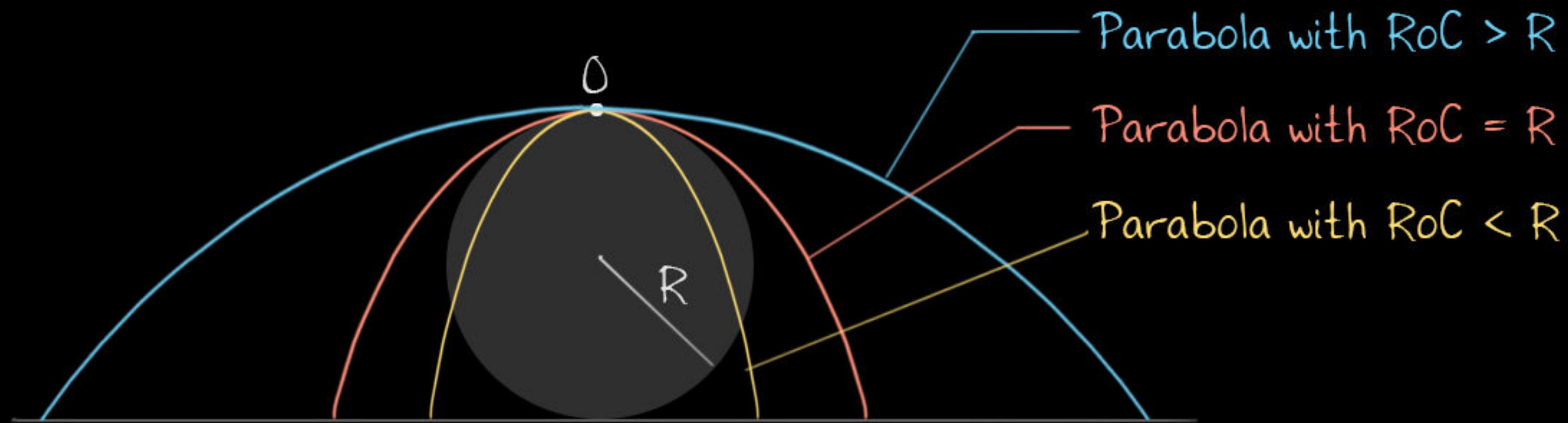
- (b) 25 m mark
 (d) 35 m mark

Concept: $\vec{s} = \vec{u}t + \frac{1}{2}\vec{g}t^2$



Concept: If we wish to encapsulate a circle with **slimmest** parabola, then at point of contact:

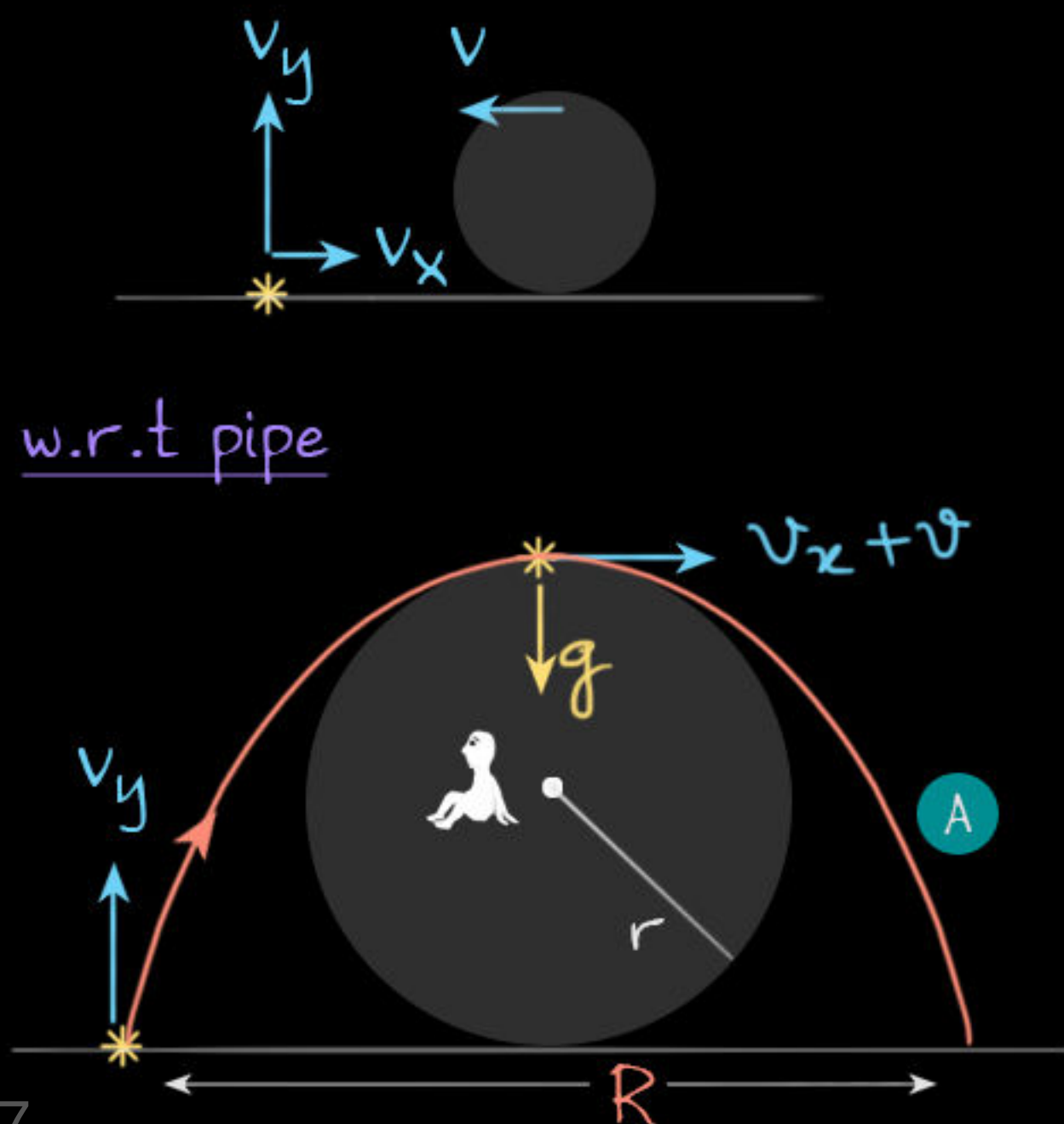
"R of that circle = Radius of curvature (RoC) of parabola "



Concept: for slimmest parabola:

$$R_{OC} = r \quad (1)$$

Let horizontal velocity of frog be v_x and v_y



12. A cylindrical pipe of radius r is rolling towards a frog sitting on the horizontal ground. Centre of the pipe is moving with a constant velocity v . To save itself, the frog jumps off and passes over the pipe touching it only at the top. Denoting air-time of the frog by T , horizontal range of the jump by R and acceleration due to gravity by g , which of the following conclusions can you make?

✓(a) $T = 4\sqrt{\frac{r}{g}}$

(b) $T \geq 4\sqrt{\frac{r}{g}}$

(c) $R = 4(\sqrt{gr} - v)\sqrt{\frac{r}{g}}$

✓(d) $R \geq 4(\sqrt{gr} - v)\sqrt{\frac{r}{g}}$

From 1: $\frac{(v_x + v)^2}{2} = g$

$\Rightarrow v_x = \sqrt{2gr} - v \quad (2)$

And $T = 2\sqrt{\frac{2h}{g}} = 4\sqrt{\frac{r}{g}} \quad (3)$

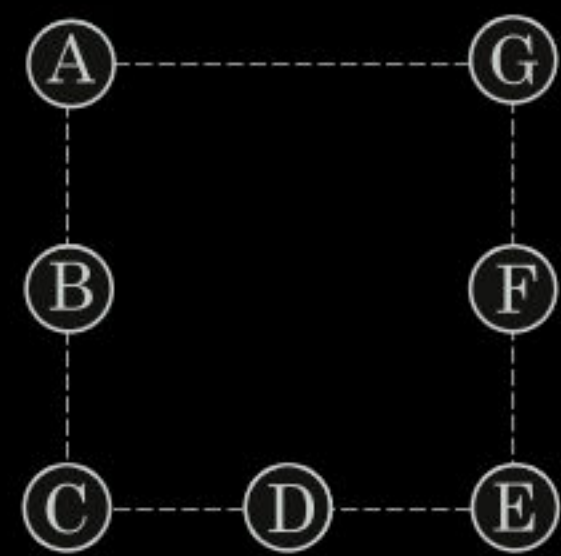
by Ankit Singhvi

From 2,3:

$R \geq v_x T$

$\Rightarrow R \geq 4r - v \cdot 4\sqrt{\frac{r}{g}}$

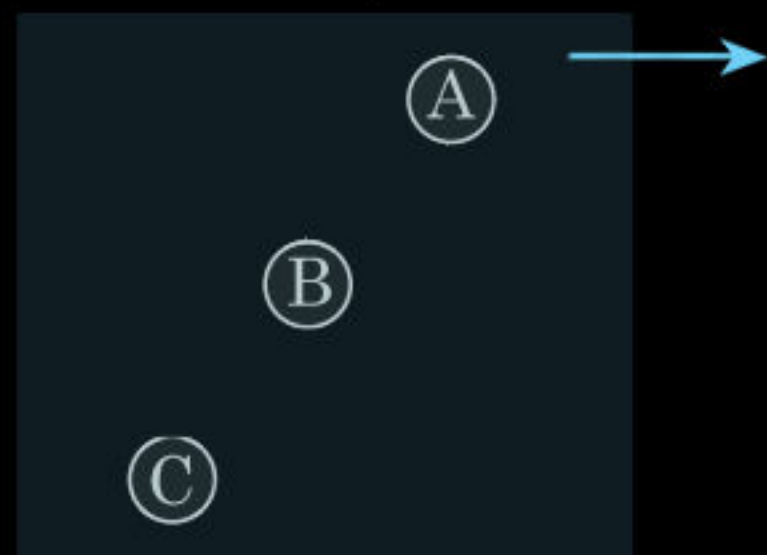
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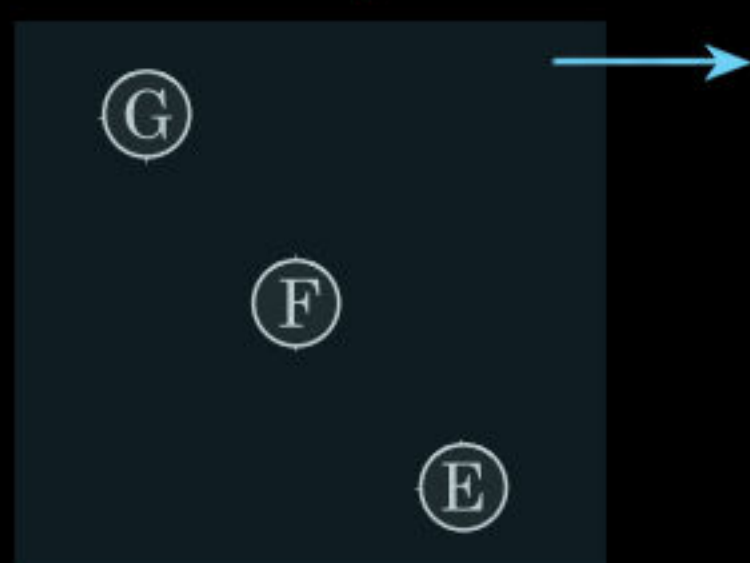
13. Seven buoys A, B, C, D, E, F and G are released in a lake at regular intervals in a manner to make a square pattern as shown in the figure. The buoys A, C, E and G are on the vertices and the buoys B, D and F are at the midpoints of the sides of the square. If the buoys were released in a uniformly flowing river in the same manner, the buoy G falls on A. What pattern would they make in the river?

Lets assume river flows towards right, so

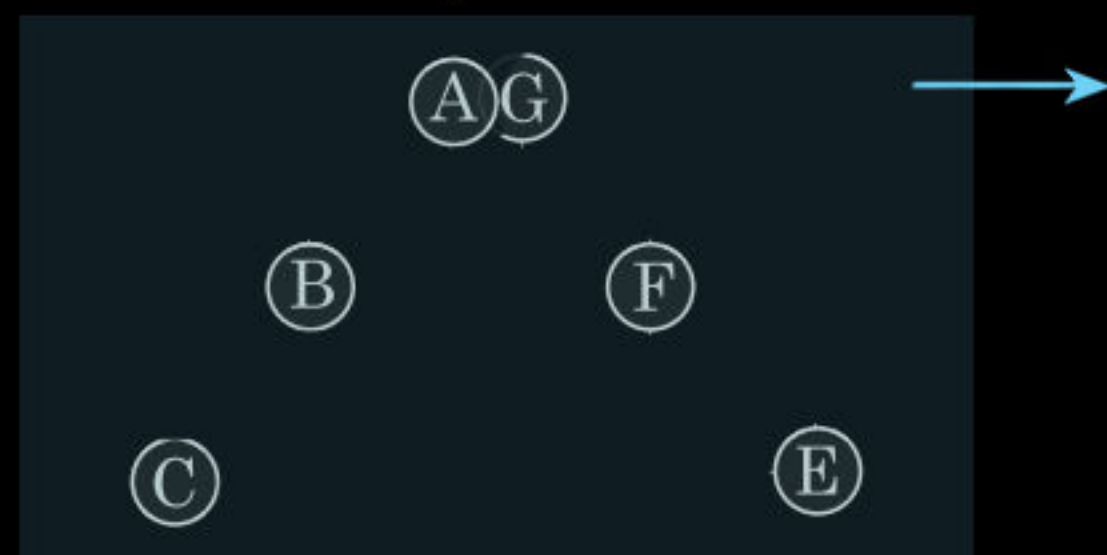
ABC will flow like



EFG will flow like

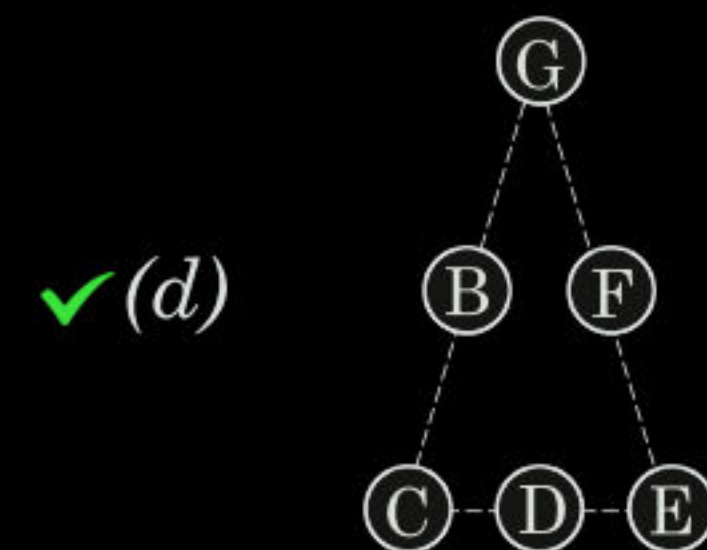
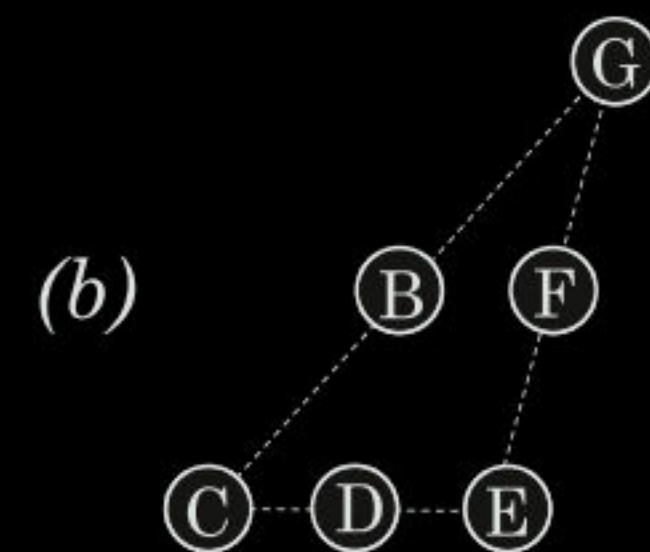
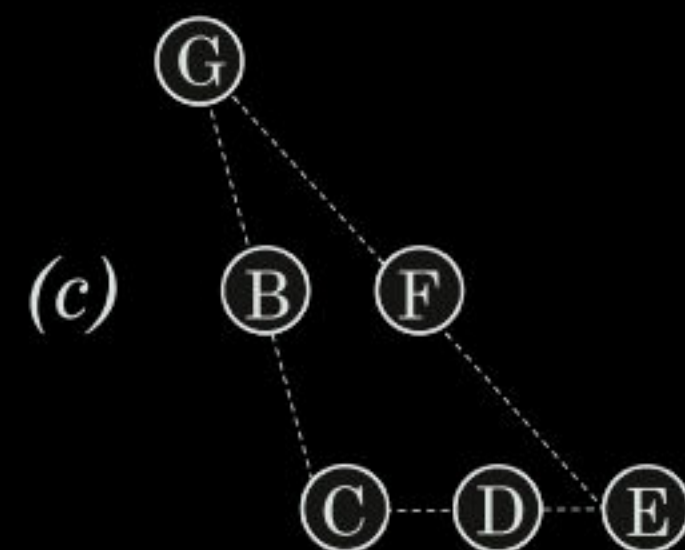
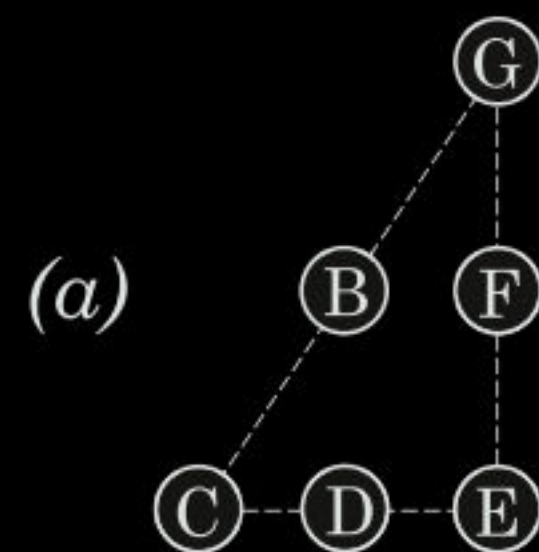


As A and G overlap:



And D will be in between C and E.

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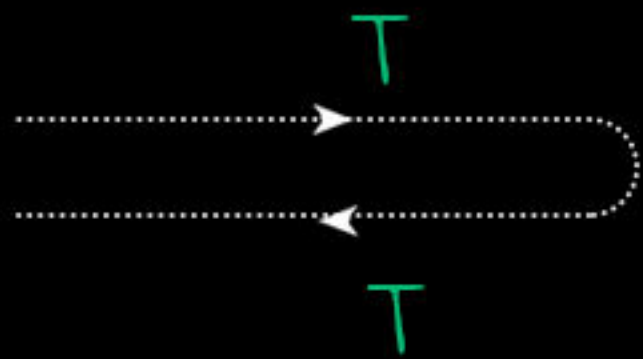


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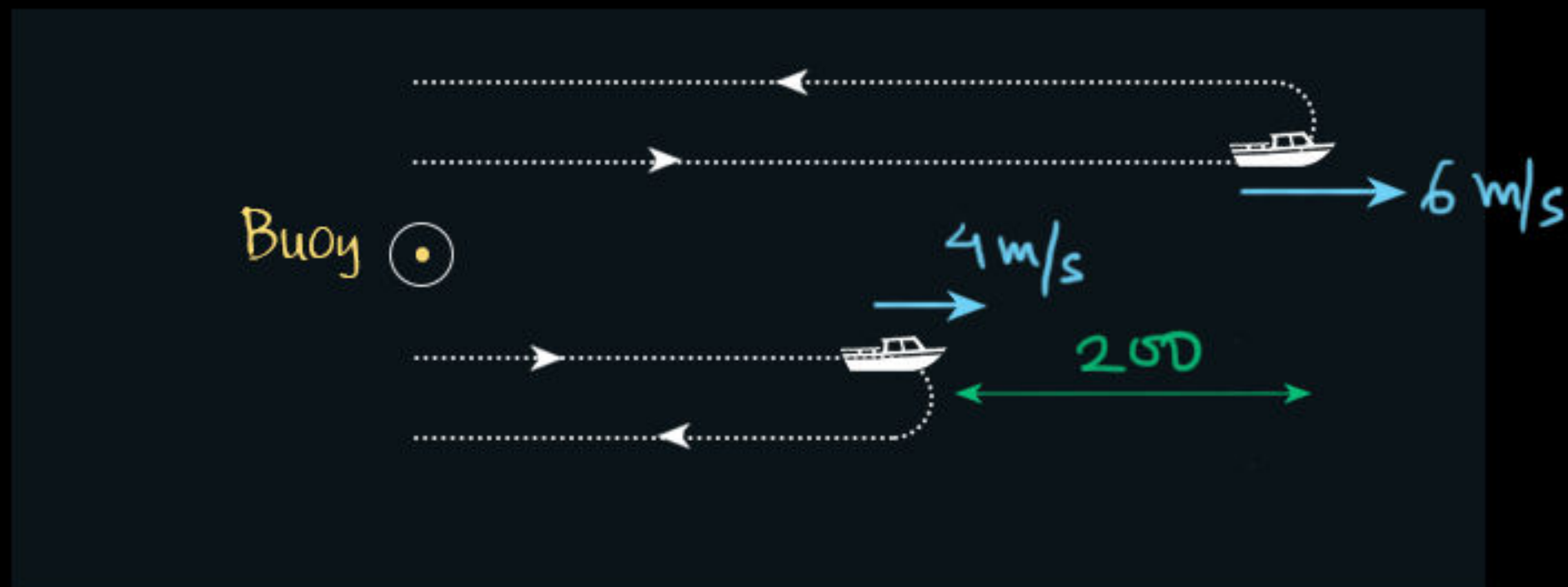
Concept: Both boats leave the buoy and turn back at the same time.

So they will reach the buoy back together too.

∴ For both boats let the time of travel be $2T$



w.r.t river



14. Two motorboats that can move with velocities 4.0 m/s and 6.0 m/s relative to water are going up-stream in a river. When the faster boat overtakes the slower boat, a buoy is dropped from the slower boat. After lapse of a time interval, both the boats turn back simultaneously and move at the same speeds relative to the water as before. Their engines are switched off when they reach the buoy again. If the maximum separation between the boats is 200 m after the buoy is dropped and water flow velocity in the river is 1.5 m/s , find distance between the places where the faster boat passes by the buoy.

- (a) 75 m
 ✓(c) 300 m

- (b) 150 m
 (d) 350 m

→ Lets assume, buoy comes to rest w.r.t river as soon as its dropped.

$$\text{Maximum gap: } 200 = (6 - 4)T$$

$$\Rightarrow T = 100$$

w.r.t ground again

∴ Between two passby events,
 Buoy would float by: $v_R (2T) = 300 \text{ m} //$

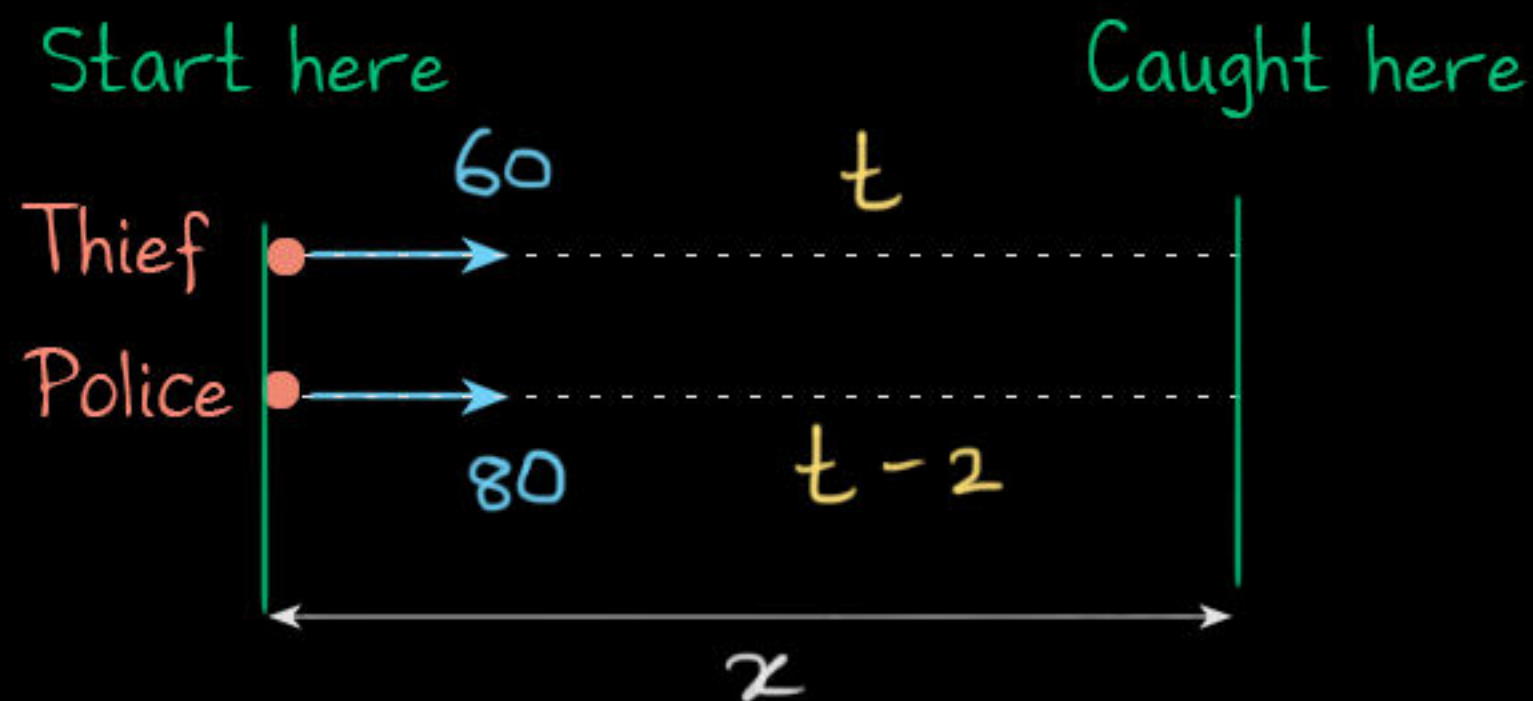
1.5 (given)

Approach: Situation is similar to a thief (60km/h) getting a head start of 2 min to run from a policeman(80km/h).

At what distance will the policeman catchup?

15. On a road going out of a city, lights of the last traffic signal glow green for 1.0 min and red for 2.0 min. After the traffic signal, the road is straight and vehicles run at their constant speeds that lie in the range 60 km/h to 80 km/h. This mode of operation of traffic lights causes the vehicles to come out of the city in groups. Some distance away from the traffic signal, these groups merge and become indistinguishable. Assuming that the vehicles acquire their constant speeds after the traffic signal in negligible time, which of the following conclusions can you make?

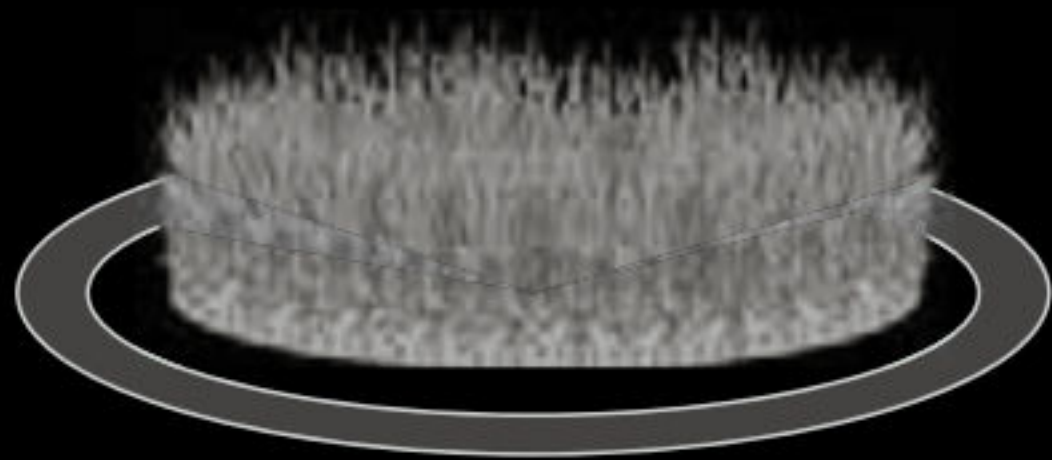
- (a) The groups become indistinguishable 4.0 km after the traffic signal.
- ✓(b) The groups become indistinguishable 8.0 km after the traffic signal.
- (c) Wider is the range of speed of the vehicles; greater is the distance where the groups become indistinguishable.
- (d) Larger is the number of fast moving vehicles, shorter is the distance where the groups become indistinguishable.



$$60t = 80(t-2)$$

$$\Rightarrow t = 8 \text{ min}$$

$$\therefore x = 60 \frac{\text{km}}{\text{hr}} \cdot \frac{8}{60} \text{ hr} = 8 \text{ km} //$$



16. Two students simultaneously start from the same place on a circular track and run for 2 min. In this time, one of them completes three and the other four revolutions. Due to thick vegetation in a circular area as shown in the figure, either of the boys can see only one third of the track at a time. How long during their run do they remain visible to each other?

(a) 20 s

✓ (b) 40 s

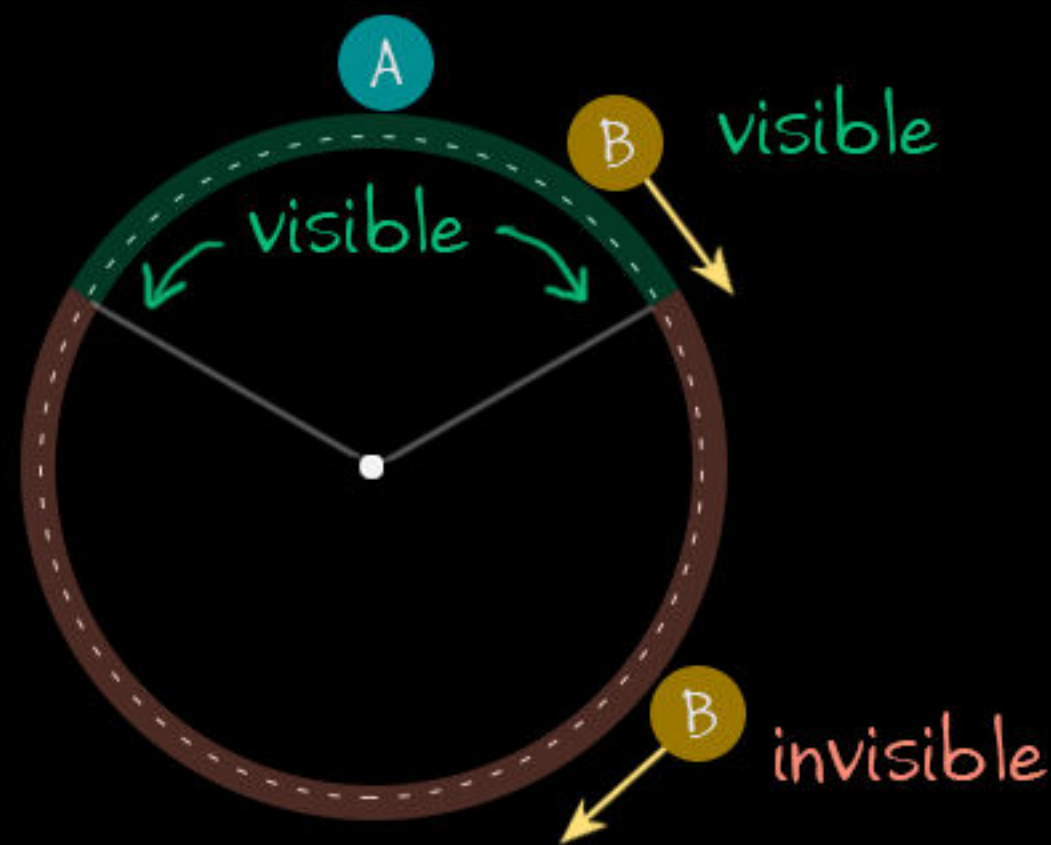
(c) 80 s

(d) 160 s

Concept: If A cant see B, then B cant see A either. So lets solve the problem w.r.t A and observe as B gets in and out of A's vision.

Lets say, B is faster

w.r.t A, B completes the circle in 2 minutes (only once)

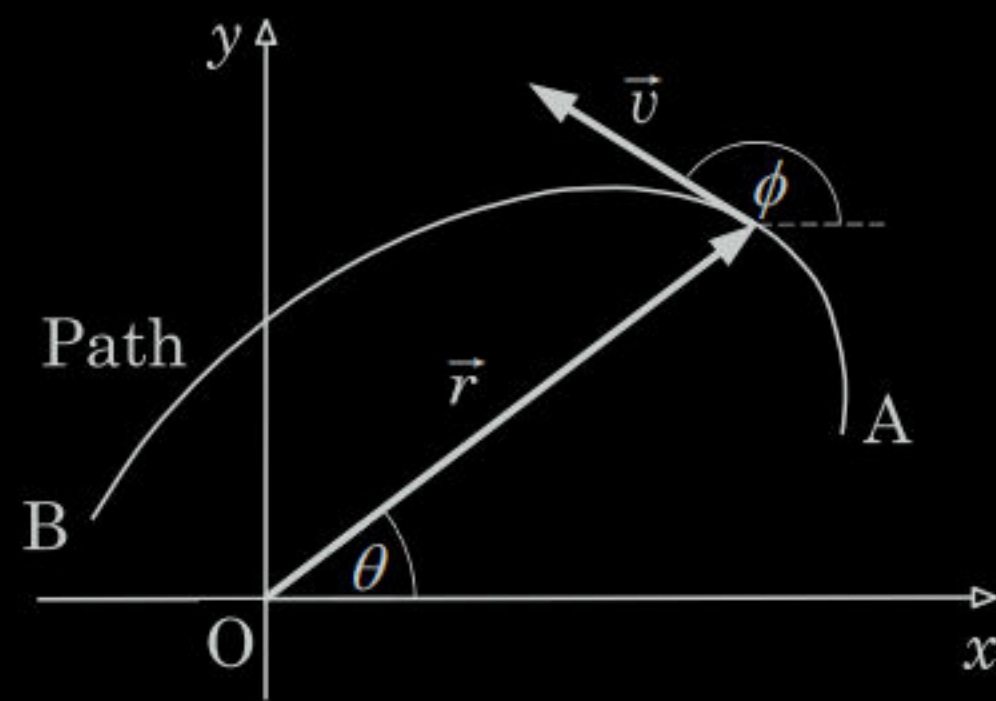


Note: Answer will be same even if they were running in opposite sense to each other

$$\therefore \text{Duration of visibility} = \frac{1}{3} \cdot 2 \text{ min} = 40 \text{ sec}$$

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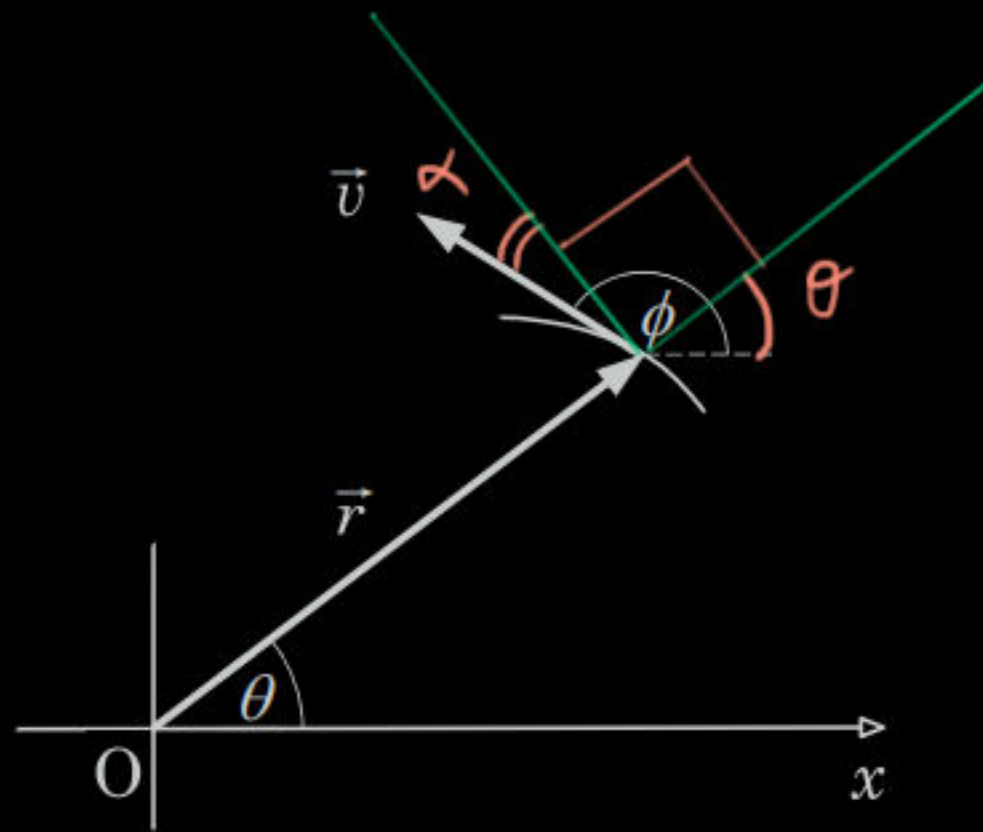
17. At a particular instant of time, position vector $\vec{r}$, velocity vector $\vec{v}$ and angular position θ of a particle traversing a path AB are shown in the figure. Here ϕ is the angle made by the velocity vector with the positive x-axis. Which of the following statements is/are correct?

✓(a) Modulus of angular velocity is $\frac{d\theta}{dt} = \frac{v \sin(\phi - \theta)}{r}$.

(b) Modulus of tangential component of acceleration is $r \frac{d^2\theta}{dt^2}$.

(c) Modulus of normal component of acceleration is $v \frac{d\theta}{dt}$.

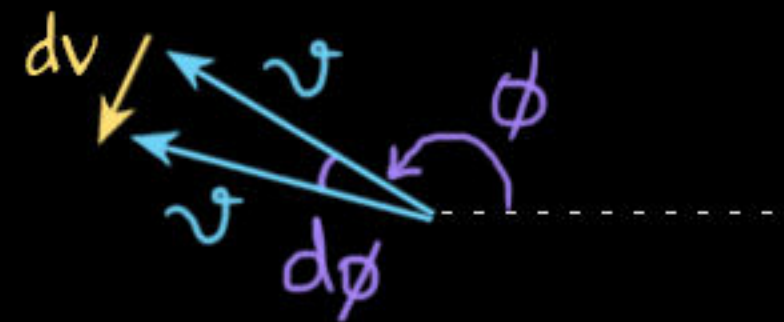
✓(d) Modulus of normal component of acceleration is $v \frac{d\phi}{dt}$.



$$\text{Angular velocity} = \frac{v_{\perp}}{r} = \frac{v \cos \alpha}{r}$$

$$= \frac{v \cos(\phi - \frac{\pi}{2} - \theta)}{r} = \frac{v \sin(\phi - \theta)}{r}$$

Tangential acceleration = $\frac{dv}{dt}$ (Independent of any angle)

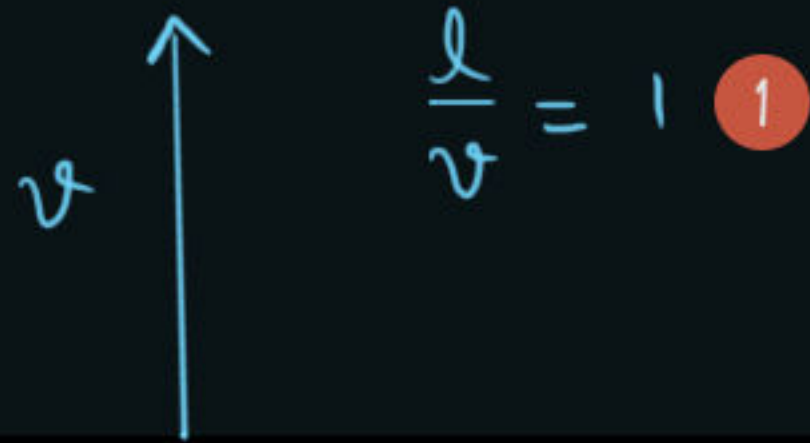


$$\text{Normal acceleration} = \frac{dv}{dt} = \frac{(v d\phi)}{dt} = v \frac{d\phi}{dt}$$

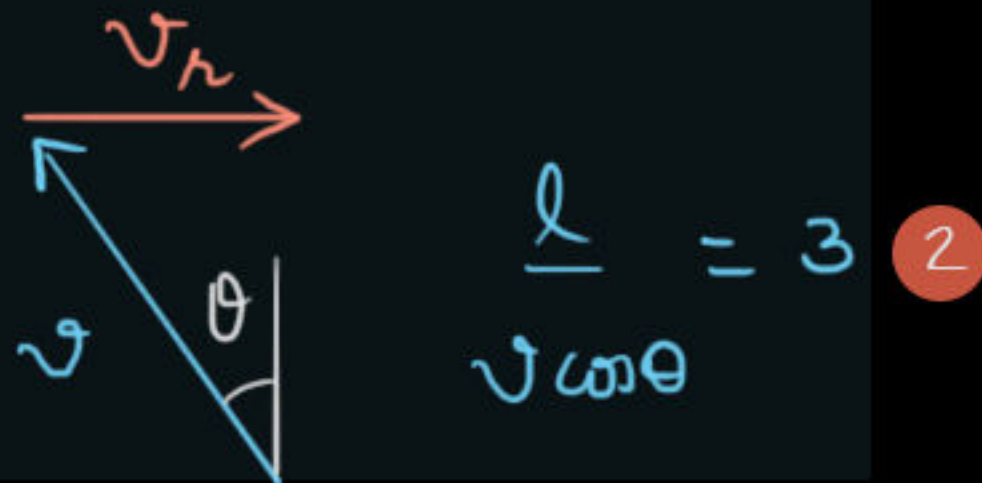
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Let width of river be l .
And velocity of boat
w.r.t water be v .



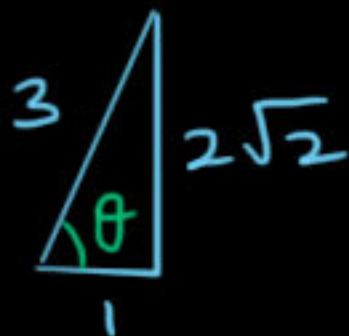
$$\frac{l}{v} = 1 \quad (1)$$



$$\frac{l}{v \cos \theta} = 3 \quad (2)$$

From 1, 2:

$$\cos \theta = \frac{1}{3}$$

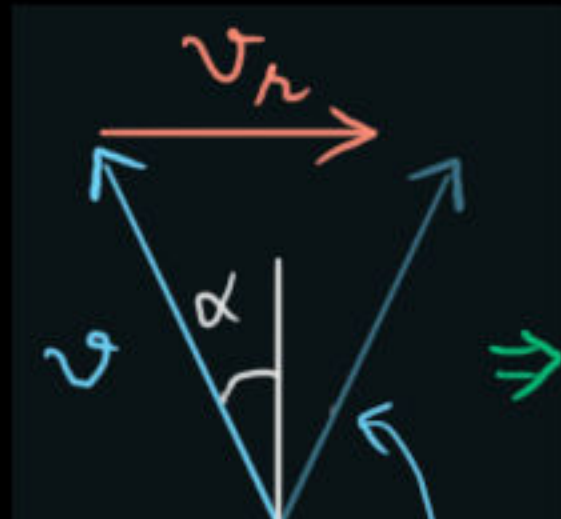


18. A boy takes 60 min to swim across a river, if his goal is to minimize time; and takes 180 min, if his goal is to minimize to zero the distance that he is carried downstream. In both these attempts, the boy swims with the same speed relative to the river current. Which of the following statements can be true?

- ✓ (a) He can swim relative to water faster than the river current.
- (b) He cannot swim relative to water faster than the river current.
- ✓ (c) If width of the river is $3\sqrt{2}$ km, speed of river current is 4 km/h.
- ✓ (d) If he crosses a $3\sqrt{2}$ km wide river in $60\sqrt{2}$ min, he will be carried $\sqrt{2}$ km downstream.

(A) $v > v_r$

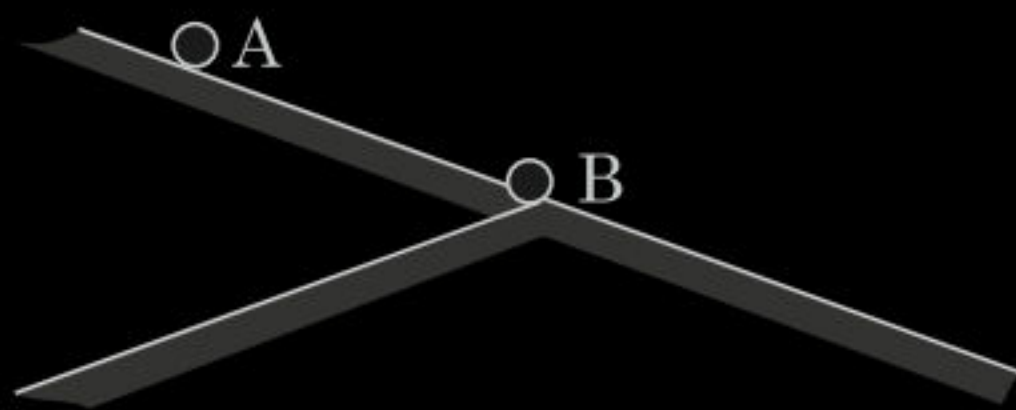
(C) $v_r = v \sin \theta = l \cdot \frac{2\sqrt{2}}{3} = \cancel{3\sqrt{2}} \frac{2\sqrt{2}}{3} = 4 //$

(D) 
$$l = (v \cos \alpha) t \quad \therefore \text{drift} = (v_r - v \sin \alpha) t$$

$$\Rightarrow \cos \alpha = \frac{1}{t} = \frac{1}{\sqrt{2}} \quad = (4 - \cancel{3\sqrt{2}} \cdot \frac{1}{\sqrt{2}}) \sqrt{2}$$

$$= \sqrt{2} //$$

Note: if v is taken towards right, drift = $(v_r + v \sin \alpha) t = 7\sqrt{2} //$
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Concept: When relative acceleration is zero, then relative velocity is unchanged.



Approach: In our case, vertical relative acceleration is zero, so relative velocity remains constant (i.e. 0).

So vertical gap between the two balls is constant.

So distance between them is shortest when horizontal gap is minimum, i.e. on top of each other.

19. Two balls A and B are simultaneously released on two frictionless inclined planes from the positions shown. The inclined planes have equal inclinations. The balls pass through a particular horizontal level 12 s and 4 s after they were released. How long after they were released will they be closest to each other?

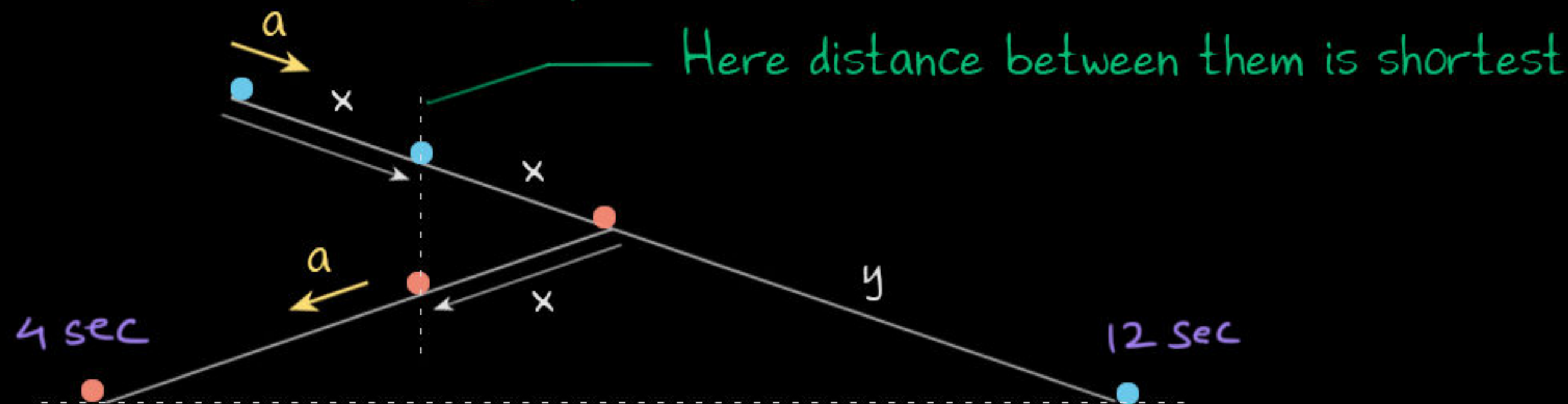
(a) 6 s

✓(b) 8 s

(c) 12 s

(d) 16 s

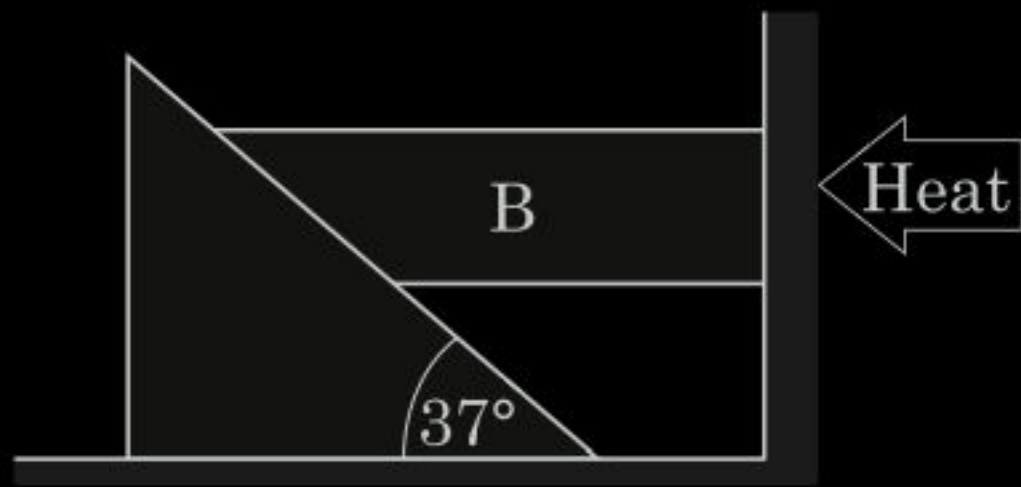
Let acceleration along slope be a .



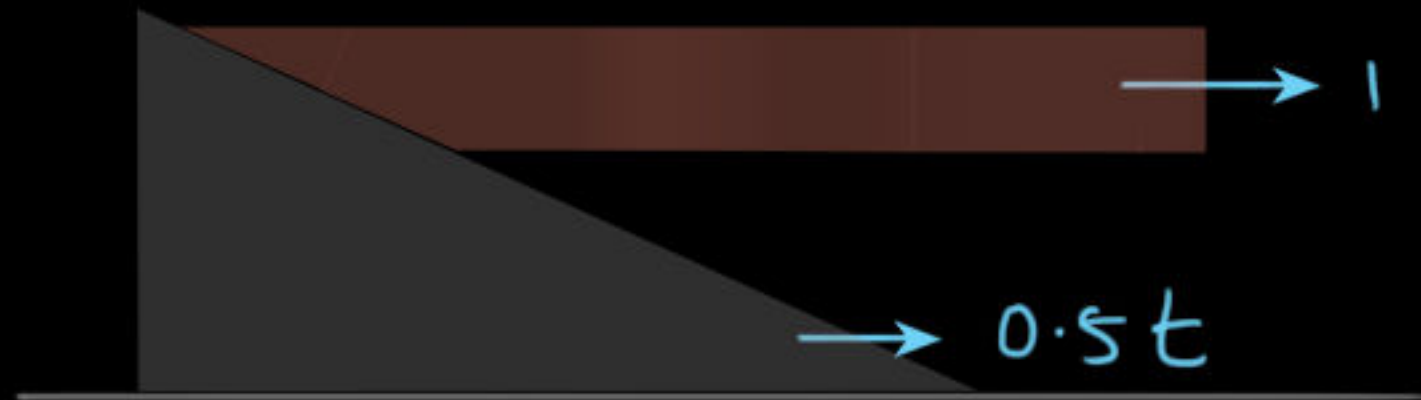
$$\begin{aligned} \text{Given: } 2x + y &= \frac{1}{2} a (12)^2 & (1) \\ y &= \frac{1}{2} a (4)^2 & (2) \\ \text{We need } t_0: x &= \frac{1}{2} a t_0^2 & (3) \end{aligned}$$

From (1) and (2): $\frac{x}{a} = 32$

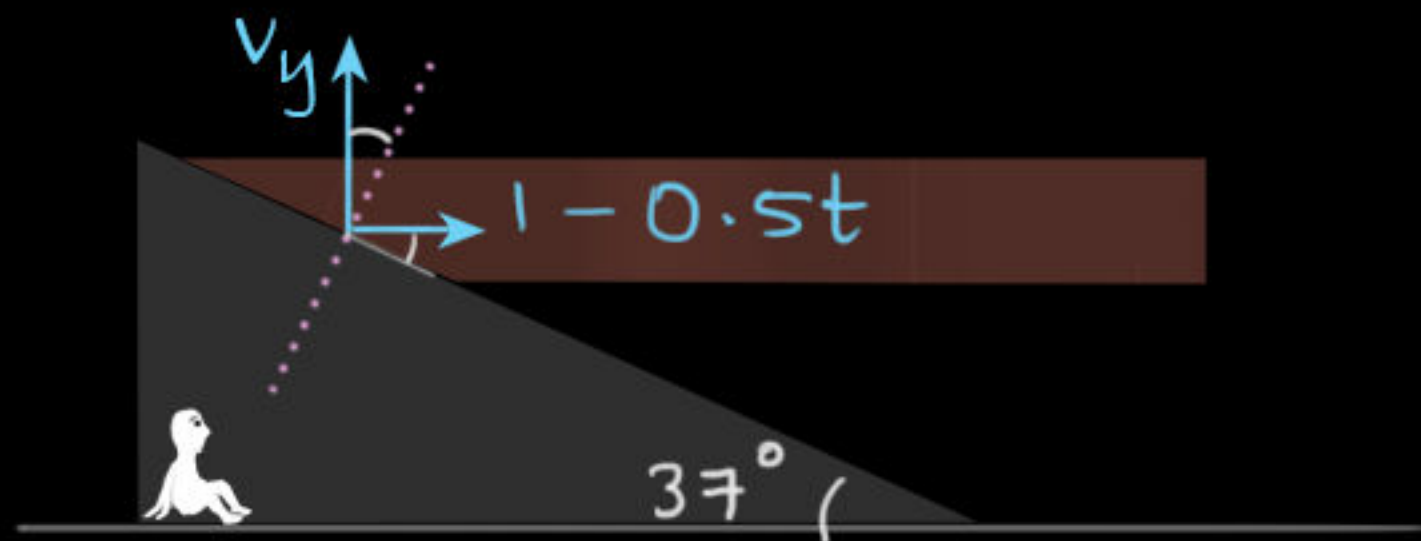
From (3): $t_0 = 8 //$



Approach: As we need to answer about vertical motion of bar, let's analyse w.r.t wedge.
w.r.t ground



w.r.t wedge



20. A horizontal wax bar B rests between a wedge and a vertical wall as shown in the figure. The wedge starts moving towards the wall with a constant acceleration 0.5 mm/s^2 . The moment the wedge starts moving, a continuous supply of heat from the wall starts melting 1.0 mm length of the wax bar per second. If the bar always remains horizontal, which of the following conclusions can you make?

- ✓(a) The bar first moves downwards and then upwards.
- ✓(b) The bar stops for a moment after 2.0 s from the beginning.
- (c) Modulus of displacement of the bar in the first 4 s is 1.5 mm .
- ✓(d) Distance travelled by the bar in the first 4 s is 1.5 mm .

Vertical

Along normal to surface velocity of wedge should be zero.

$$\therefore v_y \cos 37^\circ + (1 - 0.5t) \sin 37^\circ = 0$$

$$\Rightarrow v_y = -\frac{3}{8}(2-t)$$

Integrate ↓

$$y = -\frac{3}{16}t(4-t)$$

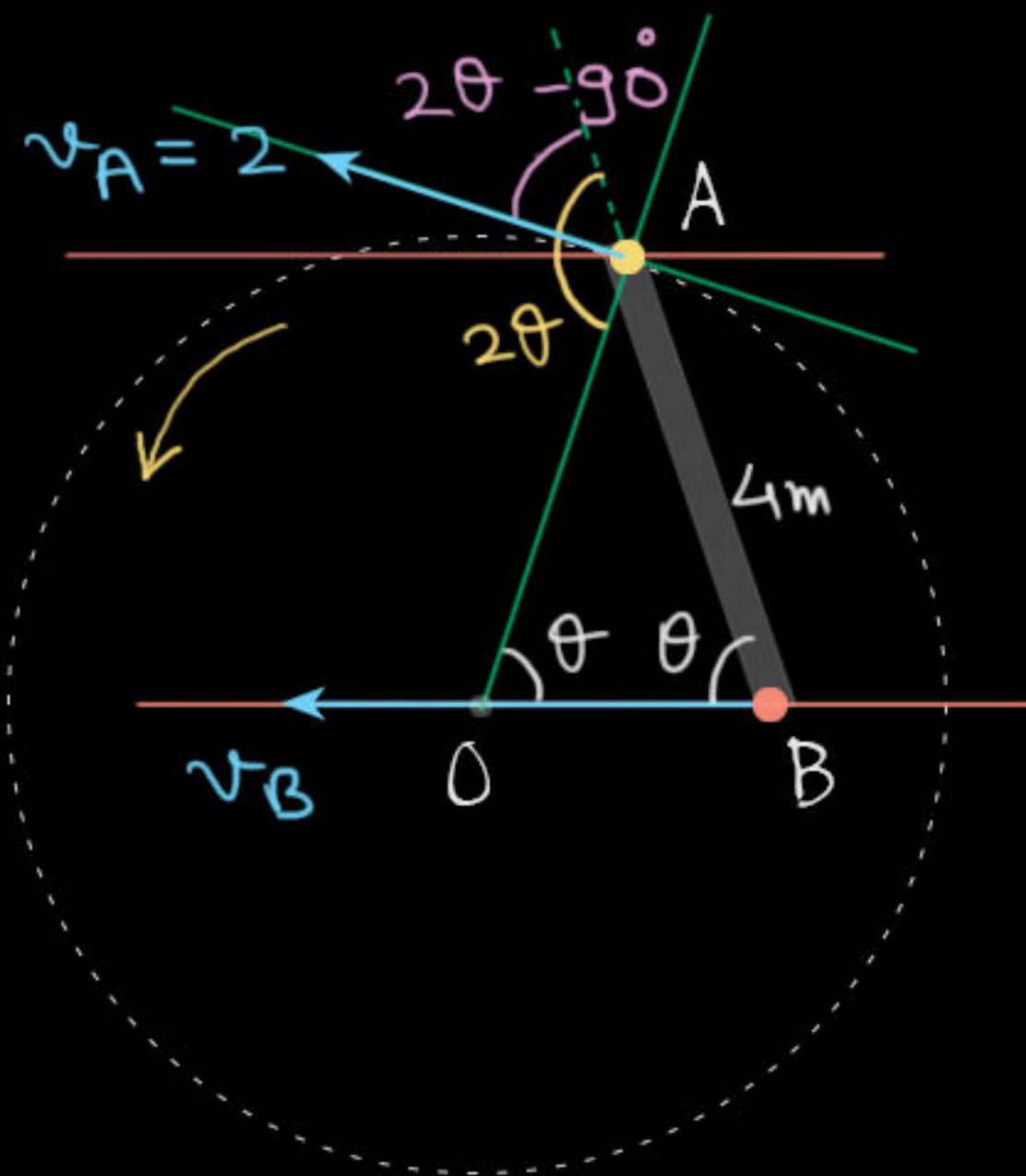
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21. Particle A moves with a constant speed 2 m/s on a circular path of radius 4 m, whereas particle B moves on a straight-line coinciding with a diameter of the circular path, maintaining a constant distance 4 m from the particle A. Which of the following conclusions can be drawn?

- ✓ (a) Maximum speed of B is 4 m/s.
- ✓ (b) Maximum acceleration of B is 2 m/s².
- ✓ (c) During one revolution of A, distance travelled by B is 32 m.
- ✓ (d) Modulus of velocity of a particle relative to the other is a constant.

Lets say at a given time AB makes θ with horizontal.



∴ distance AB is constant:

$$v_A \cos(2\theta - 90^\circ) = v_B \cos \theta$$

$$\Rightarrow 2 \quad 2 \sin \theta \quad \cancel{\cos \theta} = v_B \quad \cancel{\cos \theta}$$

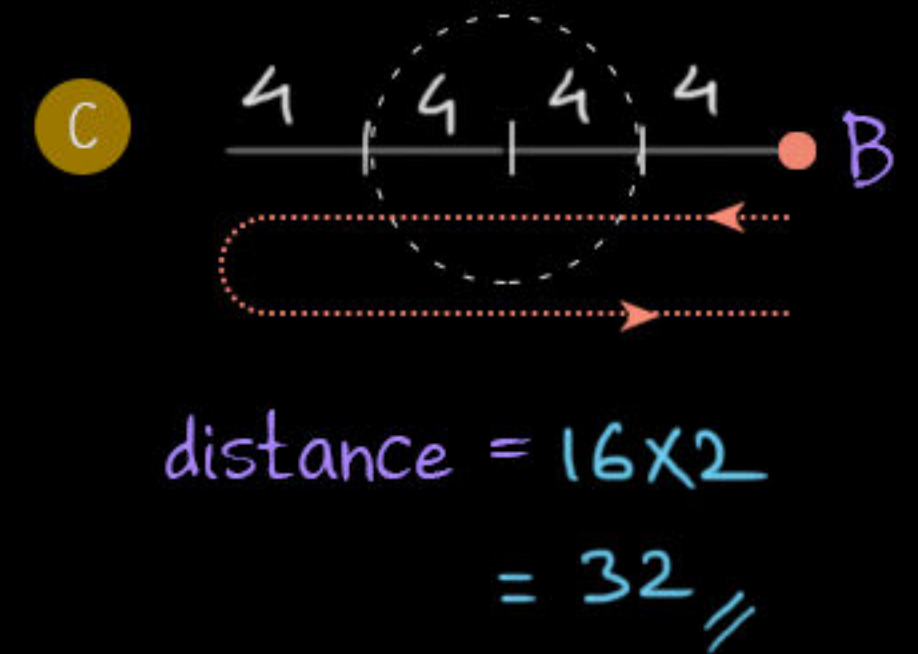
$$\Rightarrow \text{A } v_B = 4 \sin \theta$$

↓ differentiate

$$a_B = 4 \cos \theta \frac{d\theta}{dt}$$

$$\frac{v_A}{R} = \frac{2}{4}$$

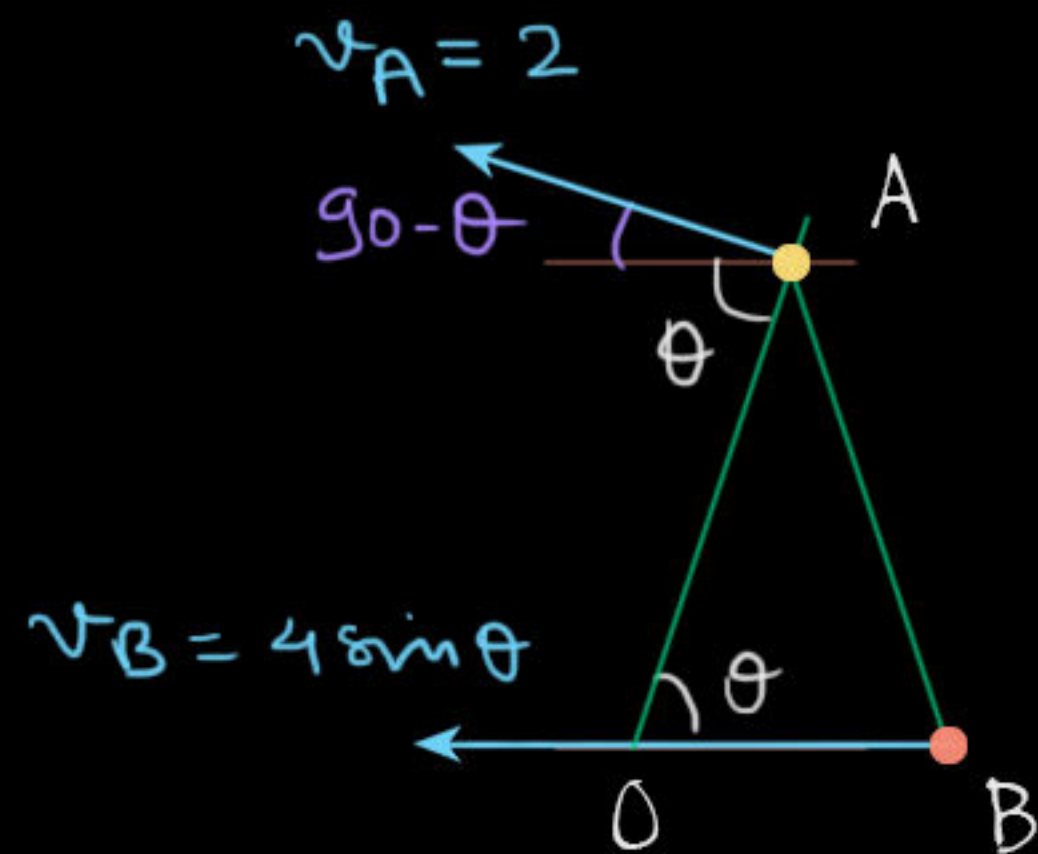
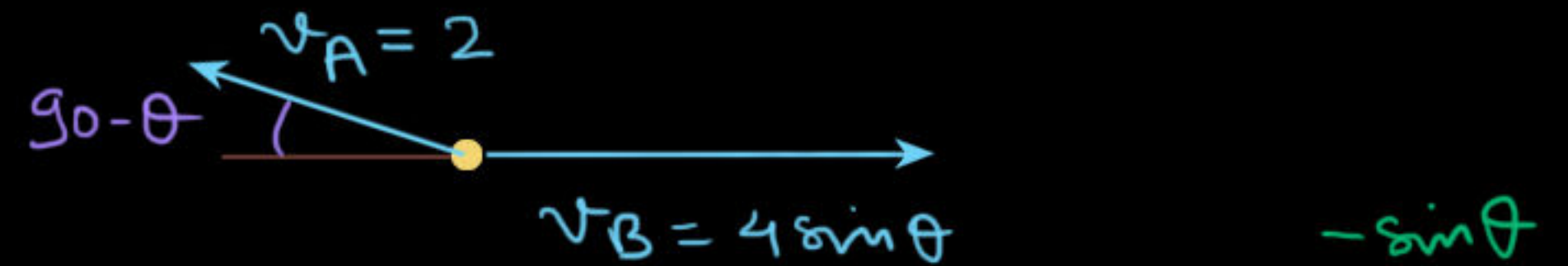
$$\text{B } \therefore a_B = 2 \cos \theta$$



$$\text{distance} = 16 \times 2$$

$$= 32 //$$

D

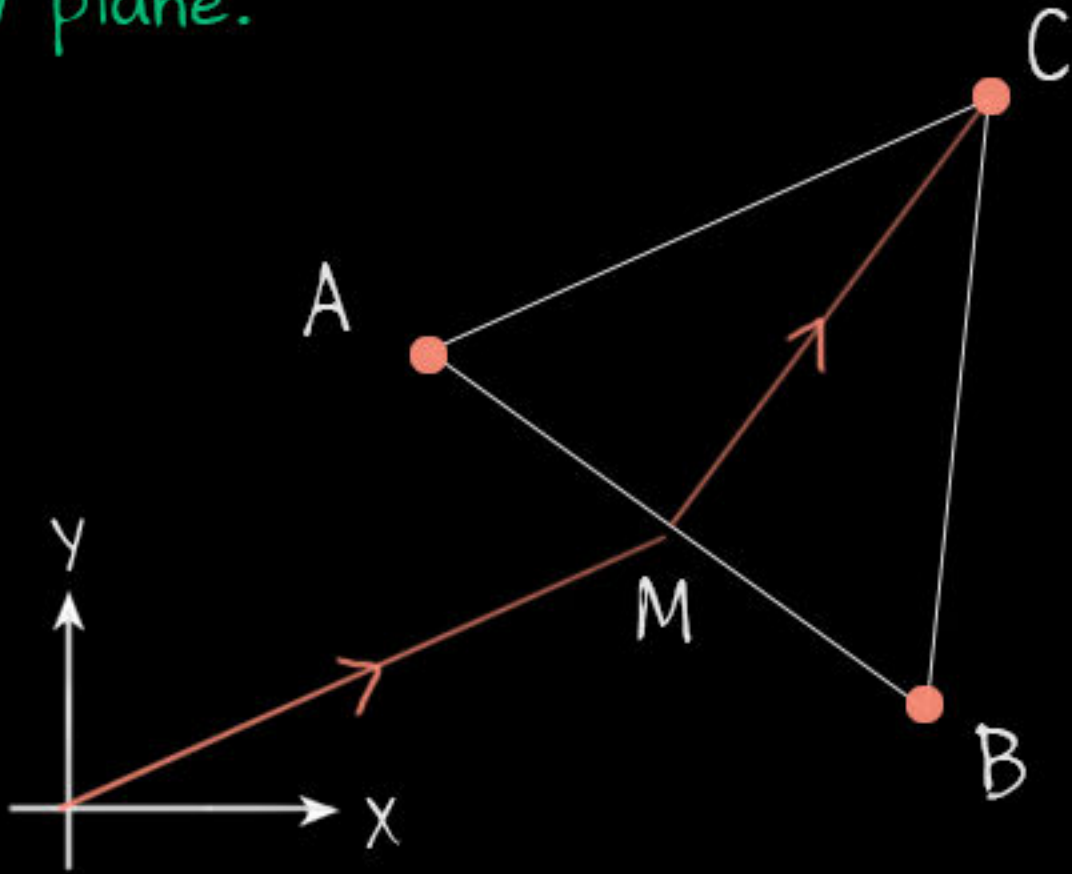
w.r.t B

$$\begin{aligned} \therefore v_{AB}^2 &= 2^2 + (4 \sin \theta)^2 + 2 \cdot 2 \cdot 4 \sin \theta \cos [180 - (90 - \theta)] \\ &= 2^2 + 16 \sin^2 \theta - 16 \sin^2 \theta \end{aligned}$$

$$\Rightarrow v_{AB} = 2 = \text{constant}$$

Approach: To get the relation between velocities, relate their positions first and differentiate it.

Lets define our coordinate axis such that triangle lies in XY plane.



22. Three ants A, B and C are crawling on a large horizontal tabletop always occupying vertices of an equilateral triangle, size of which may vary with time. If at an instant, speeds of A and B are v_A and v_B , which of the following conclusions can you make for speed v_C of C?

(a) $v_C < 0.5(v_A + v_B)$

(b) $v_C \leq 0.5(v_A + v_B)$

(c) $v_C < v_A + v_B$

✓ (d) $v_C \leq v_A + v_B$

$$\vec{r}_C = \vec{r}_M + \vec{r}_{MC}$$

$$= \frac{\vec{r}_A + \vec{r}_B}{2} + \frac{\sqrt{3}}{2} (\vec{r}_A - \vec{r}_B) \times \hat{k}$$

$$= \frac{\vec{r}_A}{2} + \frac{\sqrt{3}}{2} (\vec{r}_A \times \hat{k}) + \frac{\vec{r}_B}{2} - \frac{\sqrt{3}}{2} (\vec{r}_B \times \hat{k})$$

↓ differentiate

$$\vec{v}_C = \left[\frac{\vec{v}_A}{2} + \frac{\sqrt{3}}{2} (\vec{v}_A \times \hat{k}) \right] + \left[\frac{\vec{v}_B}{2} - \frac{\sqrt{3}}{2} (\vec{v}_B \times \hat{k}) \right]$$

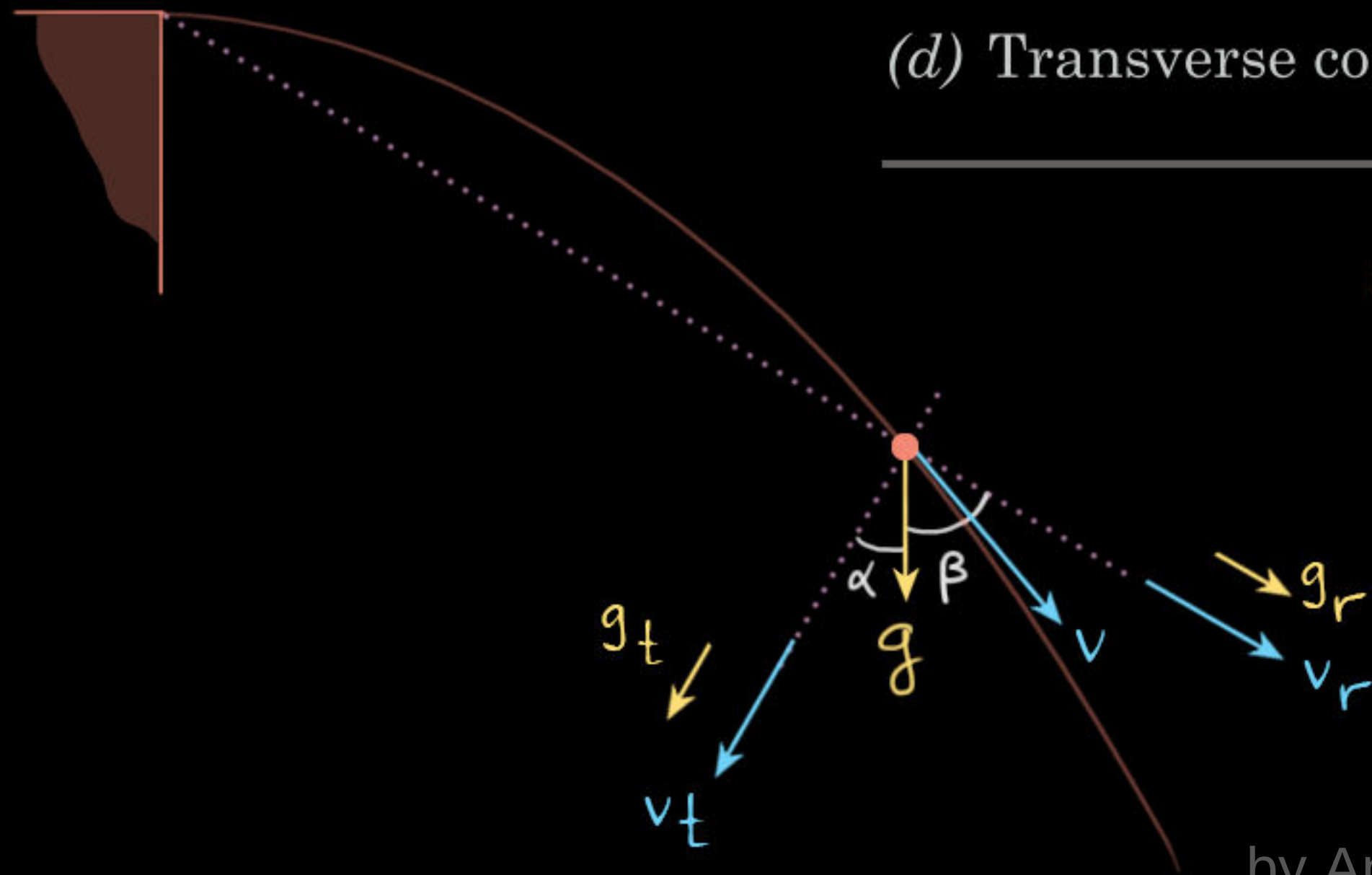
$$\vec{v}_c = v_A \left[\underbrace{\frac{\hat{v}_A}{2} + \frac{\sqrt{3}}{2} (\hat{v}_A \times \hat{k})}_{\text{A unit vector that depends on } \hat{v}_A} \right] + v_B \left[\underbrace{\frac{\hat{v}_B}{2} - \frac{\sqrt{3}}{2} (\hat{v}_B \times \hat{k})}_{\text{A unit vector that depends on } \hat{v}_B} \right]$$

$$\therefore |v_A - v_B| \leq v_c \leq v_A + v_B$$

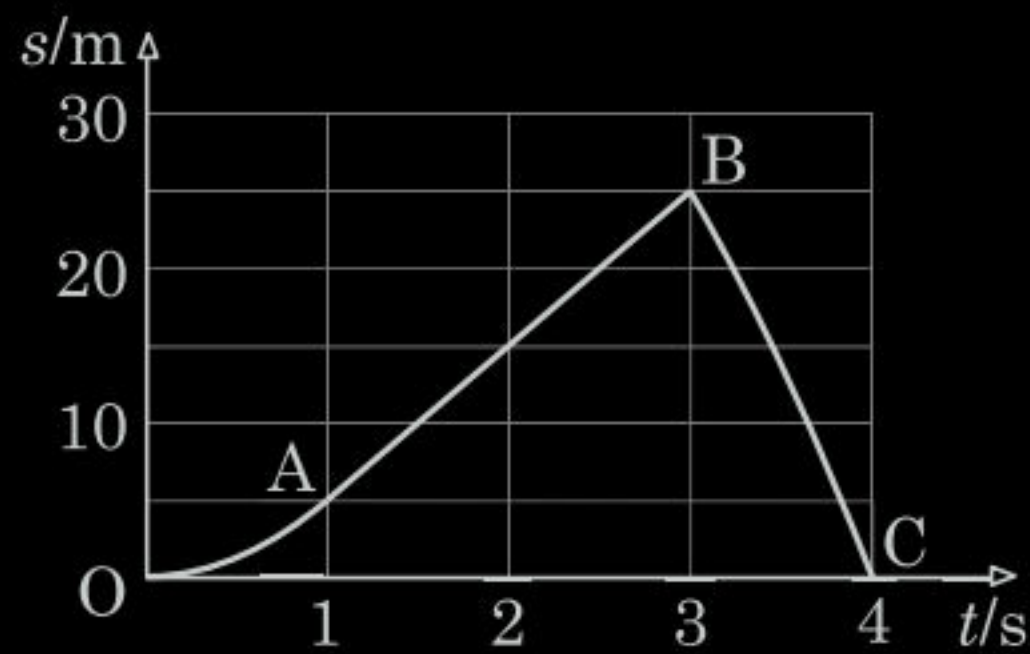
(If these unit vectors are in same or opposite directions)

23. Components along and perpendicular to a position vector are known as radial and transverse components respectively. A particle is projected horizontally from the top of a tower. Assume acceleration due to gravity to be uniform and the origin of the coordinate system at the point of projection.

Column-I	Column-II
(a) Radial component of velocity	(p) Always increases.
(b) Transverse component of velocity	(q) Always decreases.
(c) Radial component of acceleration	(r) First increases then decreases.
(d) Transverse component of acceleration	(s) First decreases then increases.



- A v_r always increases as v_r and g_r are in same direction
- B v_t always increases as v_t and g_t are in same direction
- C g_r always increases as β decreases with time.
- D g_t always decreases as α increases with time.



O: 1 is dropped

A: 2 is dropped

B: 1 reaches ground

C: 2 reaches ground



A stone is dropped from the top of a tower and before it hits the ground another stone is also dropped. Separation s between the stones is plotted against time t assuming that the stones do not bounce from the ground. Portions OA and BC of the graph are parabolic, while portion AB is a straight line. Acceleration due to gravity is 10 m/s^2 .

24. Height of the tower is

(a) 25 m

(b) 30 m

(c) 40 m

✓ (d) 45 m

25. When the first stone hits the ground, the second stone was moving with

(c) 20 m/s at a height 20 m.

✓ (d) 20 m/s at a height 25 m.

It takes 3 seconds for either of them to reach ground

$$\therefore h = \frac{1}{2}g(3)^2 = 45 \text{ m} //$$

When first stone reaches ground, second has been in air for 2 seconds.

$$\therefore v_2 = g(2) = 20 \text{ m/s} \quad \text{and} \quad s_2 = \frac{1}{2}g(2)^2 = 20 \text{ m}$$

$$\Rightarrow h_2 = h - s_2 = 45 - 20 = 25 \text{ m} //$$

Lets focus on first two cars



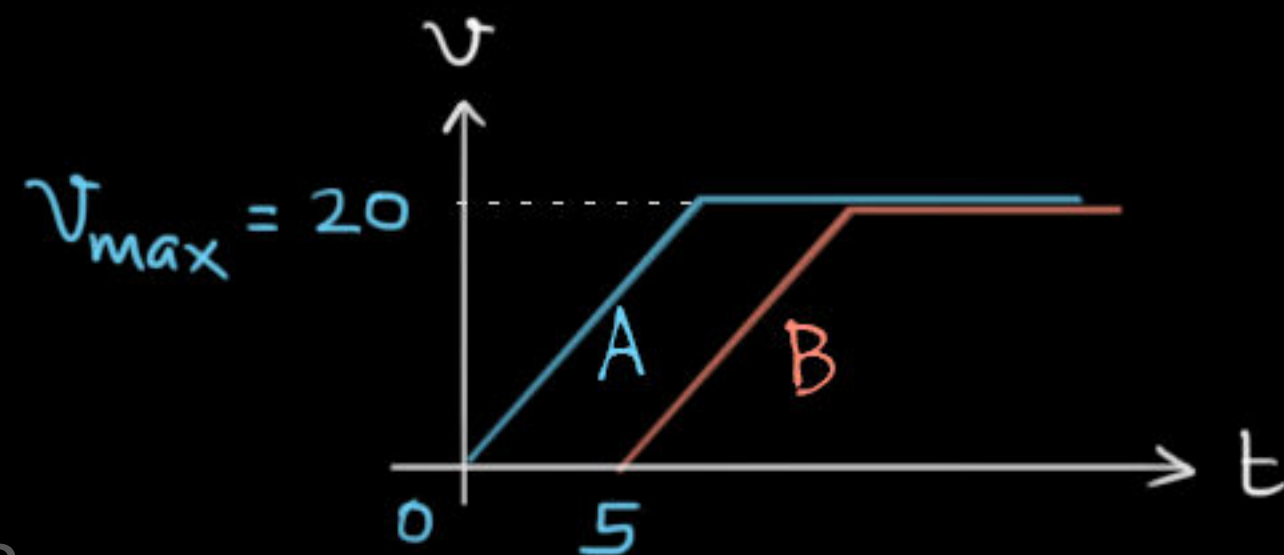
Initial gap is 10m

So, for total gap of 35m, A must travel 25m and then B starts.

$$\therefore 25 = \frac{1}{2} (2) t^2$$

$$\Rightarrow t = 5$$

Plotting v-t for both cars.



In a convoy on a long straight level road, 50 identical cars are at rest in a queue at equal separation 10 m from each other as shown.



Engine of a car provide a constant acceleration 2 m/s^2 and brakes can provide a maximum deceleration 4 m/s^2 . When an order is given to start the convoy, the first car starts immediately and each subsequent car start when its distance from a car that is immediately ahead becomes 35 m. Maximum speed limit on this road is 72 km/h. When an order is given to stop the convoy, the driver of the first car applies brakes immediately and driver of each subsequent car applies brakes with a certain time delay after noticing brake light of the front car turned red.

26. When all the cars are moving at the maximum speed, what is the separation between two adjacent cars?

(c) 100 m

✓(d) 110 m

$$\therefore \text{Final separation} = \text{Initial gap} + (\text{displacement of A} - \text{displacement of B})$$

$$= 10 +$$

$$= 10 + 5 \times 20$$

$$= 110 \text{ m} //$$

by Ankit Singhvi

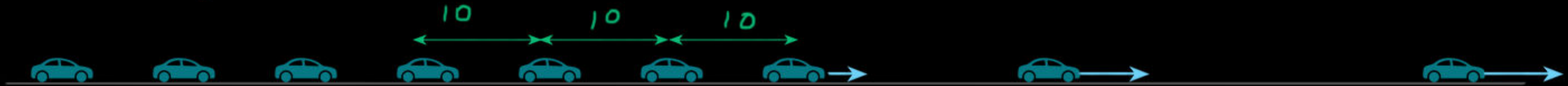
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Every 5 seconds, frontmost parked car starts and the length of parked car column get reduced by 10m.

27. During the time when motion is building up in the convoy, some of the cars are moving and the others are at rest. What is the average rate of change in length of the segment consisting of stationary cars?

✓(c) Decreasing at 2 m/s

(d) Decreasing at 5 m/s



∴ Rate of change of length = $-\frac{10\text{m}}{5\text{s}} = -2\text{ m/s}$ //

Final gap is same as initial.
So 'gap increased' should be same as 'gap decreased'.



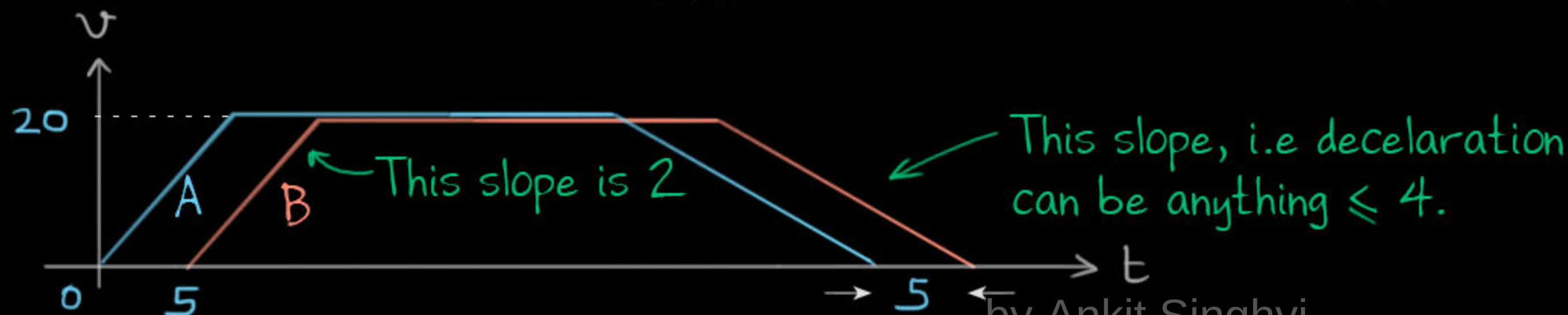
28. When all the cars are moving at the maximum speed, an order is given to stop the convoy. If all the cars decelerate at equal constant rates and separation between every two adjacent cars again becomes 10 m after the whole convoy stops, what can be the deceleration of the cars during braking?

✓(a) 2 m/s²

✓(b) 4 m/s²

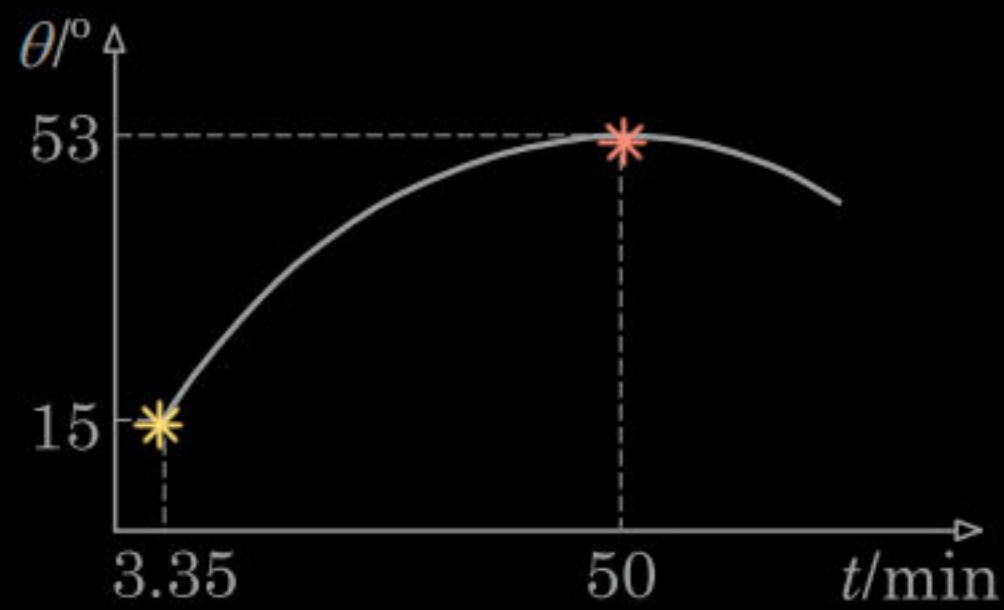
✓(c) $\leq 4\text{ m/s}^2$

(d) Insufficient information



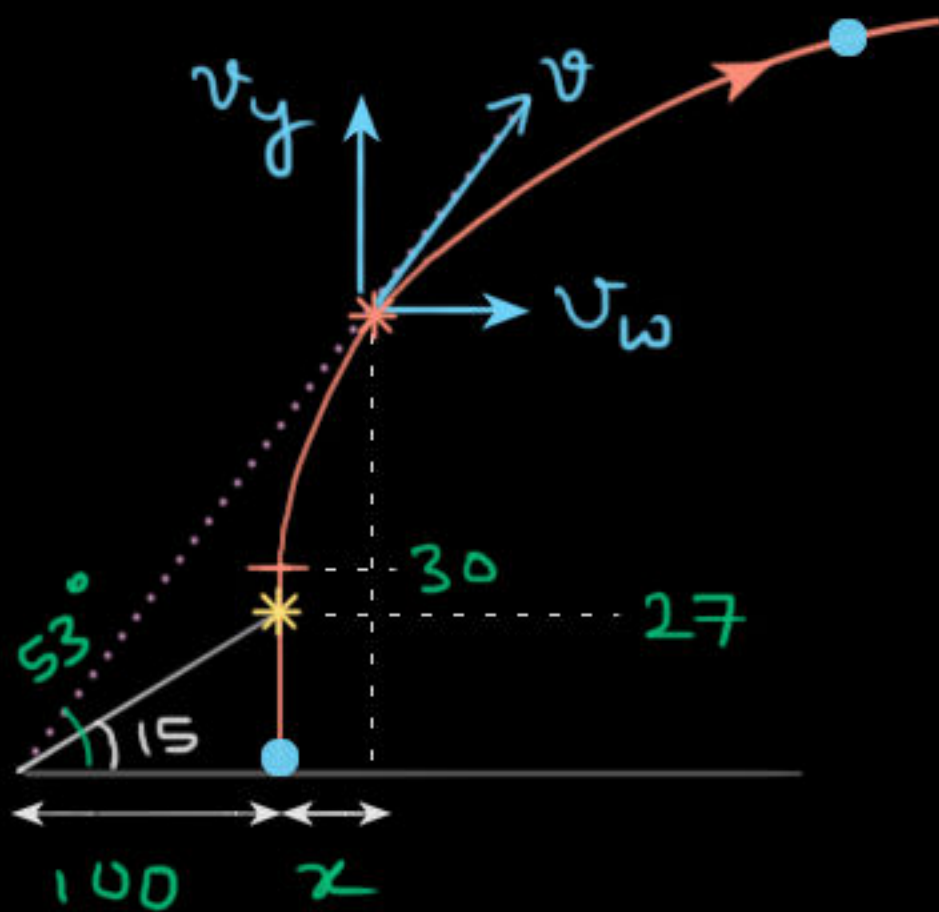
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$$\tan 15^\circ = 0.27 = \frac{27}{100}$$

⇒ Balloon is at 27m, when first observed.



A hot air balloon of constant ascent velocity can be used to investigate wind velocities at various altitudes. Employing this idea, one day a hot air balloon was released at a distance $l = 100$ m from a point, where a telescope was installed to track the balloon. Since wind velocity was almost zero up to a height of approximately 30 m that day, the balloon first rose upwards and then due to horizontal drift caused by the wind it followed a plane curvilinear path. During continuous tracking of the balloon, the telescope has been rotated in a vertical plane without change in azimuth. A graph depicting how the angle of elevation varies with time thus obtained is shown in the figure.

29. Ascent velocity of the balloon is closest to
30. Wind velocity at the altitude of the balloon, where angle of elevation acquires its maximum value is closest to
31. Horizontal drift of the balloon, when angle of elevation acquires its maximum value is closest to

$$* v_y(3.35) = 27$$

$$\Rightarrow v_y \approx 8 \text{ m/min}$$

$$* \frac{v_y}{v_w} = \tan 53^\circ$$

$$\Rightarrow v_w = 6 \text{ m/min}$$

$$* \frac{(v_y \cdot t)}{100 + x} = \tan 53^\circ$$

$$\Rightarrow x = 200$$

Lets draw circles around A and B of radii 40 and 30 respectively.

w.r.t A, B moves towards it @ 10m/s. Let P be foremost point on B's circle.

C exists as long as both circles touch each other.

Circles connect when P reaches 1 (3sec), then disconnect at 2 (9 sec), connect again at 3 (11 sec) and disconnect at 4 (17 sec).

∴ Connection @ 3 to 9 sec then @ 11 to 17 sec.

Two particle A and B are moving towards each other on a straight line with equal speeds 5 m/s. At an instant that is assumed $t = 0$ s, distance between the particles is 100 m. It is desired to move another particle C always maintaining a distance 40 m from the particle A and 30 m from the particle B.

32. When and for how long can the particle C fulfil the given condition?

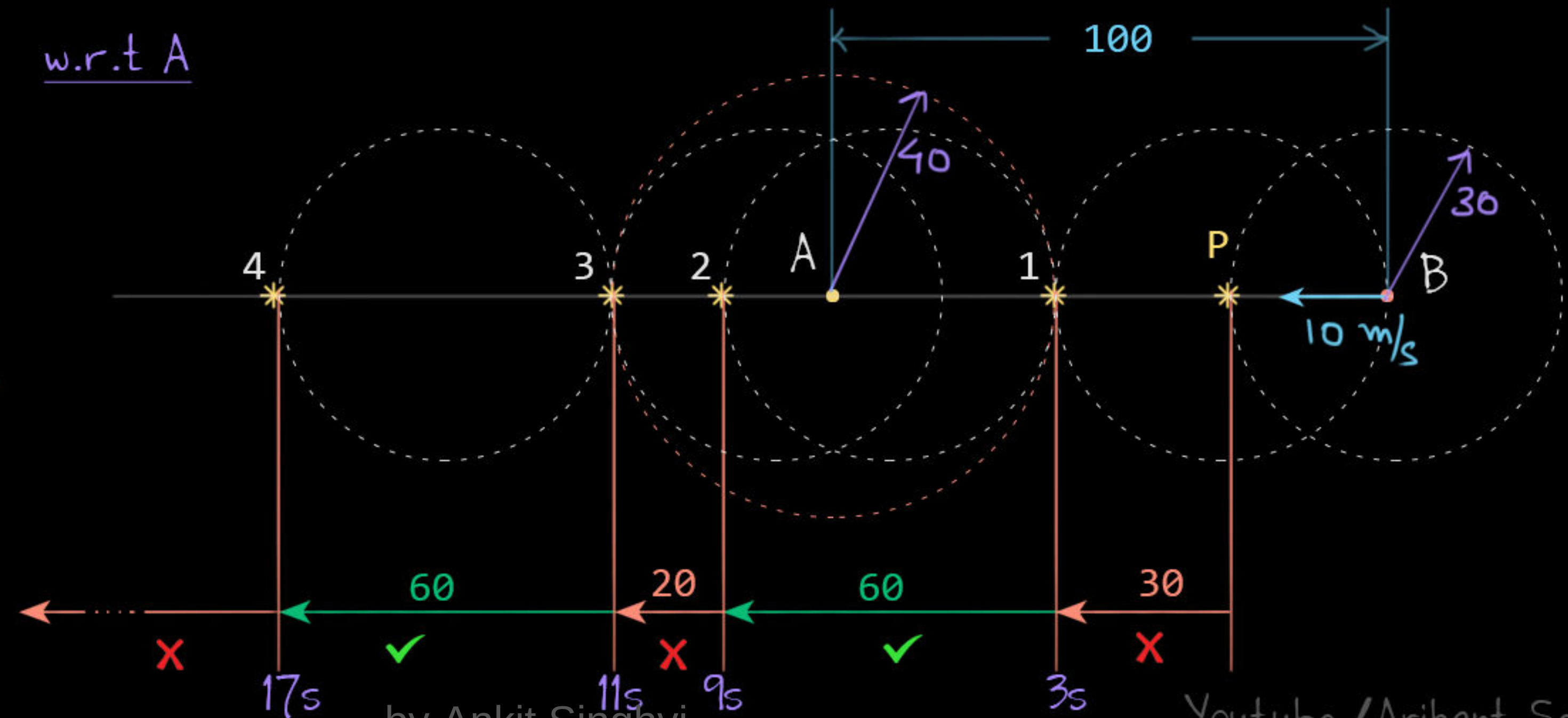
✓ (a) $3 \text{ s} \leq t \leq 9 \text{ s}$

(b) $9 \text{ s} \leq t \leq 11 \text{ s}$

✓ (c) $11 \text{ s} \leq t \leq 17 \text{ s}$

(d) $3 \text{ s} \leq t \leq 17 \text{ s}$

w.r.t A



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At 5 seconds, distance between AB is 50.
So ABC conveniently becomes a right angled triangle.

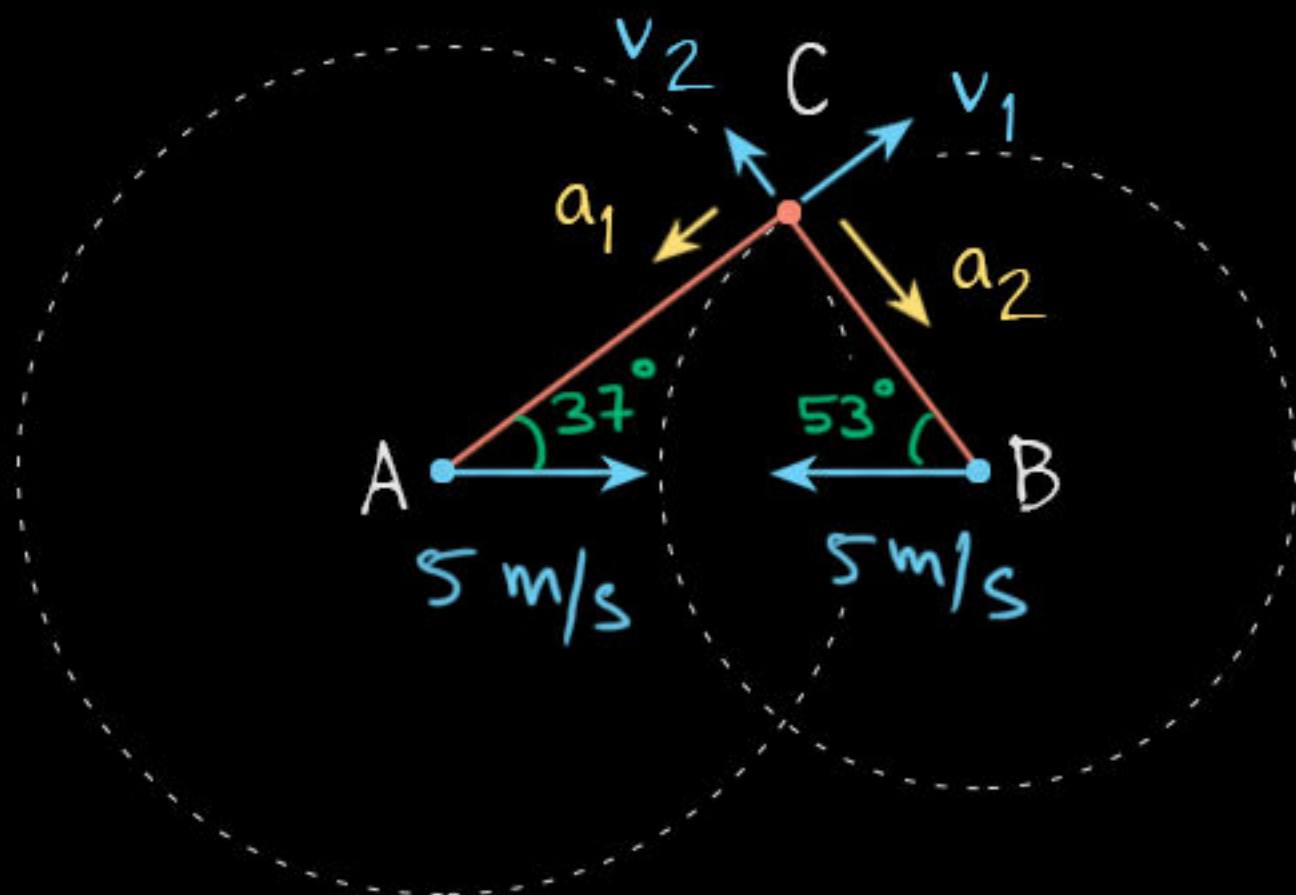
Approach: As C moves on both circles, let's treat AC and BC as rods, so we can equate velocities along these rods.

Let's assume a_1 and a_2 along CA and CB.

Approach: C moves on circle A. So w.r.t A, we can find a_1 by writing centripetal acceleration in terms of C's velocity perpendicular to AC. Similarly w.r.t B, we can find a_2 .

33. What is speed of the particle C at the instant $t = 5$ s?

(c) 5 m/s



$$AC: 5 \cos 37^\circ = v_1$$

$$\Rightarrow v_1 = 4$$

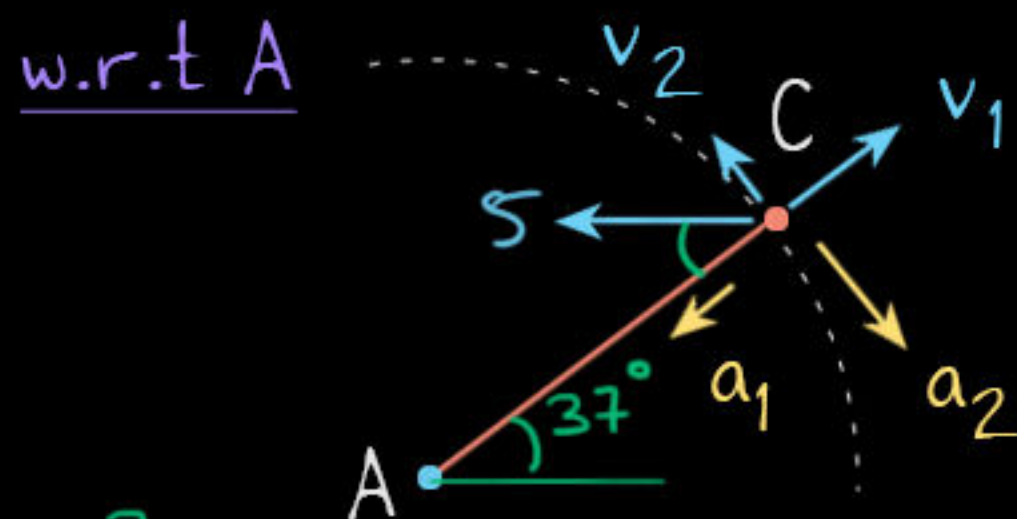
$$BC: 5 \cos 53^\circ = v_2$$

$$\Rightarrow v_2 = 3$$

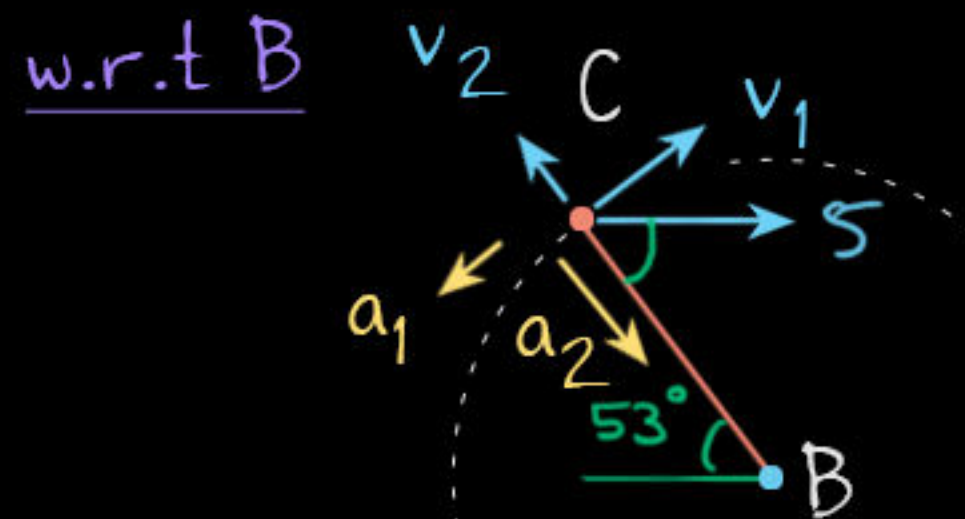
$$v_C = 5$$

34. What is modulus of acceleration of the particle C at the instant $t = 5$ s?

(a) 2.3 m/s²



$$a_1 = \frac{(v_2 + 5 \sin 37^\circ)^2}{40} = \frac{6^2}{40}$$



$$a_2 = \frac{(v_1 + 5 \sin 53^\circ)^2}{30} = \frac{8^2}{30}$$

$$\sqrt{a_1^2 + a_2^2}$$

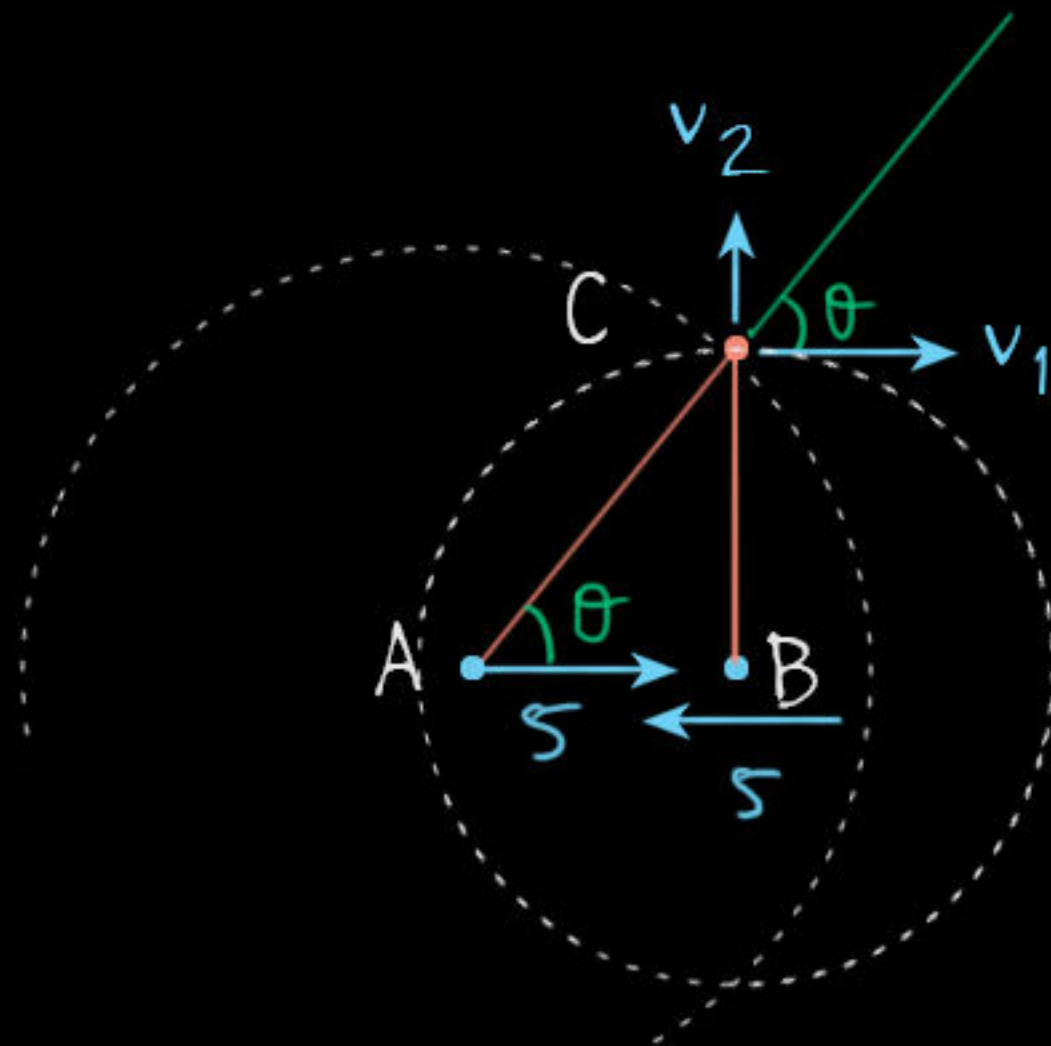
35. At the instant, when the line joining locations of A and B is perpendicular to the line joining locations of B and C, what are the magnitudes of velocities of C relative to A and B respectively?

(a) 0 m/s and 5 m/s

✓ (b) 0 m/s and 10 m/s

(c) 3.75 m/s and 0 m/s

(d) 3.75 m/s and 10 m/s



$$BC: v_2 = 0$$

$$AC: 5 \cos \theta = v_1 \cos \theta + v_2 \sin \theta$$

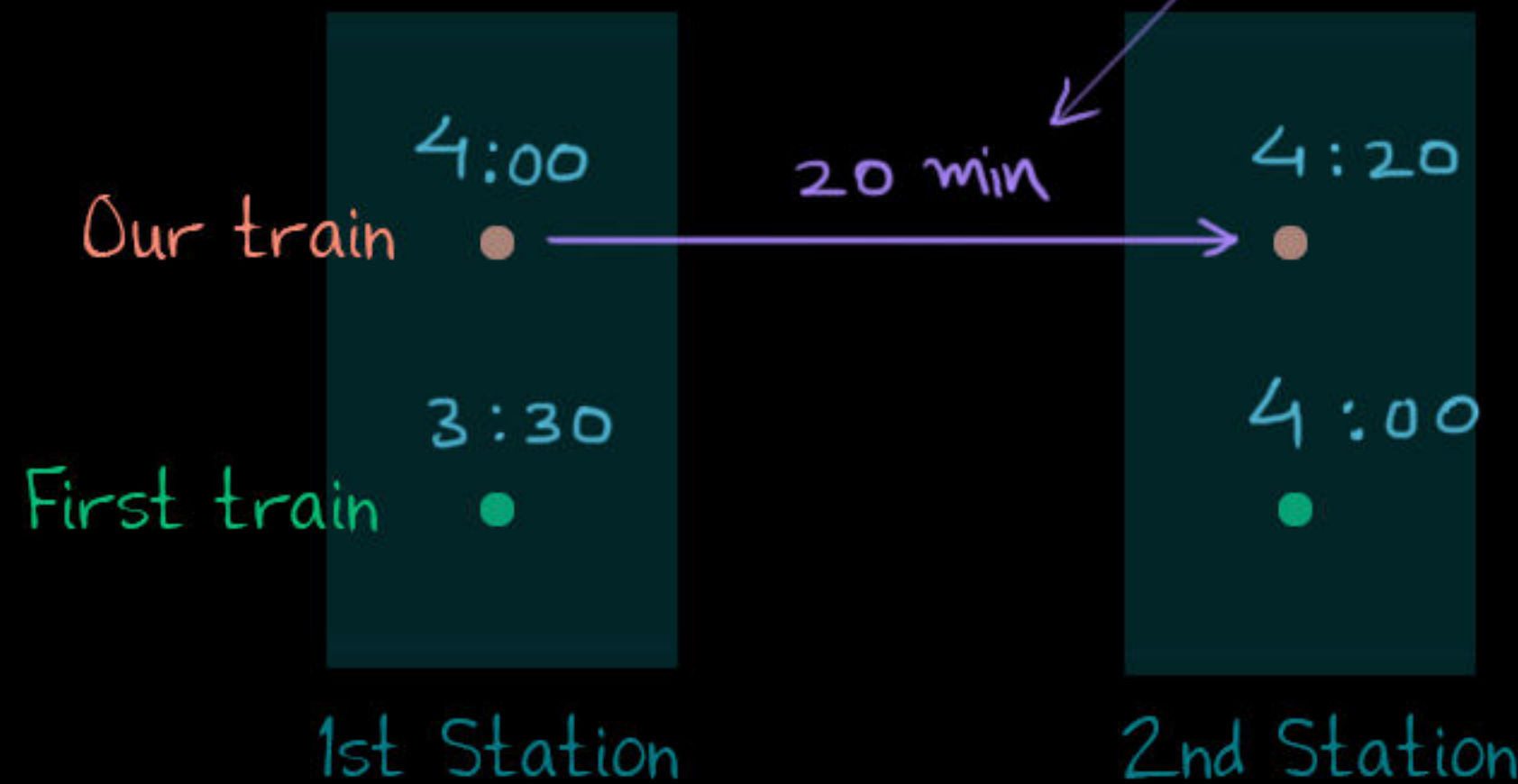
$$\Rightarrow v_1 = 5$$

$$\therefore \text{w.r.t A} \quad v_{C|A} = 0 //$$

$$\text{w.r.t B} \quad v_{C|B} = 10 //$$

Approach: Lets take a wrist watch along with us!
Now assume that initially our watch says 4:00.

1. In an announcement on a railway station, a passenger hears that the last train has passed the station $\Delta t_1 = 30$ min earlier than his train. On the next station that is $s = 20$ km away from the previous station, in another announcement he hears that the first train arrived $\Delta t_2 = 20$ min earlier than his train. Reading time from his watch, he calculates average speed of his train to be $v_p = 60$ km/h. Relying on the announcements and the passenger's calculations, determine average speed of the first train.

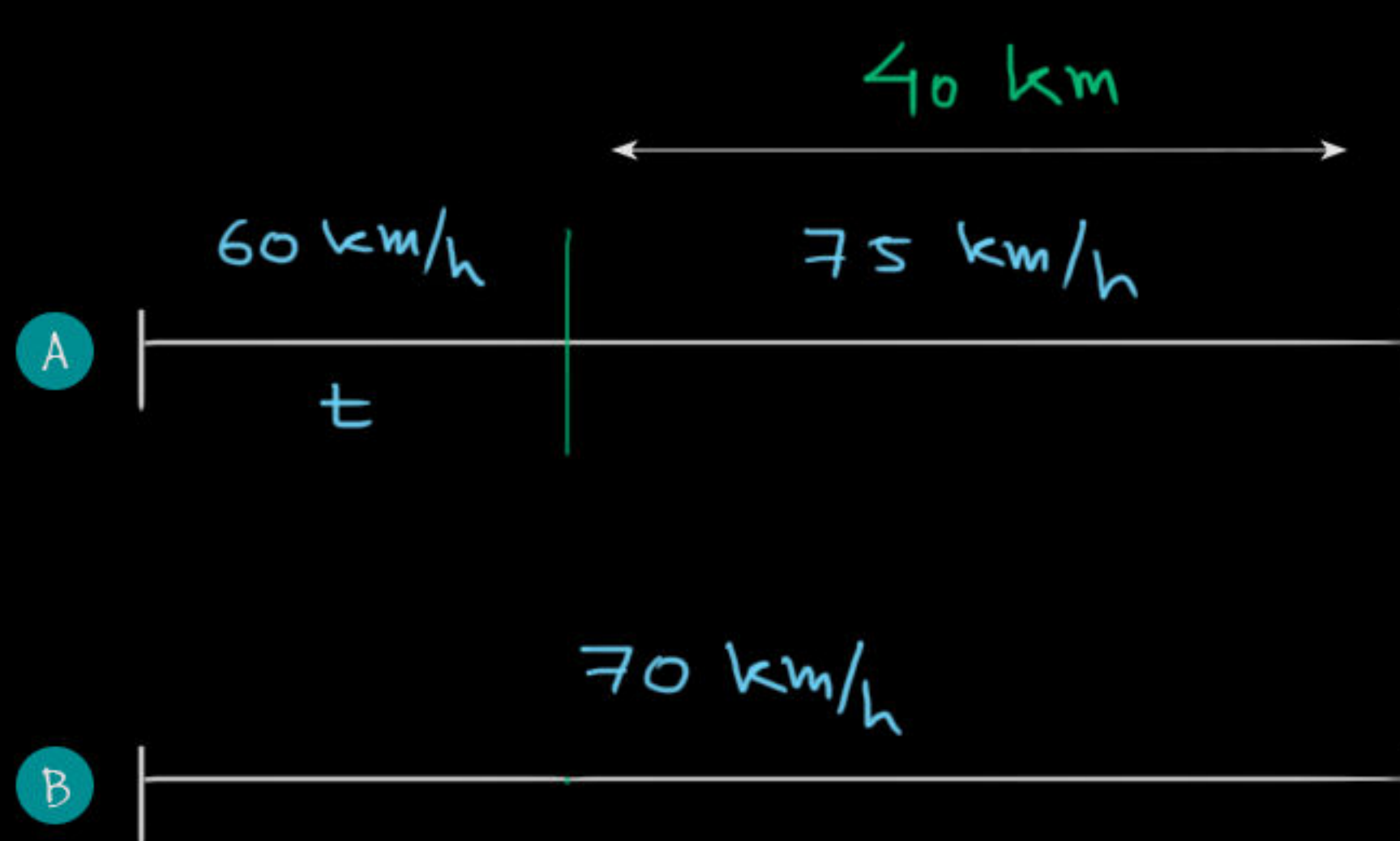


$$\therefore v_{\text{first train}} = \frac{20 \text{ km}}{\frac{1}{2} \text{ hr}} = 40 \text{ km/hr} //$$

by Ankit Singhvi

2. One day you were on a picnic with your class. During return journey from the picnic spot to your school, it began to rain, therefore the driver reduced speed of the bus and drove with an average speed $v_1 = 60 \text{ km/h}$ instead of the scheduled average speed $v_0 = 70 \text{ km/h}$. After the rain stopped, the driver drove the bus at an average speed $v_2 = 75 \text{ km/h}$ and covered the remaining $s = 40 \text{ km}$ exactly in scheduled time. How long did it rain?

Lets say it rained for t hrs.



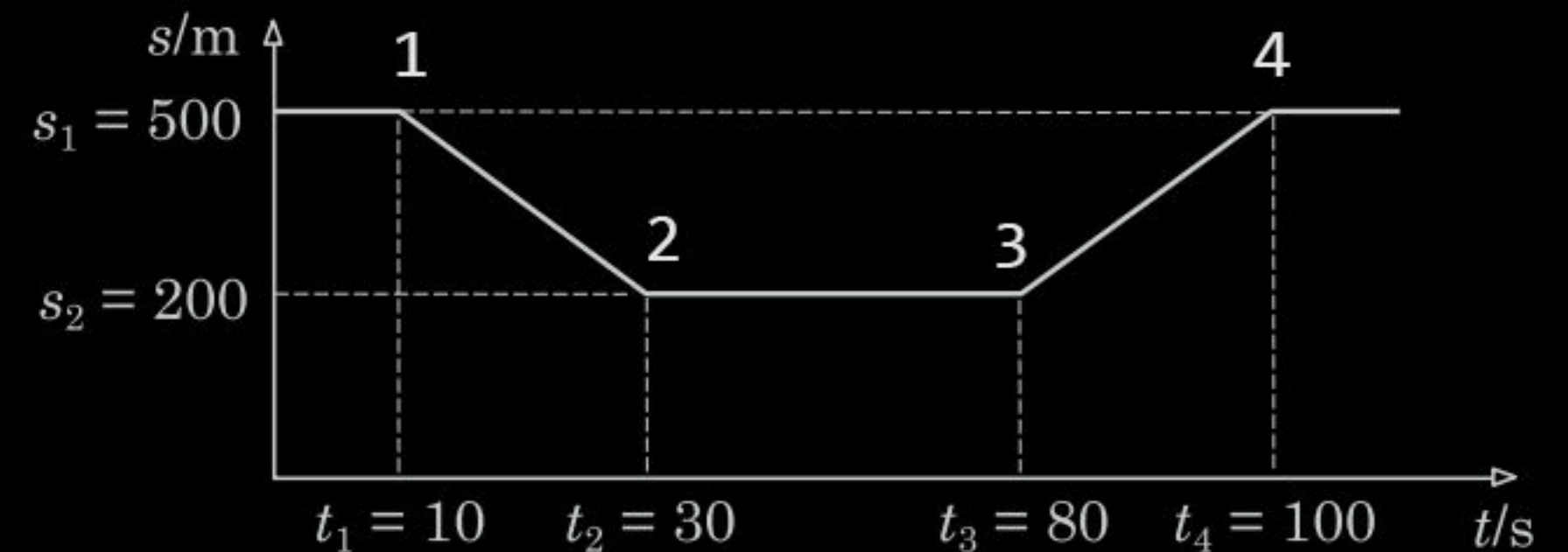
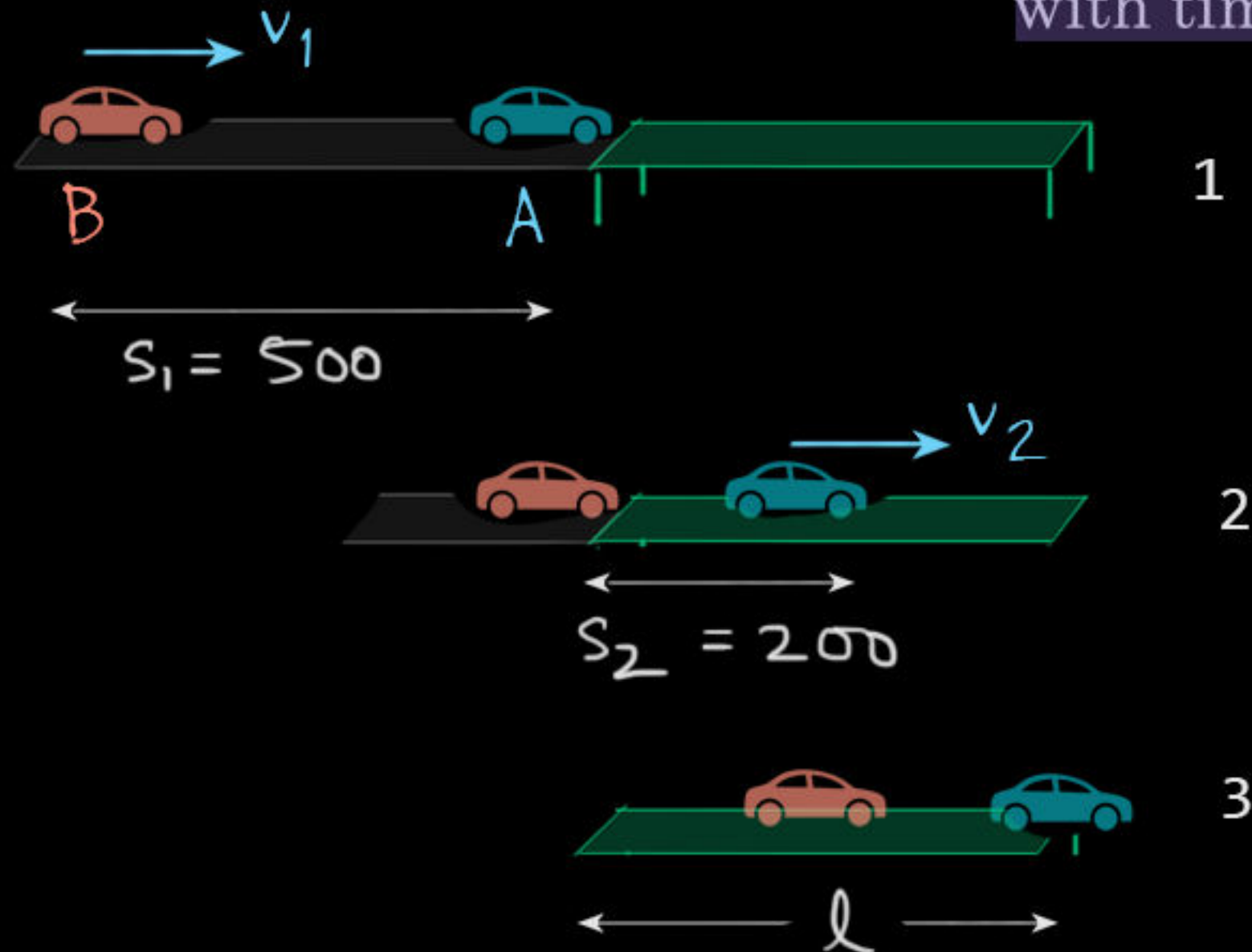
$$t_A = t_B$$

$$\Rightarrow t + \frac{40}{75} = \frac{60t + 40}{70}$$

$$\Rightarrow t = \frac{4}{15} \text{ hr} = 16 \text{ min} //$$

- 1 A enters bridge
- 2 B enters bridge
- 3 A leaves bridge
- 4 B leaves bridge

3. There is a narrow **bridge** somewhere on a road connecting two towns. Two cars travel from one of the towns to the other with a constant **speed** v_1 everywhere on the road, except **on the bridge**, where they travel with another constant speed v_2 . How the **separation s between the cars varies with time t** is shown in the following graph.



- (a) What is the speed v_1 of the cars on the road?
- (b) What is the speed v_2 of the cars on the bridge?
- (c) What is **length of the bridge**?

A $s_1 = v_1 (t_2 - t_1)$

$\Rightarrow v_1 = \frac{s_1}{t_2 - t_1} = \frac{500}{30 - 10} = 25 \text{ m/s}$

B $s_2 = v_2 (t_3 - t_2)$

$\Rightarrow v_2 = \frac{s_2}{t_3 - t_2} = \frac{200}{80 - 30} = 4 \text{ m/s}$

C $l = v_2 (t_4 - t_3)$

$= 4 (100 - 80)$

$= 80$

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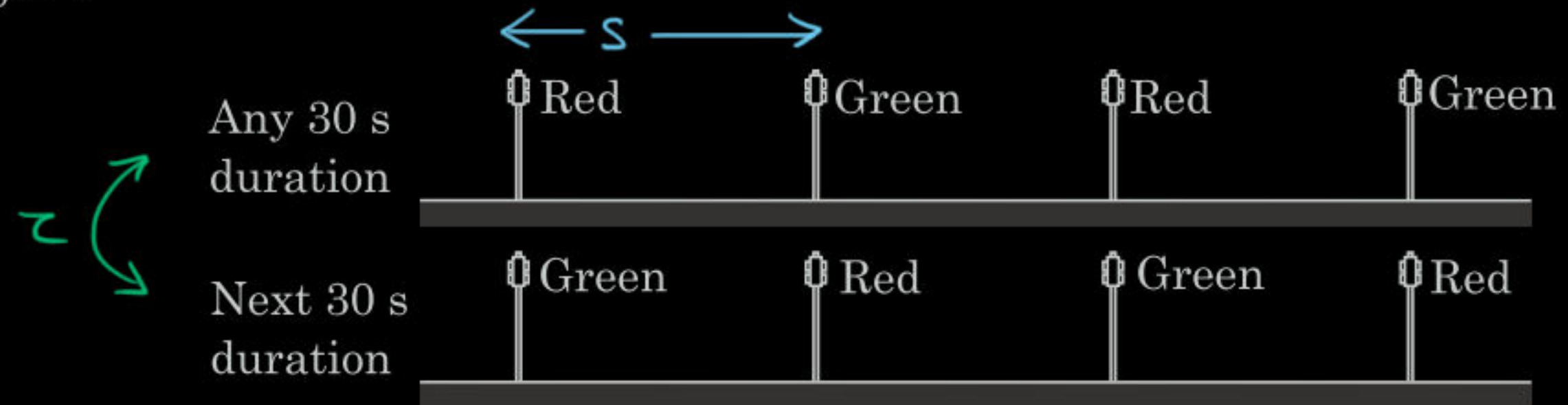
To be 100% sure to go from green to green, travel interval between signals must be odd multiple of 30 seconds.

$$\text{i.e. } \frac{s}{v} = (2n+1)\tau$$

$$\Rightarrow v = \frac{s}{(2n+1)\tau}$$

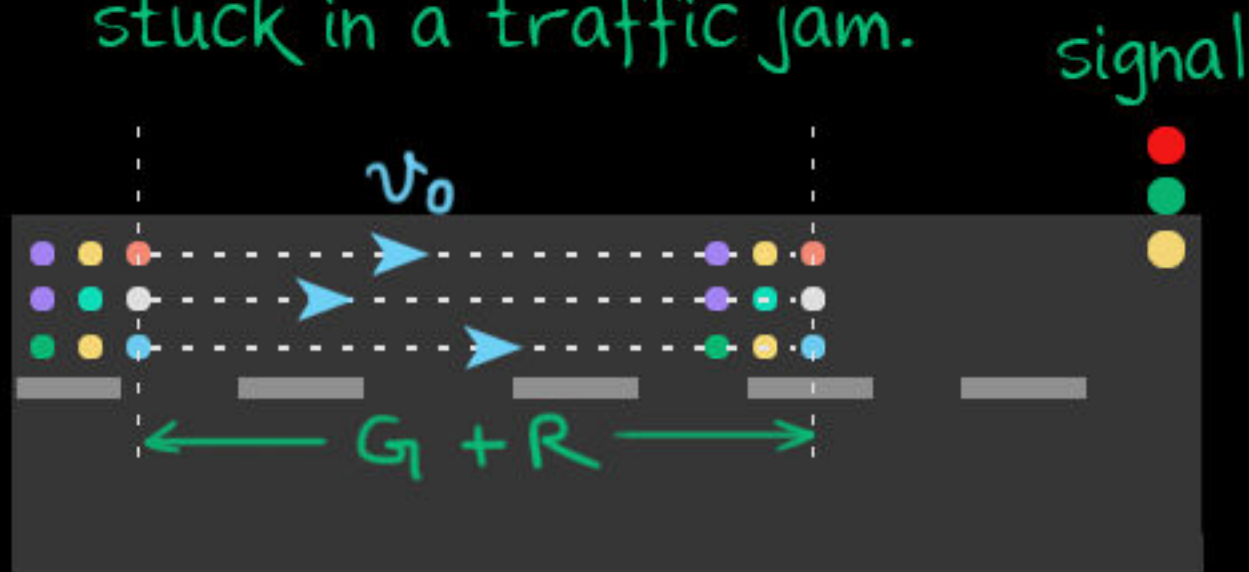
where, $n \rightarrow 0, 1, 2, \dots$

4. Traffic signals are installed at every $s = 1.00 \text{ km}$ on a long straight road. A signal remains red for $\tau = 30 \text{ s}$ and green for next $\tau = 30 \text{ s}$. The signals are synchronized in such a way that at a time, alternate signals remain red and the other remain green. The scheme is shown in the following figure.



Suggest possible constant speeds at which a vehicle can run on this road without a stop.

Shown below is one Green Red cycle for a few cars (with normal speed v_0) stuck in a traffic jam.



5. A traffic officer receives complaints on frequent traffic jams at a traffic signal on the main street of a busy market. He studied the traffic pattern and to simplify calculations made a reasonable assumptions that all the vehicles are identical in size and move with identical speeds. At present, durations of red and the green signals are equal and average speed of traffic advancement is $v_1 = 1.5 \text{ m/s}$. For improvement in the situation, if he orders to make duration of green signals $\eta = 2$ times and to leave that of red signals unchanged, what would be average speed v_2 of the traffic advancement?
- Not junction

$$\text{Average traffic advancement rate} = \frac{\text{Total length moved in a G/R cycle}}{\text{G/R cycle time}} = \frac{\text{Normal speed} \times \text{Green time}}{\text{G/R cycle time}}$$

$$\text{Initially: } v_1 = \frac{v_0 \cancel{t}}{2\cancel{t}}$$

$$\Rightarrow v_0 = 2v_1$$

$$\text{Later: } v_2 = \frac{v_0 \eta t}{(t + \eta t)}$$

$$\Rightarrow v_2 = \frac{2\eta v_1}{\eta + 1} = 2 \text{ m/s} \quad (\text{It increased from } 1.5 \text{ m/s})$$

We need to maximise vz



i.e minimise $\frac{1}{vz}$



i.e minimise the slope of line joining origin and any point on graph.

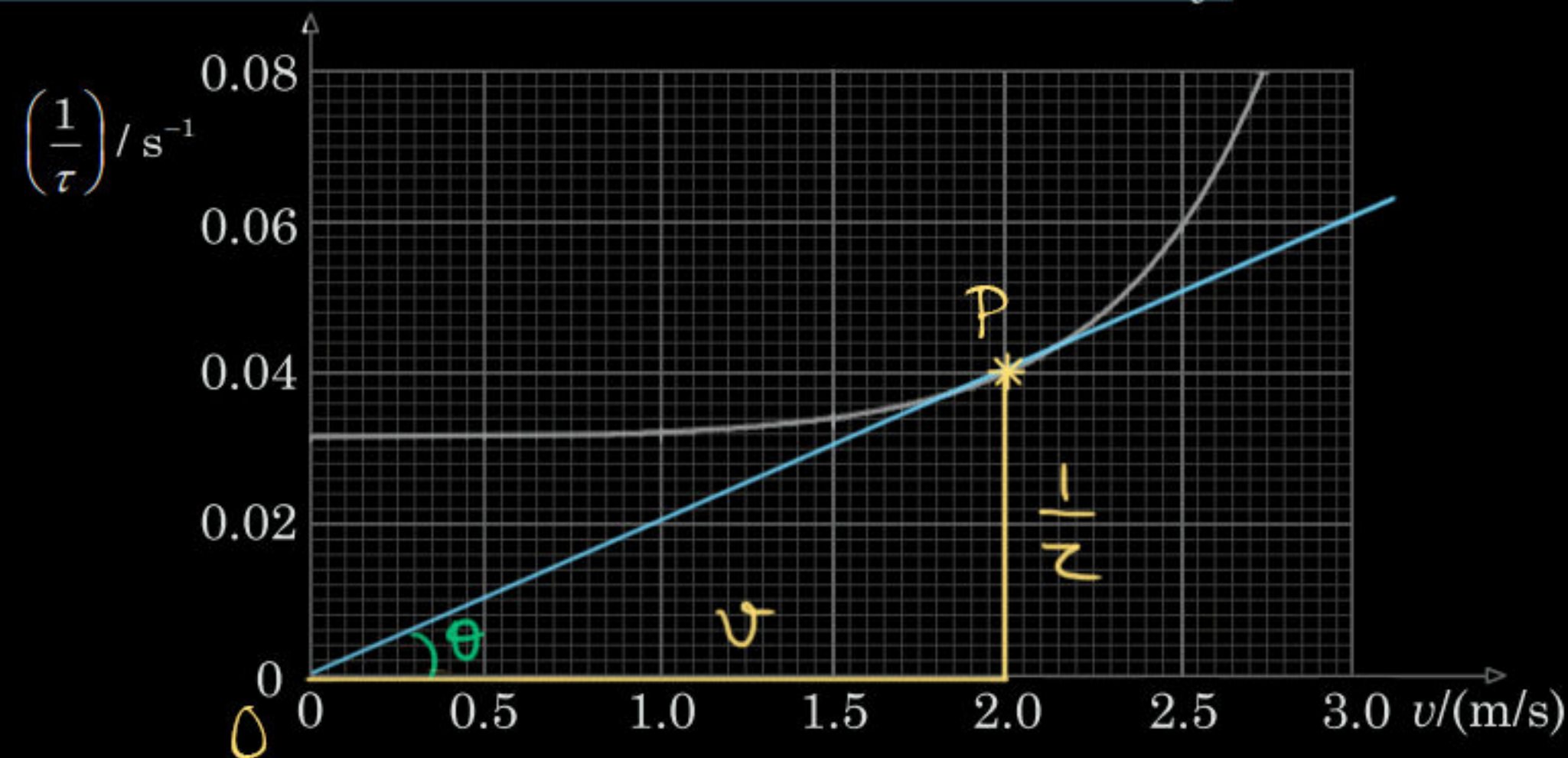


ie, at point P on graph, where OP is tangent to graph.

$\tan \theta = \frac{1}{vz}$ is minimum there

6. An engineer designs a robot that can climb stairs. If the robot climbs with a constant speed v , battery of the robot discharges completely in a time interval τ . This dependence is shown in the following graph.

With the help of the graph, determine the maximum length of a staircase, which the robot can climb with a constant velocity.



∴ Maximum distance = vz @ P

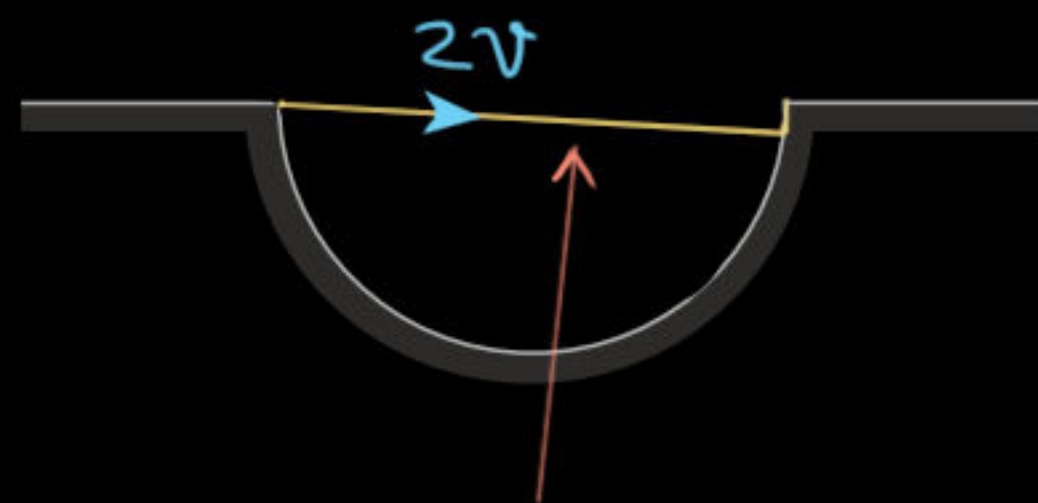
$$= 2 \left(\frac{1}{0.04} \right)$$

$$= 50 \text{ m}$$

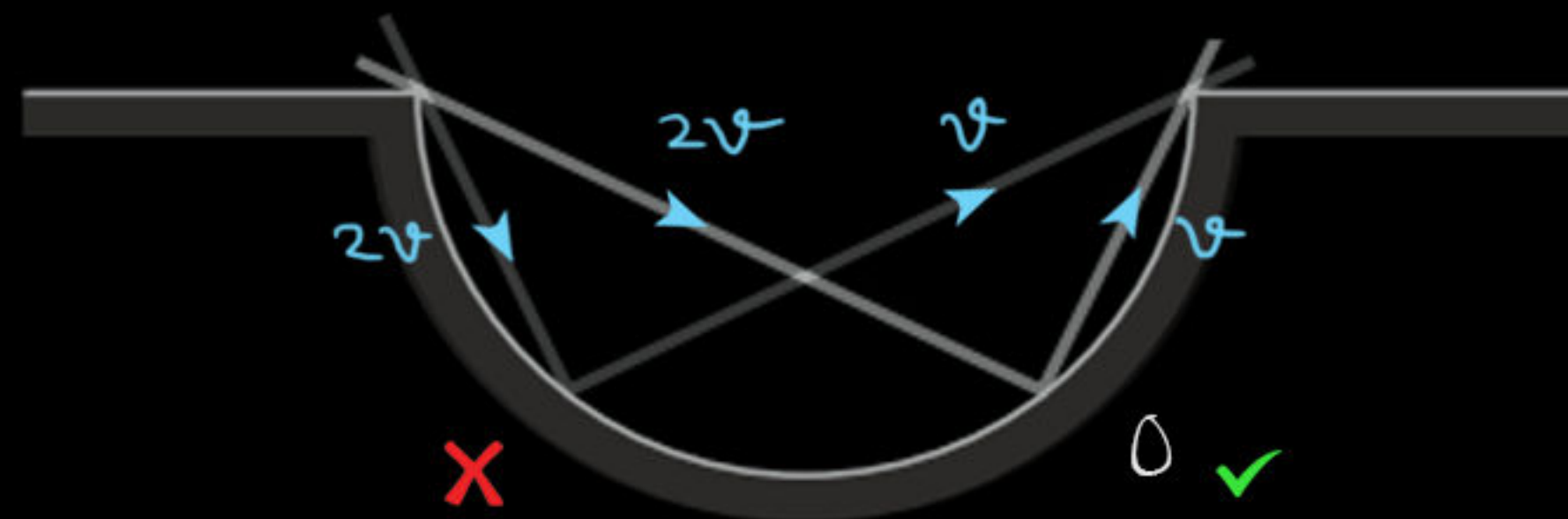
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7. A semi-cylindrical groove of radius $r = 20$ cm is made on a horizontal floor. An ant wants to cross the groove. A boy decides to help the ant making a bridge consisting of straight wire segments. But all the wires available are of length $l = 38$ cm, so the boy rigidly connects two wires at right angle and places the bridge in the groove as shown in the figure. If the ant can crawl up a wire segment at speed $v = 0.5$ cm/s and down a wire segment at speed $2v$, in what minimum time can the ant cross the groove with the help of this bridge?

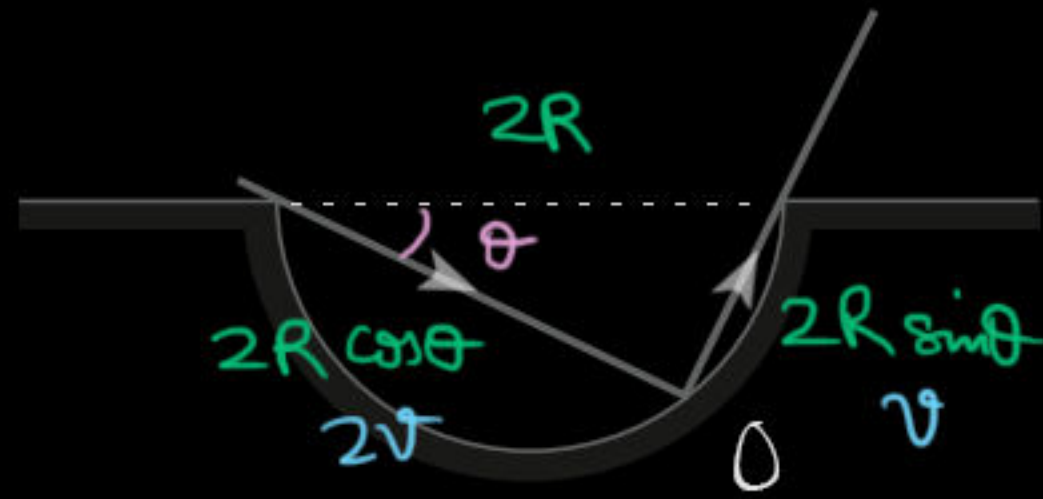


Concept 1: Shortest possible route will be going directly on other end with slightly depressed path. But rod is not that long.



Concept 2: 0 can not be on left half as symmetric position on right half will always be faster.

At a general angle θ



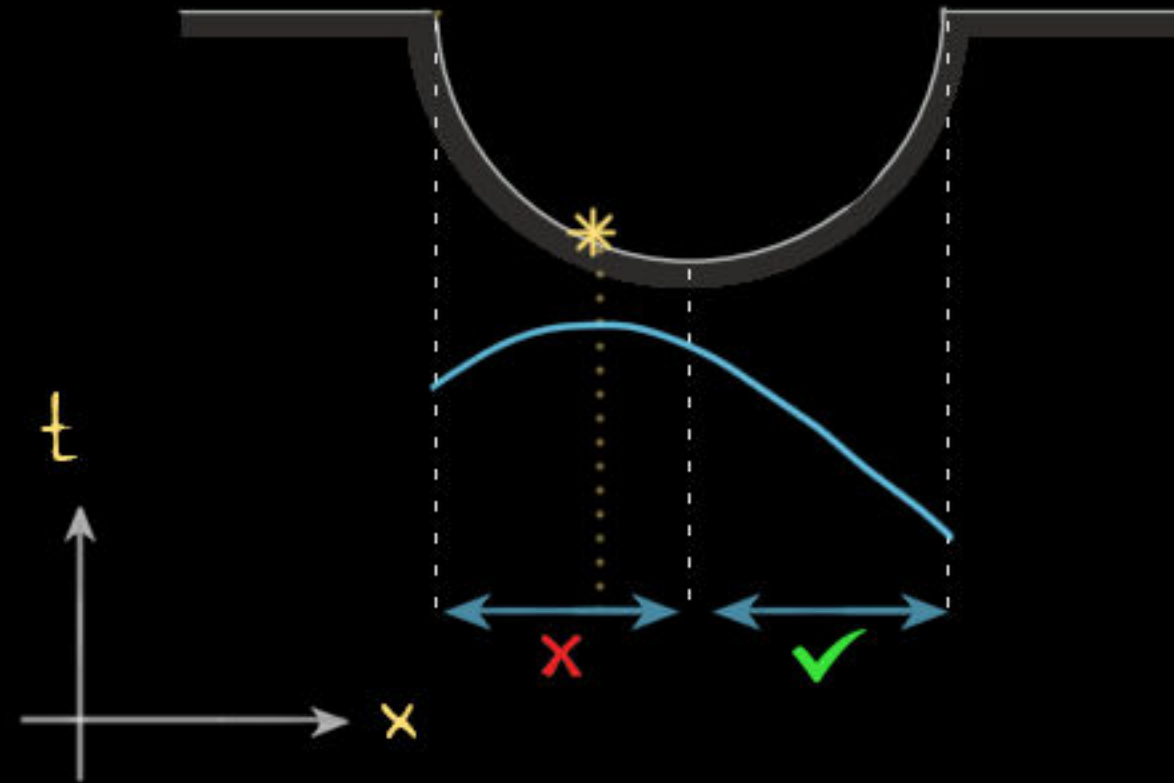
$$t = \frac{2R \cos \theta}{2v} + \frac{2R \sin \theta}{v}$$

$$\frac{dt}{d\theta} = 0 \Rightarrow -\sin \theta + 2 \cos \theta = 0$$

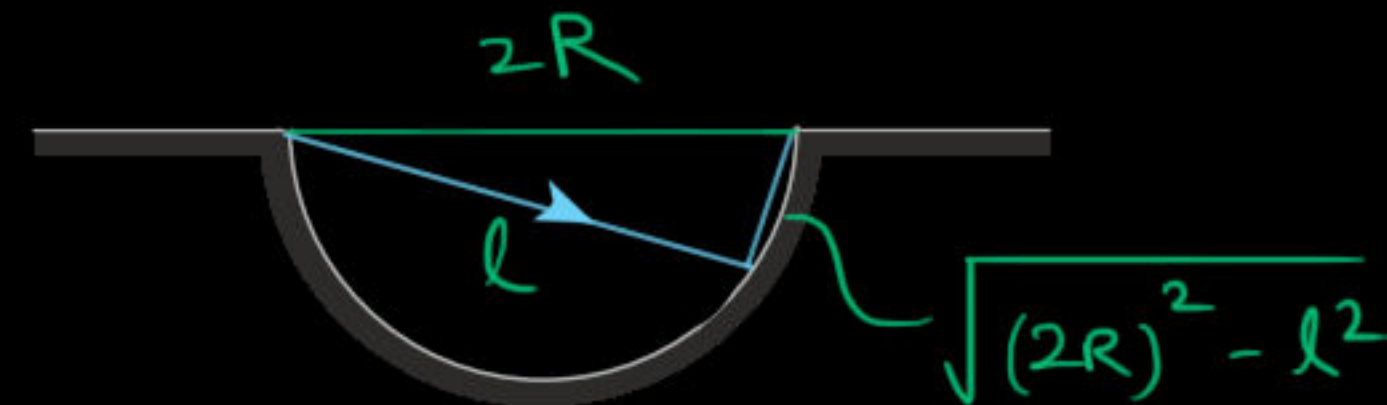
$$\Rightarrow \tan \theta = 2$$

∴ Single maxima and obviously in left half

↓
as $\frac{d^2t}{d\theta^2} < 0$



∴ For minimum time, full length l will be used going down.



$$t = \frac{l}{2v} + \frac{\sqrt{(2R)^2 - l^2}}{v}$$

Concept: velocity at midpoint is given by:

$$v_{\text{mid}} = \sqrt{\frac{u^2 + v^2}{2}} \quad (1)$$

Given: $v_{\text{av}} = \frac{u+v}{2} \quad (2)$

From 1,2:

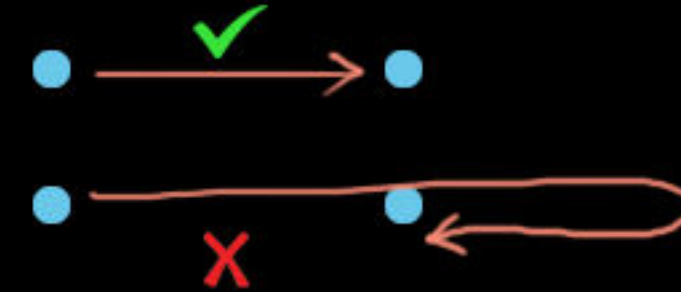
$$v_{\text{mid}}^2 = 2v_{\text{av}}^2 - uv$$

$$v_{\text{av}}^2 \leq v_{\text{mid}}^2 \leq 2v_{\text{av}}^2$$

$$\Rightarrow v_{\text{av}} \leq v_{\text{mid}} \leq \sqrt{2} v_{\text{av}}$$

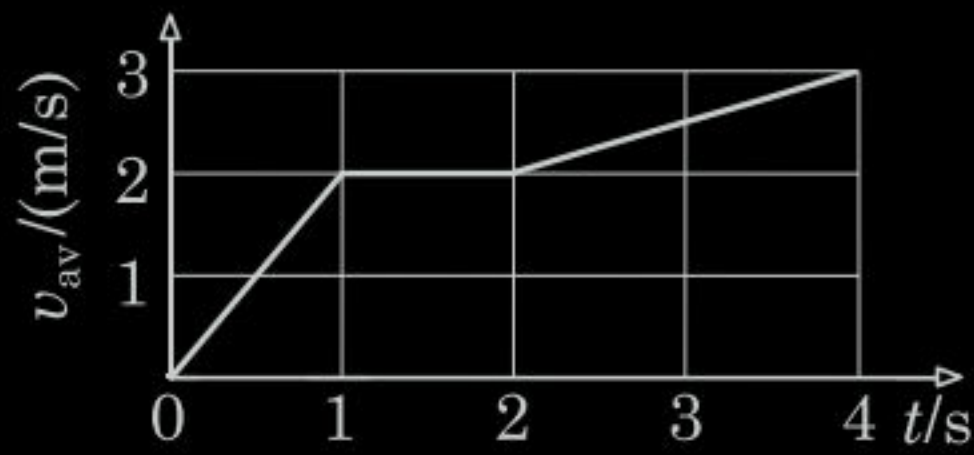
8. A particle covers a distance unidirectionally with uniform acceleration. If its average velocity is v_{av} , what could be range of modulus of its instantaneous velocity at the midpoint of the path?

i.e., u and v are both positive.



GM < AM

$$0 \leq \sqrt{uv} \leq \frac{u+v}{2} \rightarrow v_{\text{av}}$$



9. Relation between average velocity v_{av} of a body and time t is shown in the graph. If during the time interval considered, the body did not change direction of motion, draw a graph between instantaneous velocity of the body and time.

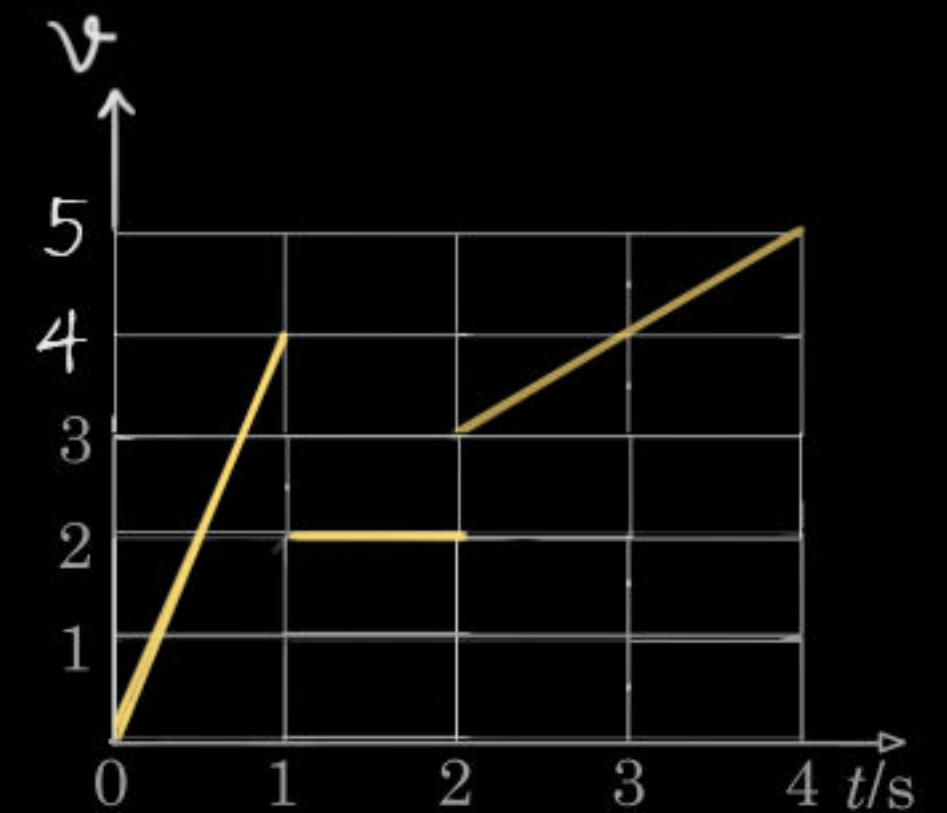
Concept: Displacement, $S = v_{av} \times t$ ① Instantaneous velocity, $v = \frac{ds}{dt}$

From graph:

$$t(0-1): v_{av} = 2t \xrightarrow{\text{①}} S = 2t^2 \xrightarrow{\text{②}} v = 4t$$

$$t(1-2): v_{av} = 2 \longrightarrow S = 2t \longrightarrow v = 2$$

$$t(2-4): v_{av} = \frac{t}{2} + 1 \longrightarrow S = \frac{t^2}{2} + t \longrightarrow v = t + 1$$



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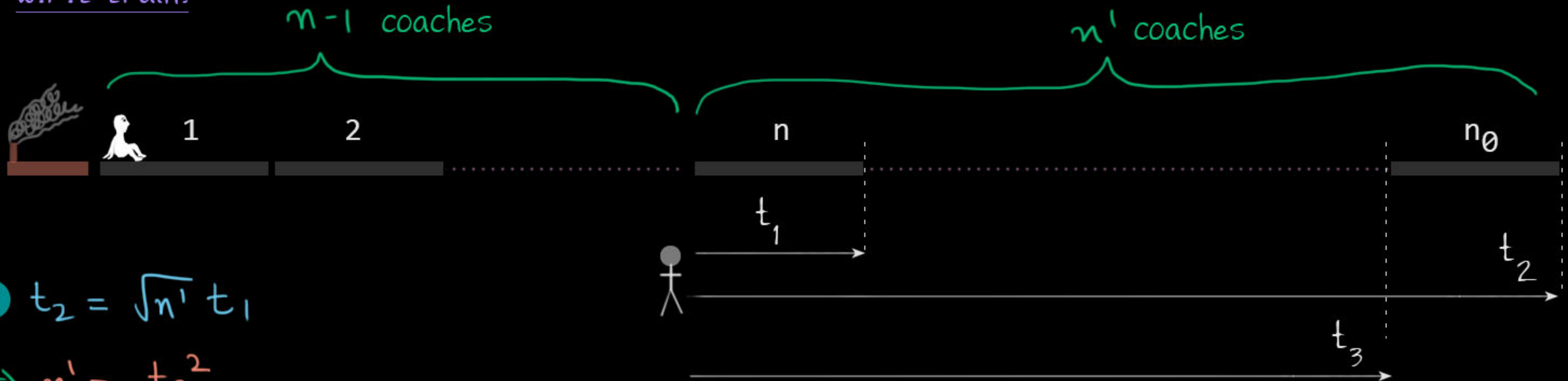
Let total coaches be n_0
and coaches that will pass by
the passenger by n'

10. A passenger is standing on the platform at the beginning of n^{th} ($= 3^{\text{rd}}$) coach of a train. If the train starts moving with constant acceleration, the third coach passes by the passenger in $\Delta t_1 = 5.0$ s and rest of the train including the 3^{rd} coach in $\Delta t_2 = 20$ s.

(a) How many coaches are in the train?

(b) In what time interval did the last coach pass by the passenger?

w.r.t train:



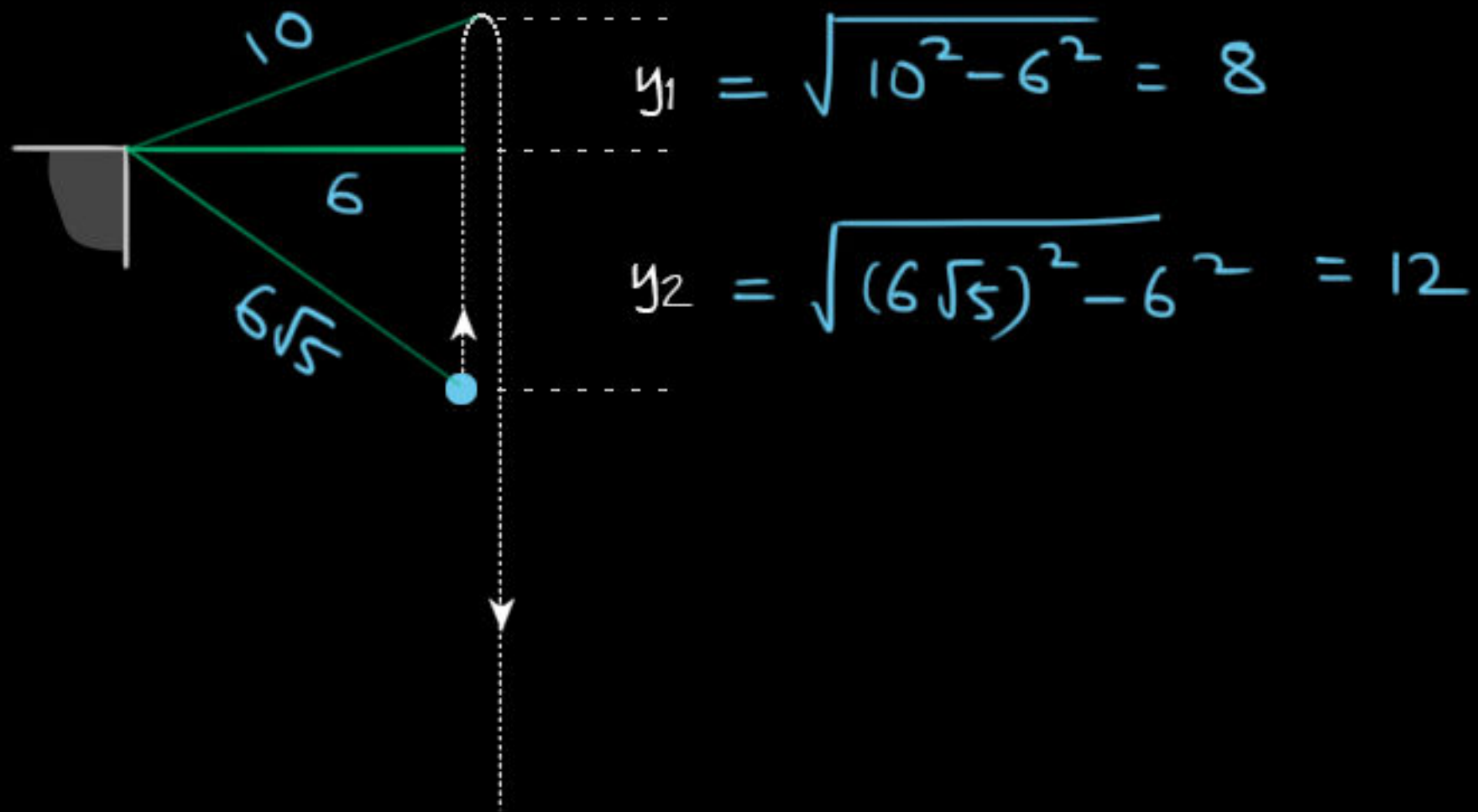
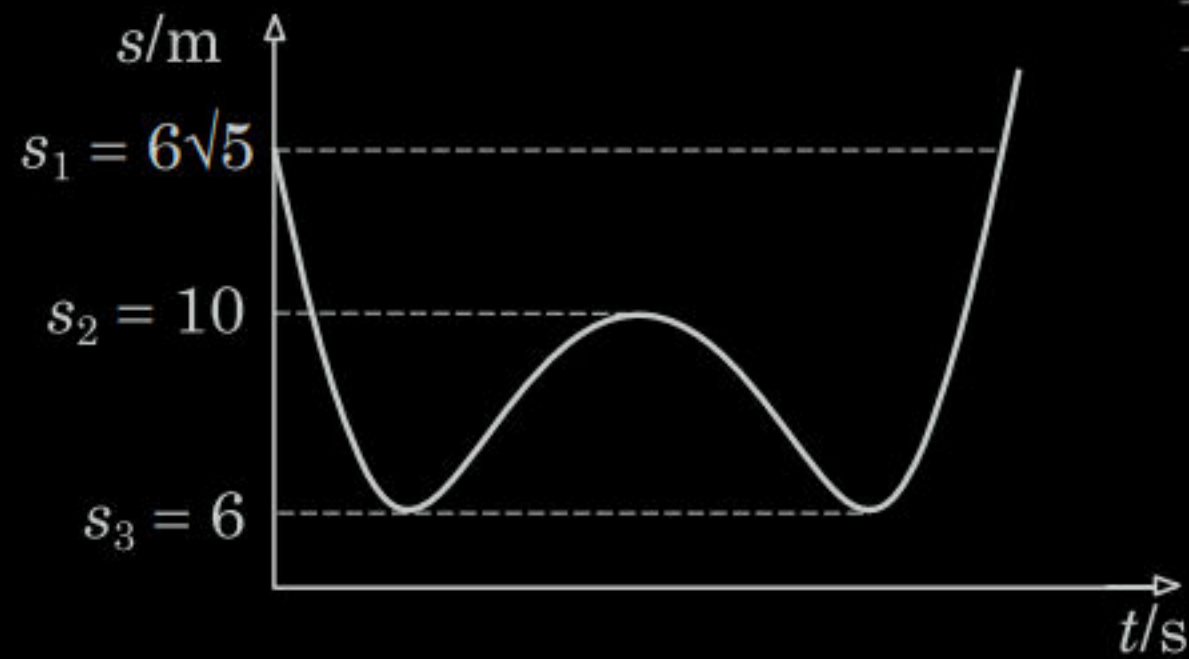
A $t_2 = \sqrt{n'} t_1$

$\Rightarrow n' = \frac{t_2^2}{t_1^2}$

Given: $n_0 = (n-1) + n'$

B $t_2 - t_3 = \sqrt{n'} t_1 - \sqrt{n'-1} t_1$
 $= t_2 - \sqrt{t_2^2 - t_1^2}$

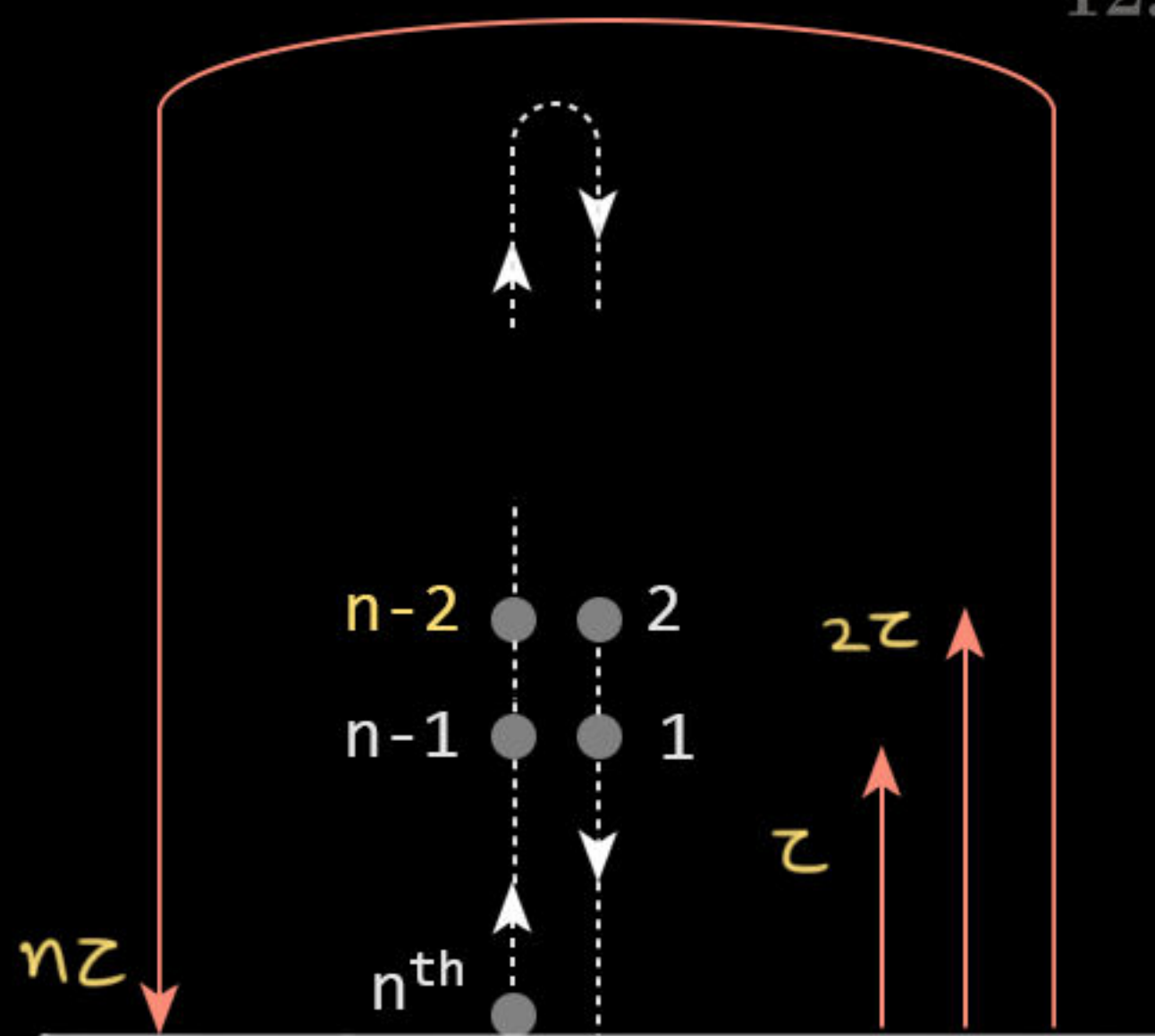
11. A ball is thrown vertically upwards. Its distance s from a fixed point varies with time t according to the following graph. Calculate velocity of projection of the ball.



$$\begin{aligned}
 v &= \sqrt{2gh} \\
 \Rightarrow v &= \sqrt{2g(y_1 + y_2)} \\
 &= \sqrt{2(10)(20)} \\
 &= 20 \text{ m/s}
 \end{aligned}$$

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by Ankit Singhvi



A Total time of flight = $n\tau$

$$\therefore v = -v + g(n\tau)$$

$$\Rightarrow v = \frac{g(n\tau)}{2}$$

$$h_i = v(i\tau) - \frac{1}{2}g(i\tau)^2$$

$$= \frac{g\tau^2}{2}(ni - i^2)$$

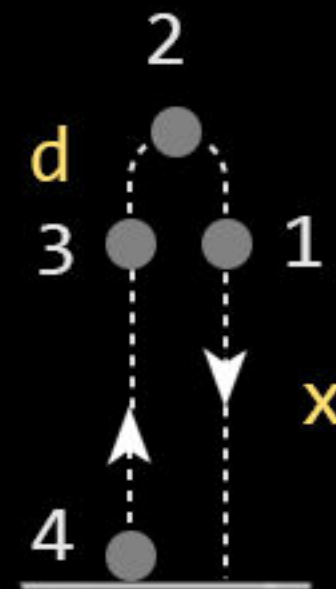
12. A clown in a circus juggles with n balls. He throws each ball vertically upwards with the same speed at equal time intervals τ . Denote acceleration of free fall by g .

(a) Find expressions for the speed of projection and height of the i^{th} ball above his hand when he throws the n^{th} ball.

If he uses $n = 4$ balls, distance between the second and third ball is $d = 50$ cm at the instant the fourth ball is projected.

(b) Where is the first ball, when the juggler throws the fourth ball?

(c) What is maximum height attained by each ball above the hands of the juggler?



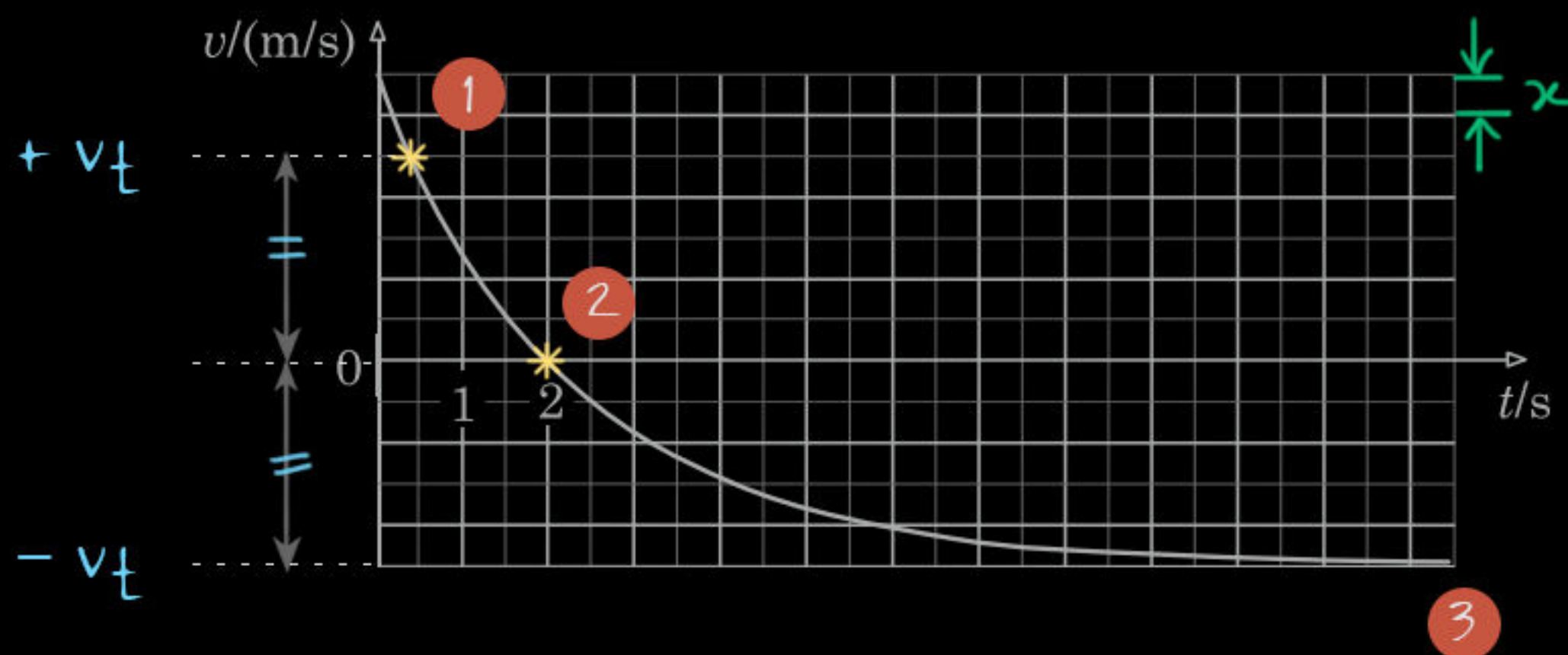
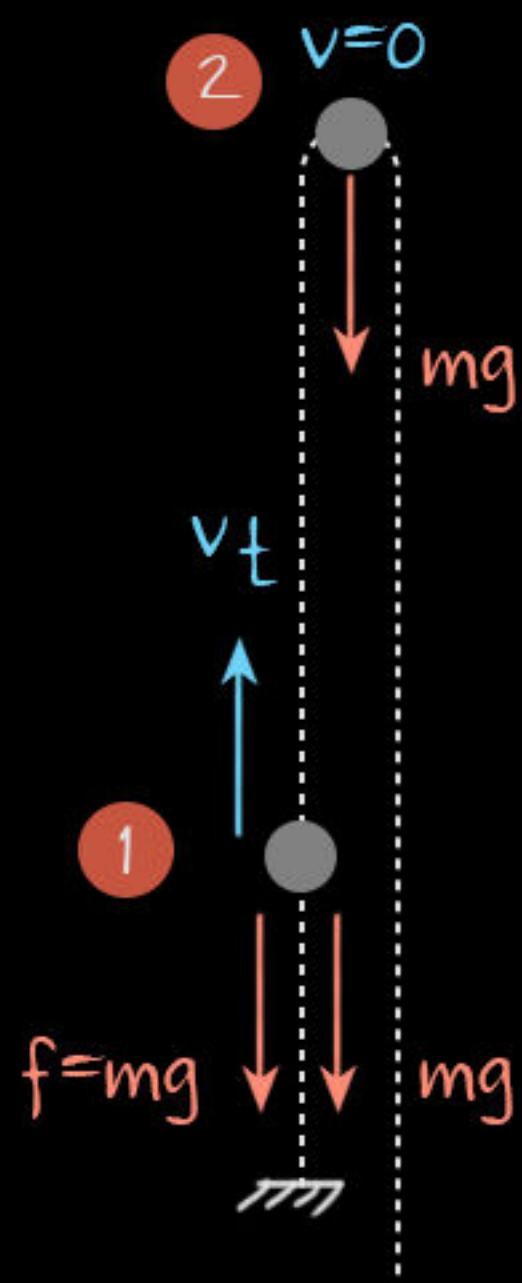
B $\frac{d}{x} = \frac{1}{3}$

$$\Rightarrow x = 3d //$$

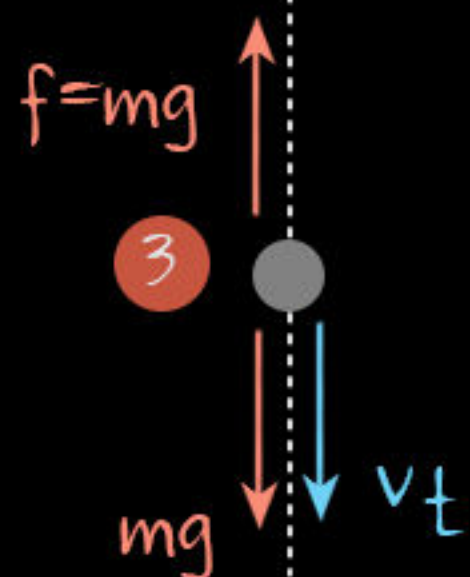
C $H = d + x = 4d //$

Let terminal velocity be v_t

13. To study effect of air resistance, a rubber ball was shot vertically upwards from a spring gun from 20th floor of a tall building. Velocity of the ball was recorded at regular intervals of time and the data obtained were plotted on a graph paper. Some of the marking on the axes are erased as shown in the following figure. With what speed did the ball strike the ground?



At terminal velocity (v_t), net force is zero
 $\therefore$ From 3: $mg = f$



Let one unit on v scale be x

Method 1 Slope at 1 should be $2g$.

$$\therefore \frac{2x}{1/2} = 2g$$

$$\Rightarrow x = 5$$

$$\Rightarrow v_t = 5x = 25 \text{ m/s}$$

Method 2 Slope at 2 should be g .

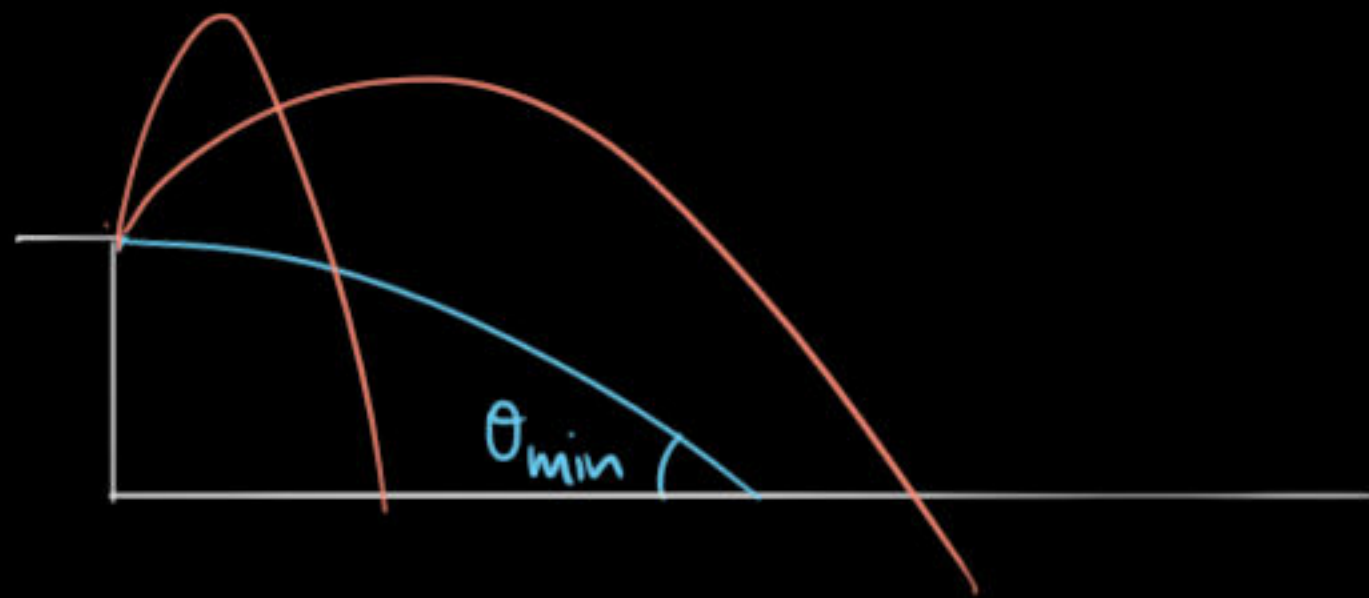
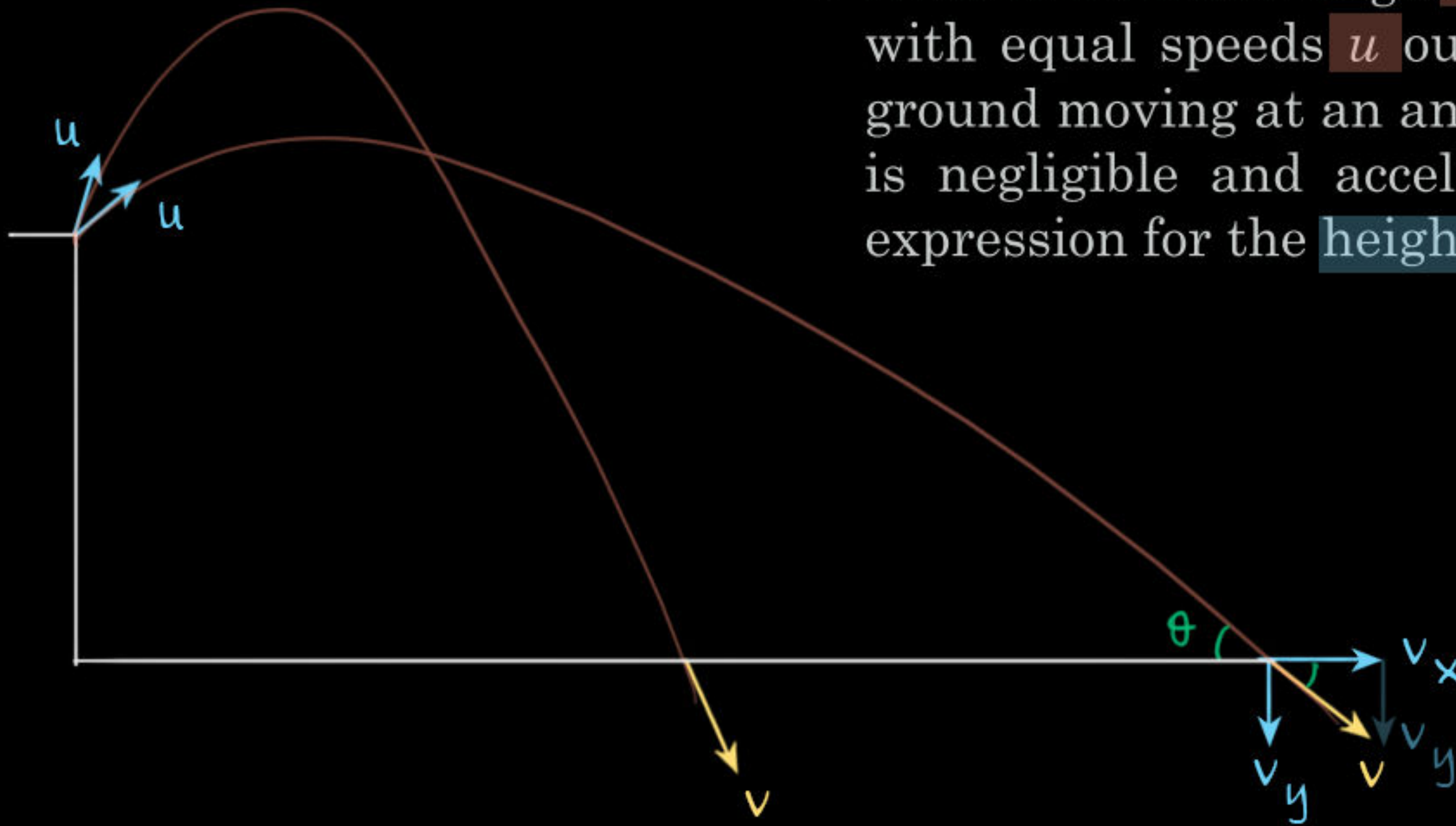
$$\therefore \frac{x}{1/2} = g$$

$$\Rightarrow x = 5$$

$$\Rightarrow v_t = 5x = 25 \text{ m/s}$$

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14. A student throws large number of small pebbles in all possible directions with equal speeds u out of a window. The pebbles hit the horizontal ground moving at an angle θ or greater with the ground. Air resistance is negligible and acceleration due to gravity is g . Deduce suitable expression for the height of the point of projection above the ground.



Note: At $\theta_{\min}$, range is NOT maximum.

u is same in all throws.

$\therefore$ By energy conservation, v is same too.

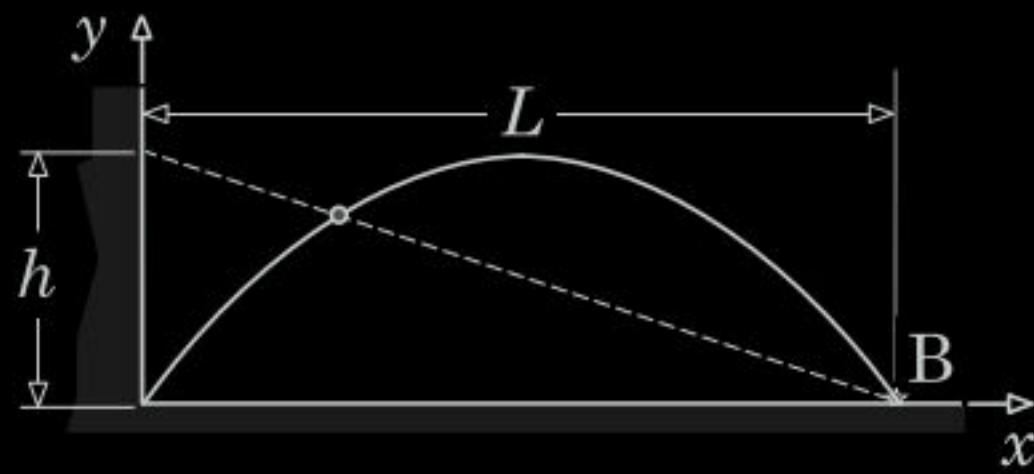
$$\tan \theta = \frac{v_y}{v_x} = \frac{\sqrt{v^2 - v_x^2}}{v_x} = \sqrt{\left(\frac{v}{v_x}\right)^2 - 1}$$

Its minimum, when v_x is maximum, which happens at horizontal throw when: $v_x = u$

$$\therefore \tan \theta = \frac{v_y}{v_x} = \frac{\sqrt{2gh}}{u}$$

$$\Rightarrow h = \frac{u^2 \tan^2 \theta}{2g}$$

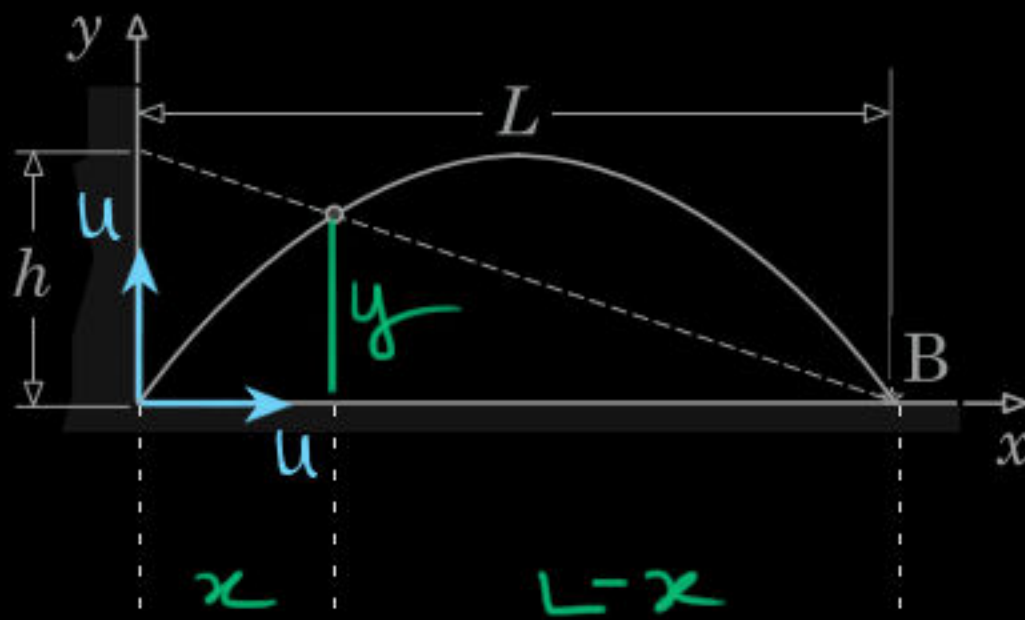
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15. A small ball is thrown from foot of a wall with the minimum possible velocity to hit a bulb B on the ground a distance L away from the wall. Find expression for height h of shadow of the ball on the wall as a function of time t . Acceleration due to gravity is g .

$\Rightarrow$ angle of projection is 45°

Method 1



Range: $L = \frac{(u\sqrt{2})^2 \sin 2\theta}{g}$

$\Rightarrow u = \sqrt{\frac{gL}{2}}$ (1)

$\frac{L-x}{y} = \frac{L}{h} \Rightarrow h = \frac{Ly}{L-x}$ (2)

$\Rightarrow h = \frac{L(u t - \frac{1}{2} g t^2)}{L - u t} = \frac{L \left(\sqrt{\frac{gL}{2}} t - \frac{1}{2} g t^2 \right)}{L - \sqrt{\frac{gL}{2}} t} = \frac{L \frac{\sqrt{gL}}{2} (\sqrt{2} L - \sqrt{g} t)}{\sqrt{\frac{L}{2}} (\sqrt{2} L - \sqrt{g} t)} = \sqrt{\frac{gL}{2}} t //$

Method 2 Concept: Equation of trajectory:

$y = x \tan \theta \left(1 - \frac{x}{L} \right)$ (3)

Rearranging 2: $y = h \left(1 - \frac{x}{L} \right)$ (4)

Comparing 3, 4: $h = x \tan \theta$

$= u t \xrightarrow{\text{From 1}} \sqrt{\frac{gL}{2}} t //$

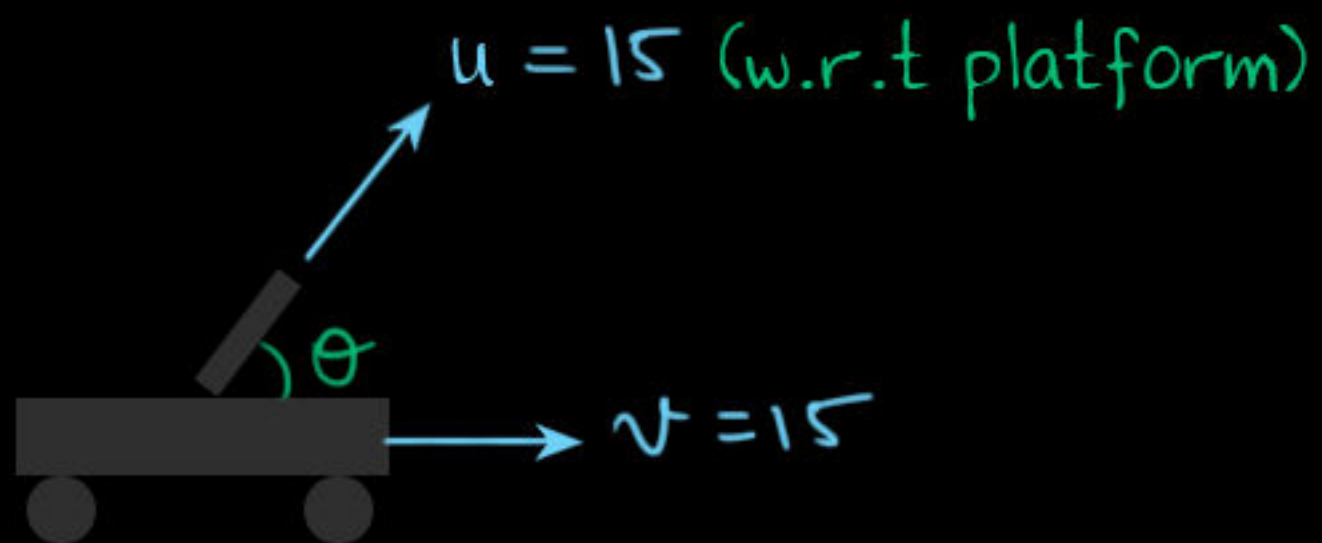
16. The maximum range of a shell fired from a gun is $R = 22.5$ m. This gun is mounted on a platform that can move horizontally with a constant speed $v = 15.0$ m/s. At what angle above the horizontal, must the gun be aimed to achieve maximum horizontal range? Neglect air resistance as well as height of the gun. Acceleration of free fall $g = 10.0$ m/s².

Let u = shell speed w.r.t gun

$$\frac{u^2}{g} = 22.5$$

$$\Rightarrow u = 15 \text{ m/s}$$

Lets say its mounted at an angle θ

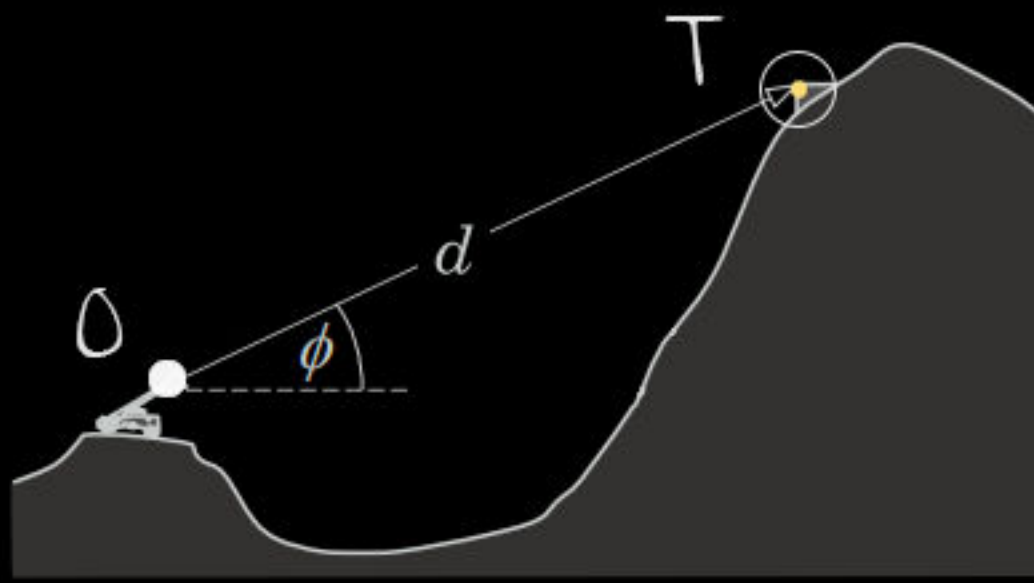


$$\begin{aligned} R &= u_x T \quad \rightarrow \quad \frac{2u_y}{g} \\ &= (u \cos \theta + v) \cdot \frac{2u \sin \theta}{g} \\ &= \frac{2u^2}{g} \sin \theta (1 + \cos \theta) \end{aligned}$$

$$@ R_{\max} \quad \frac{dR}{d\theta} = 0 \Rightarrow \frac{d}{d\theta} [\sin \theta (1 + \cos \theta)] = 0$$

$$\Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = 60^\circ$$



17. A **cannon** installed at the top of a hill **can fire shells in all directions**. There is an **enemy bunker** at an **angle of elevation ϕ** and a **distance d** from the cannon. **All the shells fired, explode in air in time T** before they reach the bunker. **At what angle to the horizontal, should a shell be fired with a speed u to explode closest to the bunker?** Acceleration due to gravity is g .

$$\vec{S} = \vec{u}T + \frac{1}{2}\vec{g}T^2$$

$\vec{u}T$

(displacement along direction of gun)

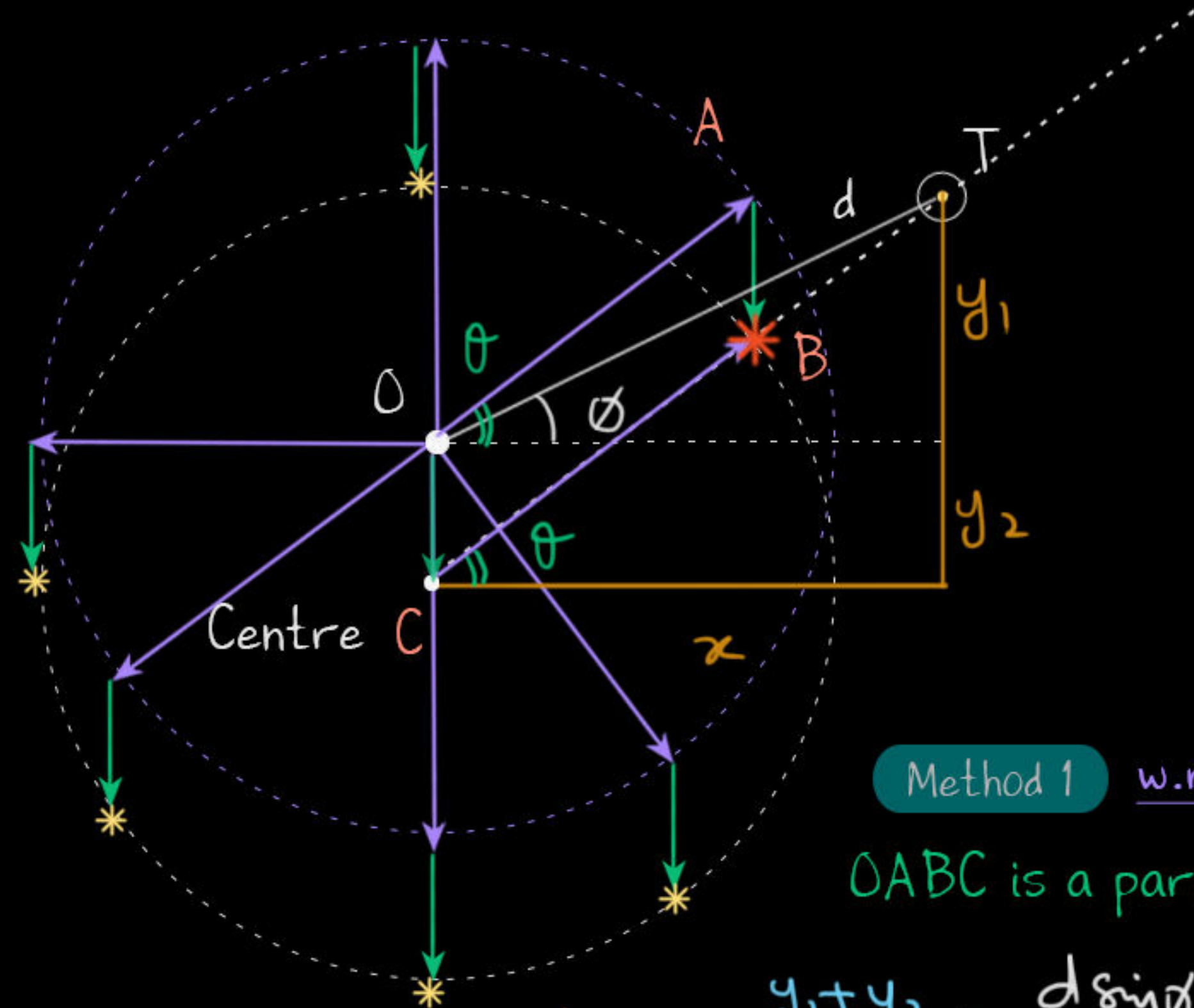
$$\frac{1}{2}\vec{g}T^2$$

(Fall due to gravity)

- O • Centre of 'blast circle' without gravity
- C • Centre of actual 'blast circle'. It clearly gets shifted downwards from O by $\frac{1}{2}gT^2$

* Possible Blast locations

B* Blast nearest to target



Method 1 w.r.t ground:

OABC is a parallelogram

$$\therefore \tan \theta = \frac{y_1 + y_2}{x} = \frac{d \sin \phi + \frac{1}{2}gT^2}{d \cos \phi}$$

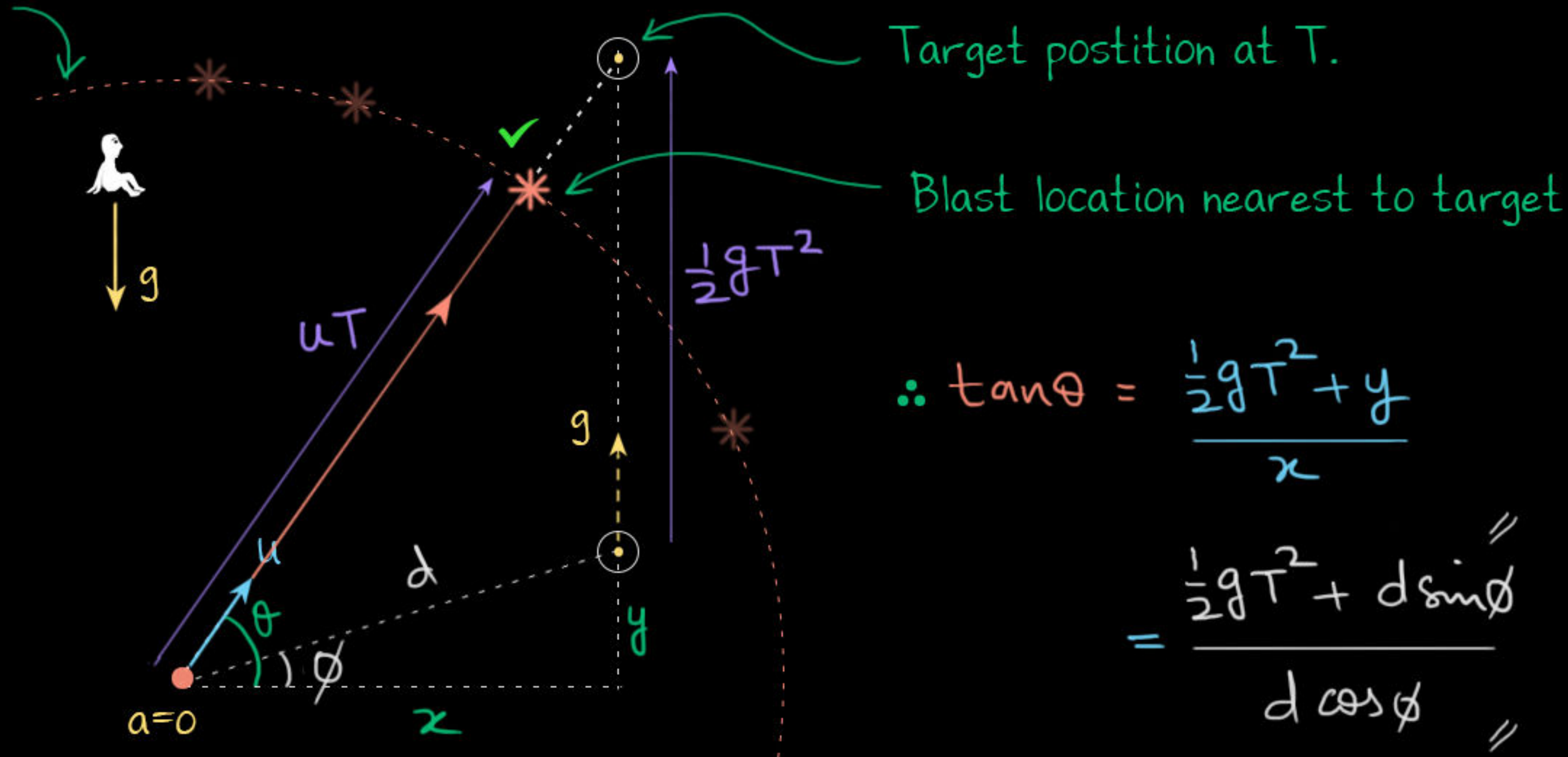
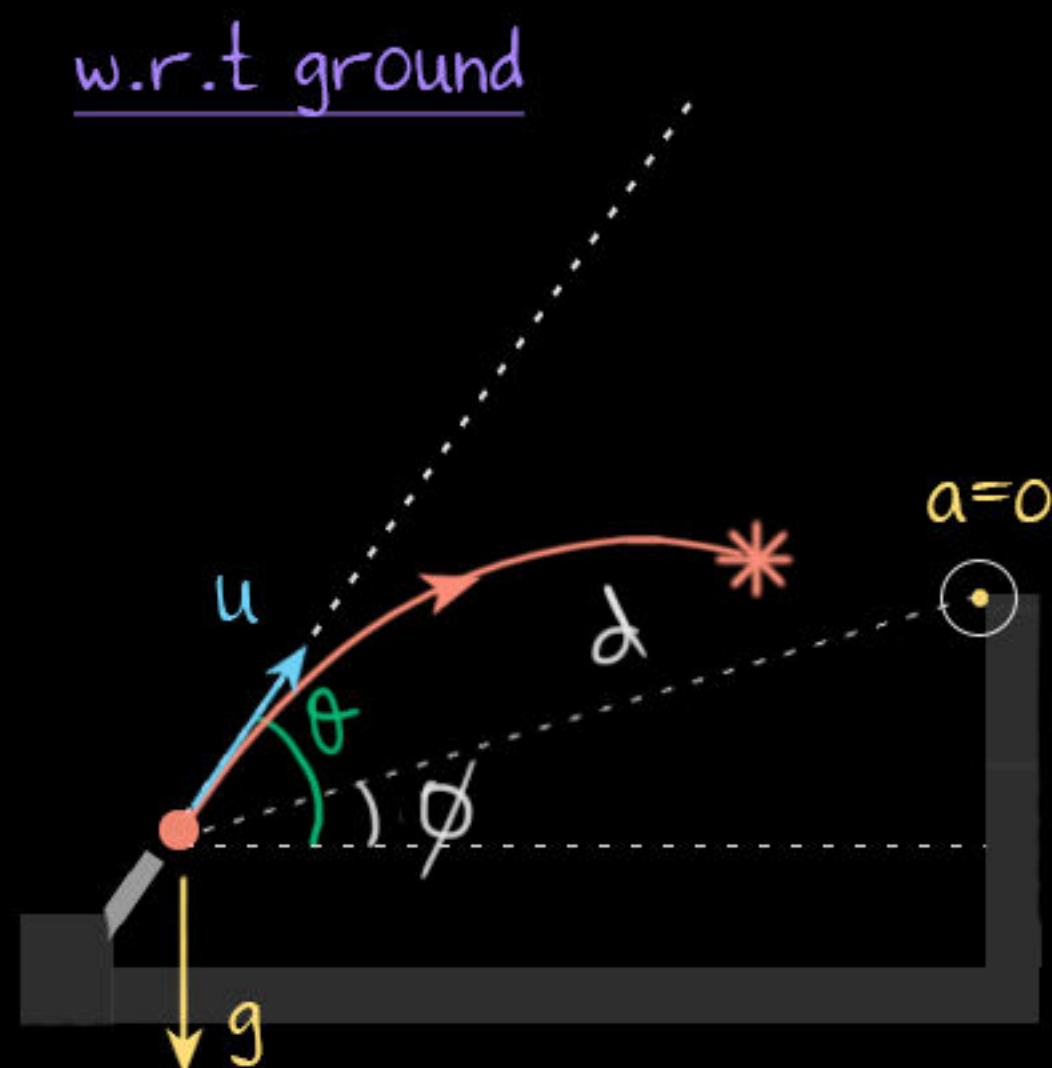
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Method 2

w.r.t an observer that starts to free fall, when gun is fired

Blast locations at T depending on different gun angles.

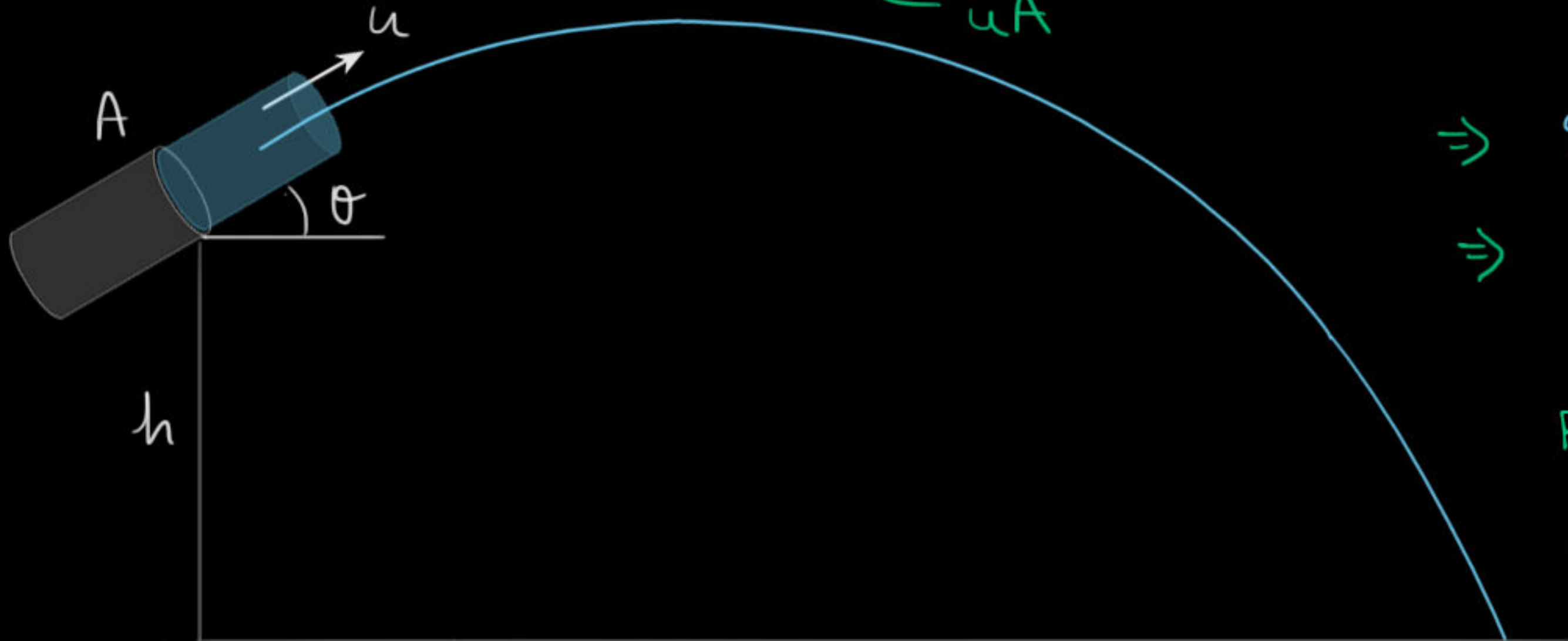


Note: Required gun angle is independent of u!

18. A boy while watering a garden keeps outlet of the hose $h = 0.8$ m above the ground. Water is continuously flowing out of the hose with a constant velocity $u = 6$ m/s at an angle $\theta = 30^\circ$ above the horizontal. Cross section area of the outlet is $A = 1.5$ cm², density of water is $\rho = 1000$ kg/m³ and acceleration of free fall is $g = 10$ m/s². Find mass of water in the water stream?

Mass in stream = mass flow rate $\times$ Time of flight = $u A \rho t$ ①

$\underbrace{\hspace{10em}}_{\text{Volume flow rate} \times \text{density}}$
 $\underbrace{\hspace{10em}}_{uA}$



Calculating time of flight t :

$$h = (-u \sin \theta) t + \frac{1}{2} g t^2$$

$$\Rightarrow g t^2 - 2 u \sin \theta t - 2 h = 0$$

$$\Rightarrow t = \frac{u \sin \theta + \sqrt{u^2 \sin^2 \theta + 2 g h}}{g} \quad ②$$

From 1,2:

$$M = \frac{u A \rho}{g} \left(u \sin \theta + \sqrt{u^2 \sin^2 \theta + 2 g h} \right)$$

We know, $R = \frac{u^2 \sin 2\alpha}{g}$

Where α is angle between u and v .

$\therefore R_{\max} = \frac{u^2}{g} @ \alpha = \pi/2$

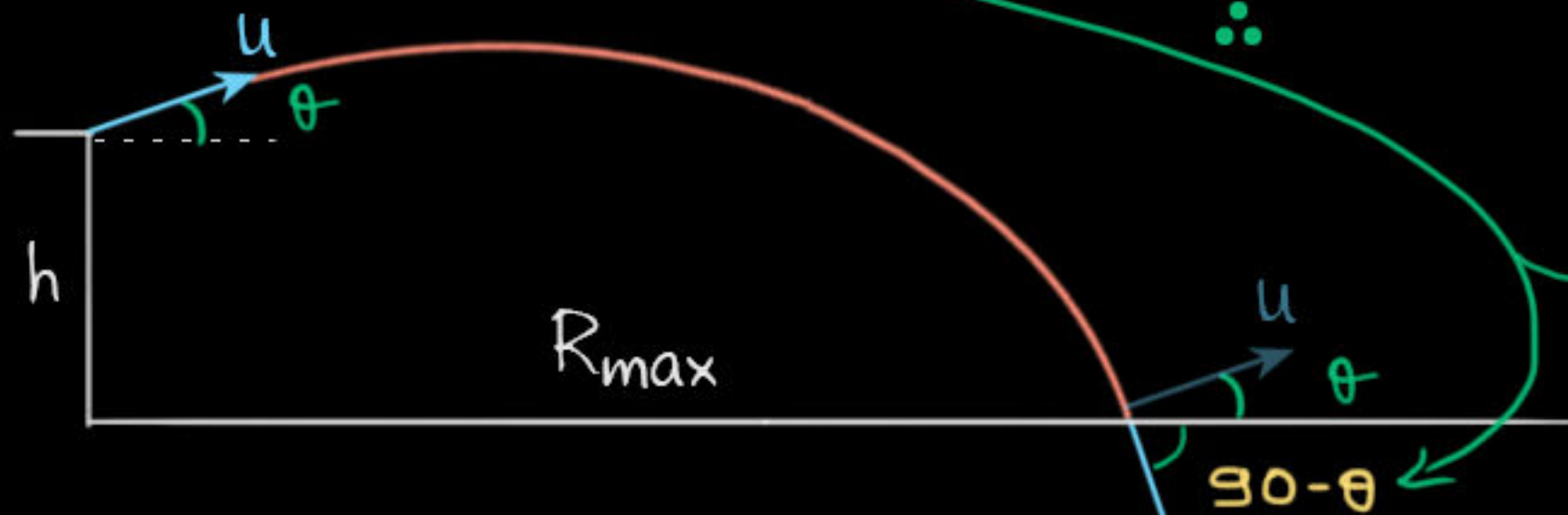
19. Angle of projection for the maximum horizontal range of a projectile is 45° , if the point of projection and the point of landing are in the same horizontal level. Determine the **angle of projection for the maximum horizontal range** of a projectile, if

(a) the point of **landing is at a height h above** the point of projection.

(b) the point of **landing is at a depth h below** the point of projection.

Get answer in terms of u

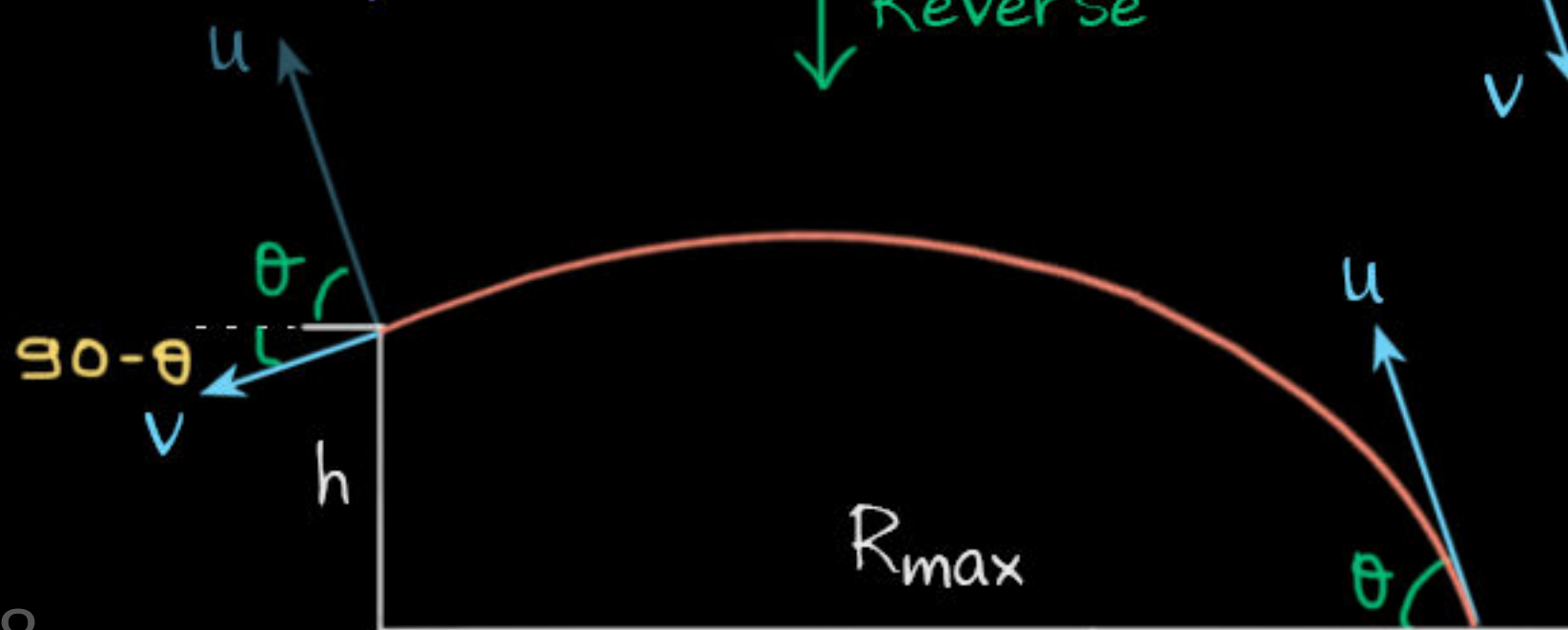
Thrown down



Finding angle of projection @ $R_{\max}$:

$u \cos \theta = v \cos (90 - \theta)$

Thrown up



$\Rightarrow \tan \theta = \frac{u}{v}$

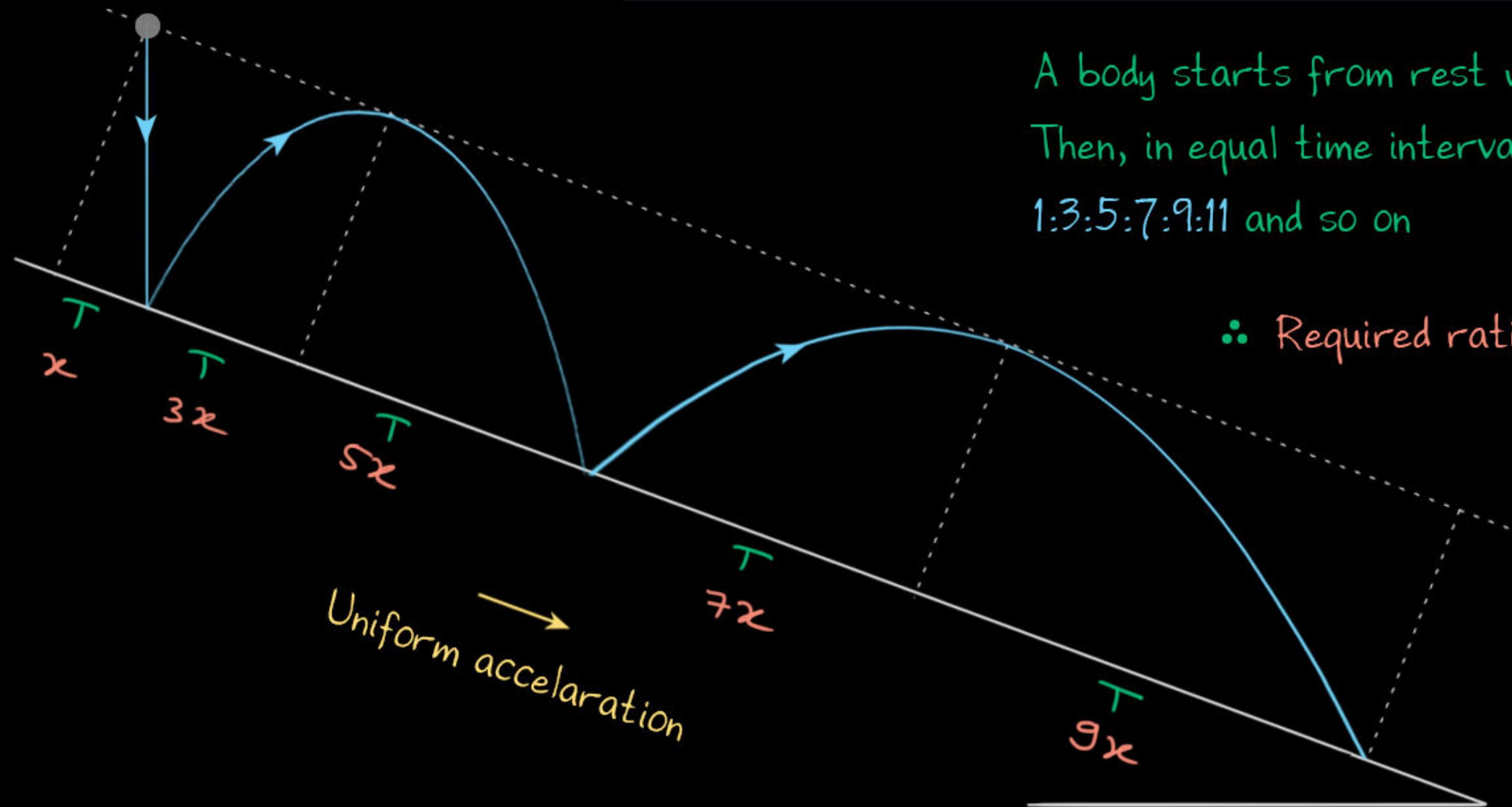
B

$= \frac{u}{\sqrt{u^2 + 2gh}}$

A

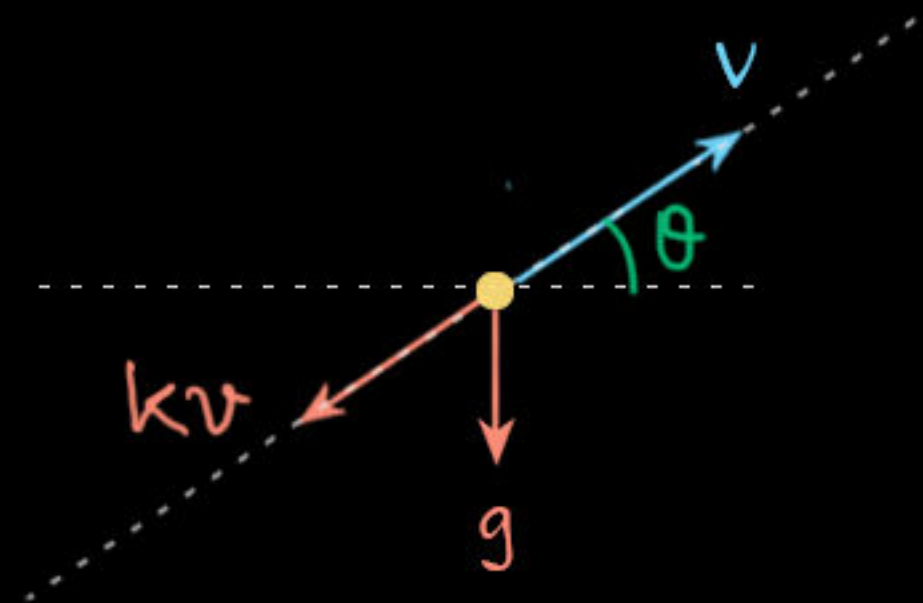
$= \frac{u}{\sqrt{u^2 - 2gh}}$

20. A ball dropped on a large inclined plane, bounces repeatedly. Every bounce is perfectly elastic i.e. there is no loss of speed and lines of motion make equal angles with the incline plane before and after the bounce. Find ratio of distance R_{12} between the first and the second bounce to distance R_{23} between the second and the third bounce.

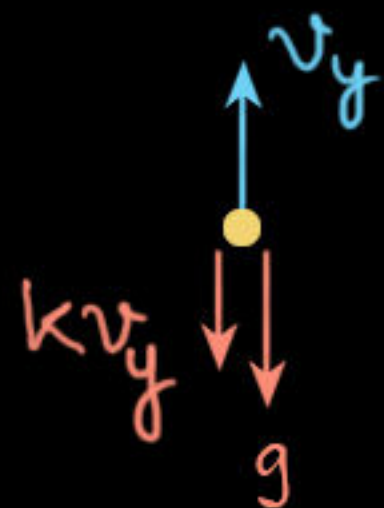


A body starts from rest with uniform acceleration. Then, in equal time intervals, gaps are in ratio of: $1:3:5:7:9:11$ and so on

$$\therefore \text{Required ratio} = \frac{3x + 5x}{7x + 9x} = \frac{1}{2} //$$



Breaking acceleration and velocity into components and writing:
 $v \cos \theta \rightarrow v_x$
 $v \sin \theta \rightarrow v_y$



21. A marble is projected in a viscous fluid, with an initial speed u at an angle θ above the horizontal. Drag force of the fluid results in an acceleration $\vec{a}_D = -k\vec{v}$ in addition to that of gravity, where k is a positive constant and $\vec{v}$ is the velocity of the marble. Determine position coordinates and x (horizontal) and y (vertical) components of velocity of the marble as functions of time t . What is the terminal velocity?

$$\frac{dv_x}{dt} = -kv_x$$

$$\Rightarrow \int_{u_x}^{v_x} \frac{dv_x}{v_x} = -k \int_0^t dt$$

$$\Rightarrow v_x = u_x e^{-kt}$$

$$\Rightarrow \frac{dx}{dt} = u_x e^{-kt}$$

$$\Rightarrow \int_0^x dx = u_x \int_0^t e^{-kt} dt$$

$$\Rightarrow x = \frac{u_x}{k} (1 - e^{-kt}) //$$

by Ankit Singhvi

$$\frac{dv_y}{dt} = -(g + kv_y)$$

$$\Rightarrow \int_{u_y}^{v_y} \frac{dv_y}{g + kv_y} = - \int dt$$

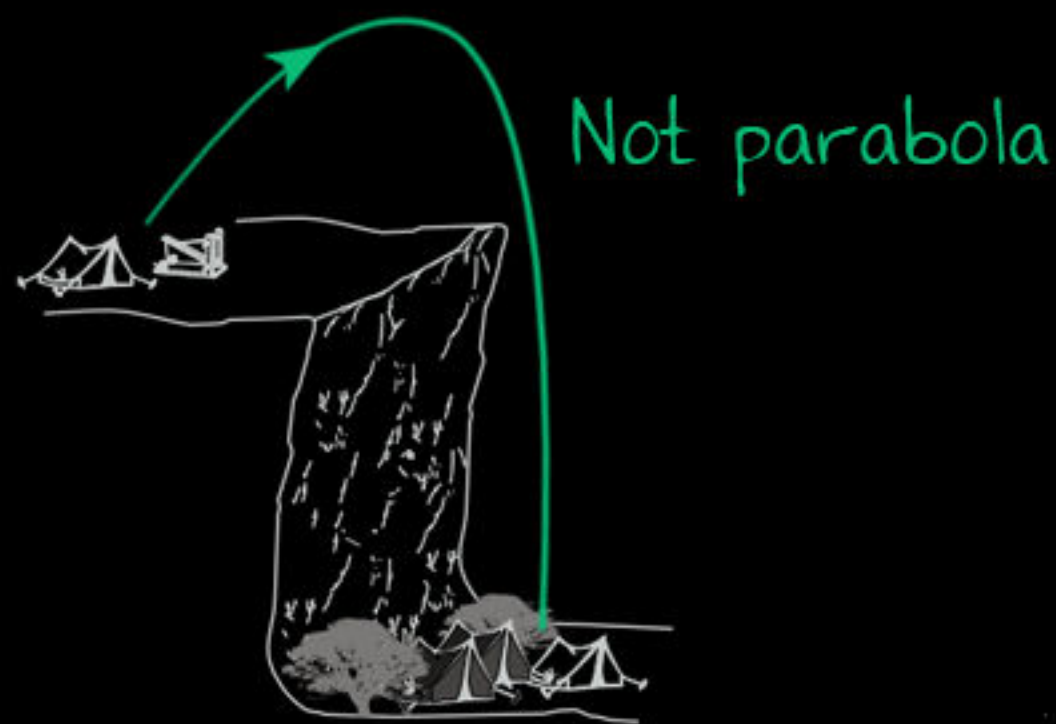
$$\Rightarrow v_y = \left(\frac{g}{k} + u_y \right) e^{-kt} - \frac{g}{k}$$

$$\Rightarrow \frac{dy}{dt} = "$$

$$\Rightarrow \int_0^y dy = \int_0^t " dt$$

$$\Rightarrow y = \frac{1}{k} \left(\frac{g}{k} + u_y \right) [1 - e^{-kt}] - \frac{gt}{k} //$$

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22. In a battle field of ancient times, a soldier with a catapult stationed on the top of a very high cliff notices camps of enemy close to the bottom of the cliff as shown in the figure. Stones can be launched from the catapult with a speed $u = 40 \text{ m/s}$ at an angle $\theta = 60^\circ$ above the horizontal. If the air resistance reduces speed of the stone at a rate $k = 0.1 \text{ (m/s)/m}$ and there is no wind, at what horizontal separation from the enemy camps should the soldier install the catapult to hit the enemy camps?

Due to air resistance alone:

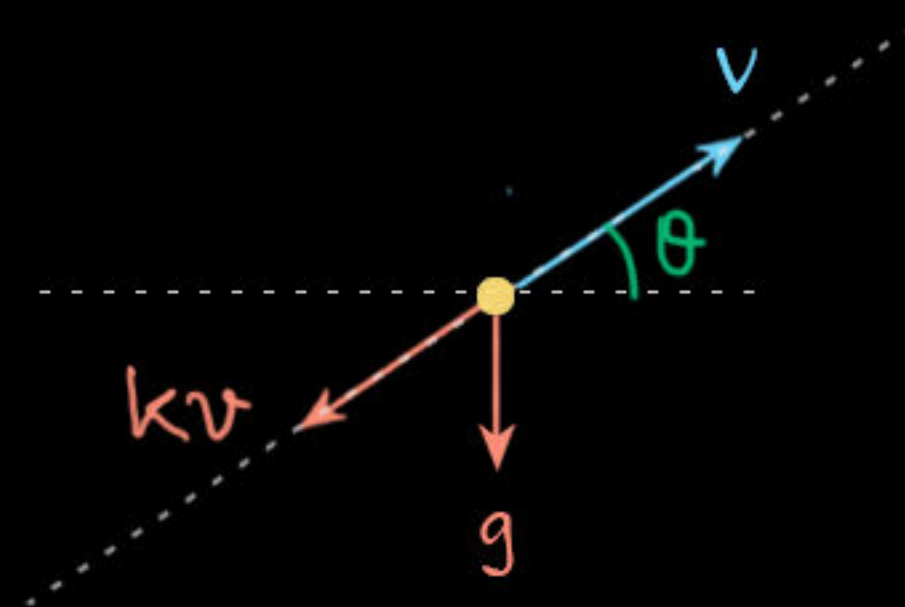
$$\frac{dv}{dl} = -k$$

$$\Rightarrow \frac{dv}{dt} \cdot \frac{dt}{dl} = -k$$

$$\Rightarrow \frac{dv}{dt} = -k \frac{dl}{dt}$$

$$\Rightarrow a = -kv$$

Due to all forces:



X-components:

$$kv \cos \theta \leftarrow \bullet \rightarrow v \cos \theta$$

|||

$$kv_x \leftarrow \bullet \rightarrow v_x$$

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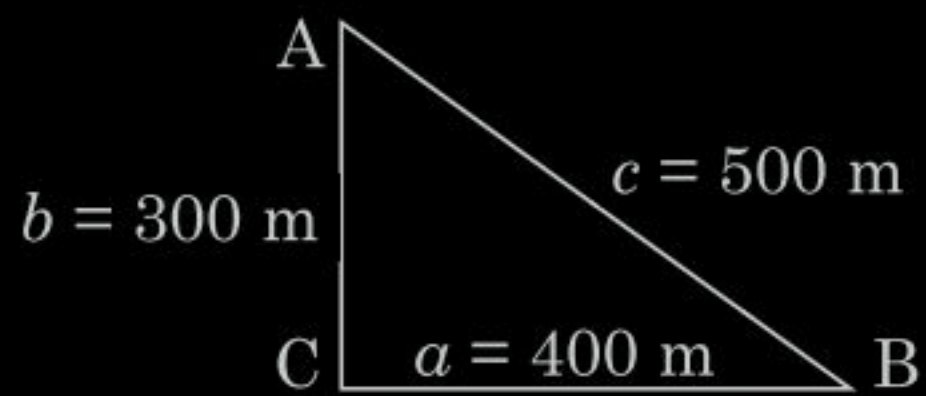
$$\therefore a_x = -kv_x$$

$$\Rightarrow \cancel{v_x} \frac{dv_x}{dx} = -k \cancel{v_x}$$

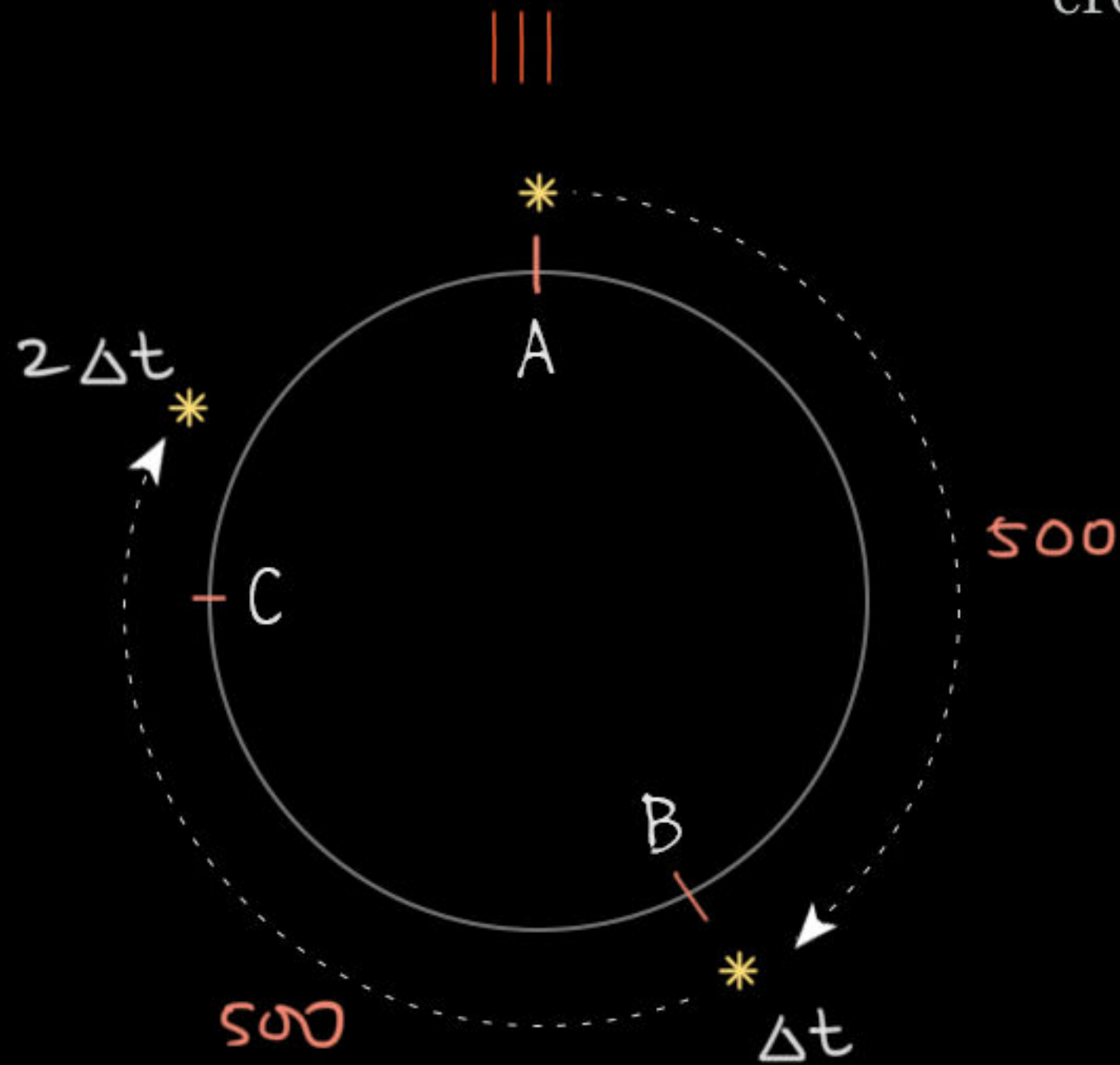
$$\Rightarrow \int_{u \cos \theta}^L dv_x = -k \int_0^L dx$$

$$\Rightarrow L = \frac{u \cos \theta}{k}$$

Youtube / Arihant Senior



23. Two bikers simultaneously start a race with constant speeds from point A to traverse a triangular track ABC, one clockwise and the other in anticlockwise sense. They simultaneously cross at B first time after a time interval $\Delta t_1 = 4$ min. If they continue the race, how long after they cross at B first time will they again simultaneously cross at B?



From any meeting point (starting from A), they meet 500 meter ahead in clockwise sense after every Δt time interval.

∴ They will meet at B again when distance traveled by 'meeting points: *' is integral multiple of circumference.

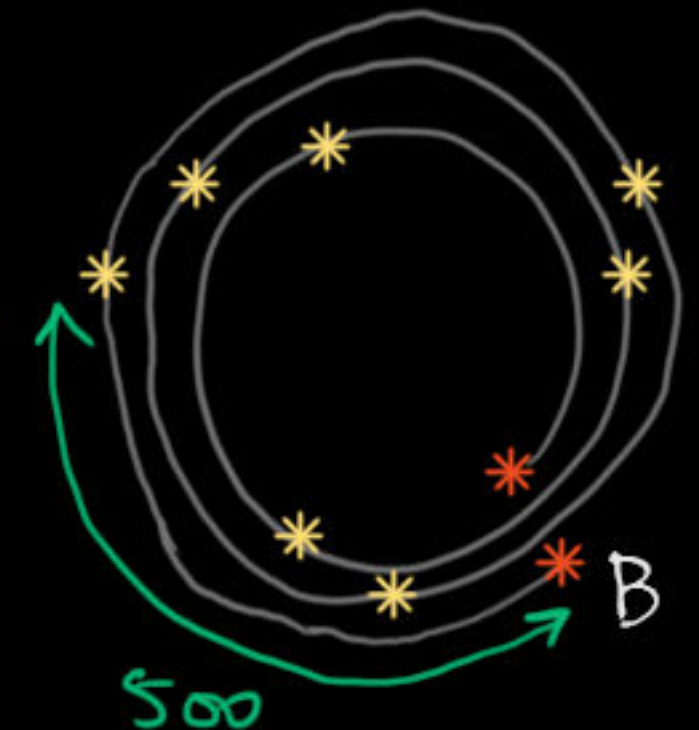
i.e when: $n_1 \cdot 500 = n_2 (500 + 400 + 300)$

$$\Rightarrow n_1 \cdot 500 = n_2 \cdot 1200$$

$\downarrow$
12

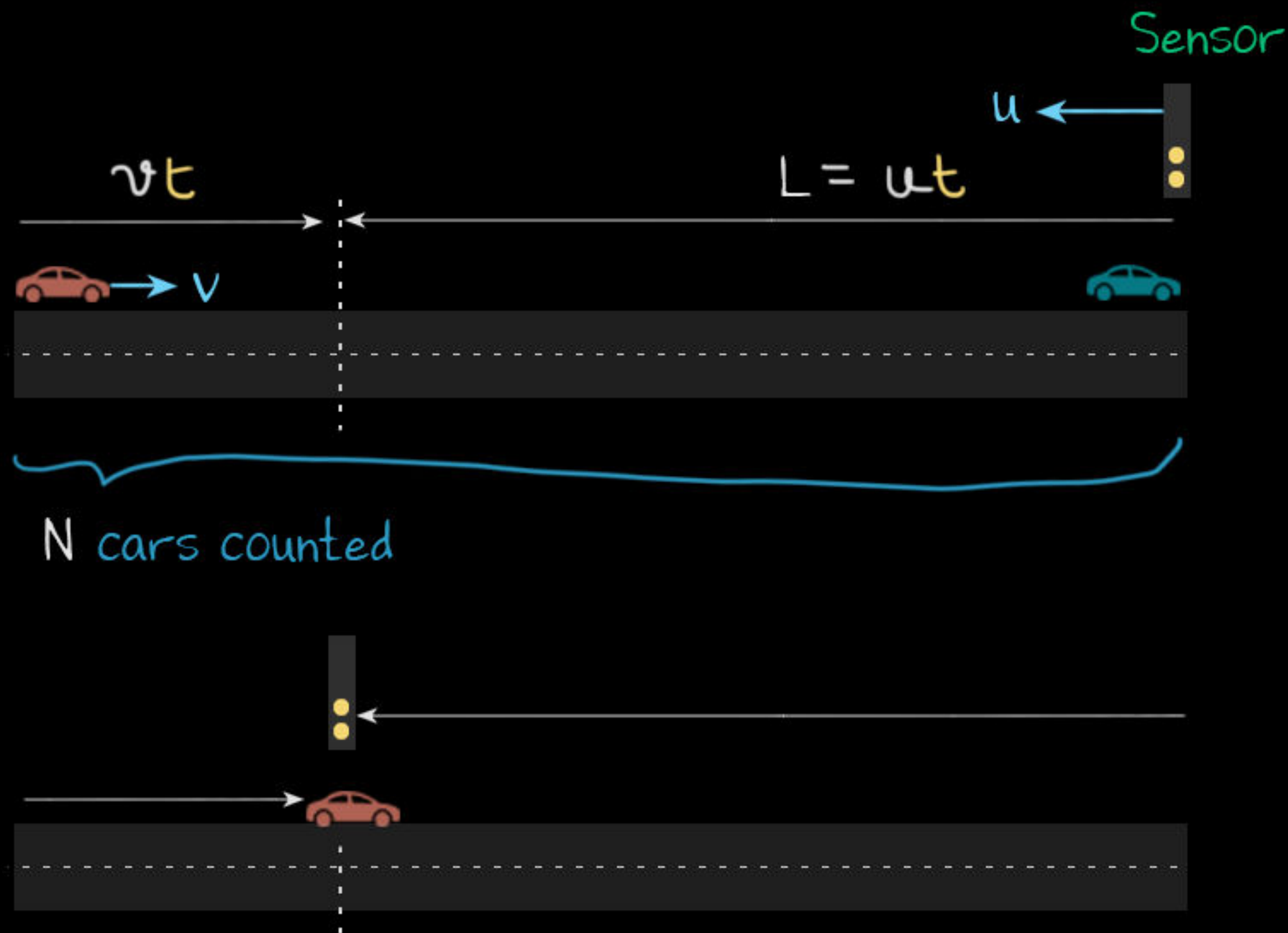
$\downarrow$
5

(smallest numbers possible)



∴ $T = n_1 \Delta t = 12 \cdot 4 \text{ min} = 48 \text{ min}$

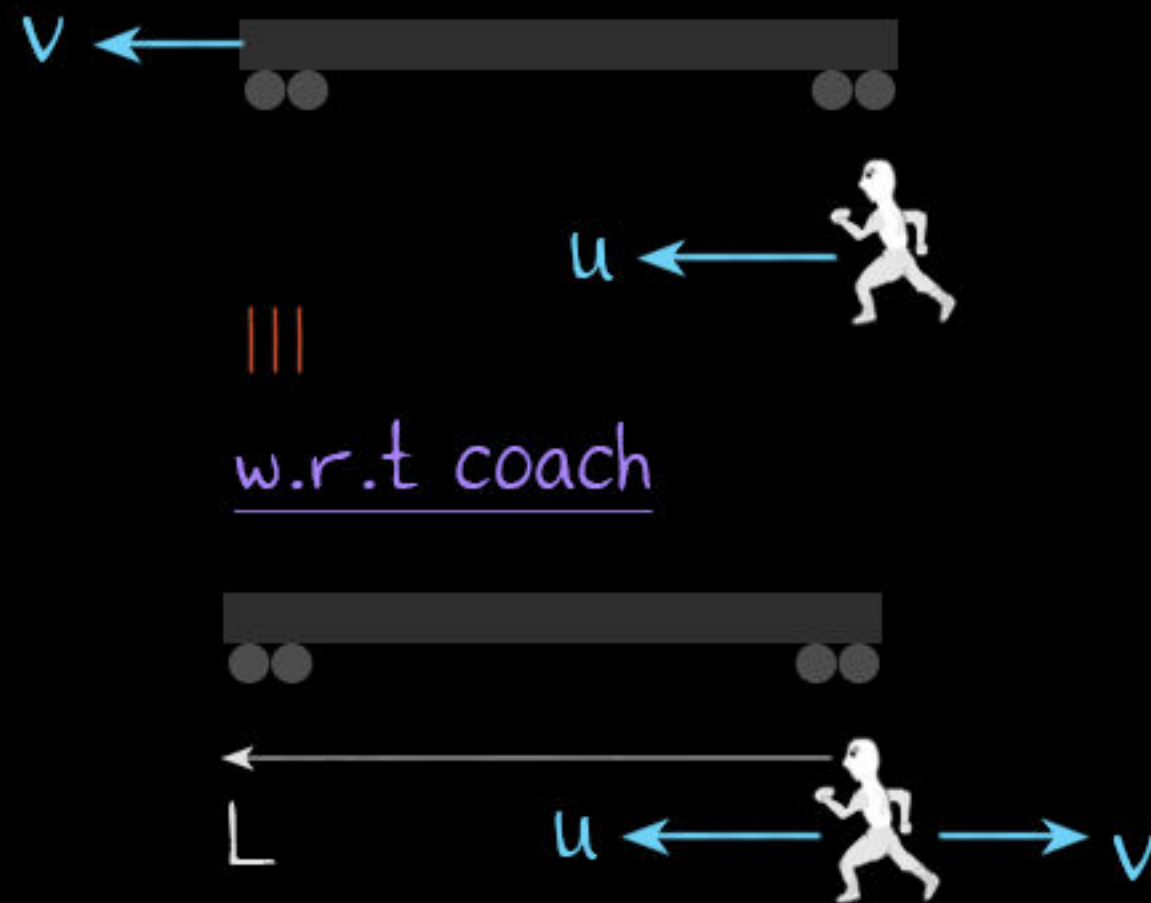
24. On a straight section of a highway, sensors are installed to measure traffic density. For each lane of the highway, a sensor runs on an overhead wire. On a particular day, a sensor running with a speed $u = 5$ km/h opposite to the flow of traffic underneath, counts $N = 360$ vehicles in a length $L = 1$ km of the highway. If all the vehicles are moving with the same constant speed $v = 40$ km/h and density of the vehicles is uniform, calculate number of vehicles per $l = 100$ m of the lane.



$$\begin{aligned}
 &\text{On } vt + ut \text{ road} \rightarrow N \text{ cars exist} \\
 &\therefore \text{On } l \text{ road} \rightarrow \frac{N}{vt + ut} l \text{ cars exist} \\
 &\quad \downarrow t = L/u \\
 &= \frac{Nul}{(u+v)L} \text{ cars exist} \\
 &\quad //
 \end{aligned}$$

Let the coach length = L .

Running along the train:



$$L = (u - v) t_1$$

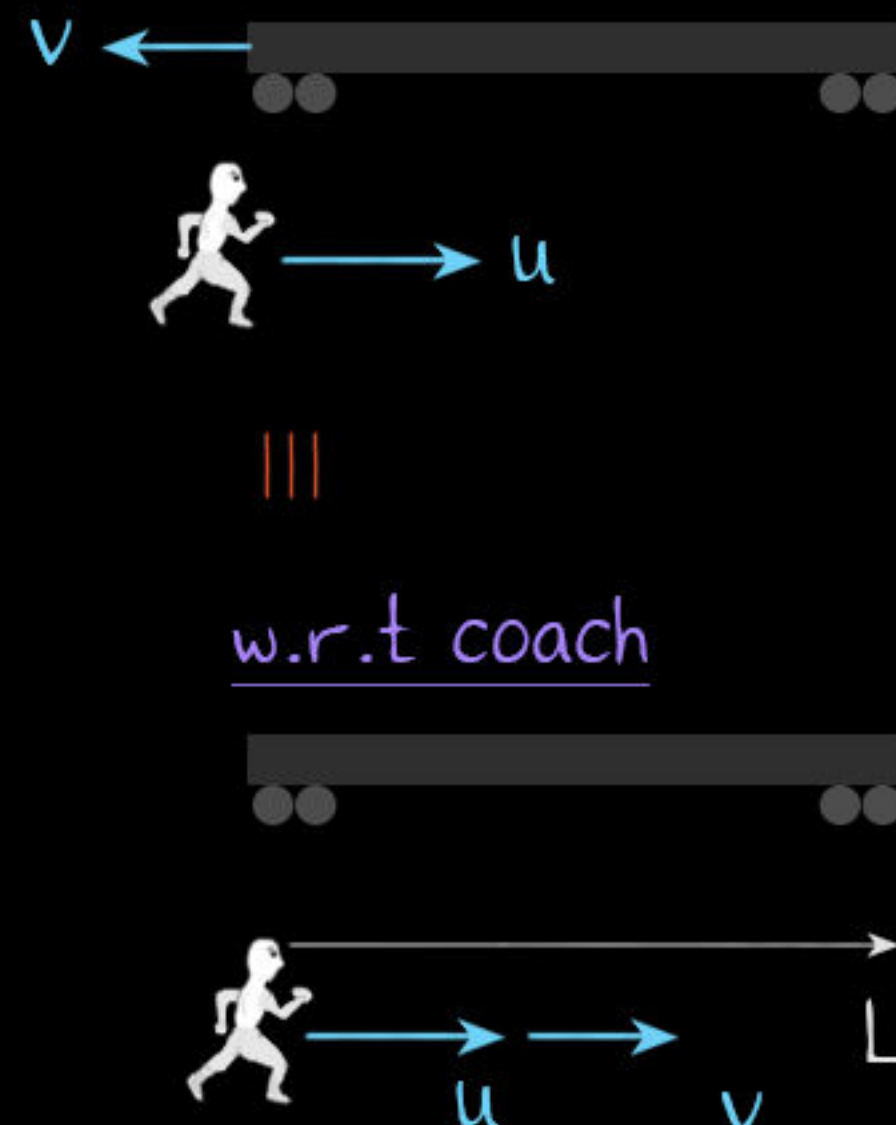
$$\Rightarrow L = (u - v) \frac{n_1 l}{u} \quad (1)$$

25. A train passes a platform with a uniform speed. A boy standing on the platform decides to estimate length of a coach and speed of the train. For this purpose, he first runs with a constant speed of $u = 10 \text{ km/h}$ in the direction of the motion of the train and passes by a coach in $n_1 = 30$ steps. Then he turns back, runs at the same constant speed and passes by a coach in $n_2 = 20$ steps. If the boy covers a distance $l = 1.0 \text{ m}$ in each step, answer the following questions.

(a) What is the speed of the train?

(b) What is the length of a coach?

Running opposite to the train:



$$L = (u + v) t_2$$

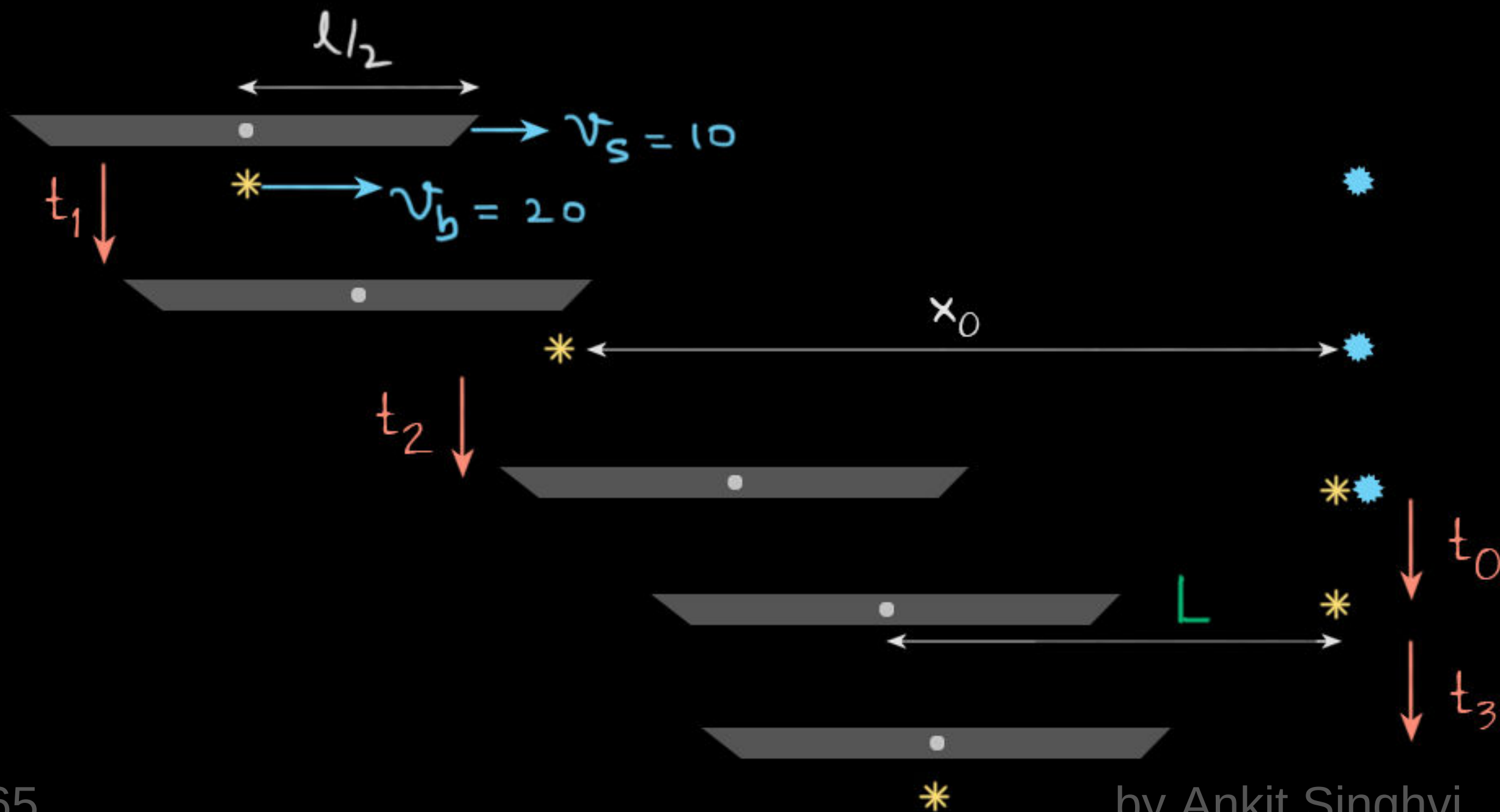
$$\Rightarrow L = (u + v) \frac{n_2 l}{u} \quad (2)$$

From 1,2:

$$v = \frac{(n_1 - n_2) u}{n_1 + n_2} //$$

$$L = \frac{2 n_1 n_2 l}{n_1 + n_2} //$$

26. A ship of length $l = 150$ m moving with velocity $v_s = 36$ km/h on the sea suddenly discovered a sinking boat straight ahead. A rescue boat has been lowered from the mid of the ship, which went to the sinking boat with speed $v_b = 72$ km/h. When the rescue boat overtakes the leading edge of the ship, the sinking boat was $x_0 = 3.0$ km away. The rescue boat reaches the sinking boat, spends $t_0 = 1.0$ min there to take the people on board and then returns with the same speed. Determine time taken in the whole rescue operation from the moment the rescue boat was lowered to the moment the rescue boat returned to the mid of the ship from where it was lowered.



$$t_1 = \frac{l/2}{v_b - v_s} = \frac{75}{10} = 7.5$$

$$t_2 = \frac{x_0}{v_b} = \frac{3000}{20} = 150$$

$$t_0 = 60$$

$$t_3 = \frac{L}{v_b + v_s} = \frac{975}{30} = 32.5$$

$$\begin{aligned} L &= x_0 + \frac{l}{2} - v_s [t_2 + t_0] \\ &= 3000 + 75 - 10(150 + 60) \\ &= 975 \end{aligned}$$

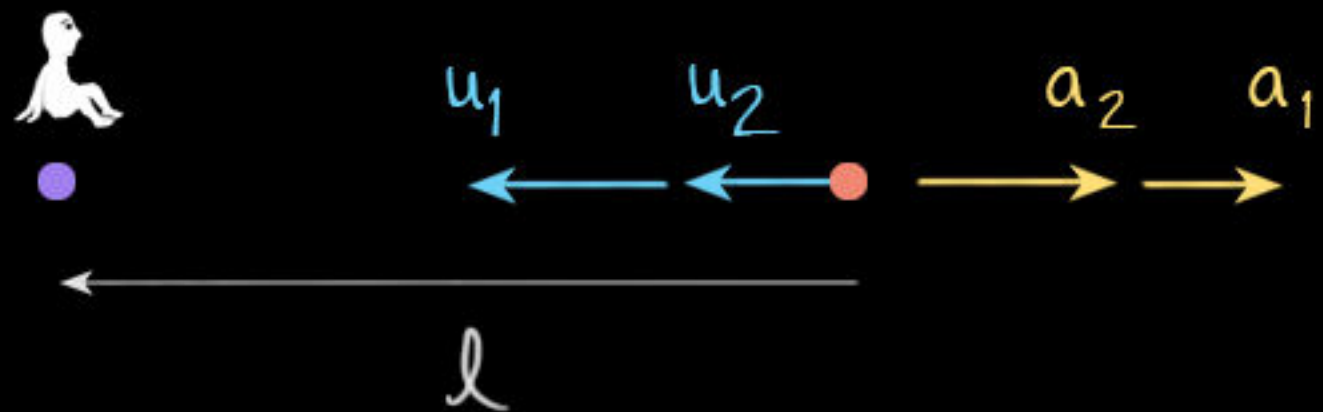
$$\begin{aligned} T &= t_1 + t_2 + t_0 + t_3 \\ &= 7.5 + 150 + 60 + 32.5 \\ &= 250 \text{ sec} // \end{aligned}$$

Youtube / Arihant Senior

27. At the initial instant, two particles are observed at different locations moving towards each other with velocities u_1 and u_2 . If they are subjected to constant accelerations a_1 and a_2 in directions opposite to their initial velocities, they will meet twice. If time interval between these two meetings is Δt , find suitable expression for their initial separation.



w.r.t 1st



$$l = (u_1 + u_2)t - \frac{1}{2}(a_1 + a_2)t^2$$

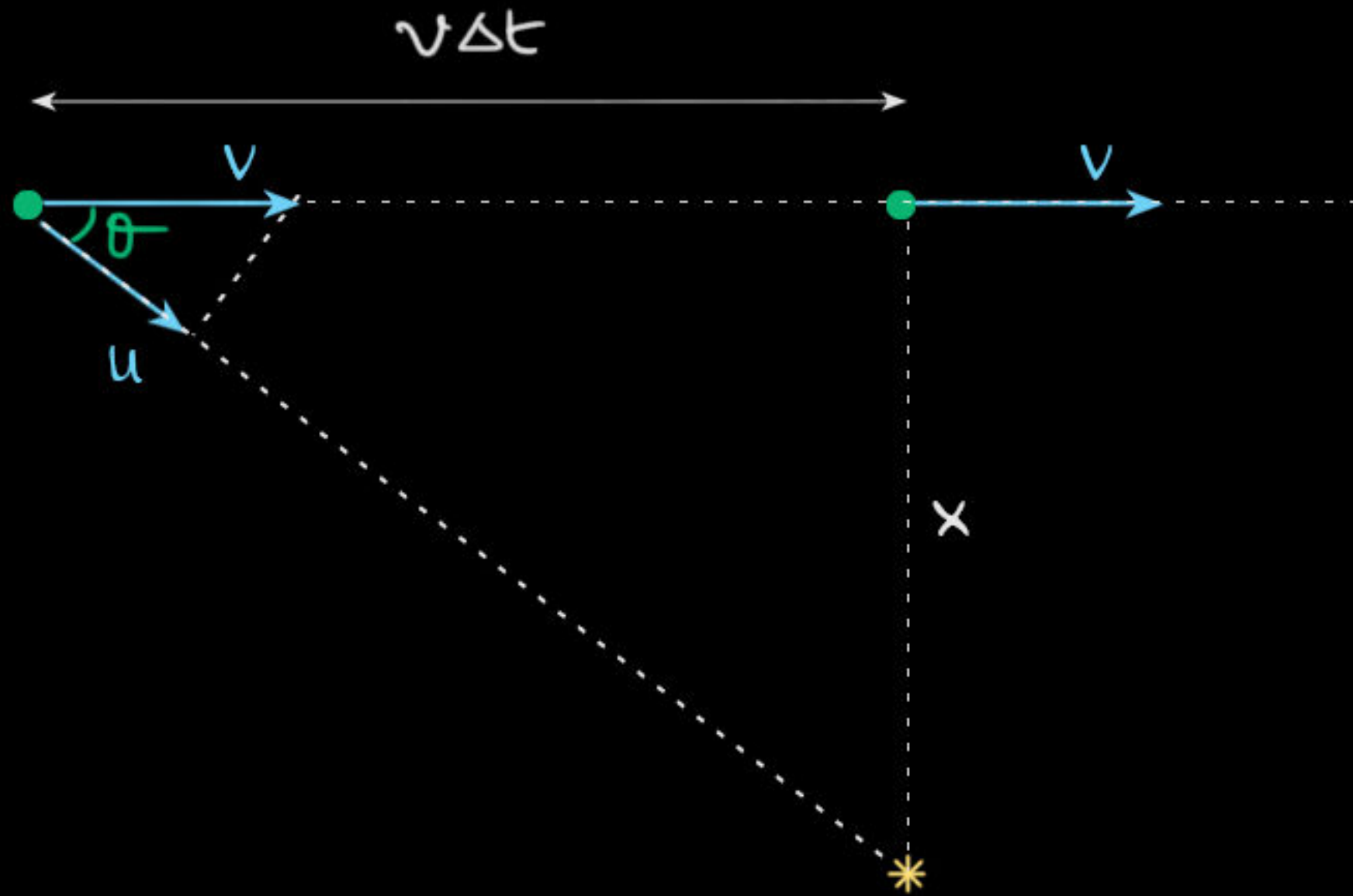
$$\text{Let the roots be } t_1 \text{ and } t_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta t = t_1 - t_2 = \frac{\sqrt{b^2 - 4ac}}{a}$$

$$\Rightarrow \Delta t = \frac{\sqrt{4(u_1 + u_2)^2 - 4(a_1 + a_2)2l}}{a_1 + a_2}$$

$$\Rightarrow l = \frac{4(u_1 + u_2)^2 - (a_1 + a_2)^2 \Delta t^2}{8(a_1 + a_2)}$$

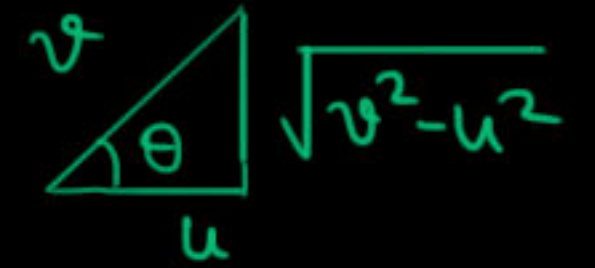
28. At a particular instant, a particle moving with a constant velocity is approaching a fixed point with a velocity $u = 3$ m/s and after a time interval $\Delta t = 6$ s the particle passes the position closest to the fixed point with a velocity $v = 5$ m/s. Find the closest distance between the fixed point and the particle.



$$v \cos \theta = u$$

and $x = v\Delta t \tan \theta$

$$= \frac{v\Delta t \sqrt{v^2 - u^2}}{u}$$



Let the gap be l

$$l^2 = x_{rel}^2 + y_{rel}^2 + z_{rel}^2$$

Approach: After 1 second, we can see x_{rel} , y_{rel} and z_{rel} are all increasing. So minimum gap has to be at $t < 1 \text{ sec}$

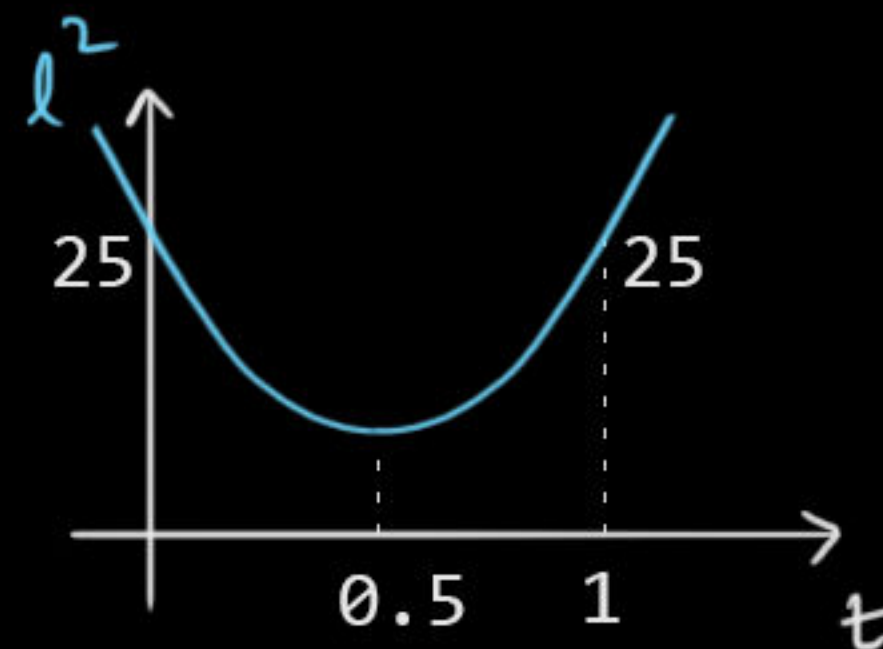
$x_{rel} = x_1 - x_2 = \text{linear function of time}$
 $\therefore$ both are linear functions

Similarly y_{rel} , and z_{rel} are linear functions too.

$\therefore l^2$ is a quadratic function,
 a parabola w.r.t time

$$l^2(t=0) = 3^2 + 4^2 + 0^2 = 5^2$$

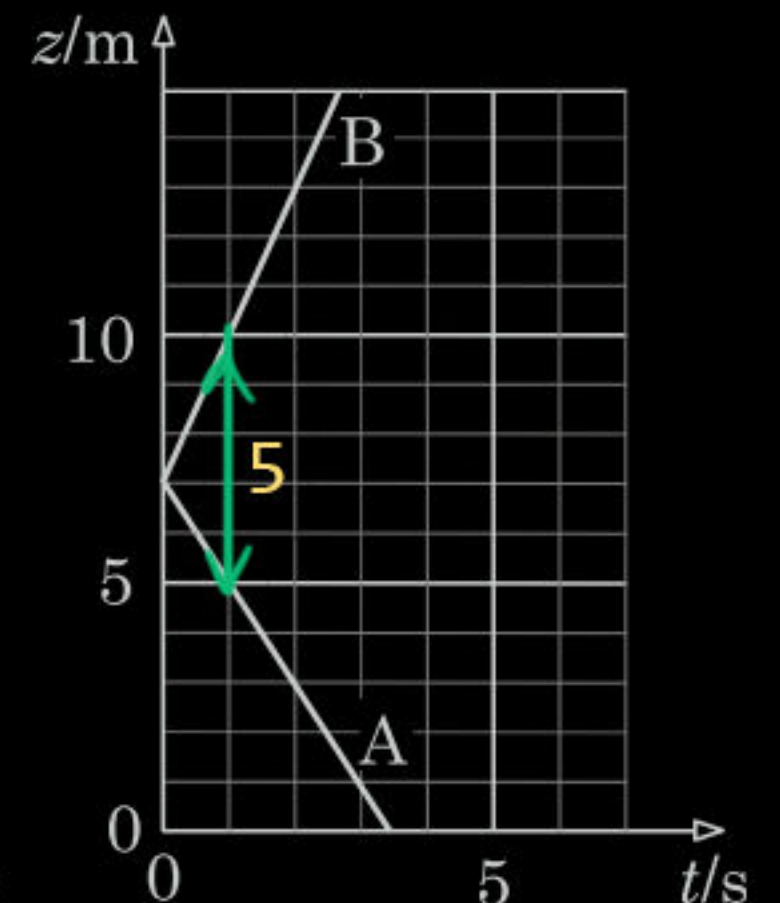
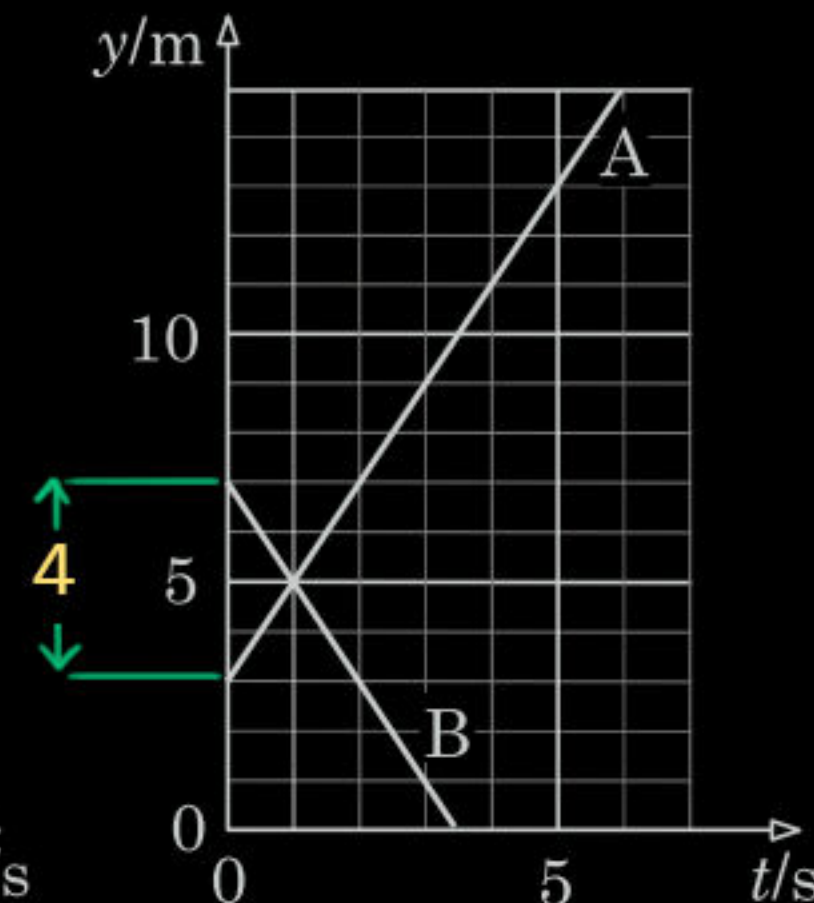
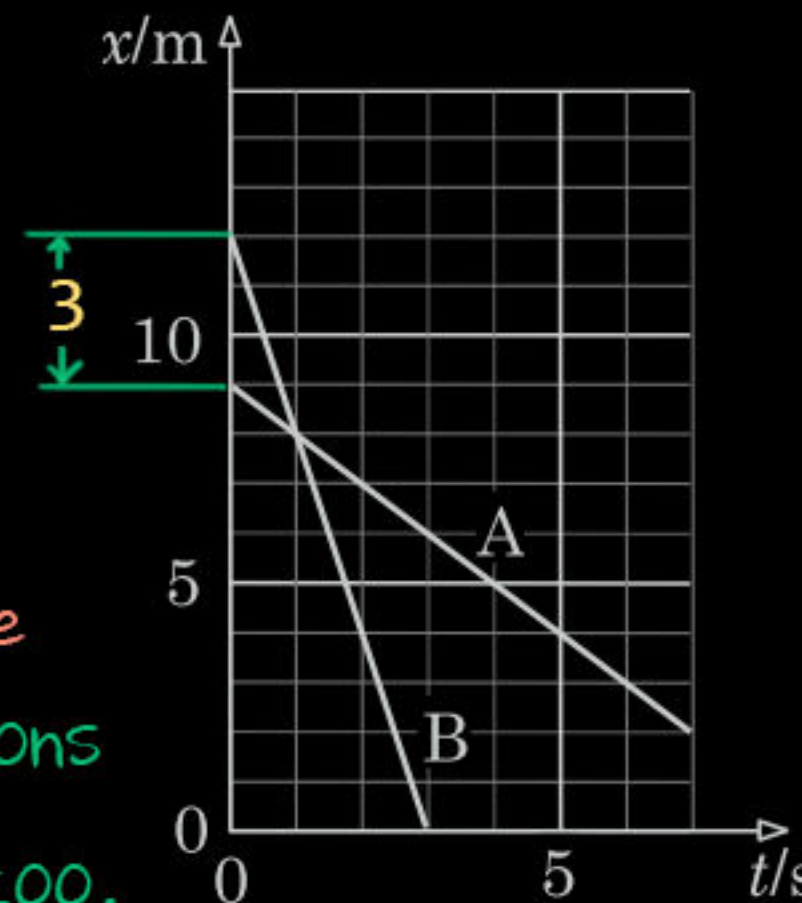
$$l^2(t=1) = 0^2 + 0^2 + 5^2 = 5^2$$



by Ankit Singhvi

29. Two material particles A and B are moving in free space. How their position coordinates x , y and z vary with time t is shown in the following graphs.

Determine at what instant of time the particles are closest to each other and the closest separation.



$\therefore$ Clearly gap is minimum at 0.5 sec.

And its value is:

$$l^2 = \left(\frac{3}{2}\right)^2 + \left(\frac{4}{2}\right)^2 + \left(\frac{5}{2}\right)^2$$

$$\Rightarrow l = \frac{5}{\sqrt{2}} = 2.5\sqrt{2} //$$

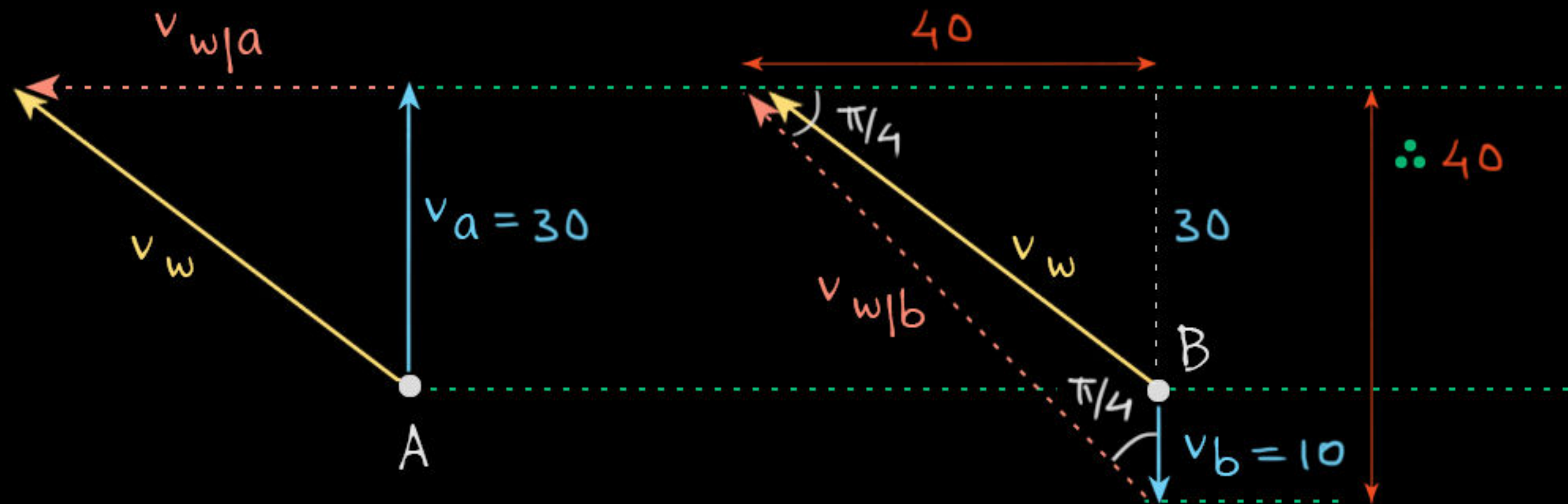
Youtube / Arihant Senior

Concept: Smoke flows with the wind as soon as its released. Velocity of chimney will have no effect on it.

$\therefore v_{\text{smoke}} \text{ is } v_{\text{wind}}$

30. Consider two steamers A and B on a calm sea. Steamer A is moving towards the north with a constant speed $v_A = 30 \text{ km/h}$ and steamer B towards the south with a constant speed $v_B = 10 \text{ km/h}$. If smoke ejected by steamer A spreads in a straight line from the steamer towards the west and smoke ejected by steamer B spreads in another straight line from the steamer towards the north-west, determine magnitude and direction of the wind velocity.

i.e w.r.t steamer



$$\therefore \vec{v}_w = -40\hat{i} + 30\hat{j} //$$

Youtube / Arihant Senior

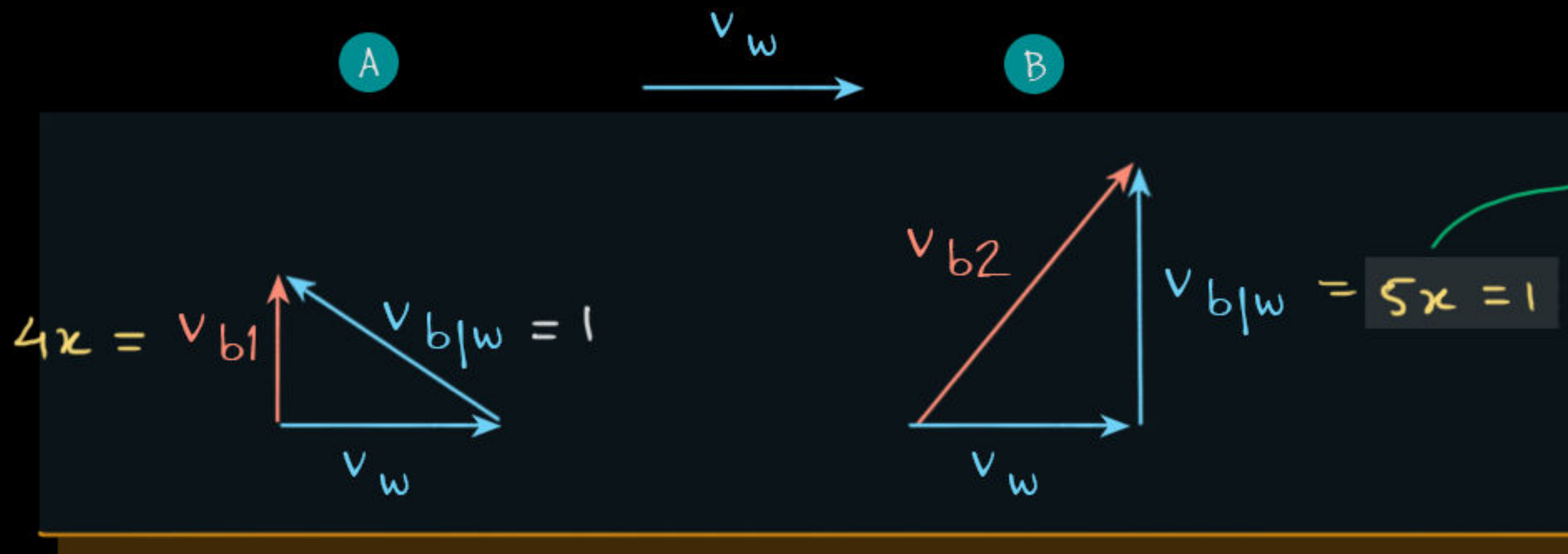
v_b perpendicular to shoreline is proportional to distance traveled perpendicular to shoreline.

Let proportionality constant be x .

Also it's given that $v_{b/w} = 1$.

31. Two identical boats are moving relative to the water current with equal speed $v_{b/w} = 1.0$ m/s. To a boy standing on the ground, the first boat appears moving perpendicular to the river current and to another boy standing on a raft in the river, the second boat appears moving perpendicular to the shoreline. In a certain time interval, distances of the boats from the shoreline increase by $\Delta y_1 = 4.0$ m and $\Delta y_2 = 5.0$ m respectively. Calculate speed of the river current.

distances from shore 'line'
NOT a fixed point on shore.



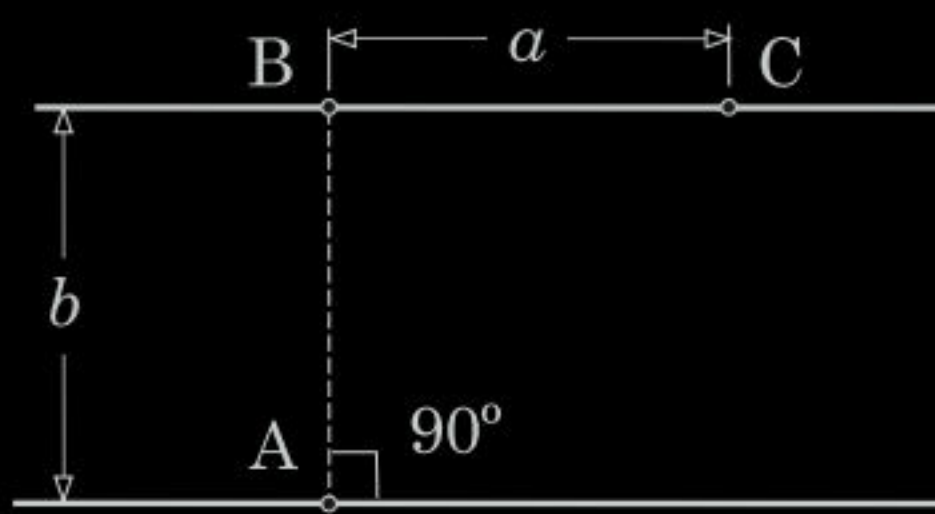
For first boat:

$$v_{b1}^2 + v_w^2 = v_{b/w}^2$$

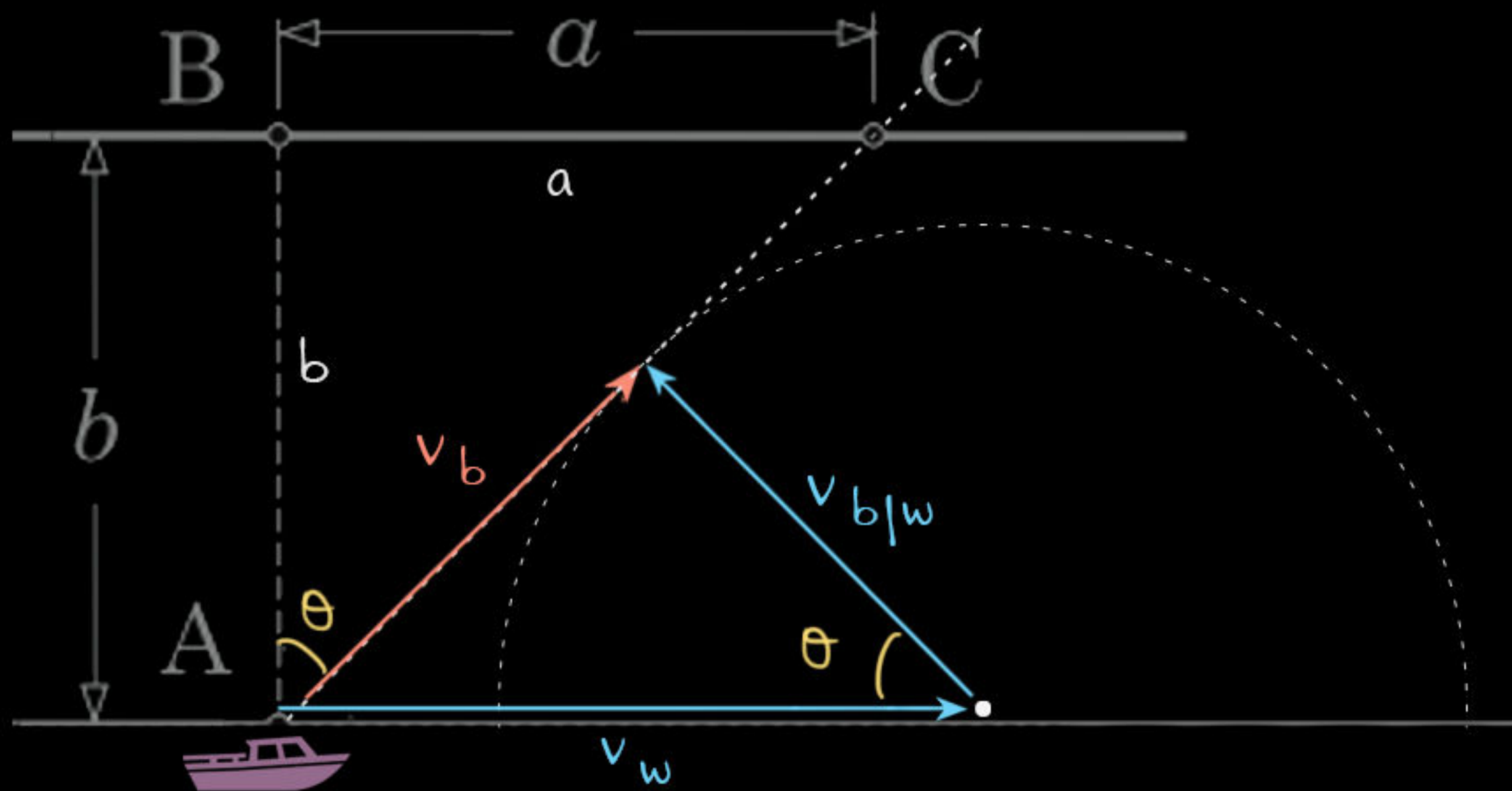
$$\Rightarrow (4x)^2 + v_w^2 = 1^2$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 + v_w^2 = 1^2$$

$$\Rightarrow v_w = \frac{3}{5} \text{ m/s}$$



32. A man in a boat starts from a point A and wants to reach a point C on the other bank of a river of width b . The point C is at distance a downstream from a point B, which is directly opposite to the point A. The water current velocity v_w is uniform everywhere. Find the minimum speed of the boat relative to the water current and corresponding direction in which the boat must be steered.



If $v_{b/w}$ is less than shown in the figure, final velocity of boat v_b (along tangent) can not be directed towards C and boat will drift right of target C.

$$\therefore \text{Minimum } v_{b/w} = v_w \cos \theta$$

$$= \frac{v_w b}{\sqrt{a^2 + b^2}}$$

Youtube / Arihant Senior

33. Three points A, B, and C are on a straight horizontal line with equal distances between adjacent points. At an instant all the three points start moving, the point A begins to move vertically upwards with a constant velocity u and the point C vertically downwards with a constant acceleration a without any initial velocity. How should the point B move vertically so that the three points always remain collinear?

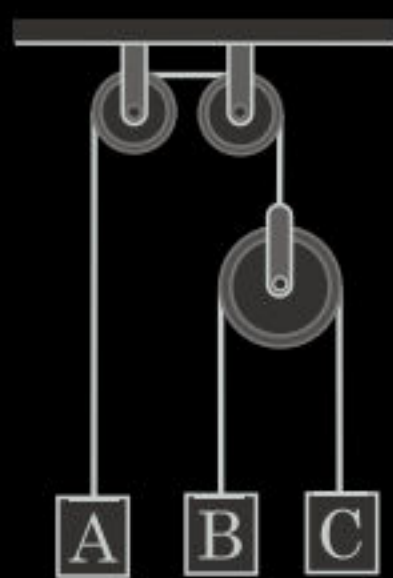
∴ B is at midpoint of coordinates A and C:

$$\begin{aligned}
 y_B &= \frac{y_A + y_C}{2} \\
 &= \frac{ut + (-\frac{1}{2}at^2)}{2} \\
 &= \frac{\frac{u}{2}t - \frac{1}{2}\left(\frac{a}{2}\right)t^2}{2} \equiv [s = ut + \frac{1}{2}at^2]
 \end{aligned}$$

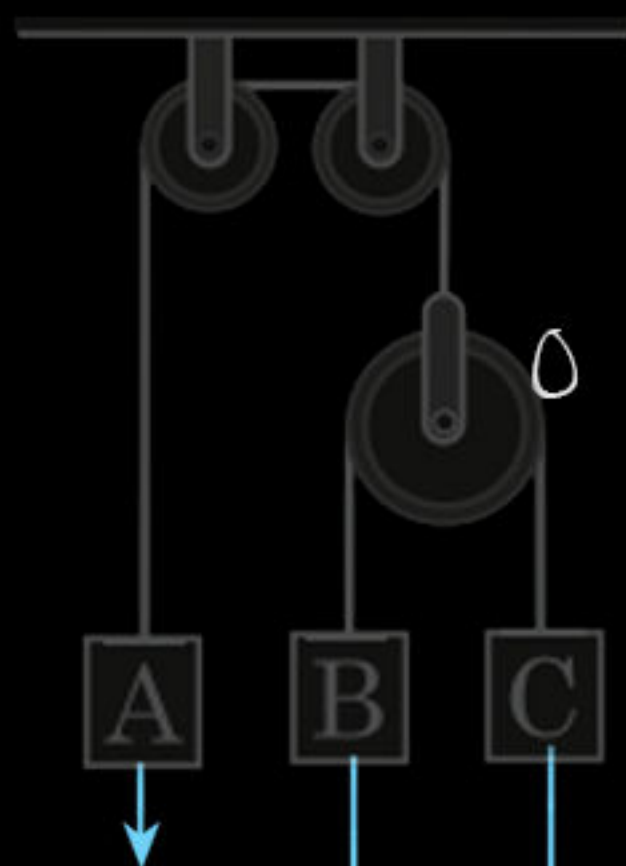
⇓

∴ With $u/2$ initial velocity and $-a/2$ acceleration

Lets assume v_A

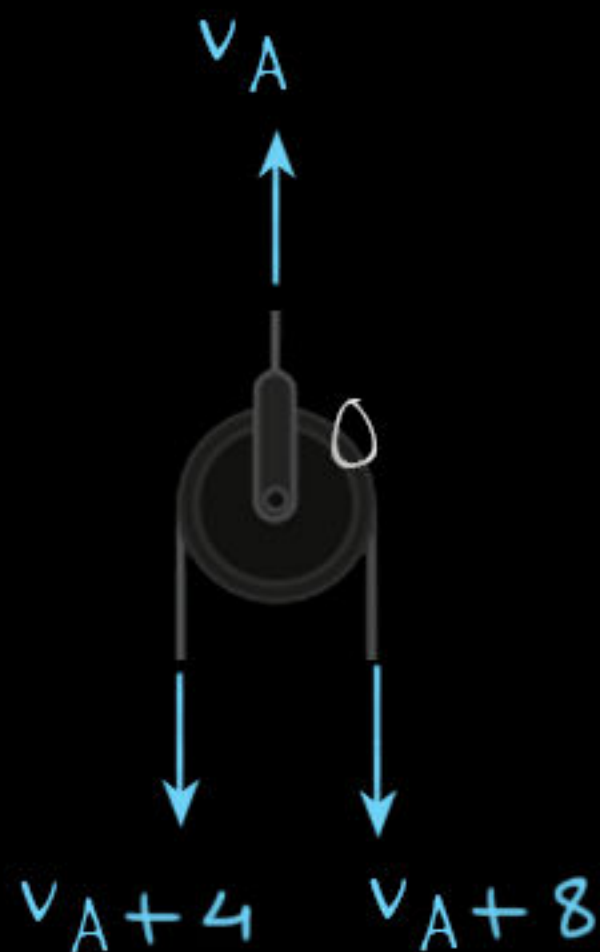


34. Three blocks A, B and C are suspended with the help of three pulleys and two threads with equal horizontal separation between adjacent blocks. Initially the blocks are held at rest at the same level and then released. The blocks move in such a way that they always remain in a straight line. If at an instant, the block B is observed moving downwards with velocity 4 cm/s relative to block A, find velocities of all the blocks at this instant.



Given v_A $v_A + 4$ $v_A + 8$

$\therefore v_B = \frac{v_A + v_C}{2}$
(being collinear)



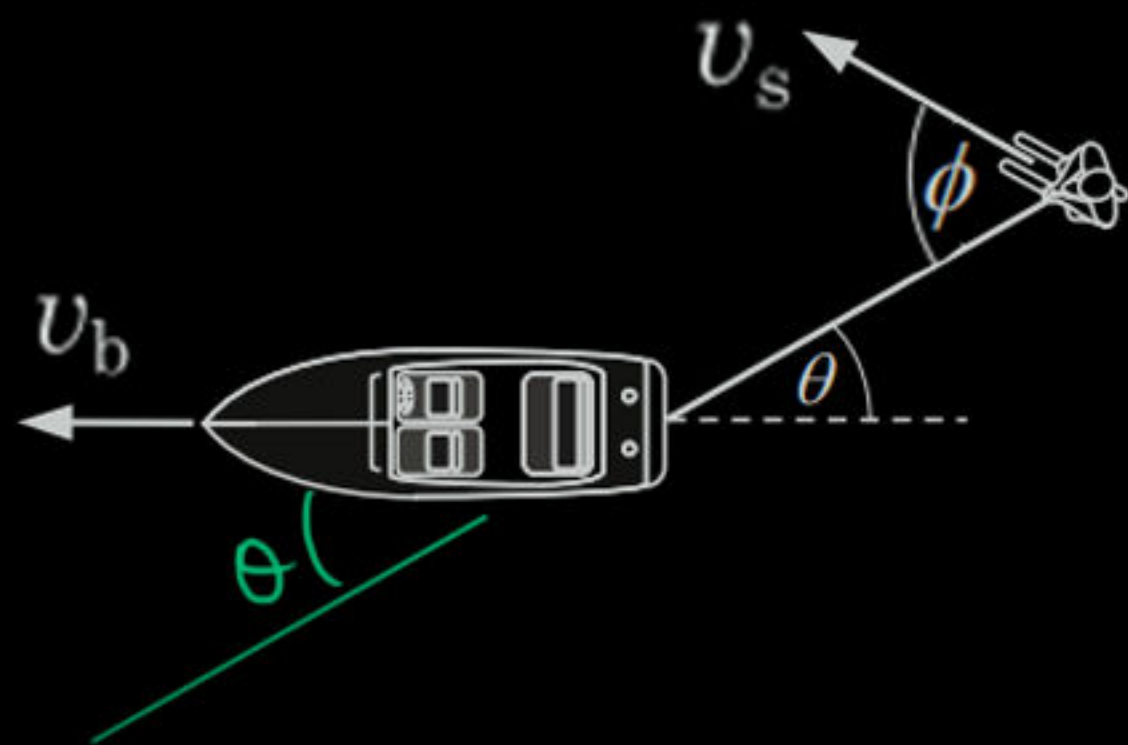
Velocity constrain on pulley O:

$$-v_A = \frac{(v_A + 4) + (v_A + 8)}{2}$$

$$\Rightarrow v_A = -3 //$$

$$\Rightarrow v_B = v_A + 4 = 1 //$$

$$\Rightarrow v_C = v_A + 8 = 5 //$$

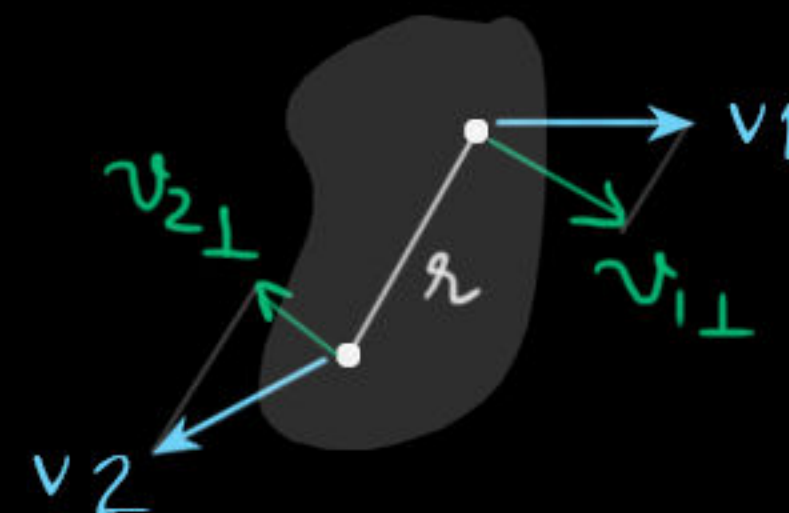


35. A boat is moving with uniform velocity $v_b = 20$ m/s pulling a water skier with the help of a tug-rope of length $l = 10$ m. To increase his speed the water skier tilts the skies slightly away from the direction of motion of the boat. As he does so, the tug rope rotates. What is the speed v_s of the skier with respect to the ground and angular velocity ω of the rope, when $\theta = 30^\circ$ and $\phi = 60^\circ$?

Velocity components of end points of rope, along the rope is equal.

$$\therefore v_b \cos \theta = v_s \cos \phi$$

$$\Rightarrow v_s = v_b \frac{\cos \theta}{\cos \phi}$$

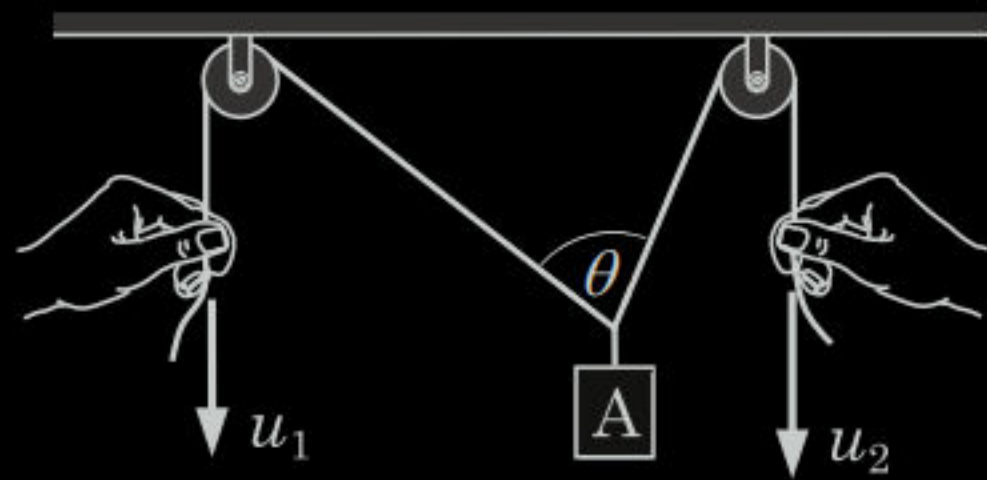


For a body we know,

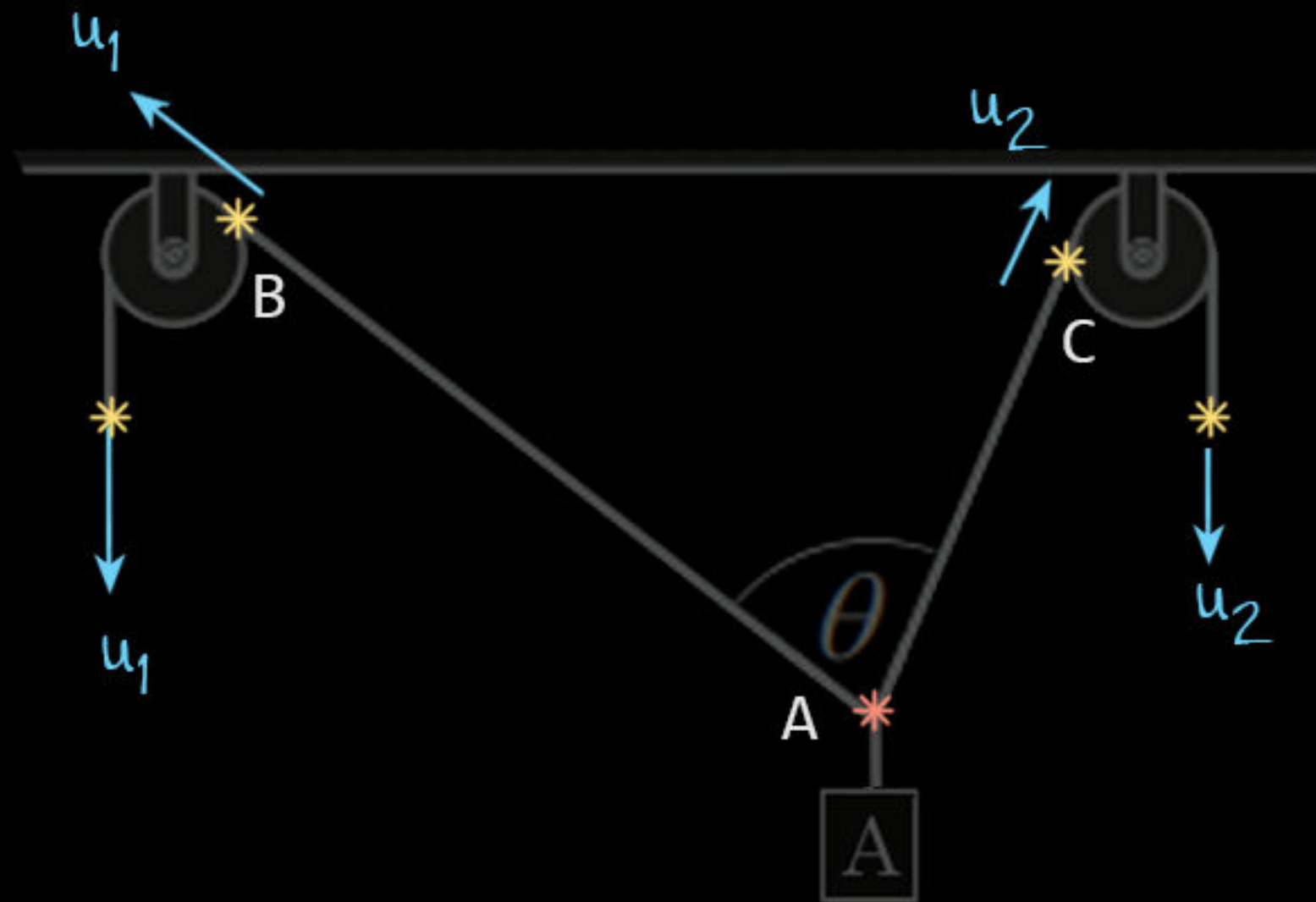
$$\omega = \frac{v_{1\perp} - v_{2\perp}}{r}$$

Here that body is rope.

$$\begin{aligned} \therefore \omega &= \frac{v_s \sin \phi - v_b \sin \theta}{l} \\ &= \frac{v_b \sin(\phi - \theta)}{l \cos \phi} \odot \end{aligned}$$



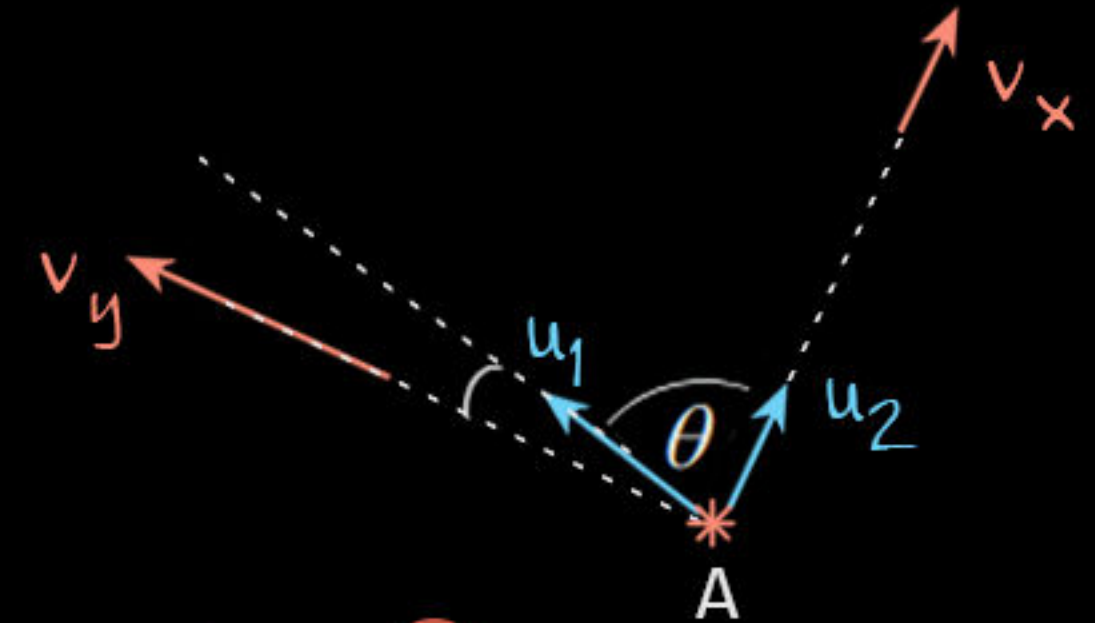
36. The load A is being pulled with the help of two inextensible strings that pass over two fixed pulleys as shown in the figure. At an instant velocities of the ends of the string being pulled are u_1 and u_2 and the angle between the strings connected to the load is θ , what is speed of the load?



We need to find the velocity of point A.

For the piece of rope AB, velocity of points A and B along the rope is equal, and its u_1 . Similarly along AC, velocity along the rope for both A and C is equal, and its u_2 .

Let the components of actual velocity (unknown) along two perpendicular directions (as shown) be v_x and v_y .



$$u_2 = v_x + v_y \cos \frac{\pi}{2} = v_x \quad (1)$$

$$\begin{aligned} 2 \quad u_1 &= v_y \cos(\frac{\pi}{2} - \theta) + v_x \cos \theta \\ &= v_y \sin \theta + v_x \cos \theta \quad (2) \end{aligned}$$

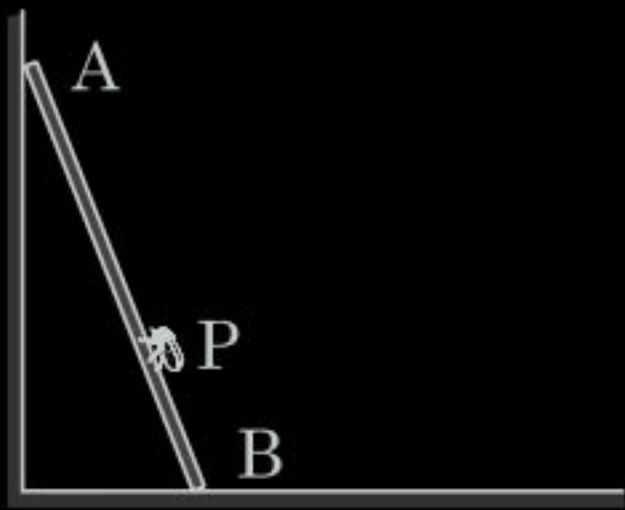
$$\text{From 1,2: } v_y = \frac{u_1 - u_2 \cos \theta}{\sin \theta}, \quad v_x = u_2$$

$$\Rightarrow v = \sqrt{v_x^2 + v_y^2} =$$

$$\frac{\sqrt{u_1^2 + u_2^2 - 2u_1 u_2 \cos \theta}}{\sin \theta}$$

by Ankit Singhvi

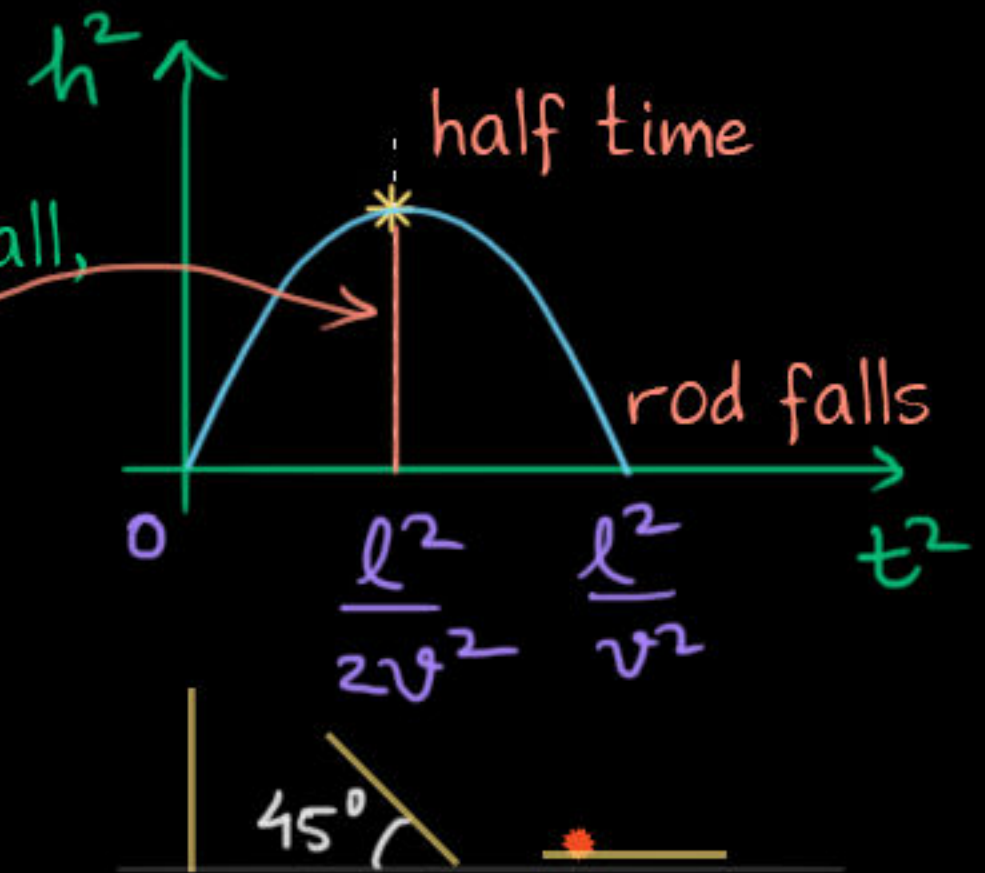
Youtube / Arihant Senior



37. A stick AB of length l stands vertically on a horizontal floor leaning on a wall. A beetle P starts climbing the stick from the floor. When the beetle starts climbing, the lower end B of the stick is made to move away from the wall with a constant velocity v . The beetle climbs the stick with a constant speed u relative to the stick. If the upper end A does not leave the wall, what maximum height can the beetle rise?

Case A

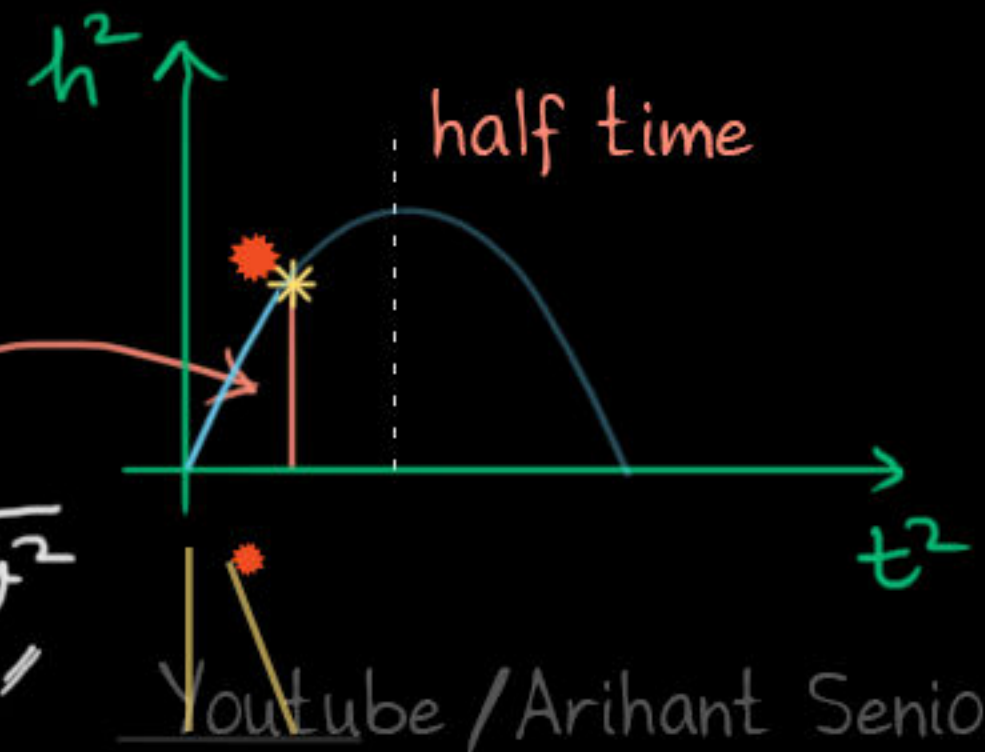
If beetle is slow and passes the 'half time' of rod fall, than regardless of u , it reaches maximum height at half time, when rod is tilted at 45°



From 1: $h_{\max} = h_{t=\frac{l}{\sqrt{2}v}} = \frac{ul}{2v} //$
 Case B if $\frac{l}{u} > \frac{l}{\sqrt{2}v}$

But if beetle is fast and reaches the top of the rod in less than half time, then its wall height is maximum height.

i.e if $\frac{l}{u} < \frac{l}{\sqrt{2}v}$, $h_{\max} = h_{t=\frac{l}{u}} = \frac{l}{u} \sqrt{u^2 - v^2} //$



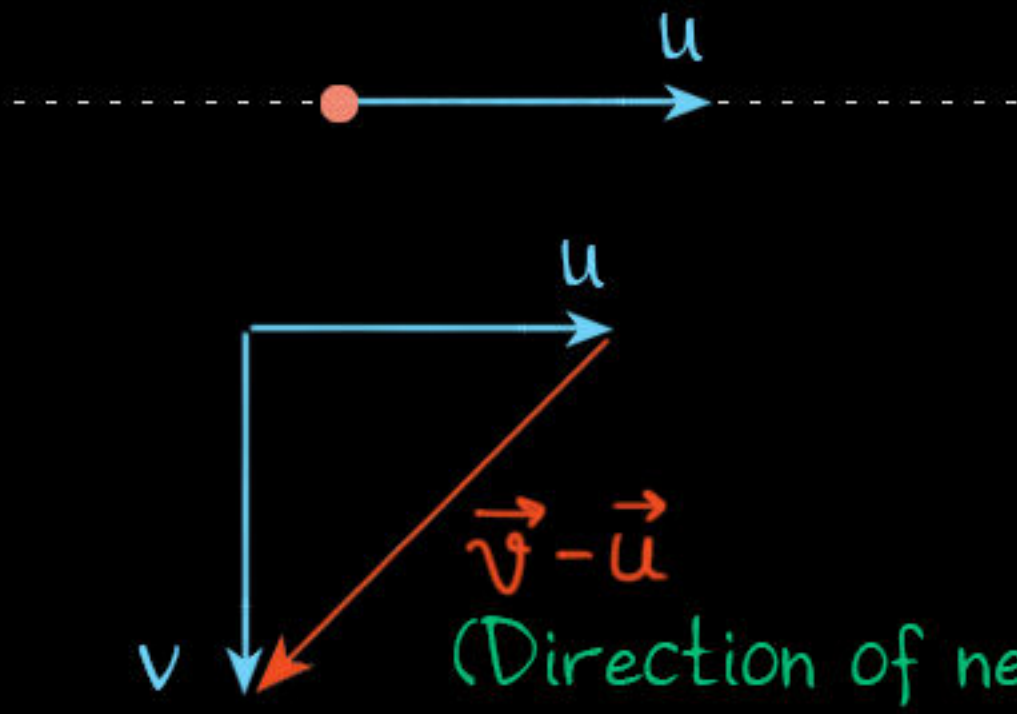
$$h = ut \sin \theta = \frac{ut \sqrt{l^2 - v^2 t^2}}{l}$$

$$\Rightarrow h^2 = \frac{u^2}{l^2} t^2 (l^2 - v^2 t^2) \quad (1)$$

by Ankit Singhvi

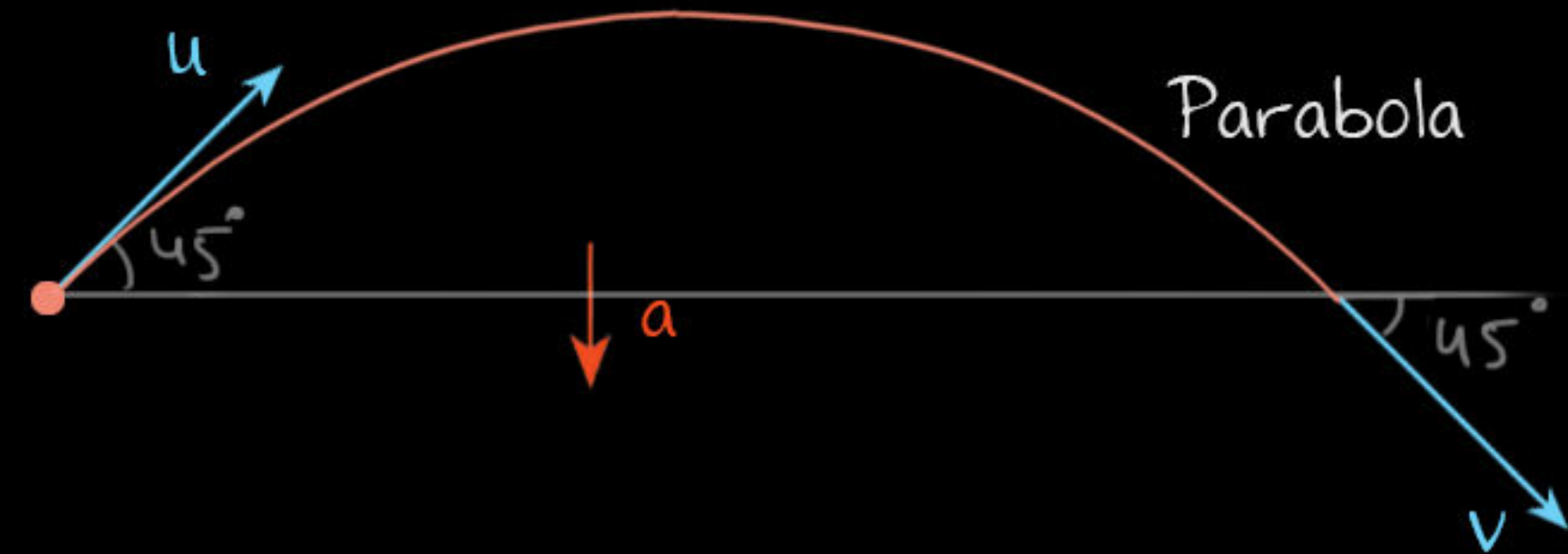
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38. A spacecraft is moving in space, where all the external forces can be neglected. Any change in its speed and direction of motion can be accomplished by rockets installed on it. At an instant when it is moving with a speed $u = 100$ m/s, the crew inside decides to take a 90° turn with an acceleration of constant modulus and then move in the new direction with the same speed u . The rockets installed can provide a maximum acceleration $a = 5\sqrt{2}$ m/s². Find the minimum time spent and shape of the path followed during the turn.



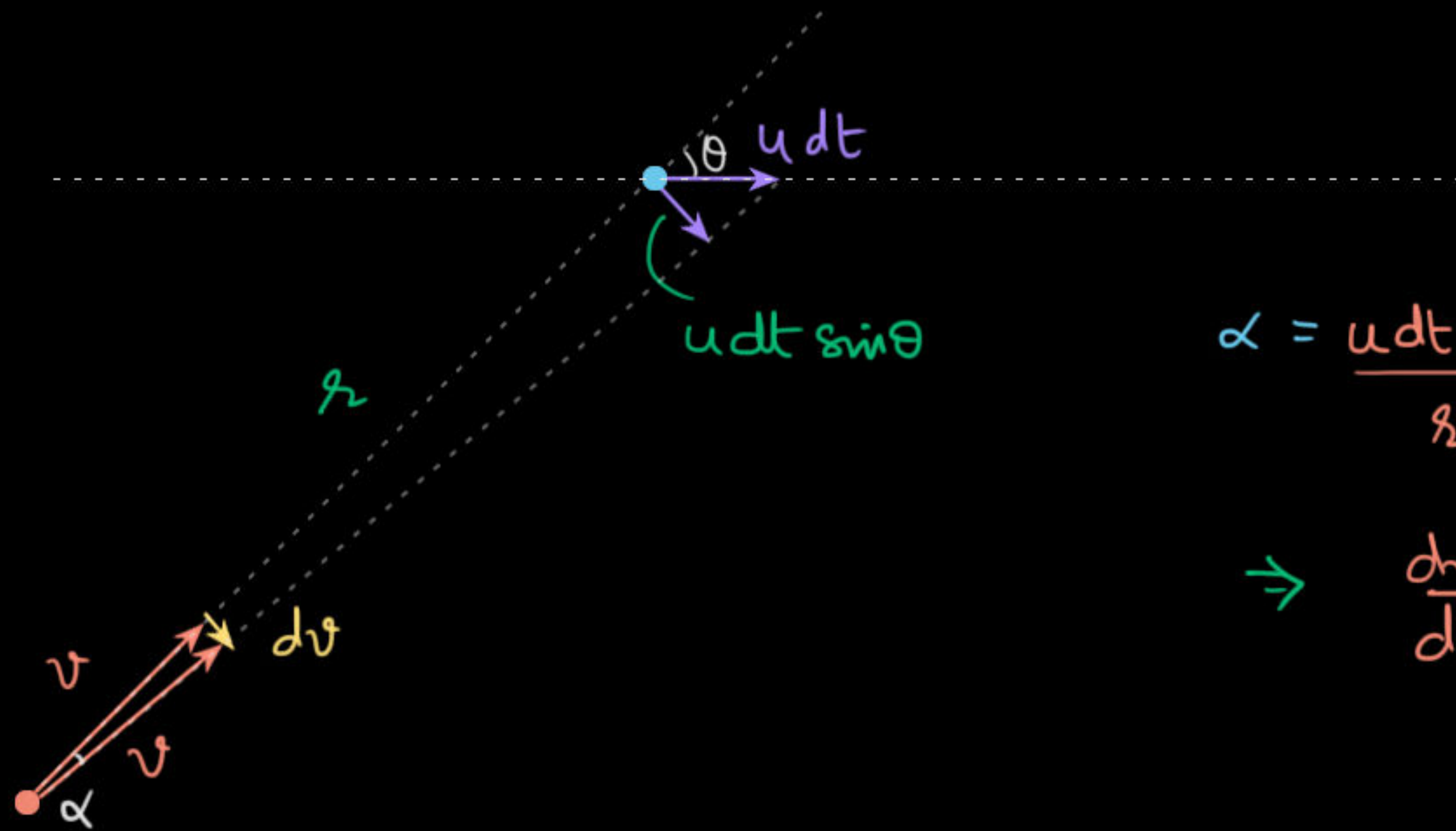
- ∴ Instantaneous acceleration must be along this direction of average acceleration during whole process, for minimum turn time.

Hint: To realise a change in velocity vector in a minimum time with an acceleration of constant modulus, in every infinitesimal time interval, changes in velocity vectors must be in the same direction. Therefore, the acceleration vector throughout the process of change in the velocity vector must be a constant and in the direction of the change in the velocity vector.



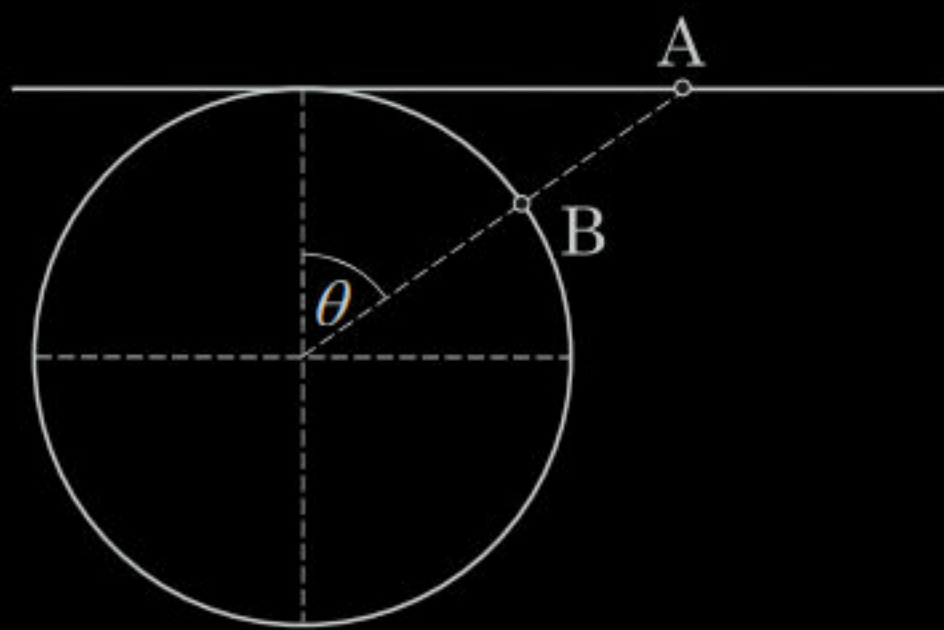
$$T = \frac{2u \sin 45^\circ}{a} = \frac{\sqrt{2}u}{a}$$

39. A dog running with a constant speed v is chasing a cat that is running with a constant velocity $\vec{u}$. During the chase, the dog always heads towards the cat. At an instant, direction of motion of the dog makes angle θ with that of the cat and the distance between them is r . Find magnitude of acceleration of the dog at this instant.

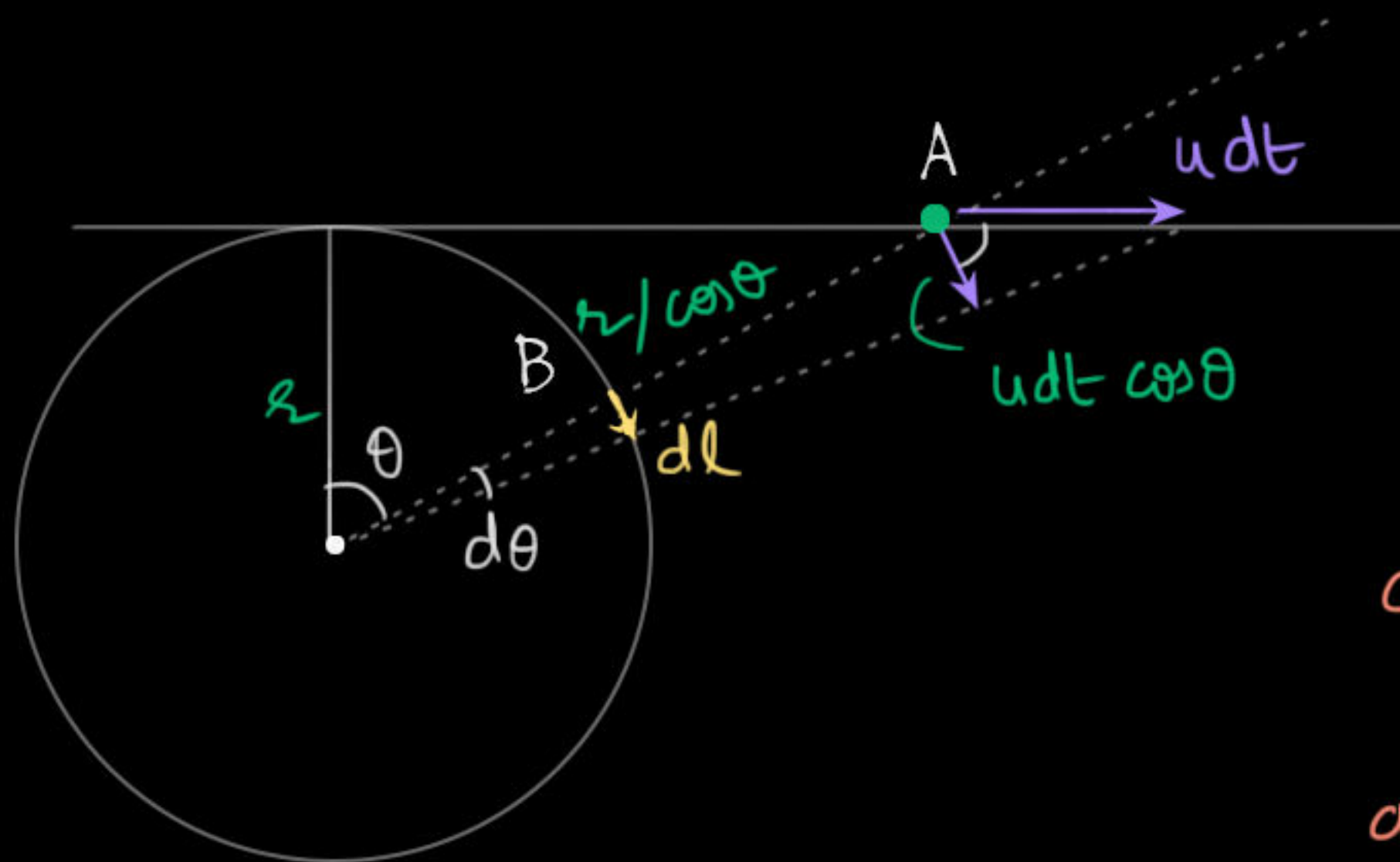


$$\alpha = \frac{u dt \sin \theta}{r} = \frac{dv}{v}$$

$$\Rightarrow \frac{dv}{dt} = \frac{uv \sin \theta}{r} //$$



40. A straight track is tangent to a circular track of radius r . Two material points A and B start simultaneously from the common point of the tracks. The point A moves with uniform velocity u on the straight track whereas the point B on the circular track always keeping itself collinear with the centre of the circular track and the point A. Find suitable expression for magnitude of acceleration of the point B when it is at angular position θ .



$$d\theta = \frac{dl}{r} = \frac{u dt \cos \theta}{r / \cos \theta}$$

$$\Rightarrow \frac{dl}{dt} = u \cos^2 \theta, \quad \frac{d\theta}{dt} = \frac{u \cos^2 \theta}{r}$$

$\downarrow$ diff

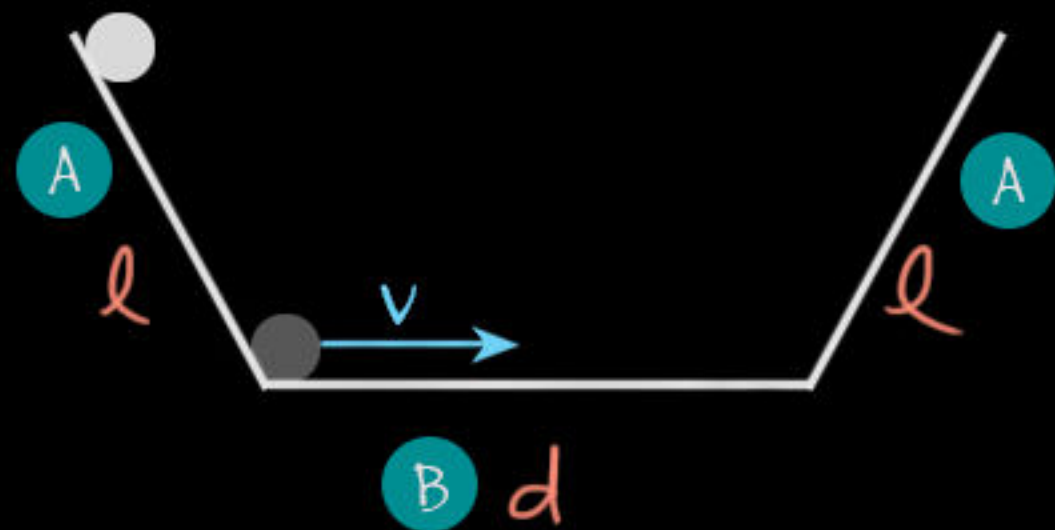
$$a_t = \frac{d^2 l}{dt^2} = -2u \cos \theta \sin \theta \cdot \frac{d\theta}{dt} = -\frac{2u^2 \sin \theta \cos^3 \theta}{r}$$

$$a_n = \frac{1}{r} \left(\frac{dl}{dt} \right)^2 = \frac{1}{r} u^2 \cos^4 \theta \quad (2)$$

$$\text{From 1,2: } a = \sqrt{a_t^2 + a_n^2} = \frac{u^2 \cos^3 \theta}{r} \sqrt{1 + 3 \sin^2 \theta}$$

by Ankit Singhvi

Youtube / Arihant Senior



1. A bead released from one end of a frictionless rigid wire frame fixed in a vertical plane slides periodically on the frame. Size of the bead is negligible as compared to the radius of curvature of the turns, which is negligible as compared to the length of straight portions of the frame. If linear dimensions of the frame are made four times, by what factor will frequency of periodic motion of the bead change?

(a) 4

(b) 2

(c) 1/4

✓(d) 1/2

Concept: Whenever dealing with dimensional changes (1x, 2x, 0.5x etc.).

Try to solve with proportionality, without worrying about constants.

∴ Uniform acceleration: $l \propto t_A^2$ ①

$$(l = \cancel{ut}^0 + \frac{1}{2}at^2)$$

$t_B = \frac{d}{v} \propto \frac{d}{\sqrt{l}}$ ②

∴ $v \propto \sqrt{l}$ ($v^2 = \cancel{u^2}^0 + 2al$)

$$T = 4t_A + 2t_B$$

↓ From 1,2

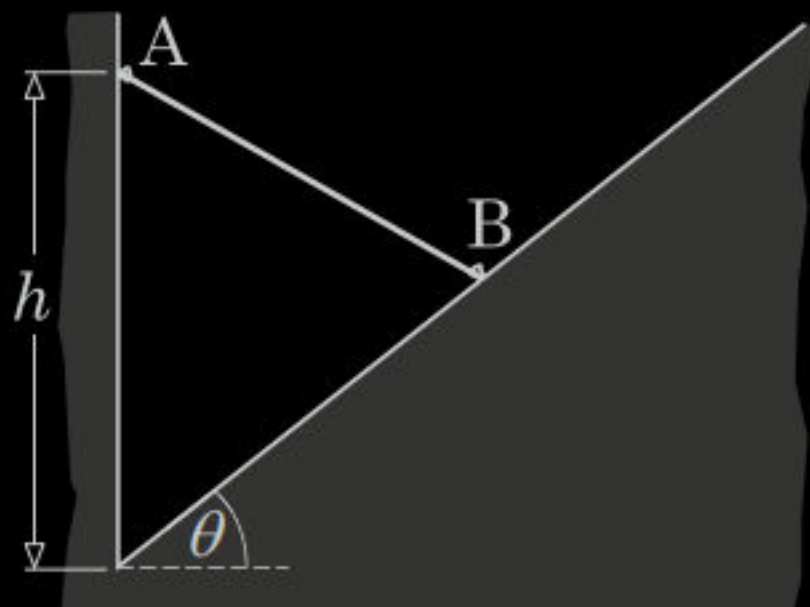
$$= c_1\sqrt{l} + c_2\frac{d}{\sqrt{l}}$$

∴ On changing l and d four times

$$T' = c_1\sqrt{4l} + c_2\frac{4d}{\sqrt{4l}} = 2T$$

$$\Rightarrow f' = \frac{f}{2}$$

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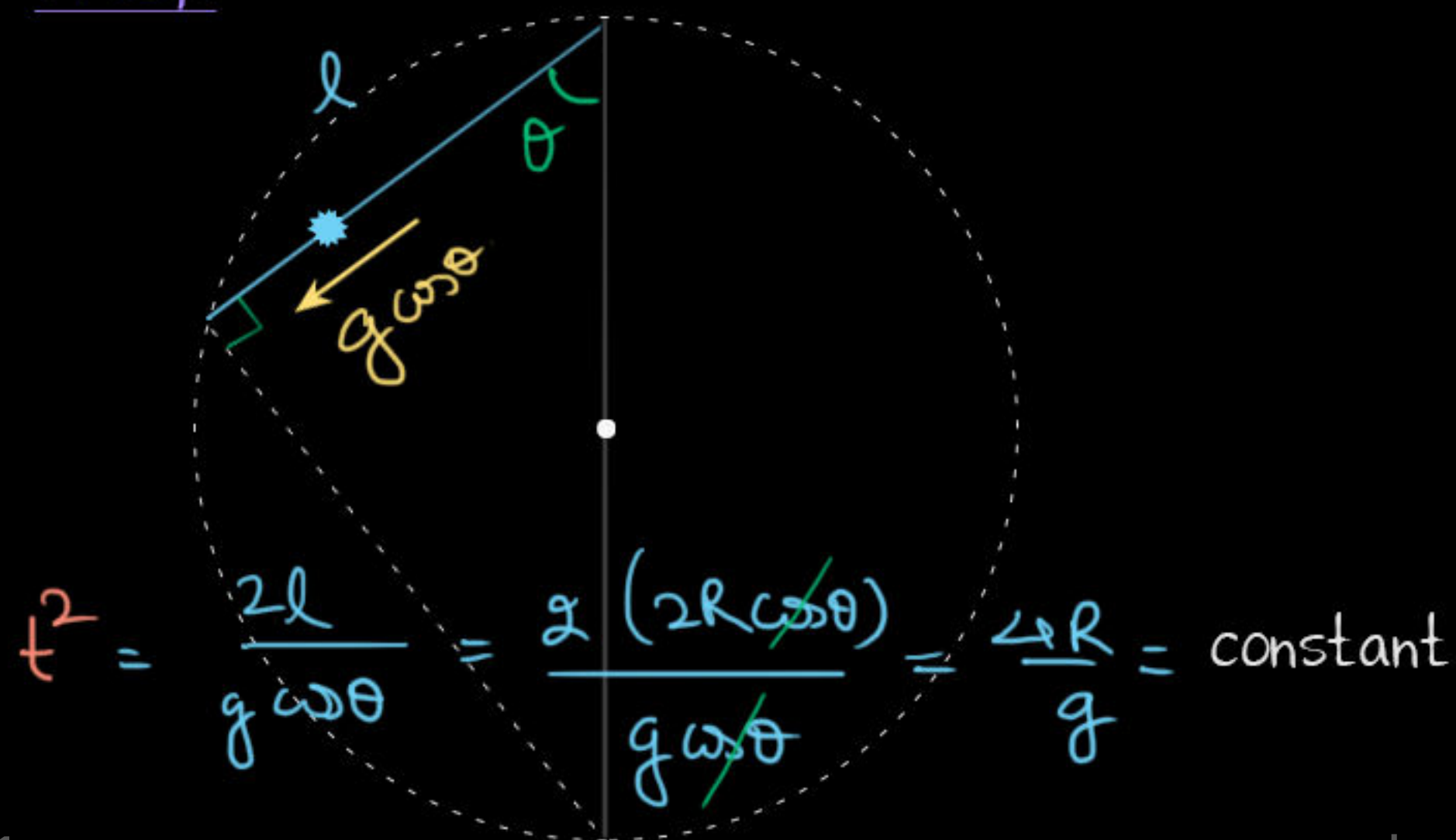


2. In the setup shown a thread is taut between a nail A on a wall and a nail B on an inclined plane as shown in the figure. You can change the length of the thread by changing location of the nail B. A bead that can slide on the thread without friction is released from the nail A. What should the length of the thread be so that the bead reaches the nail B in shortest time? $\checkmark$ (d) $h \cos \theta \sec(0.5\theta)$

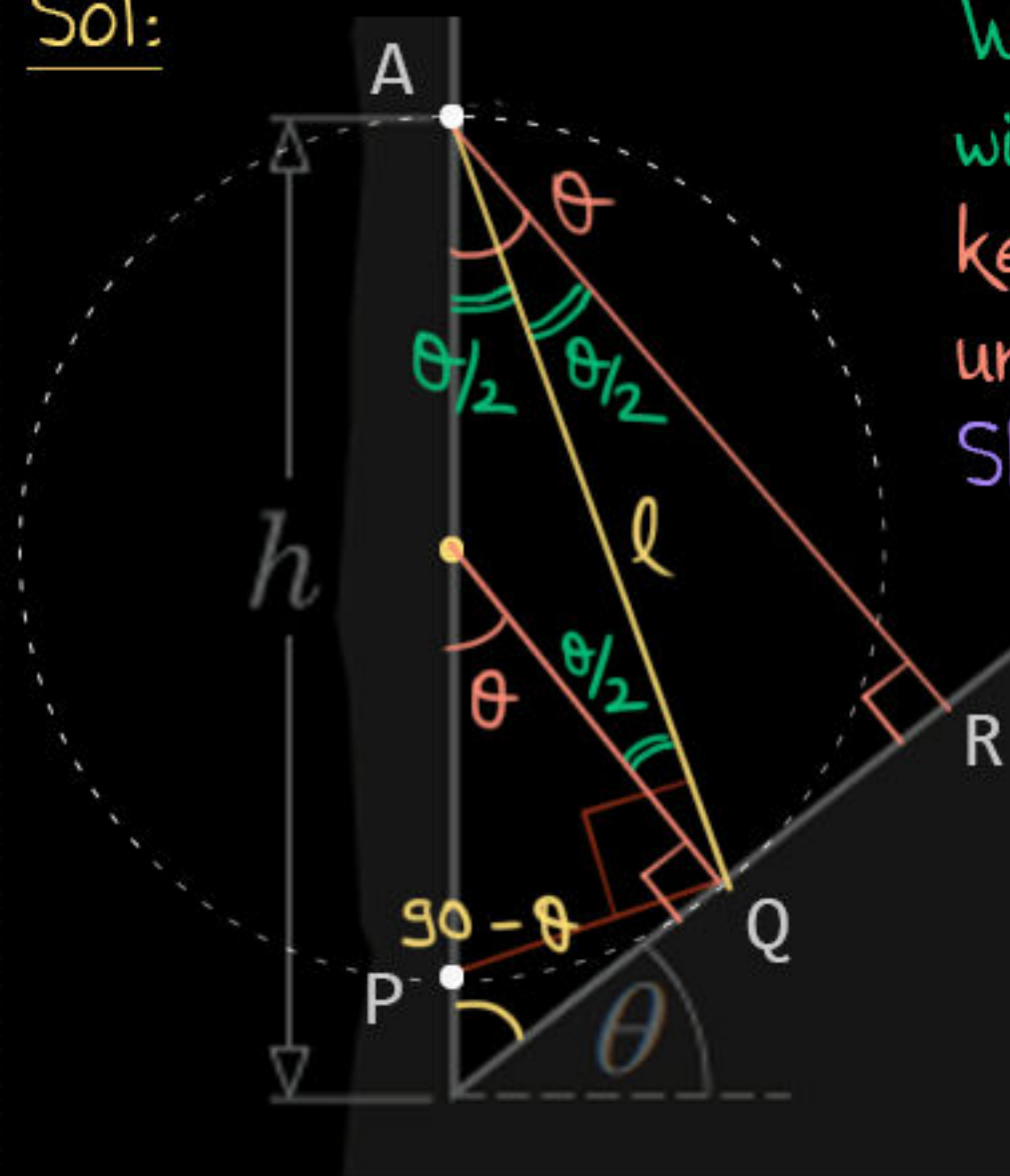
Method 1

Concept: Using a known result that: from the top of a circle, all secant paths take equal time

Proof:



Sol:



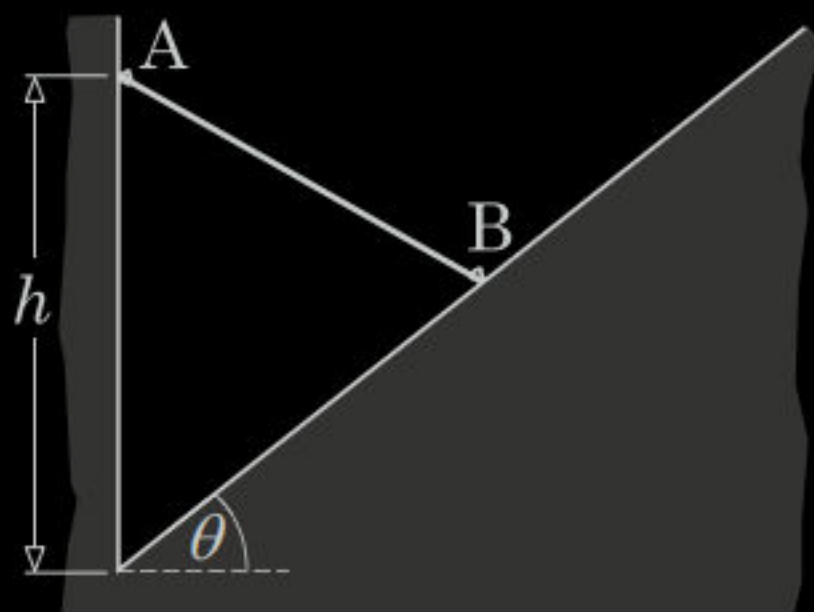
We start drawing circles with A as top point. We keep enlarging the circle until it touches the slope. Shortest time is then:

$$t_{AP} = t_{AQ}$$

and required length of thread is l.

$$l = \frac{AR}{\cos \theta/2} = \frac{h \cos \theta}{\cos \theta/2}$$

$$\therefore \text{Shortest time } t^2 = \frac{4R}{g} = \frac{2(l/\cos \theta/2)}{g} = \frac{2h \cos \theta}{g \cos^2 \theta/2}$$

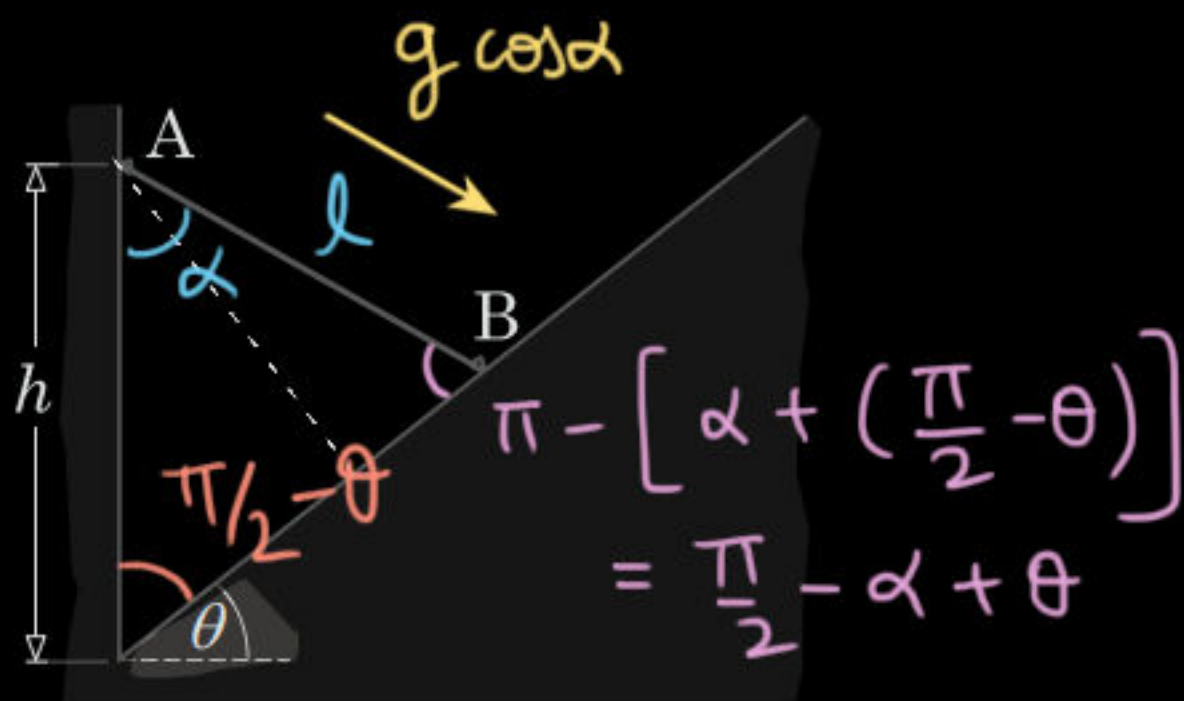


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Method 2

Let the length be l , when thread makes α with the wall.

Along the thread acceleration will be then $g \cos \alpha$



Sine law to get l :

$$t^2 = \frac{2l}{g \cos \alpha} \quad \frac{l}{\sin(\frac{\pi}{2} - \theta)} = \frac{h}{\sin(\frac{\pi}{2} - \alpha + \theta)} \quad (1)$$

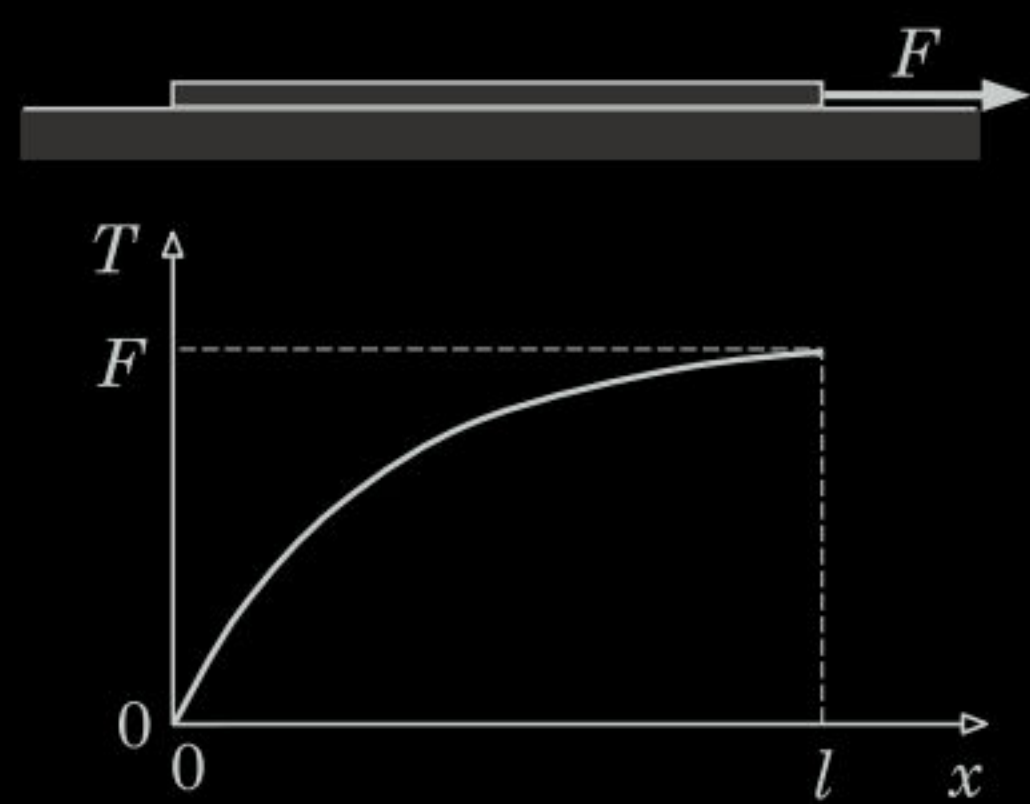
$$\Rightarrow t^2 \propto \frac{1}{\cos \alpha \cos(\alpha - \theta)} \propto \frac{1}{\cos(2\alpha - \theta) + \cos \theta}$$

$$\therefore t_{\min} @ \alpha = \frac{\theta}{2}$$

$\therefore$ At minimum time, from (1):

$$l \sin(\frac{\pi}{2} - \frac{\theta}{2} + \theta) = h \sin(\frac{\pi}{2} - \theta)$$

$$\Rightarrow l \cos \theta/2 = h \cos \theta \Rightarrow l = h \cos \theta \sec \theta/2$$



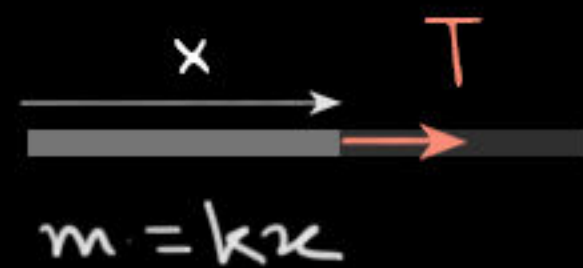
3. A rope of length l placed straight on a frictionless horizontal floor is pulled longitudinally by a force F from one of its ends. Tensile force T developed in the rope at a distance x from the rear end varies with x as shown in the graph. What can you certainly conclude about density of the rope?

✓ (b) It decreases with distance x from the rear end.

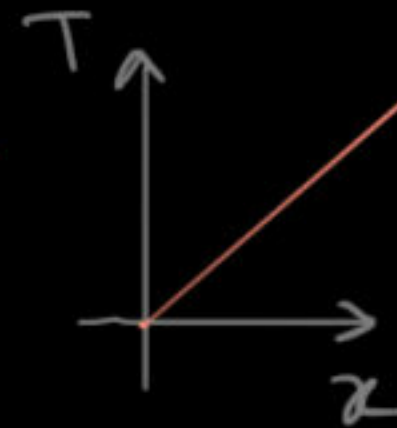
Concept: Tension is proportional to mass it has to carry.

Method 1 Generic

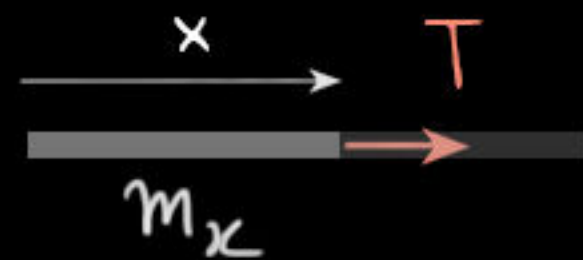
∴ For a uniform rod



$$\Rightarrow T \propto kx$$



For a non-uniform rod



$$\Rightarrow T \propto m_x$$

$$T' \propto m'_x$$

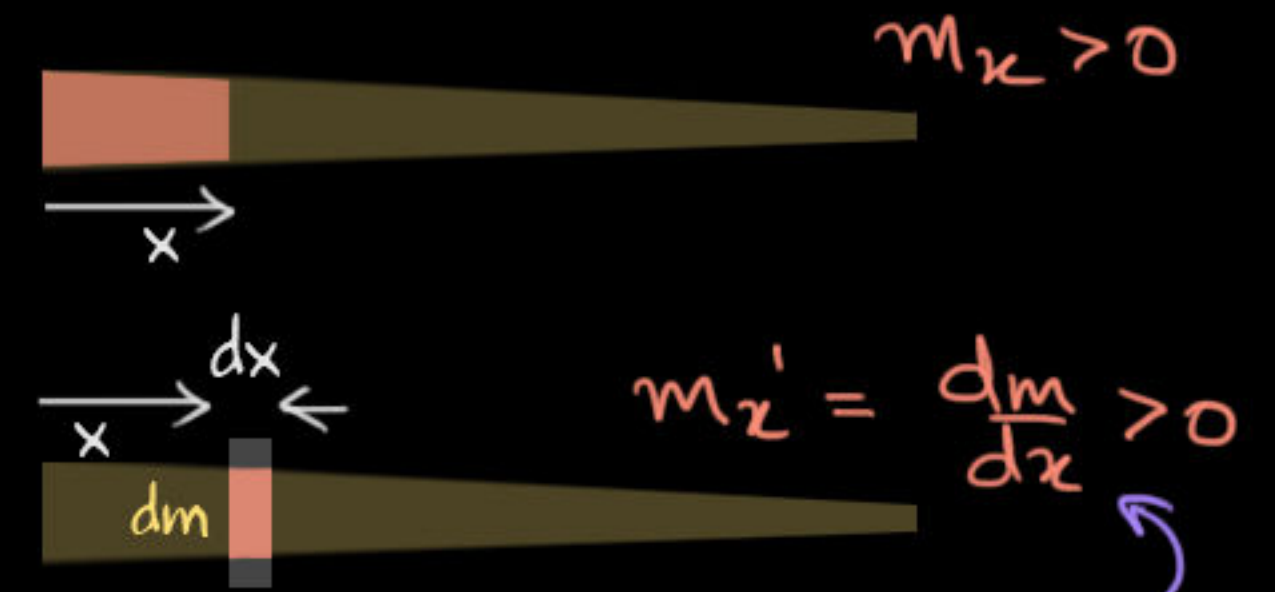
$$T'' \propto m''_x$$

Always +ve as m increases along the rod.
It's the slope of graph.

$$\downarrow \text{ -ve } \Rightarrow m''_x < 0$$

(from given graph)

by Ankit Singhvi

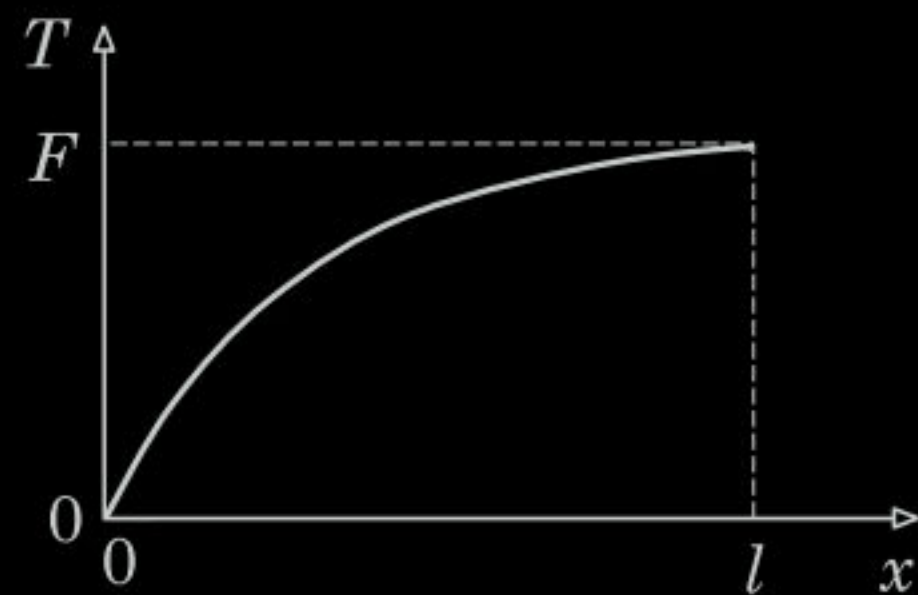


Mass is increasing with x ($m'_x > 0$), but rate of mass increase (m'_x) is reducing ($m''_x < 0$)



$$m''_x = \frac{\frac{dm_2}{dx} - \frac{dm_1}{dx}}{dx} < 0$$

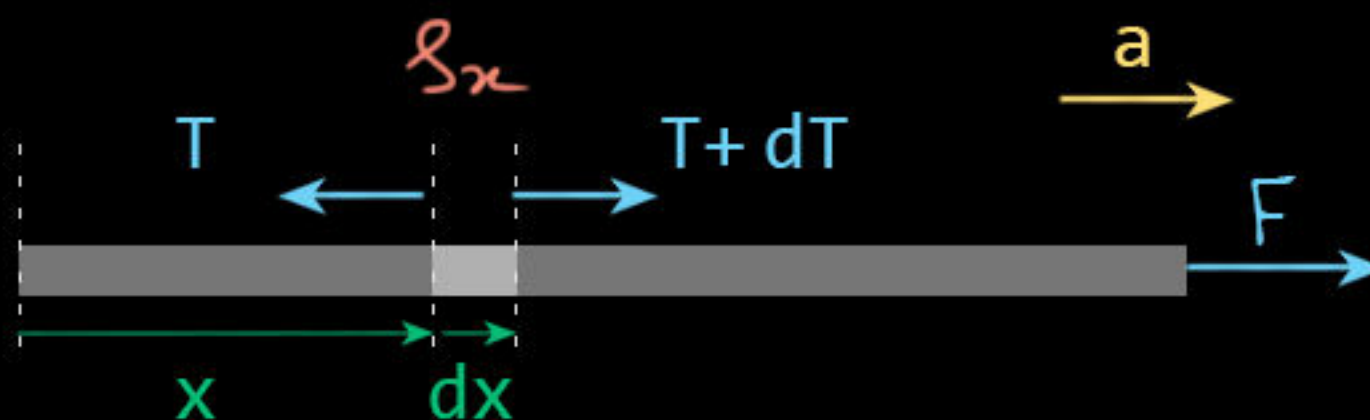
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3. A rope of length l placed straight on a frictionless horizontal floor is pulled longitudinally by a force F from one of its ends. Tensile force T developed in the rope at a distance x from the rear end varies with x as shown in the graph. What can you certainly conclude about density of the rope?

✓ (b) It decreases with distance x from the rear end.

Method 2 Relating T , x , ρ



$$F = ma$$

$$\Rightarrow dT = \rho_x (A dx) a$$

$$\Rightarrow \frac{dT}{dx} \propto \rho_x$$

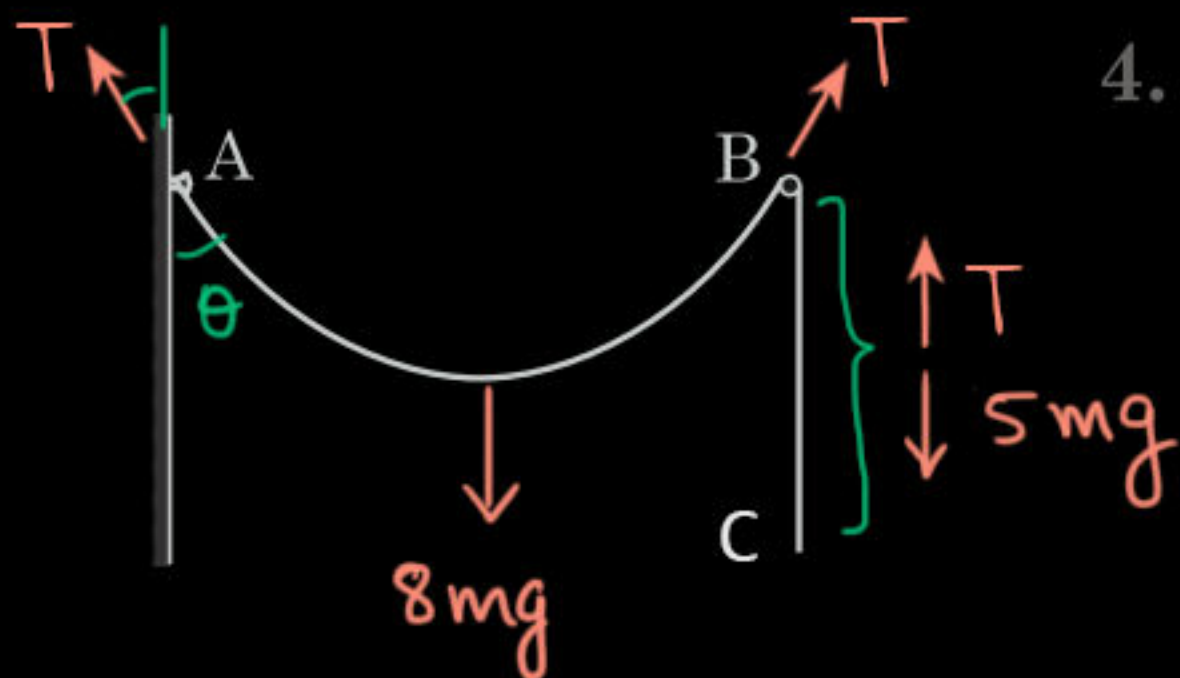
$$\Rightarrow \frac{d^2T}{dx^2} \propto \rho'_x$$

-ve (from graph)

$$\Rightarrow \rho'_x < 0$$

$$\Rightarrow \rho_x \text{ decreases with } x.$$

Youtube / Arihant Senior



4. A uniform rope of length 13 m tied on a peg A on a wall passes over a frictionless peg B fixed in level with the peg A as shown in the figure. If in equilibrium, length of the rope hanging between the pegs is 8 m, the angle which the rope makes with the wall at the peg A is closest to

(a) 30°

✓(b) 37°

(c) 45°

(d) 53°

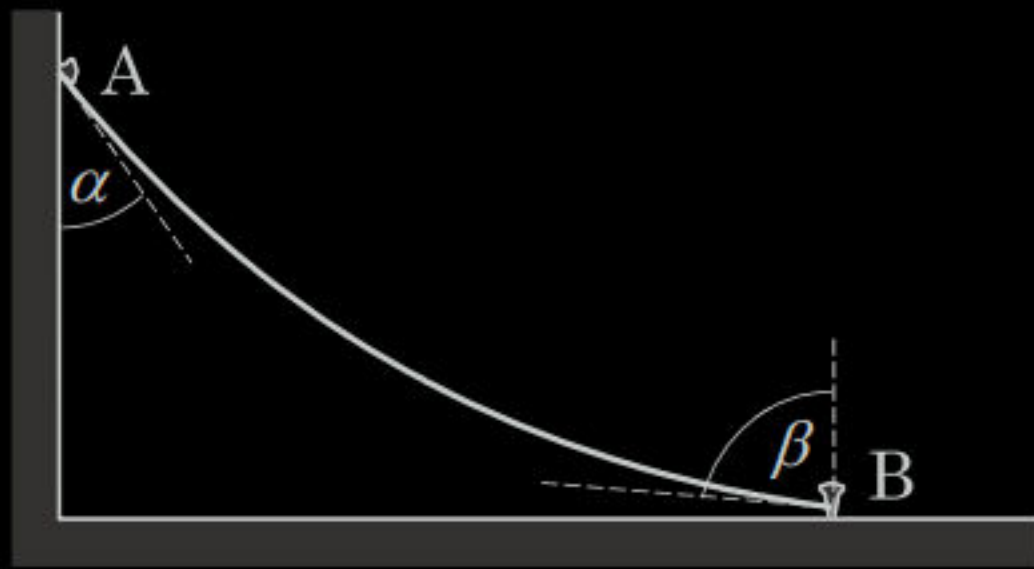
For rope hanging on right: $T = 5mg$

For rope between pegs: $2T \cos \theta = 8mg$

$$\frac{1}{2 \cos \theta} = \frac{5}{8}$$

$$\Rightarrow \cos \theta = \frac{4}{5}$$

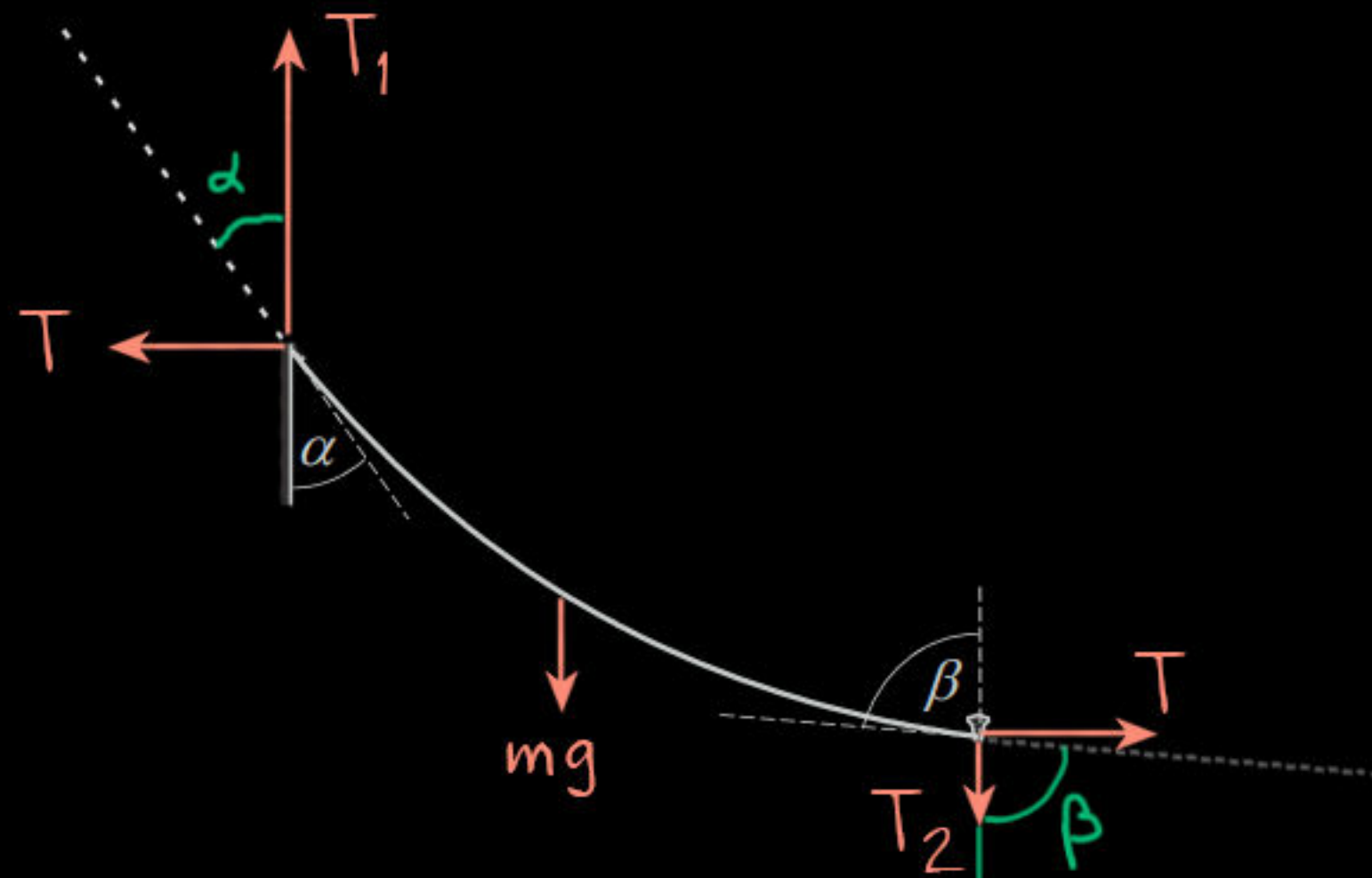
$$\Rightarrow \theta = 37^\circ //$$



5. A uniform rope is tied between a nail A on the wall and a nail B on the ground. The rope without touching the ground anywhere assumes a curved shape known as "catenary". Tangents at the ends A and B of this catenary make angles α and β with the vertical respectively. Which of the following conclusions can you make?

- ✓(a) Horizontal component of the tensile force in the rope is uniform.
- ✓(b) Vertical component of the tensile force increases with height.
- (c) Angle α can be greater than angle β .
- ✓(d) Angle α cannot assume a value of 0° .

Balancing horizontal and vertical forces, with indicative length of force vectors:



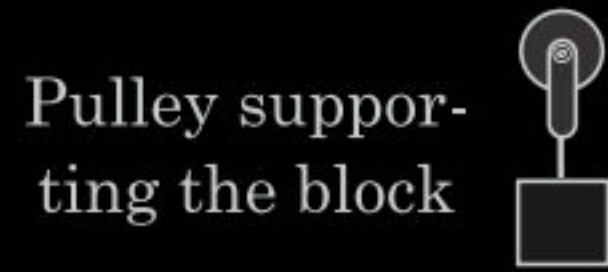
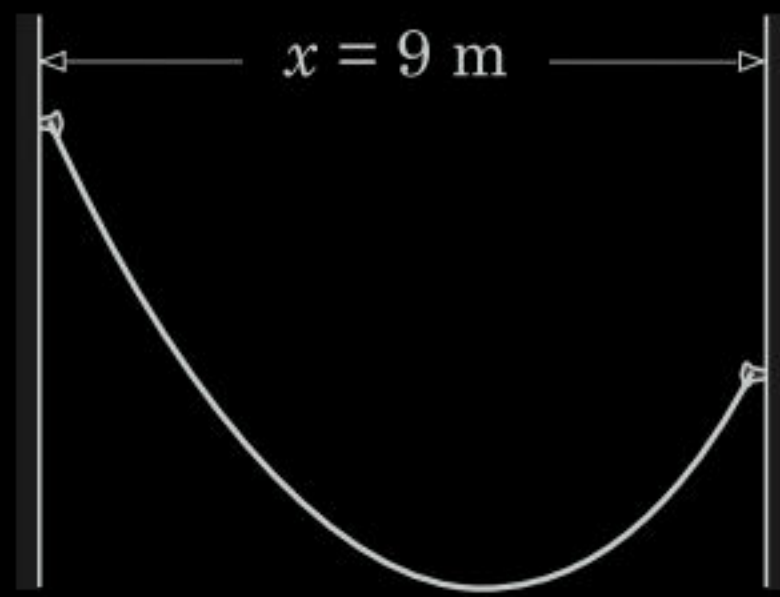
$$\tan \alpha = \frac{T}{T_1} \quad \text{and} \quad \tan \beta = \frac{T}{T_2}$$

$$\text{Also, } T_1 > T_2 \Rightarrow \alpha < \beta //$$

- ⓓ $T \neq 0$ otherwise chain will hang straight.
 $T_1 \neq 0$ to balance mg .

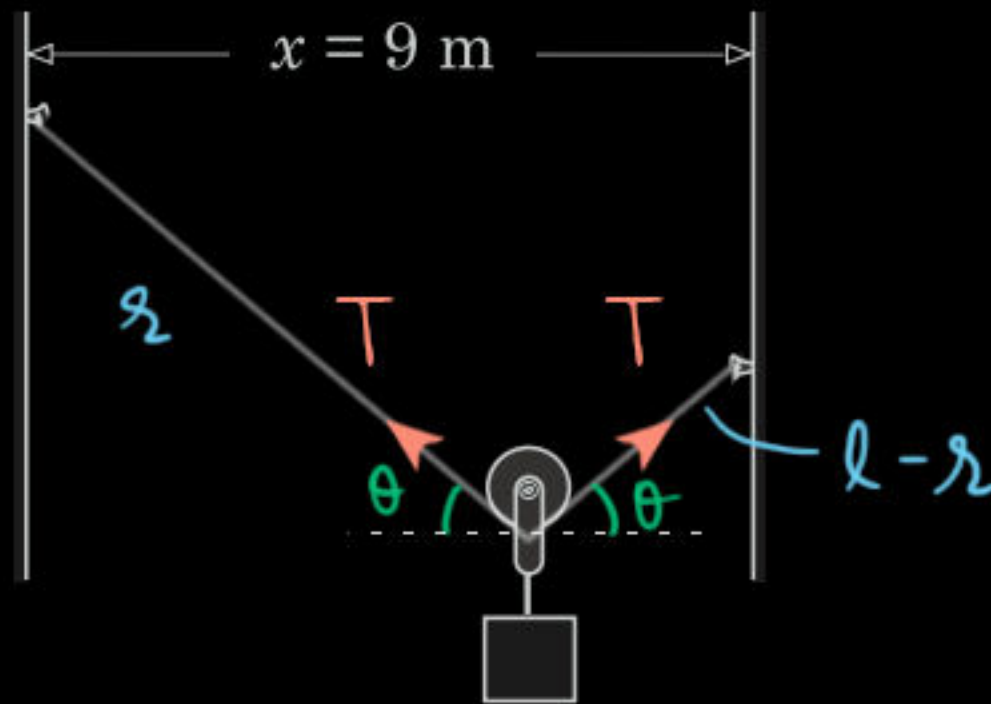
∴ If rope has to be between A and B: $\alpha \neq 0, \alpha \neq \pi/2$

However, T_2 can be zero, so β may be $\pi/2$



6. A light inextensible string of length 15 m is hanging between two pegs that are 9 m horizontally apart as shown. A small frictionless pulley of weight $W_p = 4.8 \text{ N}$ supporting a block of weight $W_b = 8.0 \text{ N}$ is gently placed on the string and allowed to move gradually to a place on the string, where it can stay in equilibrium. Radius of the pulley-wheel is negligible as compared to the length of the string. Which of the following conclusions can you make when the pulley is in equilibrium?

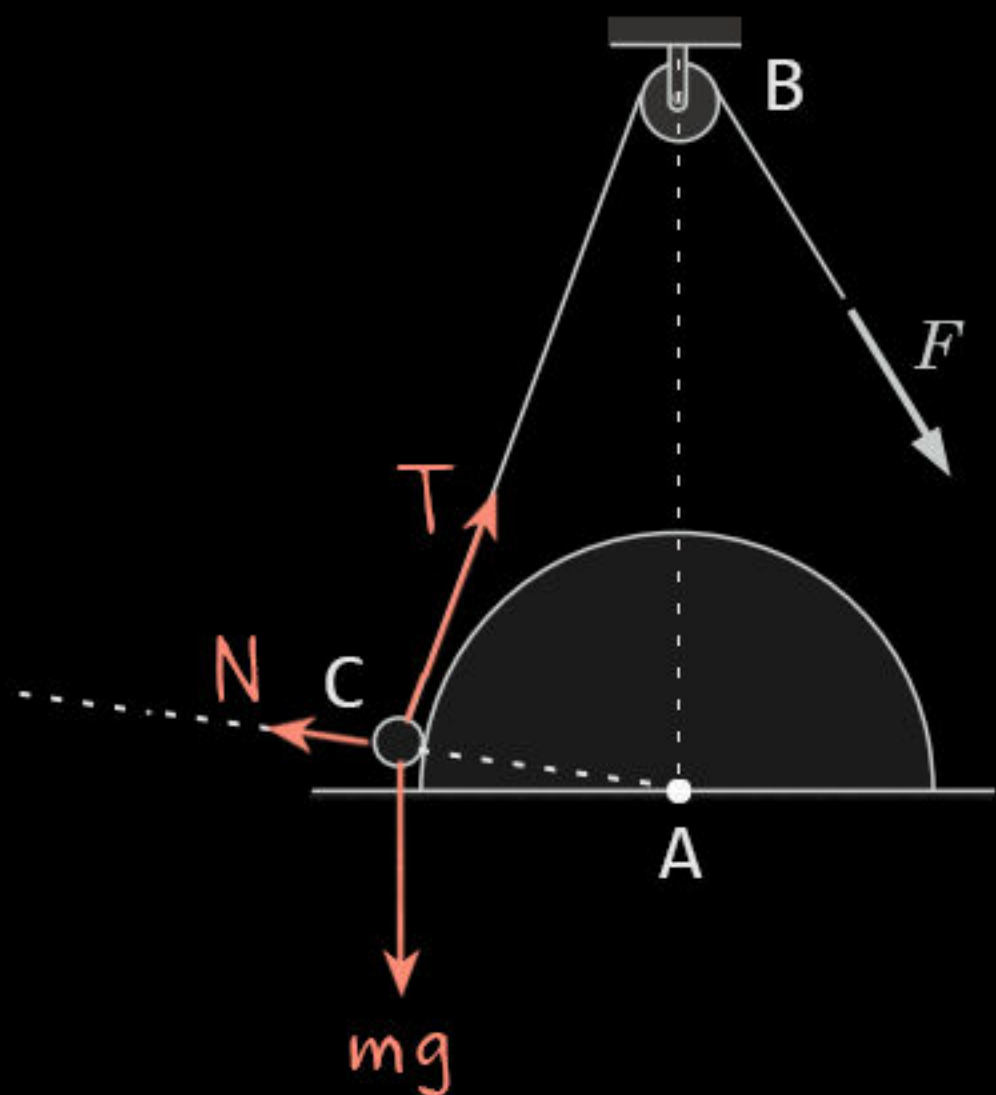
- (a) Level difference of the pegs is 4.0 m.
- (b) Tensile force in the string depends on the level difference of the pegs.
- ✓ (c) Tensile force in the string is 8.0 N irrespective of the level difference of the pegs.
- (d) Lengths of string segments on left and right sides of the pulley are 10 m and 5.0 m respectively.



$$r \cos \theta + (l - r) \cos \theta = x$$

$$\Rightarrow \cos \theta = \frac{x}{l} = \frac{9}{15} \Rightarrow \theta = 53^\circ \quad \text{Constant and independent of level difference between pegs.}$$

$$\text{C} \quad 2T \sin \theta = 12.8 \Rightarrow T = \frac{6.4}{\sin \theta} = \frac{6.4}{4/5} = 8 \text{ N} //$$

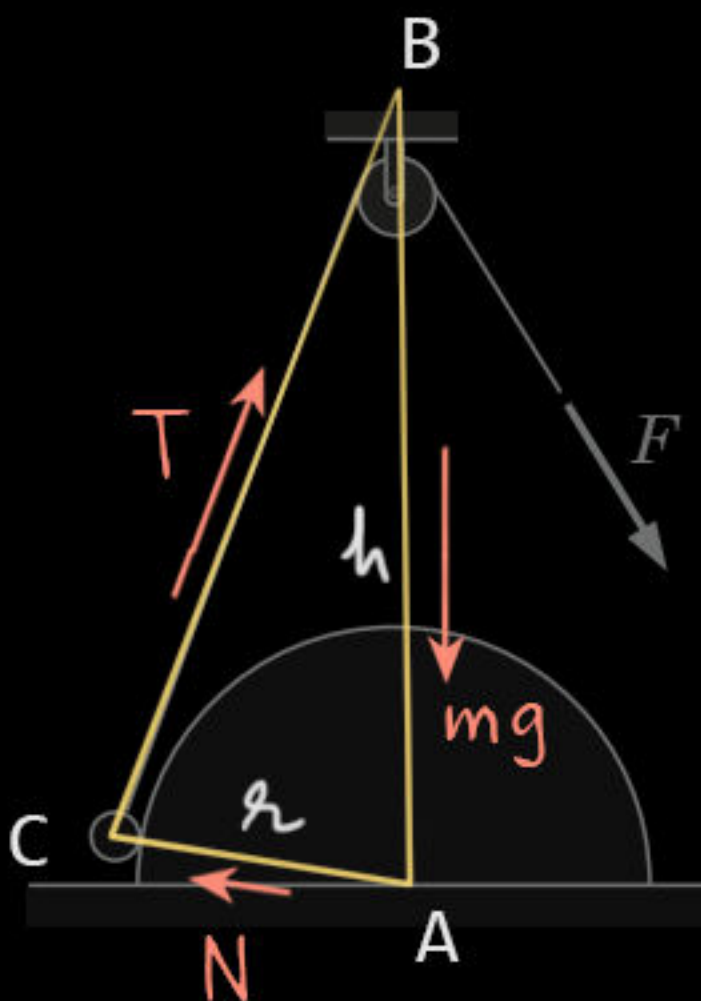


All three balanced forces form a triangle.

By AAA, this triangle is similar to geometrical triangle ABC as shown.

7. A small metal ball is being pulled gradually on a fixed frictionless hemisphere as shown in the figure. Radii of the ball and that of the pulley are much smaller than that of the hemisphere. As the ball slides from the bottom to a position close to the top of the hemisphere, how do the magnitudes of pulling force F and contact force R between the ball and the hemisphere change?

- (a) F increases and R decreases.
- (b) F decreases and R increases.
- ✓ (c) F decreases and R remains unchanged.
- (d) F remains unchanged and R decreases.

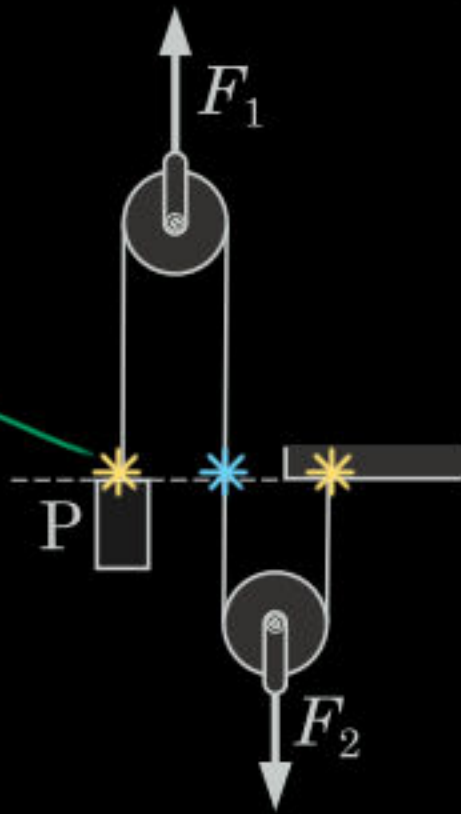


$$\therefore \frac{mg}{h} = \frac{N}{R} = \frac{T}{BC}$$

$$\Rightarrow N = \frac{mgR}{h} \rightarrow \text{Constant}$$

$$\Delta T = \frac{mg}{h} (BC) \rightarrow \text{Decreases}$$

Even though, rope is heavy, Tension at these three points is still the same.

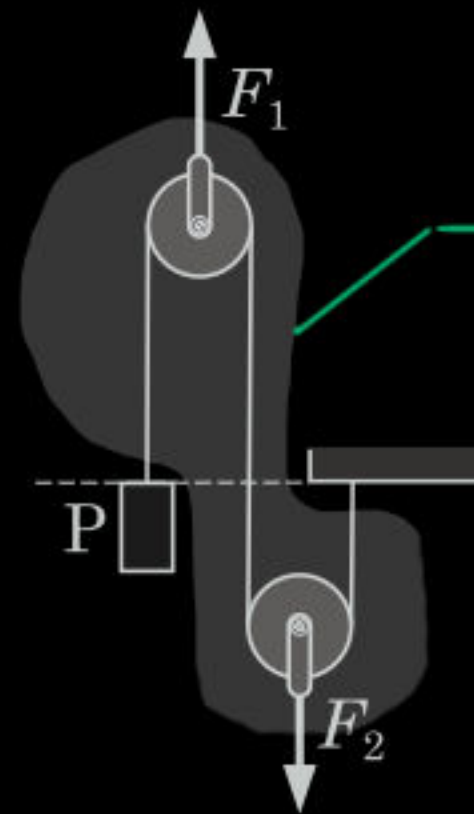
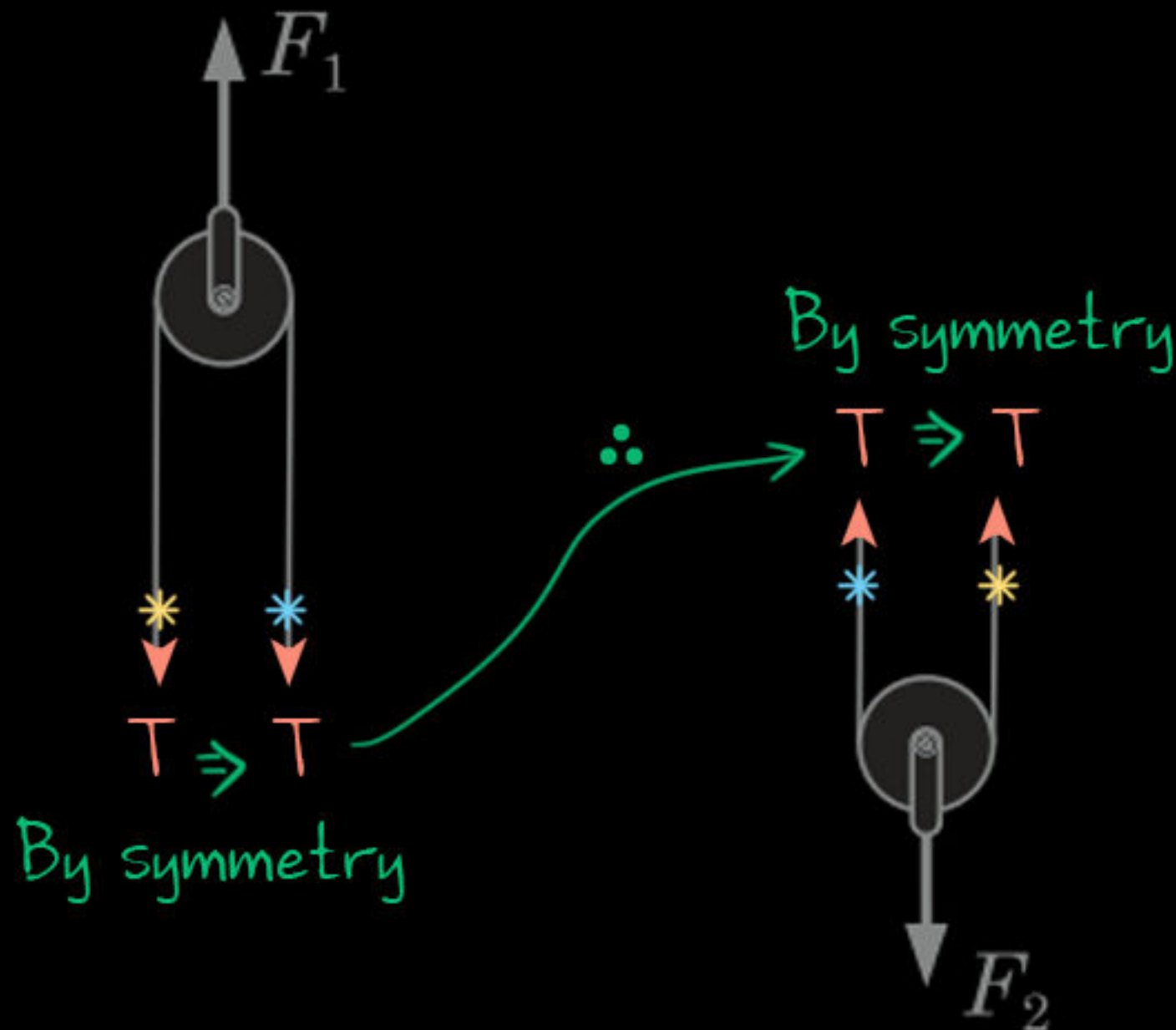


8. A block is suspended from a uniform rope that passes through two ideal pulleys. The other end of the rope is tied to a fixed support as shown in the figure. If the pulleys are pulled vertically with forces $F_1 = 110 \text{ N}$ and $F_2 = 90 \text{ N}$, the system stays at rest. Assuming linear mass density of the rope 0.25 kg/m and the acceleration of free fall 10 m/s^2 , find the length of the rope. Pulleys are not small.

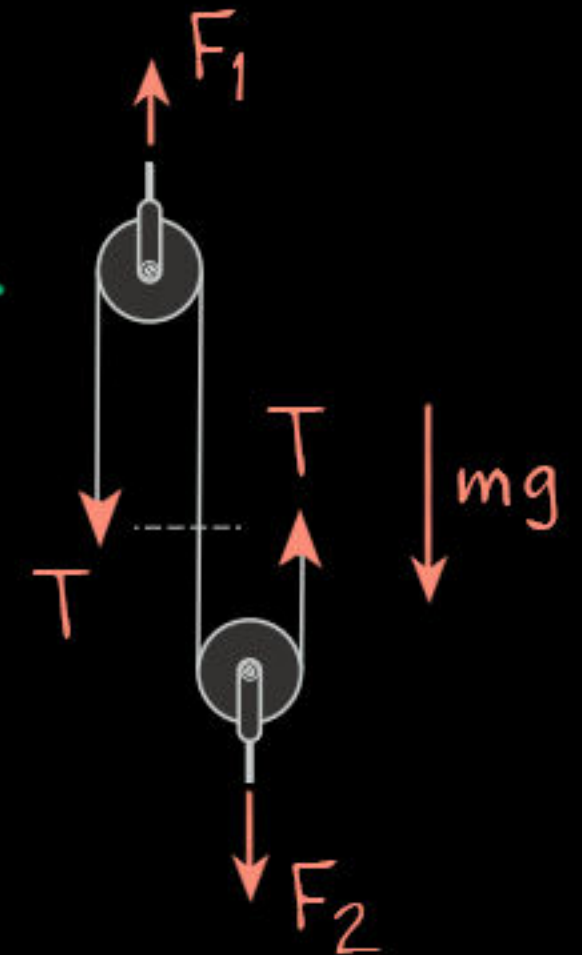
(a) 4 m
 ✓(c) 8 m

(b) 6 m
 (d) Insufficient information.

Proof:

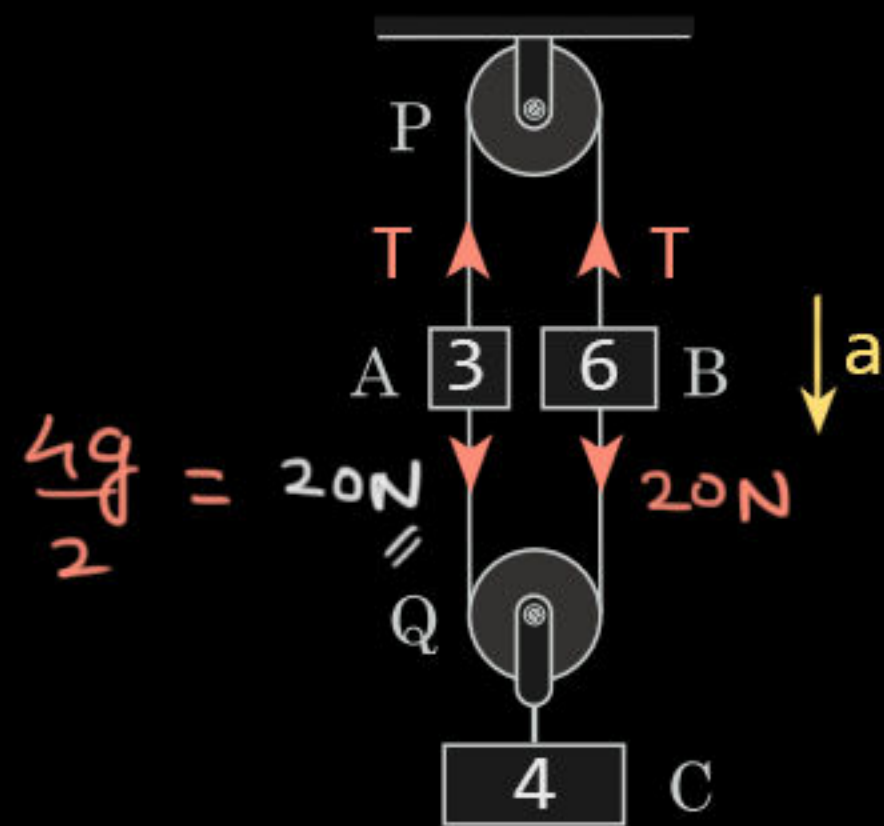


Forces on this part



$$\therefore mg = F_1 - F_2$$

$$\Rightarrow l = \frac{F_1 - F_2}{kg} = \frac{20}{(0.25)10} = 8 \text{ m}$$



System is like a belt between two pulleys. Hence, C won't move.

9. Two blocks A and B of masses 3 kg and 6 kg respectively are suspended at the ends of a light inextensible cord. The cord passes over an ideal pulley P fixed to the ceiling. Ends of another light inextensible cord are attached at the bottoms of the blocks. This cord supports another ideal pulley Q from which a block C of mass 4 kg is suspended as shown in the figure. Initially the system is held motionless and then released. Which of the following conclusions can you make? Acceleration due to gravity is 10 m/s^2 .

- (a) All the blocks remain motionless.
 ✓(b) Acceleration magnitudes of blocks A and B is $10/3 \text{ m/s}^2$ while C remains motionless.
 (c) Tensile forces in the upper and the lower cords are 40 N and 20 N respectively.
 ✓(d) Tensile forces in the upper and the lower cords are 60 N and 20 N respectively.

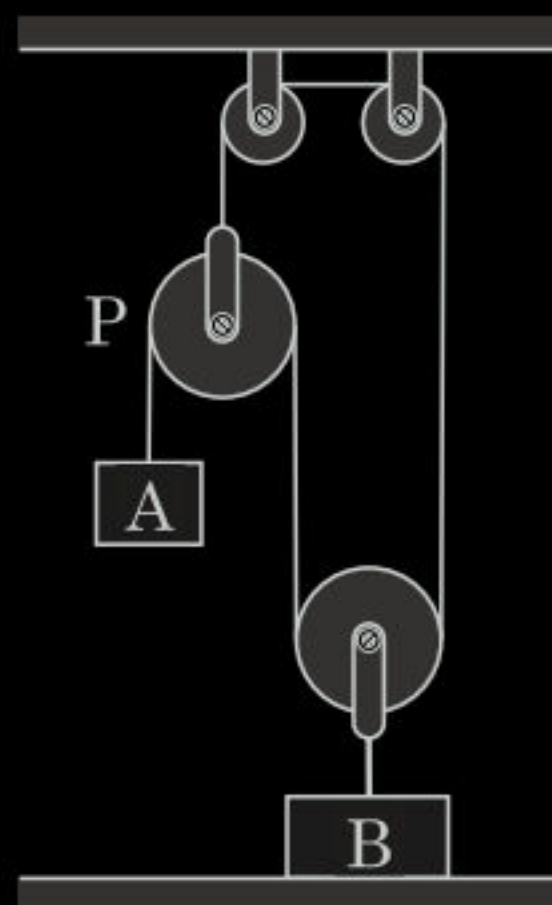


$$a = \frac{6g - 3g}{3 + 6} = g/3 //$$

$$T - 3g - 20 = 3a$$

$$\Rightarrow T = 60 \text{ N} //$$

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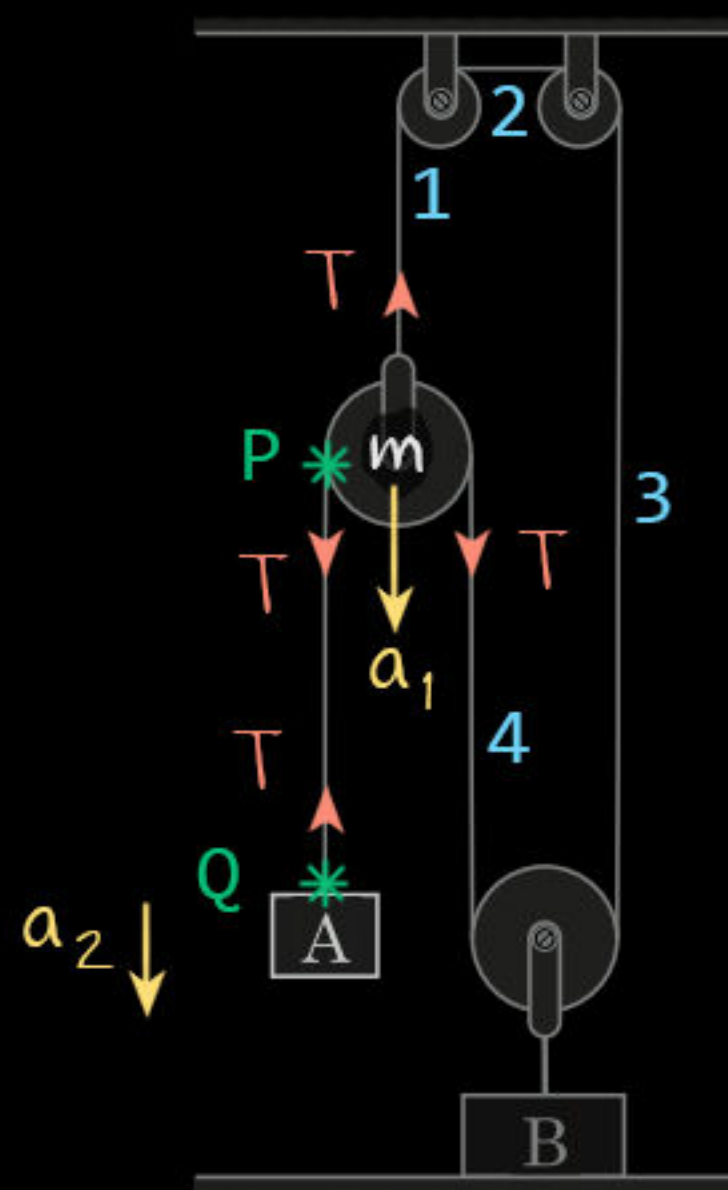


$$\therefore I = 0$$

10. In the setup shown, the string is inextensible and light, there is no friction at the axles of the pulleys but friction between the string and the pulley wheels is sufficient to prevent slipping. All the pulleys are light except the pulley P, which has whole of its mass m concentrated in its axle. Initially the block B of mass M is on the ground and the block A is held at rest. How much minimum mass should the block A have, so that after releasing it, the block B starts rising?

- ✓ (c) Whatever mass the block A has, the block B will not rise.
- ✓ (d) Pulley P moves downwards without rotation of its wheel while wheels of all the other pulleys rotate.

Torque about center of each pulley: $\tau = \vec{r} \times \vec{\alpha} = 0 = (\tau_1 - \tau_2) r \Rightarrow \tau_1 = \tau_2$
 i.e, tension is equal on both sides for all pulleys. Lets say its T .



$$a_1 = \frac{2T + mg - T}{m} = g + \frac{T}{m}$$

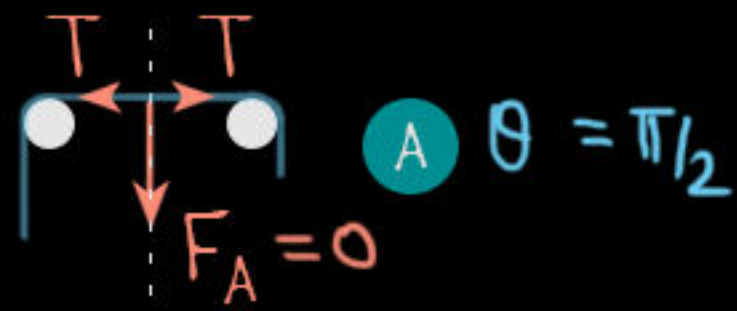
$$a_2 = \frac{m_A g - T}{m_A} = g - \frac{T}{m_A}$$

Clearly: $a_1 > a_2 \Rightarrow PQ$ will reduce with time
 $\Rightarrow$ Rope will slack (as $1+2+3+4 = \text{constant}$)
 So our assumption that T exists is wrong.

$\Rightarrow T = 0 \Rightarrow m$ and A will freefall.
 $\Rightarrow PQ$ is constant.
 $\Rightarrow m$ won't rotate, but others will.

As 1 is still being pulled down by pulley.

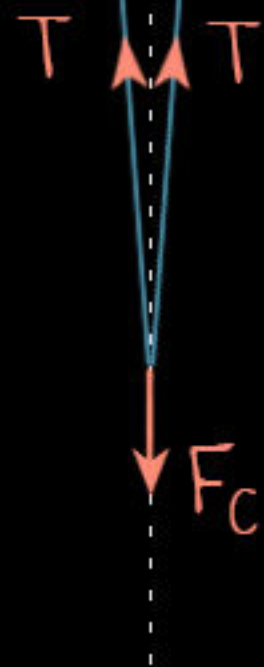
$$T = \text{constant}(mg)$$



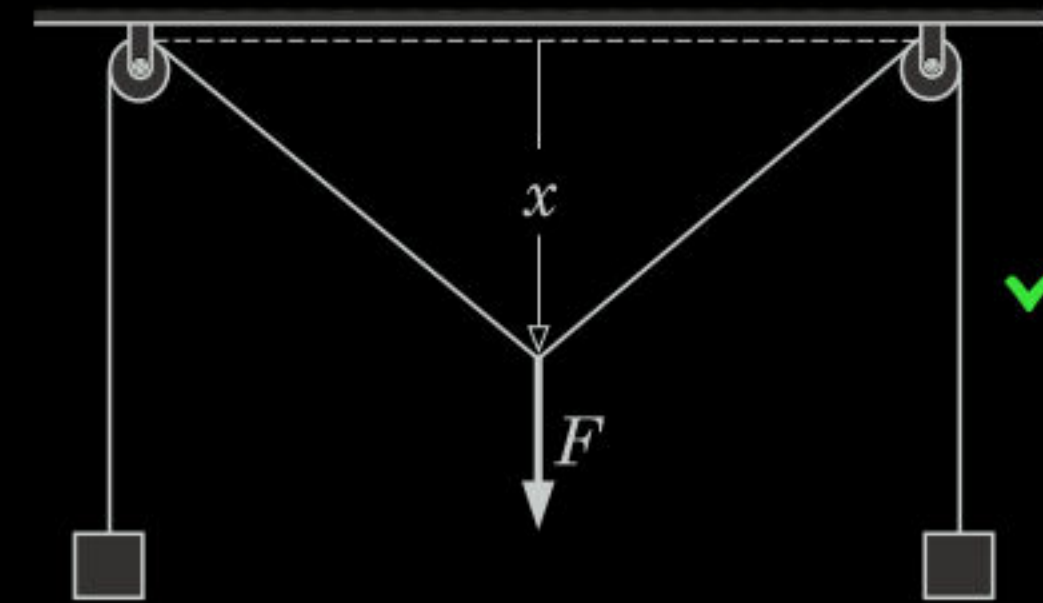
$$2T = F_C$$

$$\Rightarrow T = \frac{F_C}{2} = \frac{10}{2} = 5$$

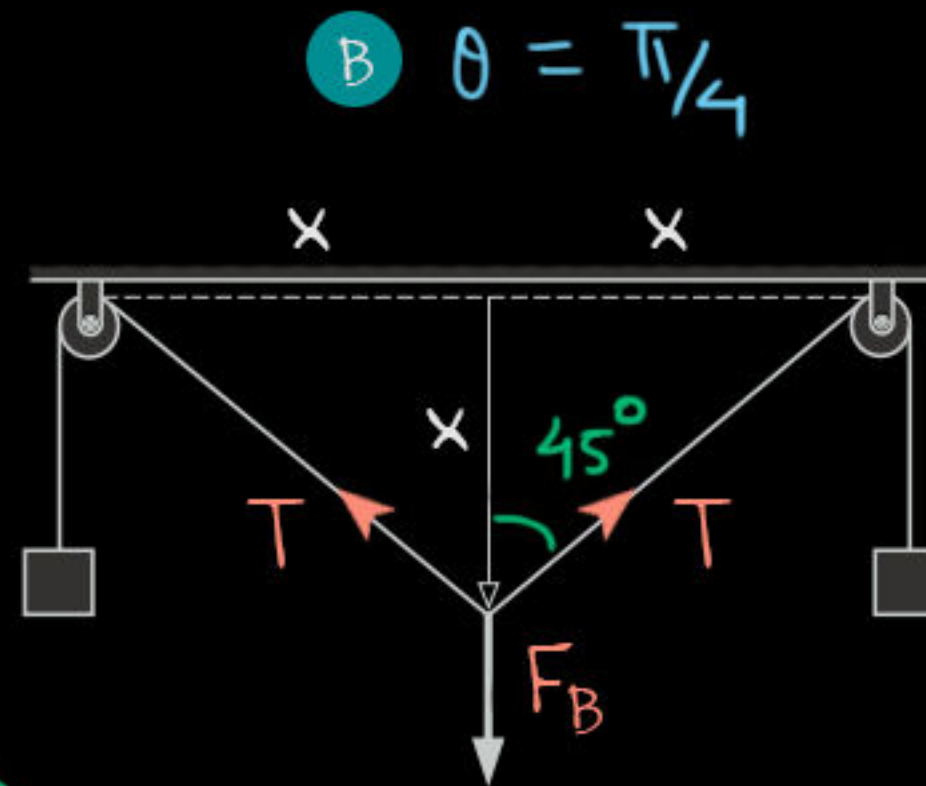
graph



11. A light inextensible cord supporting two identical loads at its ends passes over two ideal small pulleys fixed to a horizontal ceiling. If the midpoint of the cord between the pulleys is gradually pulled downwards, the pulling force F varies with displacement x of the midpoint according to the following graph. Acceleration of free fall is 10 m/s^2 . The distance between the pulleys is closest to



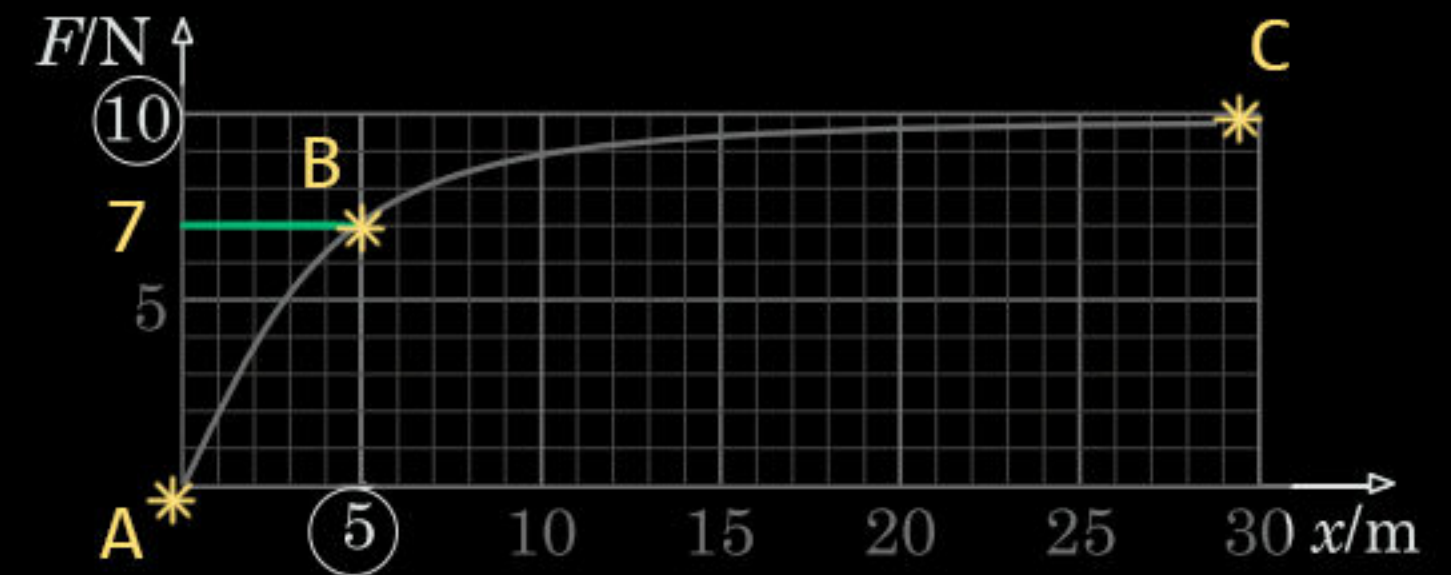
- (a) 5 m
(c) 15 m
✓ (b) 10 m
(d) 20 m



$$\therefore F_B = 2T \cos \pi/4 = 5\sqrt{2} \approx 7$$

$$\therefore \text{At B, } F_B = 7 \Rightarrow x = 5 \Rightarrow l = 2x = 10 //$$

graph
by Ankit Singhvi



Note: We solved it with 45° , but we can use any other angle too.

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Figure-1

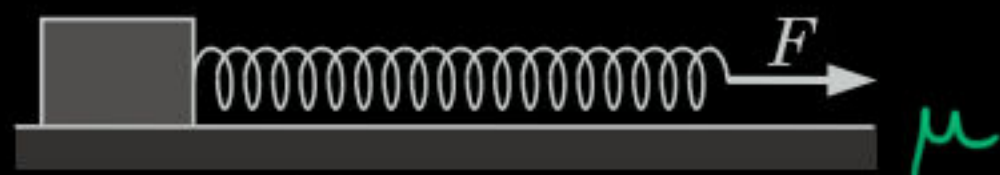


Figure-2



Figure-3

12. Consider three identical massless springs. The left ends of the first, second and the third springs are affixed to a wall, to a block of mass m placed on a horizontal floor (not frictionless) and to a block of mass m placed on a horizontal frictionless floor respectively as shown in the figures. The right end of each spring is pulled by a gradually increasing force. When magnitude of the force becomes F , extensions in these springs become x_1 , x_2 and x_3 respectively. Which of the following statements is correct?

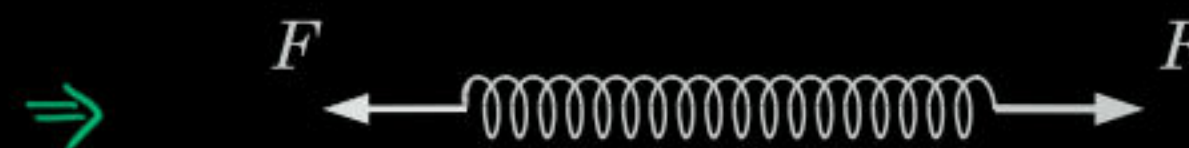
✓ (a) $x_1 = x_2 = x_3$

(b) $x_1 > x_2 > x_3$

(c) $x_1 > x_2 = x_3$

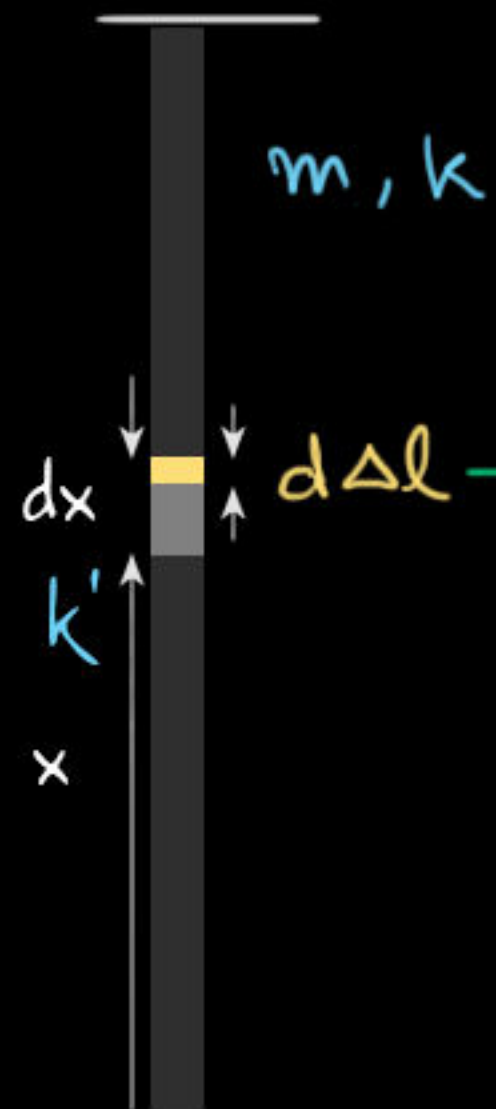
(d) $x_1 = x_2 > x_3$

Net force on each spring is zero (∵ massless)



∴ Extension for each spring will be equal and its: F/k

Extension in a heavy spring due to self weight.



13. Length of a one-metre long uniform spring of mass 50.0 g increases by 2.00 cm due to its own weight, if it is suspended from a fixed support. How much load should be suspended from the lower end of this spring, so that total extension becomes 10.0 cm.

- (a) 100 g
(c) 200 g

- (b) 125 g
(d) Insufficient information

$d\Delta l \rightarrow$ extension in dx

Let k' be spring coefficient for dx part

$$k' dx = k l$$

$$\left(\frac{x}{l} m\right) g = k' d\Delta l$$

$$\Rightarrow \frac{mg}{kl^2} \int_0^l x dx = \int_0^{\Delta l} d\Delta l$$

$$\Rightarrow \Delta l = \frac{mg}{2k}$$

Sol:

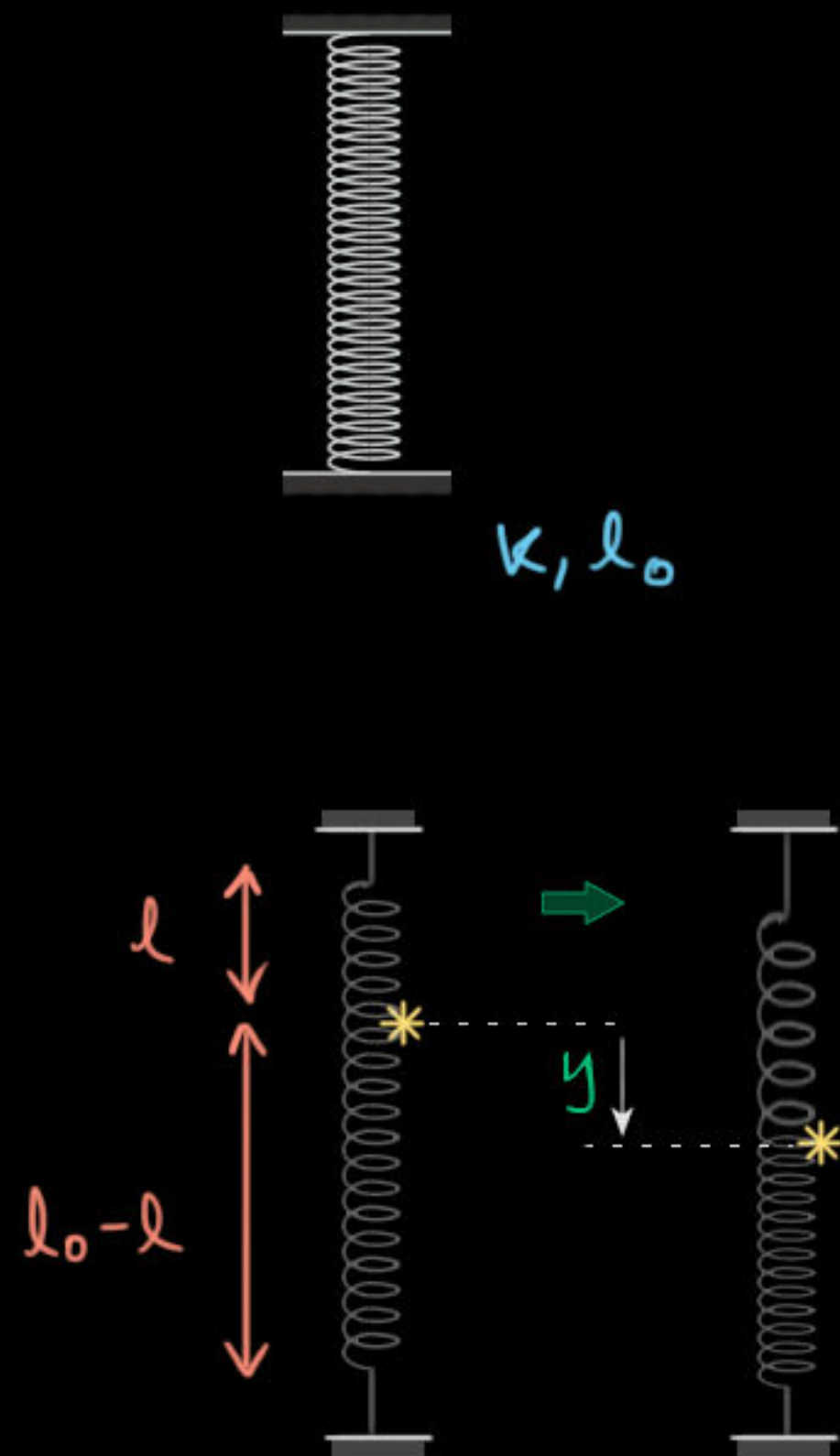
Due to self weight: $\Delta l = \frac{mg}{2k}$ (1)

Due to added weight M : $Mg = k(4\Delta l)$ (2)

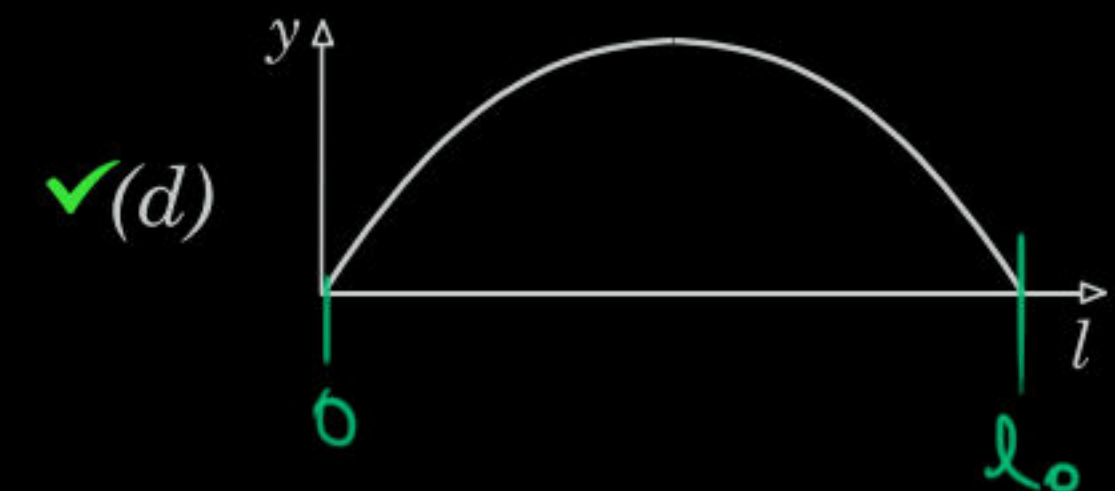
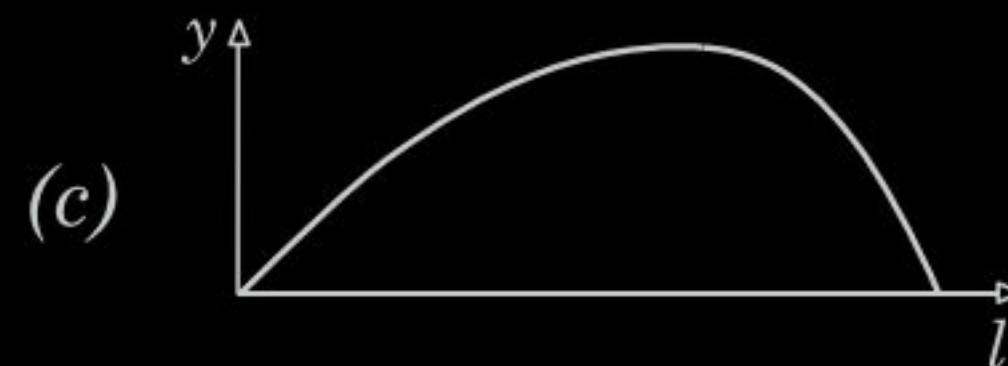
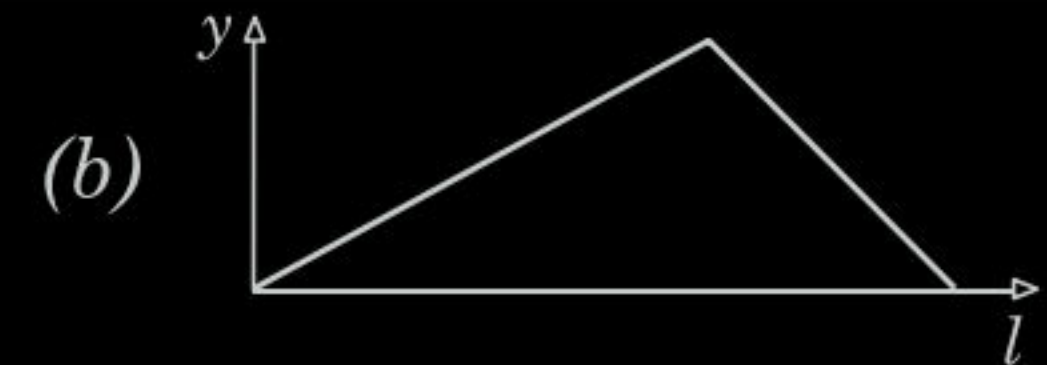
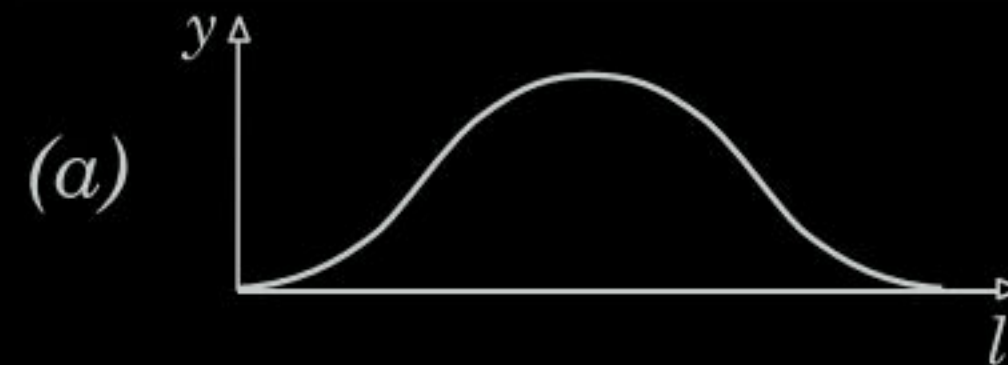
From 1,2:

$$M = 2m = 100 \text{ gm} //$$

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14. A light uniform spring is tied between the ceiling and the floor keeping the spring vertical as shown in the figure. A bead of finite mass is glued at a distance l from the upper end of the spring and then allowed to move gradually downwards. The bead shifts a distance y to come in equilibrium. This experiment is repeated for different values of l . Which of the following graph shows the best dependence of y on l ?



At equilibrium

$$mg = k_1 y + k_2 y$$

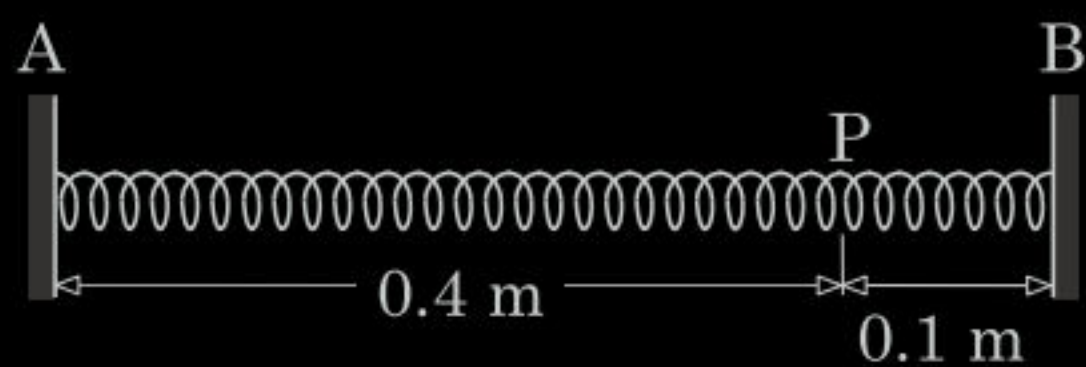
$$\Rightarrow y = \frac{mg}{k_1 + k_2} = \frac{mg}{\frac{k l_0}{l} + \frac{k l_0}{l_0 - l}} =$$

constant

$$\frac{mg}{k l_0^2} \cdot l(l_0 - l)$$

An inverted parabola
Youtube / Arihant Senior

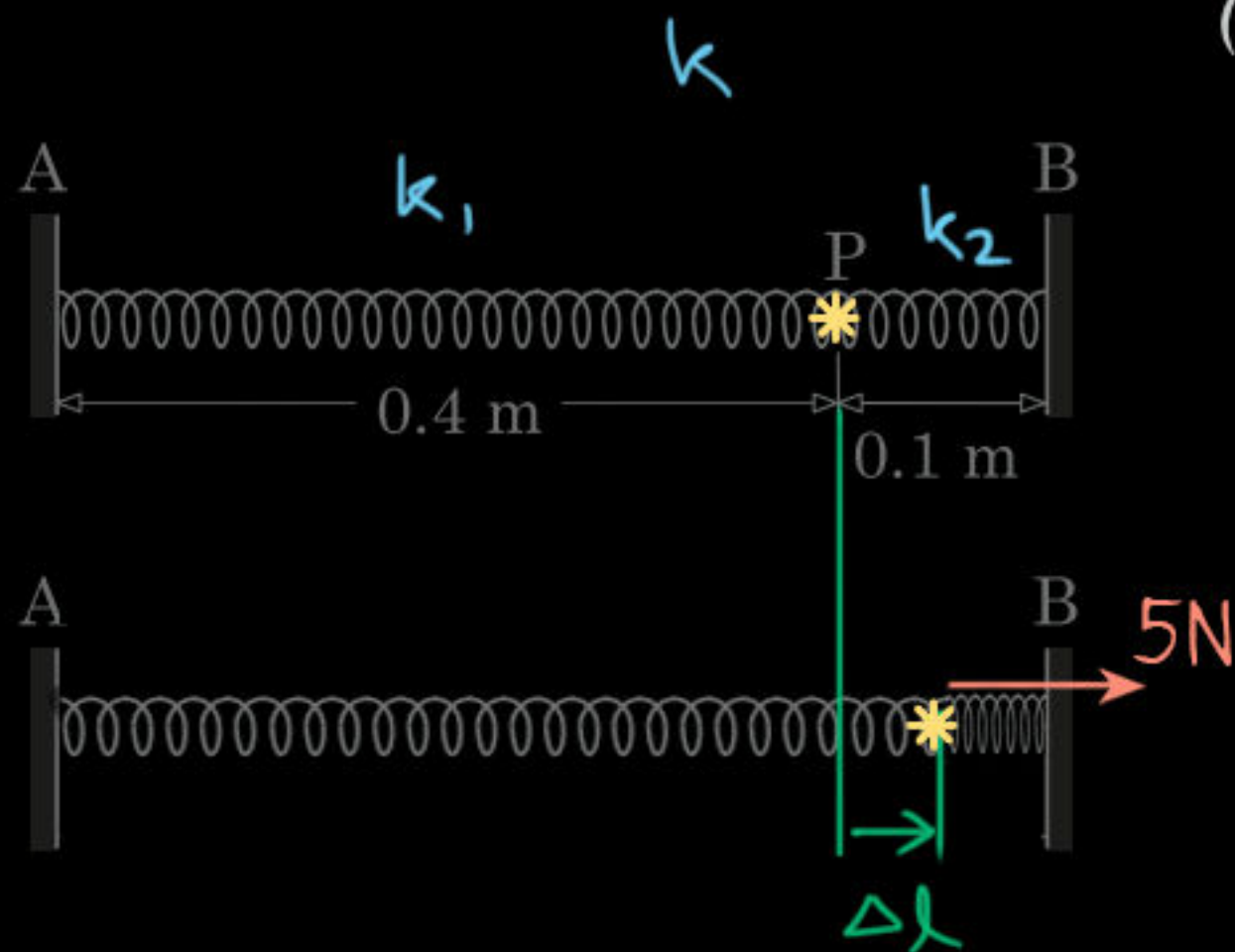
$$k l_0 = k_1 l = k_2 (l_0 - l)$$



15. A light spring of force constant 1.00 N/cm is stretched between two fixed supports A and B. A point P on the spring is pulled parallel to the spring towards the support B by a gradually increasing force. When this force becomes 5.00 N , what is magnitude of displacement of the point P?

- ✓ (a) 8.00 mm
(c) 40.0 mm

- (b) 22.2 mm
(d) Relaxed length of the spring is required.



Spring: $k(0.5) = k_1(0.4) = k_2(0.1)$ ①

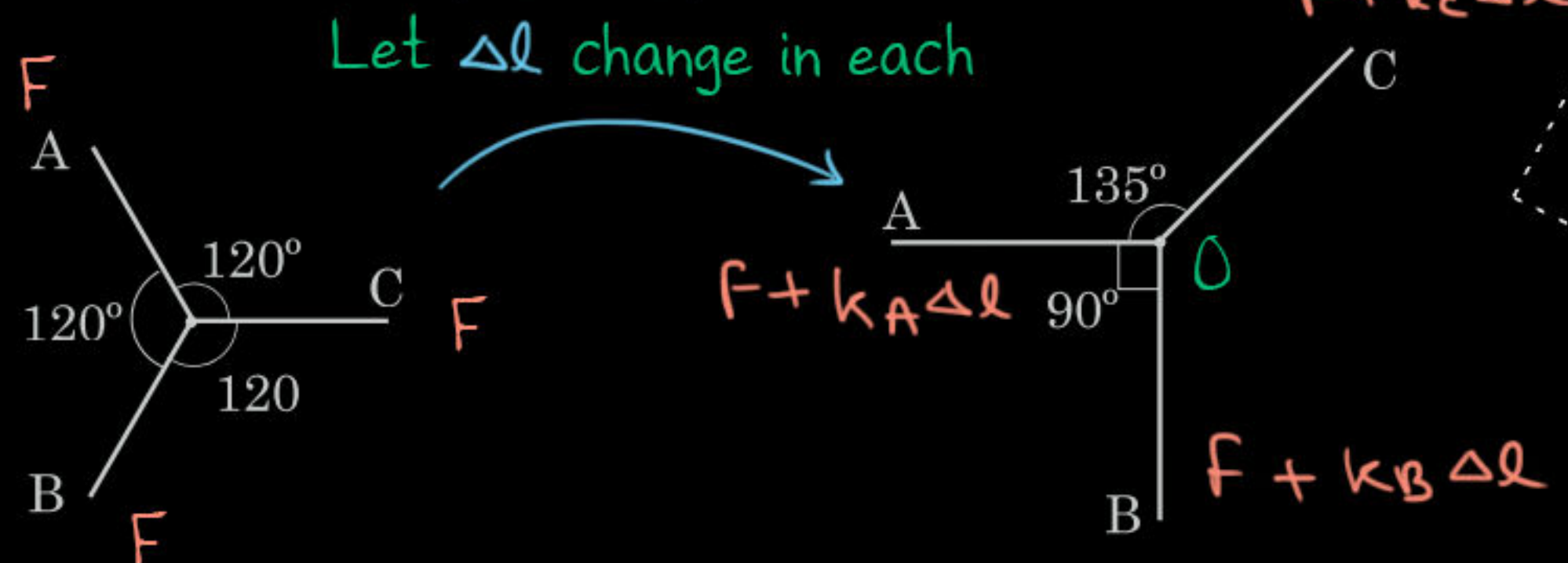
Force: $5 = k_1 \Delta l + k_2 \Delta l$

$$\Rightarrow \Delta l = \frac{5}{k_1 + k_2} \quad \text{From 1}$$

$$= \frac{5}{\frac{5k}{4} + 5k} \quad 1 \text{ N/m}$$

$$= 0.8 \text{ cm} //$$

16. Three rubber cords of force constants k_A , k_B and k_C and relaxed lengths l_A , l_B and l_C are joined with each other at one of their ends. The free ends of the cords are pulled maintaining them always in a plane. For a particular set of pulling forces, an arrangement shown in the figure-I is obtained and for another set of pulling forces, arrangement shown in the figure-II is obtained. In the arrangement of figure-I, extended length of each cord is L_1 ; and in the arrangement of figure-II, extended length of each cord is $L_2 > L_1$.



With the help of the given information, check validity of the following statements.

- ✓(a) $l_A = l_B < l_C$ and $k_A = k_B < k_C$
- (b) $l_A = l_B < l_C$ and either $k_A = k_B > k_C$ or $k_A = k_B < k_C$
- (c) $l_A = l_B < l_C$ if $k_A = k_B > k_C$ or $l_A = l_B > l_C$ if $k_A = k_B < k_C$
- (d) $l_A < l_B < l_C$ if $k_A > k_B > k_C$ or $l_A > l_B > l_C$ if $k_A < k_B < k_C$

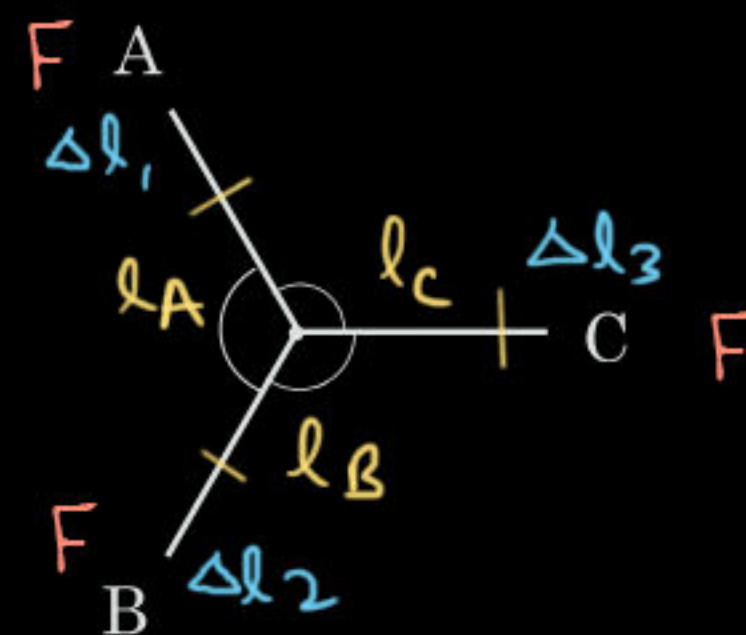
Forces perpendicular to OC:

$$\Rightarrow k_A = k_B$$

Forces along OC:

$$\frac{1}{\sqrt{2}} (F + k_A \Delta l + F + k_B \Delta l) = F + k_C \Delta l$$

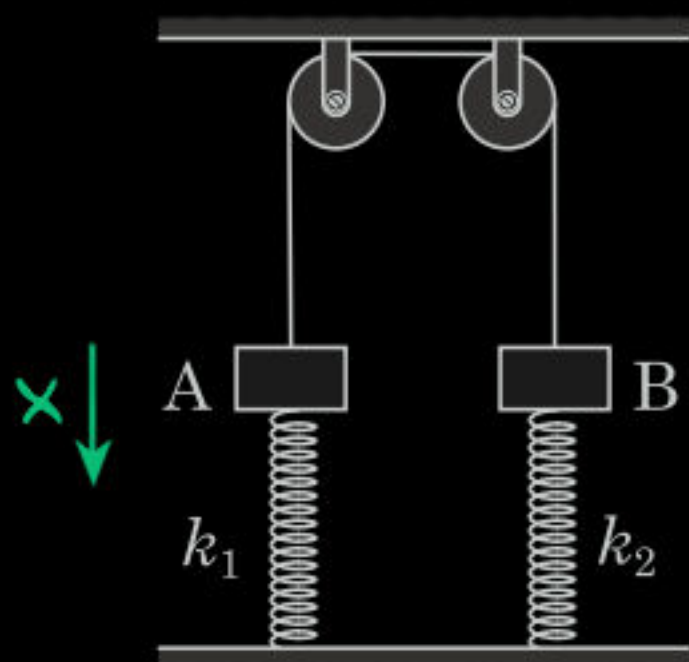
$$\Rightarrow \sqrt{2} = \frac{F + k_C \Delta l}{F + k_A \Delta l} \Rightarrow k_A = k_B < k_C \quad (1)$$



$$k_A \Delta l_1 = k_B \Delta l_2 = k_C \Delta l_3 \quad (2)$$

From 1,2: $\Delta l_1 = \Delta l_2 > \Delta l_3$

$$\Rightarrow l_A = l_B < l_C$$



17. Two blocks A and B each of mass m are connected by an ideal string that passes over two fixed ideal pulleys. The blocks are also connected with the ground by springs of force constants k_1 and k_2 ($k_1 > k_2$). When both the springs are relaxed, the block A is pulled down a distance x and released. Acceleration magnitudes of the blocks A and B immediately after the release are a_1 and a_2 respectively. Mark the correct options.

(a) $a_1 > a_2$

(b) $a_1 \leq a_2$

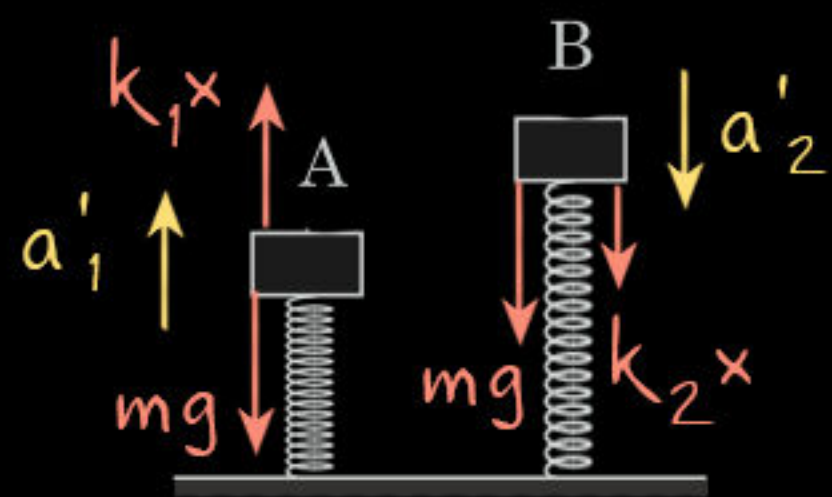
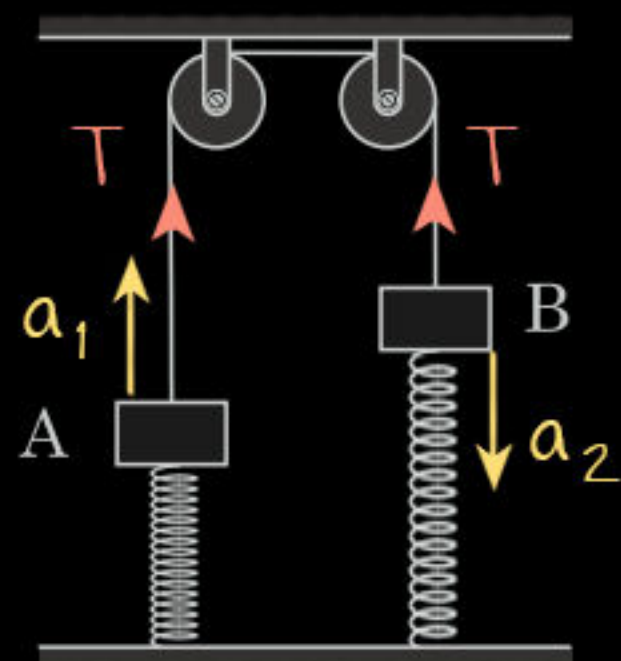
✓(c) $a_1 > a_2$ if $k_1 > k_2 + 2mg/x$

✓(d) $a_1 = a_2$ if $k_1 \leq k_2 + 2mg/x$

Its moved by x .

Lets say, accelerations are a_1 and a_2 , and tension be T .

Let a'_1 and a'_2 be accelerations without taking tension into account.



Case A $T \geq 0 \Rightarrow a_1 = a_2$

condition

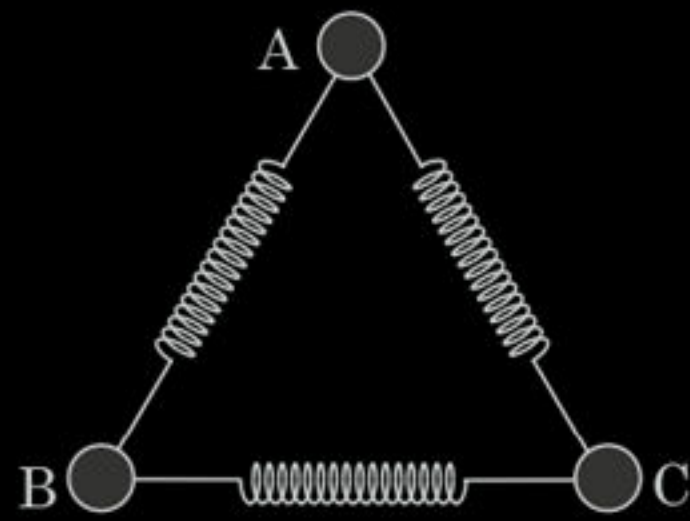
$a'_2 \geq a'_1$

i.e: $\frac{mg + k_2x}{m} \geq \frac{k_1x - mg}{m} \Rightarrow k_1 \leq k_2 + \frac{2mg}{x}$ ①

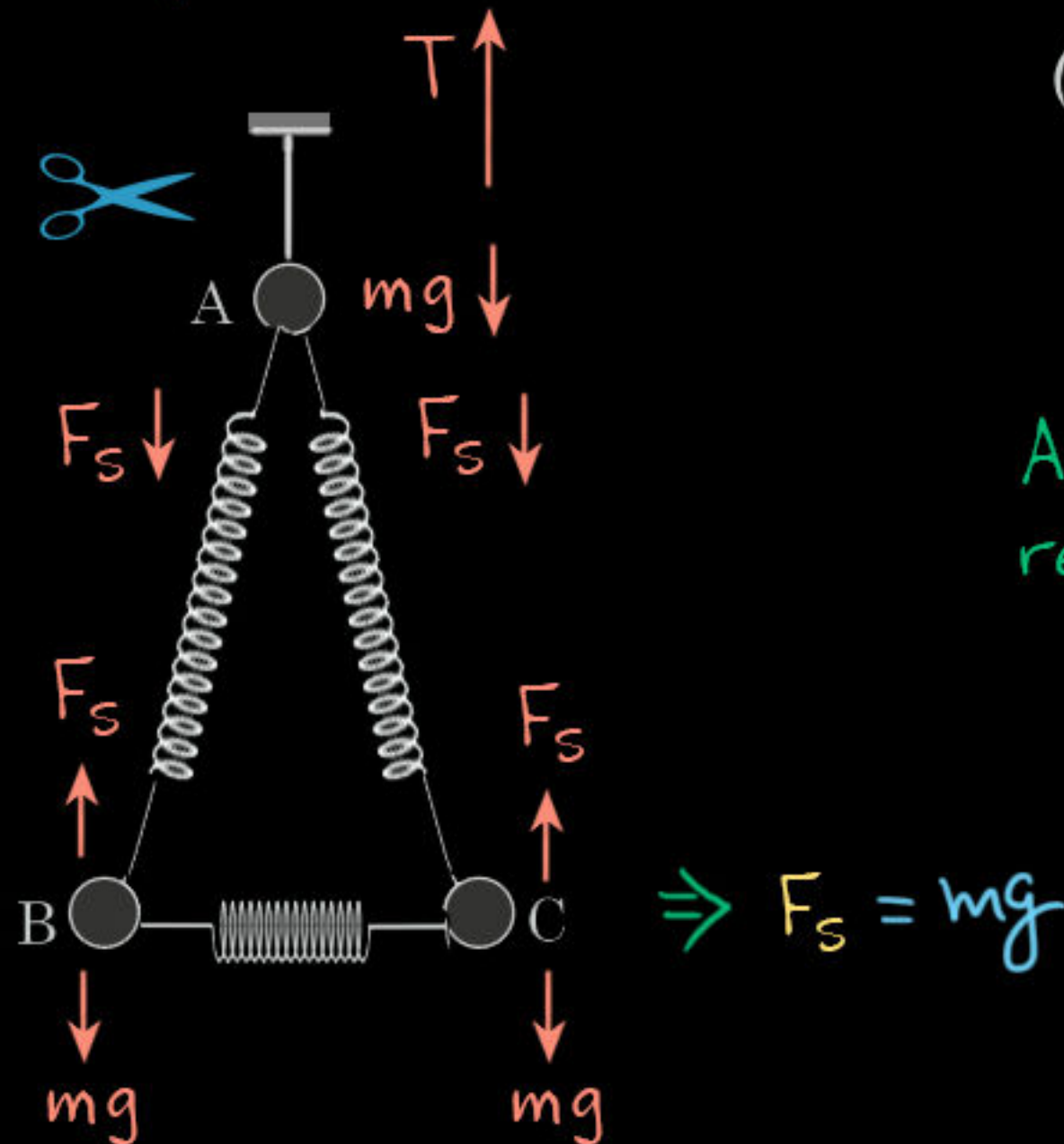
Case B String is slack $\Rightarrow a_1 > a_2$

condition

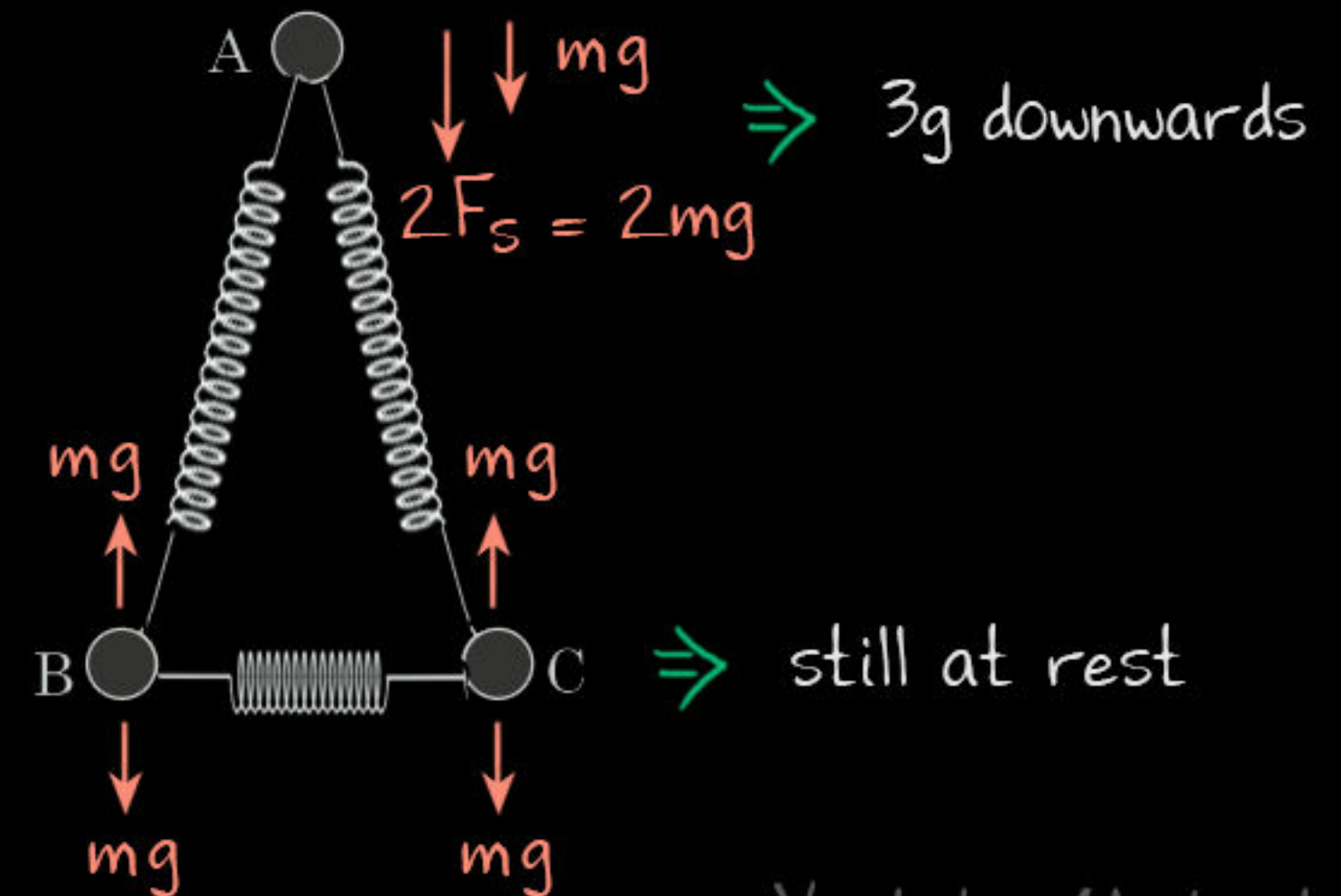
$a'_2 < a'_1 \Rightarrow k_1 > k_2 + \frac{2mg}{x}$
From 1



Horizontal forces don't change on cutting the cord, so let's forget about them and write only vertical forces on each ball.



After cutting, only T is gone, rest of the forces are same



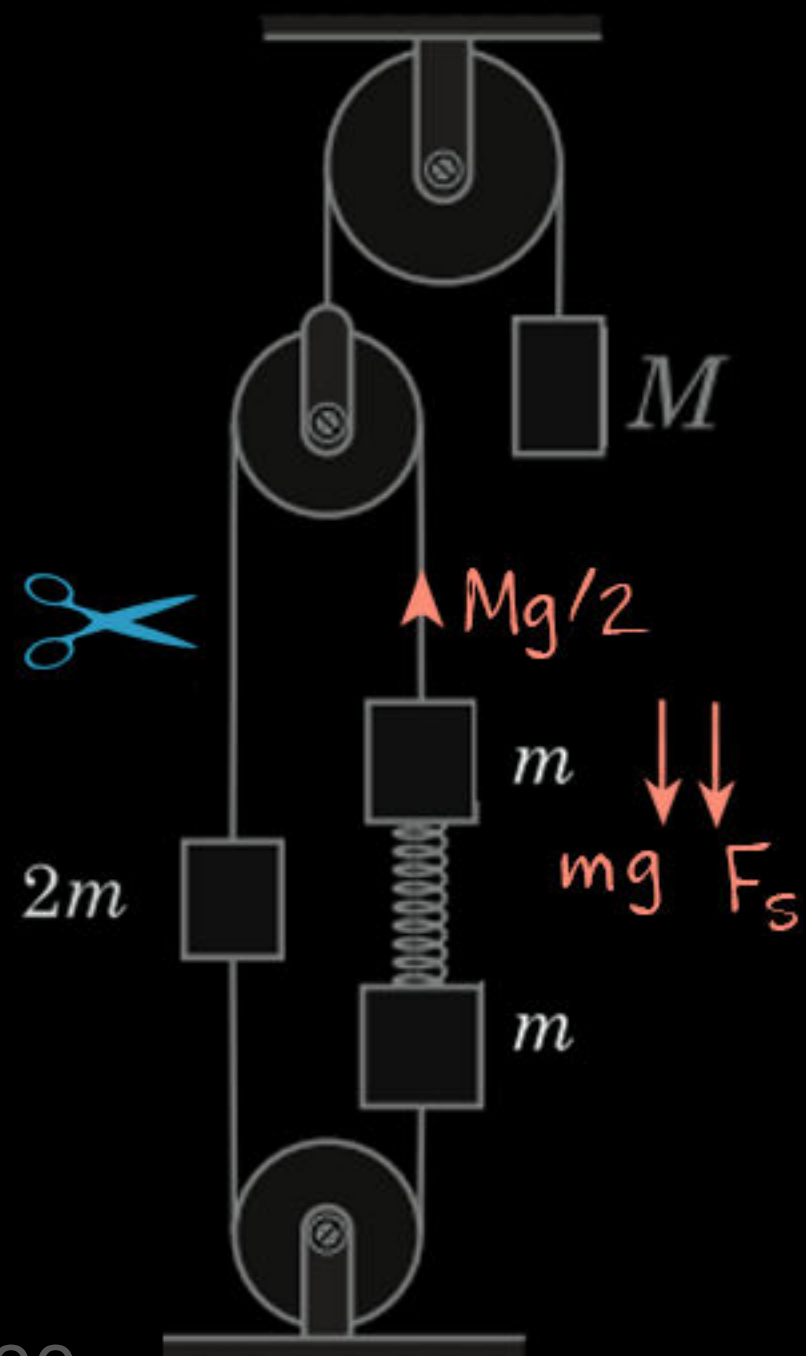
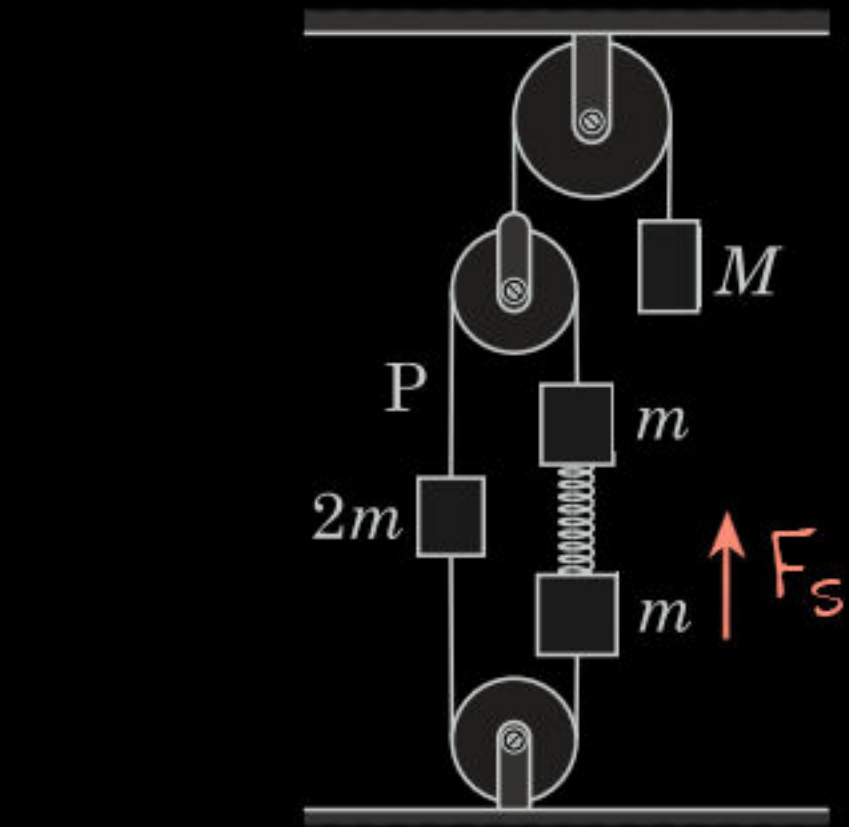
18. Three identical balls A, B and C each of mass m connected by three identical springs when placed on a frictionless horizontal floor occupy corners of an equilateral triangle as shown in the figure. This assembly is suspended by attaching the ball A to the ceiling with the help of a light thread. What can you predict regarding accelerations of the balls immediately after the thread is cut? Acceleration of free fall is g .

- (a) All the balls will have downward acceleration g .
- (b) All the balls have acceleration greater than g in different directions.
- ✓(c) Acceleration of the upper ball is $3g$ downwards and the accelerations of the lower balls are vanishingly small.
- (d) Acceleration of the upper ball is $3g$ downwards and the lower balls move apart with finite accelerations.

19. In the setup shown, the pulleys, the cords and the spring are ideal and masses of the loads are indicated in the figure. Initially the system is in equilibrium. What should be the range of mass M so that acceleration of the load of mass $2m$ becomes greater than acceleration of free fall immediately after the cord is cut at point P?

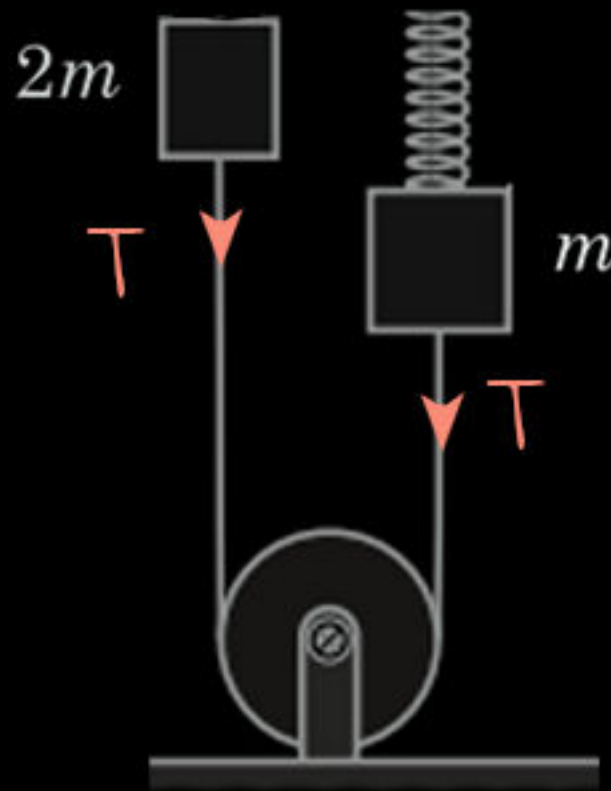
- (a) $M > 4m$
(c) $M > 8m$

- ✓(b) $M > 6m$
(d) $M > 14m$



@ equilibrium
(before cutting)

$$F_s = \frac{Mg}{2} - mg$$



Once the cord is cut, for $2m$ to accelerate more than g , T must be positive



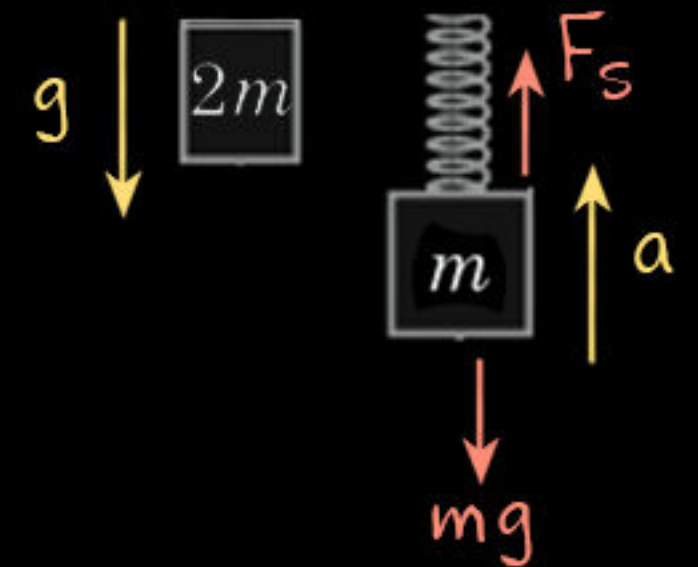
W/O taking T into account: $a > g$

$$F_s - mg = ma > mg$$

$$F_s > 2mg$$

$$\Rightarrow \frac{Mg}{2} - mg > 2mg$$

$$\Rightarrow M > 6m //$$

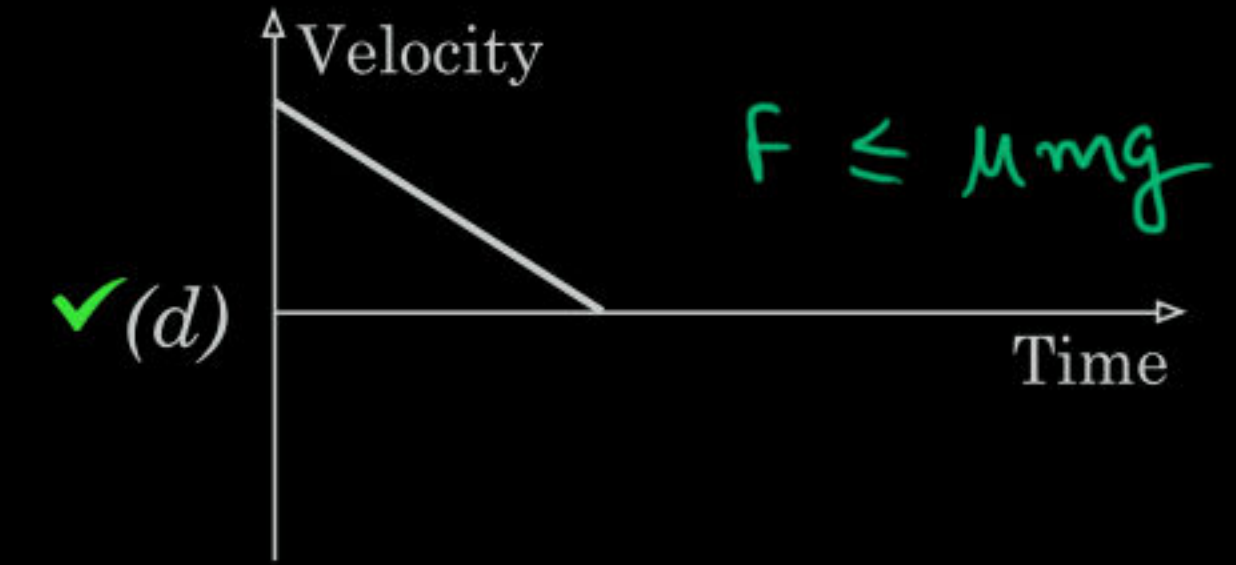
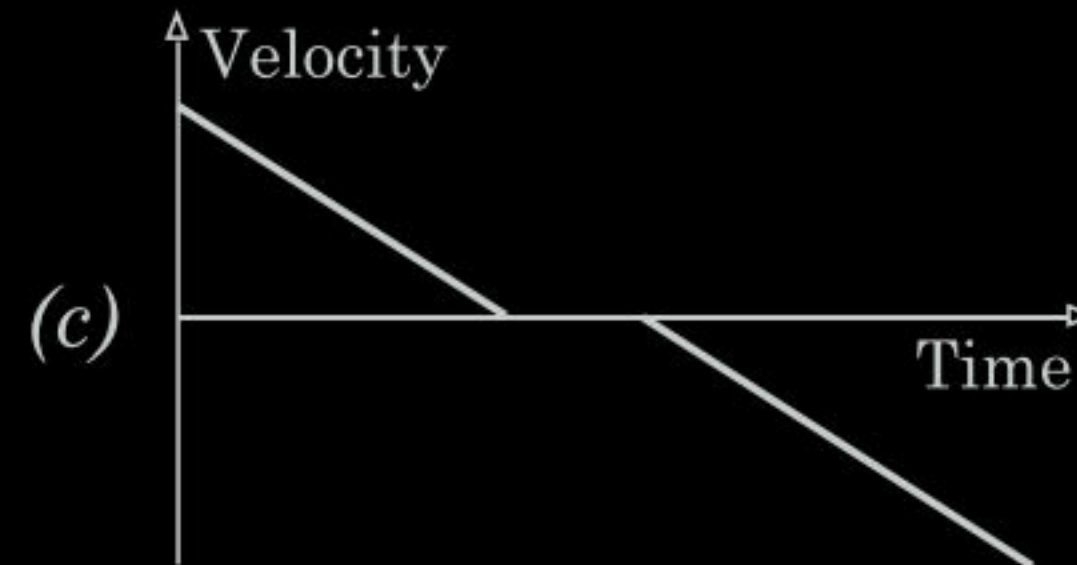
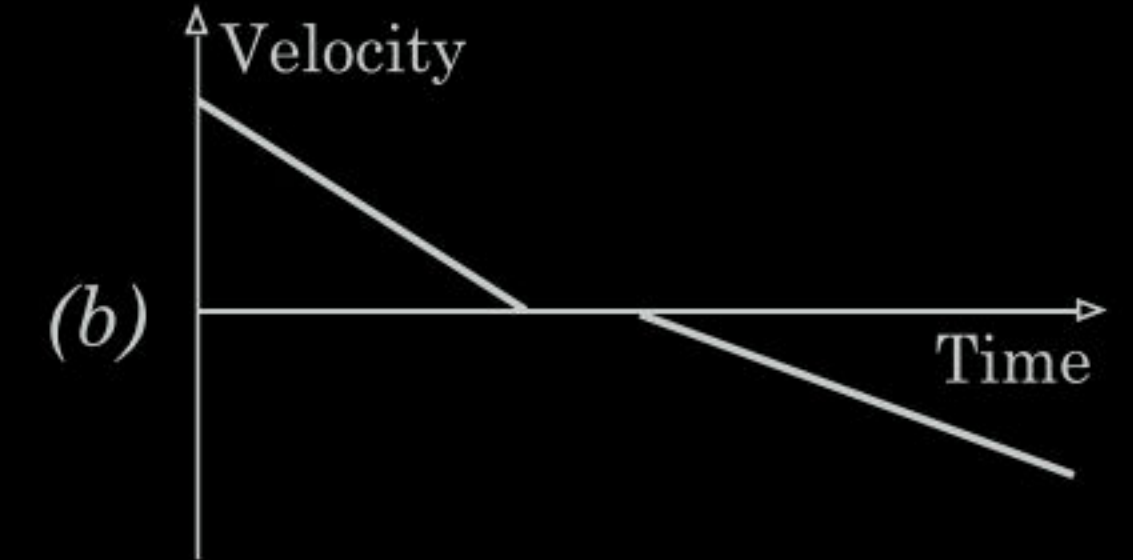
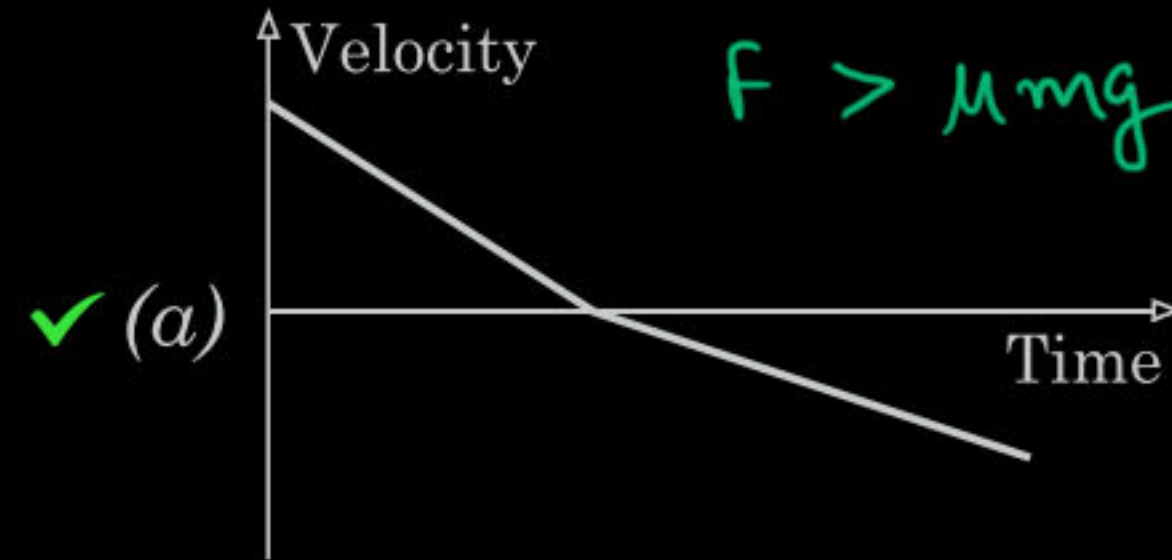
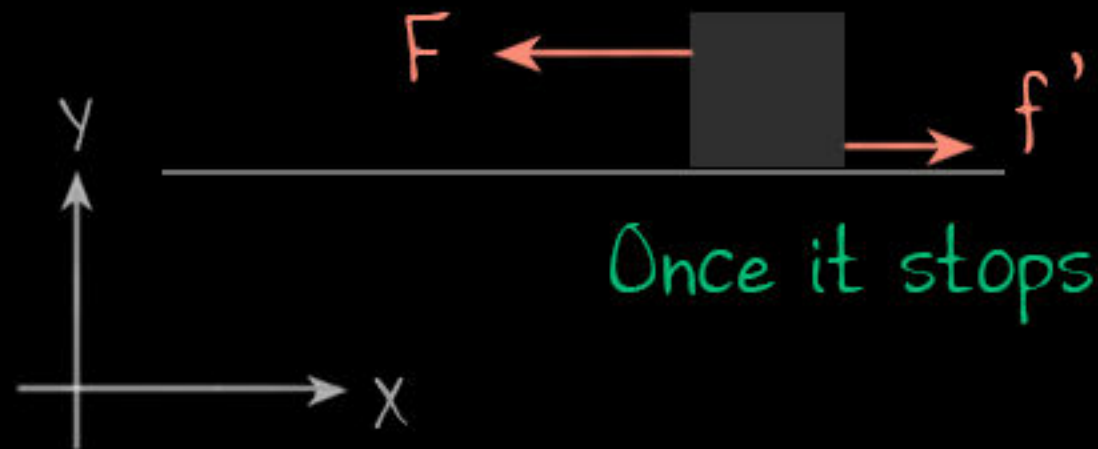


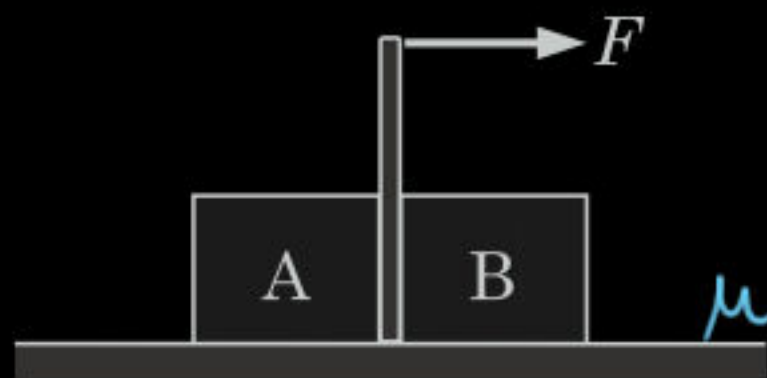
20. While a block is sliding on a horizontal floor, a constant horizontal force is applied on it opposite to its direction of motion. Which of the following can be a correct velocity-time graph for the ensuing motion of the block?

More acceleration $\Rightarrow$ More slope



Less acceleration $\Rightarrow$ Less slope

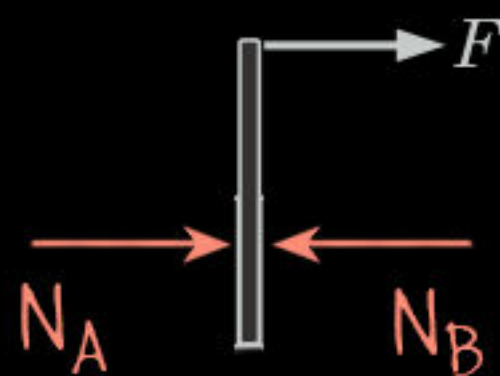




21. A rod is inserted between two identical blocks A and B placed close to each other on a horizontal floor, which is not frictionless. If the upper end of the rod is pulled horizontally by a gradually increasing force F , which of the blocks will start sliding first?

- (a) Block A
- ✓ (b) Block B
- (c) Both will start sliding simultaneously.
- (d) It is a matter of chance and hence either one may start sliding first.

Let the normal reaction on rod be N_A and N_B .



When a block just moves, acceleration of rod will be zero. So, net force on it must be zero too.

$$\Rightarrow F + N_A = N_B$$

$$\Rightarrow N_A < N_B$$

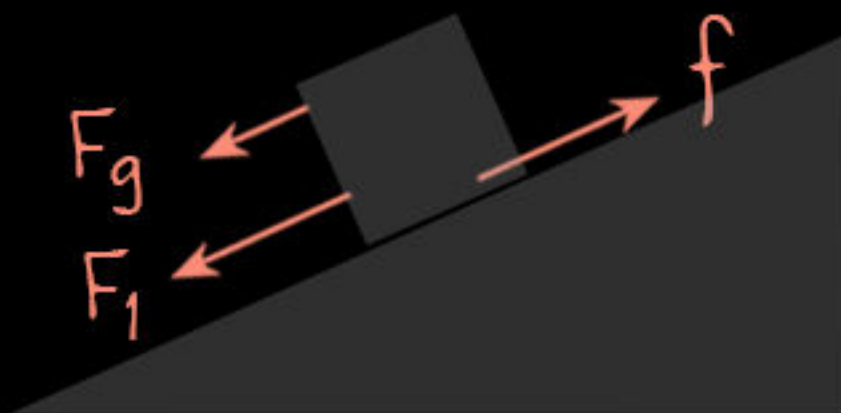
$$\Rightarrow \text{B will move first.}$$



22. A force of 20 N along the line of the greatest slope is required to slide a block upwards with constant speed and a force of 8 N along the line of the greatest slope is required to slide the block downwards with constant speed on an inclined plane. The force of kinetic friction between the block and the plane is

(a) 8 N
 ✓(c) 14 N

(b) 12 N
 (d) None of these

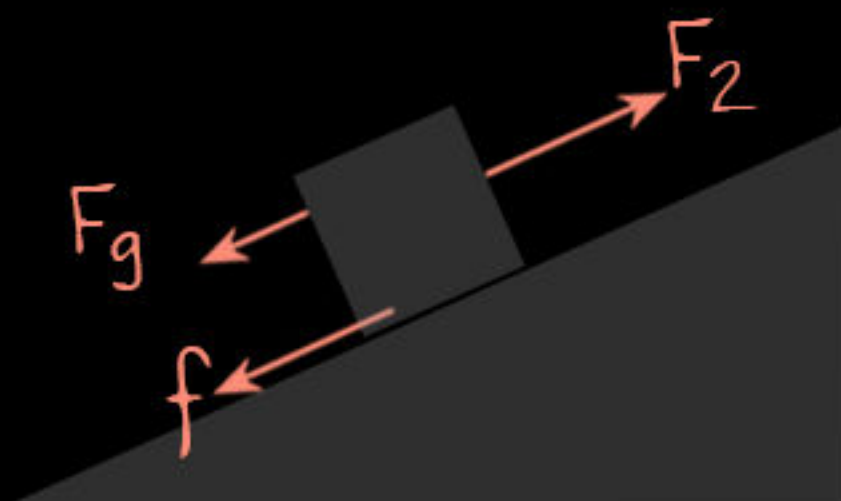


$$F_g + f_1 = f$$

$$F_g + f = f_2$$

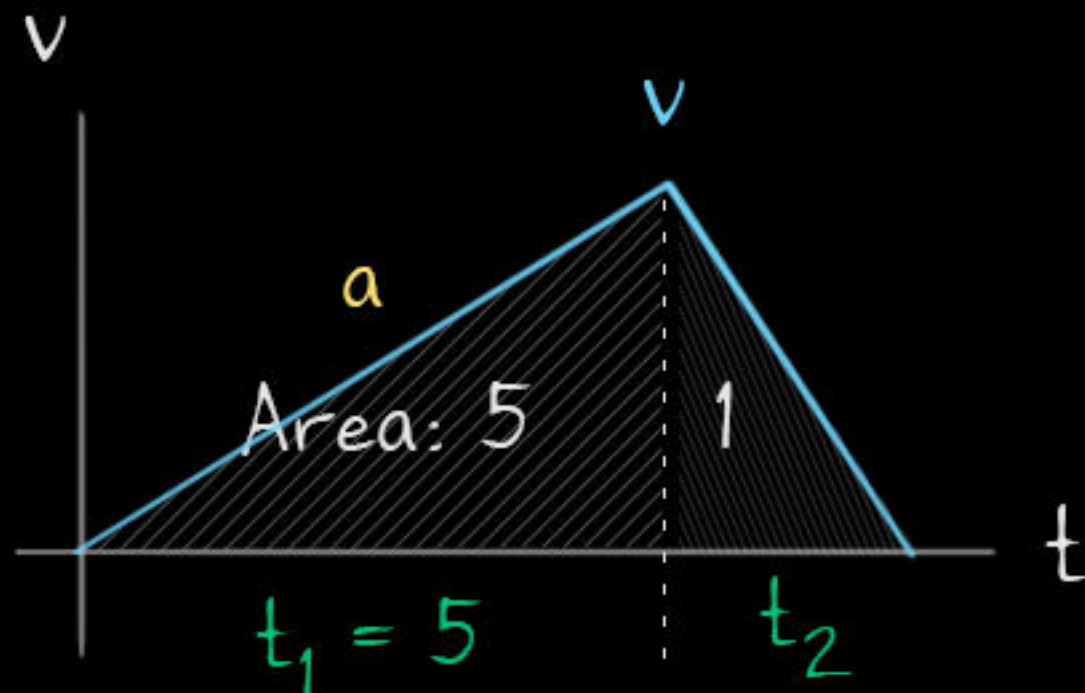
Subtract: $f_1 + f_2 = 2f$

$$\Rightarrow f = \frac{f_1 + f_2}{2} = 14 \text{ N}$$



23. A block of mass 5 kg rests on a horizontal floor. When a constant horizontal force is applied on the block for a time interval 5 s, the block slides on the floor a distance 5 m under the action of the force and after the removal of the force, it further slides a distance 1 m before coming to a stop. Acceleration due to gravity is 10 m/s^2 .

- ✓(a) Magnitude of the applied force is 12 N.
- ✓(b) The maximum speed acquired by the block is 2 m/s.
- ✓(c) Coefficient of friction between the block and the floor is 0.2.
- ✓(d) The block continues to move for 1 s after the removal of the force.

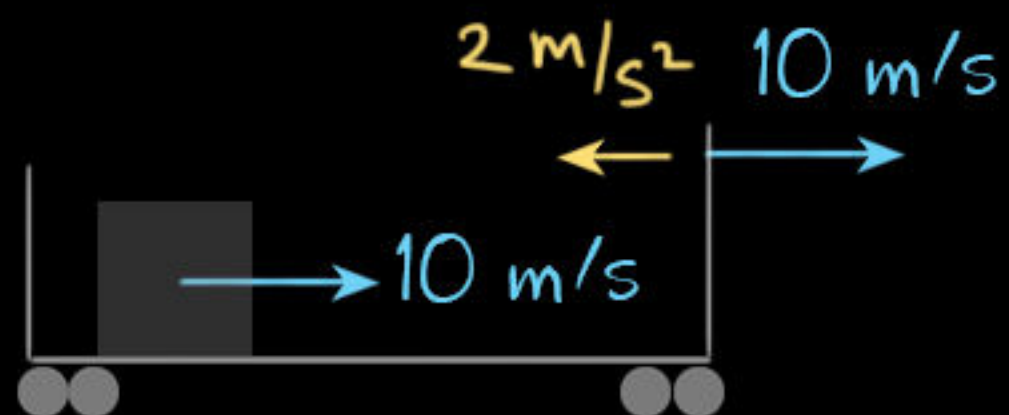


$$\frac{1}{2} \cdot 5 \cdot v = 5 \Rightarrow v = 2$$

$$\frac{1}{2} v \cdot t_2 = 1 \Rightarrow t_2 = 1$$

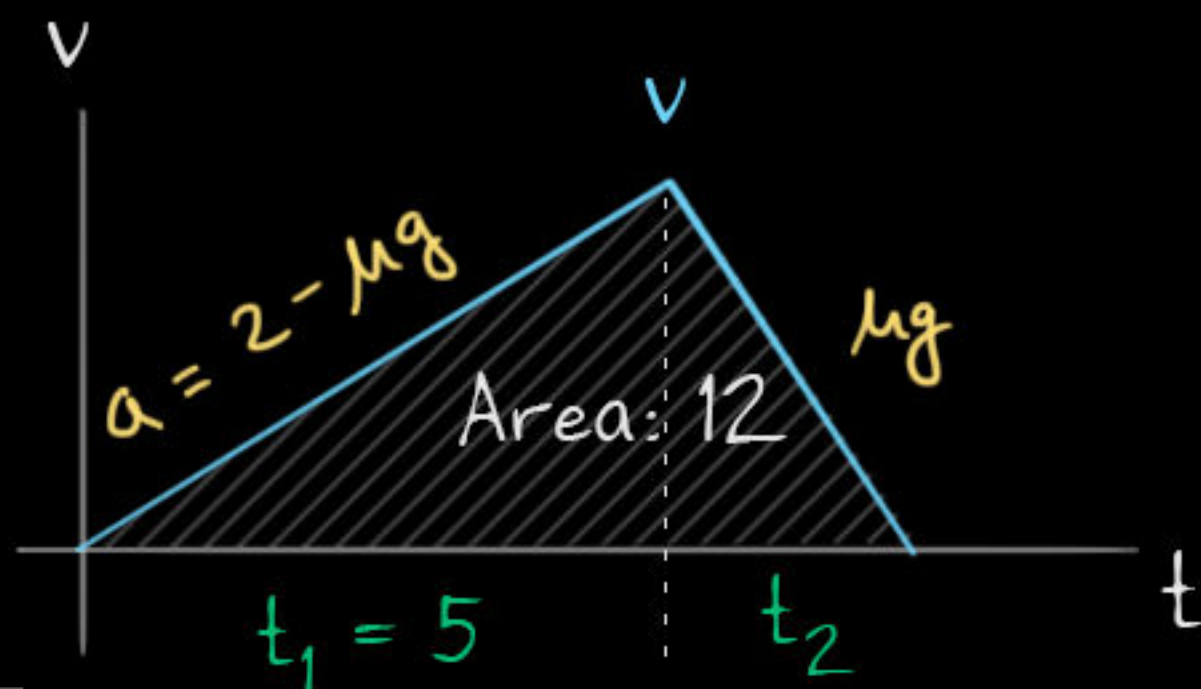
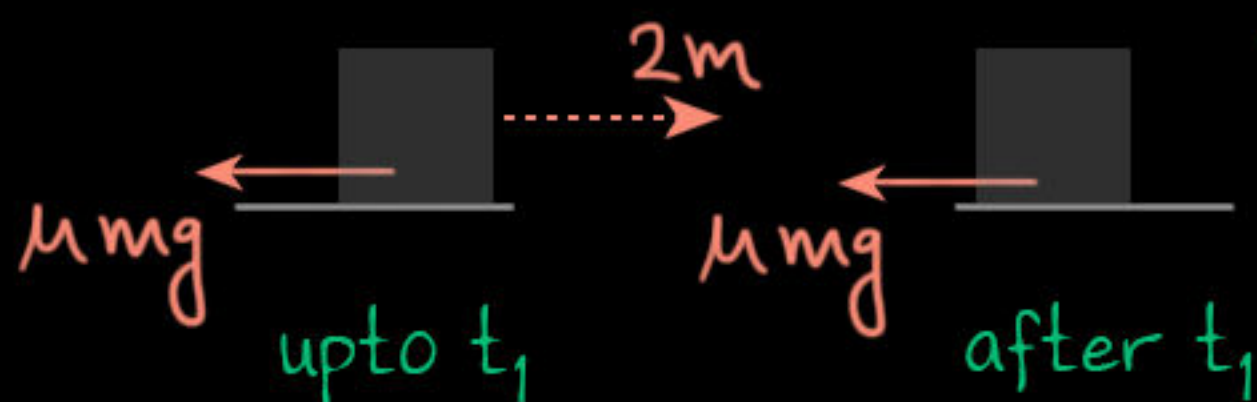
$$v = \mu g t_2 \Rightarrow \mu = 0.2$$

$$v = \frac{F - \mu m g}{m} t_1 \Rightarrow F = 12 \text{ N}$$



Lets say train stops in t_1
 $\Rightarrow 10 = 2t_1 \Rightarrow t_1 = 5$

w.r.t train:



24. Near a station, a train is retarding at 2.0 m/s^2 to stop. When its speed is 36 km/h a passenger standing in the corridor of a bogie, puts his suitcase on the floor. The floor was a little bit slippery so the suitcase began to slide and finally stopped in the train after sliding 12 m on the floor relative to the train. According to this situation, which of the following statements are correct?

- ✓ (a) The train stopped before the suitcase stopped sliding.
- (b) Speed of the suitcase relative to the bogie decreases monotonically.
- ✓ (c) Speed of the suitcase relative to the bogie first increases then decreases.
- ✓ (d) Coefficient of friction between the suitcase and the floor of the bogie is close to 0.135.

$$\frac{1}{2} (t_1 + t_2) v = 12 \quad (1)$$

$$v = (2 - \mu g) t_1 \quad (2)$$

$$v = \mu g t_2 \quad (3)$$

From 1,2,3: $\mu = \frac{100}{74} = 0.135 //$

by Ankit Singhvi

Youtube / Arihant Senior



25. A block rests on a long plank that is moving with a constant velocity 2.0 m/s on a horizontal floor. Coefficient of friction between the block and the plank is 0.10. If the plank starts decelerating uniformly and stops in 0.50 s, what is the distance slid by the block on the plank? Acceleration of free fall is 10 m/s².

(a) 0.50 m

(b) 0.75 m

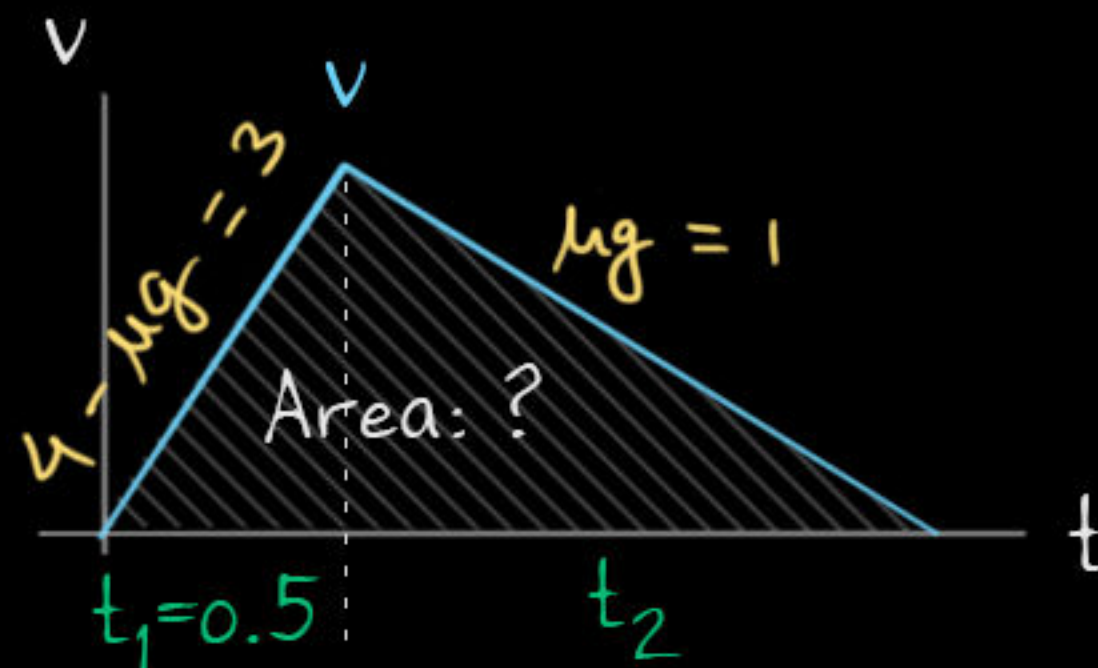
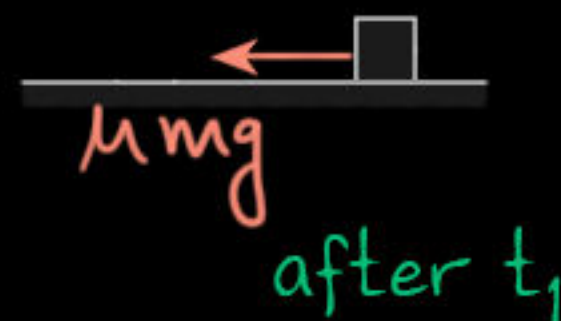
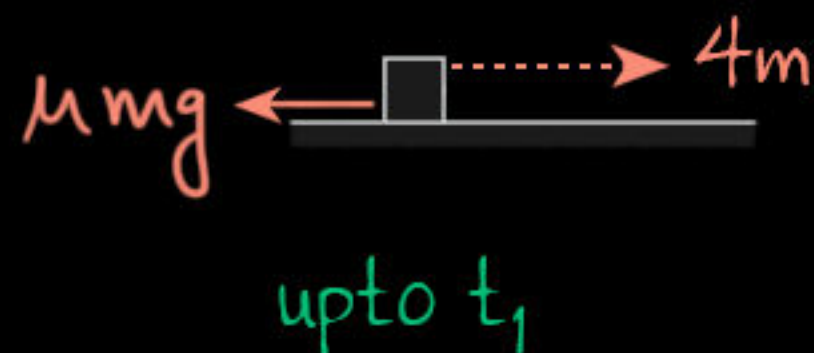
(c) 1.0 m

✓ (d) 1.5 m

Lets say planks acceleration is a .

$$2 = a(0.5) \Rightarrow a = 4$$

w.r.t plank:



$$v = 3t_1 = 1.5$$

$$v = 1t_2 = t_2$$

$$\Rightarrow t_2 = 1.5$$

$$\text{Distance slid by block on the plank:} = \frac{1}{2}(t_1 + t_2)v$$

$$= \frac{1}{2}(0.5 + 1.5)(1.5)$$

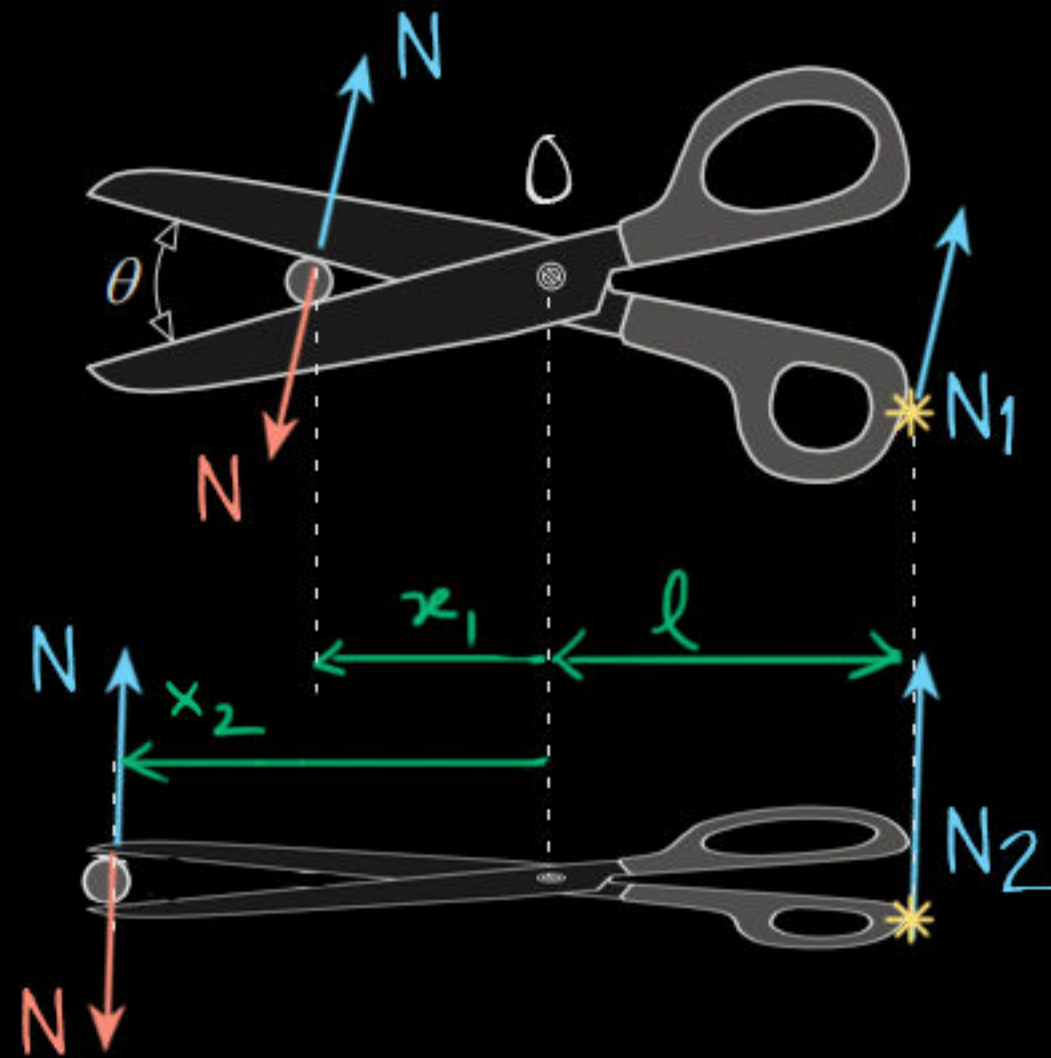
$$= 1.5 \text{ m}$$

For least effort, wire should be as close to hinge as possible.

Proof:

Lets say Normal reaction needed to cut the wire is N .

Showing forces on top of wire (N) and the arm touching it (N and N_1/N_2).



$\Sigma \tau_o$ balance

$$Nx_1 = N_1 l$$

$$Nx_2 = N_2 l$$

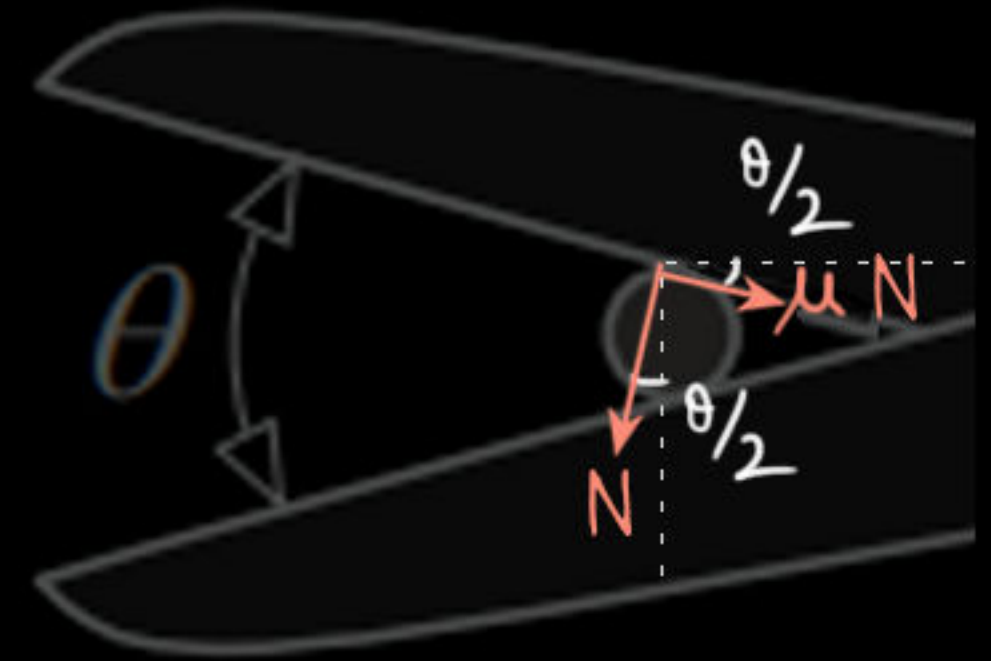
$$N_2 = \frac{x_2}{x_1} N_1 \rightarrow \therefore \text{As } x_2 > x_1, \text{ hence } N_2 > N_1.$$

26. A pair of scissors is used to cut a wire of circular cross section held vertically. To reduce required force the wire must be placed close to the hinge; but if it is placed close to the hinge, it slides away from the hinge until the angle between the blades becomes θ . Find the coefficient of friction between the blades and the wire.

✓(b) $\tan(0.5\theta)$

Sol:

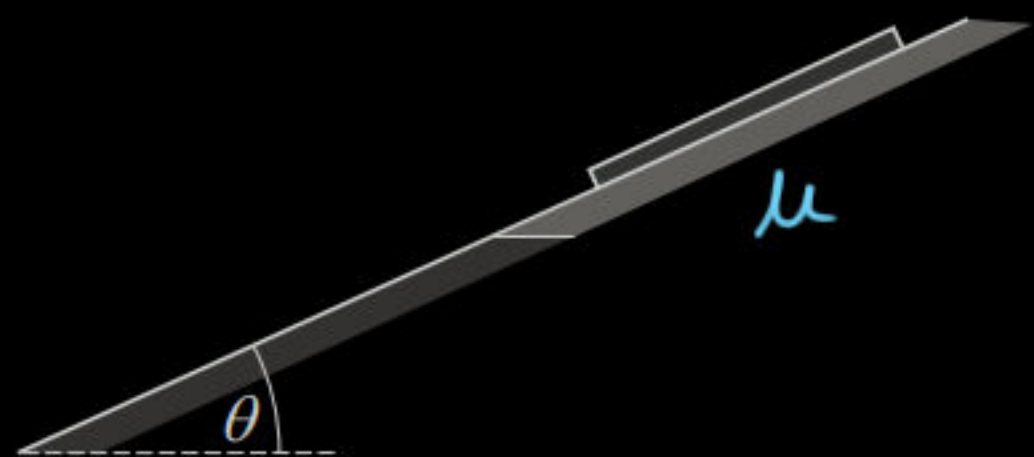
When just slipping:



For both surfaces that touch the scissors.

$$F_x: 2 \times (\mu N \cos \theta/2) = 2 \times (N \sin \theta/2)$$

$$\Rightarrow \mu = \tan \theta/2 //$$



27. A homogeneous bar of mass m is released from rest on a fixed inclined plane of inclination θ above the horizontal as shown. The upper part of the plane shown in dark grey is not frictionless and the lower part shown in light grey is frictionless. Coefficient of friction between the bar and the upper part of the plane is μ . What will be the maximum tensile force in the bar while it is sliding on the plane? Acceleration of free fall is g .

(a) Zero

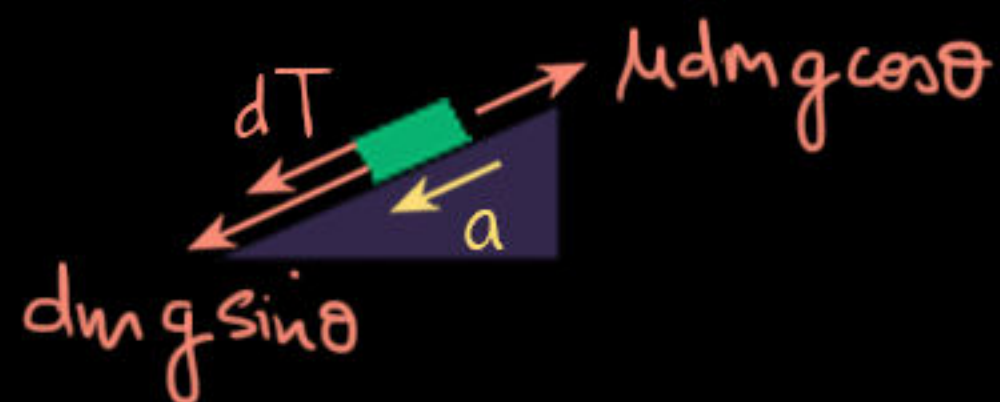
✓(b) $0.25\mu mg \cos \theta$

(c) $0.5\mu mg \cos \theta$

(d) $\mu mg \cos \theta$

Concept: Tension will increase linearly from either end and will be maximum at the junction *.

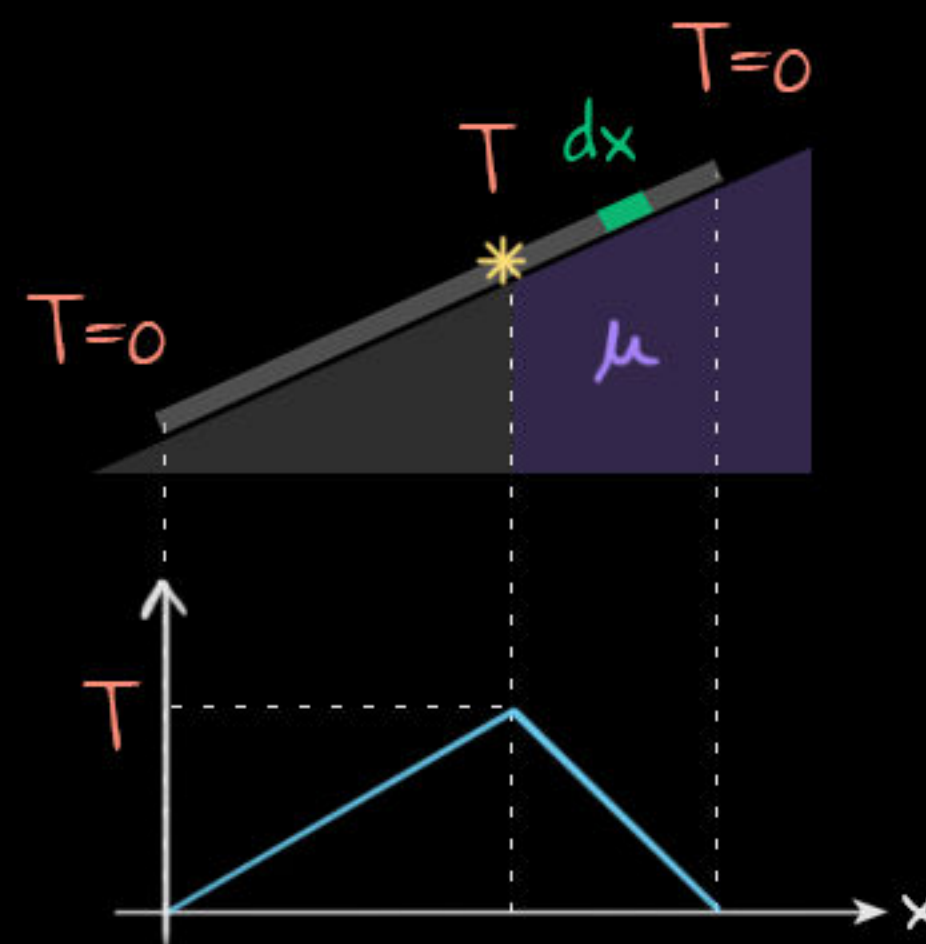
Proof: For a random element dx :



$$dT + dmgs \sin \theta - \mu dmgs \cos \theta = \lambda dx a$$

$$\Rightarrow \frac{dT}{dx} = \lambda (a - g \sin \theta + \mu g \cos \theta) : \text{a constant!}$$

∴ Slope of $T-x$ is a straight line.



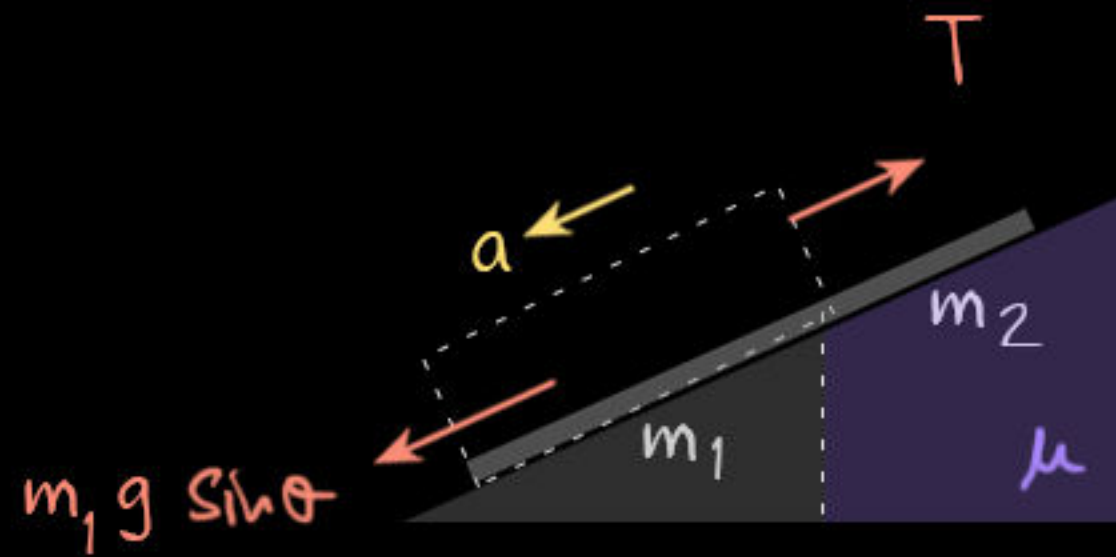
Note:

- 1 If frictional slope was below instead, T would be compressive, but same variation.
- 2 If it were pulled by a Force F at an end. Then at that end tension will be F instead of zero. But again, linear variation.

Approach: We will find T , when junction at a random position. And then, find $T_{\max}$

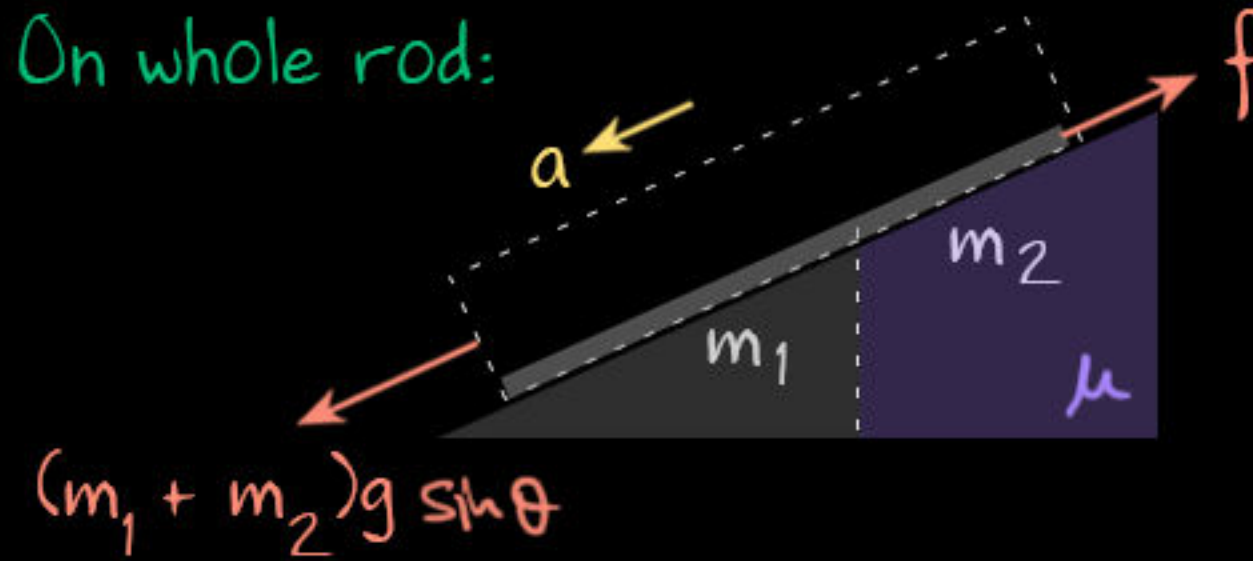
Sol:

On lower section:



$$\frac{m_1 g \sin \theta - T}{m_1} = a \quad (1)$$

On whole rod:



$$\frac{(m_1 + m_2) g \sin \theta - \mu m_2 g \cos \theta}{m_1 + m_2} = a \quad (2)$$

From 1,2:

$$g \sin \theta - \frac{T}{m_1} = g \sin \theta - \frac{\mu m_2 g \cos \theta}{m_1 + m_2}$$

$$\Rightarrow T = \frac{\mu m_1 m_2 g \cos \theta}{m_1 + m_2} = \frac{\mu g \cos \theta}{M} m_1 (M - m_1)$$

max @ $m_1 = \frac{M}{2}$

$$\Rightarrow T_{\max} = \frac{\mu g \cos \theta}{M} \left(\frac{M}{2} \right) \left(\frac{M}{2} \right) = \mu \frac{M g \cos \theta}{4}$$

Both viscous and buoyant forces will apply upwards and will be constant and equal for both balls at terminal velocity.

Lets call them f .

Also, heavier ball will be at the bottom.

As heavier ball will accelerate down faster with slack rope.

$$a = g - \frac{f}{m}$$

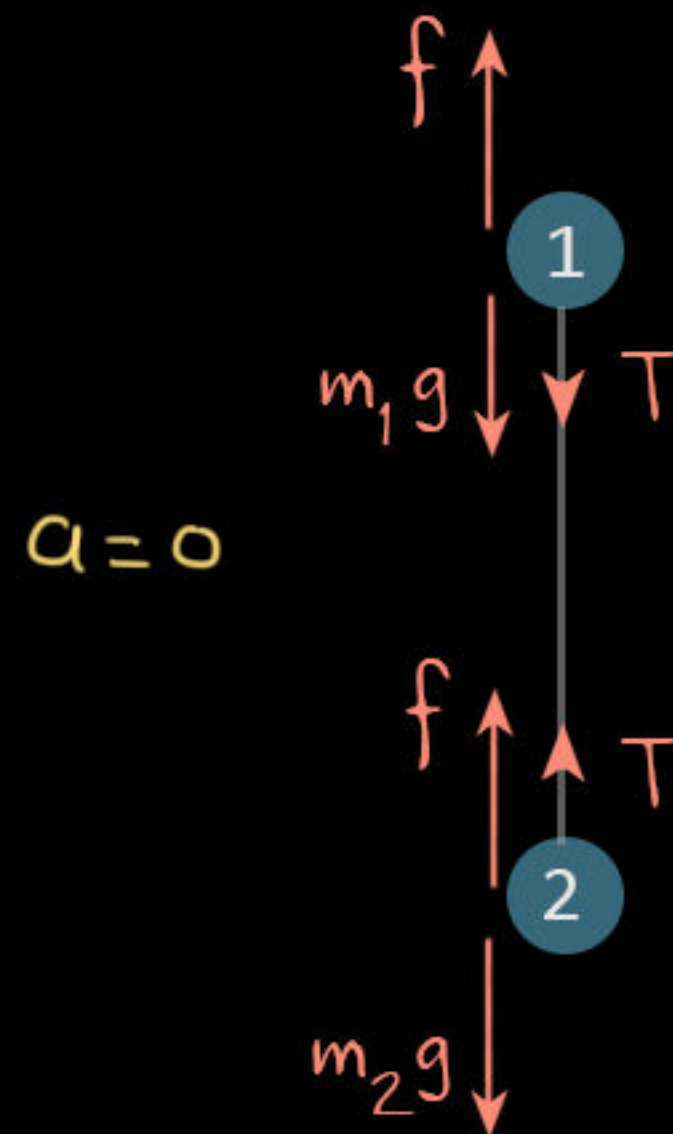
28. Two balls of equal volume and masses m_1 and m_2 ($m_2 > m_1$) connected with a thin light thread are dropped from a certain height. Viscous drag of air on a ball depends on its velocity and buoyant force is equal to the weight of air displaced by the ball. When the balls acquire a uniform velocity after a sufficient time from the instant they were dropped, what is the tensile force in the thread? Acceleration due to gravity is g .

(a) Zero

(b) $(m_2 - m_1)g$

(c) $0.5(m_2 + m_1)g$

✓ (d) $0.5(m_2 - m_1)g$



$$1: \quad f = m_1g + T$$

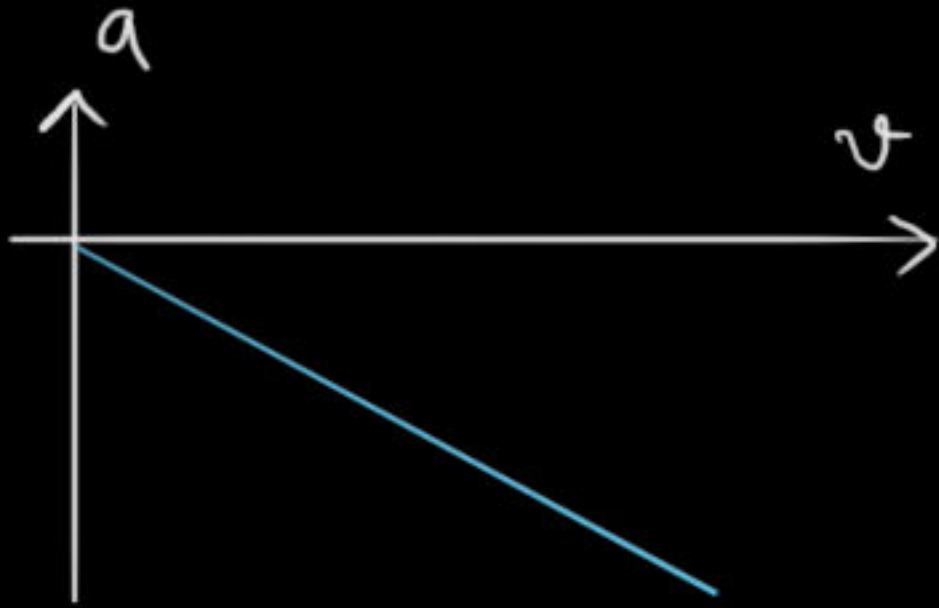
$$2: \quad f + T = m_2g$$

$$\text{Subtract: } 2T = (m_2 - m_1)g$$

$$\Rightarrow \quad T = \frac{(m_2 - m_1)g}{2} //$$

Approach: Lets turn this random graph into proper a-v graph. (by flipping twice!)

As the force is retarding, slope of a-v graph is -ve.

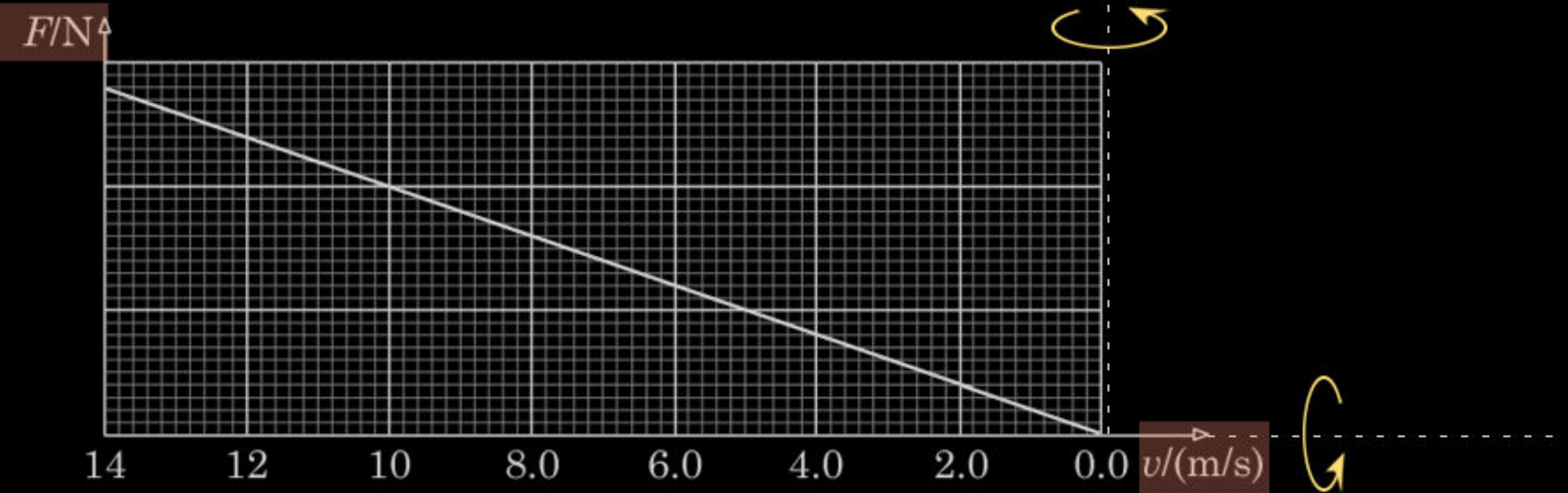


Lets say, $a = -kv$
where k is +ve constant.

$$\therefore v \frac{dv}{dx} = -kv$$

$$\Rightarrow dv = -k dx$$

29. A block of mass 1.0 kg is given an initial velocity 20 m/s by a sharp hit on a horizontal floor lubricated with oil. The block moves in a straight line and stops due to viscous drag of the oil film. How the force F of viscous drag varies with velocity v of the block is shown in the following graph for a velocity interval [14 m/s, 0.0 m/s]. However, the experimenter forgot to label the ordinate.



After an instant, when the block was moving with 8 m/s, it slides 20 m and stops. How far does the block slide from the beginning to the instant it stops sliding?

Given

$$\int_0^8 dv = -k \int_{20}^0 dx \Rightarrow 8 = 20k$$

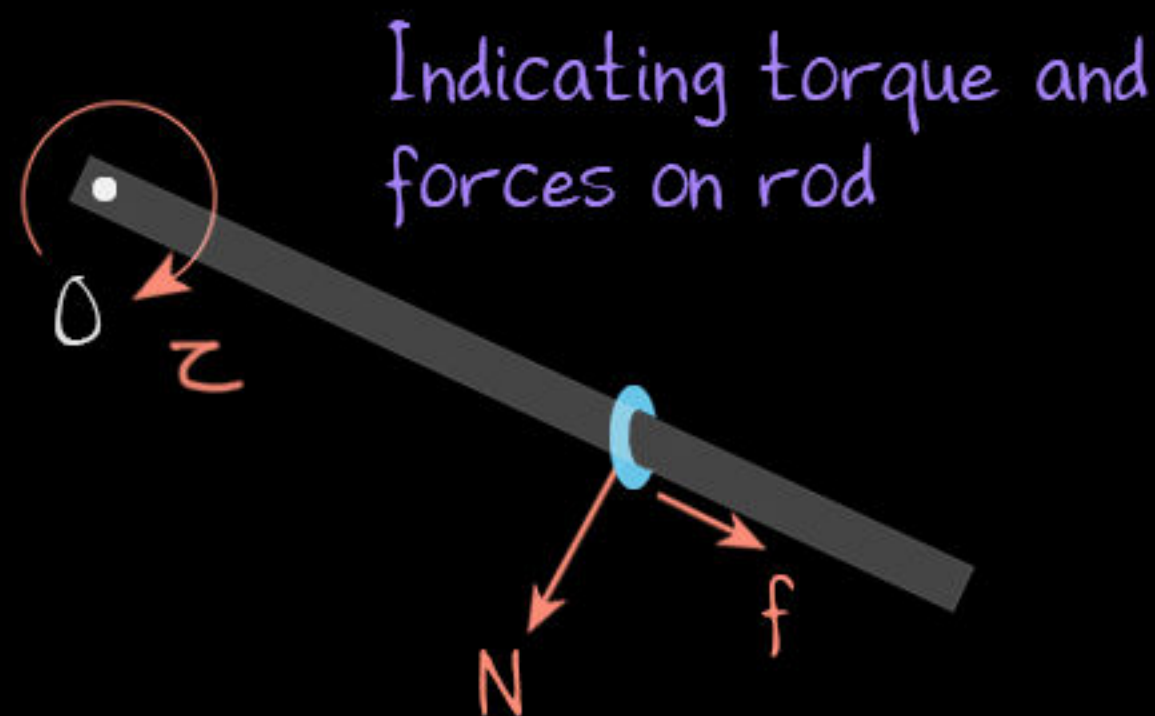
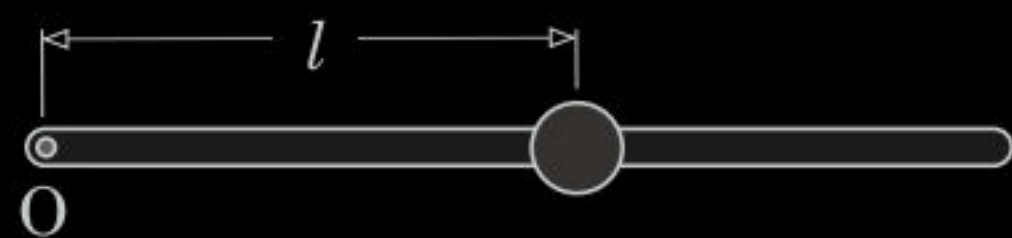
To find

$$\int_0^{20} dv = -k \int_x^0 dx \Rightarrow 20 = xk$$

divide $x = 50$

Note: mass 1kg is irrelevant

by Ankit Singhvi



Concept: Torque on massless rod is zero, as I is zero.

$$\tau = 0$$



$$N = 0$$



$$f = \mu N = 0 \text{ (Regardless of } \mu \text{)}$$

30. A massive bead is threaded on a long light rod, one end of which is pivoted to a fixed point O. Initially, the rod is held horizontally and the bead is at a distance l from the pivot. Coefficient of friction between the rod and the bead is μ . Which of the following statements correctly describe relation between angle θ that the rod makes with the horizontal after the rod is released and time t ?

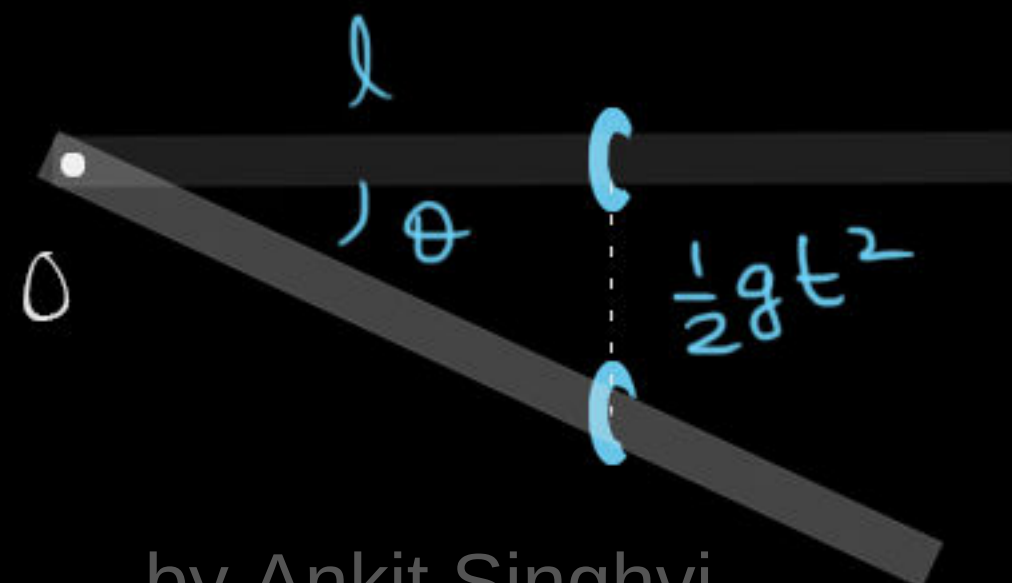
✓(a) If μ is negligible, $\theta \rightarrow \tan^{-1} \left(\frac{gt^2}{2l} \right)$.

✓(b) If μ is finite, $\theta \rightarrow \tan^{-1} \left(\frac{gt^2}{2l} \right)$.

(c) If μ is finite, θ is smaller than $\tan^{-1} \left(\frac{gt^2}{2l} \right)$ by a finite amount.

(d) None of the above statements is correct.

∴ Both N and f are zero, so sleeve will free fall.



$$\therefore \tan \theta = \frac{\frac{1}{2}gt^2}{l}$$

by Ankit Singhvi

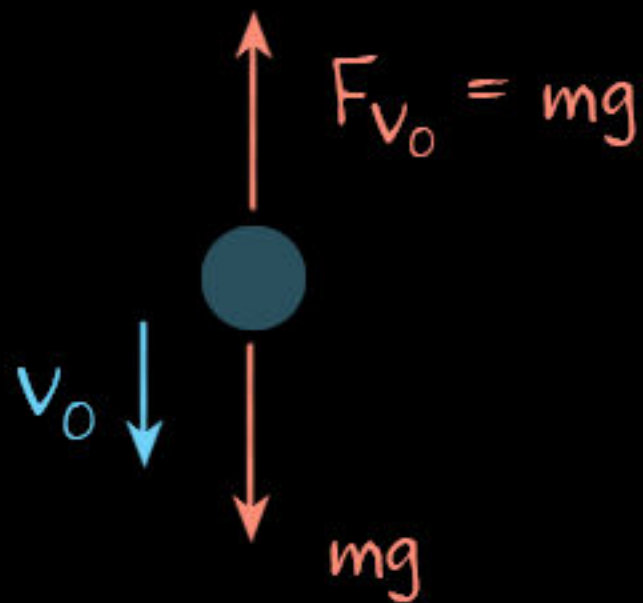
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31. A ball dropped from a high altitude acquires a terminal velocity before hitting the ground, where it bounces off elastically. If air resistance depends on the speed of the ball, what will its acceleration be immediately after the first bounce?

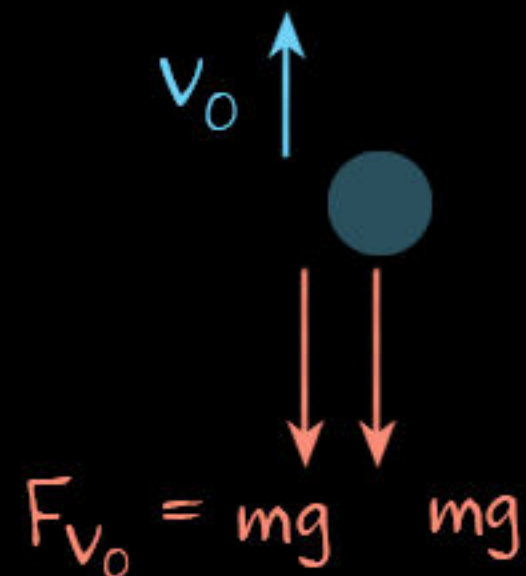
- (a) Zero
✓(c) $2g \downarrow$
(b) $g \downarrow$
(d) $3g \downarrow$

Concept: On bouncing, air resistance changes direction, but not magnitude.

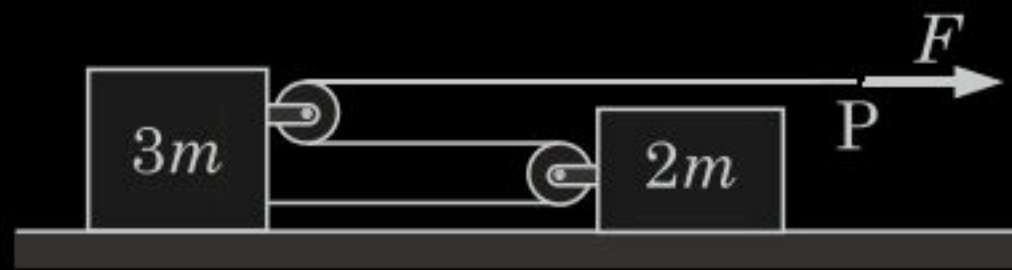
Before bouncing



Immediately after bouncing



$$a = \frac{2mg}{m} = 2g \downarrow //$$

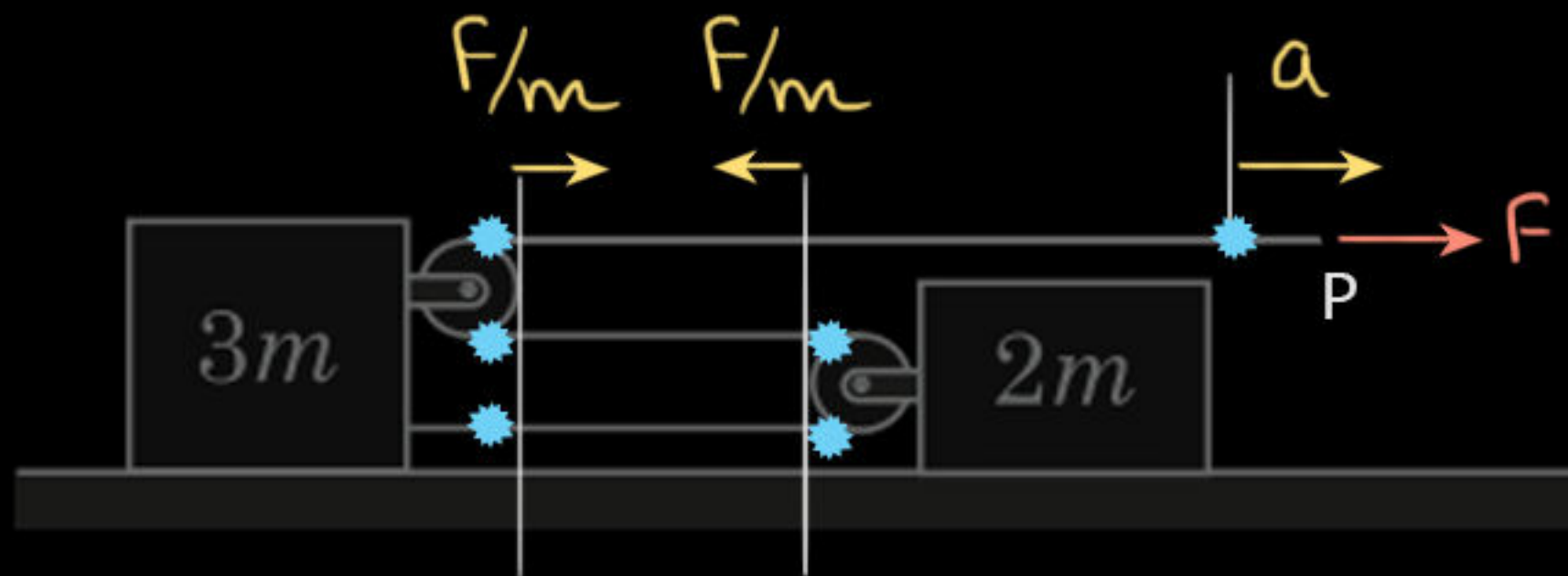


32. In the setup shown, blocks of masses $3m$ and $2m$ are placed on a frictionless horizontal ground and the free end P of the thread is being pulled by a constant force F . Find acceleration of the free end P.

- (a) $F/(5m)$
(c) $3F/m$

- (b) $2F/m$
✓ (d) $5F/m$

Concept: Total gain in a string is zero.

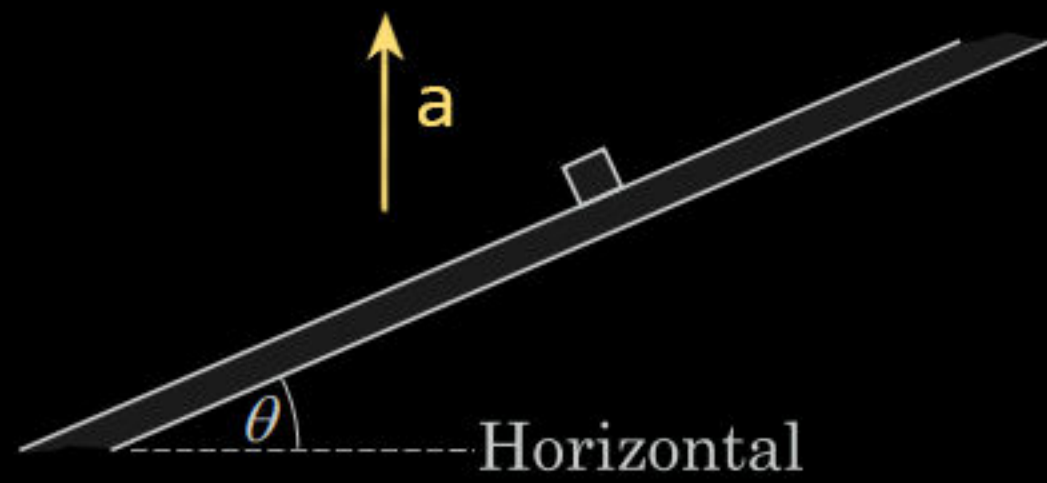


Acceleration of bodies

Total gain in string = 0

$$\Rightarrow -3 \times \frac{F}{m} - 2 \times \frac{F}{m} + a = 0$$

$$\Rightarrow a = 5F/m //$$



33. A small block is sliding on a frictionless inclined plane that is moving upward with a constant acceleration. If the block remains at a level height, what is the acceleration of the inclined plane? Acceleration due to gravity is g .

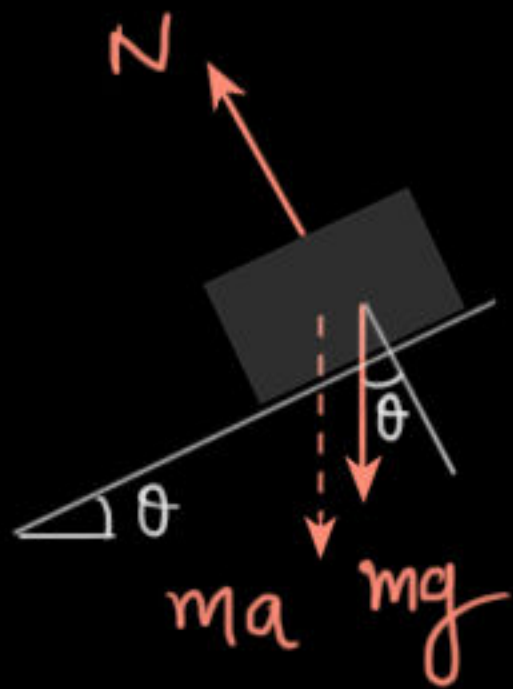
(a) $g \tan \theta$

(b) $g \cot \theta$

(c) $g \sin^2 \theta$

✓(d) $g \tan^2 \theta$

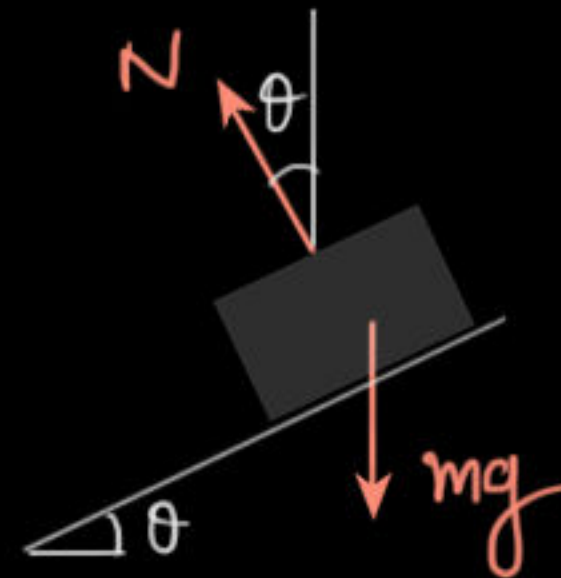
w.r.t plane



Net force normal to plane is zero.

$$N = (mg + ma) \cos \theta \quad (1)$$

w.r.t ground

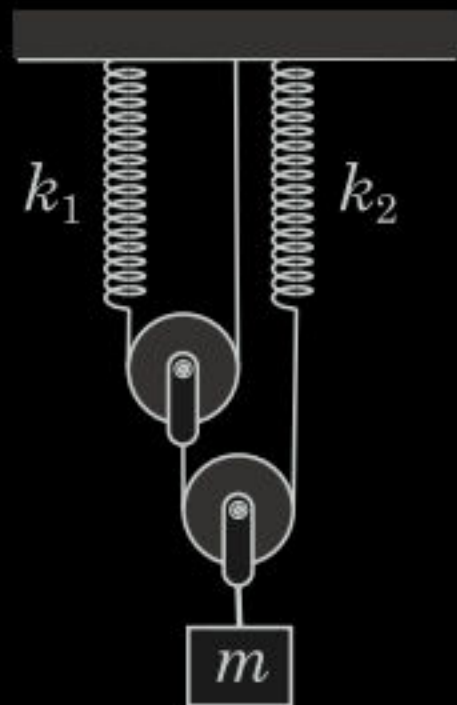


Net vertical force is zero.

$$N \cos \theta = mg \quad (2)$$

From 1,2:

$$a = g \tan^2 \theta //$$



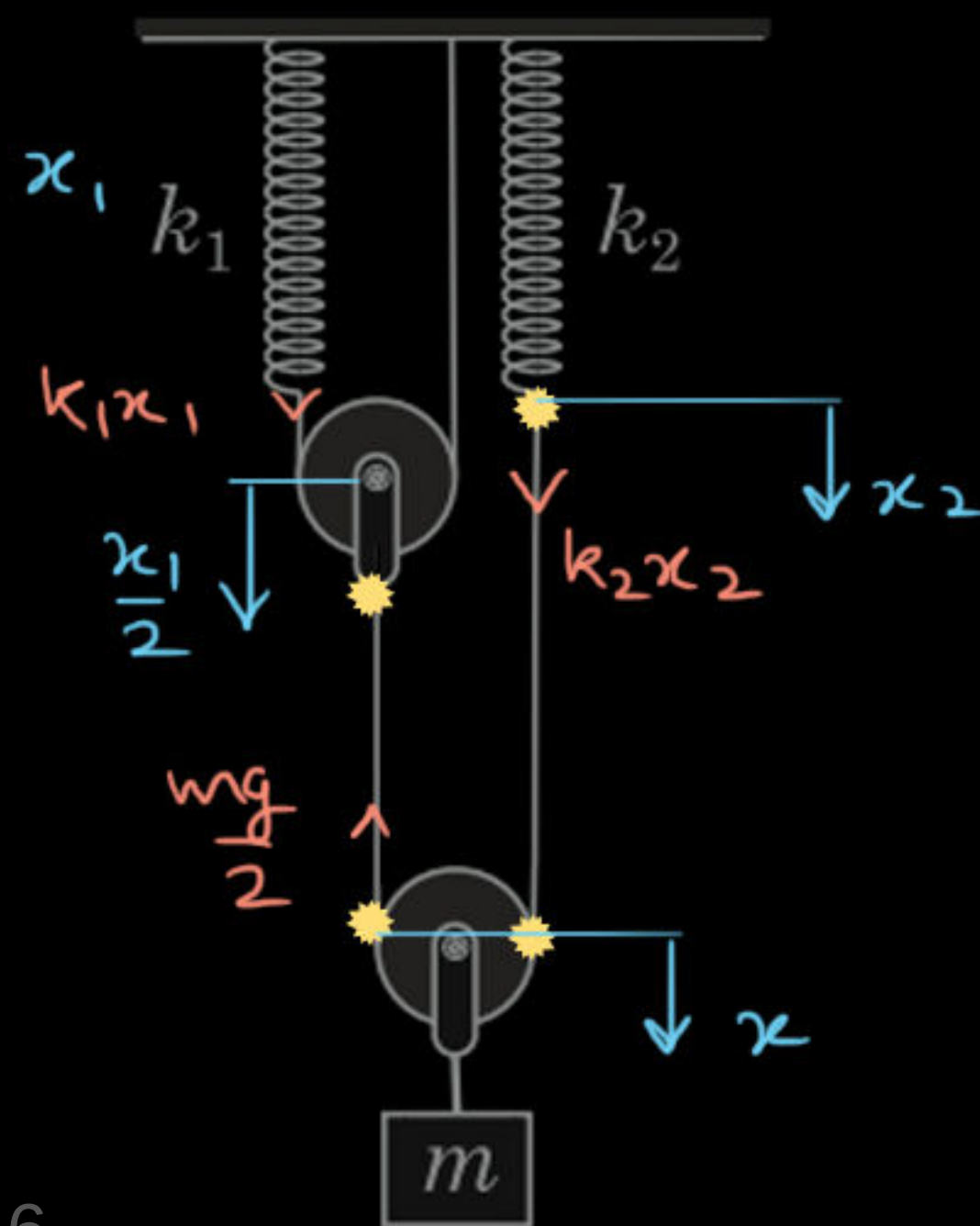
34. In the arrangement shown, the springs are light and have stiffness $k_1 = 100 \text{ N/m}$ and $k_2 = 200 \text{ N/m}$ and the pulleys are ideal. An 8.0 kg block suspended from the lower pulley is initially held at rest maintaining the strings straight and springs relaxed. Now the force supporting the block is gradually reduced to zero. How far does the block descend during the process of reduction of the force? Acceleration due to gravity is 10 m/s^2 .

(a) 10 cm

✓(b) 15 cm

(c) 1.6 m

(d) None of these



Total gain in string = 0

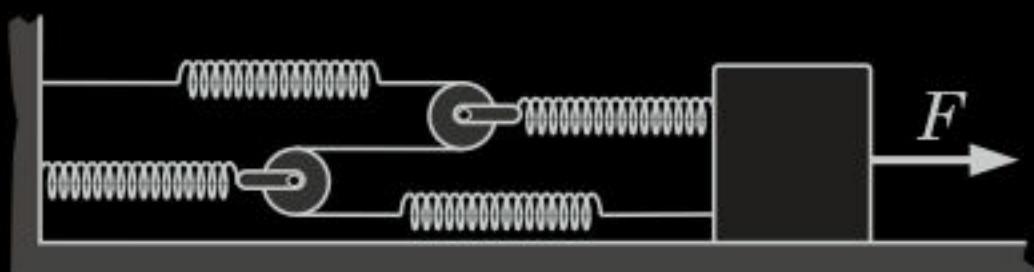
$$\Rightarrow -\frac{x_1}{2} + 2x - x_2 = 0 \quad (1)$$

Force balance:

$$\frac{mg}{2} = k_2 x_2 = 2k_1 x_1 \quad (2)$$

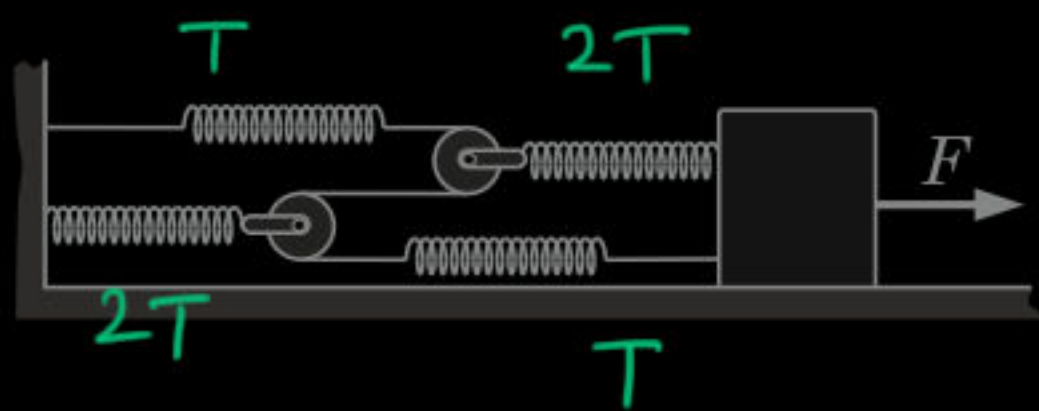
From 2

$$-\frac{1}{2} \left(\frac{mg}{4k_1} \right) + 2x = \frac{mg}{2k_2} \Rightarrow x = 15 \text{ cm} //$$



35. In the setup shown, a block is placed on a frictionless floor, the cords and pulleys are ideal and each spring has stiffness k . The block is pulled away from the wall. How far will the block shift, while the pulling force is increased gradually from zero to a value F ?

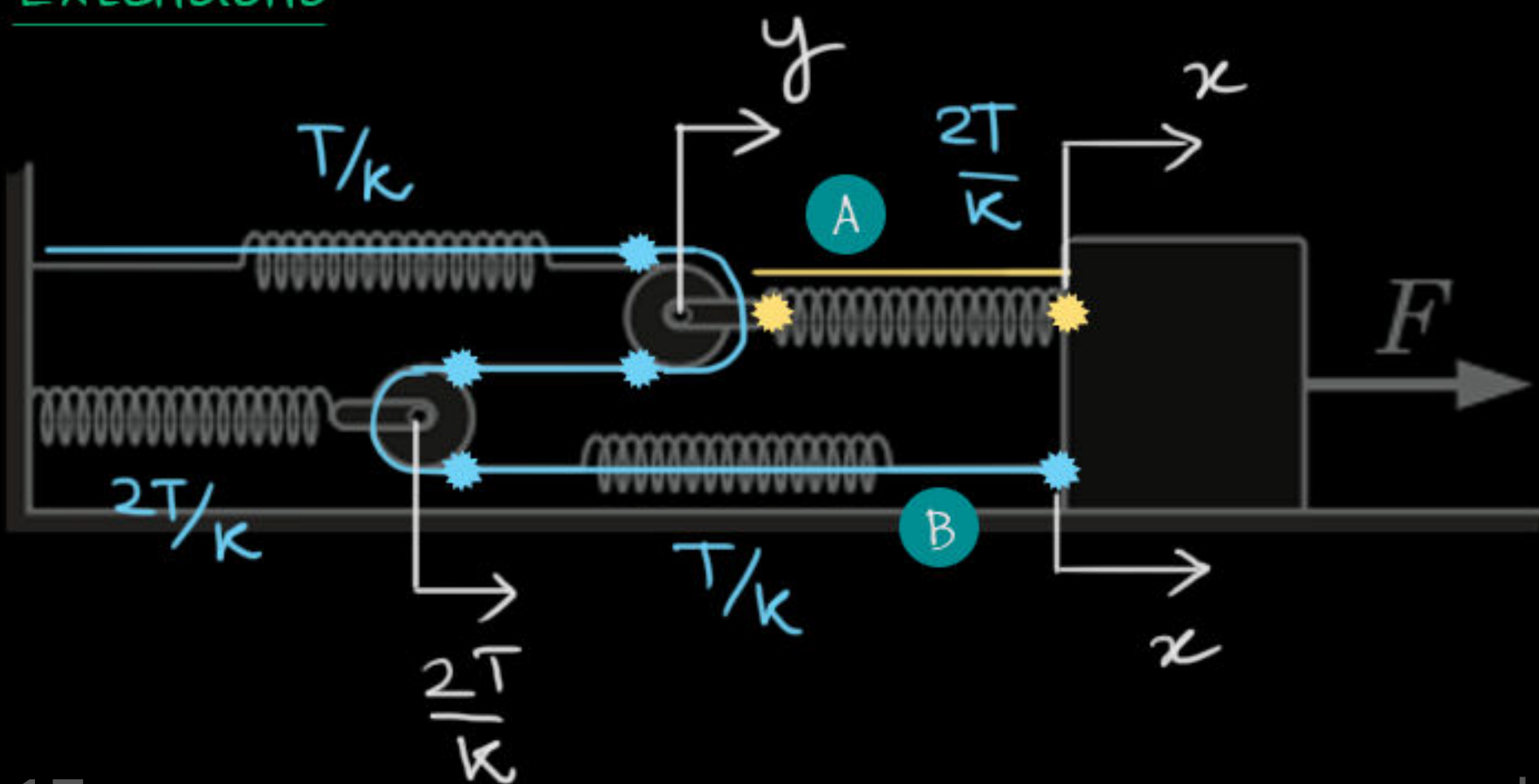
Forces



$$3T = F$$

$$T = F/3$$

Extensions



In A total gain in string spring system = $\frac{2T}{k}$

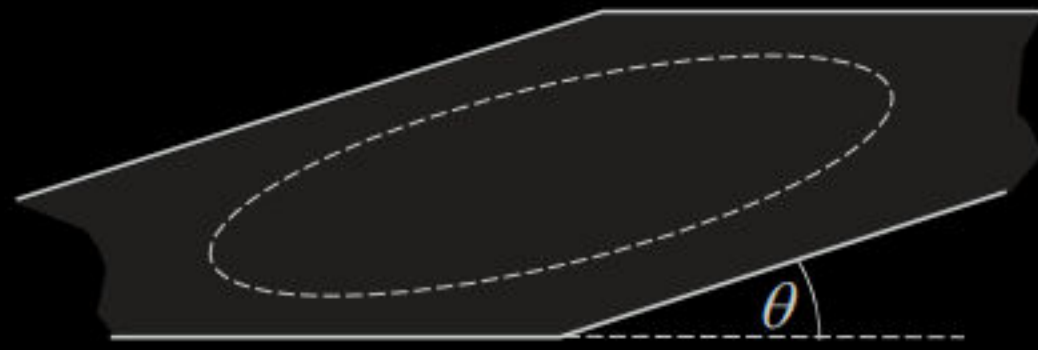
$$\Rightarrow x - y = \frac{2T}{k} \quad (1)$$

In B total gain in string spring system = $\frac{T}{k} + \frac{T}{k}$

$$\Rightarrow 2y - 2\left(\frac{2T}{k}\right) + x = \frac{T}{k} + \frac{T}{k} \quad (2)$$

From 1,2:

$$x = \frac{10T}{3k} = \frac{10}{3k} \left(\frac{F}{3} \right) = \frac{10F}{9k} //$$



36. A large parking place has uniform slope of angle θ with the horizontal. A driver wishes to drive his car in a circle of radius R , at constant speed. Coefficient of static friction between the tyres and the ground is μ . What greatest speed can the driver achieve without slipping? Assume entire load of the car on the front wheels.

(a) $\sqrt{gR \tan \theta}$

(b) $\sqrt{gR \cot \theta}$

(c) $\sqrt{gR(\sin \theta + \mu \cos \theta)}$

✓ (d) $\sqrt{gR(\mu \cos \theta - \sin \theta)}$

Concept: If it can turn without slipping at bottom, it can complete the circle.

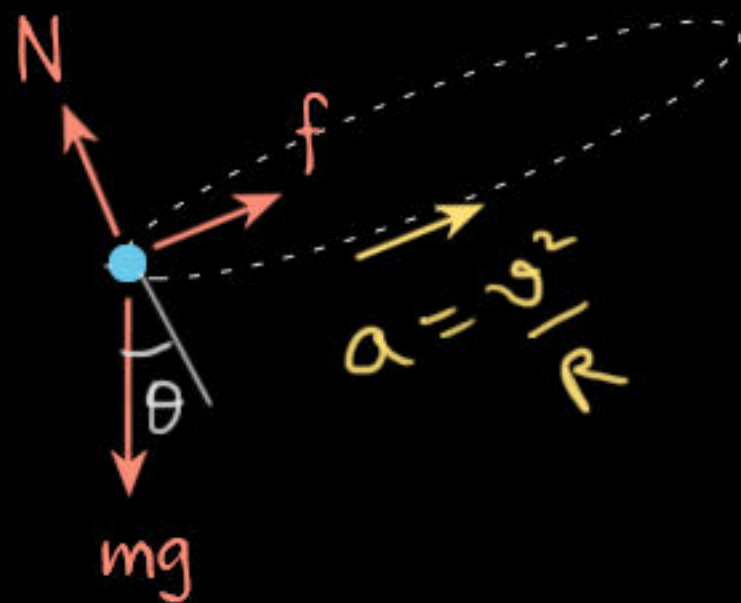
As friction there does all the heavy lifting to provide the constant acceleration.

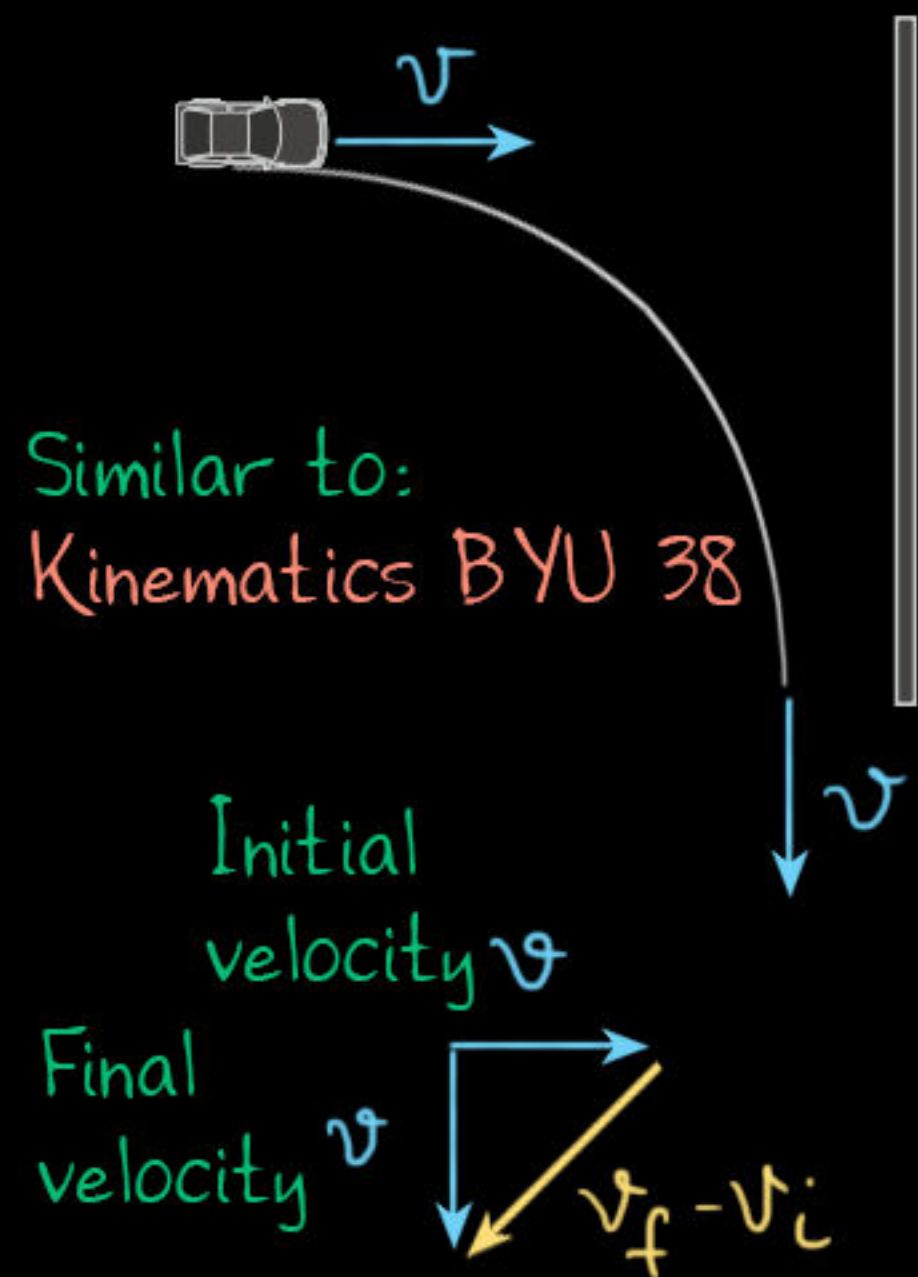
At bottom, $f - mg \sin \theta = \frac{mv^2}{R}$

For $v_{\max}$, $f = f_{\max} = \mu mg \cos \theta$

$\therefore \mu mg \cos \theta - mg \sin \theta = \frac{mv_{\max}^2}{R}$

$\Rightarrow v_{\max} = \sqrt{gR(\mu \cos \theta - \sin \theta)}$





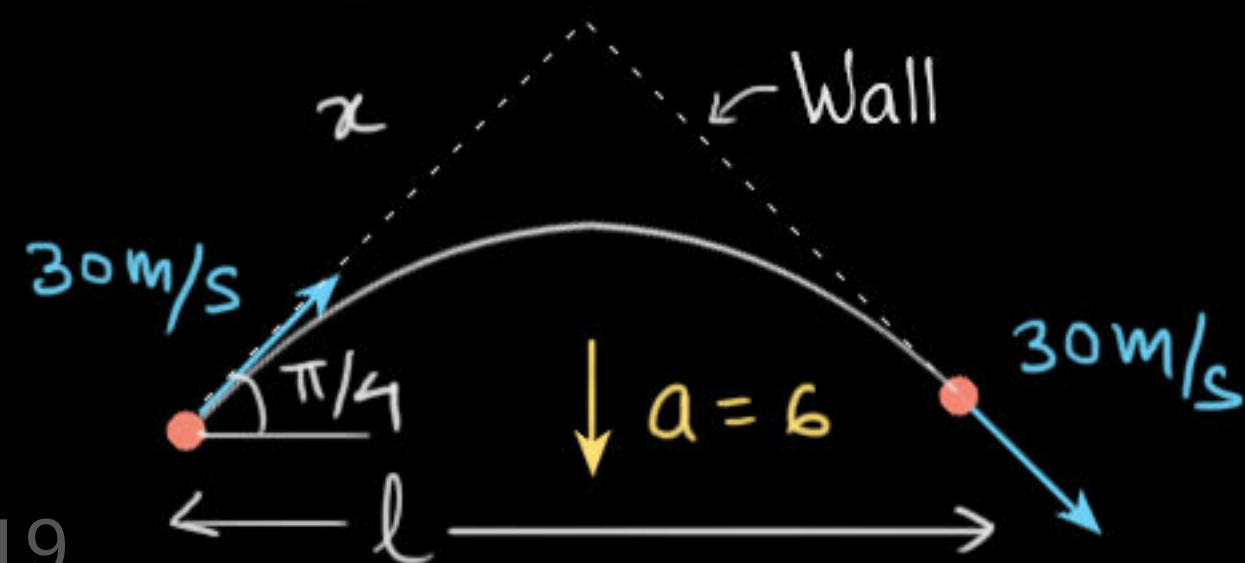
38. A car is moving with speed 108 km/h on a large uniform horizontal pavement perpendicularly towards a wall. To avoid collision, the driver takes a turn and finally runs the car parallel to the wall with the same speed. Coefficient of friction between the tyres and the pavement is 0.6, acceleration of free fall is 10 m/s^2 and all the load of the car is concentrated on the front axle. If the turning process is the fastest one on this pavement, check validity of the following statements.

- (a) The car follows a circular path during the turn.
- ✓ (b) The car follows a parabolic path during the turn.
- ✓ (c) Minimum radius of curvature of the path is 75 m.
- ✓ (d) Initial distance of the wall must be greater than $75\sqrt{2} \text{ m}$.

Concept: Acceleration should be consistently in direction of average acceleration, if turn needs to be achieved in minimum time.

Maximum a possible = $\mu g = 6$.

Like a projectile with $g = 6$!

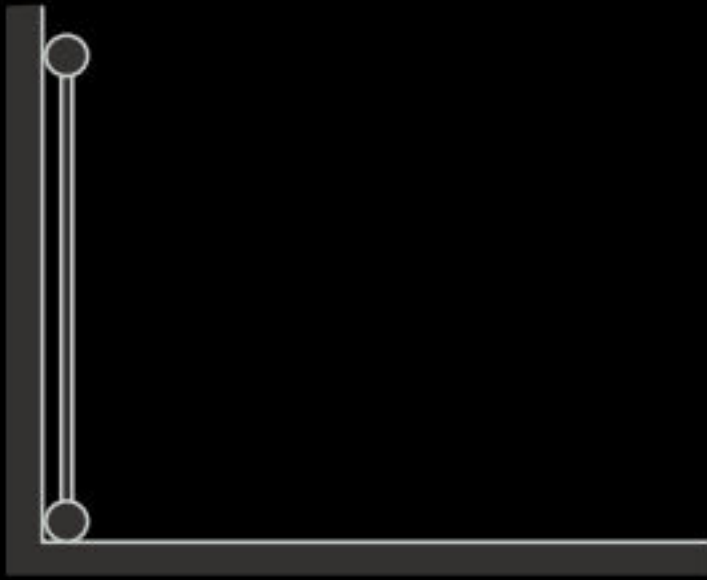


∴ Parabolic path

✚ RoC is minimum at top. Its = $\frac{v_{\text{top}}^2}{a_{\perp}} = \frac{(30 \cos \pi/4)^2}{6} = 75 \text{ m}$

✚ Minimum distance from wall x :

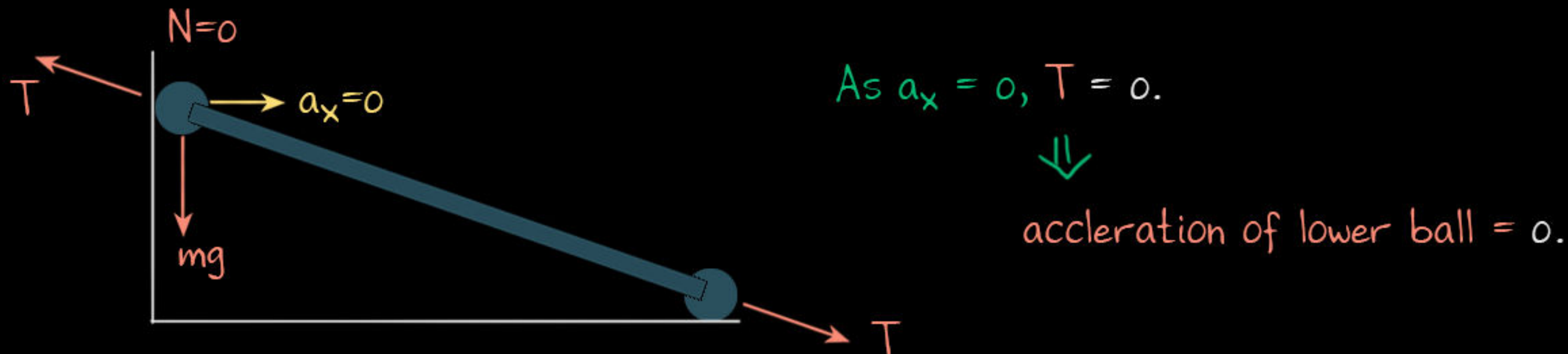
$$x = 30 \cdot t_{1/2} = 30 \cdot \frac{4 \sin \theta}{6} = \frac{30 \cdot 30}{6} \cdot \frac{1}{\sqrt{2}} = 75\sqrt{2} \text{ m}$$



39. A dumbbell is constructed by fixing small identical balls at the ends of a light rod of length l . The dumbbell stands vertically in the corner formed by a frictionless wall and frictionless floor. After the bottom end is slightly pushed towards the right, the dumbbell begins to slide. Value of which of the following quantities vanish, when the upper ball is leaving the wall.

- ✓ (a) Tensile force in the rod
- ✓ (b) Acceleration of the lower ball
- (c) Acceleration of the upper ball
- (d) Normal reaction from floor on the lower ball

Concept: Just after upper ball loses contact, $N=0$ and its a_x is still zero.



40. A particle is moving with constant angular velocity on a circular path of radius R in the x - y plane. If observed from a reference frame moving with constant velocity along the z -axis, the particle will appear moving on a helical path of constant pitch h . Making use of the given information, what expression can be deduced for radius of curvature of the helical path.

Concept: Constant pitch implies velocity of reference frame is constant. Lets say its v_z

w.r.t reference frame:

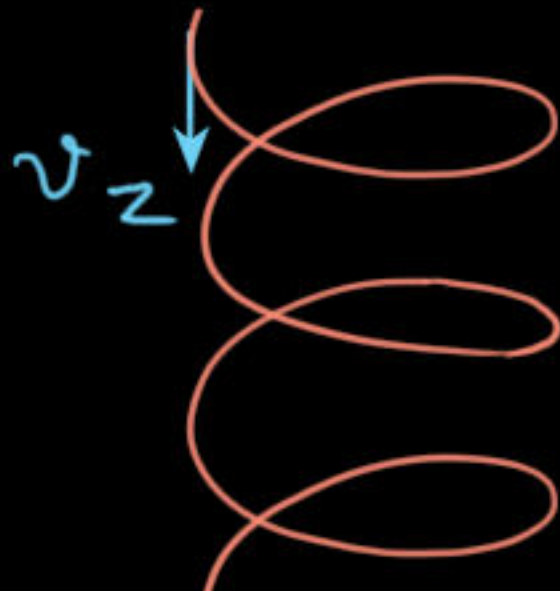


Diagram showing a helical path with pitch h and vertical velocity v_z . The pitch is defined as the vertical distance between two consecutive loops.

$$h = v_z \cdot \frac{2\pi}{\omega}$$

$$\Rightarrow v_z = \frac{h\omega}{2\pi}$$

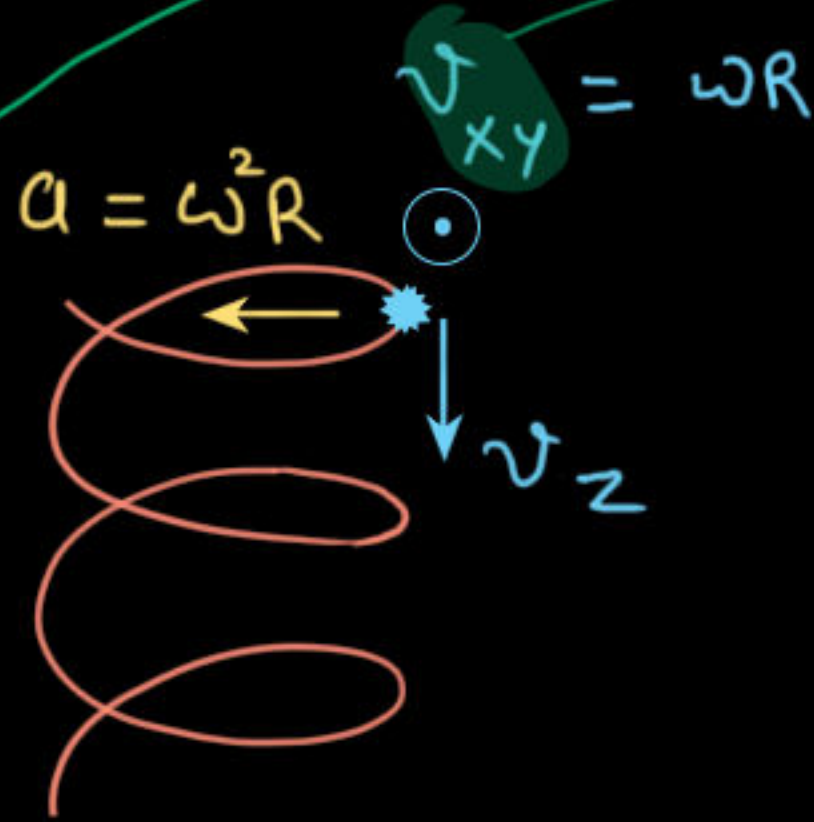


Diagram showing a helical path with radius R , angular velocity ω , and vertical velocity v_z . The radius of curvature is labeled as R_oC .

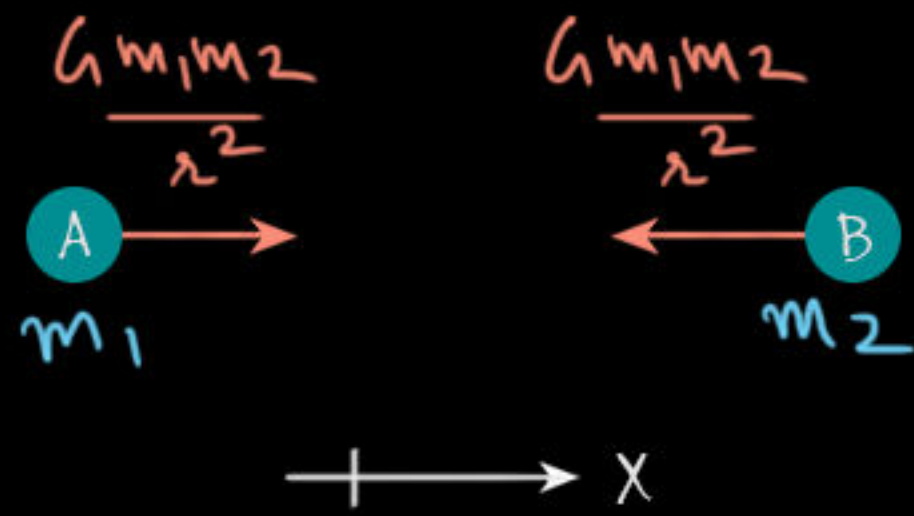
$$a = \omega^2 R$$

$$v_{xy} = \omega R$$

$$R_oC = \frac{v^2}{a_{\perp}} = \frac{v_{xy}^2 + v_z^2}{\omega^2 R}$$

$$= \frac{\cancel{\omega^3 R^2} + \frac{h^2 \cancel{\omega^2}}{4\pi^2}}{\cancel{\omega^2} R}$$

$$= R + \frac{h^2}{4\pi^2 R}$$



$$F_1 = \cancel{m_1} a_1 = \frac{G \cancel{m_1} m_2}{r^2} \hat{r}$$

$$F_2 = \cancel{m_2} a_2 = -\frac{G m_1 \cancel{m_2}}{r^2} \hat{r}$$

$$\Rightarrow \begin{aligned} a_1 &= k m_2 \hat{r} \\ a_2 &= -k m_1 \hat{r} \end{aligned}$$

where, k is +ve constant

41. Imagine that mass, which governs acceleration of the bodies and their mutual gravitational interaction, might sometimes be negative. Two particles A and B of masses m_1 and m_2 are initially at rest some distance apart in free space relative to an inertial frame. What would happen after they are released?

The masses	Observation
(a) $m_1 < 0$ and $m_2 < 0$	(p) They move towards each other.
(b) $m_1 > 0$, $m_2 < 0$ and $ m_1 = m_2 $	(q) They move away from each other.
(c) $m_1 > 0$, $m_2 < 0$ and $ m_1 > m_2 $	(r) Eventually B will escape away from A.
(d) $m_1 < 0$, $m_2 > 0$ and $ m_1 > m_2 $	(s) B follows A and finally collides with A.
	(t) B follows A with a constant separation.

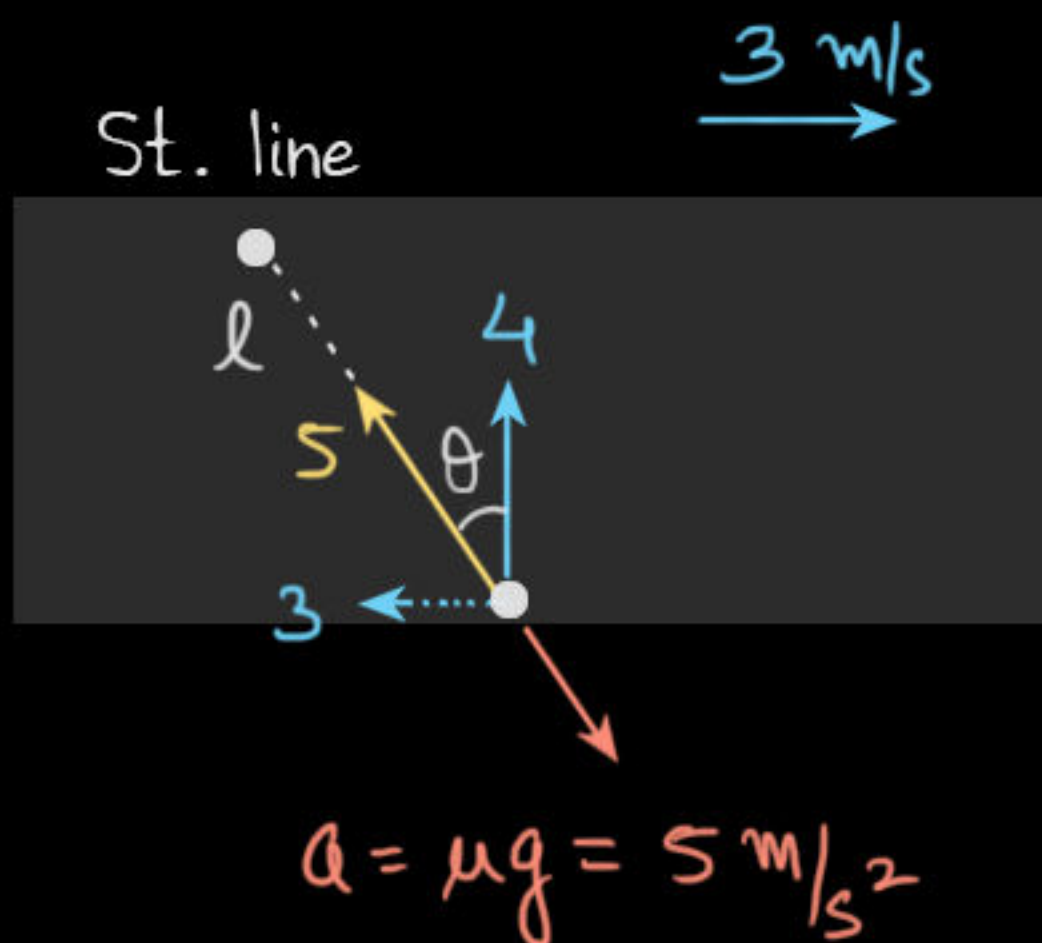
A $a_1 = -|k m_2| \hat{r}, a_2 = +|k m_1| \hat{r} \Rightarrow r, r //$

B $a_1 = -|k m| \hat{r}, a_2 = -|k m| \hat{r} \Rightarrow t //$

C $a_1 = -|k 0| \hat{r}, a_2 = -|k 0| \hat{r} \Rightarrow s //$

D $a_1 = +|k 0| \hat{r}, a_2 = +|k 0| \hat{r} \Rightarrow r //$

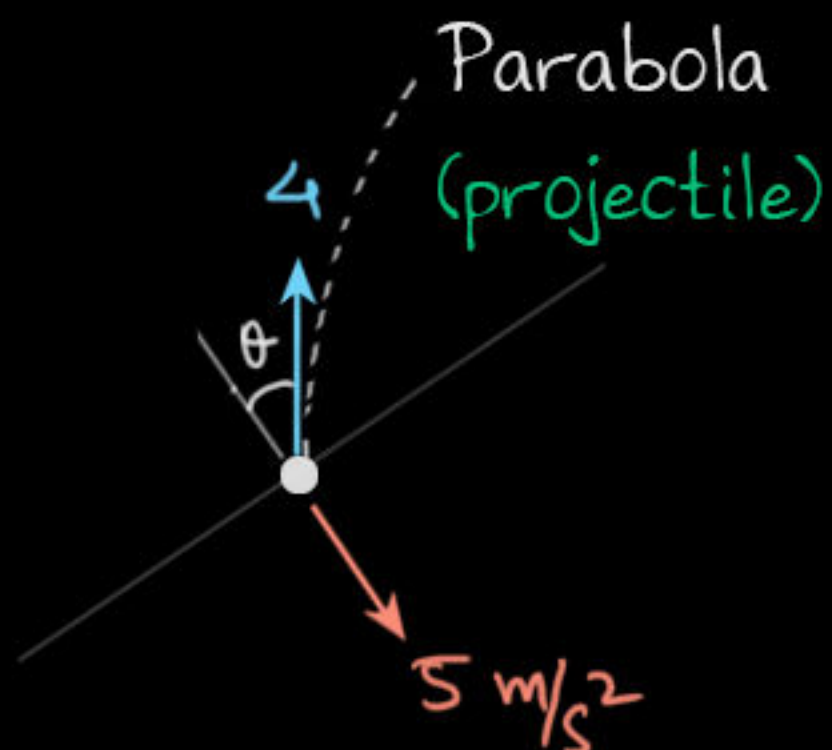
w.r.t conveyor belt



A horizontal conveyor belt is running at a constant speed $v_b = 3.0 \text{ m/s}$. A small disc enters the belt moving horizontally with a velocity $v_0 = 4.0 \text{ m/s}$ that is perpendicular to the velocity of the belt. Coefficient of friction between the disc and the belt is 0.50.

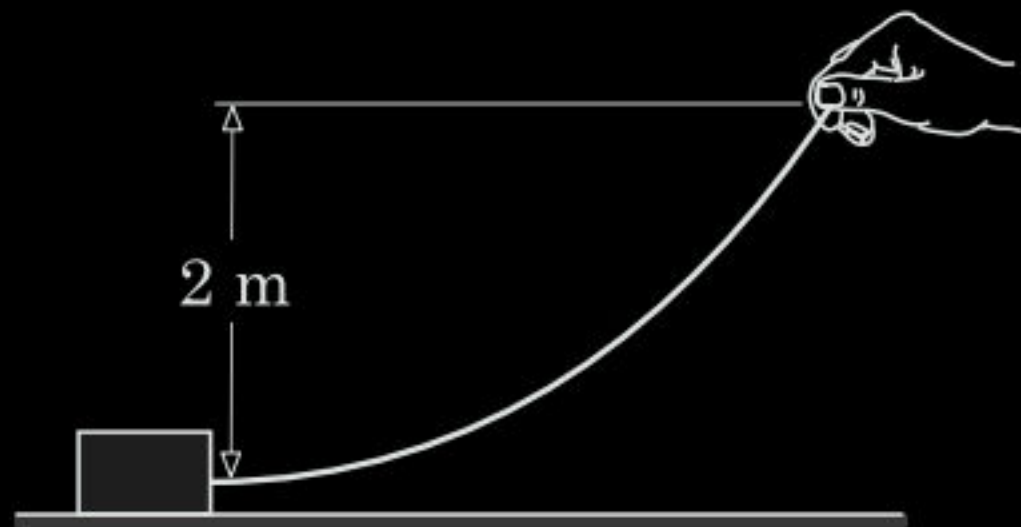
42. What can you predict regarding the path of the disc?
- (a) It is a parabola relative the belt.
 - ✓ (b) It is a straight line relative to the belt.
 - ✓ (c) It is a parabola relative to the ground.
 - (d) It is a straight line relative to the ground.
43. What should the minimum width of the belt be so that the disc always remains on the belt?
44. What is the minimum speed of the disc relative to the ground?

w.r.t ground



$$\text{Minimum width} = l \cos \theta = \left(\frac{v^2}{2a} \right) \cos \theta = \frac{5^2}{2 \cdot 5} \cdot \frac{4}{5} = 2 \text{ m} //$$

$$\text{Minimum speed is at top of projectile} = 4 \sin \theta = 4 \cdot \frac{3}{5} = 2.4 //$$



Similar problems:

Irodov 1.93, 1.107

Concept: Relating the small variation of tension along the rope with small mass of the element.

Lower end of a uniform inextensible rope of mass 2 kg and length 4 m is attached to a block of mass 7.5 kg placed on a horizontal floor. Coefficient of friction between the block and the floor is 0.5. The upper end of the rope is held 2 m above the lower end so that the tangent at the lower end remains horizontal as shown in the figure. In this situation, the block stays standstill on the floor. Acceleration due to gravity is 10 m/s^2 .

45. The upper end must be pulled at an angle that is closest to
 (a) 60° above the horizontal ✓ (b) 53° above the horizontal
 (c) 45° above the horizontal (d) Insufficient information
46. Frictional force between the block and the floor is closest to
 ✓ (a) 15 N (b) 20 N
 (c) 30 N (d) 37.5 N
47. The upper end of the rope is now slowly shifted downwards and simultaneously away from the block maintaining the tangent at lower end horizontal. When the block begins sliding, at what height above the lower end is the upper end?
 (a) 0.5 m (b) 0.75 m
 ✓ (c) 1.0 m (d) 1.5 m

∴ The element (dl) is at rest, F_t and F_N are balanced.

F_N (irrelevant to this problem): $T d\theta = dm g \cos\theta$

$$F_t: dT = dm g \sin\theta$$

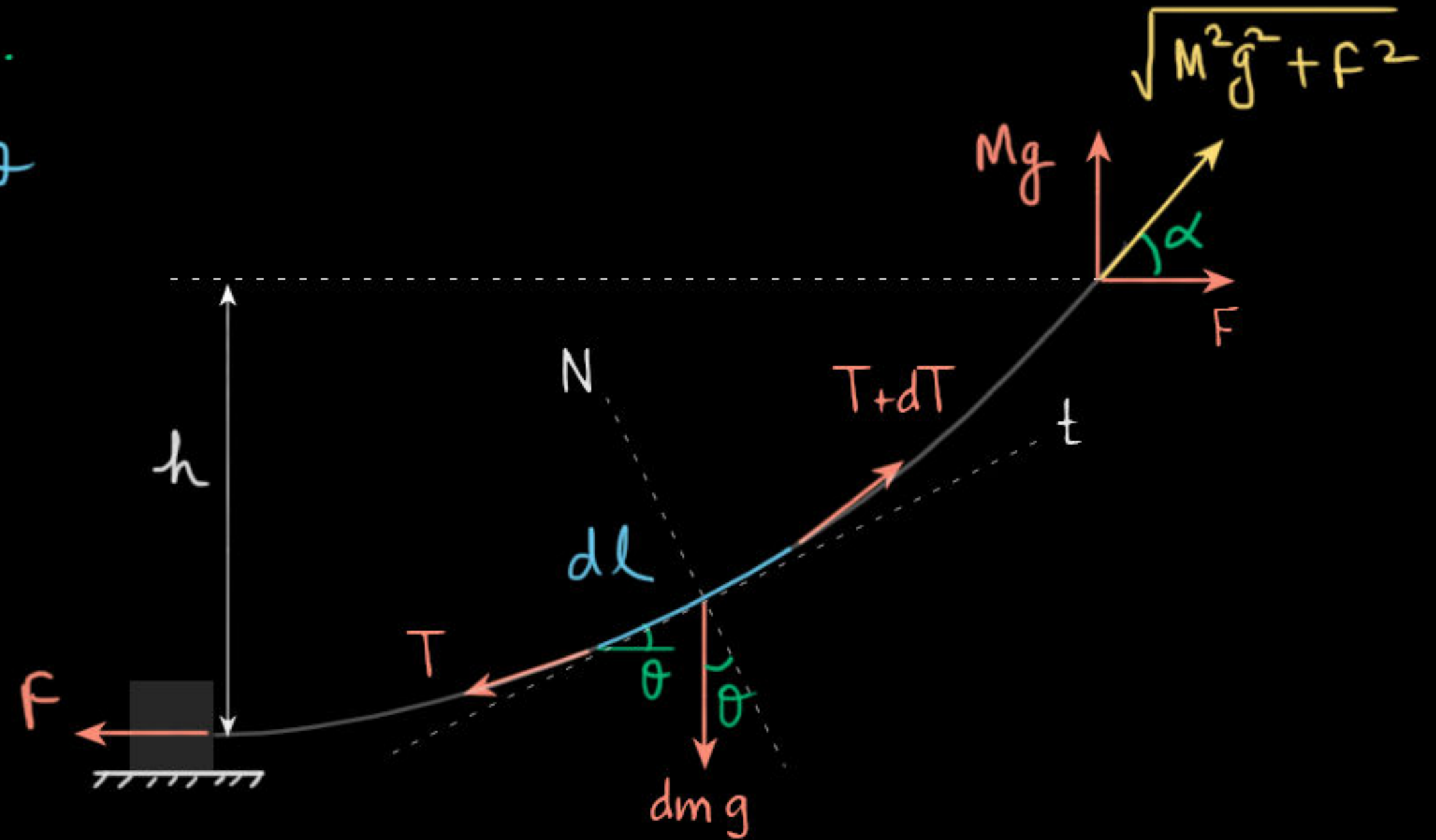
$$= \left(\frac{M}{L} dl\right) g \sin\theta$$

$$\Rightarrow \int_F^{\sqrt{M^2 g^2 + F^2}} dT = \frac{Mg}{L} \int \underbrace{dl \sin\theta}_h$$

$$\Rightarrow \sqrt{M^2 g^2 + F^2} - F = \frac{Mg}{L} h^2 \quad (1)$$

$$\Rightarrow F = 15 \text{ N} \quad (F = \text{friction, as block is at rest})$$

$$\text{Also, } \alpha = \tan^{-1} \frac{Mg}{F} = \tan^{-1} \frac{4}{3} = 53^\circ$$



When block slides $F = \mu mg$

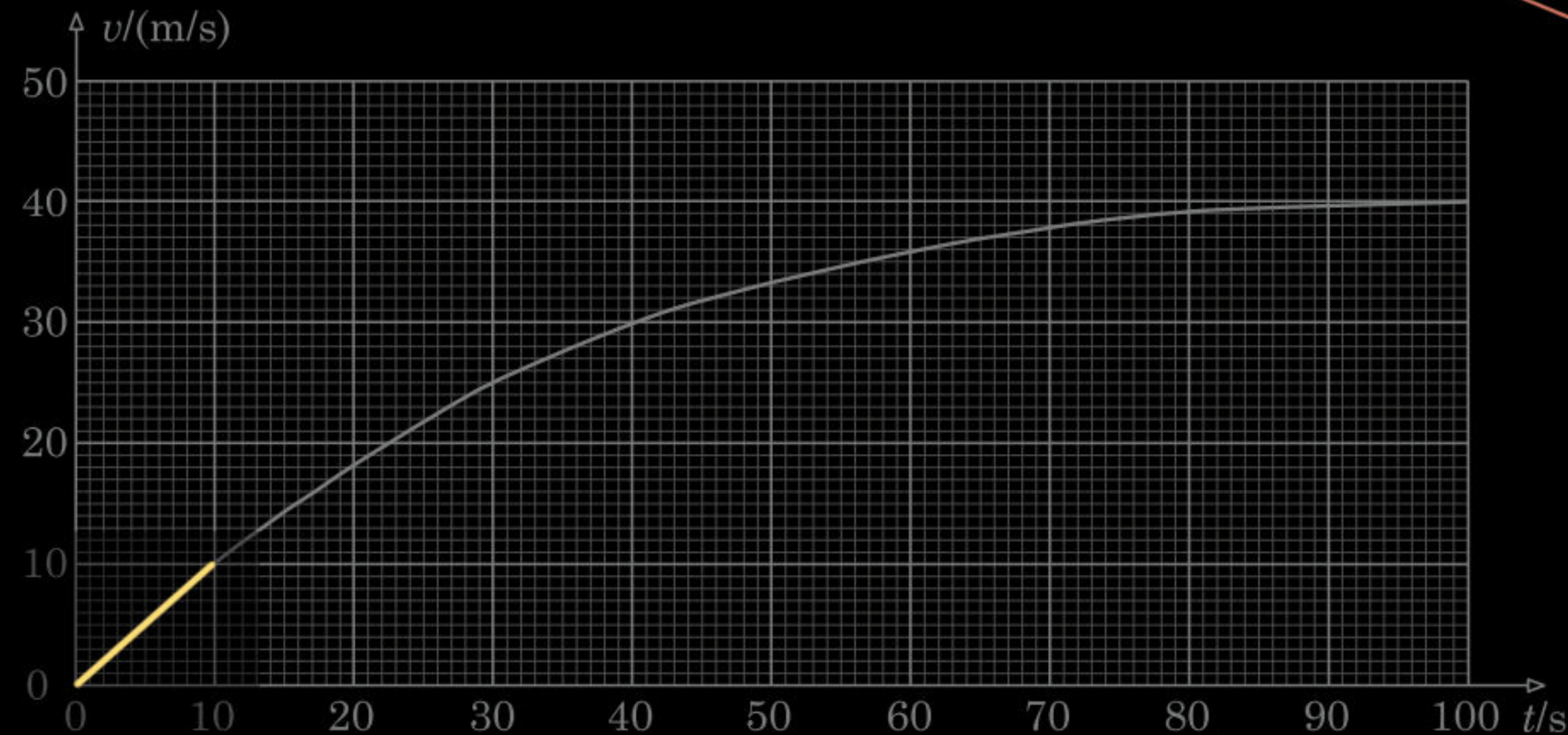
Corresponding h from equation (1) is given by:

$$h = \frac{L}{M} \left(\sqrt{M^2 + \underbrace{\mu^2 m^2}_{0.5}} - \underbrace{\mu m}_{7.5} \right)$$

$$= 1 \text{ m}$$

Velocity-time relation of a four-wheel drive car running without any gearshift on a straight level road is shown in the following graph. The engine is running at constant throttle and the wheels are maintained always at the verge of slipping. Air drag on the car depends on its speed; therefore, it can be neglected for initial few seconds. The graph can satisfactorily be treated as a straight line in every 10 s interval.

→ Force is constant = μmg



→ First 10 seconds, acceleration is only μg

$\Rightarrow \mu g = \text{Slope of graph}$

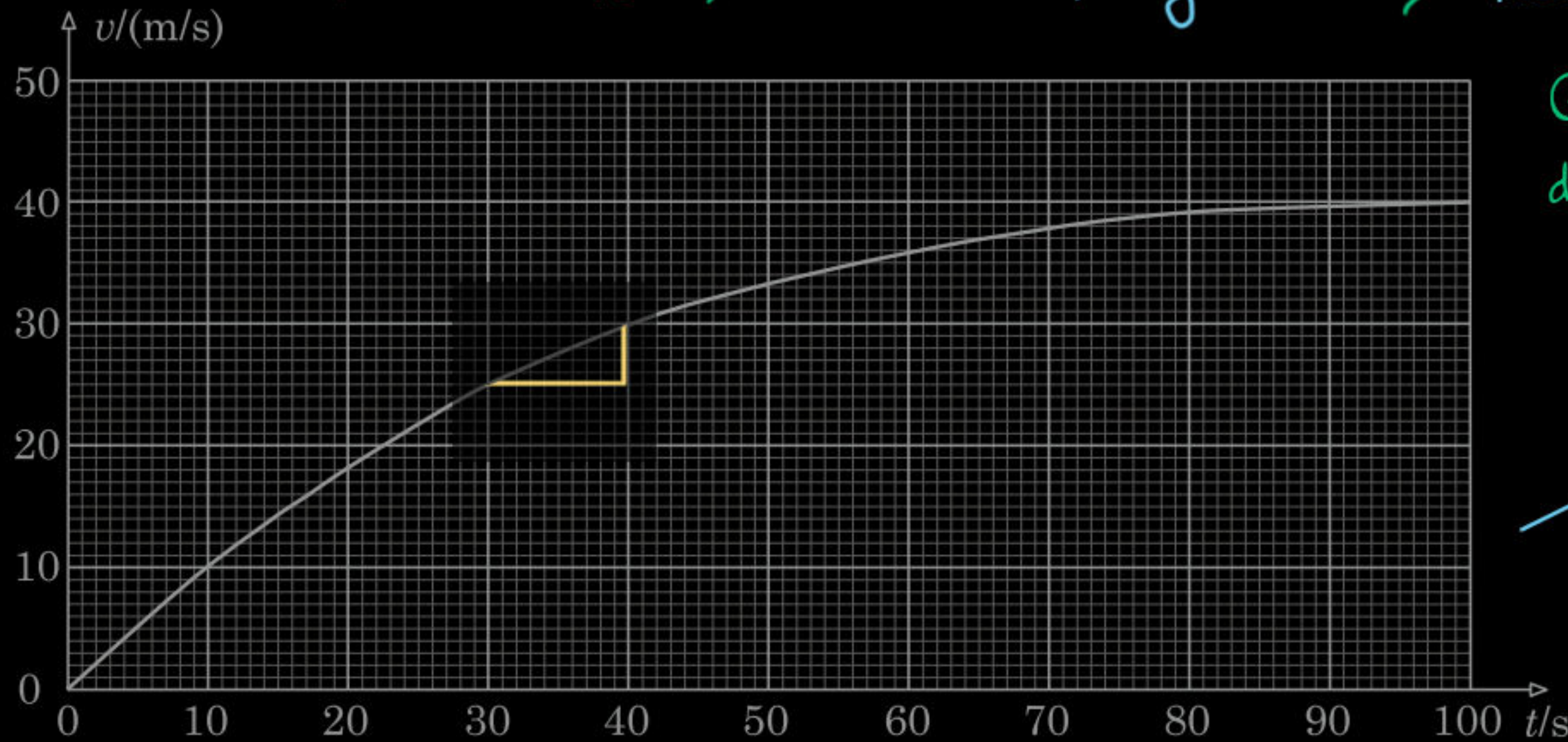
$$\Rightarrow \mu g = \frac{10}{10}$$

$$\Rightarrow \mu = \frac{1}{g} = 0.1 //$$

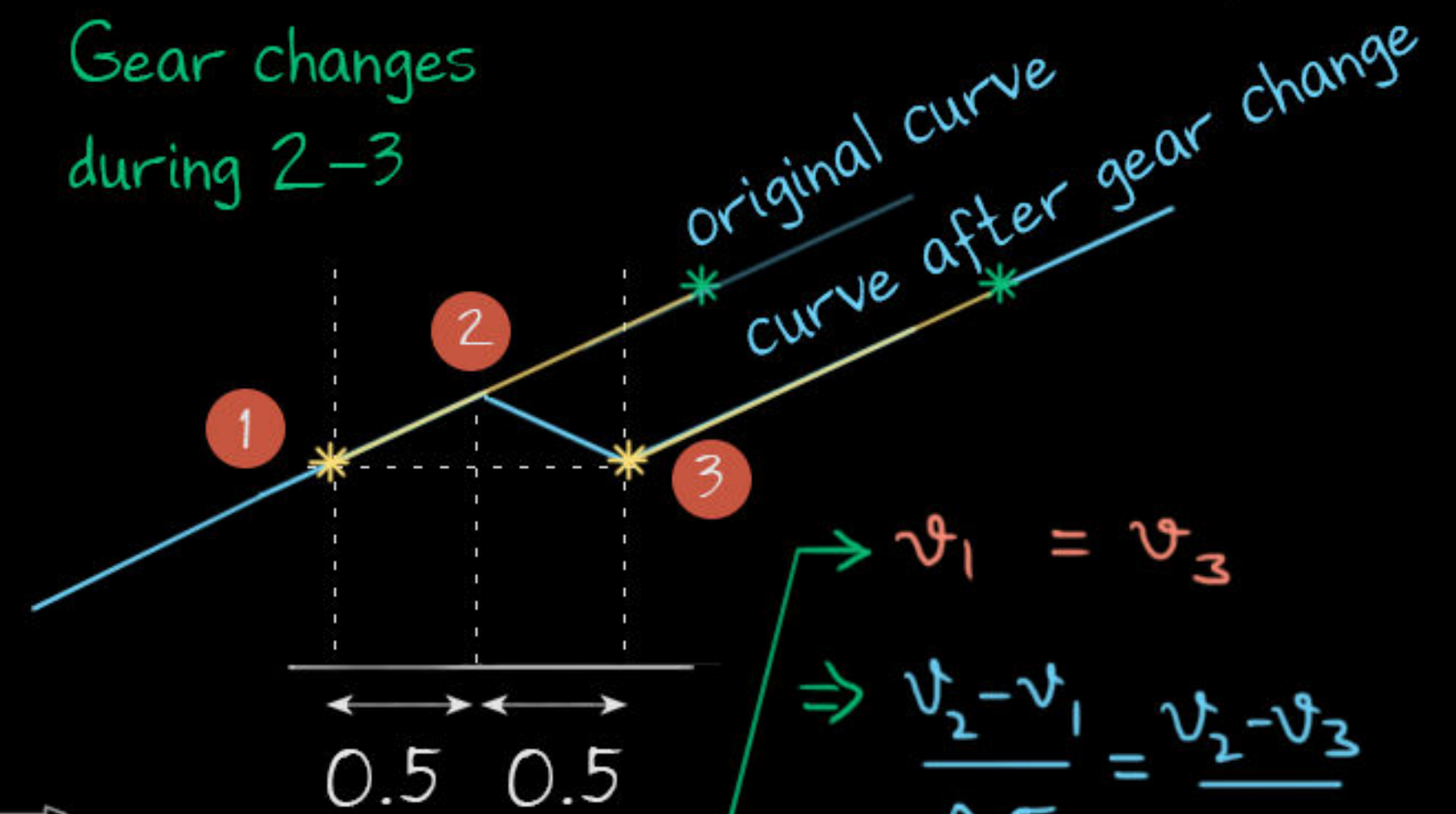
48. Coefficient of friction between the tyres and the road is closest to

- (a) 0.04 (b) 0.05
 ✓ (c) 0.10 (d) 0.20

Let viscous force be F_v . $\Rightarrow$ Net Force = $\mu mg - F_v \Rightarrow$ Acceleration = Slope of $v-t = \mu g - \frac{F_v}{m}$



Gear changes during 2-3



$$v_1 = v_3$$

$$\Rightarrow \frac{v_2 - v_1}{0.5} = \frac{v_2 - v_3}{0.5}$$

$$\therefore \text{Slope } 1-2 = \text{Slope } 2-3$$

$$\mu g - \frac{F_v}{m} = \frac{F_v}{m}$$

$$\Rightarrow \text{Slope} = \frac{F_v}{m} = \frac{\mu g}{2} = \frac{1}{2}$$

The driver shifts gears once after acquiring some speed. Gear shifting requires a period of 0.5 s, during which the engine remains declutched so that the car decelerates due to air drag. After the gearshift, clutches are released and the car follows the same velocity profile as shown in the graph with a delay of 1.0 s. Therefore, the car acquires the final velocity 1.0 s later than that acquired without a gearshift. Acceleration of free fall is 10 m/s^2 .

49. Minimum time after the car starts, when the gear was shifted is closest to (a) 10 s (b) 25 s ☒ (c) 30 s (d) 35 s

This occurs at 30 seconds.

50. Due to gearshift, how much lesser distance does the car cover during the first 100 s, as compared to the case of no gearshift?

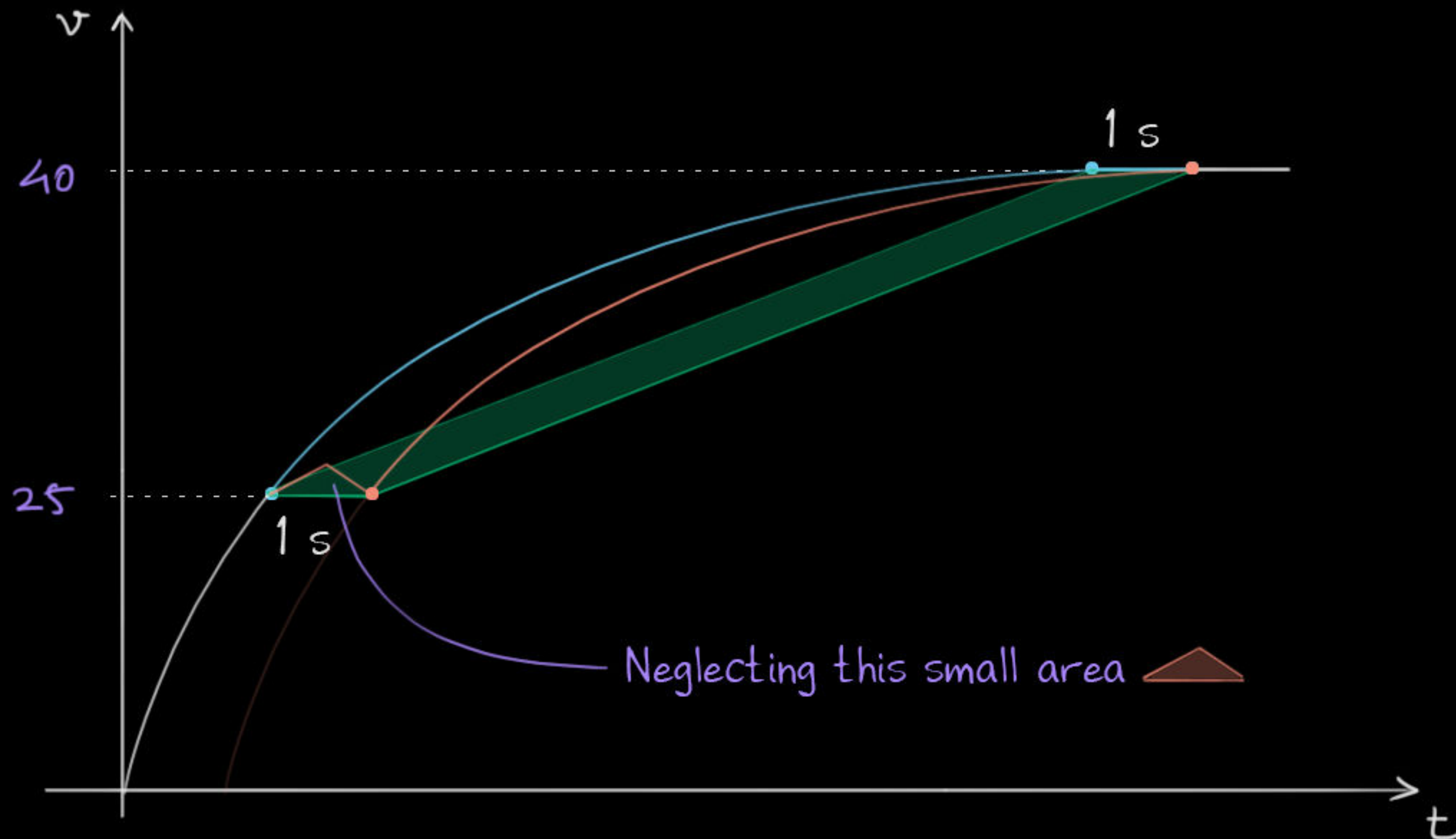
(a) 12.5 m

✓(b) 15 m

(c) 18 m

(d) 30 m

Please refer to Pathfinder chapter 1 (kinematics) mcq 8 for details of the following graph method.



$$\Delta S = \Delta \text{Area}$$

=

= base $\times$ height

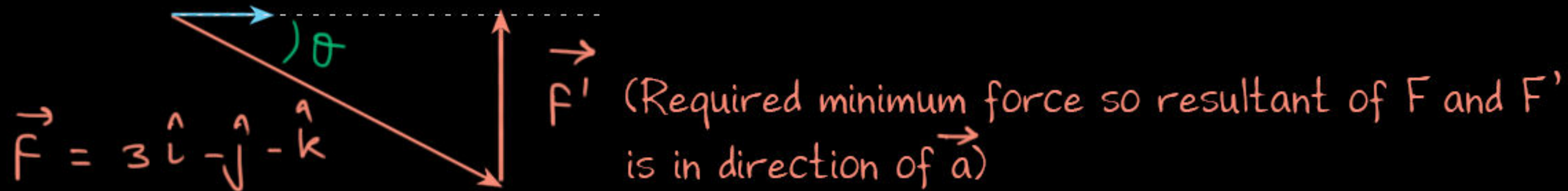
$$= 1 \times (40 - 25)$$

$$= 15 \text{ m}$$

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1. Under simultaneous action of two forces, a stationary particle starts moving parallel to a vector $\hat{i} - \hat{j}$. If one of the force is $(3\hat{i} - \hat{j} - \hat{k})$ N and the other has smallest possible magnitude, find the other force.

$$\vec{a} = \hat{i} - \hat{j}$$

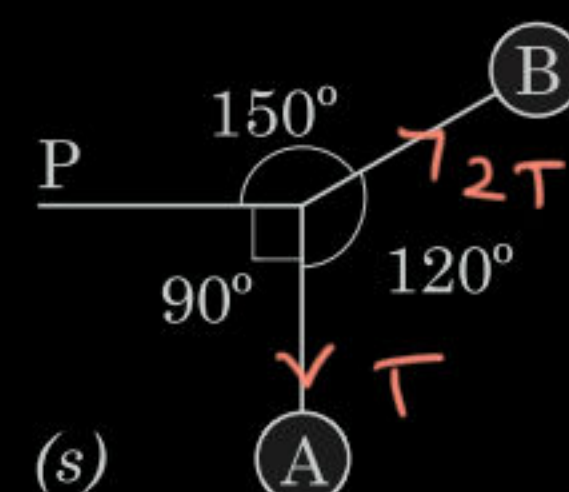
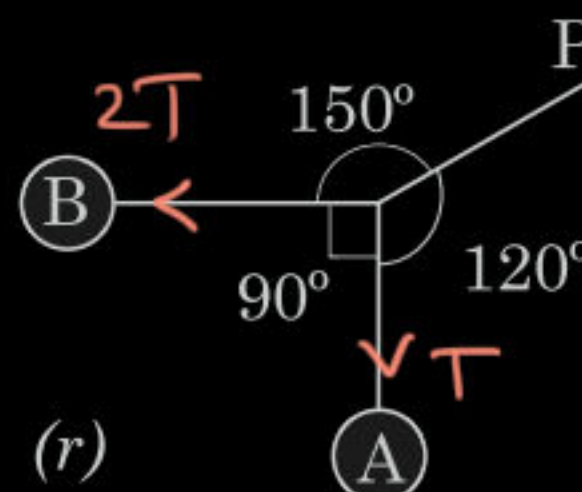
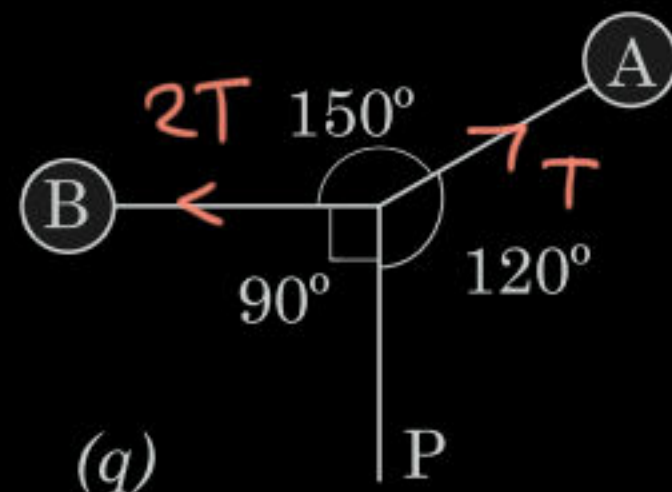
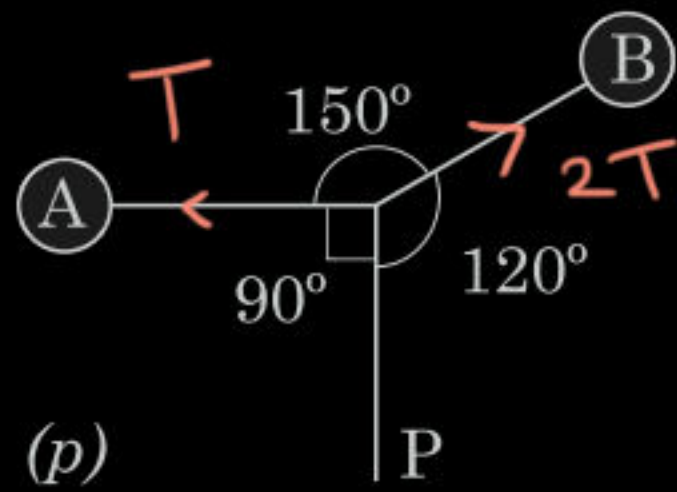


$$\begin{aligned} \therefore \vec{F}' &= (F \cos \theta) \hat{a} - \vec{F} \\ &= \left(\frac{\vec{F} \cdot \vec{a}}{a} \right) \frac{\vec{a}}{a} - \vec{F} = \frac{(3+1)}{2} (\hat{i} - \hat{j}) - (3\hat{i} - \hat{j} - \hat{k}) = -\hat{i} - \hat{j} + \hat{k} \end{aligned}$$

2. Two small discs A and B of masses 1 kg and 2 kg are connected by a light cord that is connected at its midpoint to another light cord. This assembly is placed on a frictionless horizontal floor in such a way that the segments of the cord connecting the disc and the new cord remain straight making angles of 90° , 120° and 150° . For this description, following four arrangements are suggested.

The free end P of the new cord is pulled by a 10 N force and both the discs begin to move with accelerations of equal magnitudes.

- (a) Which of the above arrangements satisfy the given condition?
 (b) What is the magnitude of the acceleration?



As junction is massless, net force on it must be zero.
 $\Rightarrow$ Perpendicular to the direction of P forces must be balanced.

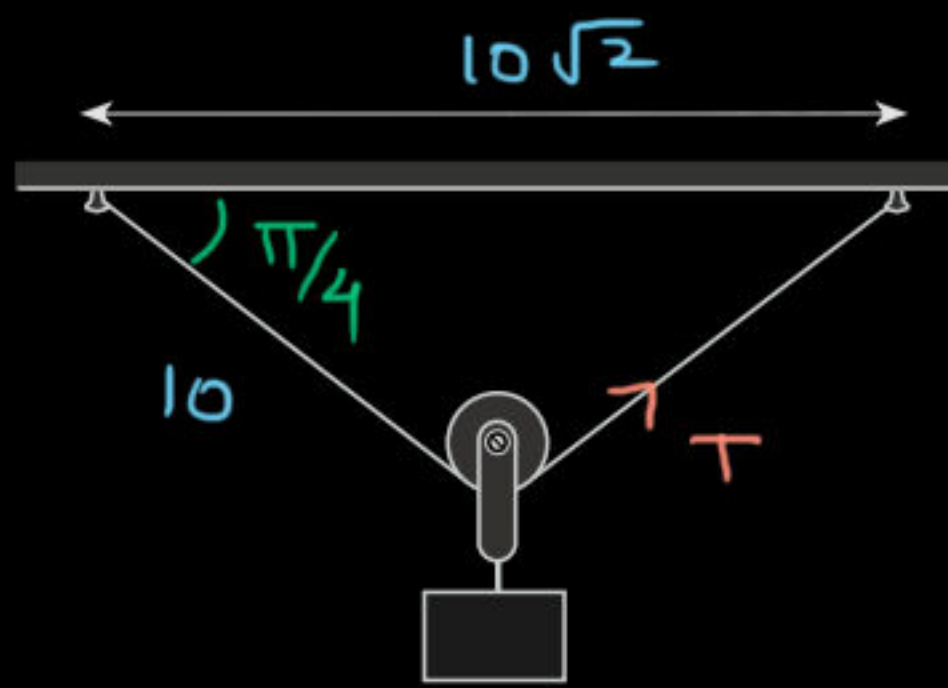
$$T \neq 2T \cos 30^\circ \quad 2T \neq T \cos 30^\circ \quad 2T \sin 30^\circ \neq T \sin 60^\circ \quad T = 2T \cos 60^\circ //$$

Forces along P must be balanced too.

$$\therefore 10 = 2T \cos 30^\circ$$

$$\Rightarrow T = 10/\sqrt{3} = ma$$

$$\Rightarrow a = 10/\sqrt{3} //$$



3. A light and inextensible string of length $l = 20 \text{ m}$ tied between two nails, supports a frictionless pulley of weight $W_p = 10\sqrt{2} \text{ N}$ from which a block of weight $W_b = 10\sqrt{2} \text{ N}$ is also suspended as shown in the figure. The nails are fixed in a level a distance $x = 10\sqrt{2} \text{ m}$ apart. Radius of the pulley is $r = 10 \text{ cm}$. How much normal reaction per unit of its length does the string apply on the pulley?

$$2T \cos \frac{\pi}{4} = (10\sqrt{2} + 10\sqrt{2})$$

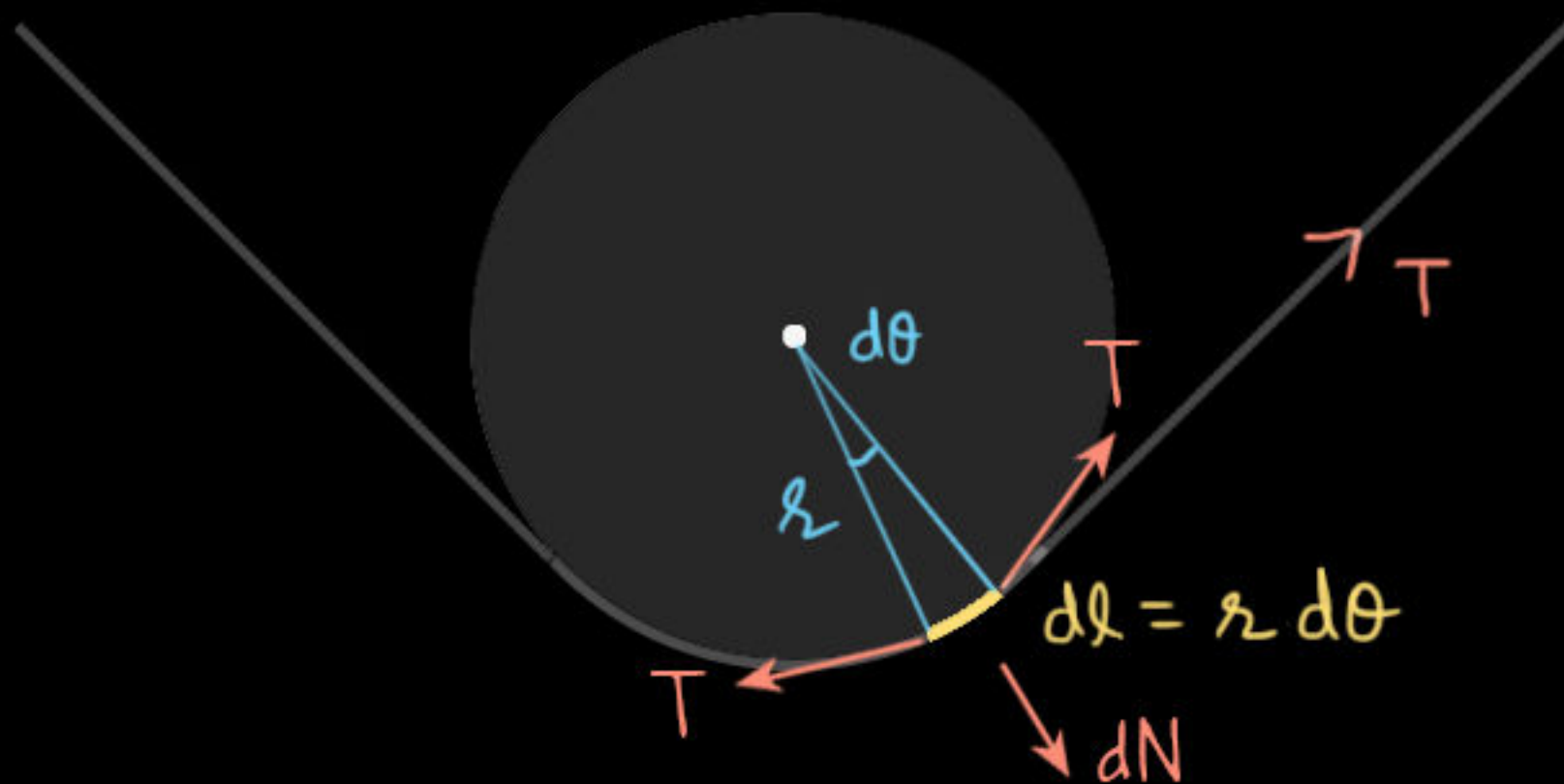
$$\Rightarrow T = 20 \text{ N}$$

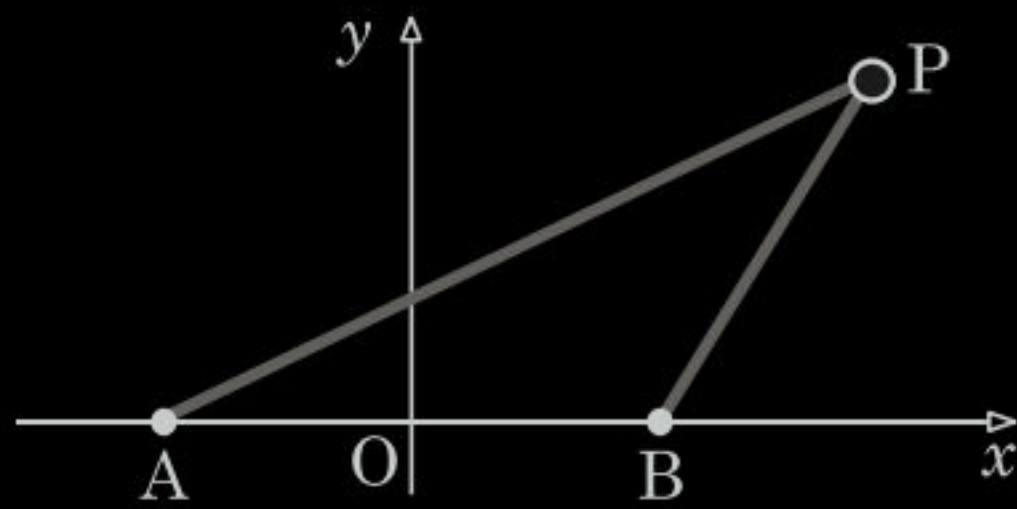
- The pulley is frictionless and rope is light, Tension in rope is same everywhere.

Balancing forces on rope element:

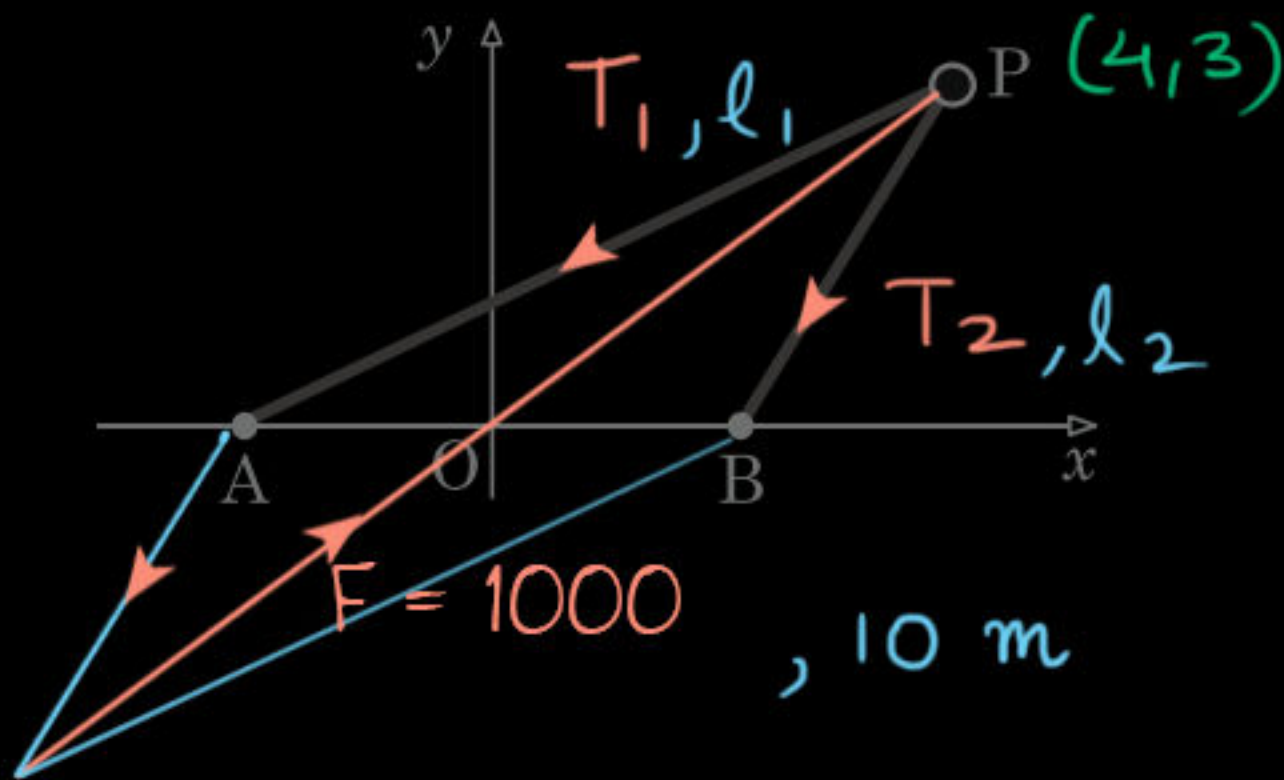
$$dN = T d\theta = T \frac{dl}{r}$$

$$\Rightarrow \frac{dN}{dl} = \frac{T}{r} = \frac{20}{0.1} = 200 \text{ N/m}$$





4. Two identical elastic cords of negligible relaxed lengths are tied at one of their ends to fixed nails A and B that are equidistant from the origin O. The other ends of the strings are tied to a small ball. To hold the ball in equilibrium at a point P (4 m, 3 m), a force of magnitude $F = 1000$ N is required. Assuming free space conditions, find force constant of the cords?



∴ The relaxed lengths are zero, Tensions in cords are proportional to their lengths.

We complete the parallelogram, such that F will balance T_1 and T_2 . Length of diagonal will be 10m.

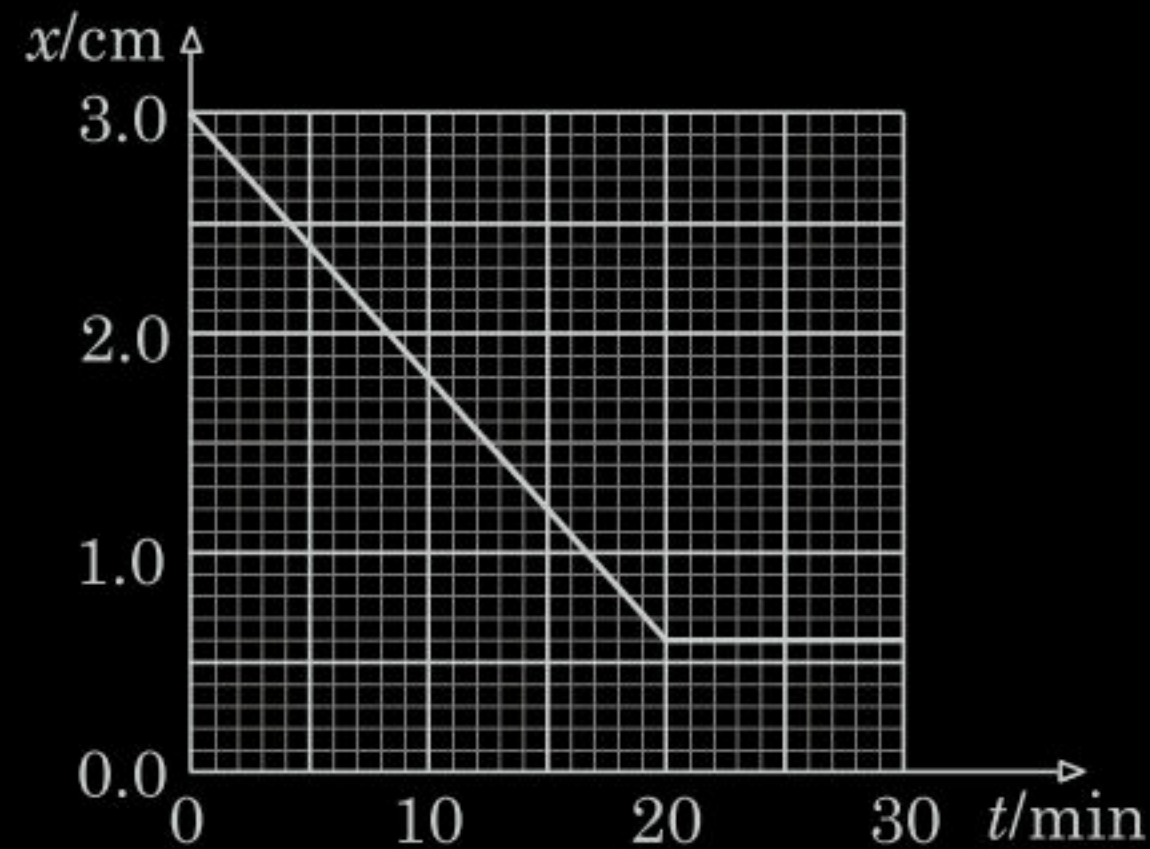
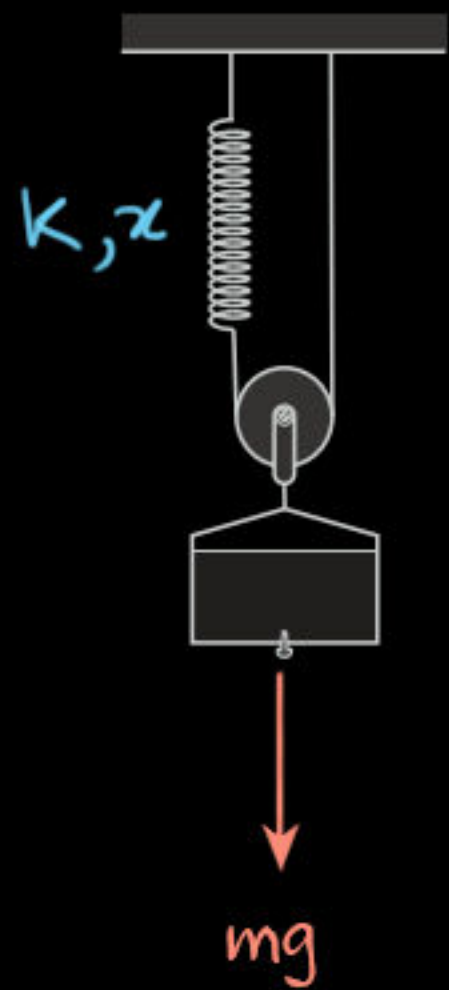
$$\therefore \frac{1000}{10} = \frac{T_1}{l_1} = \frac{T_2}{l_2} = k = 100 \text{ N/m}$$

Similar to NLM MCQ 7

There we completed the triangle

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5. A wide container filled with water is suspended with the help of a light spring of stiffness 1000 N/m , a light inextensible cord and an ideal pulley. Initially, when the system is in equilibrium, the plug inserted in an orifice at the centre of the bottom of the container is pulled out. The extension in the spring is recorded and shown in the adjoining graph. Find the time rate of flow of water from the orifice.



At a general time, let extension be x and mass of container be m .

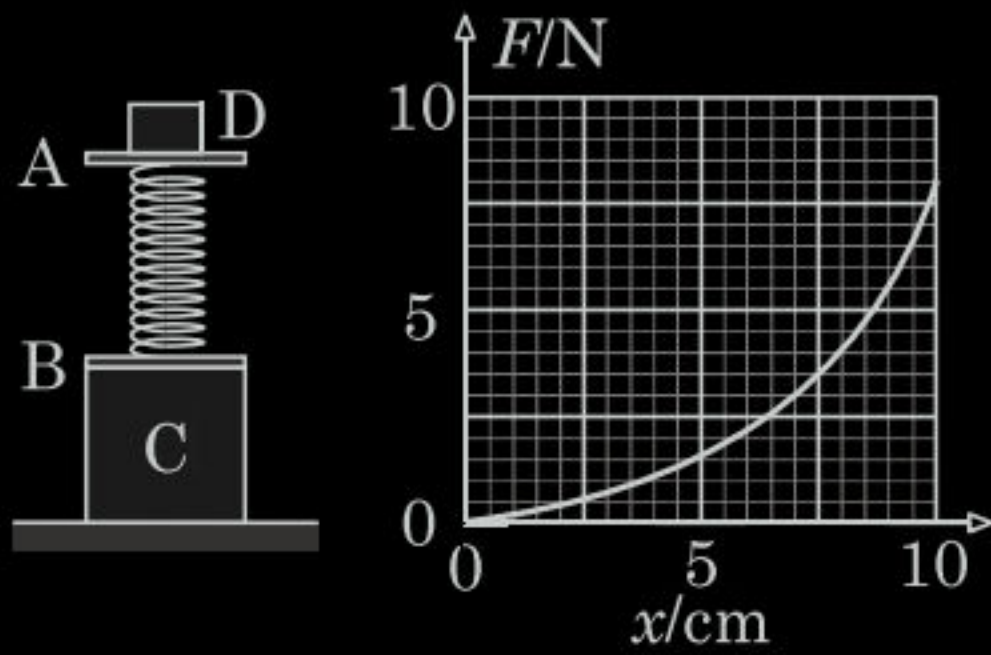
$$mg = 2kx$$

differentiate:

$$\frac{dm}{dt} g = 2k \frac{dx}{dt}$$

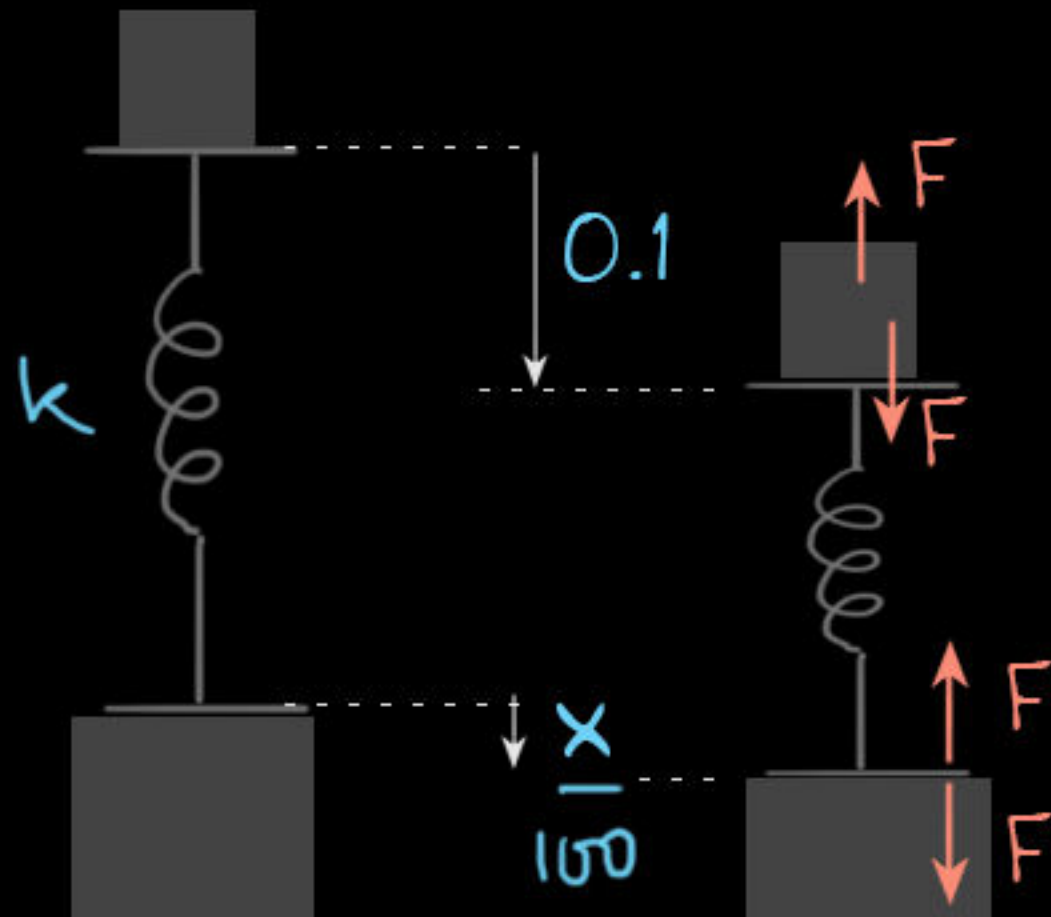
$$\Rightarrow \frac{dm}{dt} = \frac{2k}{g} \left(\frac{3 - 0.6}{20 \times 60} \right) \times 10^{-2}$$

$$= 4 \text{ gm/s}$$



Let the spring exerts the force F .

$$\therefore F = mg = k \Delta x$$

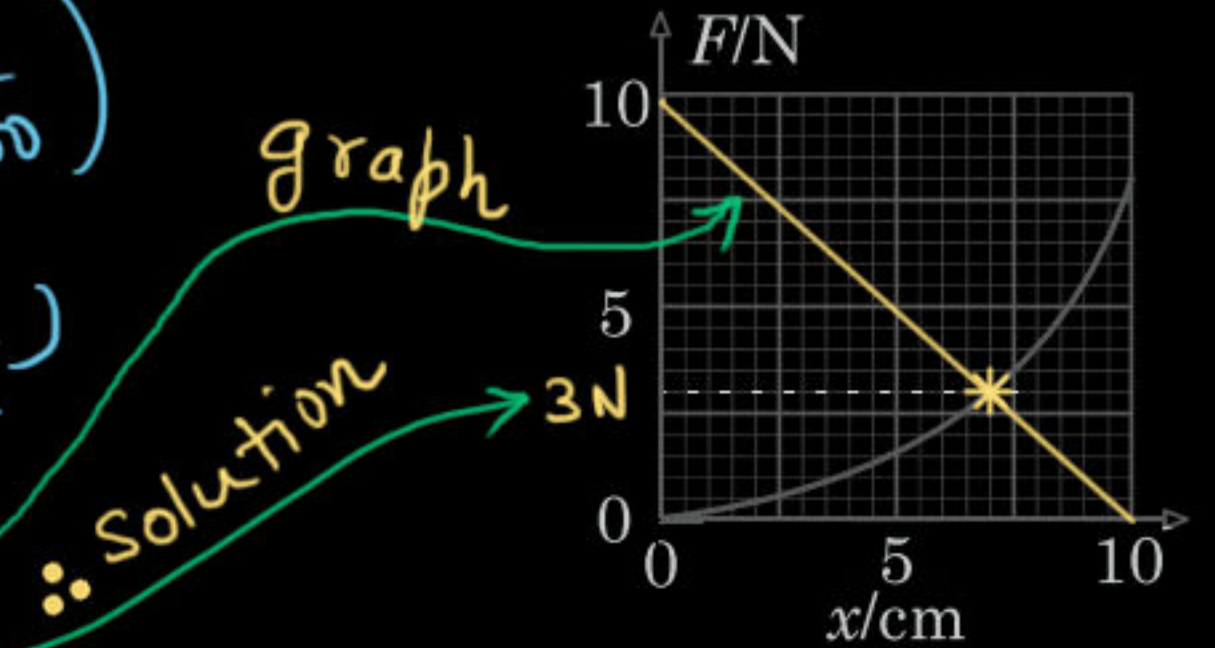


6. Two light discs A and B are attached at the ends of a spring of force constant 100 N/m . The assembly is placed on a rubber pad C, which is placed on the floor. When a load D is placed on the disc A, the disc shifts downwards and stays there in equilibrium simultaneously the rubber pad is compressed and the disc B shifts downwards by a distance x . The restoring force F of the rubber pad varies with its compression x according to the given graph. Acceleration due to gravity is 10 m/s^2 . Find mass of the load, so that when equilibrium is established, the disc A shifts 10 cm downwards.

$$\text{On spring: } F = k \left(0.1 - \frac{x}{100} \right)$$

$$\Rightarrow F = 100 \left(\frac{10 - x}{100} \right)$$

$$\Rightarrow F = 10 - x$$



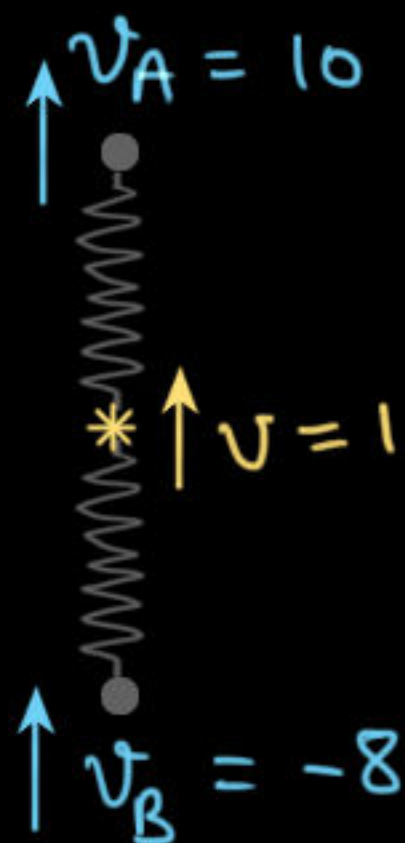
As plates and springs are massless,
 F will be transmitted everywhere unchanged.

$$\therefore F = 3 = mg \Rightarrow m = 0.3 \text{ kg}$$

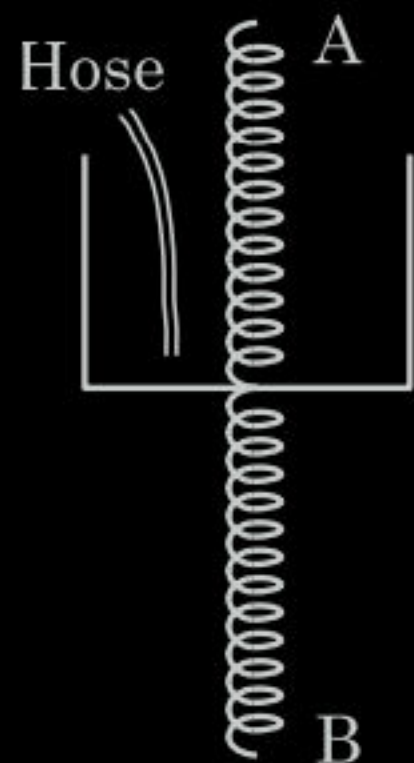
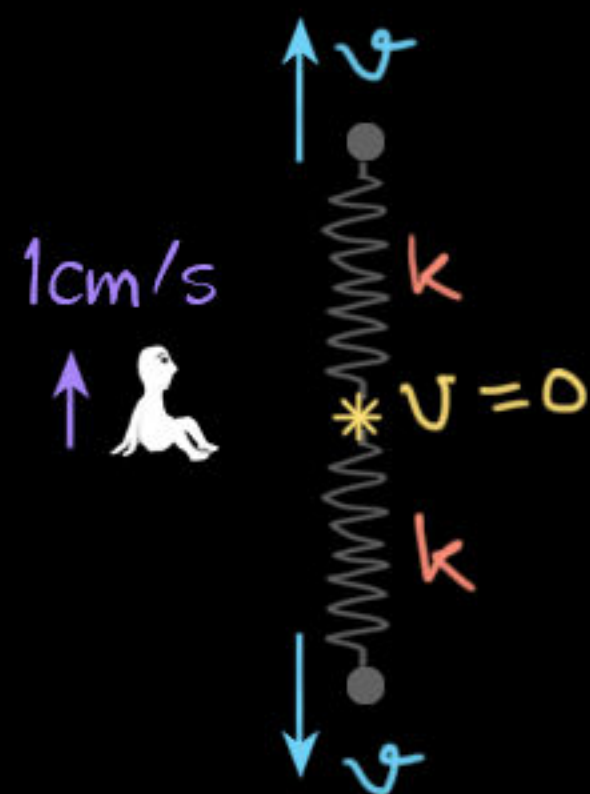
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gnd frame:



Moving frame:



7. Two identical springs each of force constant $k = 10 \text{ N/m}$ are attached at the midpoint of the bottom of an inertia-less cup, one from the inside and the other from the outside. Free ends A and B of the springs are held maintaining the springs vertical. The ends A and B of the springs are made to move simultaneously with constant velocities $v_A = 10.0 \text{ cm/s}$ upwards and $v_B = 8.0 \text{ cm/s}$ downwards respectively. The moment when the ends start moving, the cup starts collecting water at the rate $r = 1.0 \text{ g/s}$ from a hose with a negligible velocity relative to the cup. Find velocity of the cup. Acceleration due to gravity is $g = 10 \text{ m/s}^2$.

In gnd frame:

Without the cup, Tension, and hence extension in both springs will be equal. Therefore the junction * will always be at midpoint of system. And hence it will be traveling at:

Assume acceleration of cup to be zero.

$$v^* = \frac{v_A + v_B}{2} = 1 \text{ cm/s} \uparrow$$

$$mg - 2kx = ma$$

$$\text{Unsolvable: } mg - 2kx = m \frac{d^2x}{dt^2}$$

∴ In a reference frame moving up with v^*

Junction is at rest and for parallel springs, equivalent spring constant $= 2k$

∴ With the cup: $mg = 2kx$

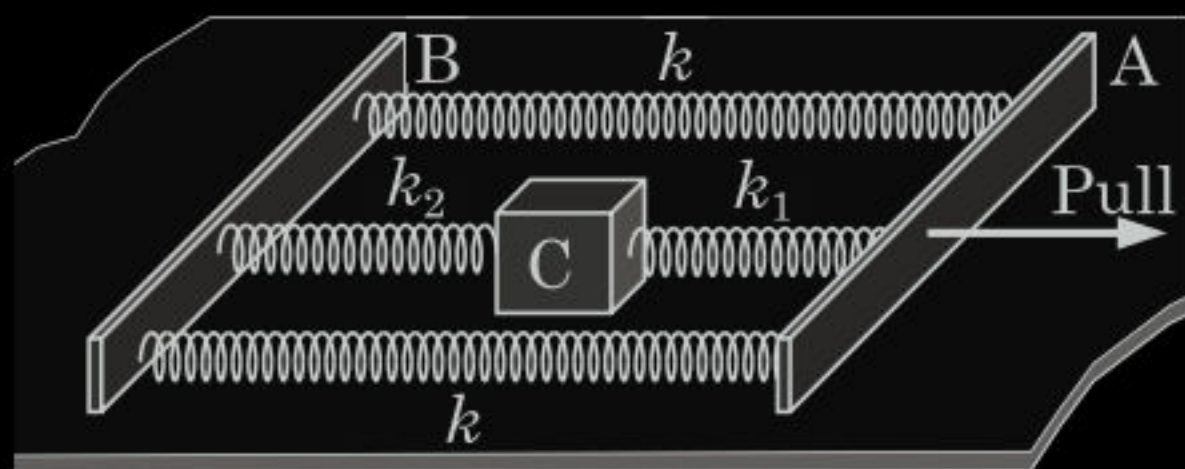
$$\Rightarrow \frac{dm}{dt}g = 2k \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{mg}{2k} = 0.05 \downarrow$$

v^* in reference frame

by Ankit Singhvi

$$\begin{aligned} \vec{v}_{*/G} &= \vec{v}_{*/f} + \vec{v}_{f/G} \\ &= 0.05 \downarrow + 1 \uparrow = 0.95 \text{ cm/s} \uparrow \end{aligned}$$

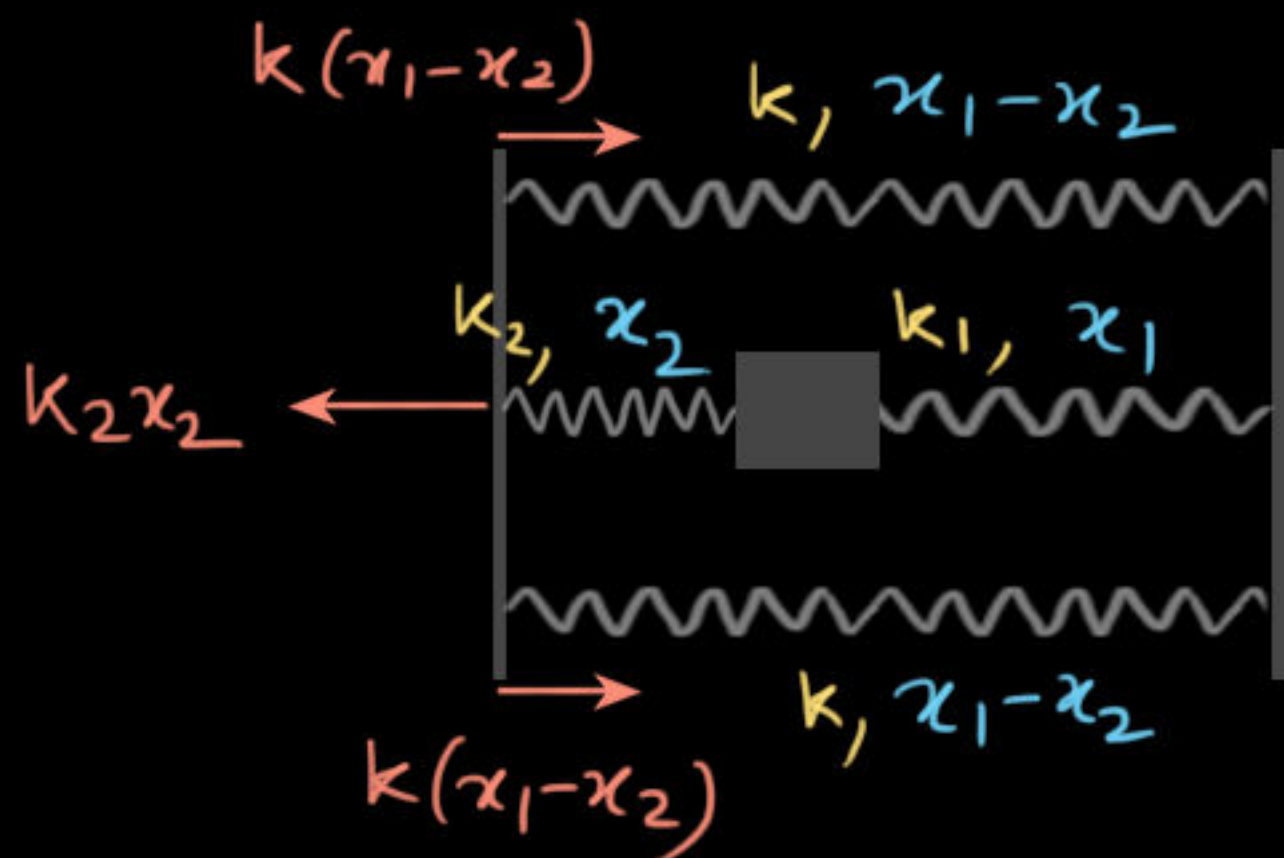
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8. An assembly placed on a frictionless floor, consists of two light bars A and B, four light springs and a block C of mass m . Stiffness of each springs is shown in the figure. Initially all the springs are relaxed and the bars are parallel to each other. The bar A is pulled from its midpoint towards the right by a force that increases gradually from zero. When the block acquires an acceleration a , by what amount will the separation between the bars increase?

Let the extension in first spring be x_1
and compression in second spring be x_2 .

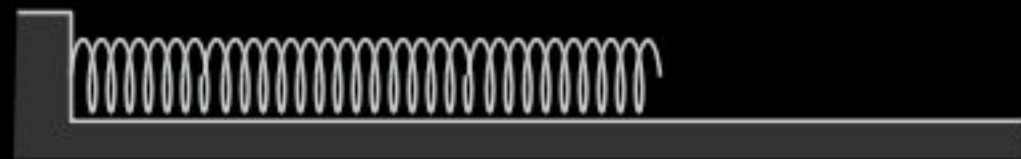
∴ Extension in big springs will be: $x_1 - x_2$



On block: $k_1 x_1 + k_2 x_2 = ma$ ①

On left plate: $k_2 x_2 = 2k(x_1 - x_2)$ ②

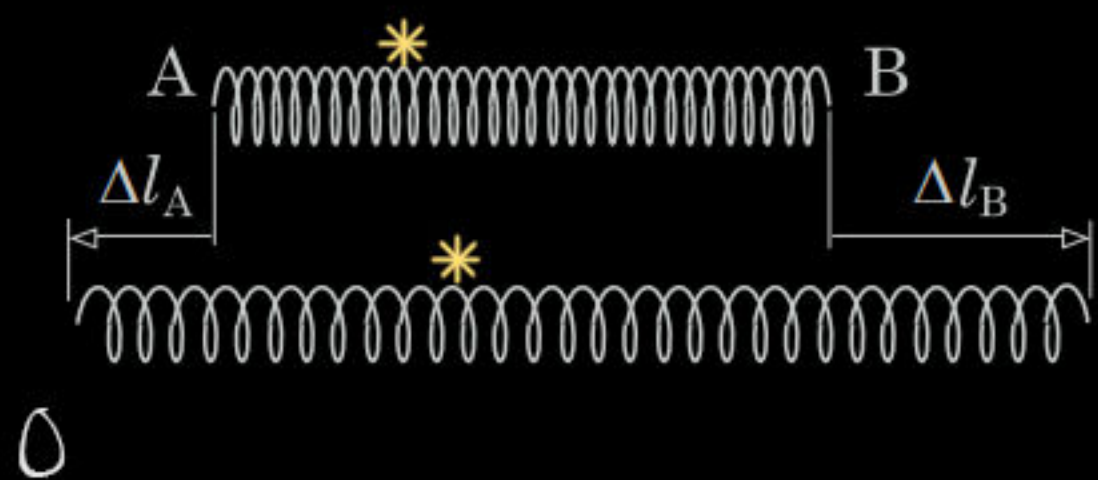
From 1,2: $x_1 - x_2 = \frac{mk_2 a}{k_1 k_2 + 2k(k_1 + k_2)}$



9. A uniform massless spring if extended or compressed, distance between every two consecutive turns (pitch) changes by the same amount. A spring obeying this property and Hooke's law is called a linear spring.

(a) A linear spring of relaxed length $l_0 = 30$ cm is attached at one end to a wall. If the other end is pulled away from the wall with a force $F = 60$ N, $p = 6^{\text{th}}$ turn from the wall reaches a position, where $q = 8^{\text{th}}$ turn of relaxed spring was. Find force constant of the spring.

(b) The ends A and B of a linear spring of relaxed length l_0 are so pulled that the ends shift by distances $\Delta l_A = 5$ cm and $\Delta l_B = 25$ cm as shown in the figure. Find shift of a point of the spring that was at a distance l_0/n ($n = 3$) from the end A in relaxed spring.



6^{th} turn reaches 8^{th} turn.

$$\therefore \text{Fractional change in length} = \frac{8-6}{6} = \frac{1}{3}$$

Given: $l_0 = 30$

$$\Rightarrow \Delta l_0 = \frac{30}{3} = 10$$

$$\text{Given: } 60 \text{ N} = k \Delta l_0 = k \left(\frac{10}{100} \right)$$

$$\Rightarrow k = 600 \text{ N/m} //$$

$$\text{Initial position of mark w.r.t } O = \Delta l_A + \frac{l_0}{n}$$

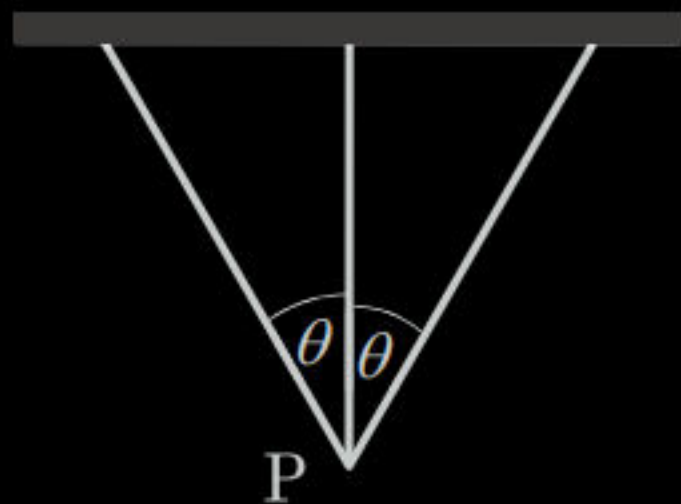
$$\text{Shift} = \text{Final position} - \text{initial position}$$

$$= \frac{\Delta l_A + \cancel{l_0} + \Delta l_B}{n} - \left(\Delta l_A + \frac{\cancel{l_0}}{n} \right)$$

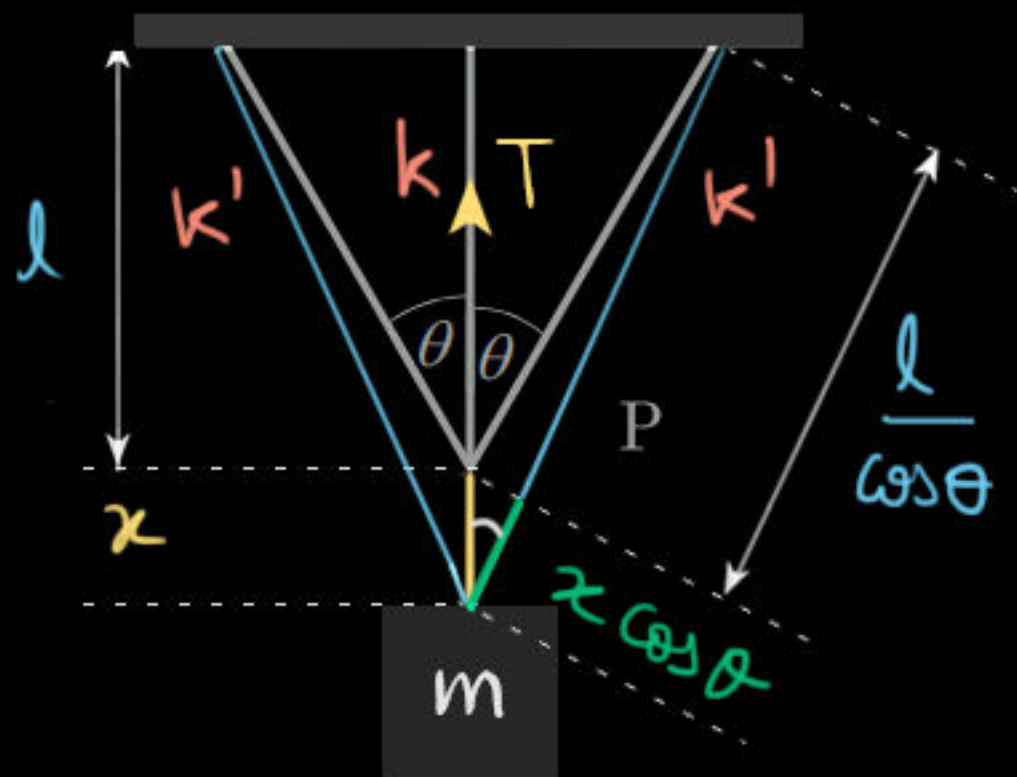
$$= \frac{\Delta l_A(1-n) + \Delta l_B}{n} //$$

by Ankit Singhvi

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10. Three segments cut from a long elastic light cord are knotted at point P. The other ends of the cords are attached to the ceiling so that all the segments are in a vertical plane and angles between the outer and the middle segments each being θ as shown in the figure. A load of mass m is suspended from the knot P. If extensions in the cords are negligible as compared to their relaxed lengths, find the tensile force T developed in the middle cord. Acceleration due to gravity is g .

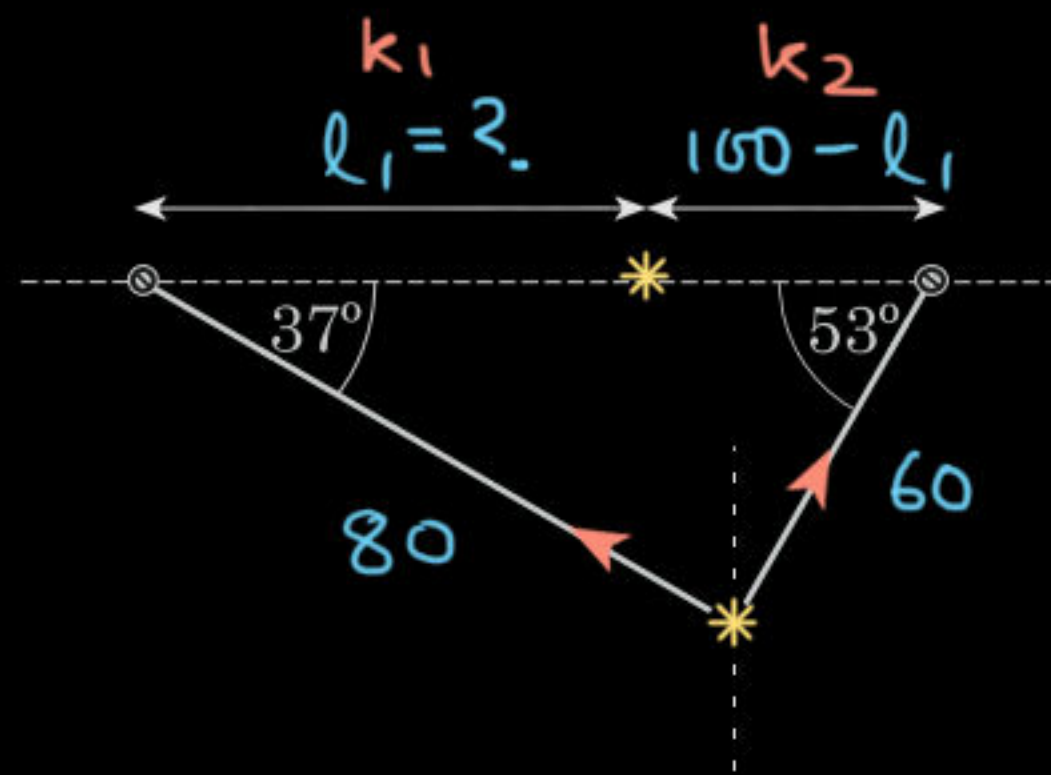


Let the middle cord's spring coefficient be k and length l .
And it is pulled down by negligible distance x .

$$\text{For the cords: } kx = k' \left(\frac{x}{\cos \theta} \right) \Rightarrow k' = k \cos \theta$$

$$\begin{aligned} \text{Force on block: } mg &= kx + 2 \left[(k' x \cos \theta) \cos \theta \right] \\ &= kx + 2kx \cos^3 \theta \end{aligned}$$

$$\Rightarrow T = kx = \frac{mg}{1 + 2 \cos^3 \theta}$$



11. A light elastic cord is tied between two nails in the same level **100 cm apart**. Distance between the nails is equal to the relaxed length of the cord. **A bead is glued somewhere on the cord and then released**. When equilibrium is established, elongated sections of the cord make angles **37° and 53°** with the horizontal. At **what distance from the left nail was the bead glued?**

For the cords:

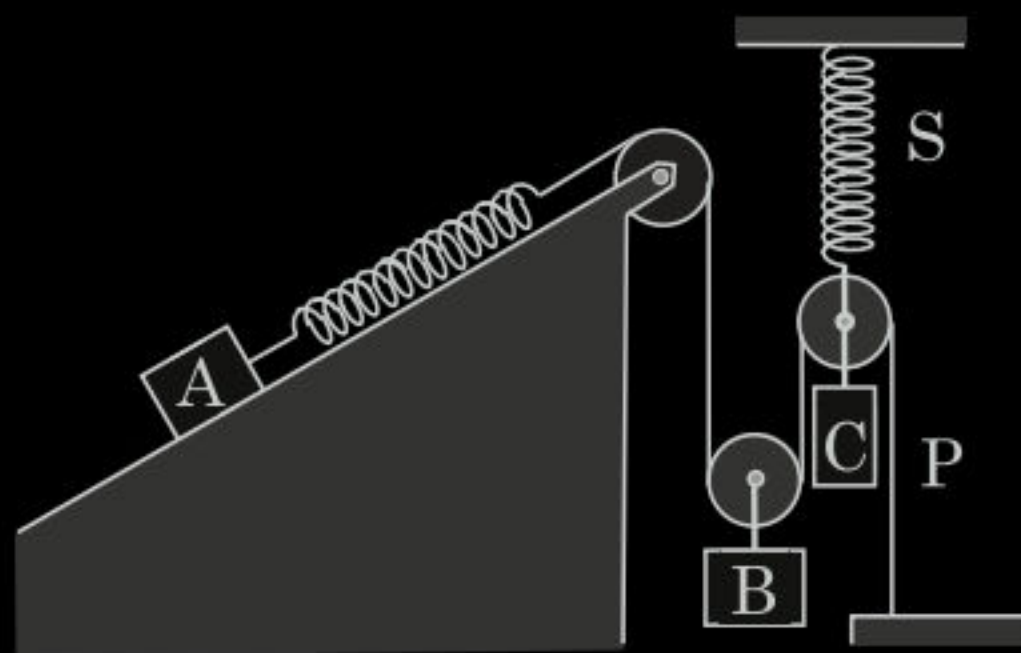
$$k_1 l_1 = k_2 (100 - l_1) \quad (1)$$

Horizontal forces are balanced.

$$\therefore k_1 (80 - l_1) \cos 37^\circ = k_2 [60 - (100 - l_1)] \cos 53^\circ \quad (2)$$

Dividing 2 and 1:

$$\frac{(80 - l_1) \frac{4}{5}}{l_1} = \frac{(l_1 - 40) \cdot \frac{3}{5}}{100 - l_1} \xrightarrow{\text{Solving quadratic}} l_1 = 59.17 //$$

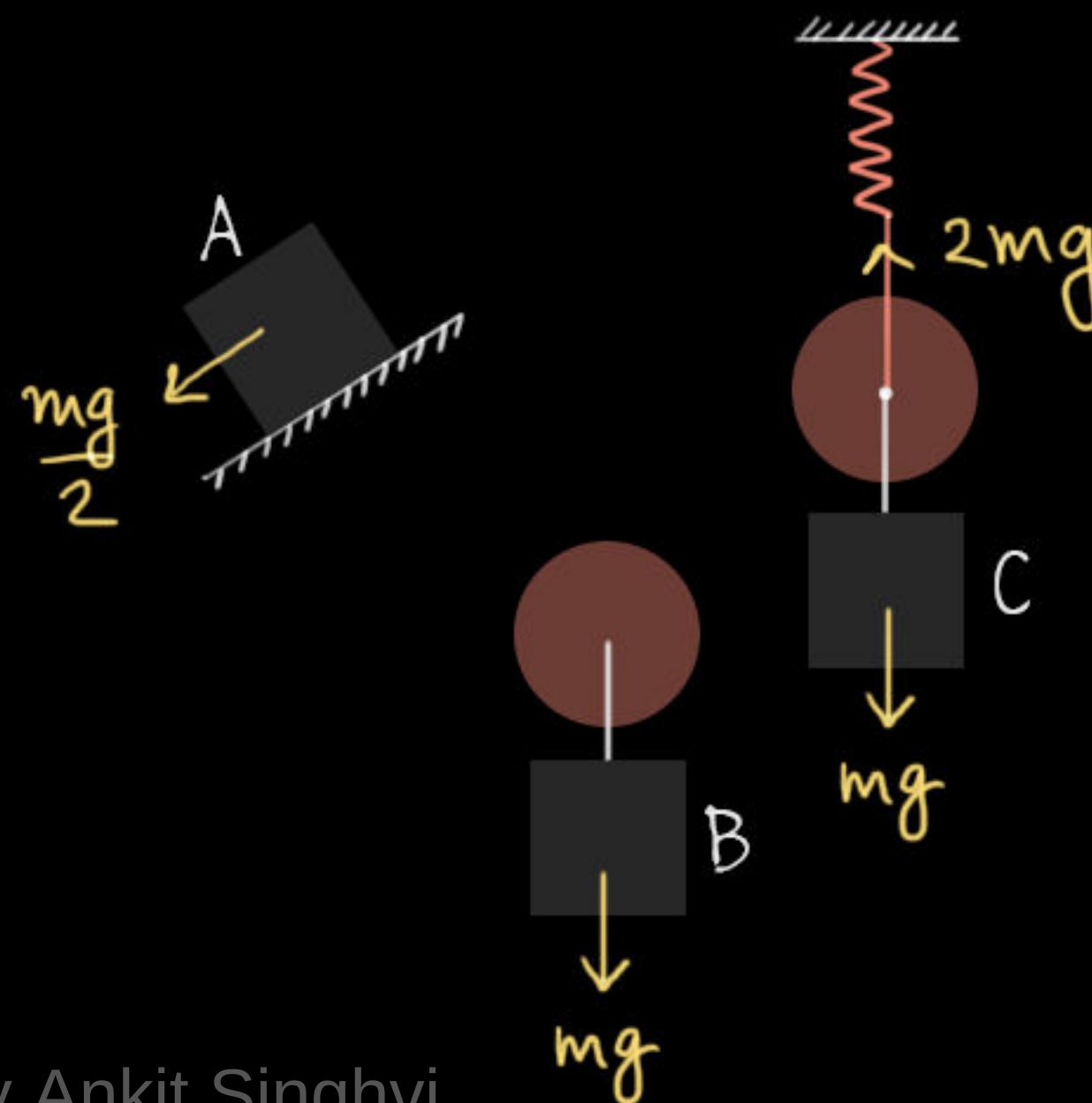
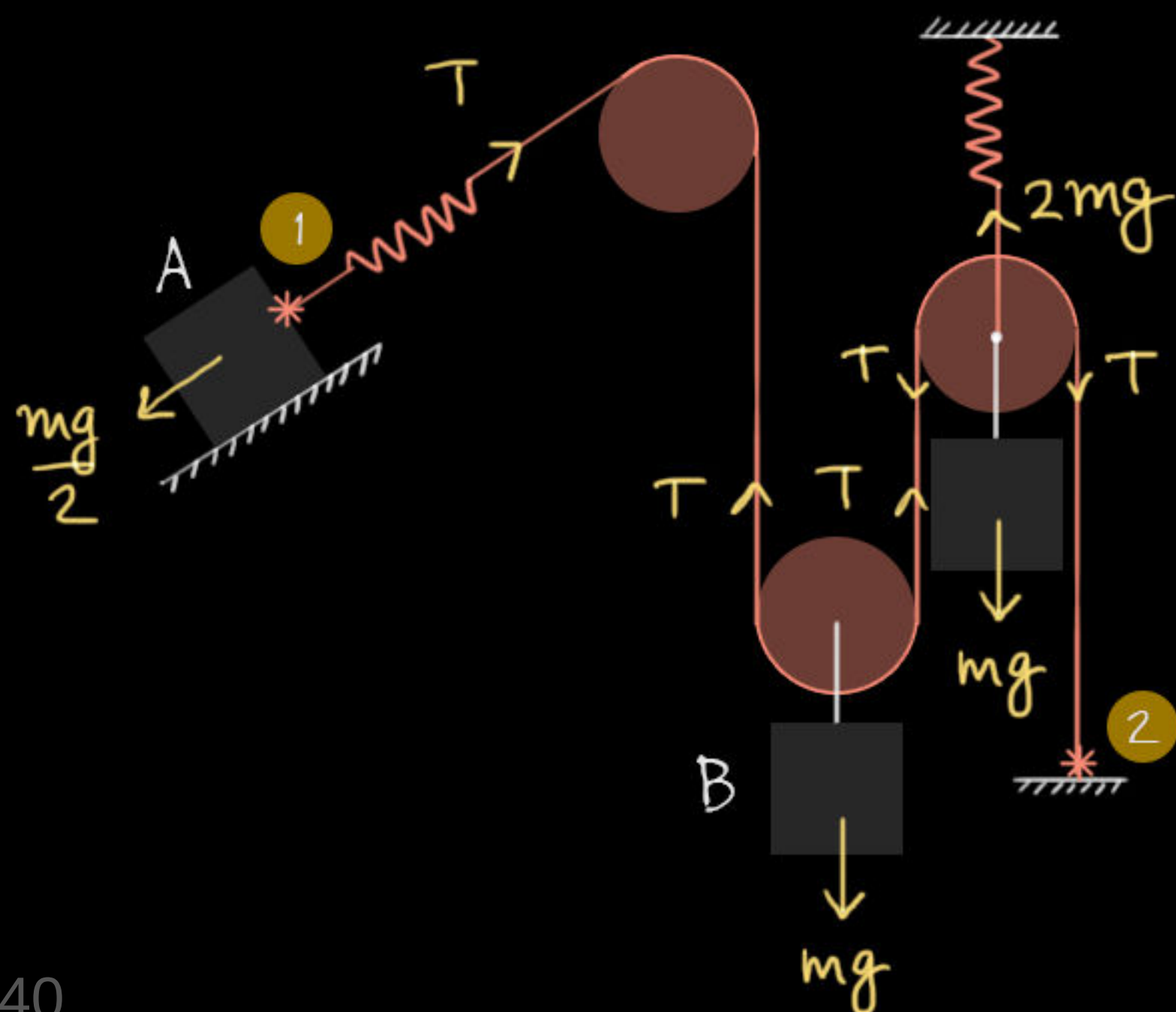


12. The system shown in the figure is in equilibrium. The blocks A, B and C are of equal masses, the inclined face is frictionless, the pulleys are light and frictionless, the thread is light and inextensible, the springs are light and the blocks B and C are connected to the axles of the respective pulleys by rigid links. Find accelerations of the blocks A, B and C, immediately after

- (a) the thread is cut at point P. (b) the spring S is cut.

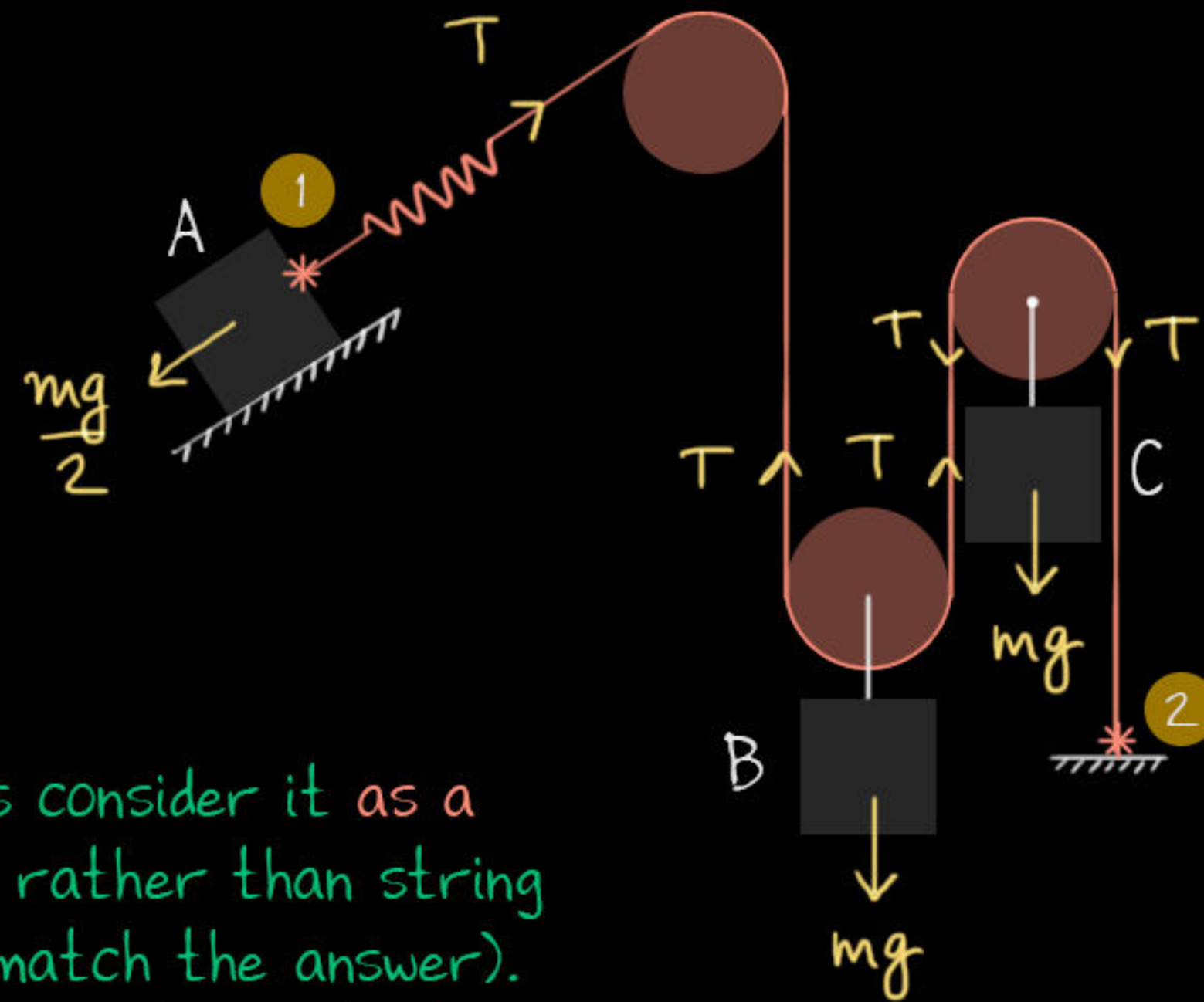
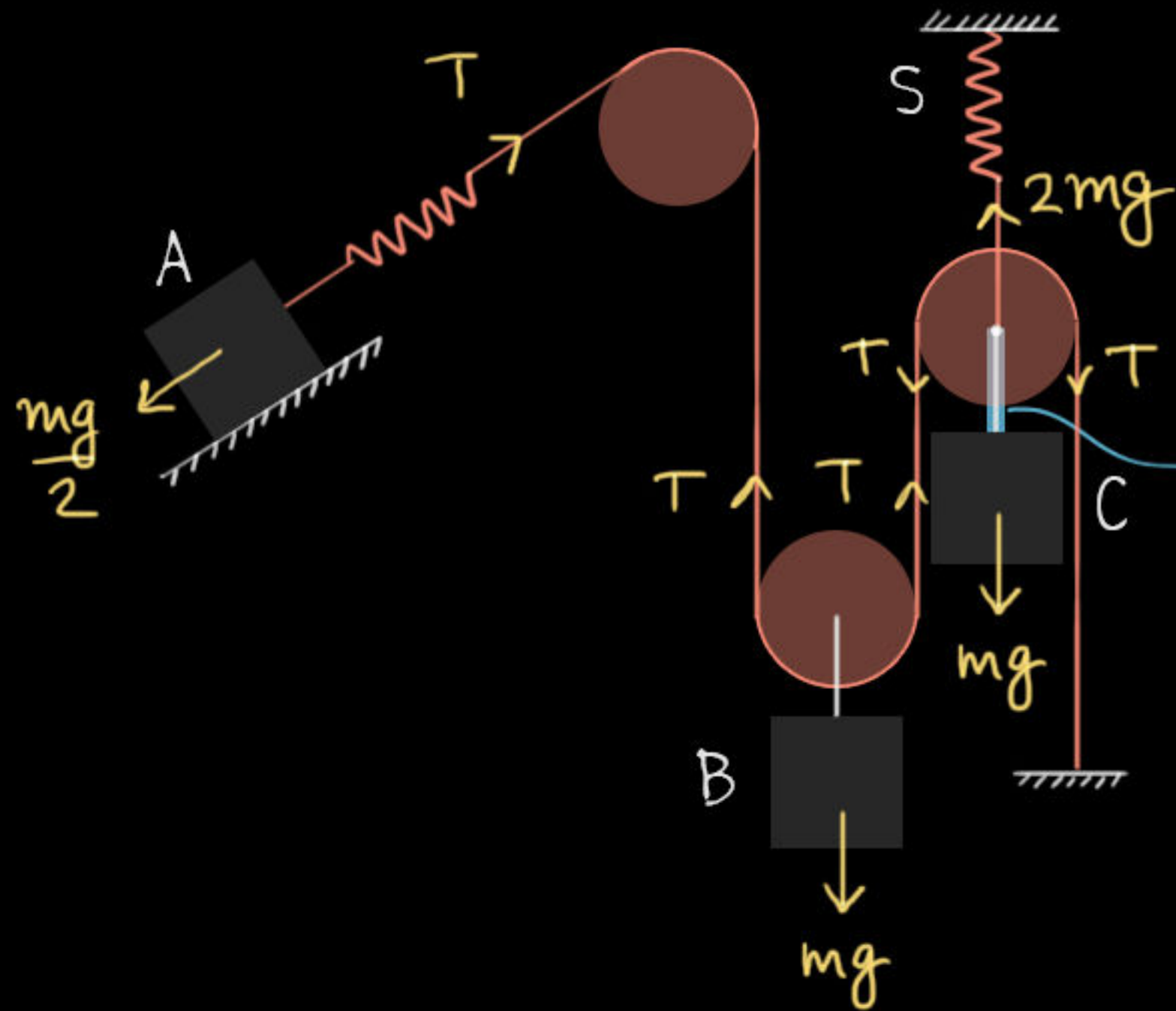
$T = mg/2$ (As B is supported by $2T$)

A When string is cut anywhere, whole string from 1 to 2 will disappear



$\therefore a_A = g/2 \swarrow$
 $a_B = g \downarrow$
 $a_C = g \uparrow$

- B When spring S is cut, Block C cant come down immediately.
- Length of string from 1 to 2 is momentarily unchanged.
 - 1st spring's extension too is unchanged and it will keep the string taut with tension maintained at $kx = mg/2$.

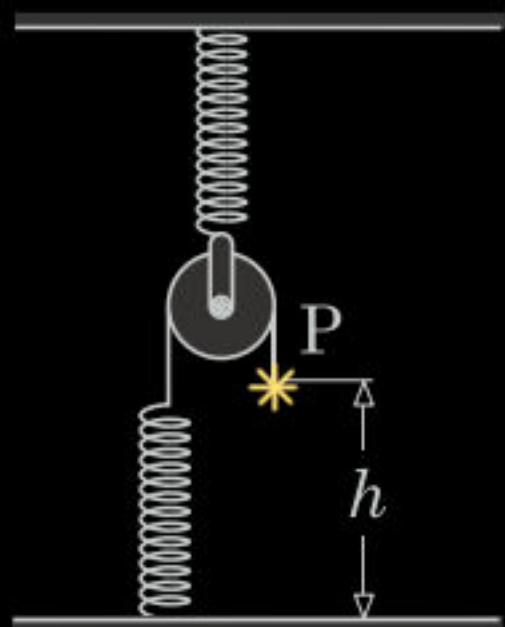


Lets consider it as a rod, rather than string (to match the answer). If its a string, C will free fall with g .

$$\therefore a_A = 0$$

$$a_B = 0$$

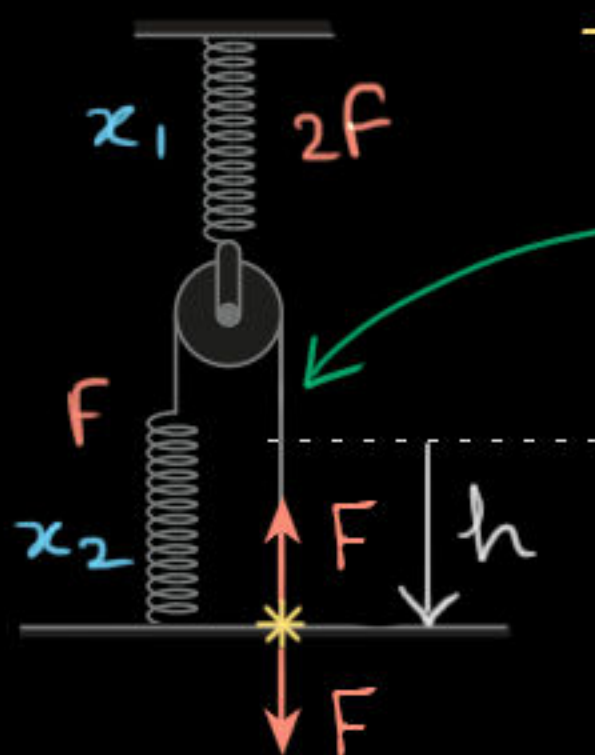
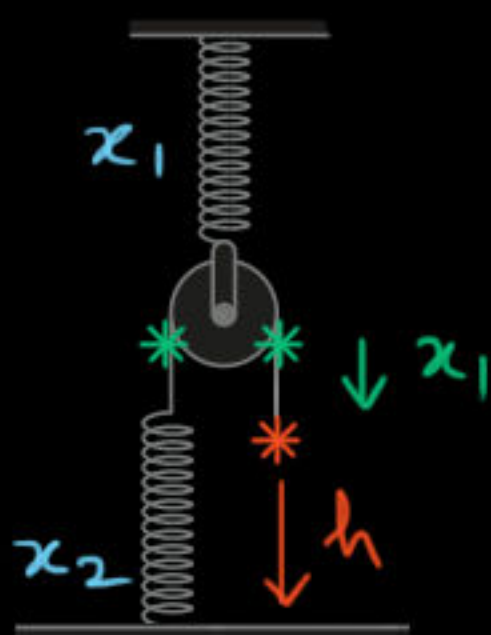
$$a_C = 2g \downarrow$$



13. A light frictionless pulley is suspended with the help of a light spring. One end P of a light inextensible thread that passes over the pulley is free and the other end is tied to another light spring that is affixed to the ground at its lower end as shown in the figure. Stiffness of both the springs is $k = 500 \text{ N/m}$. The free end P is $h = 10.0 \text{ cm}$ above the ground. What minimum pull at the end P will bring it to the ground?

Concept: As the pulley is massless, if a finite force is applied at point P downwards, rope will move with infinite velocity, and suddenly come to rest in infinitesimally small time when that force is balanced.

- When P just reaches the ground, forces should be balanced.



Total gain in this string spring system = x_2

$$\Rightarrow -2x_1 + h = x_2 \quad (1)$$

$$\text{Spring force: } F = \frac{kx_1}{2} = kx_2 \quad (2)$$

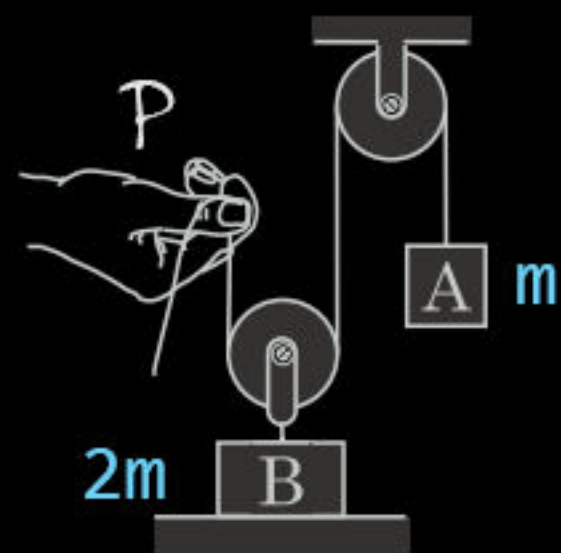
From 1,2:

$$-2 \left(\frac{2F}{k} \right) + h = \frac{F}{k}$$

$$\Rightarrow F = \frac{kh}{5}$$

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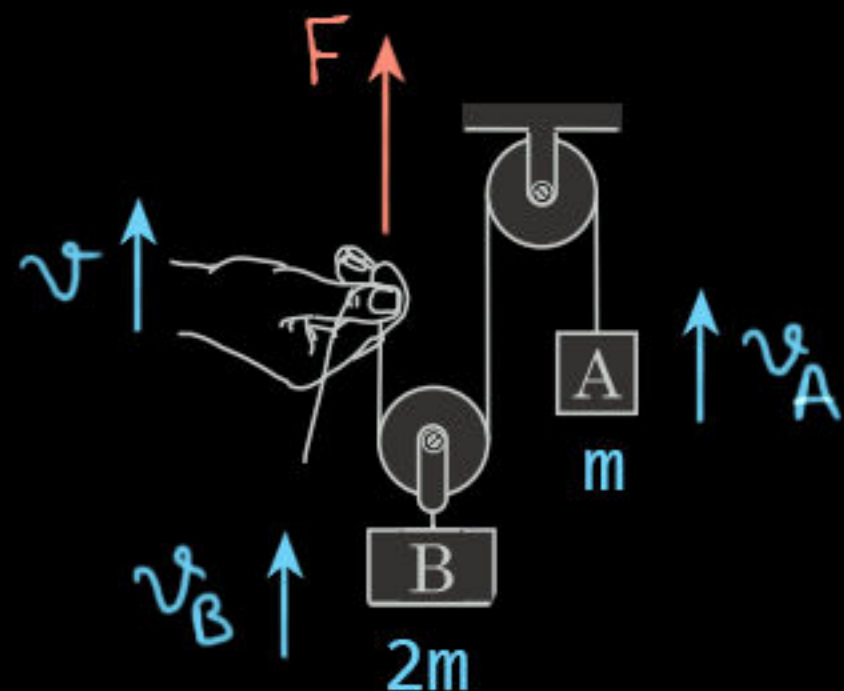
by Ankit Singhvi



14. In the system shown, initially the block A of mass m is hanging at rest and the block B of mass $2m$ is on the ground. The pulleys have negligible masses and negligible friction, and the thread is extremely light and almost inextensible. Acceleration due to gravity is g . The free end of the thread is pulled upwards with a constant force. When it acquires a speed v , find speeds of both the blocks.

Concept (not relevant to problem): A will move upwards. As $T > mg$ is a must to move B up.

Sol:



Net gain in rope = 0.

$$\Rightarrow v - 2v_B - v_A = 0 \quad (1)$$

$$\text{Block A: } F - mg = ma_A$$

$$\text{Block B: } 2F - 2mg = 2ma_B$$

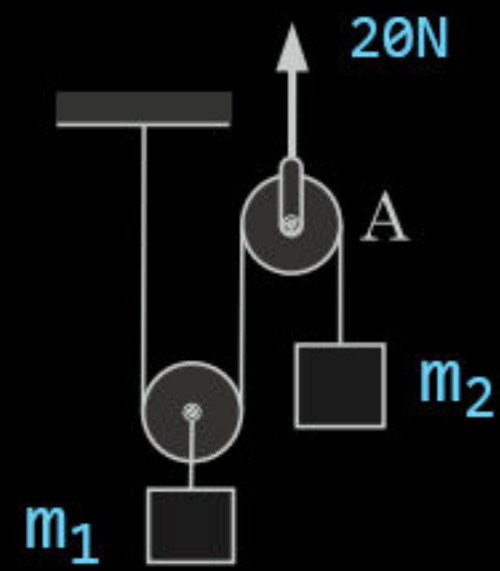
$$a_A = a_B$$

integrate $\downarrow$

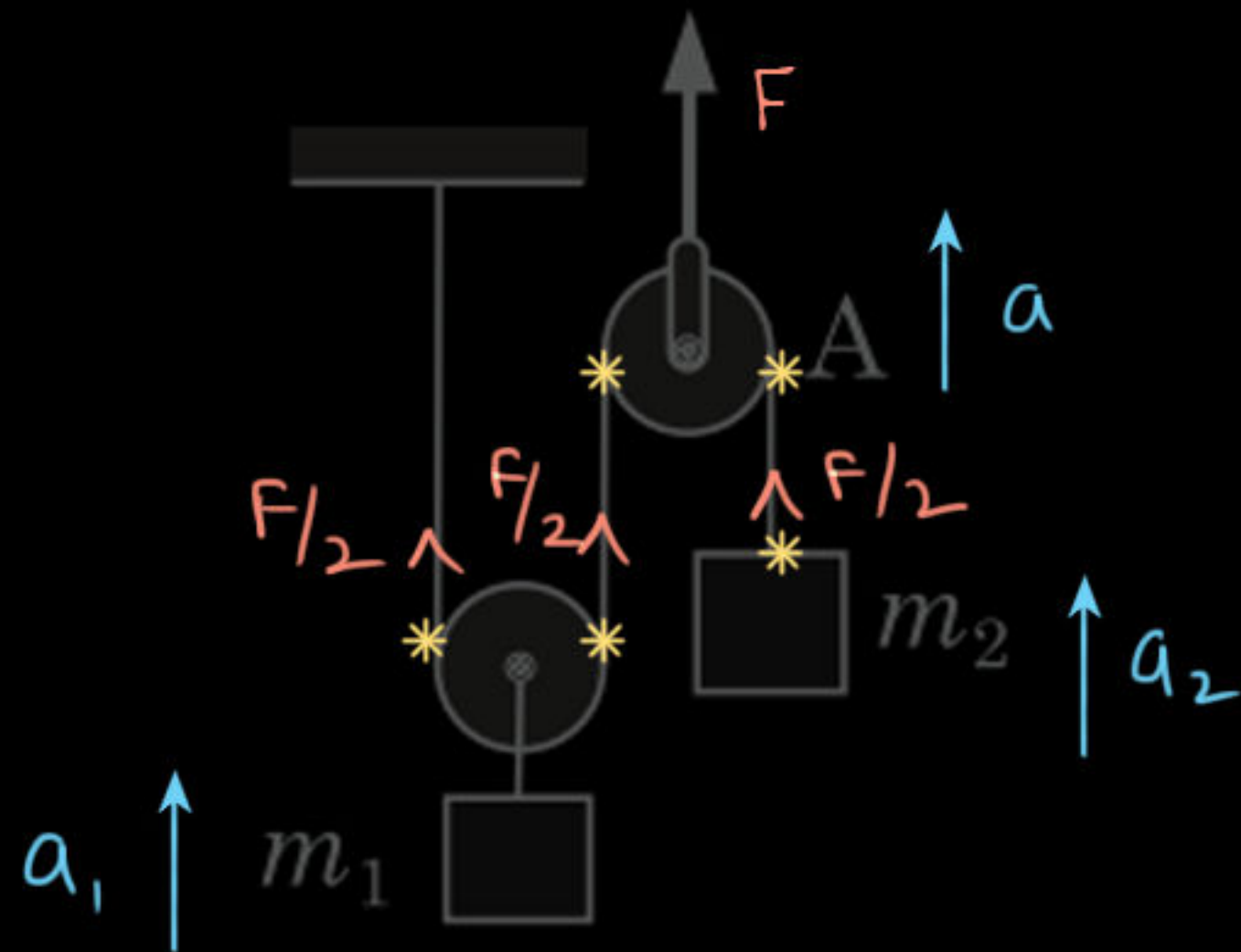
$$v_A = v_B \quad (2)$$

From 1,2:

$$v_A = v_B = v/3 \quad \uparrow //$$



15. In the system shown, thread is inextensible, masses of the thread and that of the pulleys are negligible as compared to the loads to be lifted and friction at the axles of the pulleys is absent. Masses of the loads are $m_1 = 1.0 \text{ kg}$ and $m_2 = 2.0 \text{ kg}$. Acceleration of free fall is $g = 10 \text{ m/s}^2$. If axle of the pulley A is pulled upwards with a force $F = 20 \text{ N}$, how much acceleration will it acquire?



$$F - m_1 g = m_1 a_1 \quad (1)$$

$$F/2 - m_2 g = m_2 a_2 \quad (2)$$

Total gain in string = 0

$$-2a_1 + 2a - a_2 = 0 \quad (3)$$

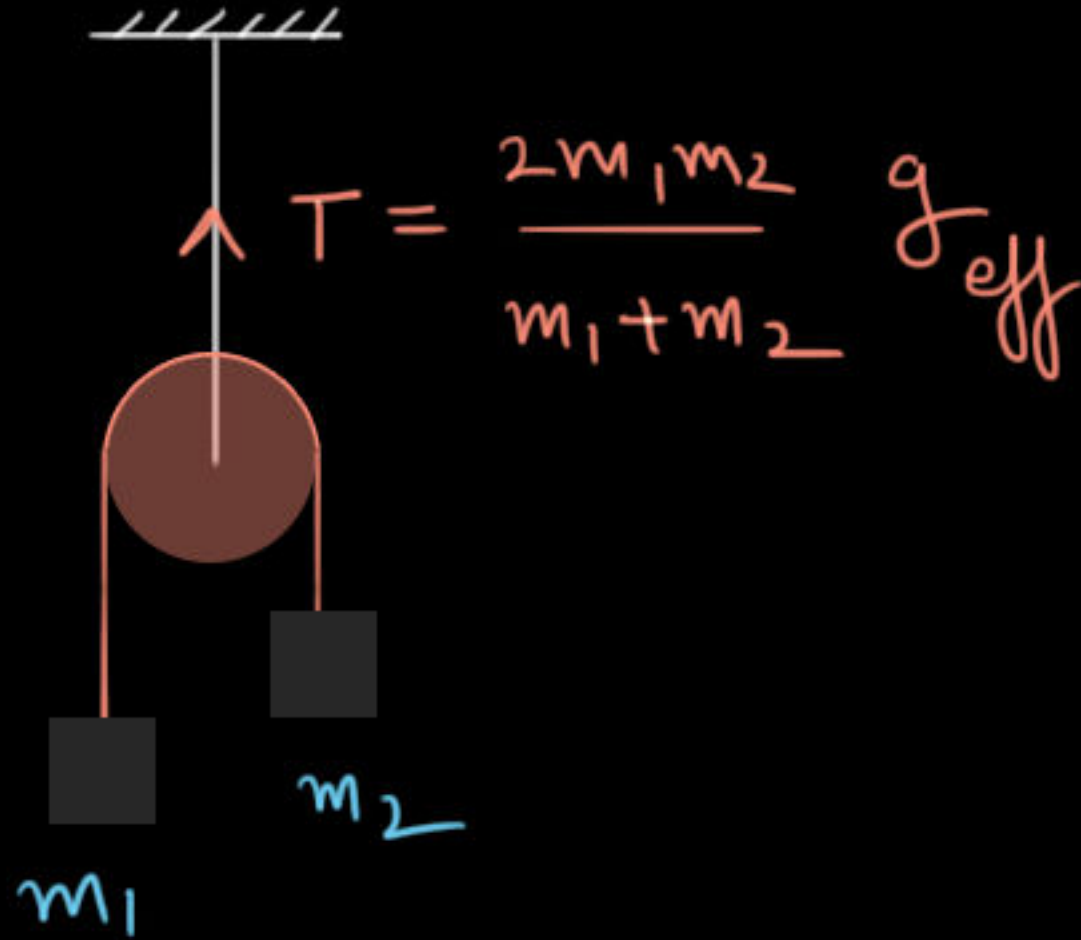
From 1, 2, 3:

$$a = \frac{F(4m_2 + m_1)}{4m_1 m_2} - \frac{3g}{2} \uparrow = 0$$

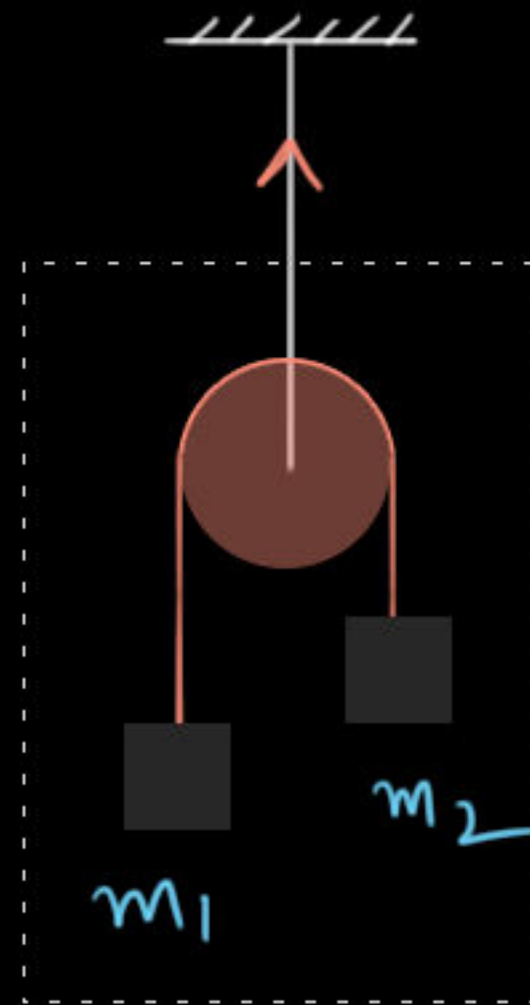
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Atwood machine equivalent mass

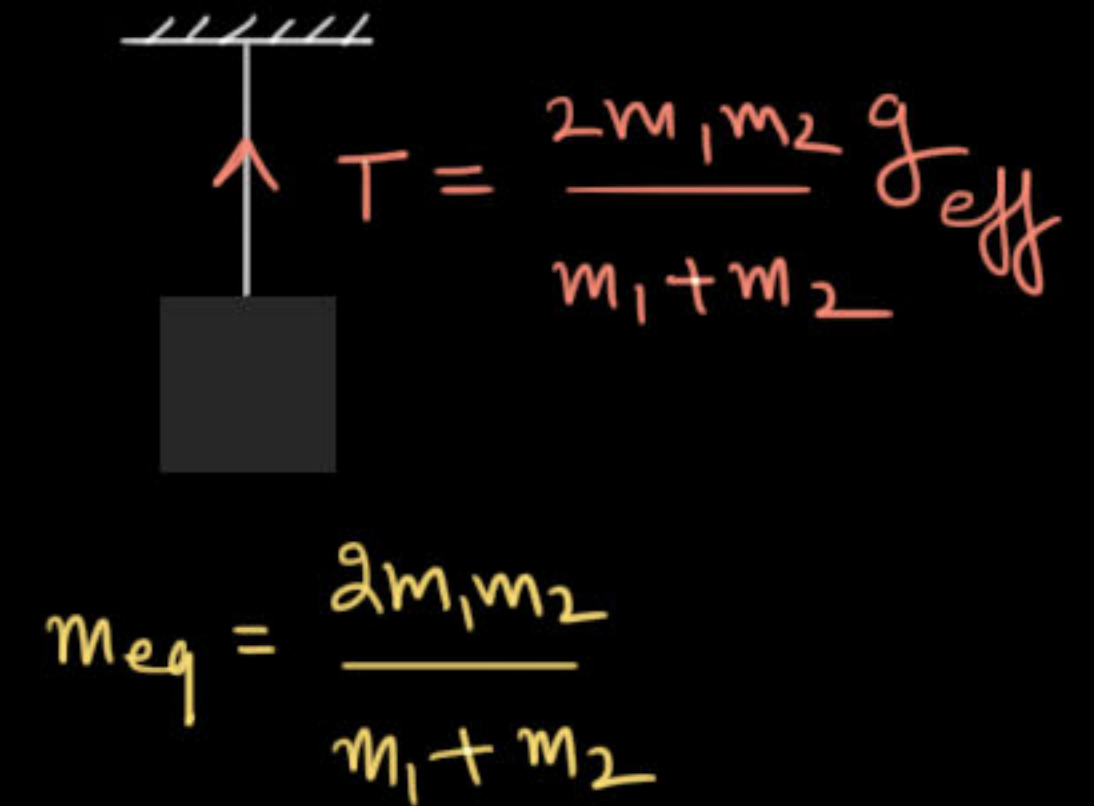
Concept: In an atwood machine, that may be accelerating, we can calculate the tension as:

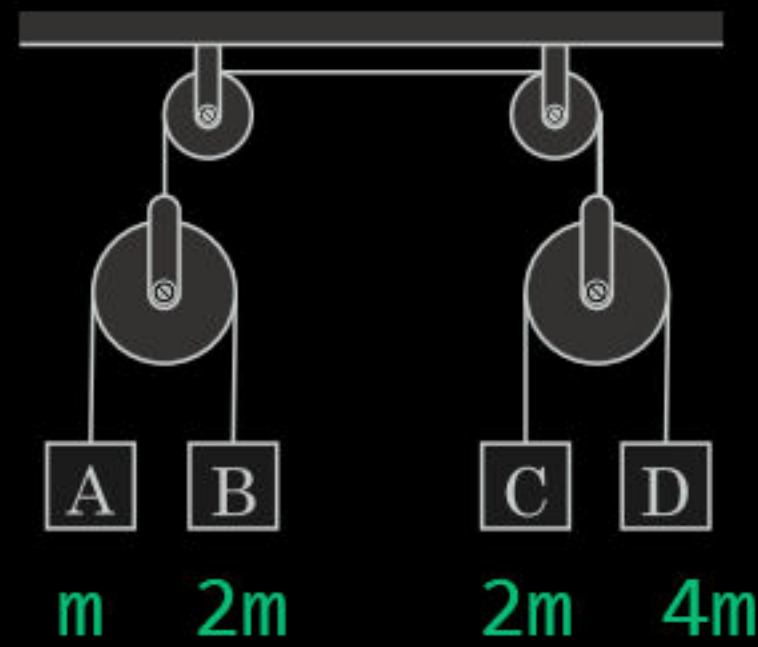


∴ As far as just the top string is concerned, we can replace the below system by a single mass m_{eq} .



≡





16. The system shown in the figure consists of four blocks A, B, C and D of masses m , $2m$, $2m$ and $4m$ respectively. The threads are inextensible, masses of the threads and the pulleys are negligible, and friction is absent at the axles of the pulleys. Initially the system is held motionless. Find accelerations of all the blocks after the system is set free. Acceleration of free fall is g .

Diagram showing the effective forces on the blocks A and B (left side) and C and D (right side) after release. The acceleration of the pulley is $g/3$ upwards.

For block A (mass m):

$$a_A = \frac{4g}{3} + \frac{g}{3} = \frac{5g}{3}$$

For block B (mass $2m$):

$$a_B = \frac{g}{3} - \frac{4g}{3} = -\frac{g}{3}$$

For block C (mass $2m$):

$$a_C = \frac{2g}{3} - \frac{g}{3} = \frac{g}{3}$$

For block D (mass $4m$):

$$a_D = -\frac{g}{3} - \frac{2g}{3} = -g$$

Diagram showing the effective forces on the blocks A and B (left side) after release. The acceleration of the pulley is $g/3$ upwards.

For block A (mass m):

$$a_A = \frac{4g}{3} + \frac{g}{3} = \frac{5g}{3}$$

For block B (mass $2m$):

$$a_B = \frac{g}{3} - \frac{4g}{3} = -\frac{g}{3}$$

Diagram showing the effective forces on the blocks C and D (right side) after release. The acceleration of the pulley is $g/3$ downwards.

For block C (mass $2m$):

$$a_C = \frac{2g}{3} - \frac{g}{3} = \frac{g}{3}$$

For block D (mass $4m$):

$$a_D = -\frac{g}{3} - \frac{2g}{3} = -g$$

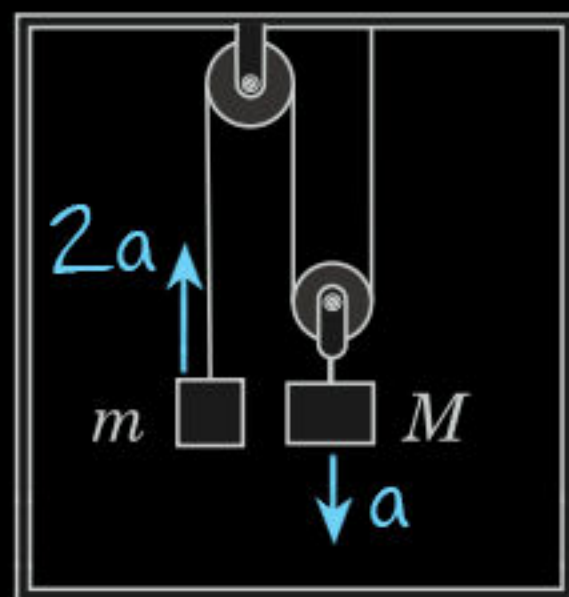
∴ Upward accelerations:

$$a_A = \frac{4g}{3} + \frac{g}{3} = \frac{5g}{3}$$

$$a_B = \frac{g}{3} - \frac{4g}{3} = -\frac{g}{3}$$

$$a_C = \frac{2g}{3} - \frac{g}{3} = \frac{g}{3}$$

$$a_D = -\frac{g}{3} - \frac{2g}{3} = -g$$



17. An arrangement setup inside a lift is shown in the figure. The pulleys and threads are ideal and masses of the blocks are m and M ($M > 2m$). Find minimum acceleration of the lift for which the thread remains taut and both the blocks accelerate in the same direction relative to the ground. Acceleration due to gravity is g .

Concept: As $M > 2m$, w.r.t lift, M will always accelerate down, and m will always accelerate up (solve and get it!). Except of course when lift is moving down with g or more, then $T=0$ and it's freefall.

w.r.t lift

$$T - mg_{\text{eff}} = m(2a) \quad (1)$$

$$Mg_{\text{eff}} - 2T = Ma \quad (2)$$

$$\Rightarrow a = \frac{(M-2m)g_{\text{eff}}}{4m+M}$$

(For string to remain taut)

When lift goes down, M will always go down, but m needs to go down too.

$$\therefore a_l > 2a$$

$$\Rightarrow a_l > \frac{2(M-2m)(g-a_l)}{4m+M}$$

$$\Rightarrow g > a_l > \frac{2(M-2m)g}{3M}$$

by Ankit Singhvi

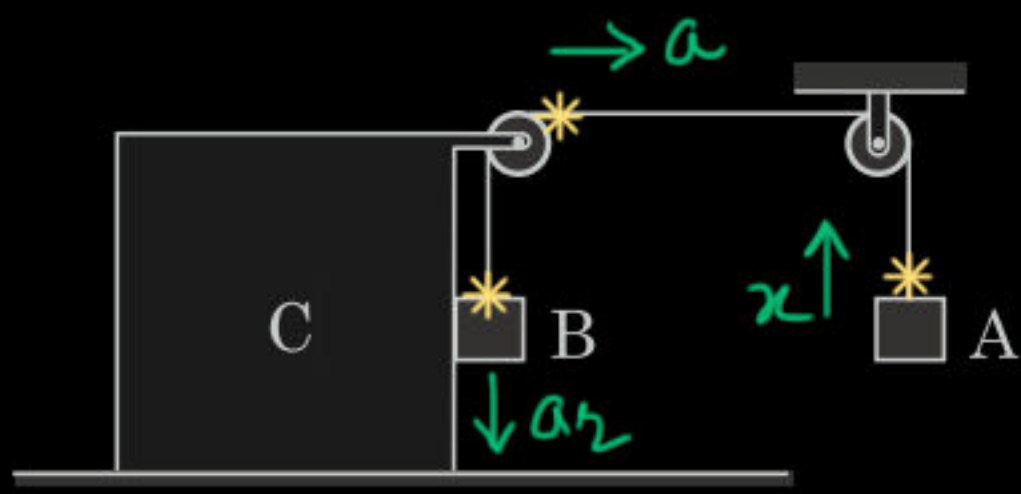
When lift goes up, m will always go up, but M needs to go up too.

$$\therefore a_l > a$$

$$\Rightarrow a_l > \frac{(M-2m)(g+a_l)}{4m+M}$$

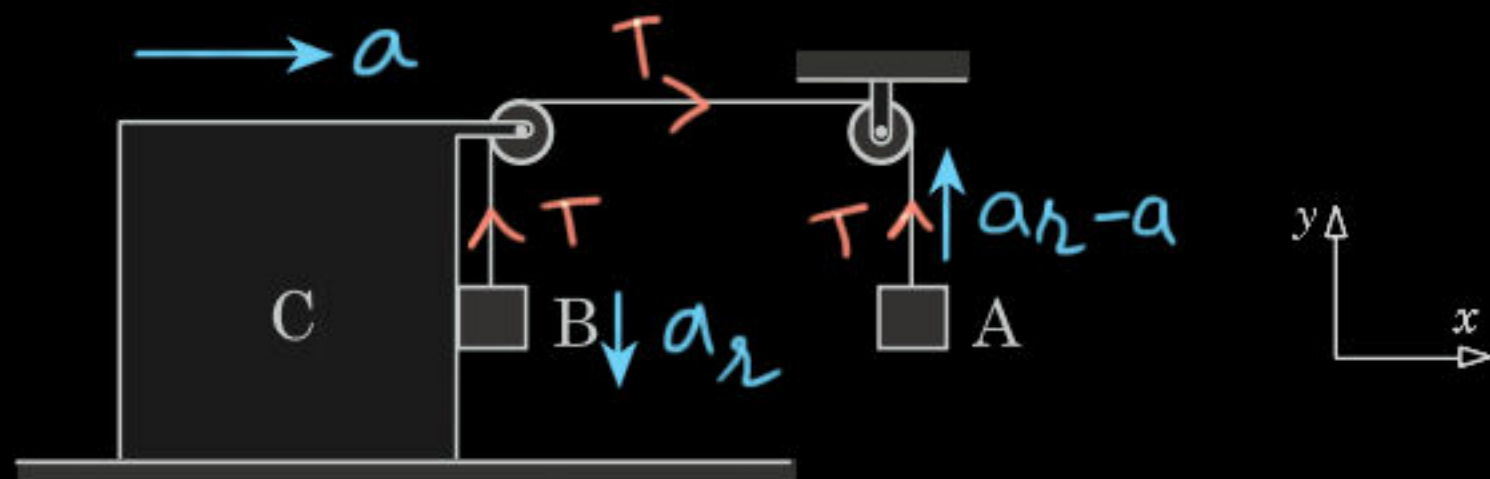
$$\Rightarrow a_l > \frac{(M-2m)g}{6m}$$

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18. In the system shown, the blocks A, B and C are of equal mass, the pulleys are ideal and the cord is light and inextensible. There is no friction between the blocks B and C as well as between the horizontal floor and the block C. If the system is set free, find acceleration vectors of all the blocks. Acceleration of free fall is $g = 10 \text{ m/s}^2$.

∴ Total string gain is zero
 i.e. $a_2 - a - x = 0$
 $\Rightarrow x = a_2 - a$



For B + C in X direction:

$$T = 2ma \quad (1)$$

For B in Y direction:

$$mg - T = ma_2 \quad (2)$$

For A in Y direction:

$$T - mg = m(a_2 - a) \quad (3)$$

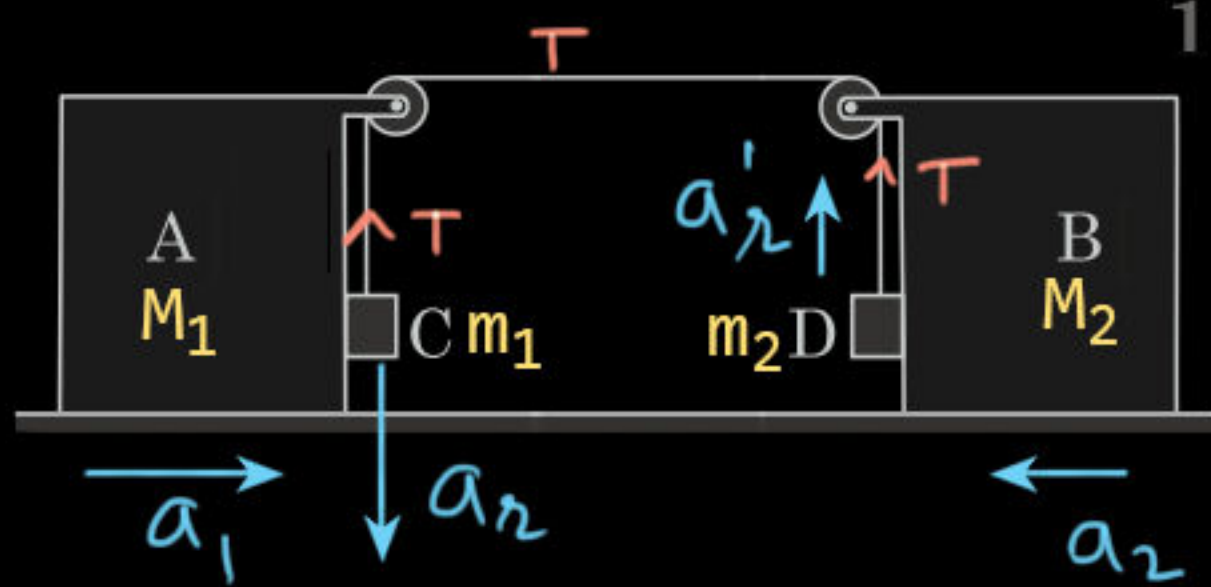
From 1, 2, 3:

$$a_2 = g/5, \quad a = 2g/5$$

$$\therefore a_A = (a_2 - a) \hat{j} = -\frac{g}{5} \hat{j} //$$

$$a_B = a \hat{i} - a_2 \hat{j} = \frac{2g}{5} \hat{i} - \frac{g}{5} \hat{j} //$$

$$a_C = a \hat{i} = \frac{2g}{5} \hat{i} //$$



19. In the given arrangement, masses of the blocks A, B, C and D are M_1 , M_2 , m_1 and m_2 respectively, pulleys are ideal and cords are light and inextensible. There is no friction between any pair of surfaces in contact. If the system is set free, find x-components of accelerations of all the blocks. Acceleration of free fall is $g = 10 \text{ m/s}^2$.

$$T = (M_1 + m_1)a_1 \quad (1)$$

$$T = (M_2 + m_2)a_2 \quad (2)$$

$$m_1g - T = m_1a_2 \quad (3)$$

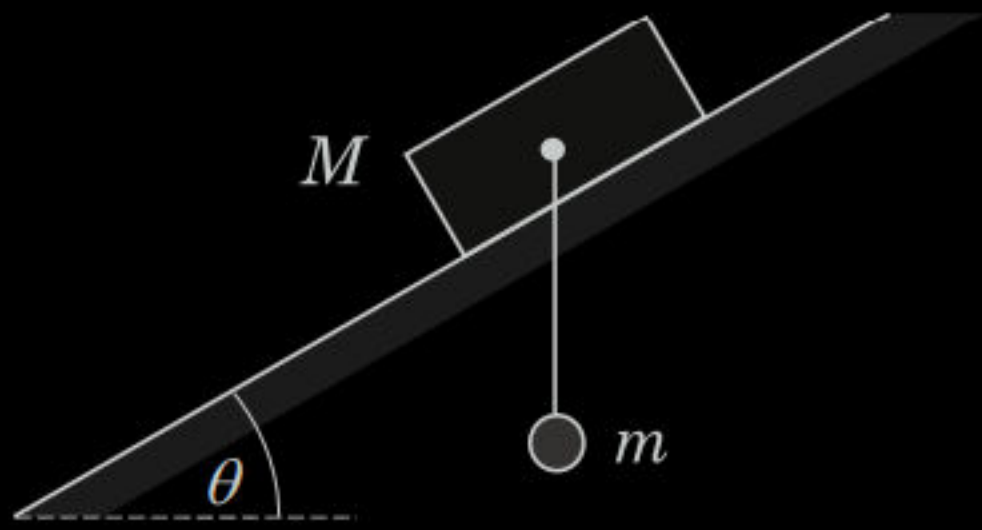
$$T - m_2g = m_2a_2' \quad (4)$$

$$\text{Constraint equation: } a_2 - a_1 - a_2 - a_2' = 0 \quad (5)$$

From 1, 2, 3:

$$a_1 = \frac{2g}{(M_1 + m_1) \left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{M_2 + m_2} + \frac{1}{M_1 + m_1} \right)}$$

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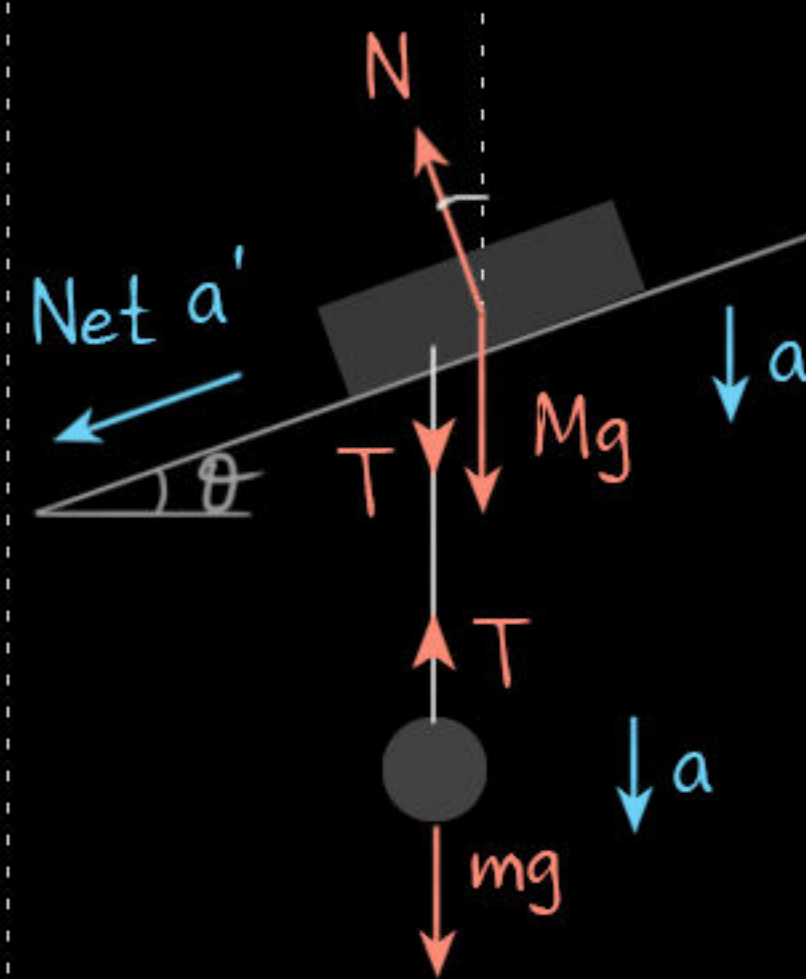


20. A ball of mass m is suspended from a bar of mass M with a light inextensible cord. The bar can slide on a frictionless slope of inclination θ . Initially the bar is held motionless so that the ball also stays motionless as shown in the figure. Find accelerations of the bar and of the ball relative to the ground immediately after the bar is released. Acceleration due to gravity is g .

Concept: The string will remain taut. Because for it to be slack, acceleration of M must be greater than g downwards, which is not possible.

As the cord is taut, acceleration of both bodies along the thread must be equal.

Acceleration of m will be vertical, as both mg and T are vertical immediately afterwards.

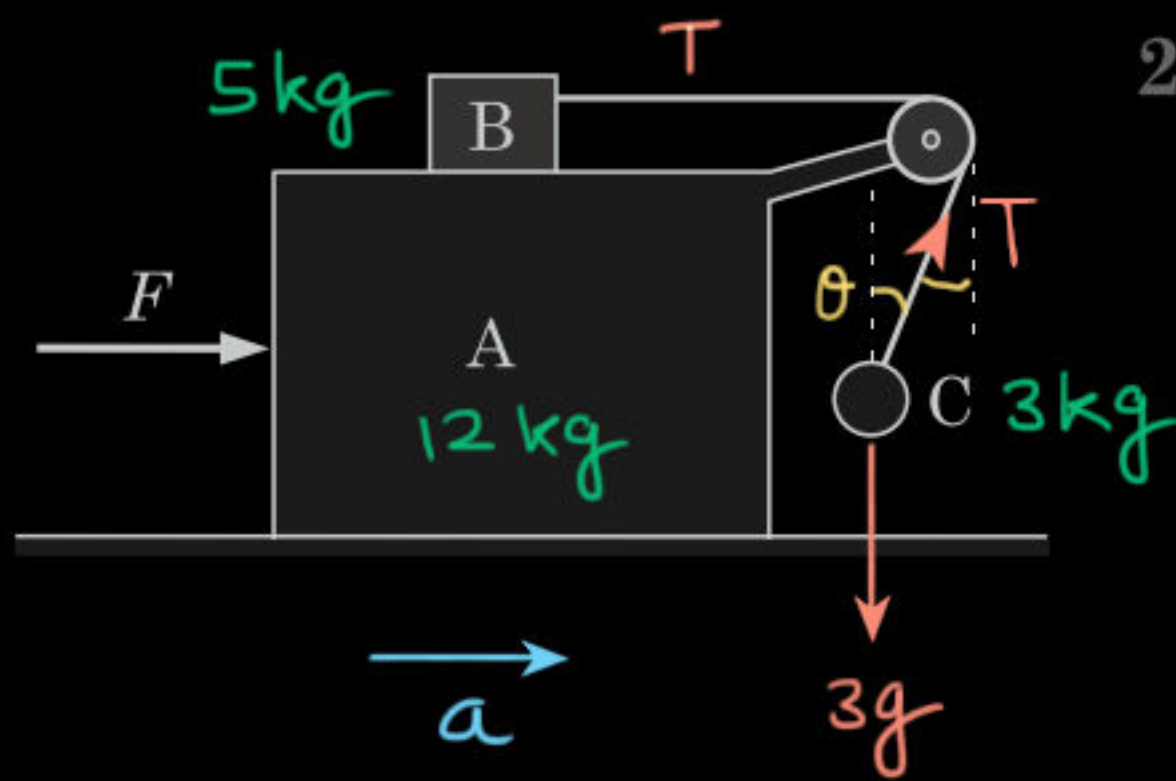


$$\begin{aligned} & (Mg + T) \cos \theta \\ & Mg + T - N \cos \theta = Ma \\ \Rightarrow & (Mg + T) \sin^2 \theta = Ma \quad (1) \end{aligned}$$

$$mg - T = ma \quad (2)$$

$$\text{From 1,2: } a = \frac{(M+m)g \sin^2 \theta}{(M+m \sin^2 \theta)}$$

$$\therefore \text{Acceleration of bar along the slope} = a' = \frac{a}{\sin \theta}$$



21. In the setup shown, magnitude of the force F exerted on the block A is so adjusted that the block B and the ball C remain motionless relative to the block A without contact between A and C. All surfaces in contact are frictionless. Masses of these bodies are $m_A = 12 \text{ kg}$, $m_B = 5 \text{ kg}$ and $m_C = 3 \text{ kg}$ respectively. Acceleration of free fall is $g = 10 \text{ m/s}^2$. Find expression for the necessary force F .

System: $F = (5 + 12 + 3)a = 20a$ ①

Block B: $T = 5a$ ②

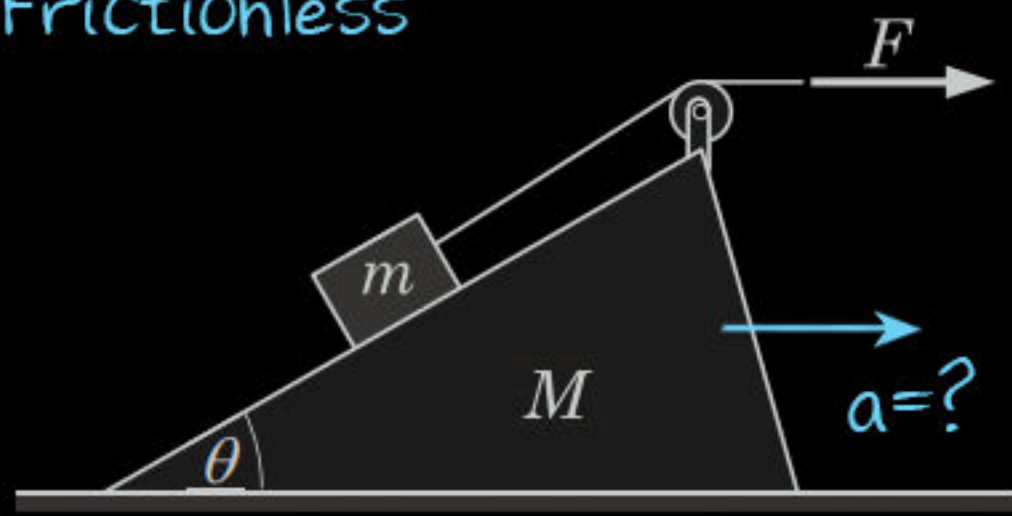
Block C: $T \sin \theta = 3a$ ③

$T \cos \theta = 3g$ ④

From 1, 2, 3, 4:

$F = 150 \text{ N}$ //

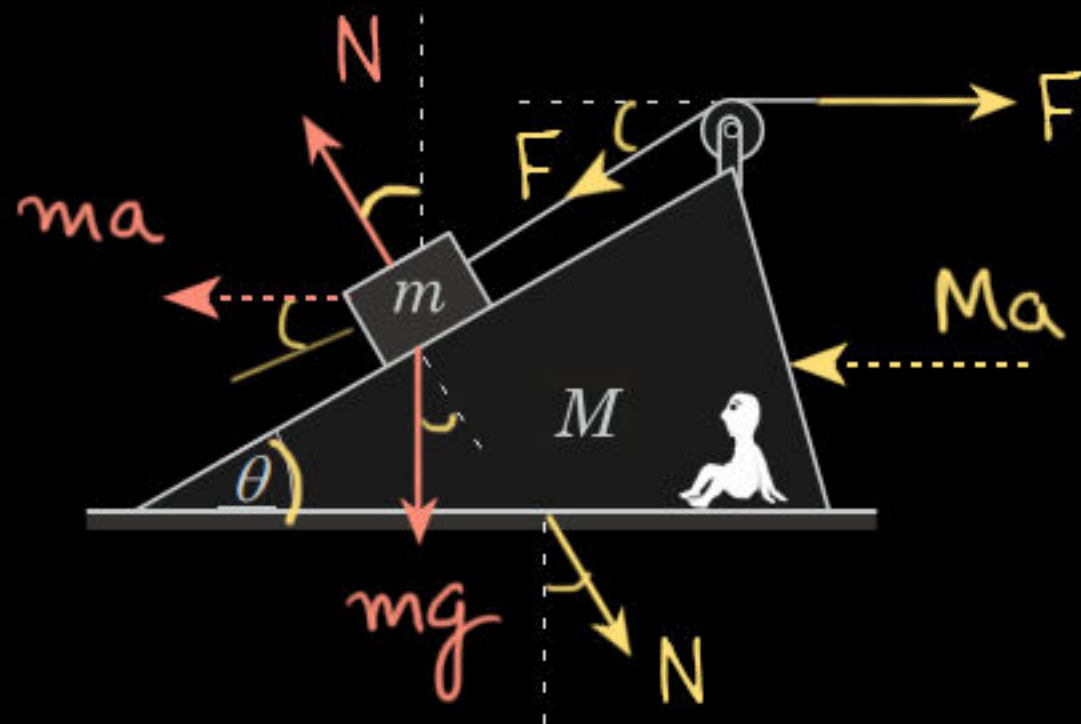
Frictionless



22. A block of mass m placed on a wedge of mass M is attached at one end of a light inextensible cord that passes over an ideal pulley affixed on the top of the wedge as shown in the figure. The free end of the thread is pulled by a constant horizontal force F . If friction between the block and the wedge as well as between the wedge and the floor is absent, find acceleration of the wedge. Acceleration due to gravity is g .

w.r.t observer on wedge

Wedge will be at rest now.



Balancing forces on m normal to plane:

$$N + ma \sin \theta = mg \cos \theta \quad (1)$$

Balancing forces on M in X direction:

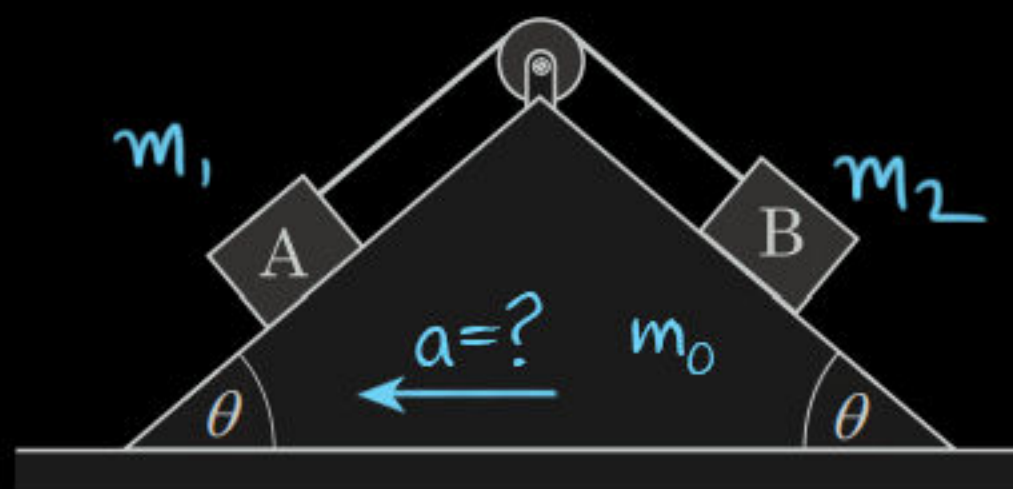
$$F + N \sin \theta = F \cos \theta + Ma \quad (2)$$

From 1,2:

$$a = \frac{mg \sin \theta \cos \theta + F(1 - \cos \theta)}{M + m \sin^2 \theta} //$$

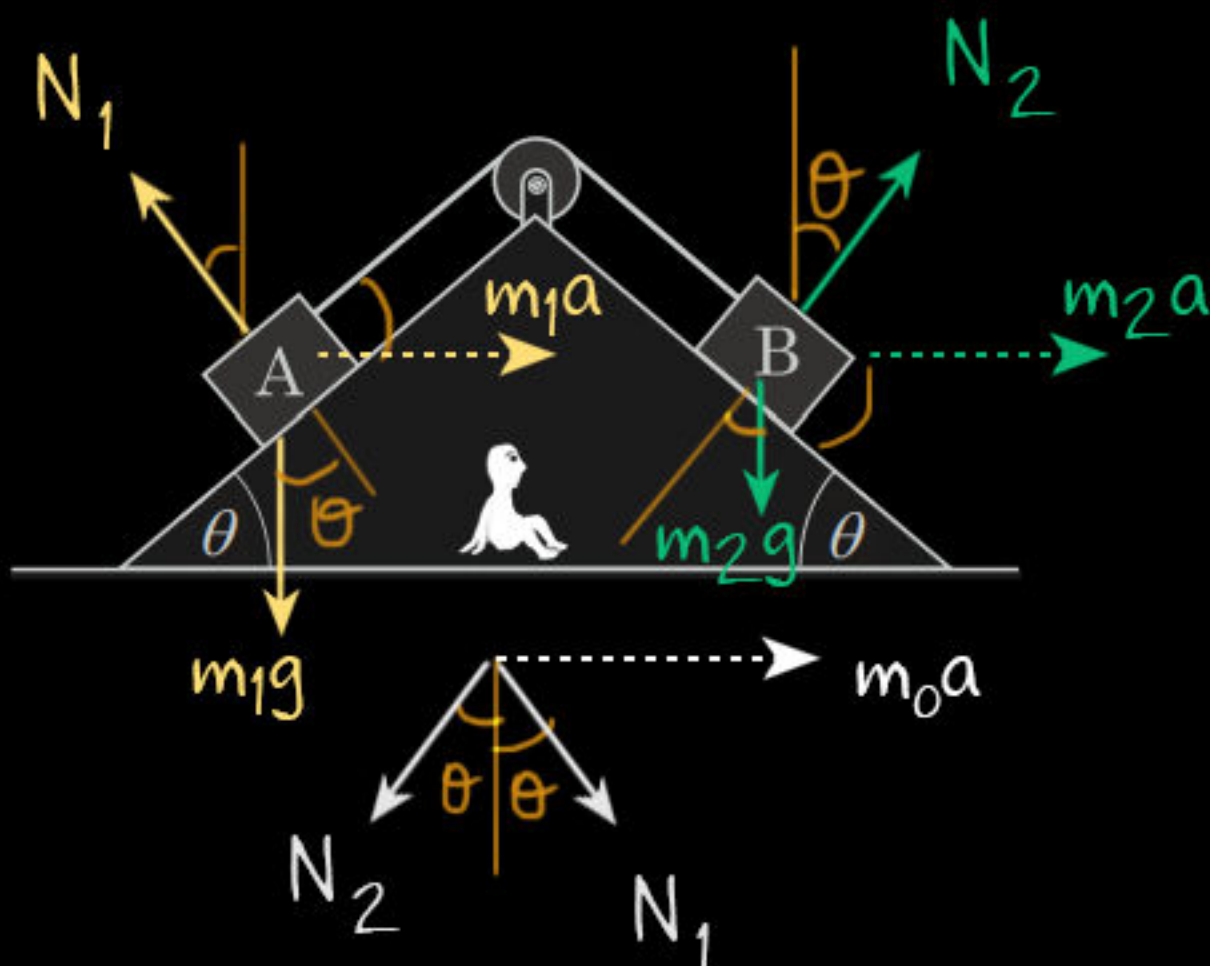
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23. In the system shown, blocks A and B of masses m_1 and m_2 ($m_2 > m_1$) are placed on the frictionless inclined surfaces of a triangular wedge of mass m_0 . The blocks are connected by a light inextensible cord that passes over an ideal pulley affixed at the top of the wedge. The wedge is placed on a horizontal frictionless floor. Initially the system is held motionless and then set free. Find acceleration of the wedge. Acceleration due to gravity is g .

w.r.t observer on wedge
Wedge will be at rest now.



Balancing forces on A and B normal to plane:

$$N_1 = m_1 a \sin \theta + m_1 g \cos \theta \quad (1)$$

$$N_2 + m_2 a \sin \theta = m_2 g \cos \theta \quad (2)$$

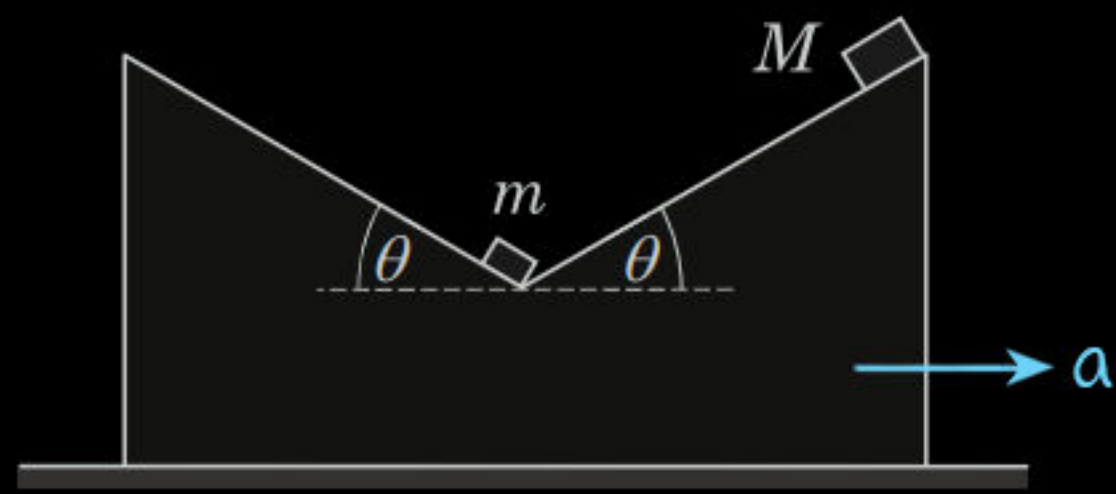
Balancing forces on wedge in X direction:

$$m_0 a + N_1 \sin \theta = N_2 \sin \theta \quad (3)$$

From 1,2,3: $a = \frac{(m_2 - m_1) g \sin \theta \cos \theta}{m_1 \sin^2 \theta + m_0 + m_2 \sin^2 \theta}$

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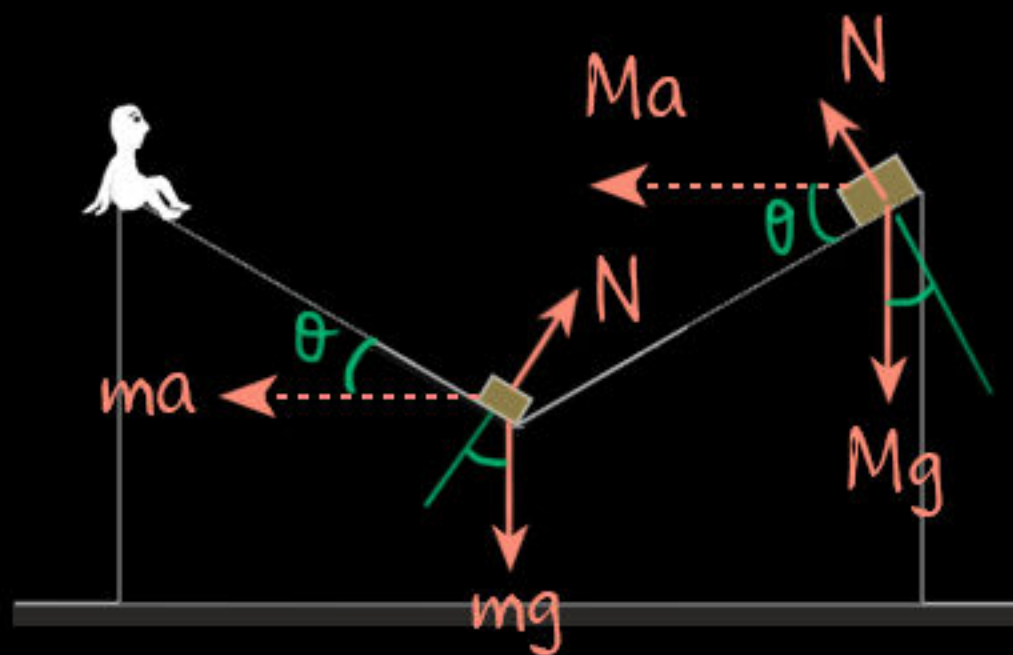
24. Top of a wedge made of a very light material has two frictionless inclined planes. A block of mass m is placed at the bottom of the inclined planes and another block of mass M is held motionless at the top of one of the inclined planes as shown in the figure. Find range of values of m in terms of M and the angle of inclination θ so that when the upper block is released, the lower block starts sliding up the wedge.

Concept: Force on a massless body is zero.

Concept: When sliding up, m loses touch with right slope. So no reaction force from there.

∴ For F_x to be zero on wedge, N is equal on both masses.

Sol: w.r.t observer on wedge
Wedge will be at rest now.



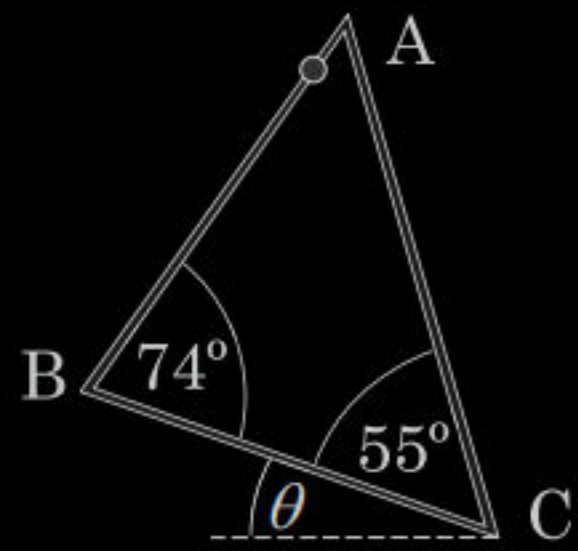
For m to slide up: $ma \cos \theta - mg \sin \theta > 0$ (1)

Forces on m normal to plane: $N = mg \cos \theta + ma \sin \theta$ (2)

Forces on M normal to plane: $N + Ma \sin \theta = Mg \cos \theta$ (3)

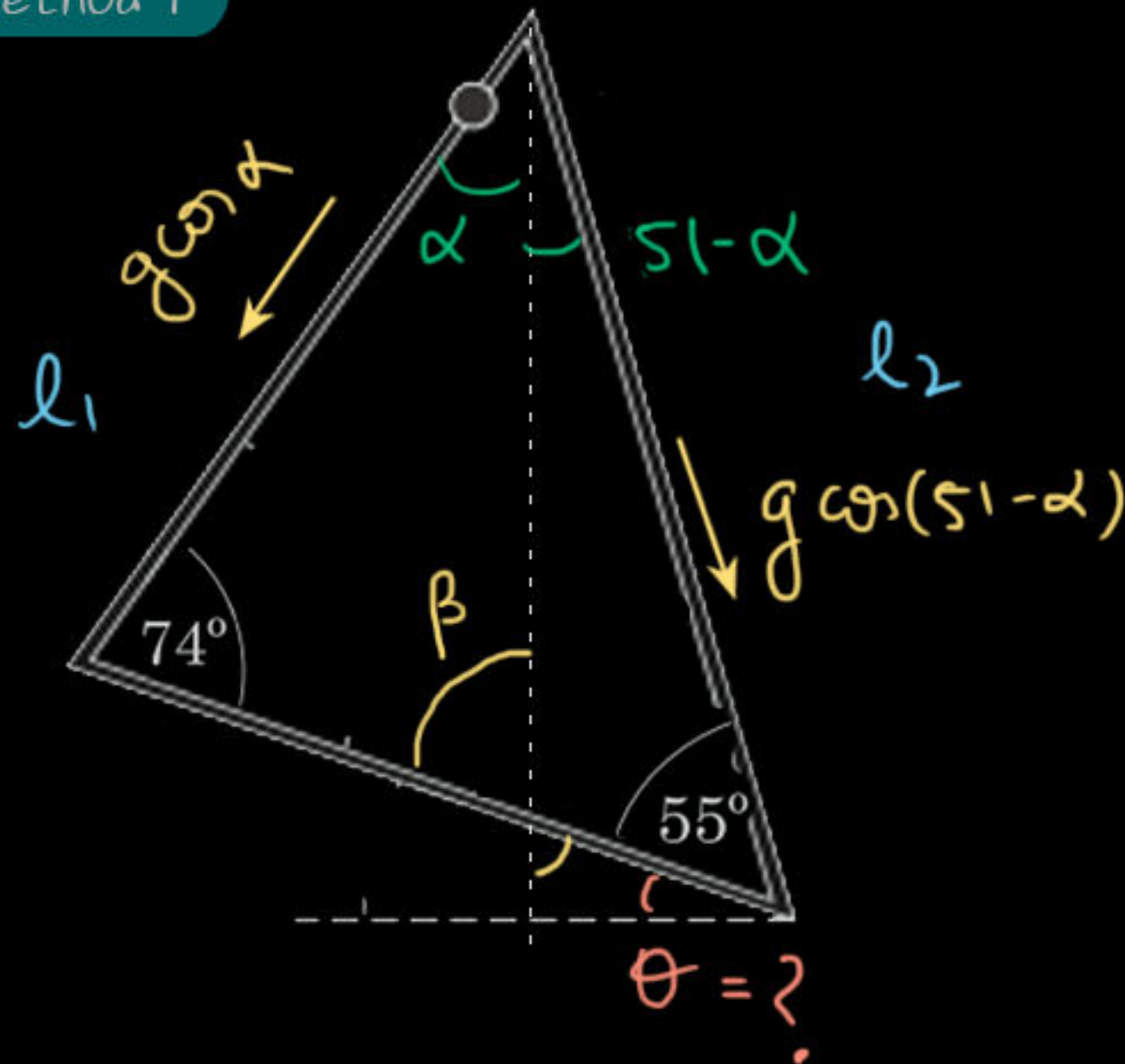
From 1, 2, 3: $m < M \cos 2\theta$

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25. A rigid triangular frame ABC made of a thin rod is fixed in a vertical plane. Angles between the rod segments at the corner B and C are 74° and 55° respectively. A small bead starting from rest from corner A takes equal time to slide down the arms AB and AC. There is no friction between the bead and the arms. Find angle θ , which the arm BC makes with the horizontal.

Method 1



$$\text{Equal time for both paths: } t^2 = \frac{2l_1}{g \cos \alpha} = \frac{2l_2}{g \cos(51 - \alpha)} \quad (1)$$

$$\text{Sine rule: } \frac{l_1}{\sin 55^\circ} = \frac{l_2}{\sin 74^\circ} \quad (2)$$

Dividing and solving a difficult trigonometric equation.

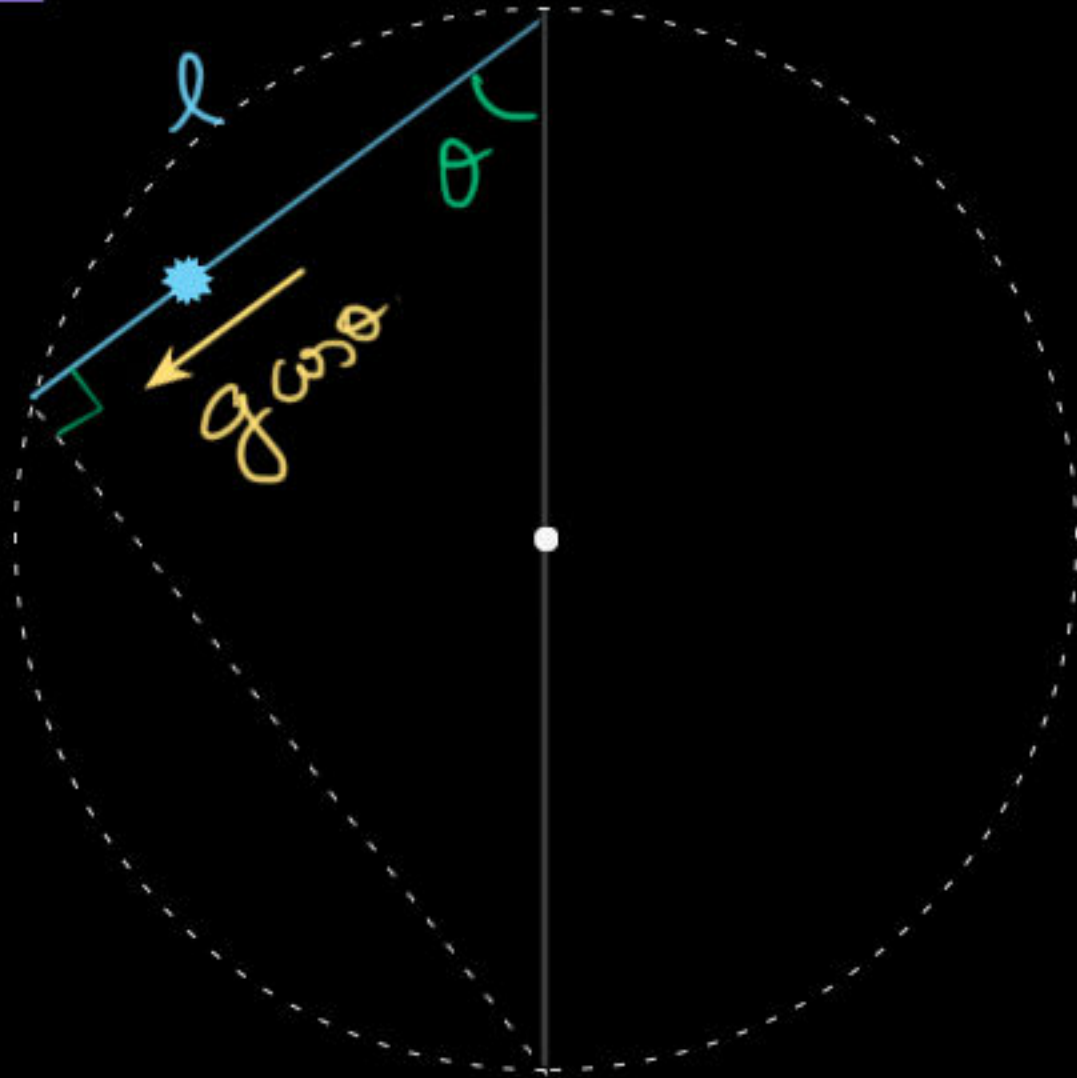
$$\alpha = 35^\circ$$

$$\therefore \theta = 19^\circ //$$

$$\theta = 90 - \beta = 90 - [\pi - (74 + \alpha)] = \alpha - 16$$

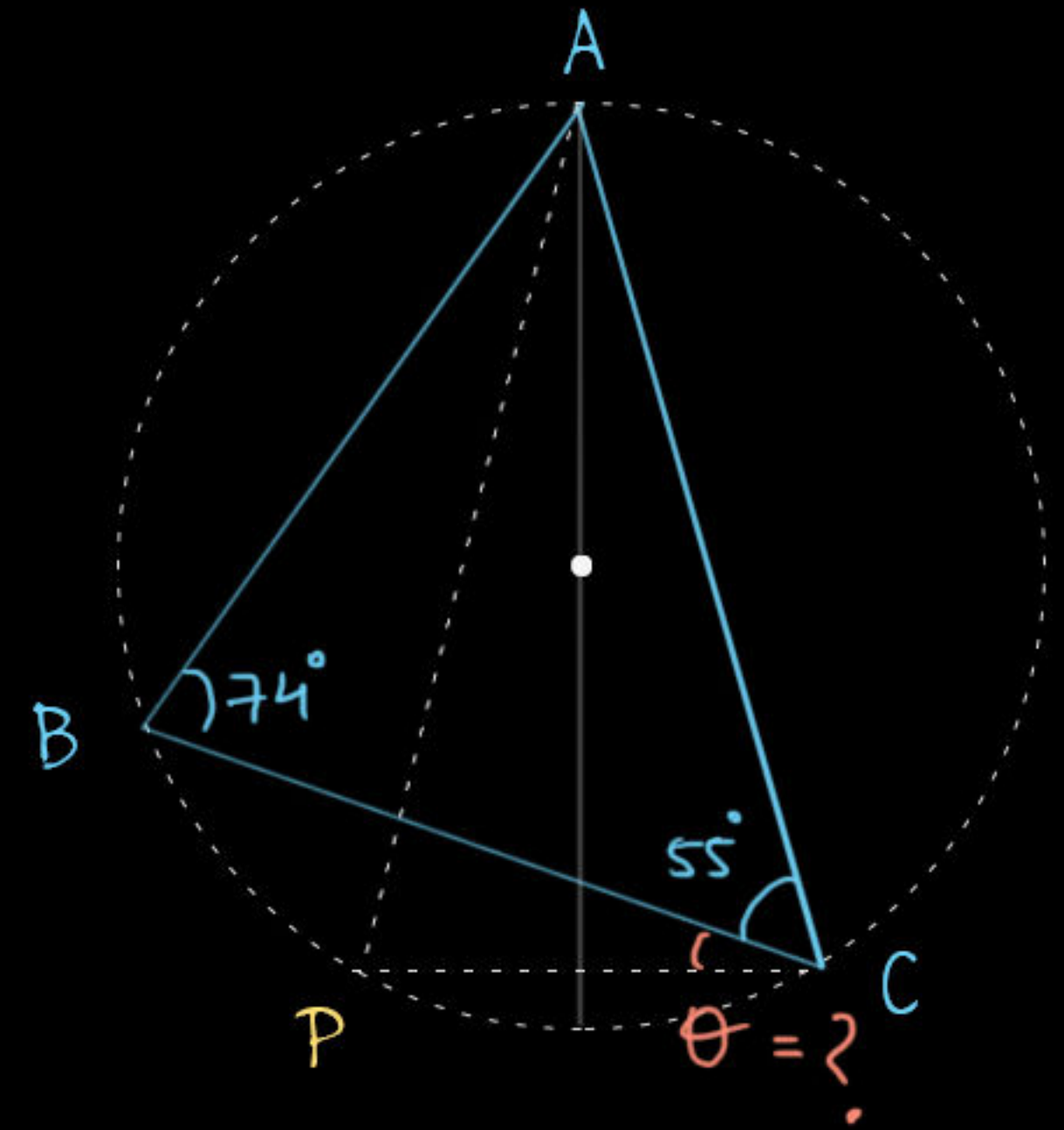
M 2 Using a known result that: from the top of a circle, all secant paths take equal time

Proof:



$$t^2 = \frac{2l}{g \cos \theta} = \frac{2(2R \cos \theta)}{g \cos \theta} = \frac{4R}{g} = \text{constant}$$

Sol:



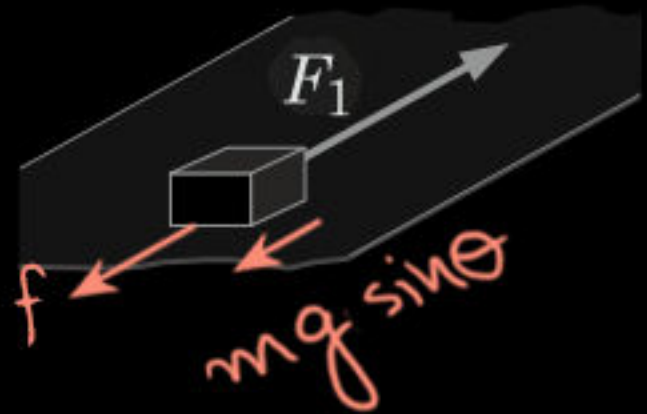
$$74^\circ = \angle B = \angle P = \angle C = \theta + 55^\circ$$

$$\Rightarrow \theta = 19^\circ //$$

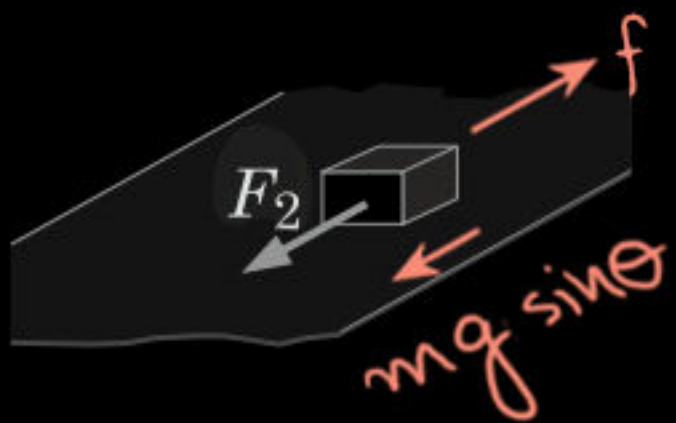
Let limiting friction be f .

As the block is traveling with constant velocity, net force on it will be zero.

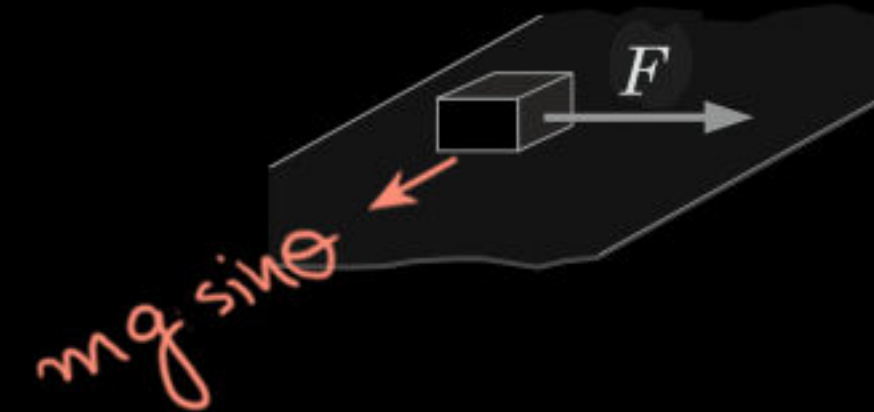
26. To drag a block up an inclined plane with a constant speed, a force F_1 has to be applied along the line of fastest descent, whereas to drag the block down the plane with a constant speed, a force F_2 has to be applied along the line of fastest descent as shown in the following figure. Find the minimum horizontal force F parallel to the plane required to slide the block.



$$f + mg \sin \theta = F_1 \quad (1)$$



$$F_2 + mg \sin \theta = f \quad (2)$$



Friction will act opposite to resultant of F and $mg \sin \theta$

For minimum F to just slide the block, friction will reach limiting value while forces will be still balanced.

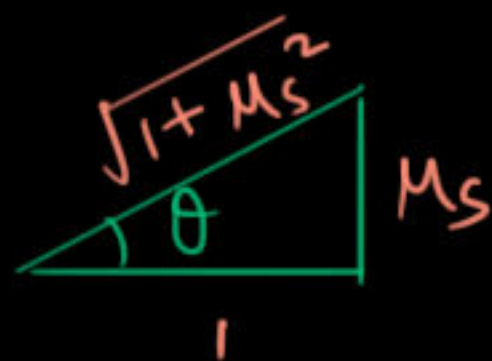
$$\therefore f = \sqrt{F^2 + (mg \sin \theta)^2} \quad (3)$$

From 1,2,3: $F = \sqrt{F_1 F_2}$

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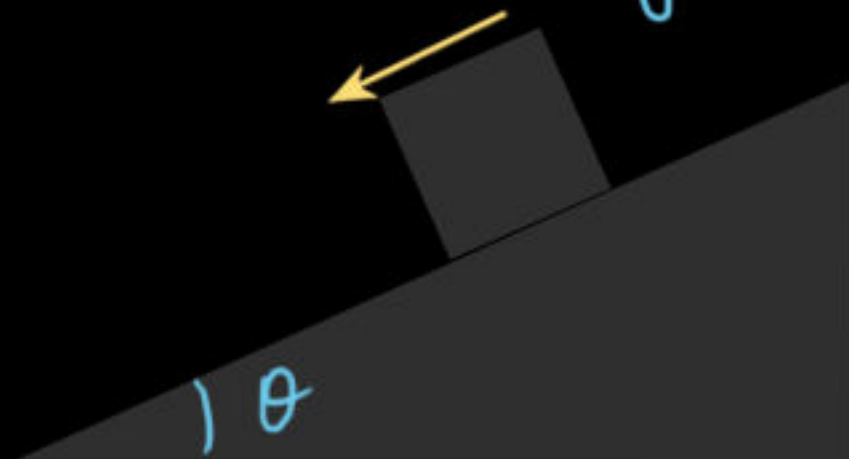
Concept: When angle reaches angle of repose, it will slide.

$$\therefore \tan \theta = \mu_s \quad (1)$$



Once it starts sliding:

$$a = g \sin \theta - \mu_k g \cos \theta \quad (2)$$



27. A small block is kept on an inclined plane of adjustable inclination. Coefficients of static and kinetic frictions between the block and the plane are μ_s and μ_k respectively. The angle of inclination of the plane is gradually increased from zero until the block starts sliding. How much speed will the block acquire in sliding down a distance l ? Acceleration due to gravity is g .

From 1,2:

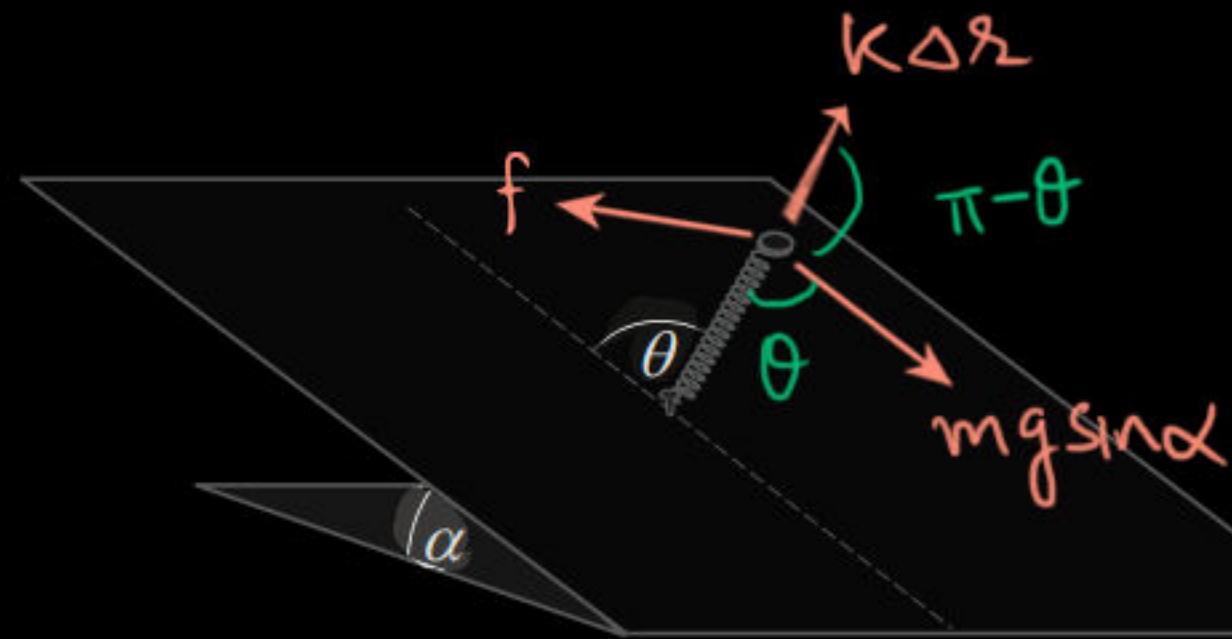
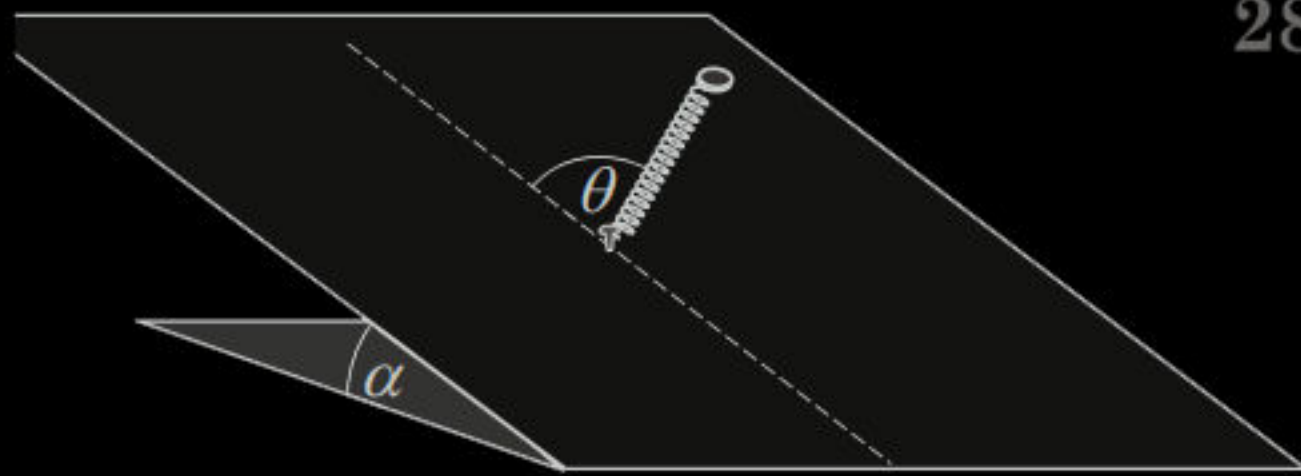
$$a = \frac{(\mu_s - \mu_k)g}{\sqrt{\mu_s^2 + 1}}$$

Required velocity when it slides by l :

$$v = \sqrt{2al}$$

$$= \sqrt{\frac{2gl(\mu_s - \mu_k)}{\sqrt{\mu_s^2 + 1}}}$$

28. One end of a spring of stiffness k is attached to a nail O on an inclined plane and the other end to a small disc of mass m placed on the inclined plane. Inclination of the plane with the horizontal is α and coefficient of friction between the disc and the plane is slightly less than $\tan \alpha$. Find suitable expression for deformation Δr in the spring when the disc is in equilibrium at an angular position θ from the line of greatest slope as shown in the figure. Acceleration due to gravity is g .



$$\therefore \mu < \tan \alpha$$

$$\Rightarrow \mu \cos \alpha < \sin \alpha \quad (1)$$

Friction will act opposite to resultant of $k \Delta r$ and $mg \sin \alpha$

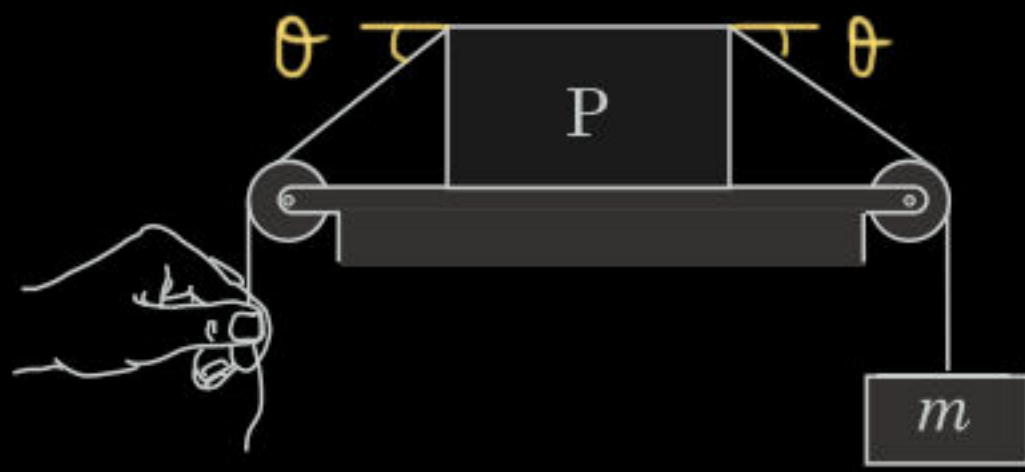
$$\therefore \text{For equilibrium: Resultant} \leq (f_{\max} = \mu mg \cos \alpha) < mg \sin \alpha \quad (\text{From 1})$$

$$\Rightarrow (\cancel{mg \sin \alpha})^2 + (k \Delta r)^2 - 2(\cancel{mg \sin \alpha})(k \Delta r) \cos \theta < (\cancel{mg \sin \alpha})^2$$

$$\Rightarrow 0 < \Delta r < \frac{2mg \sin \alpha \cos \theta}{k} //$$

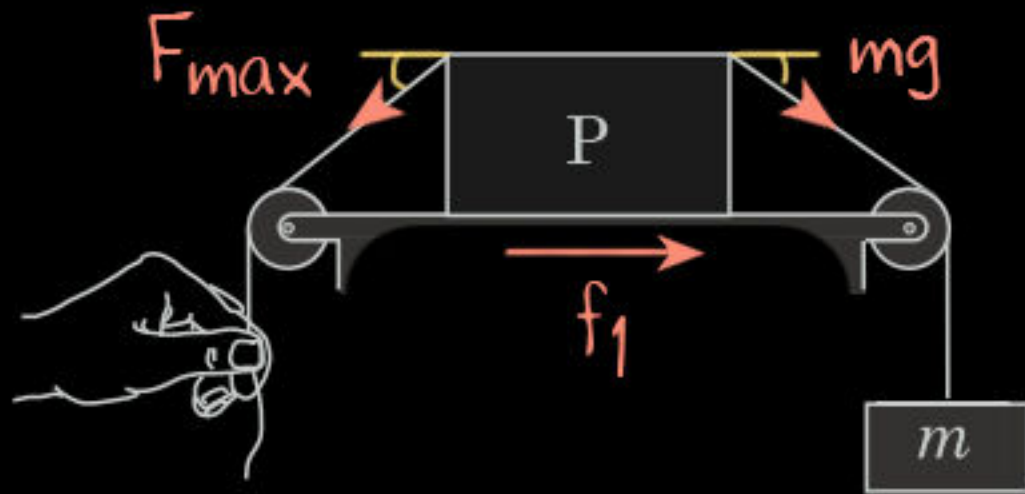
Note: spring can not pull, as disc slides down anyway without spring.
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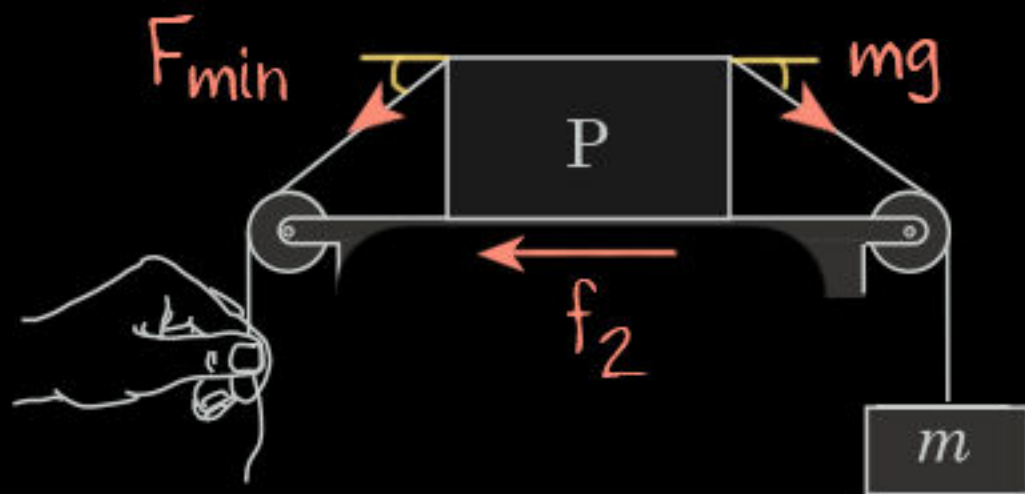


30. A light block P on a horizontal tabletop placed equidistant from the pulleys supports a load with the help of a cord as shown in the figure. If the minimum and the maximum forces applied by hand to keep the system in equilibrium are $F_{\min} = 40 \text{ N}$ and $F_{\max} = 90 \text{ N}$, calculate mass m of the load. Acceleration due to gravity $g = 10 \text{ m/s}^2$.

Let the cord make θ with the horizontal.



$$(F_{\max} - mg) \cos \theta = f_1 = \mu [(mg + F_{\max}) \sin \theta] \quad (1)$$



$$(mg - F_{\min}) \cos \theta = f_2 = \mu [(mg + F_{\min}) \sin \theta] \quad (2)$$

Dividing 1,2: $m = \frac{\sqrt{f_1 f_2}}{g}$

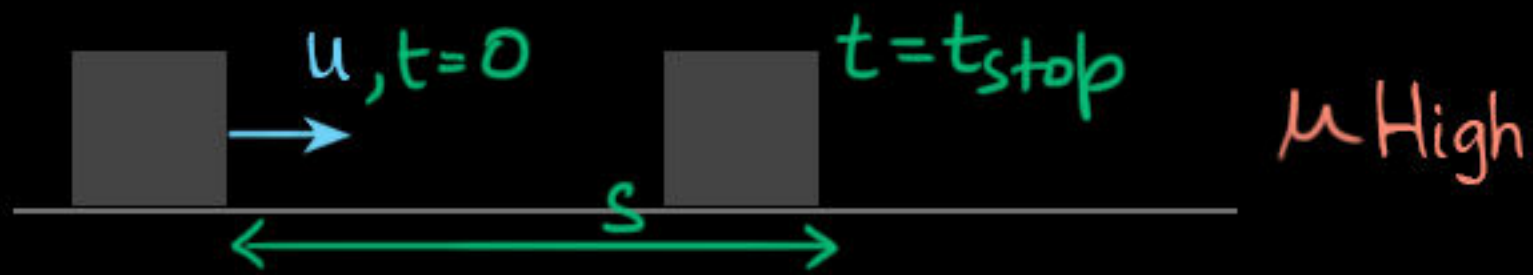
31. A block given a velocity u on horizontal ground (not frictionless) is observed a distance s away a time τ later. Find coefficient of kinetic friction between the floor and the block. Acceleration of free fall is g .

u, s, τ $a = -\mu g$ $v = 0, \neq 0$
 Given To find Cases A B

$$\therefore s < \frac{u\tau}{2}$$

distance traveled on frictionless ground

A Stopped So τ is irrelevant in finding μ



$$\left(\frac{u+v}{2}\right) t_{\text{stop}} = s \quad \& \quad \tau > t_{\text{stop}}$$

$$\tau > \frac{2s}{u}$$

$$\Rightarrow 0 < s < \frac{u\tau}{2}$$

$\therefore s$ must be +ve.

$$v^2 = u^2 - 2(\mu g)s$$

$$\Rightarrow \mu = \frac{u^2}{2gs}$$

B Not stopped



$$\left(\frac{u+v}{2}\right) \tau = s \quad \& \quad v > 0$$

$$\therefore \frac{2s}{\tau} - u > 0$$

$$\Rightarrow \frac{u\tau}{2} < s < u\tau$$

$\therefore$ friction exists

$$s = u\tau - \frac{1}{2}(\mu g)\tau^2$$

$$\Rightarrow \mu = \frac{2(u\tau - s)}{g\tau^2}$$

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Concept: Even while wearing down, the acceleration μg is unchanged.



Case A It stops due to friction.

$$v^2 = u^2 - 2\mu g s$$

$$\Rightarrow s = \frac{u^2}{2\mu g}$$

32. A block of mass m made of chalk is projected with velocity u along a horizontal floor. Coefficient of friction between the block and the floor is μ . If due to wear, mass of the block decreases at a constant rate r (a unit of mass in every unit of distance travelled), find expression for total distance travelled by the block. Acceleration due to gravity is g .



Case B Chalk is finished.

$$\frac{dm}{dx} = -r$$

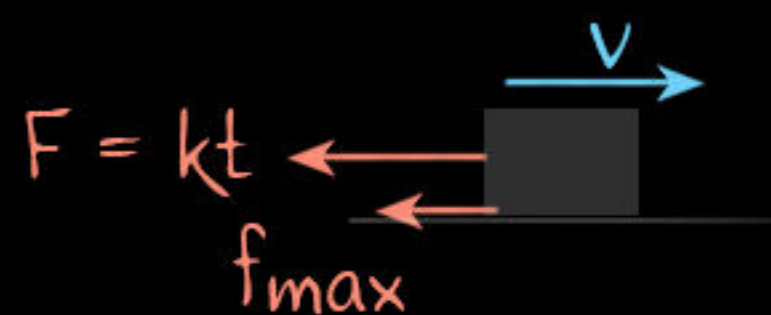
$$\Rightarrow \int_m^0 dm = -r \int_0^s dx$$

$$\Rightarrow s = \frac{m}{r}$$

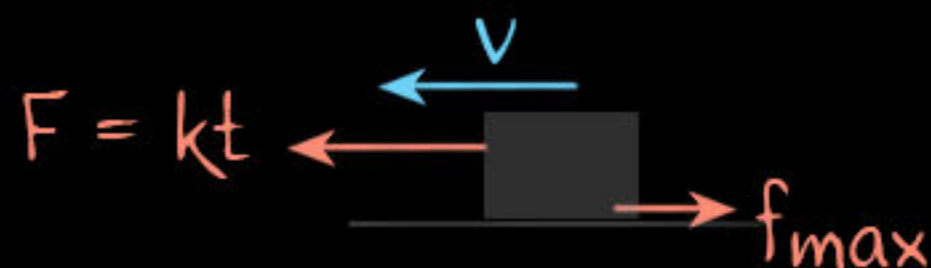
$\therefore s = \text{whichever is less of } \frac{u^2}{2\mu g} \text{ or } \frac{m}{r}$

Concept: $F=kt$ will help the friction in stopping the block from going eastwards. But as soon as block stops, friction will oppose F , until it can no more and block will slide westwards.

$$f_{\max} = \mu mg = 15 \text{ N}$$



Rest: $t_1 - t_2$



33. A block of mass $m = 5.0 \text{ kg}$ is sliding eastwards on a horizontal floor. When its velocity is $u = 8.0 \text{ m/s}$, in addition to frictional force from the floor a westward force F starts acting on it. Magnitude of this force varies with time according to equation $F = kt$, where $k = 5.0 \text{ N/s}$ and t is time in seconds. Coefficient of friction between the block and the floor is $\mu = 0.3$. Draw a graph to show how the frictional force f on the block varies with time. Acceleration of free fall is $g = 10 \text{ m/s}^2$.

It stops at: t_1

$$m \frac{dv}{dt} = -kt - \mu mg$$

$$-\int_u^0 dv = \int_0^{t_1} (kt - \mu mg) dt$$

$$\Rightarrow u = \frac{kt_1}{2} + \mu g t_1$$

$$\text{Solving: } t_1 = 2 \text{ sec}$$

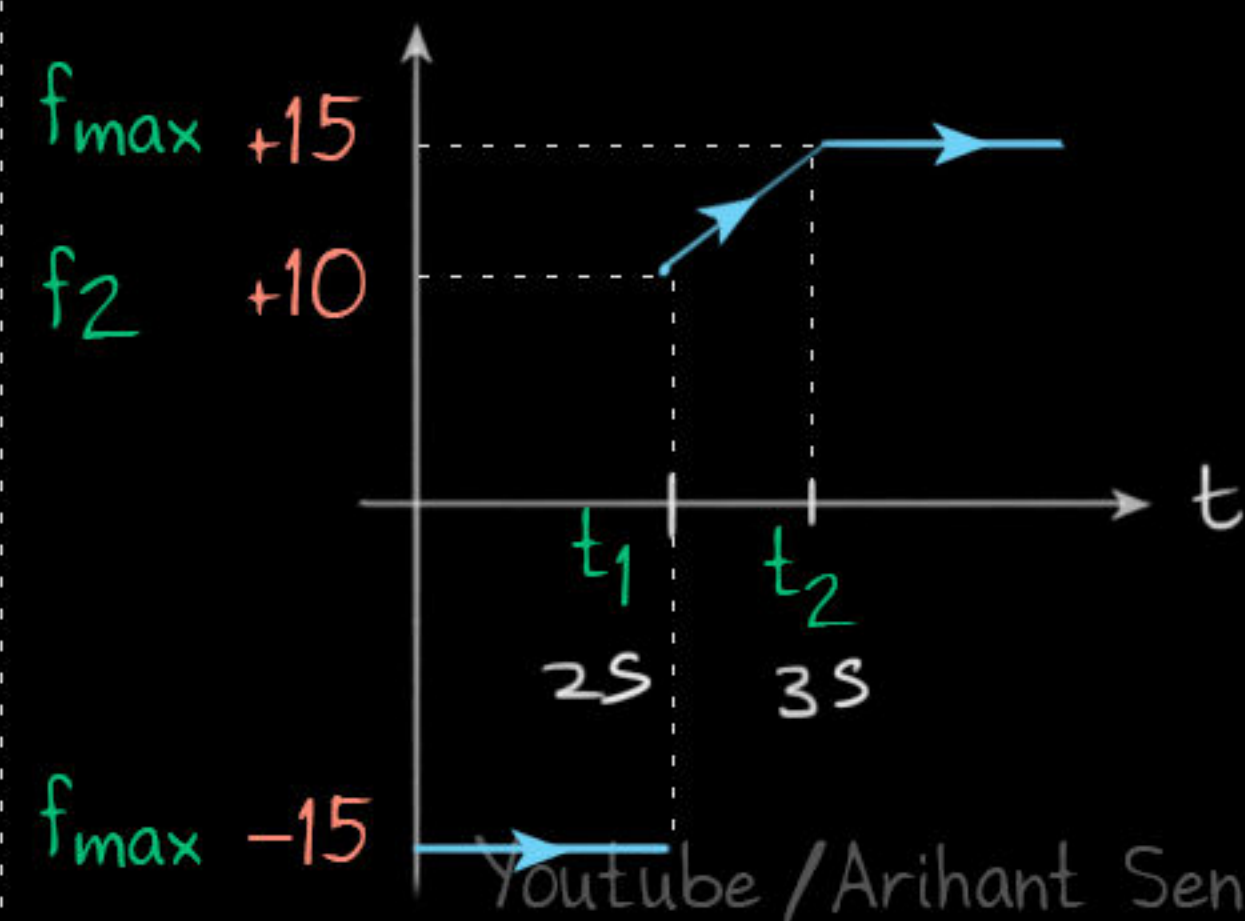
$$F \text{ at that time: } f_2 = kt = 10 \text{ N}$$

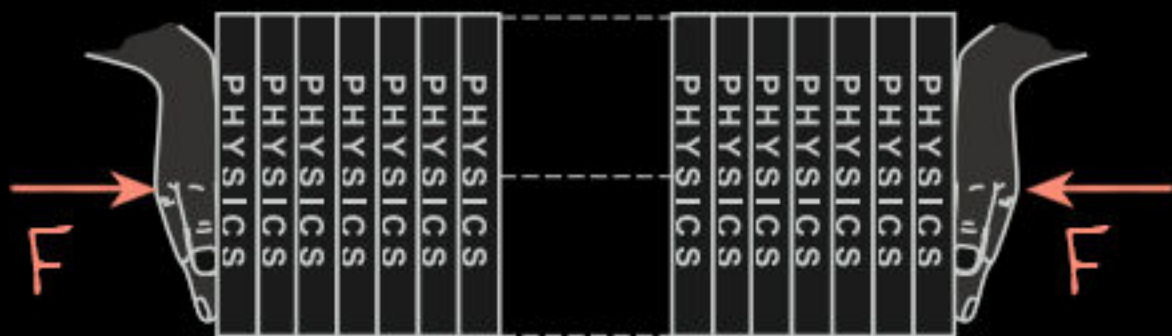
by Ankit Singhvi

It restarts at: t_2

$$kt_2 = \mu mg$$

$$\Rightarrow t_2 = \frac{\mu mg}{k} = 3 \text{ sec}$$

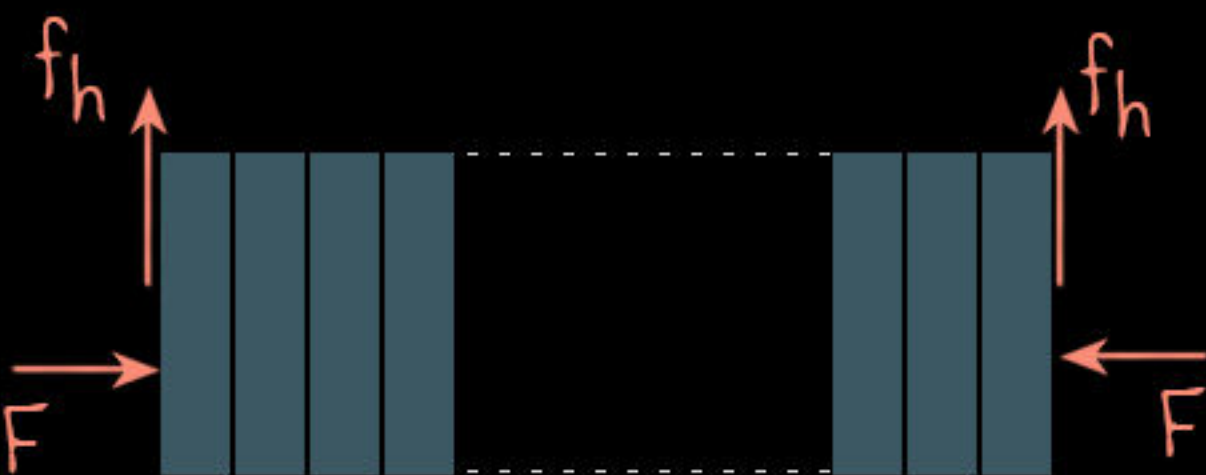




Concept: F is same everywhere, so slipping occurs either at hands or after 1st book from either side.

34. A boy lifts a stack of several identical books pressing hard with his hands. Coefficient of static friction between hand and a book is $\mu_{hb} = 0.40$, between the books is $\mu_{bb} = 0.25$ and mass of a book is $m = 400$ g. Now the boy starts decreasing the pressure gradually. When the horizontal component of the force applied by the boy becomes $F = 120$ N, the books are about to fall. Acceleration of free fall is $g = 10$ m/s².

- (a) How many books are there in the stack?
 (b) Find frictional force between the third and the fourth book.

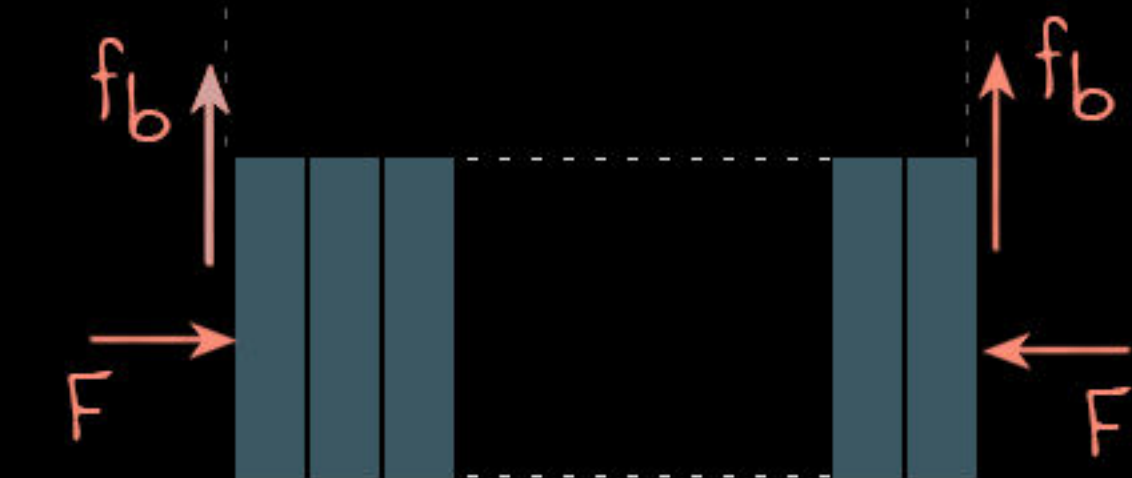


Hands supports n books.

$$\therefore 2f_h = nmg \rightarrow$$

For eq. to break at hand at given F :

$$2\mu_h F < nmg \Rightarrow n > 24 \quad (1)$$



1st & last book support $n-2$ books. For eq. to break between end books at F :

$$\therefore 2f_b = (n-2)mg \rightarrow 2\mu_b F < (n-2)mg \Rightarrow n > 17 \quad (2)$$

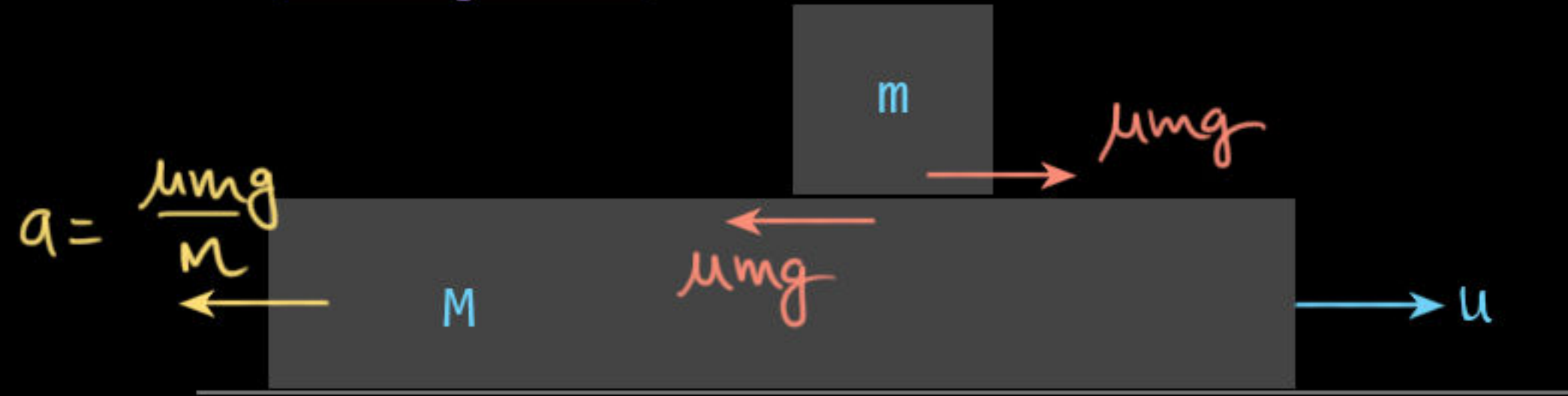
A From 1,2: for equilibrium to be maintained at both surfaces at given F : $n = 17$

B $2f_b = (n-2)mg \Rightarrow f_b = 11mg/2 = 22$ N //

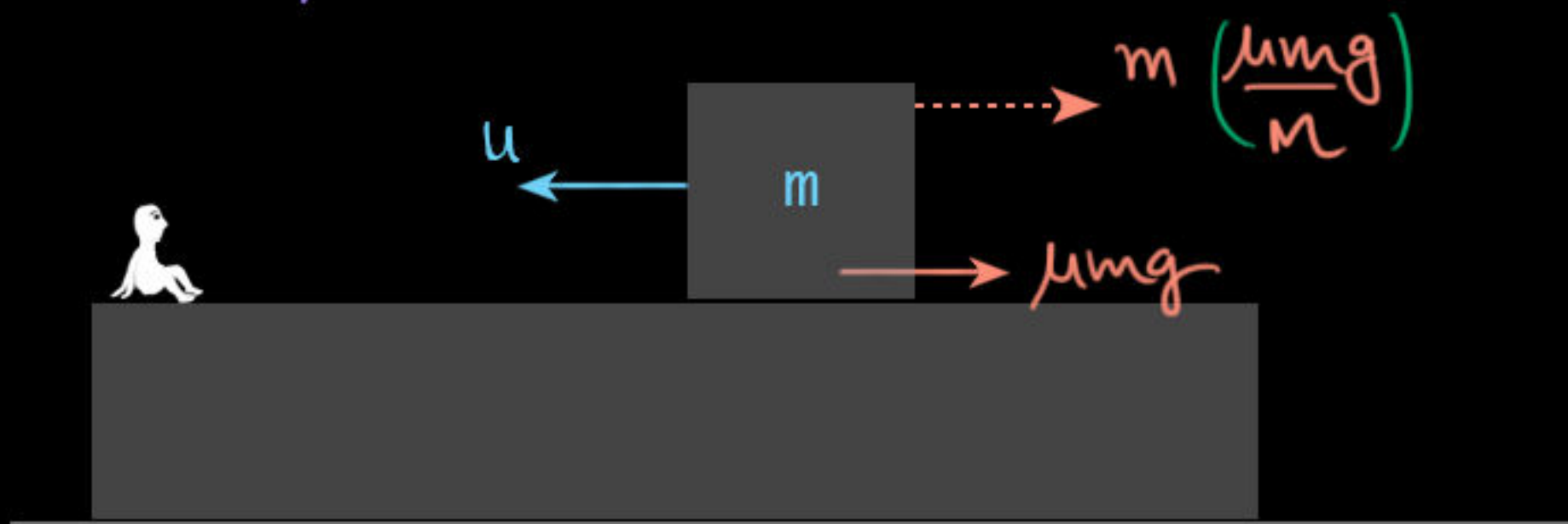
35. A large plank of mass M is moving with a velocity u on a horizontal frictionless floor. A block of mass m is gently placed on the plank without any velocity. If the block slides a distance l on the plank before it stops sliding, find coefficient of friction between the block and the plank.



w.r.t ground:



w.r.t plank:



$$v^2 = u^2 + 2as$$

$$\Rightarrow 0 = u^2 - 2 \left[\frac{\mu mg}{M} + \mu g \right] l$$

$$\Rightarrow \mu = \frac{u^2 M}{2lg(m+M)} //$$

Algorithm to solve multiple layer slipping problems.

Step 1

Best case scenario
(Everything moves together)

Substeps

1 Figure out 'most slippery' surface.
And then...



2 Solve for situation when
its just about to slip.

Step 2

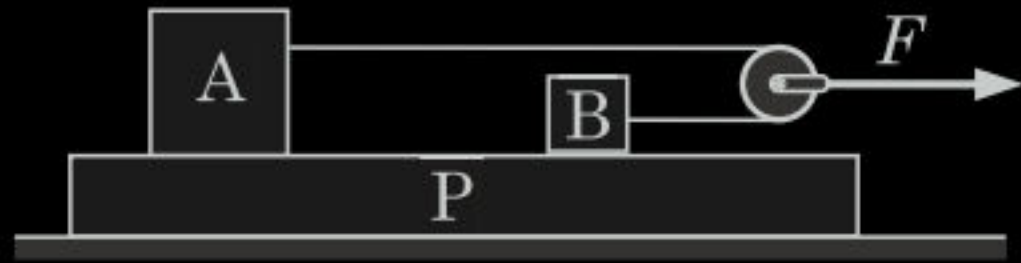
Worst case scenario
(Slipping everywhere)

1 Figure out the 'least slippery' surface.
And then...

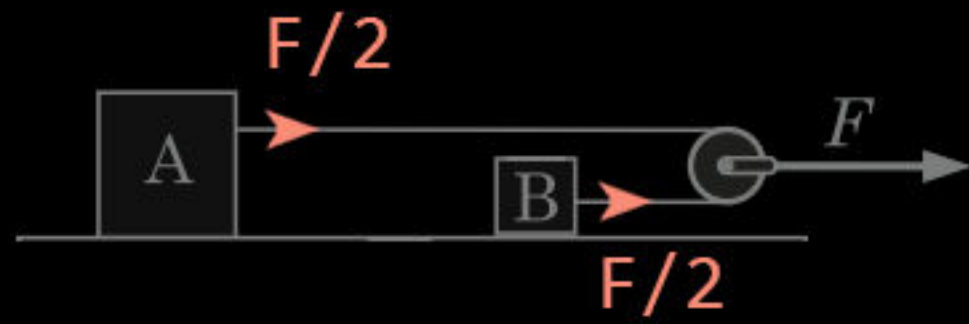


2

Only μ is given	Solve for situation when its just about to slip.
μ_k is given too	Solve for situation when its slipped.



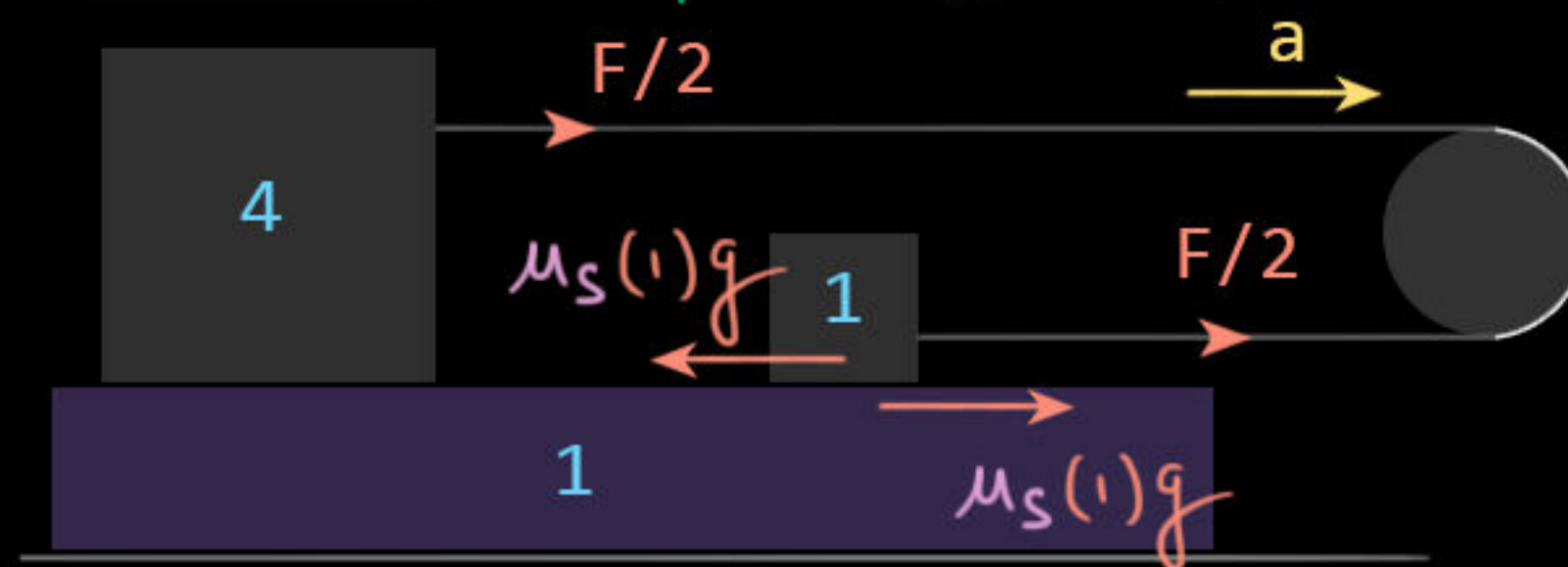
36. In the setup shown, blocks A and B of masses 4.0 kg and 1.0 kg are placed on a platform P of mass 1.0 kg that is placed on a horizontal frictionless floor. The blocks are connected by a light inextensible string that passes round an ideal pulley. Coefficients of static and kinetic friction between the blocks and the platform are 0.16 and 0.10. On applying a horizontal force F on the pulley, acceleration of the platform becomes 2.0 m/s^2 . Find magnitude of the force F , frictional forces on the blocks and their accelerations. Acceleration due to gravity is 10 m/s^2 .



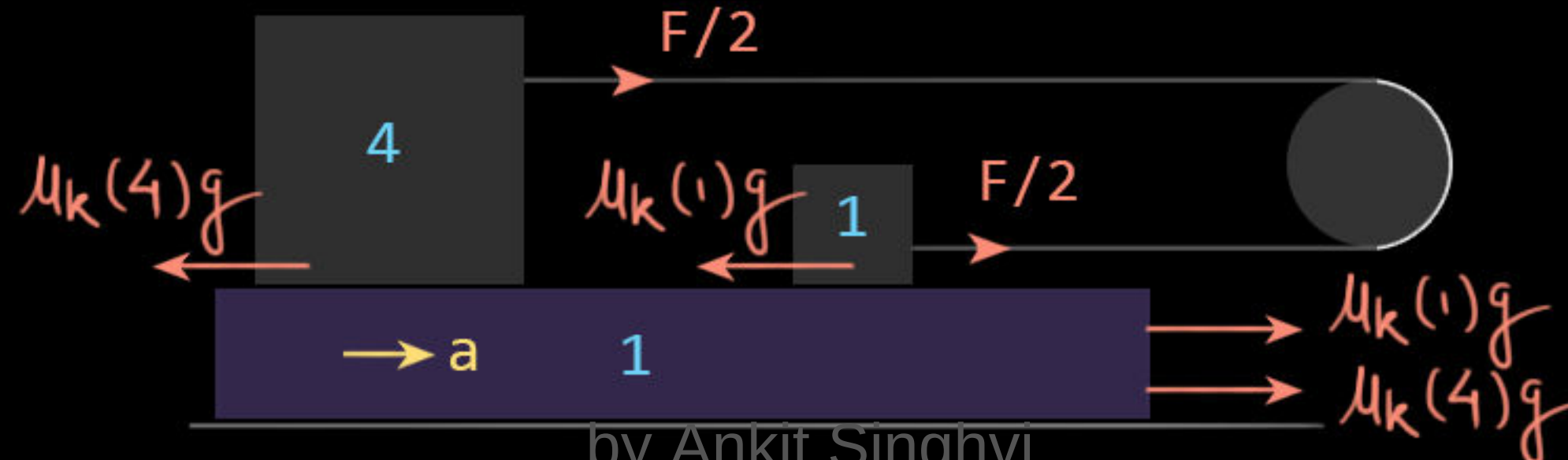
As same force $F/2$ is applied on both blocks, lighter block B will slip first.

(As rest of the conditions like touching the platform are same for both blocks)

Best case (no slip limiting case):



Worst case (slip everywhere with kinetic friction):



$$a = \frac{F}{4+1+1} = \frac{F}{6} \quad (1)$$

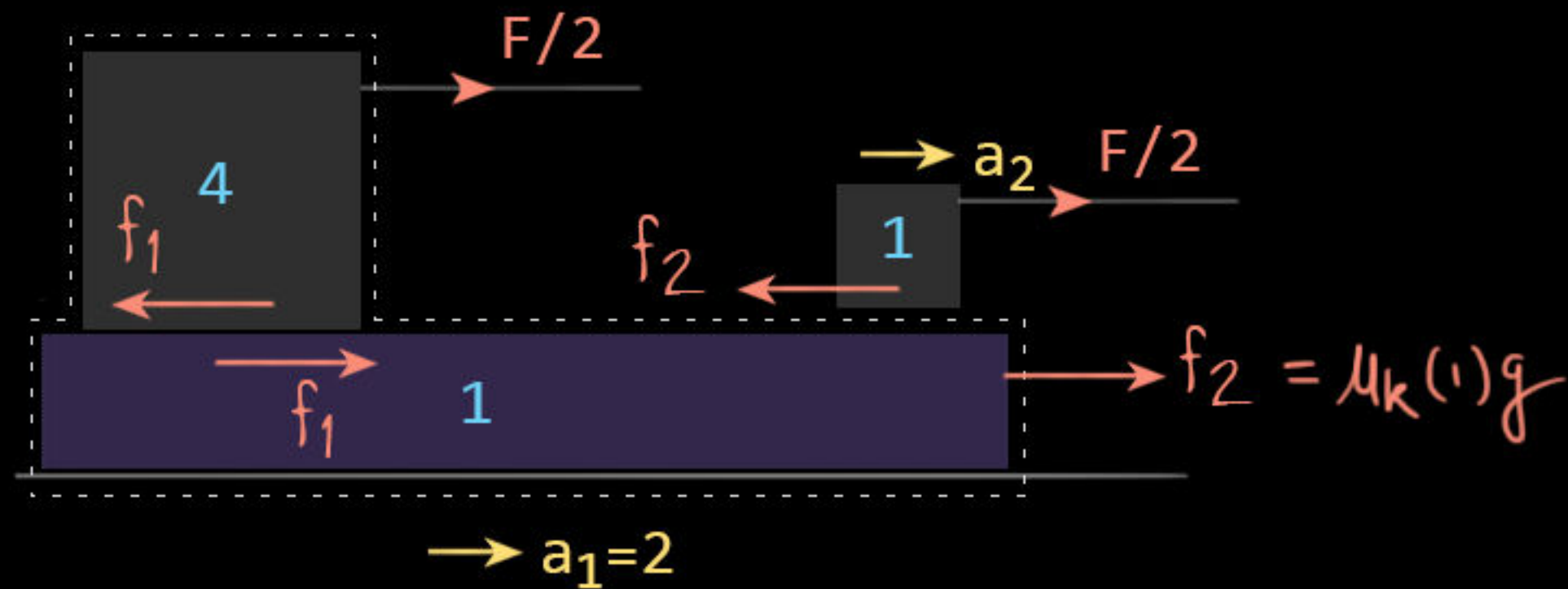
$$\frac{F}{2} - \mu_s(1)g = (1)a \quad (2)$$

From 1,2: $a = 0.8$
Reject

$$a = \frac{\mu_k(1)g + \mu_k(4)g}{1}$$

$$= 5 \text{ Reject}$$

- Given case where acceleration of plank is 2 is the..
Intermediate case (smaller block slips, bigger doesn't)



$$a_1 = 2 \text{ (given)}$$

$$F/2 + \mu_k(1)g = (4+1)2$$

$$\Rightarrow f = 18 //$$

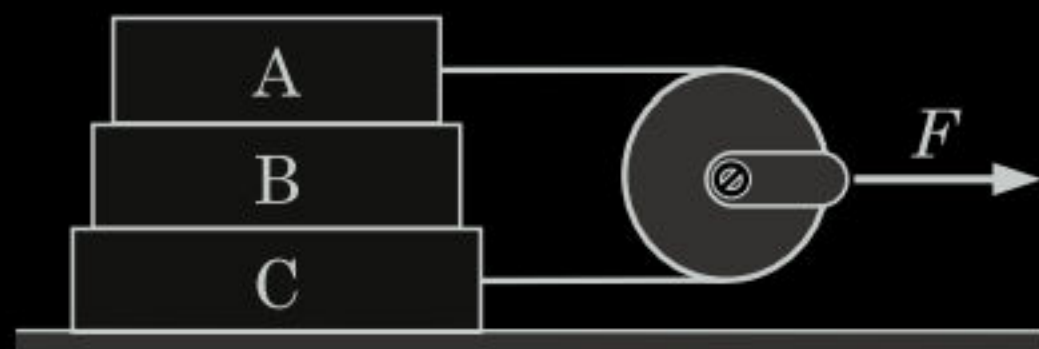
$$F/2 - f_1 = 4(2)$$

$$\Rightarrow f_1 = 1 //$$

$$f_2 = \mu_k(1)g = 1 //$$

$$F/2 - \mu_k(1)g = (1)a_2$$

$$\Rightarrow a_2 = 8 //$$

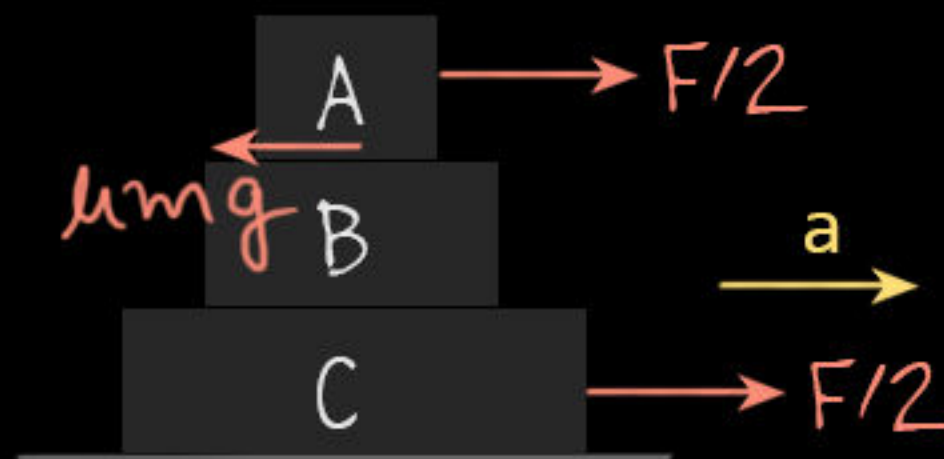


37. In the given setup, a bar B is sandwiched between bars A and C that are connected by a light inextensible thread, which passes round an ideal pulley. Mass of each bar is m , coefficient of friction between the bars is μ and the floor is frictionless. Acceleration due to gravity is g . If a horizontal force F is applied on the pulley, find acceleration of the bar B.

As equal force $F/2$ is applied on equal masses A and B, top block B will slip first.

As limiting friction on surface AB (μmg) is less than limiting friction on surface BC ($\mu(2m)g$)

Best case (no slip limit, i.e A is about to slide):

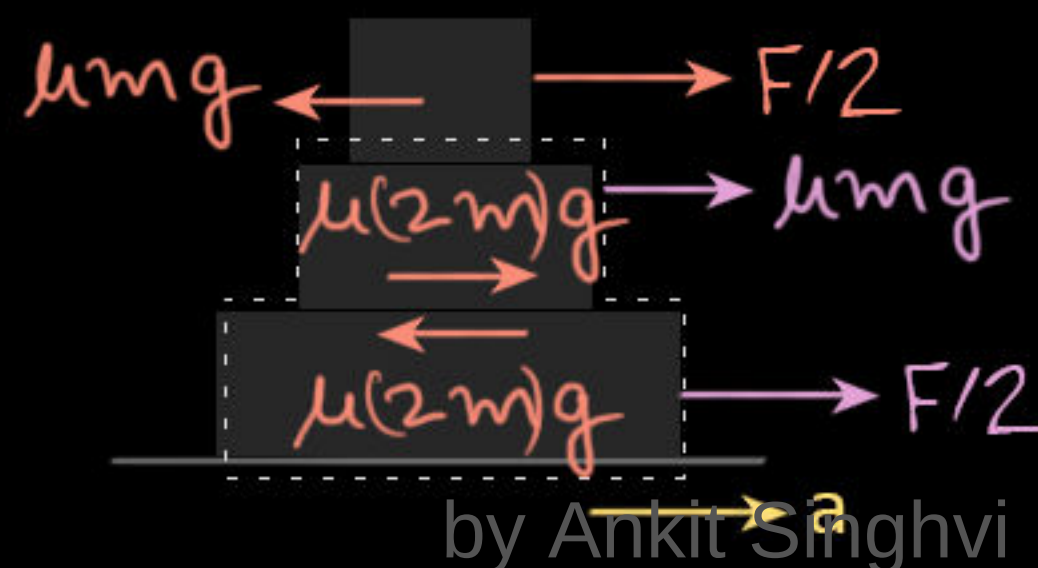


$$F = 3ma \quad (1)$$

$$F/2 - \mu mg = ma \quad (2)$$

$$\text{From 1,2: } a = F/3m, \quad F \leq 6\mu mg$$

Worst case (all slip limit, i.e C is about to slide too):



$$\text{B and C: } F/2 + \mu mg = (m+m)a$$

$$\text{B: } \mu mg + 2\mu mg = ma$$

From 1,2:

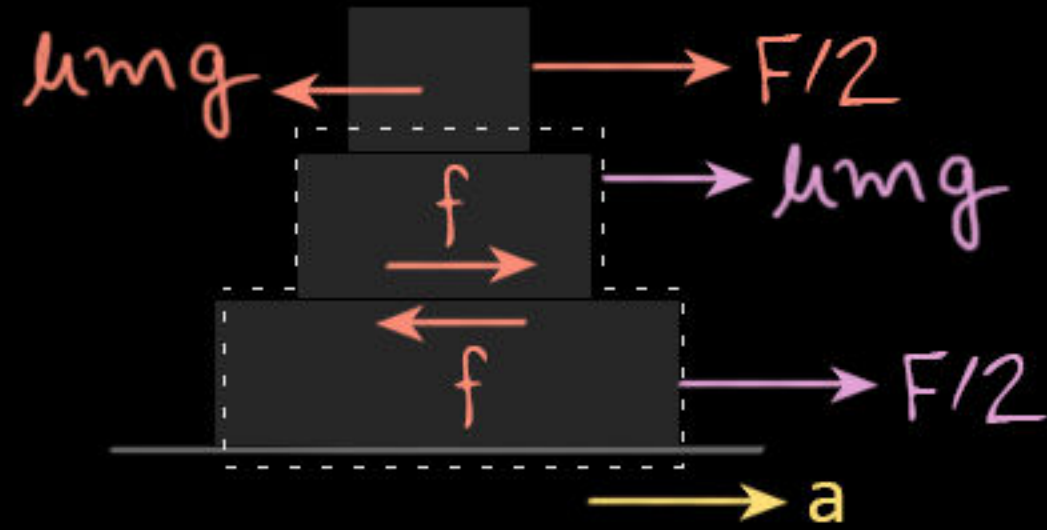
$$a = 3\mu g, \quad F \geq 10\mu mg$$

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Intermediate case (top block slips, bigger doesn't)

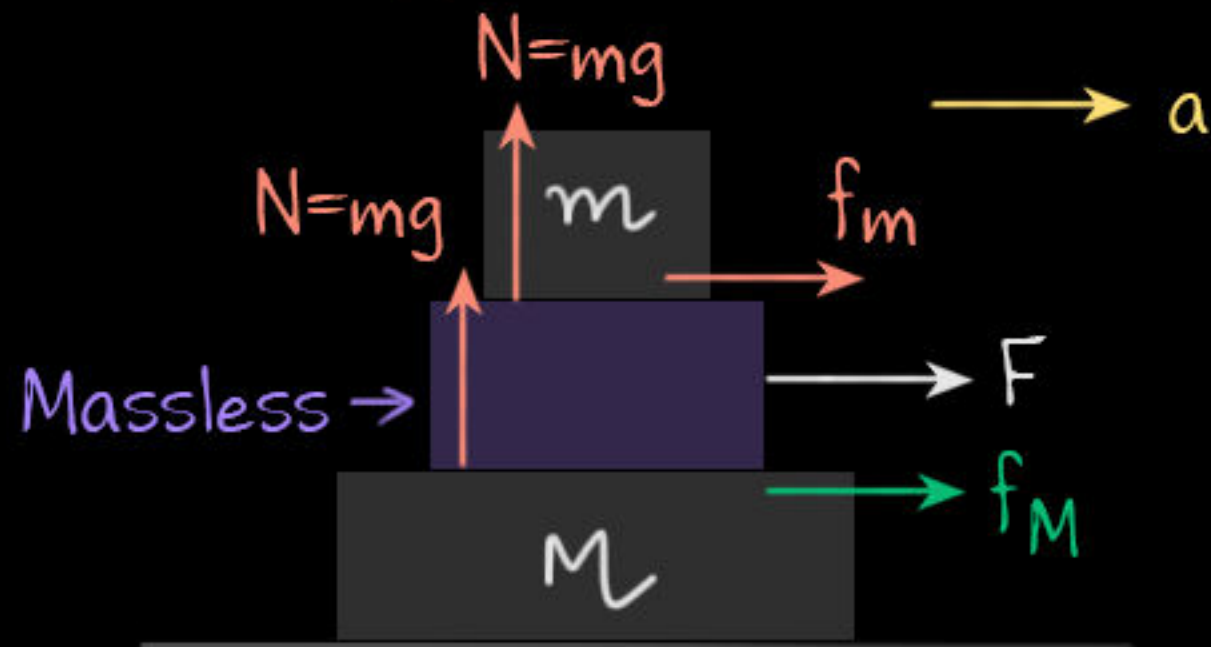
i.e $6\mu mg \leq F \leq 10\mu mg$, $a = \frac{F/2 + \mu mg}{2m}$





Normally, we like the objects to stay to avoid slipping. But here we want them to move with paper. B is heavier, so difficult to move. So it will slip first.

Proof When not slipping:



$$f_m = ma \quad f_M = Ma$$

Limiting value of f_m and f_M is same (μmg),

∴ As 'a' increases, f_M will reach the limit first (∵ $Ma > ma$)

∴ M will slide w.r.t paper, while m will move along.

38. On a horizontal frictionless floor, a block A of mass m rests on another stationary block B of mass M ($m < M$) with a thin paper sheet of negligible mass inserted between them. The paper sheet is wide enough to prevent direct contact between the blocks. Coefficient of friction between each block and the paper sheet is μ . Find necessary horizontal force F applied on the paper sheet as shown in the figure to fulfil the following conditions. Acceleration due to gravity is g .

(a) The block B does not slide relative to the paper sheet.

(b) The paper sheet slides relative to B but does not slide relative to A.

∴ Best case (no slip limit, i.e. when B is about to slide):

$$a = \frac{F}{m+M} = \frac{f_{M_{\max}}}{M} \Rightarrow F = \frac{\mu mg (m+M)}{M}$$

Worst case (all slip limit, i.e. when A is about to slide too):

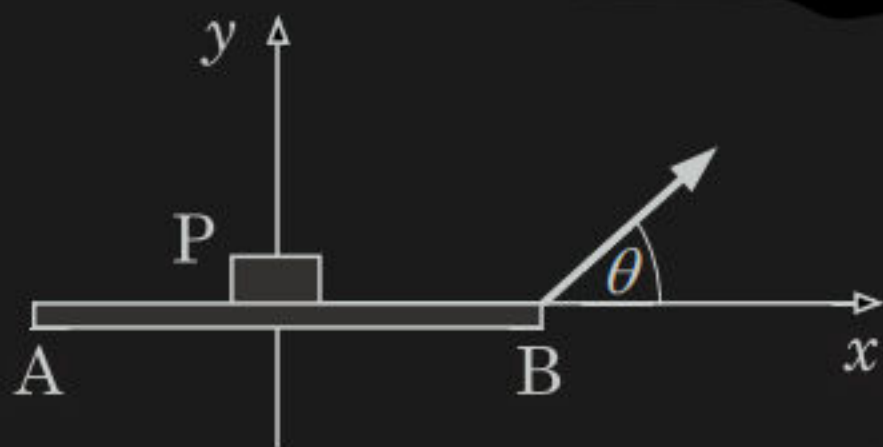


$$F = f_{m_{\max}} + f_{M_{\max}} = \mu mg + \mu mg$$

$$= 2\mu mg$$

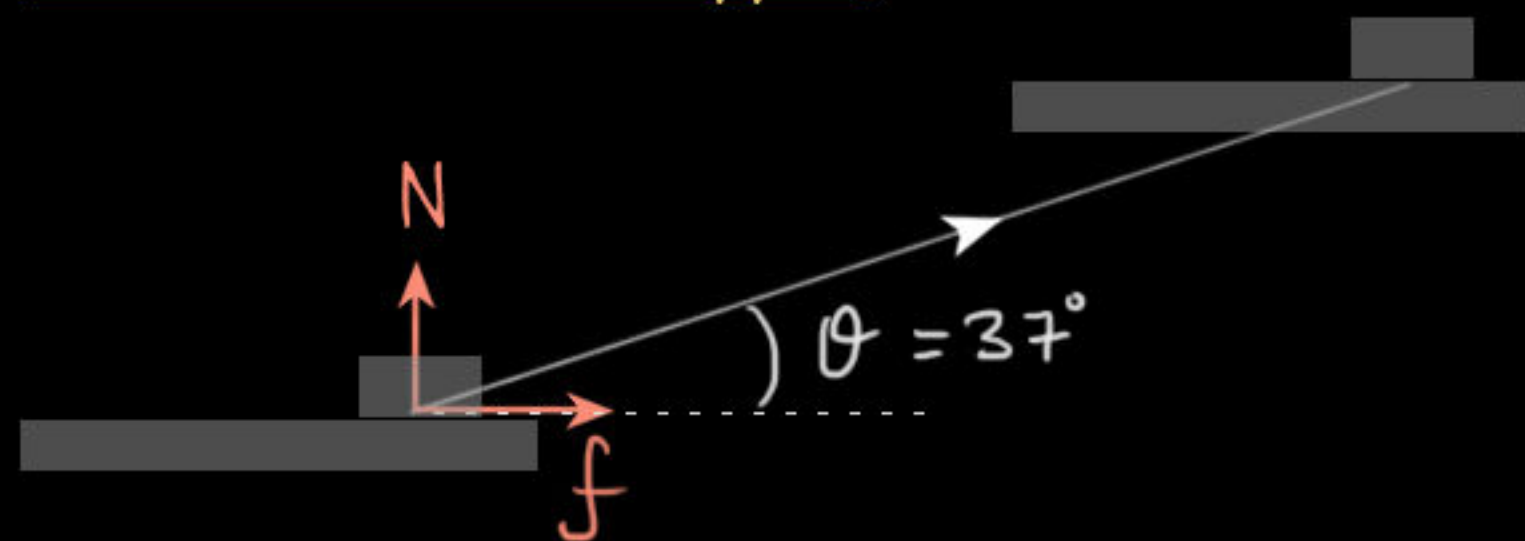
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39. A bar AB and a block P are placed on a frictionless horizontal floor with their adjacent vertical faces in contact as shown in the figure by a top view. Here the x - y plane of a coordinate frame is in the plane of the floor. The bar is made to move horizontally along the floor in a straight line with a **constant acceleration** without rotation. Direction of motion of the bar makes an angle $\theta = \sin^{-1}(0.6)$ with the positive x -axis. Coefficient of friction between the block and the bar is $\mu = 0.75$. **Find distance travelled by the block until the plank moves a distance $l = 40$ cm.**

Lets assume no slipping

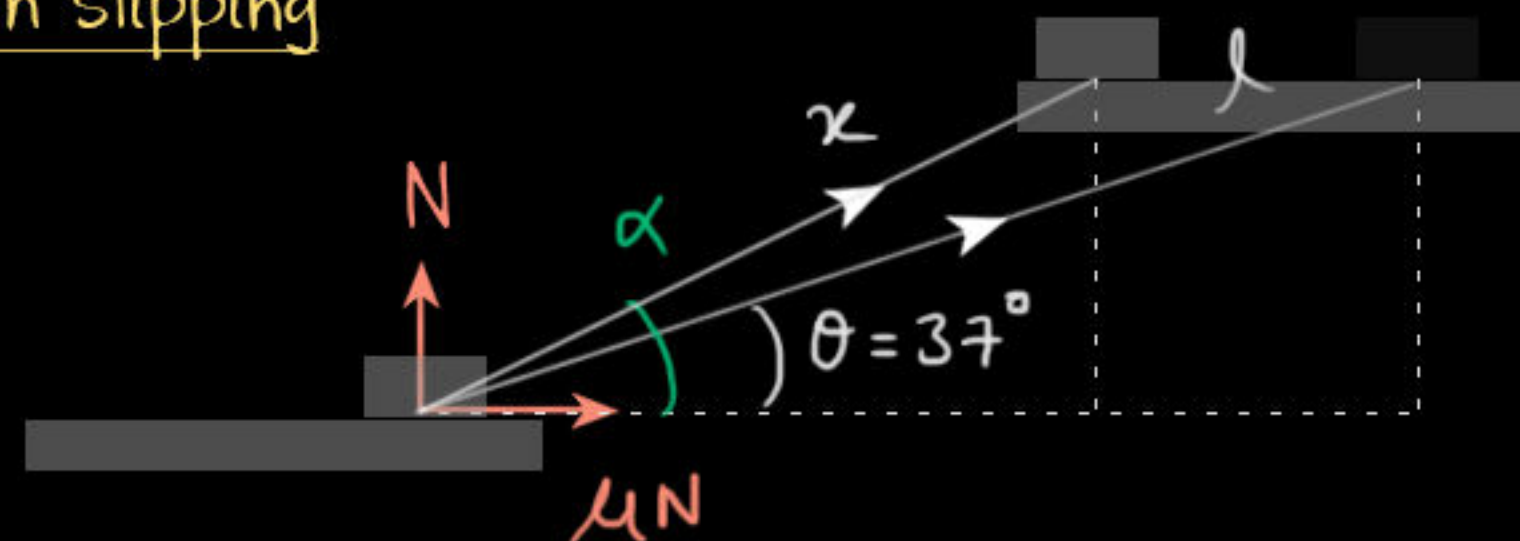


$$\frac{N}{f} = \tan 37^\circ = \frac{3}{4}$$

$$\Rightarrow f = \frac{4}{3}N > \mu N \quad \therefore \text{Not possible}$$

Our assumption is wrong.
Block will slide.

With slipping

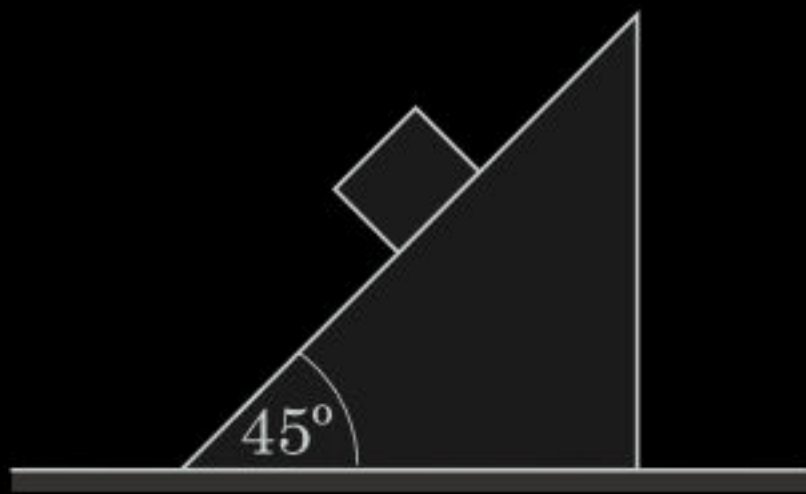


$$\tan \alpha = \frac{N}{\mu N} = \frac{1}{\mu} \Rightarrow \alpha = \tan^{-1}(\mu) \quad (1)$$

$$x \sin \alpha = l \sin \theta \quad (2)$$

$$\text{From 1,2: } x = l \sin \theta \sqrt{\mu^2 + 1}$$

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40. A block is released on the slant face of a wedge of equal mass placed on a horizontal floor. The slant face of the wedge makes an angle of 45° with the floor. Coefficient of friction between all surfaces in contact is the same. Find range of coefficient of friction μ for the following cases.

- Both the bodies remain motionless.
- The wedge remains motionless and the block slides down.
- The block slides down the wedge and the wedge slides on the floor.
- The block does not slide on the wedge but the wedge slides on the floor.

From no motion, to when block just slides:

$$\mu > \tan \theta = 1 //$$

(Angle of repose)

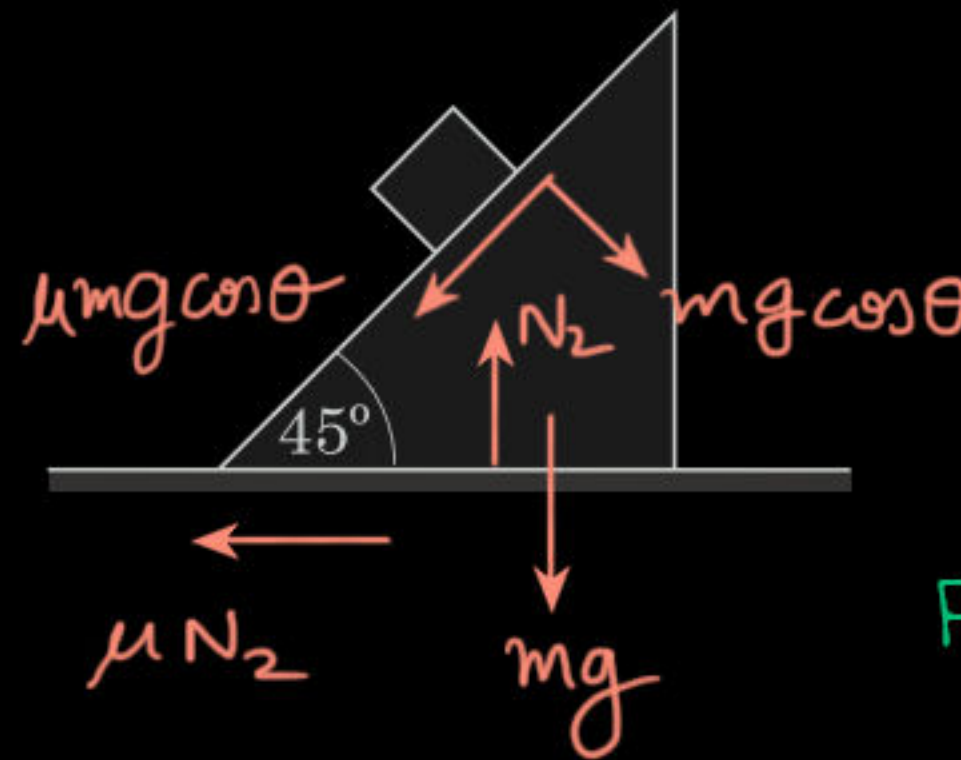
- $\therefore$
- $\mu > 1$
 - $\sqrt{5} - 2 < \mu < 1$
 - $\mu < \sqrt{5} - 2$

D Not possible, as the equivalent fused block will never slide, even if the floor is frictionless.



From block sliding to when wedge too just slides:

Forces on wedge:



Concept: Acceleration of wedge will be zero, while its bottom will reach limiting value.

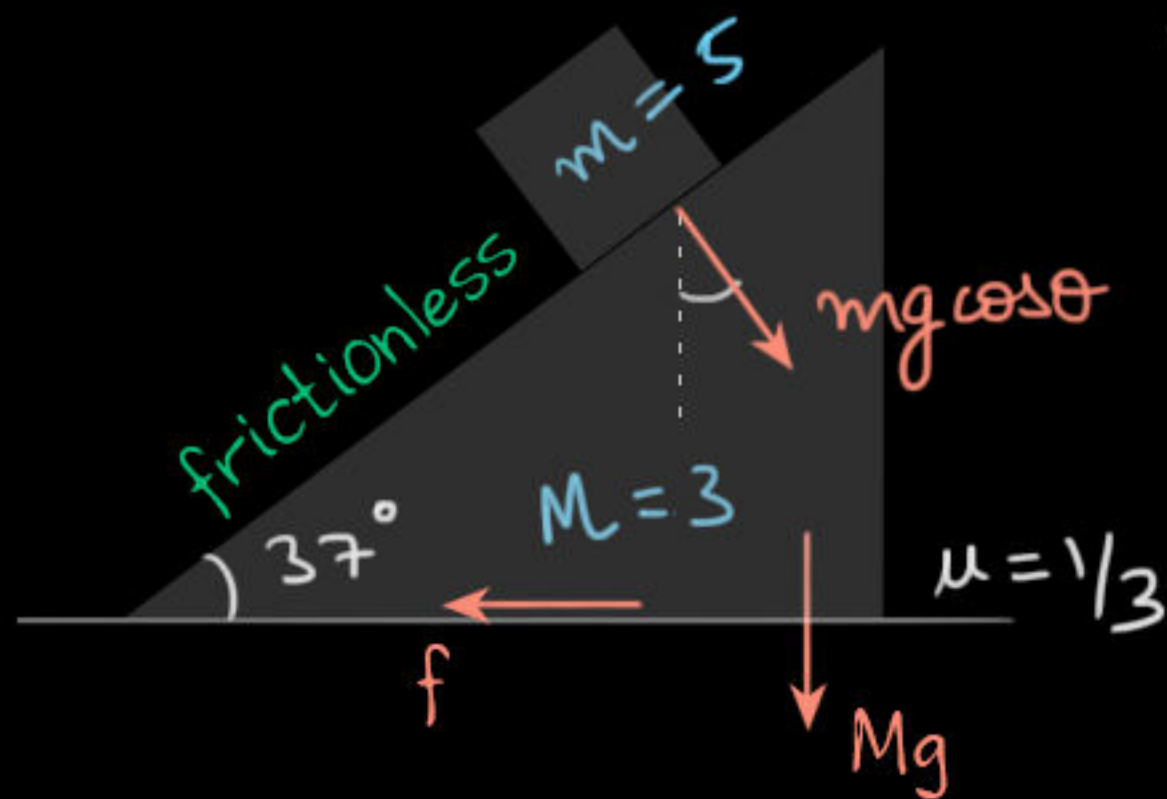
$$F_y: N_2 = \frac{\mu mg}{2} + \frac{mg}{2} + mg \quad (1)$$

$$F_x: \mu N_2 + \frac{\mu mg}{2} = \frac{mg}{2} \quad (2)$$

$$\text{From 1, 2: } \mu = \sqrt{5} - 2 //$$

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41. A wedge of mass $M = 3$ kg rests on a horizontal floor. Slant face of the wedge makes an angle $\theta = \sin^{-1}(0.6)$ with the horizontal. A block of mass $m = 5$ kg is released on the slant face. Coefficient of friction between the floor and the wedge is $\mu = 1/3$ and the slant face is friction-less. Find the force of normal reaction between the block and the slant face. Acceleration of free fall is $g = 10$ m/s².

Sol:

Assume wedge is at rest:

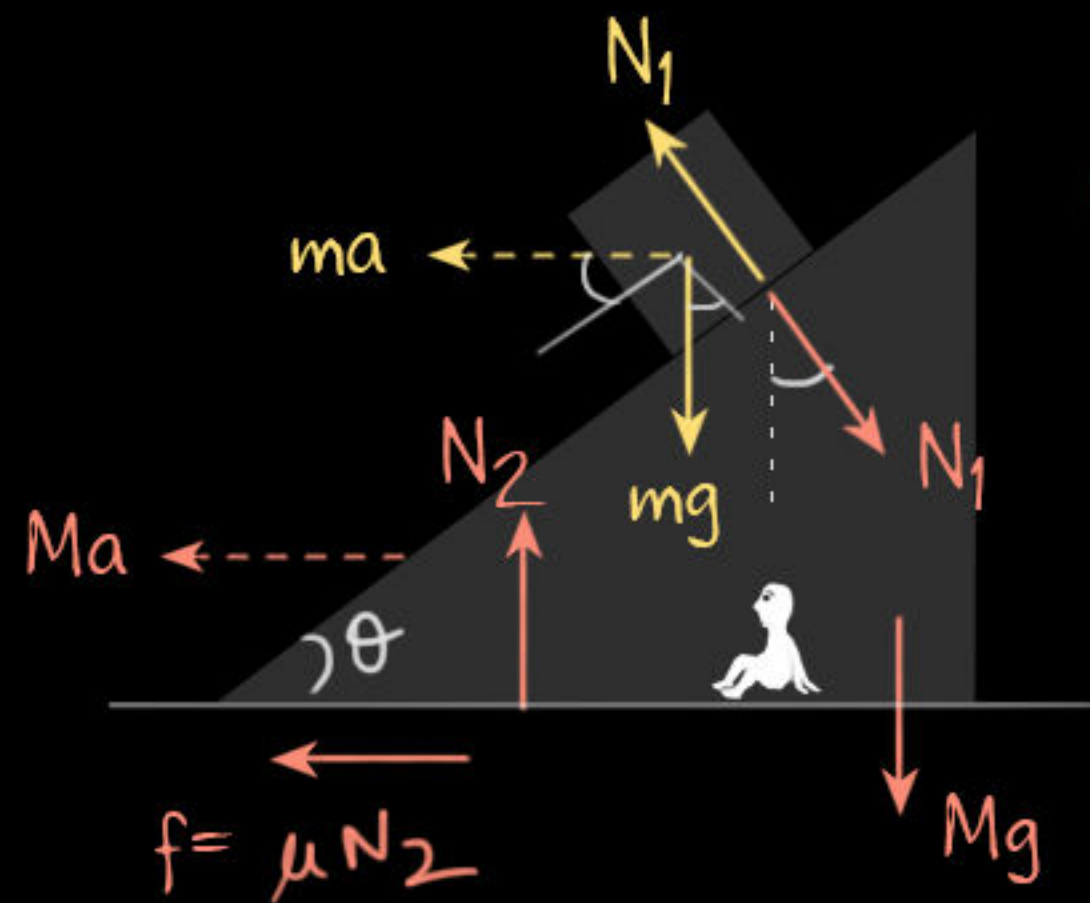
If we are lucky, answer would be simply $mg \cos \theta$

$$mg \cos \theta \sin \theta = f \leq \mu (Mg + mg \cos \theta \cos \theta)$$

$$\Rightarrow 5 \cdot \frac{4}{5} \cdot \frac{3}{5} \leq \frac{1}{3} \left(3 + 5 \cdot \frac{4}{5} \cdot \frac{4}{5} \right)$$

$$\Rightarrow 12 \leq \frac{31}{3} \quad \times \quad \therefore \text{wedge slides}$$

Let the acceleration of wedge be $a \rightarrow$
w.r.t observer on wedge



F_m normal to surface:

$$N_1 + ma \sin \theta = mg \cos \theta \quad (1)$$

F_M in Y and X direction:

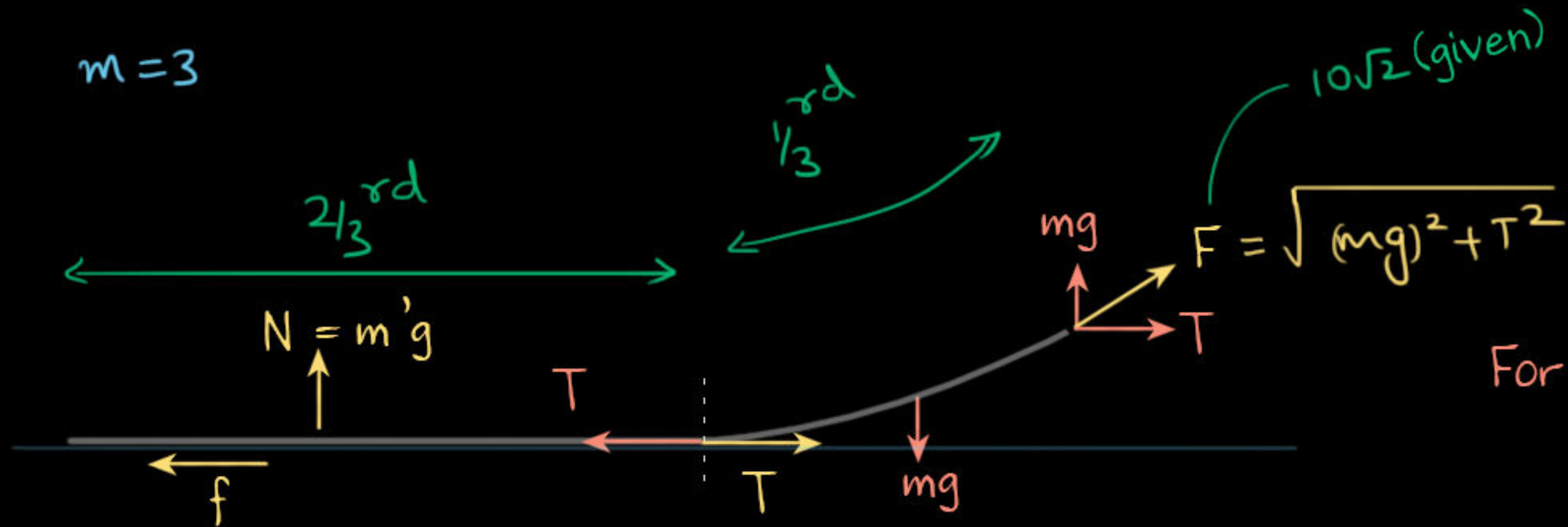
$$N_2 = N_1 \cos \theta + Mg \quad (2)$$

$$N_1 \sin \theta = \mu N_2 + Ma \quad (3)$$

From 1,2,3:
$$N_1 = \frac{mMg (\cos \theta + \mu \sin \theta)}{M + m \sin \theta (\sin \theta - \mu \cos \theta)}$$

Concept: Rope is horizontal when it starts to bend. So tension too be horizontal there.

42. One end of a uniform rope of mass $m = 3.0$ kg placed on the ground is pulled by a force $F = 10\sqrt{2}$ N in such a way that $\eta = 2/3$ of length of the rope remains at rest on the ground in a straight line and rest of the rope in the air in the vertical plane containing the portion of the rope on the ground. Find range of values of coefficient of friction between the rope and the ground. Acceleration of free fall is $g = 10$ m/s².



For rope in air: $10\sqrt{2} = \sqrt{\left(\frac{1}{3} \cdot 3g\right)^2 + T^2}$

$\Rightarrow T = 10$ N

For rope on ground: $T \leq f_{\max}$

$\Rightarrow 10 \leq \mu \left(\frac{2}{3} \cdot 3g \right)$

$\Rightarrow \mu \geq \frac{1}{2}$

Let $f = km$

Initially, with balanced forces

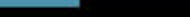




 $F = 1000k$

$f = 800k$


 $a = \frac{f}{m} = k$

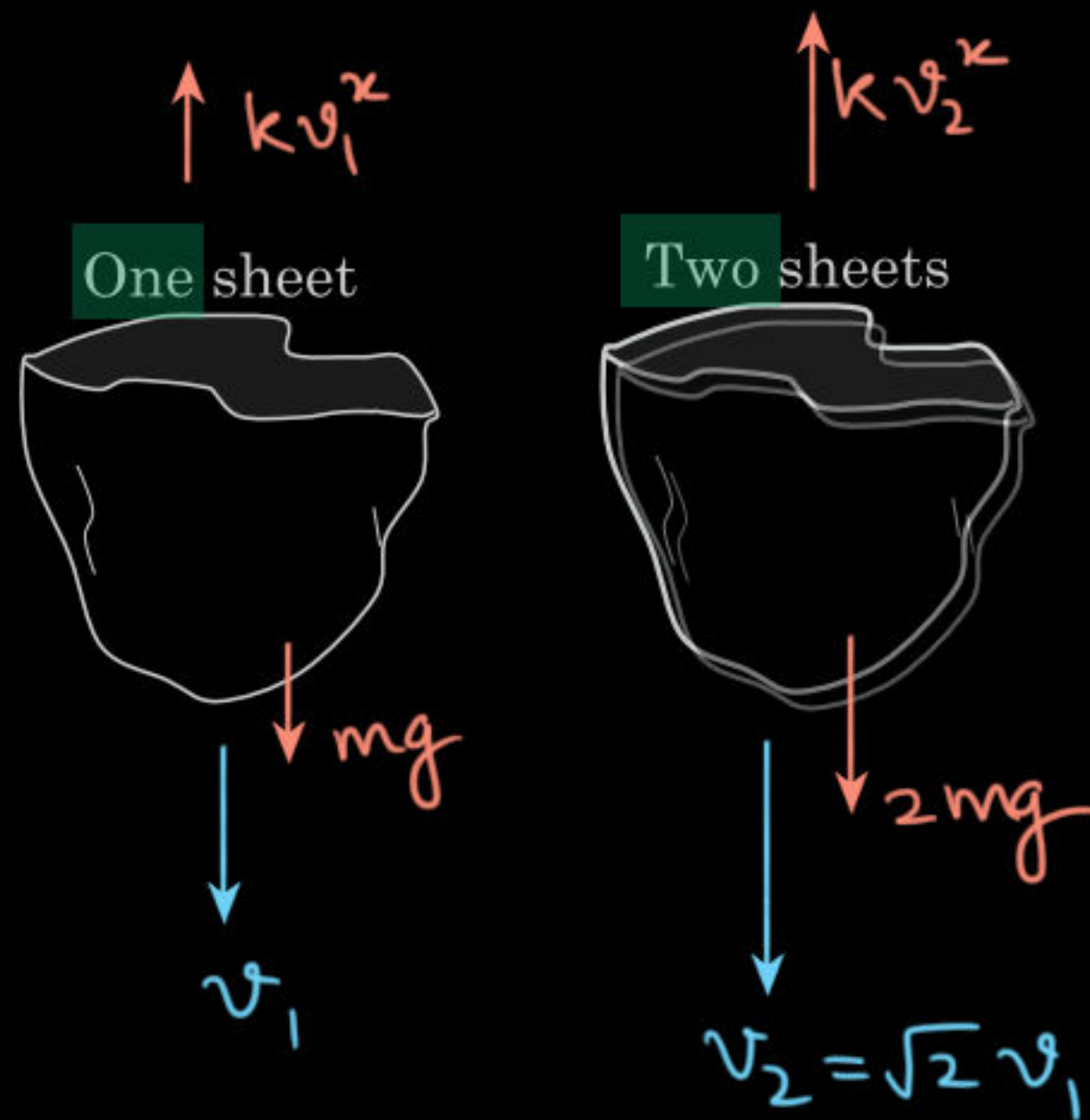

 $a = \frac{200k}{800} = \frac{k}{4}$



$a = \frac{f}{m} = k$



$$= ABCQ + CPEQ$$
$$= \frac{1}{2} (a+b+b) 4c + \frac{1}{2} (a+b+b) c$$
$$= l + \frac{l}{4} = \frac{5l}{4} = 125 \text{ m}$$



44. A student starts an experiment with a stack of three identical sheets of paper. He crumples the stack against his fist as shown in the figure, then separates a single sheet from the other two without altering their crumpled (pseudo-conical) shape. This yields two objects of the same shape, but of different masses. He releases these objects simultaneously, and records their motion in still air. After the two objects acquire their terminal velocities, the heavier one moves $\eta = \sqrt{2}$ times farther than the lighter one in a particular time interval. If the force of air resistance is proportional to v^x , where v is velocity and x is a constant exponent, what should be value of the exponent x ?

$$\therefore v_2 = \sqrt{2} v_1$$

At terminal velocity, forces are balanced.

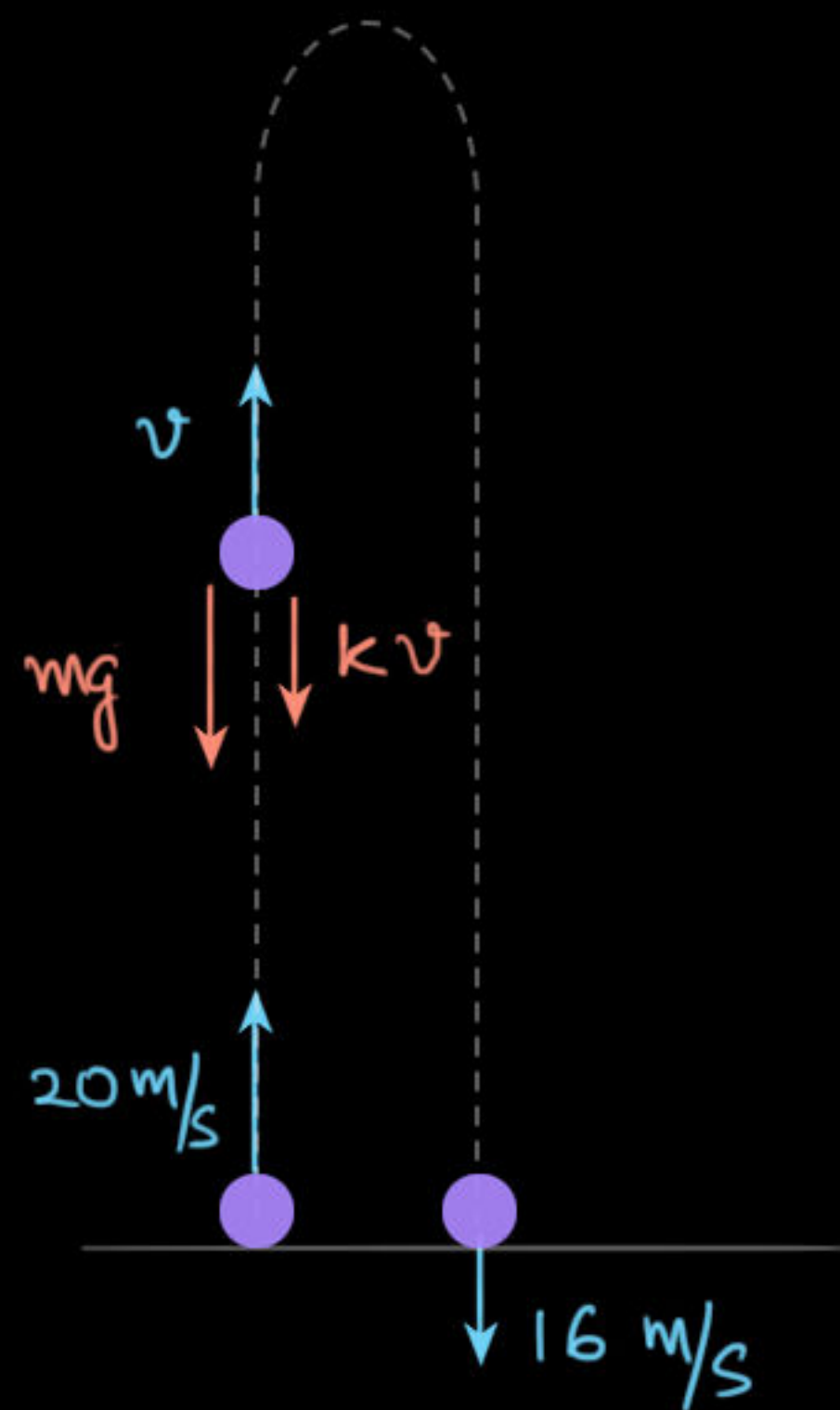
$$\begin{aligned} \therefore mg &= kv_1^x \\ 2mg &= kv_2^x \end{aligned}$$

$$\text{Dividing } \frac{1}{2} = \left(\frac{v_1}{v_2}\right)^x = \left(\frac{1}{\sqrt{2}}\right)^x = \frac{1}{2^{x/2}}$$

$$\Rightarrow x = 2 //$$

45. A ball projected vertically upwards with a velocity 20 m/s returns back on the ground with a velocity 16 m/s. If the air resistance is proportional to the speed of the ball, find airtime of the ball. Acceleration of free fall is 10 m/s².

Method 1



$$m \frac{d\vec{v}}{dt} = m\vec{g} - k\vec{v}$$

← superimpose

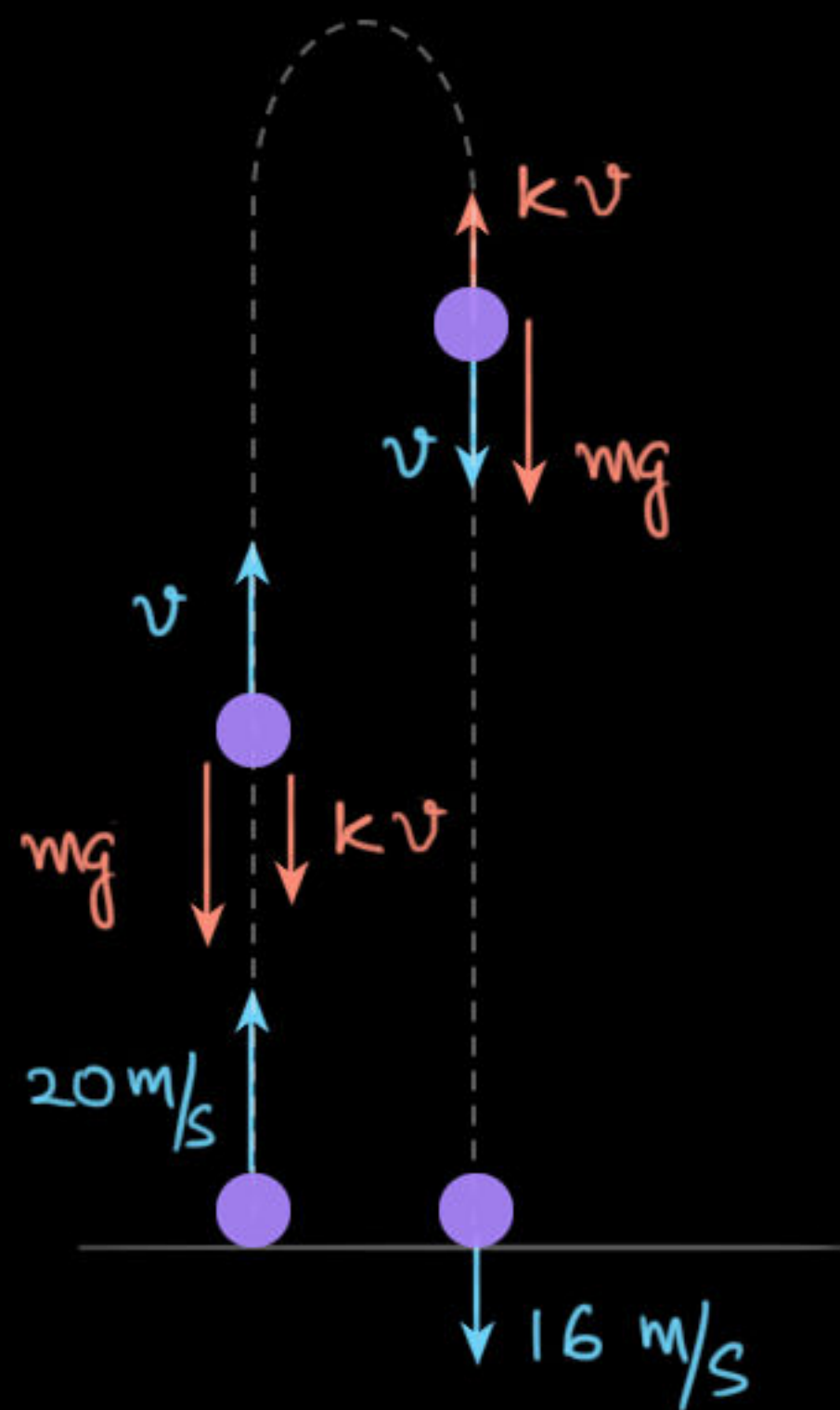
$$\begin{aligned} m \frac{d\vec{v}}{dt} &= m\vec{g} \\ m \frac{d\vec{v}}{dt} &= -k\vec{v} \end{aligned}$$

$$\Rightarrow \int_{20\hat{j}}^{-16\hat{j}} d\vec{v} = \int_0^{t_0} m\vec{g} dt - k \int_0^{t_0} \vec{v} dt$$

$$\Rightarrow -36\hat{j} = -10t_0\hat{j}$$

$$\Rightarrow t_0 = 3.6 \text{ sec} //$$

Method 2



While going up: +ve $\uparrow$

$$m \frac{dv}{dt} = -mg - kv$$

$$\Rightarrow \int_{20}^0 m dv = \int_0^{t_2} -mg dt - k \int_0^h v dt$$

$$\Rightarrow -20m = -mgt_1 - kh \quad (1)$$

While going down: +ve $\downarrow$

$$m \frac{dv}{dt} = mg - kv$$

$$\Rightarrow \int_0^{16} m dv = \int_0^{t_2} mg dt - k \int_0^h v dt$$

$$\Rightarrow 16m = mgt_2 - kh \quad (2)$$

Subtracting 1 from 2:

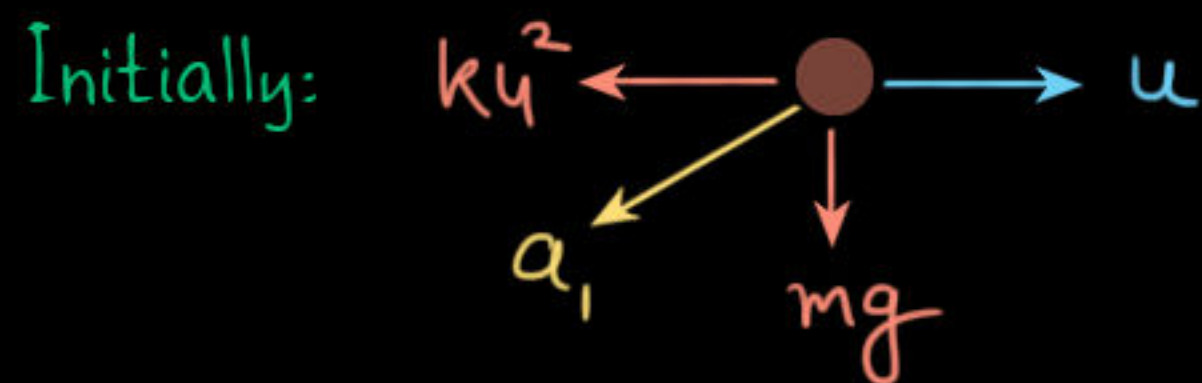
$$\Rightarrow 36m = mg(t_1 + t_2)$$

$$\Rightarrow t_1 + t_2 = \frac{36}{g} = 3.6 \text{ sec}$$

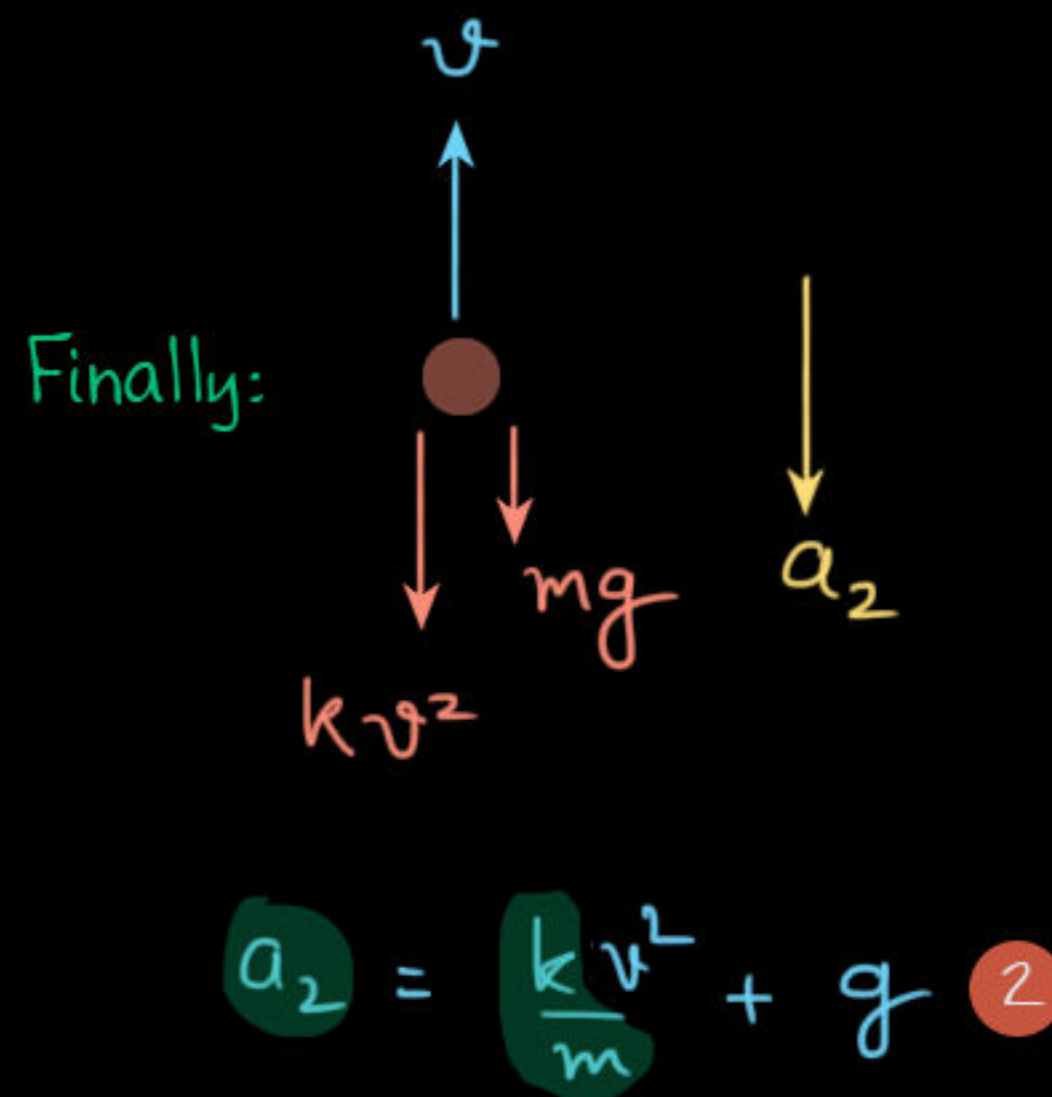
Youtube / Arihant Senior

46. A cricket ball during its motion in the air experiences a drag force proportional to square of its speed relative to the air. Immediately before a batsman hits a ball, the ball was moving horizontally in the air with speed $u = 20 \text{ m/s}$ and acceleration $a = \sqrt{164} \text{ m/s}^2$. After the hit, the ball starts rising vertically upwards with an initial velocity $v = 10 \text{ m/s}$. Neglecting buoyant force, calculate acceleration of the ball immediately after the hit? Assume acceleration of free fall $g = 10 \text{ m/s}^2$.

Let: $f = k v^2$

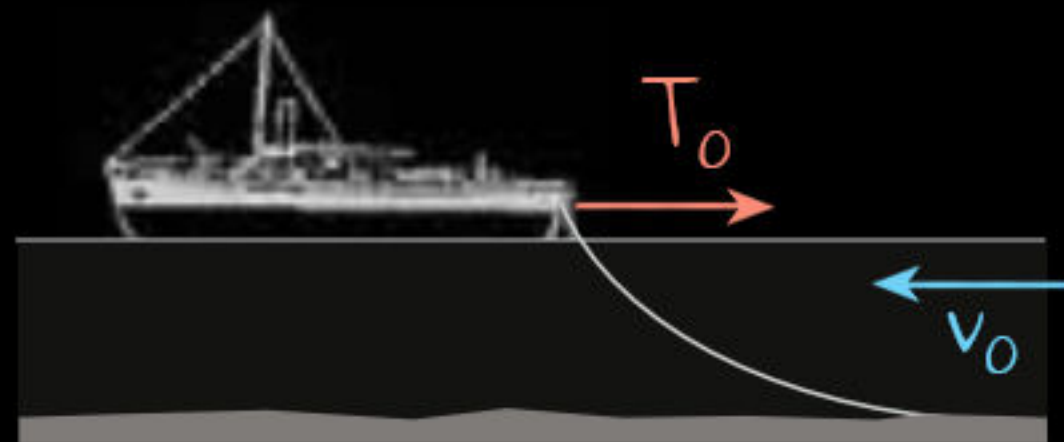


$$a_1 = \sqrt{\left(\frac{ku^2}{m}\right)^2 + g^2} \quad (1)$$



From 1,2:

$$a_2 = \left(\frac{v}{u}\right)^2 \sqrt{a_1^2 - g^2} + g //$$



47. A boat of mass m is anchored in the middle of a river, which is flowing with a constant velocity v_0 . Horizontal component of the force exerted on the boat by the anchor chain is T_0 . If the anchor chain suddenly breaks, determine time required for the boat to attain a velocity equal to $0.5v_0$. Assume that the drag force of the water is proportional to the velocity of the boat relative to the water and neglect air drag.

Let viscous force be:

$$f = kv_r$$

Proportionality constant

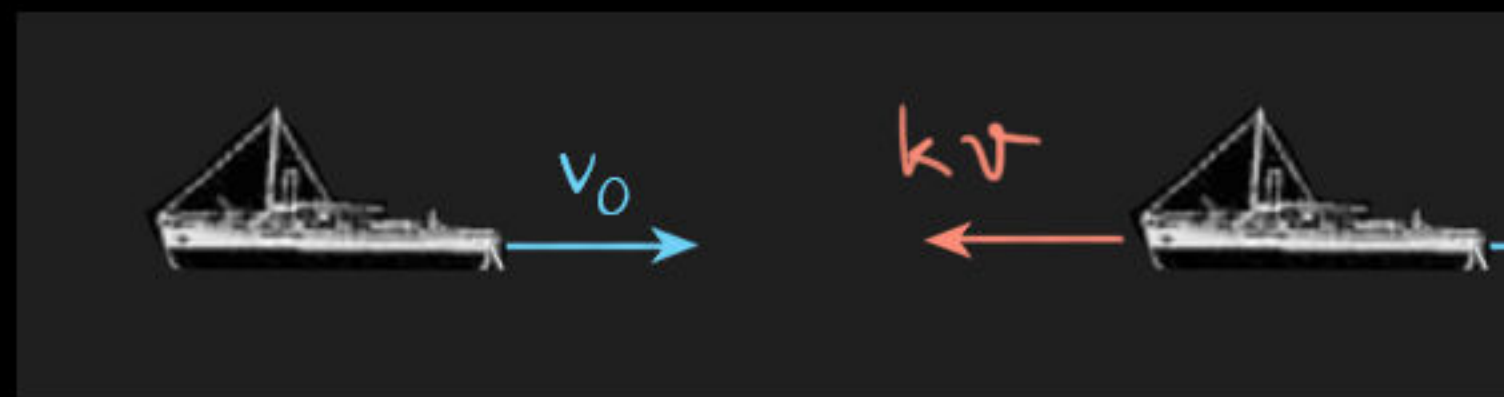
Velocity of boat w.r.t water

∴ Initially when boat is anchored:



$$T_0 = kv_0$$

w.r.t water (when chain breaks)

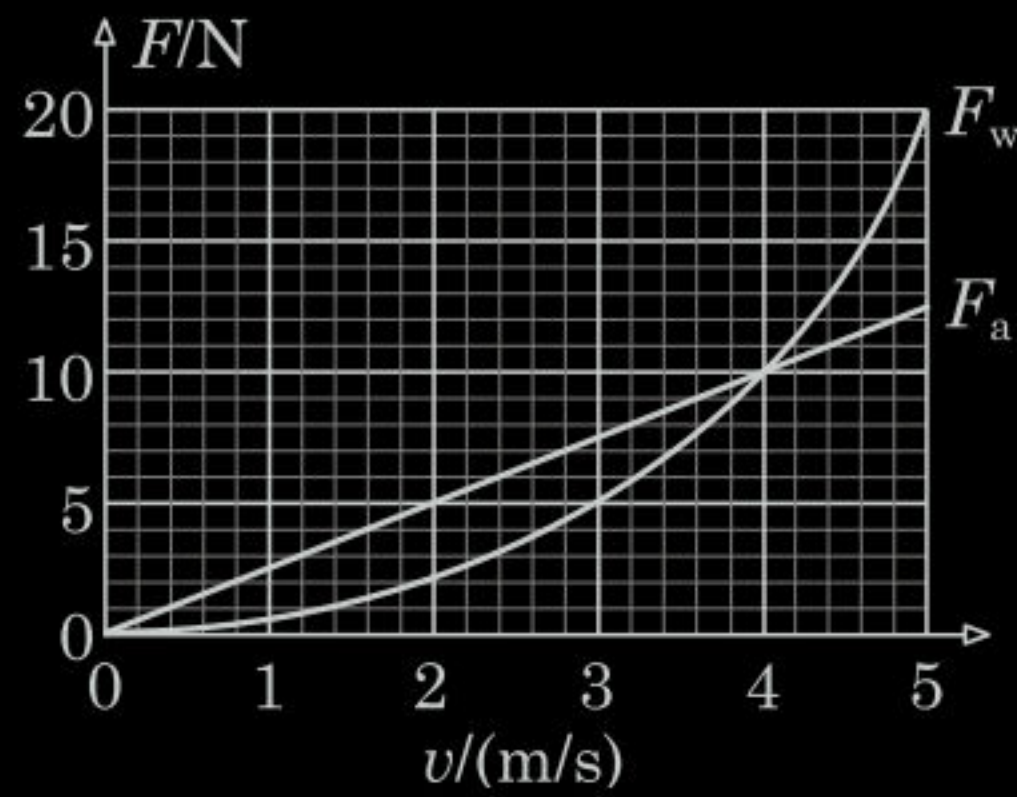


$$\frac{dv}{dt} = -\frac{kv}{m} \quad (\text{This equation too is w.r.t river})$$

this given velocity is w.r.t ground

$$v_0 - 0.5v_0 \quad \xrightarrow{T_0/v_0} \quad \Rightarrow \int_{v_0}^{0.5v_0} \frac{dv}{v} = -\frac{k}{m} \int_0^t dt$$

$$\Rightarrow t = \frac{mv_0 \ln 2}{T_0}$$

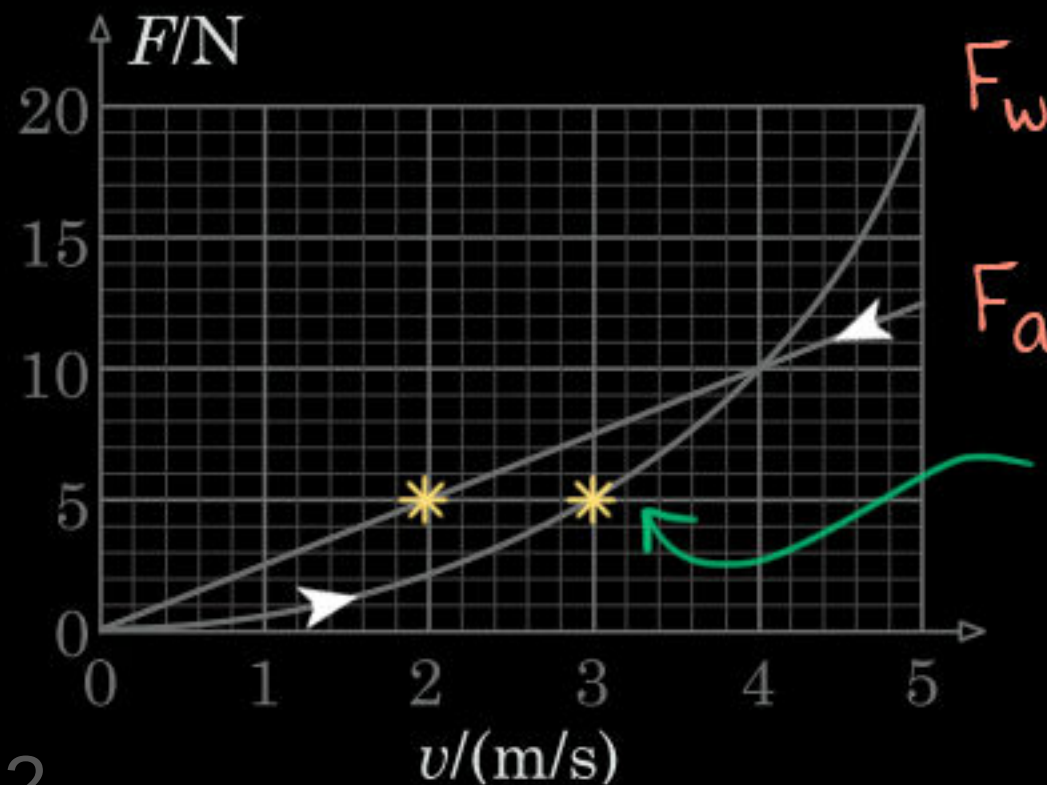


At terminal velocity: $F_a = F_w$

48. The given figure shows how forces of air resistance F_a and water resistance F_w on a sailboat vary with velocity v of the boat relative to the respective media.

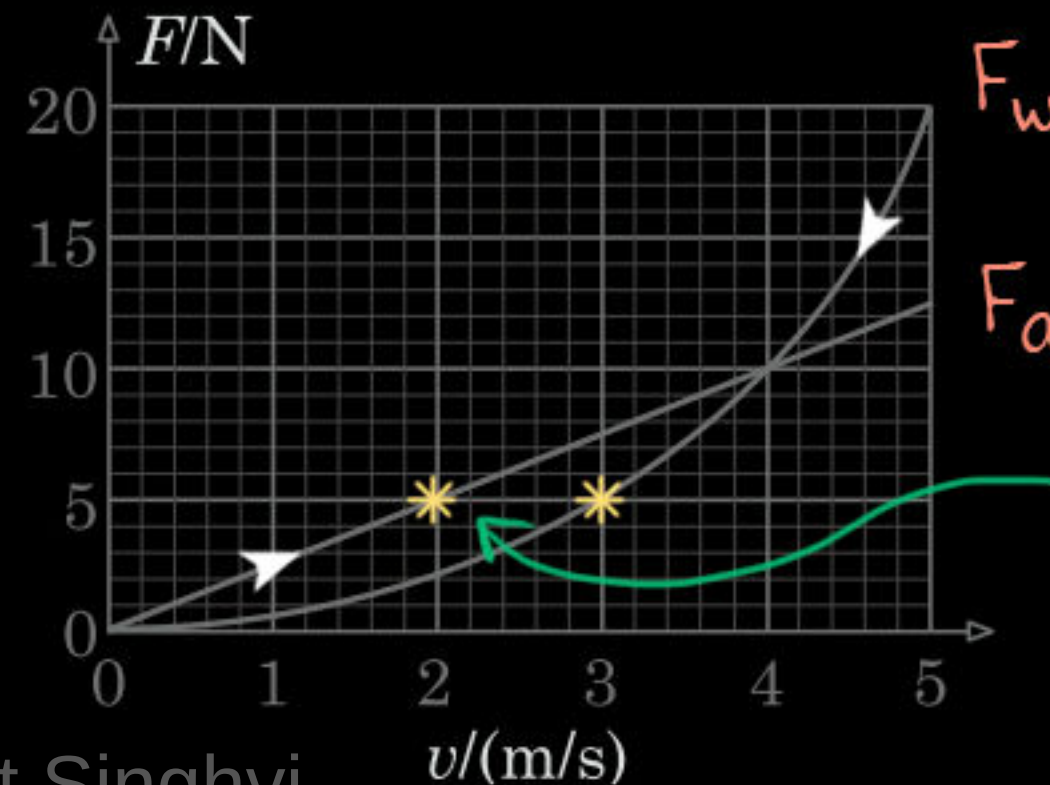
- (a) What speed a sailboat will acquire in stagnant water, when the wind speed is 5 m/s everywhere? The direction of motion of the boat coincides with the direction of the wind.
- (b) What speed a sailboat will acquire in stagnant air, when the water is flowing with velocity 5 m/s everywhere? The direction of motion of the boat coincides with the direction of the water current.

Water is stagnant and wind blows with 5m/s.
So w.r.t water velocity of boat will increase from 0 and w.r.t air, velocity of boat will reduce from 5.
Until after a long time, $F_a = F_w$. Also $v_a + v_w = 5$ always.

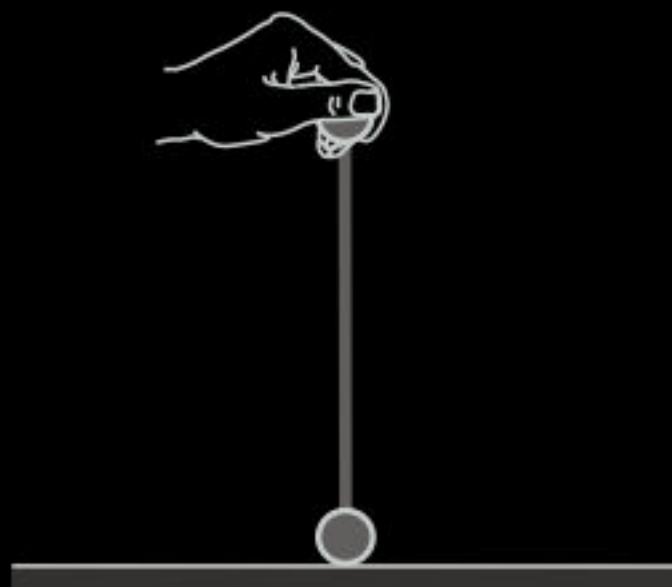


∴ w.r.t stagnant water, boats velocity will be 3m/s **A**

Air is stagnant and water flows with 5m/s.
So w.r.t air velocity of boat will increase from 0 and w.r.t water, velocity of boat will reduce from 5.
Until after a long time, $F_a = F_w$. Also $v_a + v_w = 5$ always.



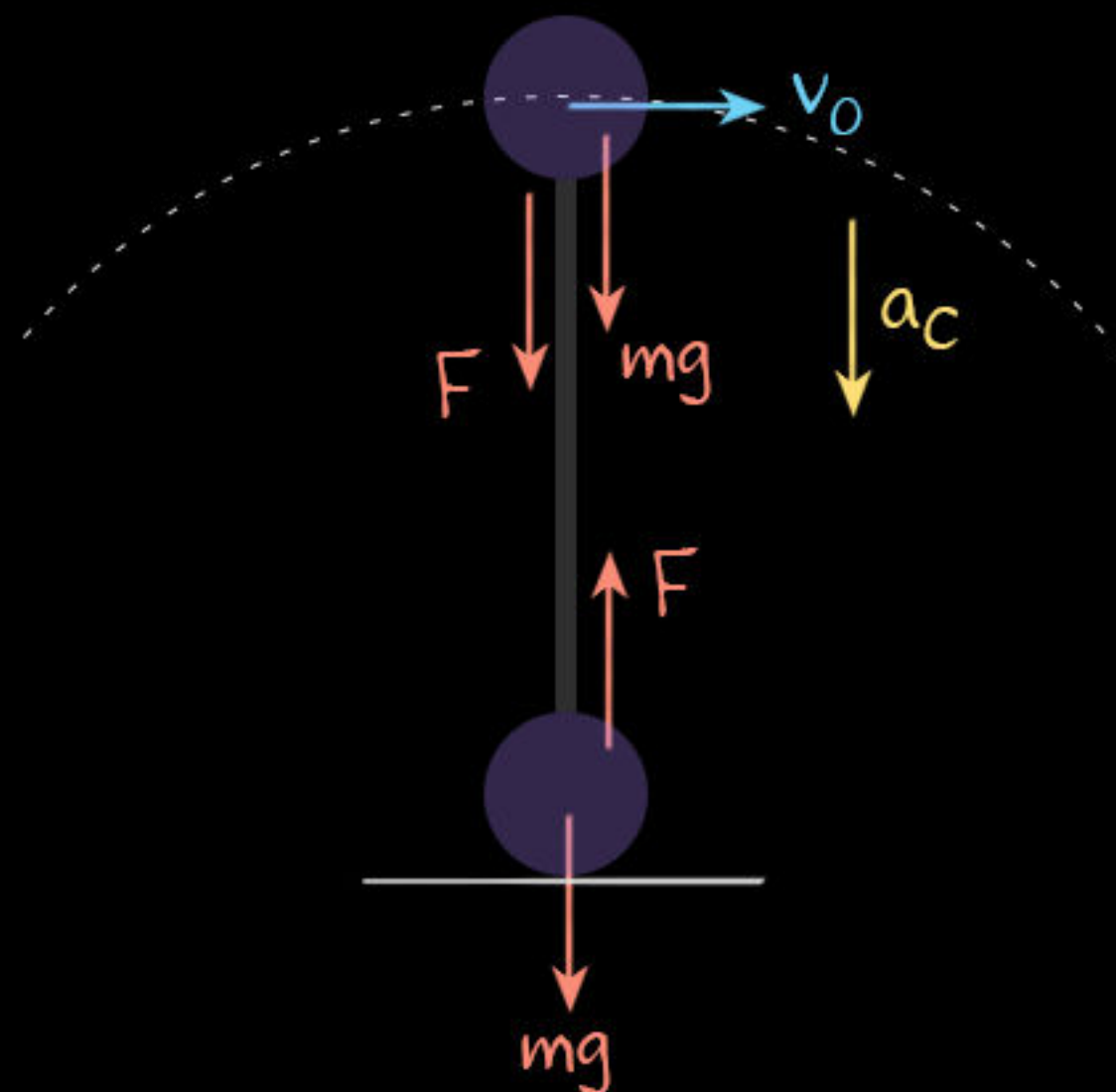
∴ w.r.t stagnant air, boats velocity will be 2m/s **B**



49. A dumbbell is constructed by affixing small balls each of mass m at the ends of a light rod of length l . The dumbbell is held vertically with the lower ball resting on a horizontal floor as shown in the figure. The upper ball is released and given a horizontal velocity v_0 by a sharp hit. If the bottom ball immediately loses contact with the floor, what should the maximum length l of the dumbbell be? Acceleration due to gravity is g .

Concept: With the critical length of dumbbell, Normal reaction of bottom ball just becomes zero, but is still at rest.

Approach: Upper ball will undergo circular motion w.r.t stationary bottom ball.



Let the tension in rod be F .

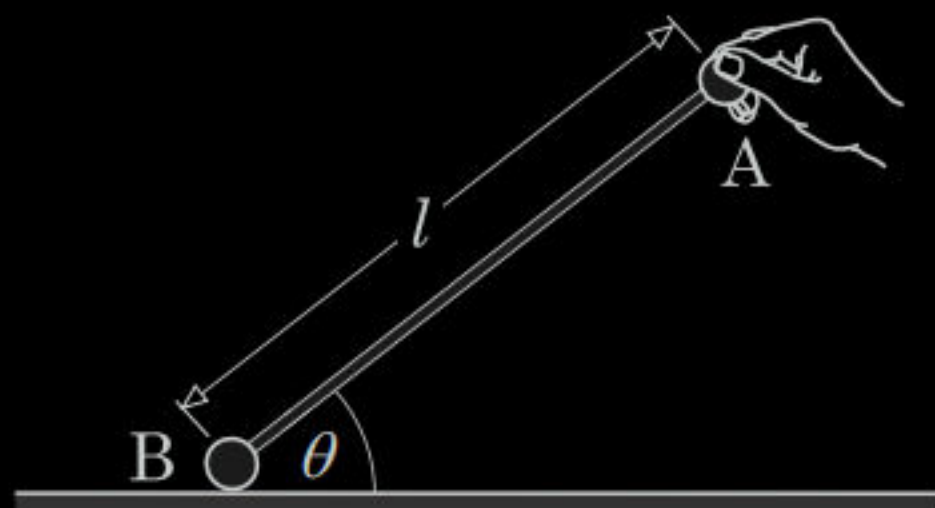
Bottom ball: $F = mg$ ①

Top ball, centripetal acceleration:

$$a_c = \frac{v^2}{R}$$

$$\Rightarrow \frac{F + mg}{m} = \frac{v_0^2}{l} \quad \text{②}$$

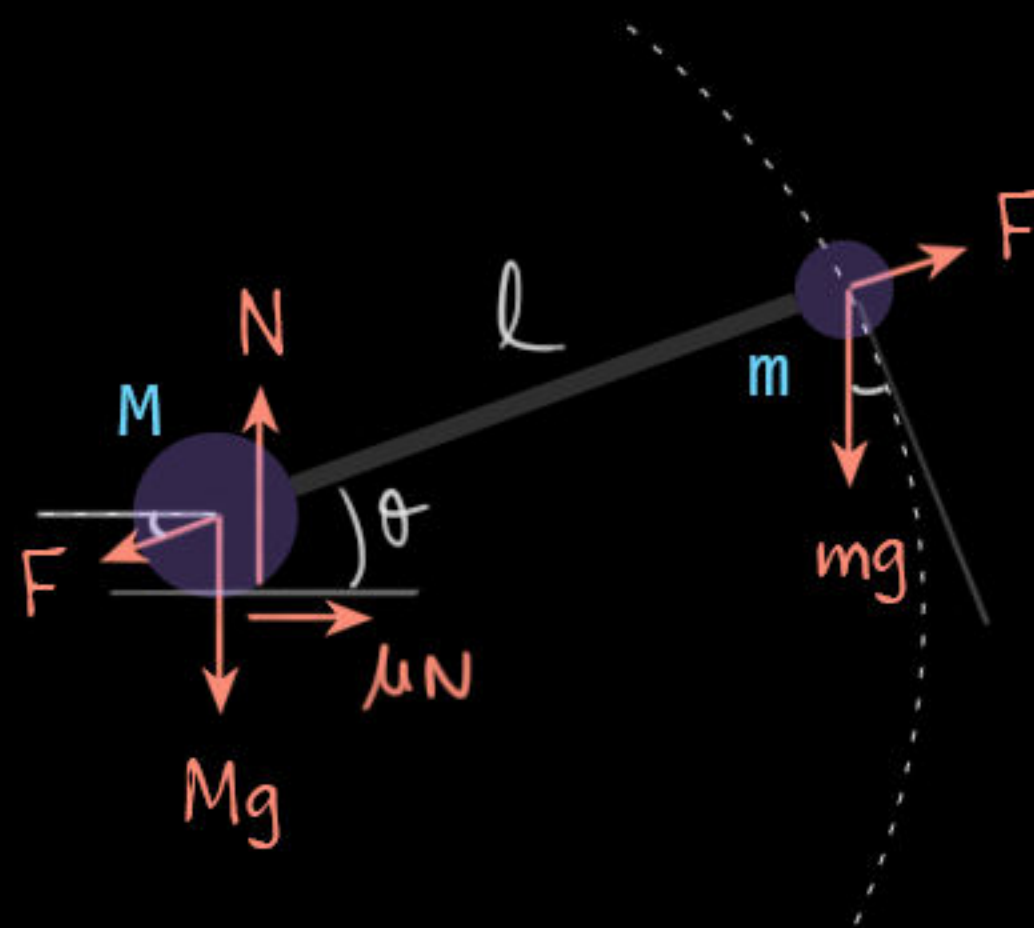
$$\text{From 1,2: } l \geq \frac{v_0^2}{2g}$$



50. A dumbbell consists of two very small size balls A and B of masses m and M fixed at the ends of a light rigid rod of length l . The dumbbell is held motionless on a horizontal floor making an angle θ with the floor as shown in the figure. Find range of coefficient of friction μ between the floor and the ball B, so that the ball starts sliding immediately after the dumbbell is released.

Concept: With the critical value of μ , friction at bottom ball reaches limiting value, but is still at rest.

Approach: In critical case m will undergo circular motion, and as its initial velocity is zero, centripetal force is zero too. So forces on it along the rod will be balanced. 3



Let the tension in rod be F .

As M is at rest:

$$N = F \sin \theta + Mg \quad 1$$

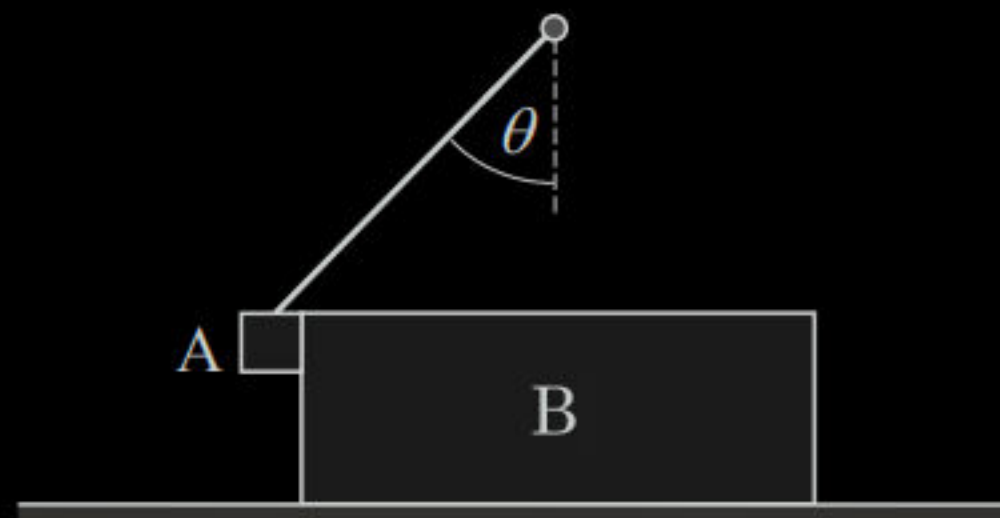
M is about to slide at F .
And we want M to slide.

$$\therefore F \cos \theta > \mu N \quad 2$$

$$\because v_m = 0, a_c = 0.$$

$$\Rightarrow F = mg \sin \theta \quad 3$$

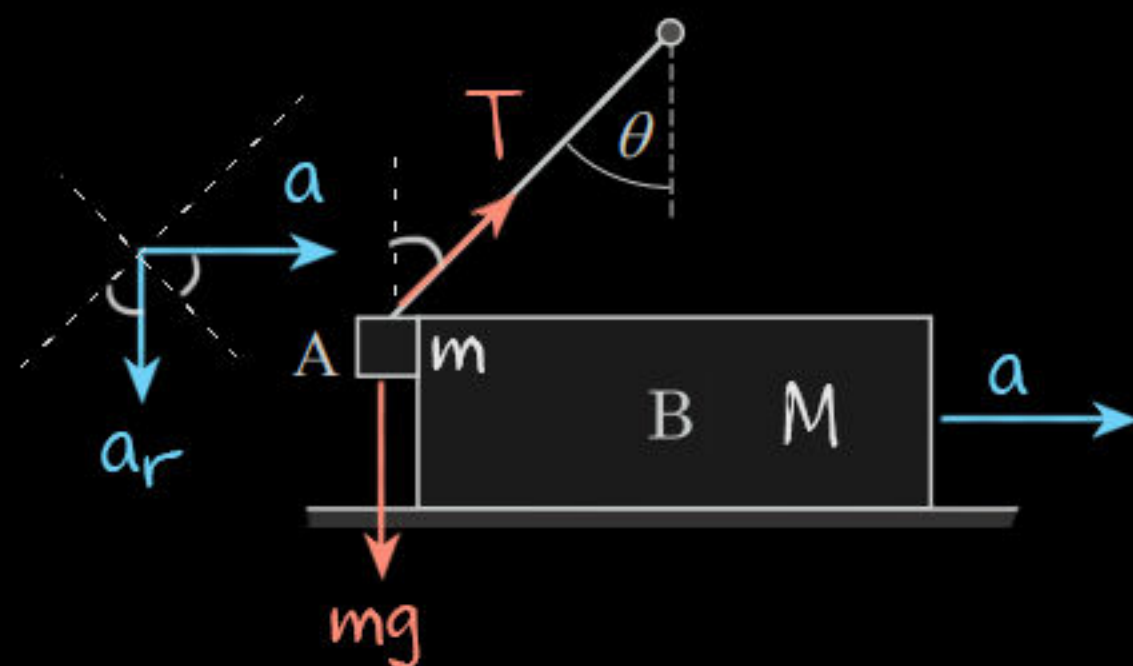
From 1, 2, 3: $\mu < \frac{m \sin \theta \cos \theta}{M + m \sin^2 \theta}$
 Youtube / Arihant Senior



51. In the setup shown, a small block A of mass m suspended from a peg with the help of a light inextensible cord rests on a vertical face of a block B of mass M , which is held motionless on the horizontal ground. Inclination of the cord with the vertical is θ and all the surfaces in contact are frictionless. Find acceleration of the block B immediately after it is released. Acceleration due to gravity is g .

Approach: A will undergo circular motion, with initial velocity zero. So its acceleration along the thread will be zero.

Let the acc. of B be a .
And acc. of A w.r.t B be a_r



F_x on system:

$$T \sin \theta = (m + M) a \quad (1)$$

F_y on m :

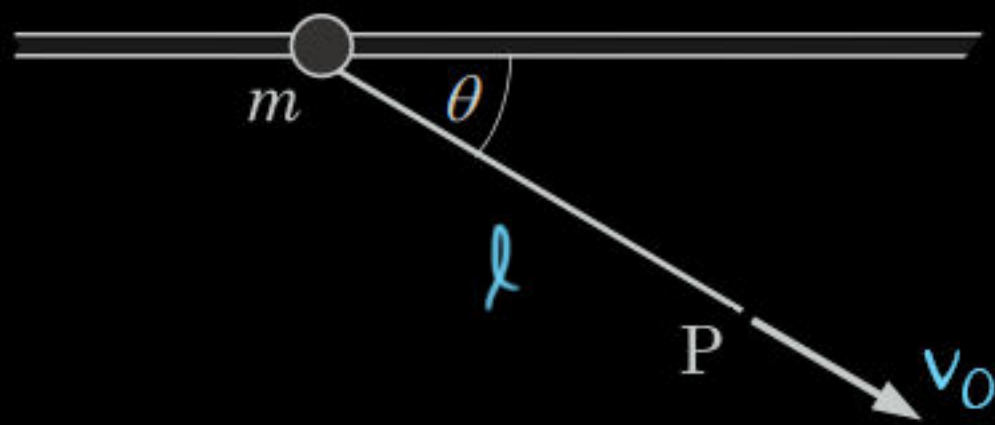
$$mg - T \cos \theta = m a_r \quad (2)$$

acc. of A along thread is zero:

$$a_r \cos \theta = a \sin \theta \quad (3)$$

From 1,2,3:
$$a = \frac{mg \sin \theta \cos \theta}{m + M \cos^2 \theta} //$$

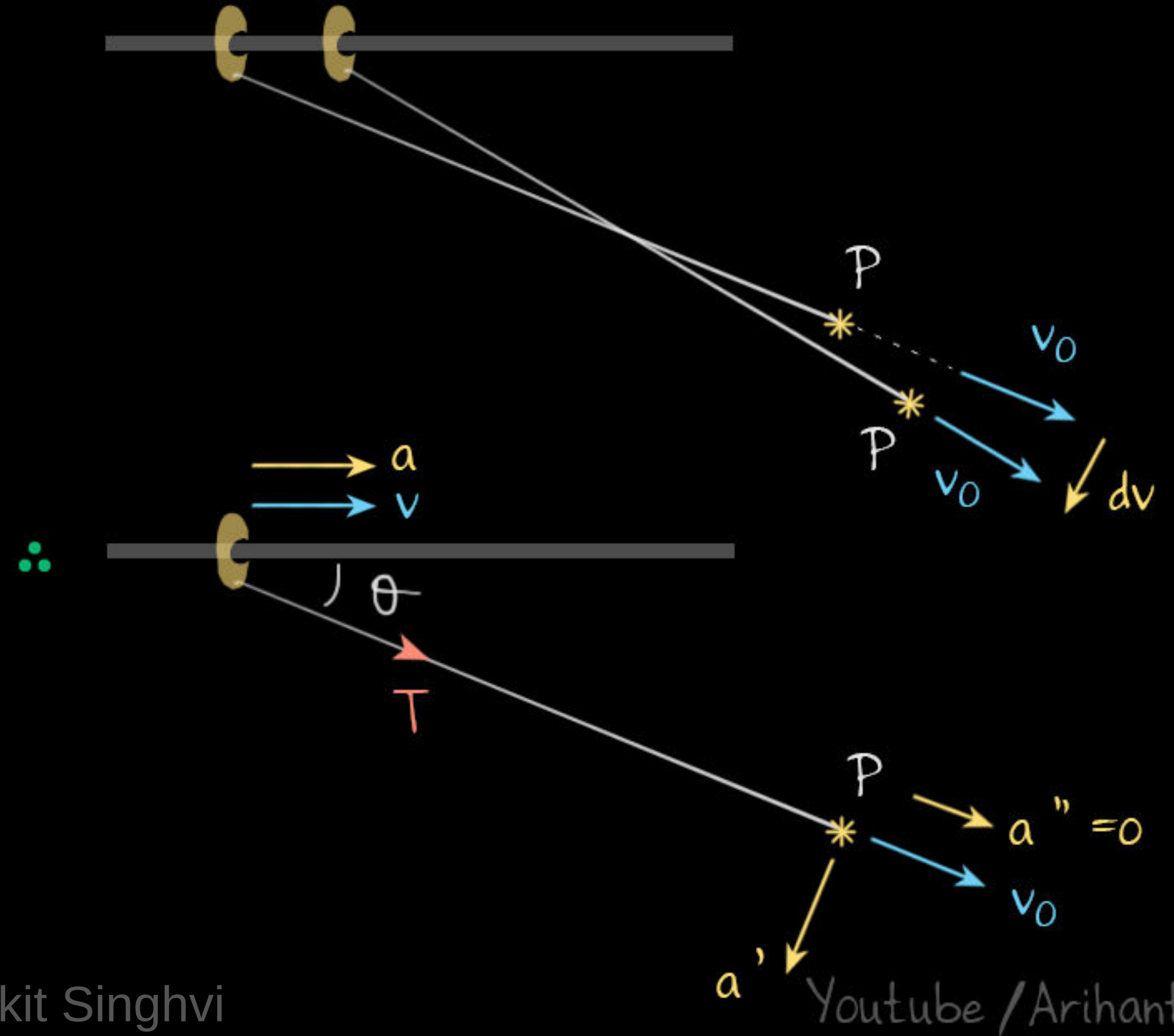
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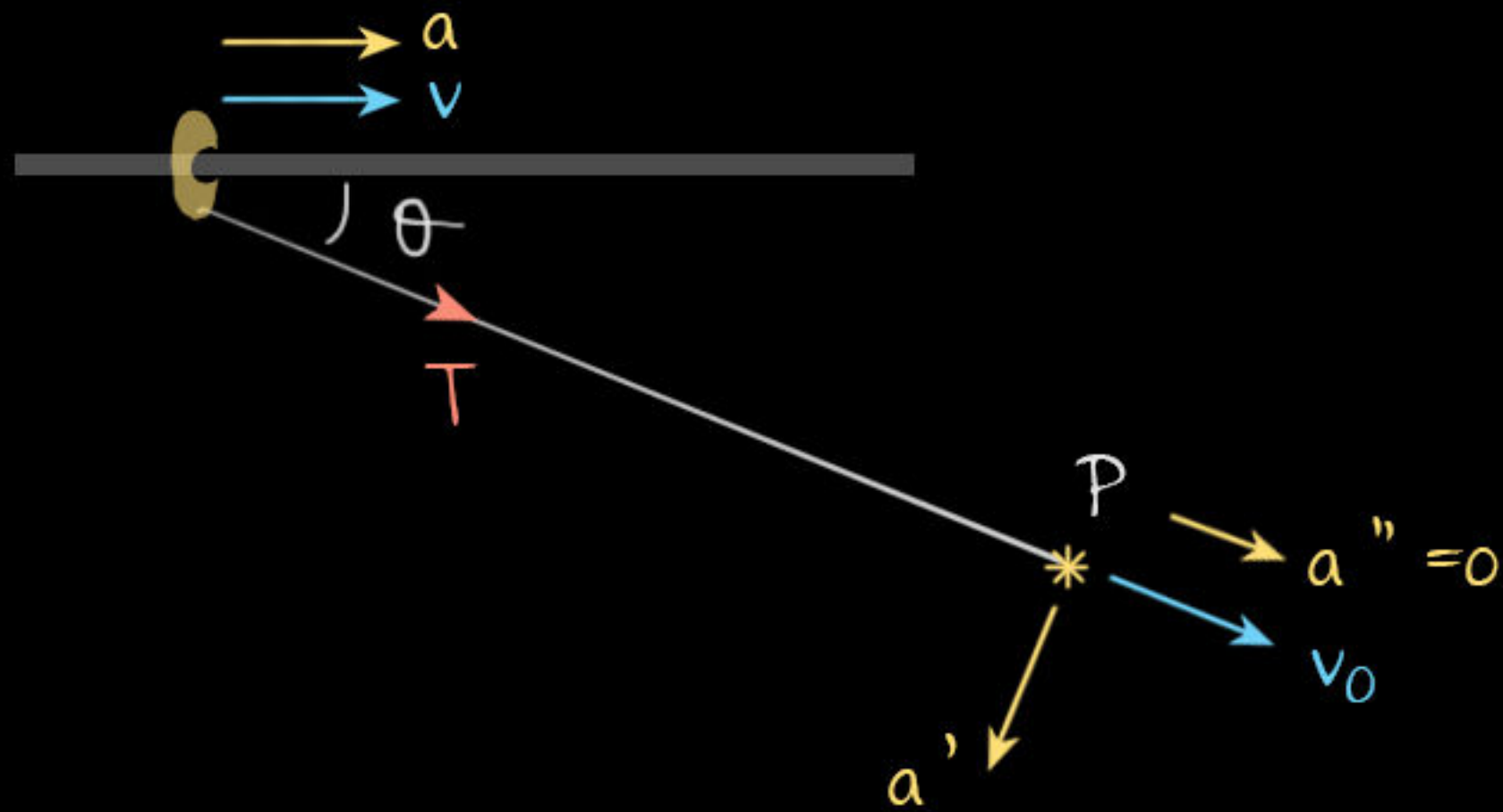
52. A bead of mass m , which can slide on a straight horizontal frictionless rod, is being pulled with the help of a light inextensible thread of length l in such a manner that the end P of the thread, which is being pulled moves with a constant speed v_0 always directed along the thread. Find an expression for the tensile force T in the thread at an instant when the thread makes angle θ with the rod.

Concept: Acceleration of P along the rope (a'') is zero as its velocity along the rope is constant (v_0).
Let's say acceleration of P perpendicular to rope is a' .

Let bead velocity be v , acceleration a .



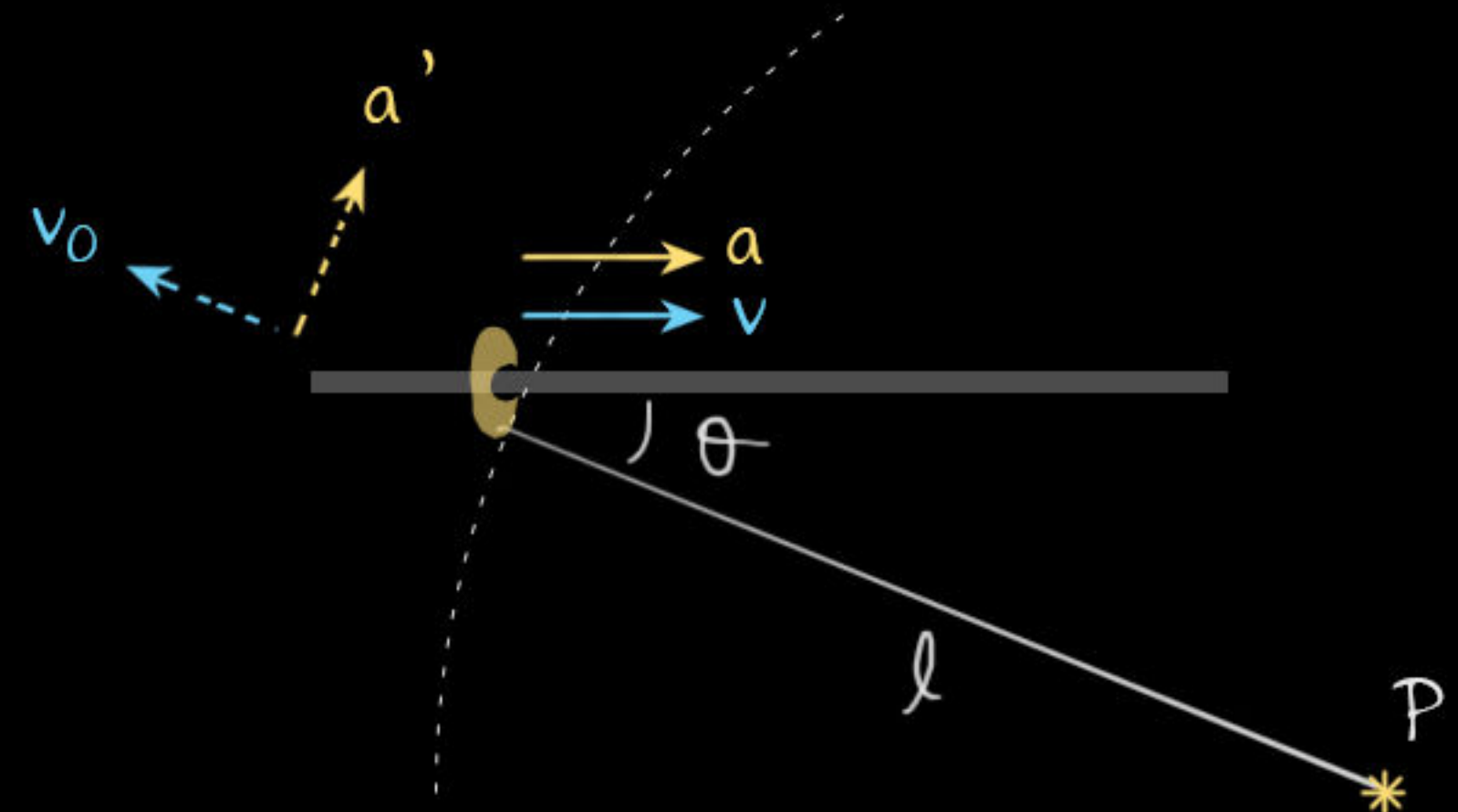
w.r.t ground



Force on bead in X direction: $T \cos \theta = ma$ (1)

∴ Rope is inextensible: $v \cos \theta = v_0$ (2)

w.r.t P (We can also do w.r.t bead)

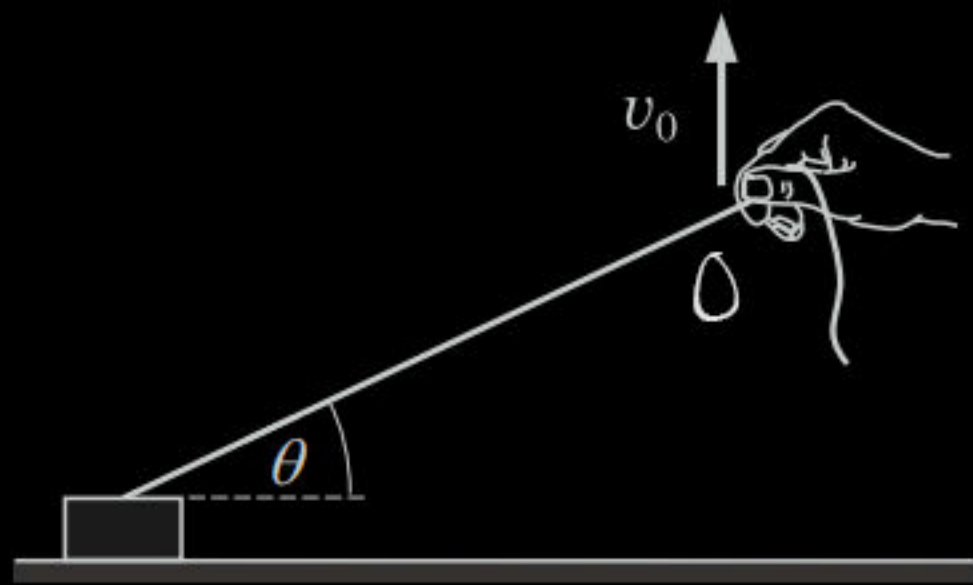


Acceleration of bead is: $\vec{a} + \vec{a}'$

$$a_c = \frac{v_{\perp}^2}{R} \Rightarrow a \cos \theta = \frac{(v \sin \theta)^2}{l} \quad (3)$$

From 1,2,3: $T = \frac{mv_0^2 \sin^2 \theta}{\cos^4 \theta}$

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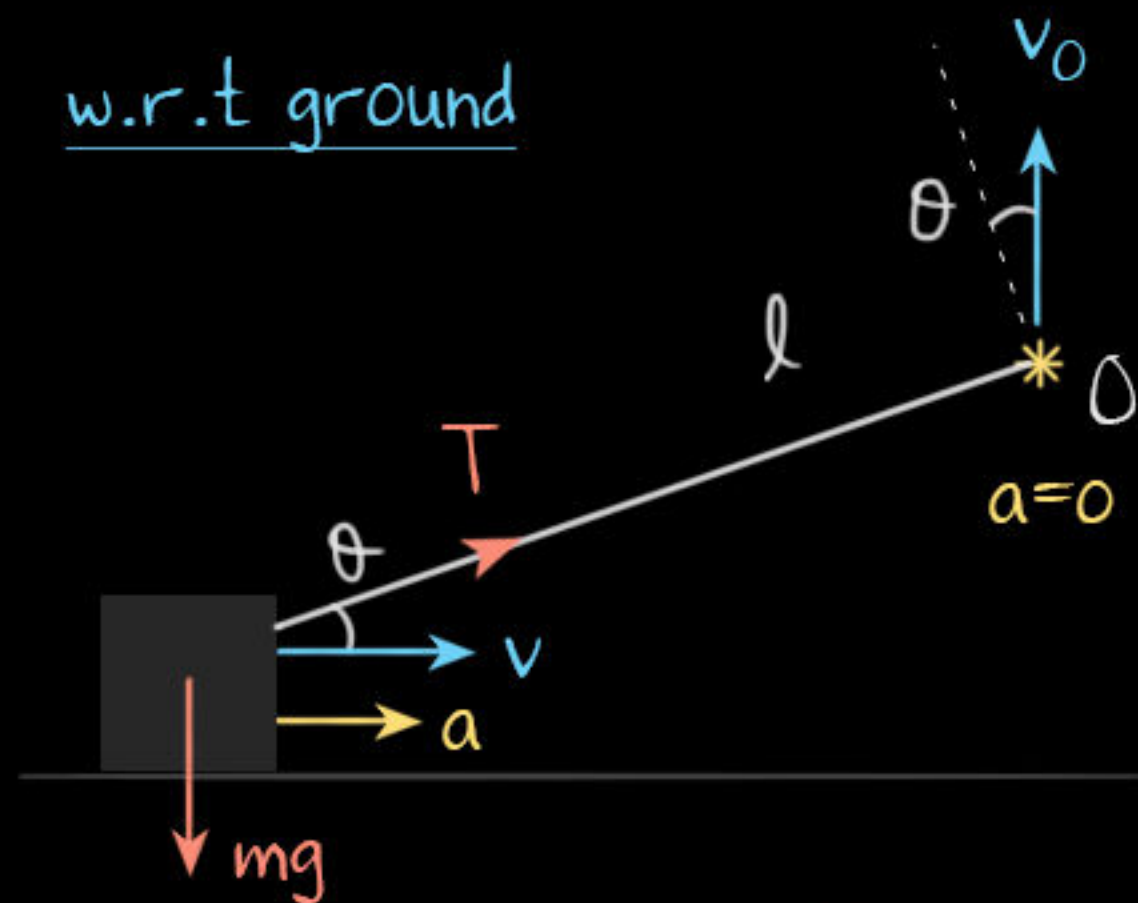
53. A light inextensible straight thread of length $l = 3.2$ m is placed on a frictionless horizontal floor. A small block of mass $m = 3$ kg placed on the floor is attached at one end of the thread. With how much constant speed v_0 the free end of the thread must be lifted vertically upwards so that the block leaves the floor, when the thread makes an angle $\theta = 30^\circ$ with the floor. Acceleration of free fall is $g = 10$ m/s².

Concept: With respect to O block is performing circular motion.

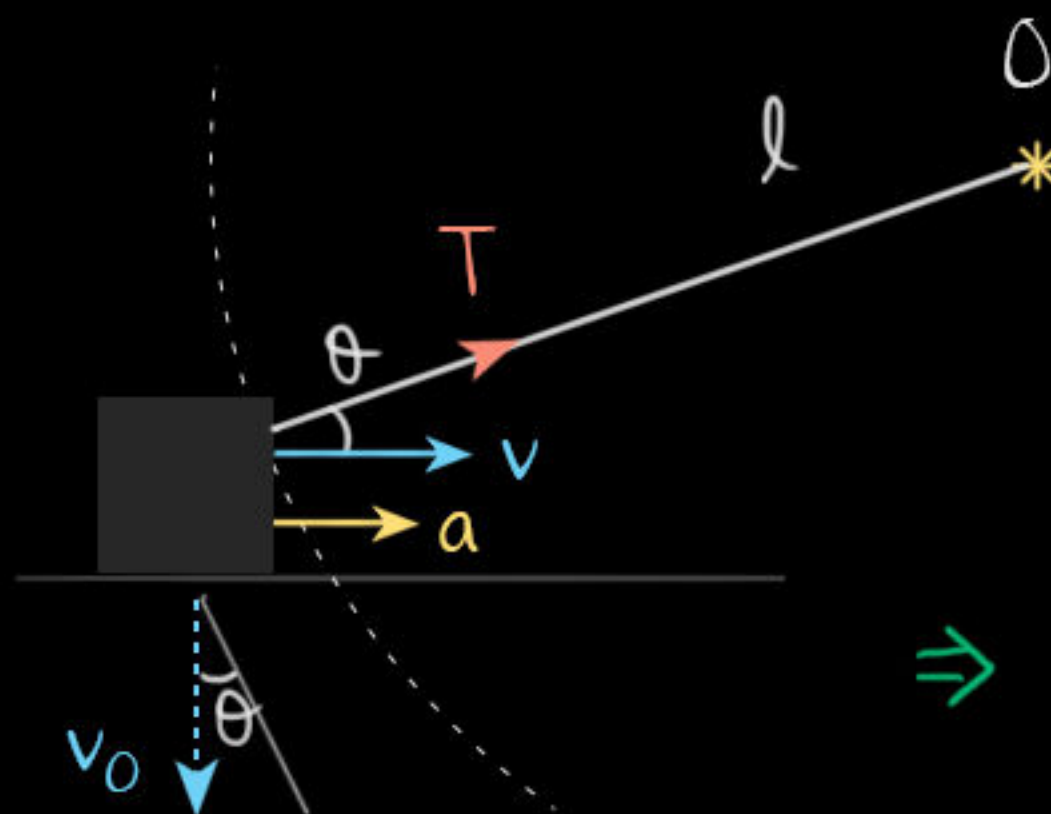
constant speed, constant direction.

$\therefore O$'s acceleration = 0

w.r.t ground



w.r.t O



$$a_c = \frac{v_{\perp}^2}{R}$$

$$\Rightarrow a \cos \theta = \frac{(v \sin \theta + v_0 \cos \theta)^2}{l} \quad (4)$$

Force in X direction: $T \cos \theta = ma \quad (1)$

Rope is inextensible: $v \cos \theta = v_0 \sin \theta \quad (2)$

Normal reaction is zero: $T \sin \theta = mg \quad (3)$

From 1-4: $v_0 = \omega^2 \sqrt{\frac{gl}{\sin \theta}}$

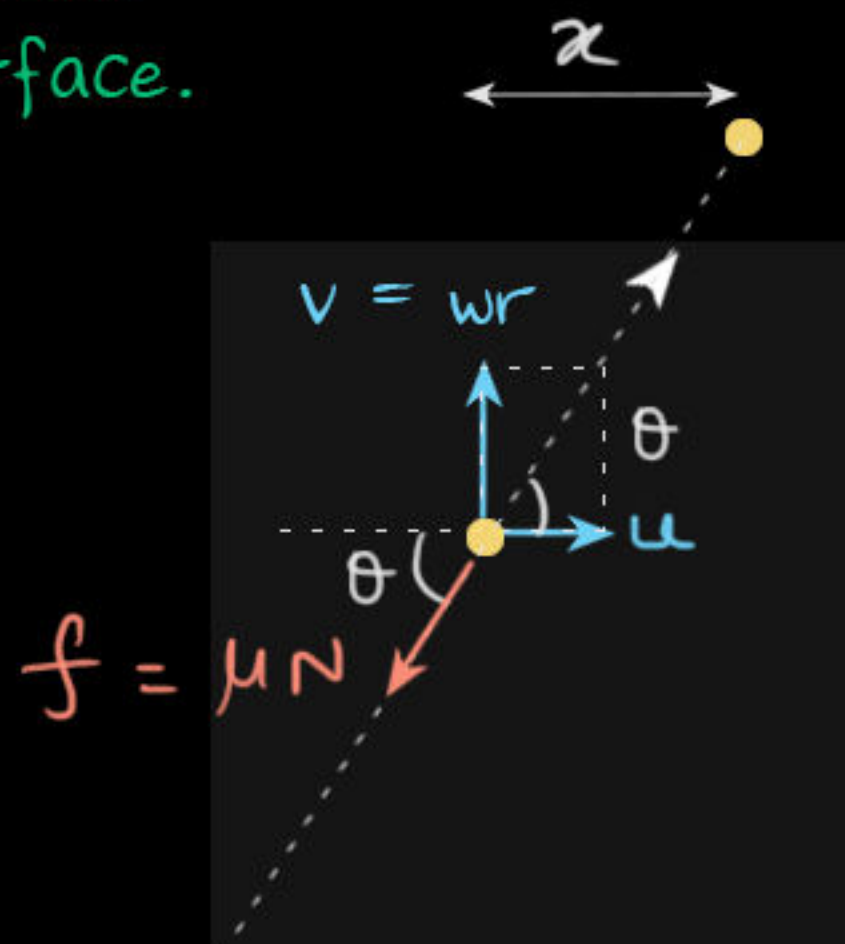
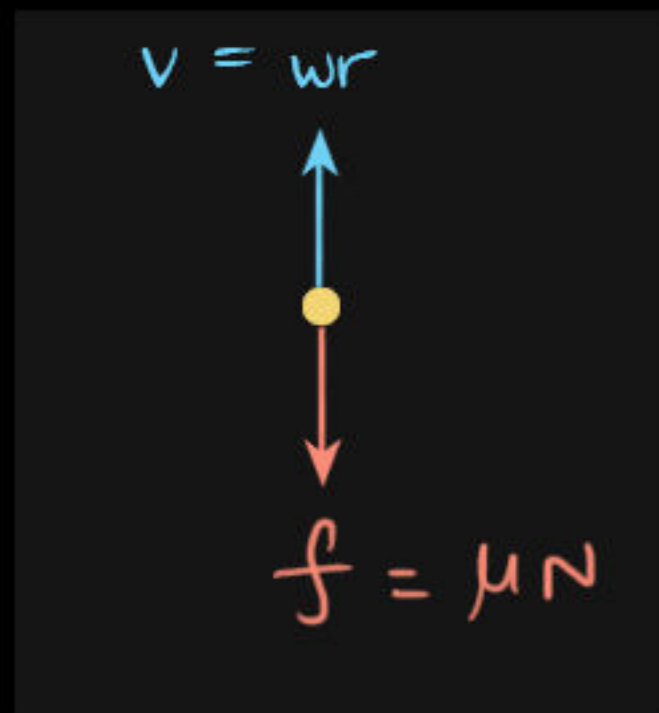
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54. In a gravity free space a long uniform cylinder of radius $r = 30$ cm wears a thin uniform ring that can slide on the cylinder. When the ring is given an angular velocity $\omega = 10$ rad/s, it stops in $t_0 = 3.0$ s. Now the ring is given the same angular velocity and simultaneously projected along the rod with speed $u = 4.0$ m/s. How far will the ring move along the rod before it stops?

Concept: Lets reduce ring to a point object, and cylinder to a flat surface.



$$\tan \theta = \frac{\omega r}{u} \quad (2)$$



$$v_x^2 = u_x^2 + 2a_x x$$

$$\Rightarrow u^2 = 2 \left(\frac{\mu N}{m} \cos \theta \right) x \quad (3)$$

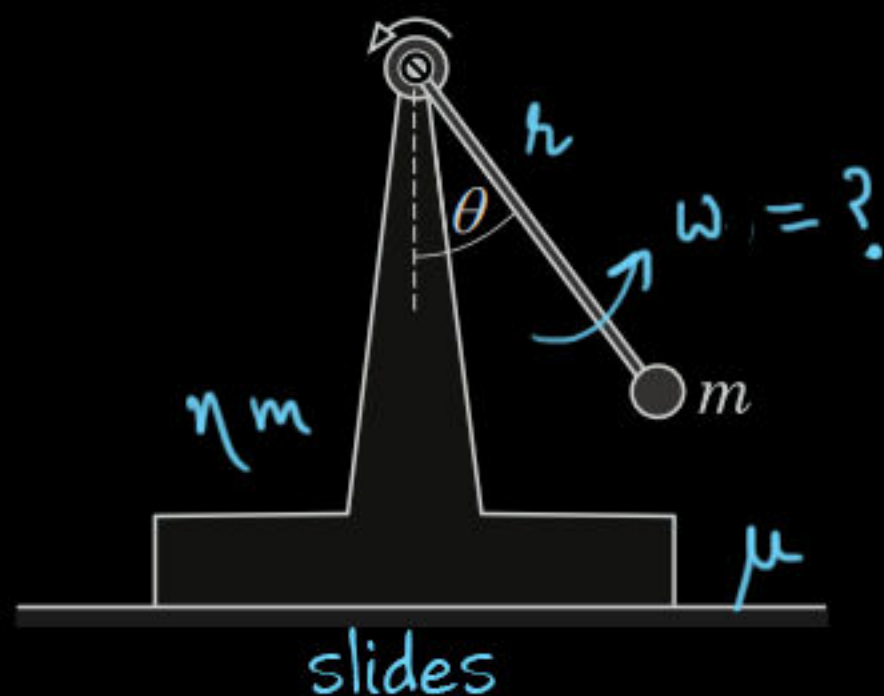
$$v = u + at$$

$$\Rightarrow \omega r = \left(\frac{\mu N}{m} \right) t_0 \quad (1)$$

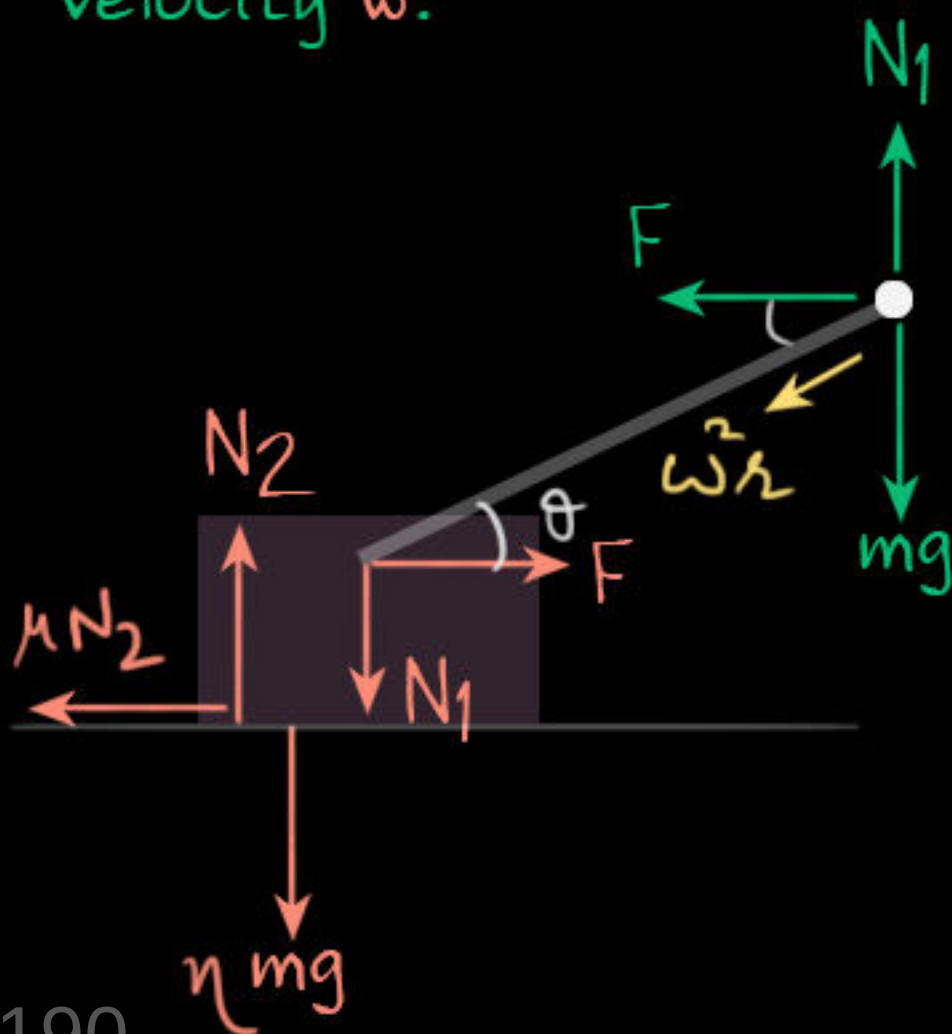
$$\text{From 1, 2, 3: } x = \frac{ut_0}{2\omega r} \sqrt{\omega^2 r^2 + u^2}$$

Note: Path shown is total displacement of a mark on the ring on the cylindrical/flat surface.





Let N_1 and F be reaction forces from rod.
Let the block just slide at an angle θ and angular velocity ω .



55. A motor is installed at the top of a pole rigidly fixed on a platform. A light rod of length $r = 1$ m is rigidly attached to the motor shaft at its one end and at the other end a small ball of mass m is attached. The rod can be rotated in a vertical plane with the help of the motor. Total mass of the platform, the pole and the motor is $\eta = 4$ times the mass of the ball. The motor rotates the rod at a constant angular velocity. The platform is placed on a horizontal surface, where coefficient of friction is $\mu = 1/\sqrt{3}$. At which minimum angular velocity ω_0 of the rod, will the platform start sliding?

On ball m
 F_y and F_x :

$$mg - N_1 = m\omega^2 r \sin\theta \quad (1)$$

$$F = m\omega^2 r \cos\theta \quad (2)$$

On block F_x : $F = \mu[\eta mg + N_1] \quad (3)$

From 1,2,3:

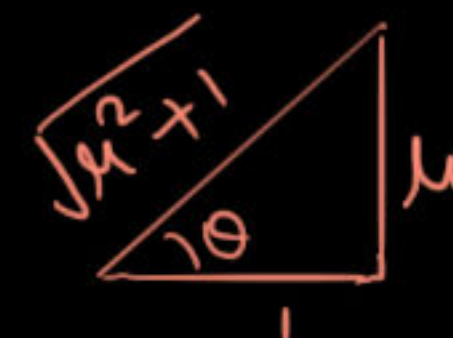
$$\omega^2 = \frac{\mu g (\eta + 1)}{r (\mu \sin\theta + \cos\theta)} \quad (A)$$

by Ankit Singhvi

∴ For minimum ω at given θ

$$\frac{d}{d\theta} (\mu \sin\theta + \cos\theta) = 0$$

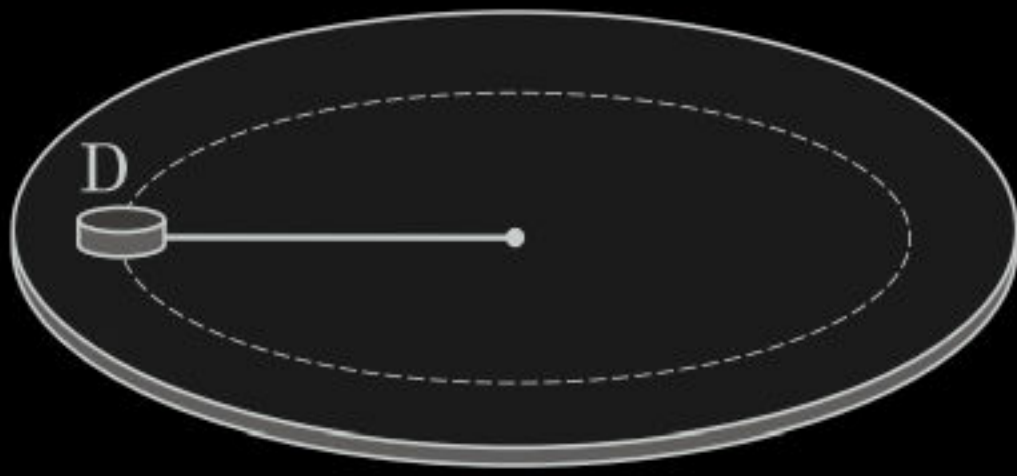
$$\Rightarrow \tan\theta = \mu \quad (B)$$



From A,B:

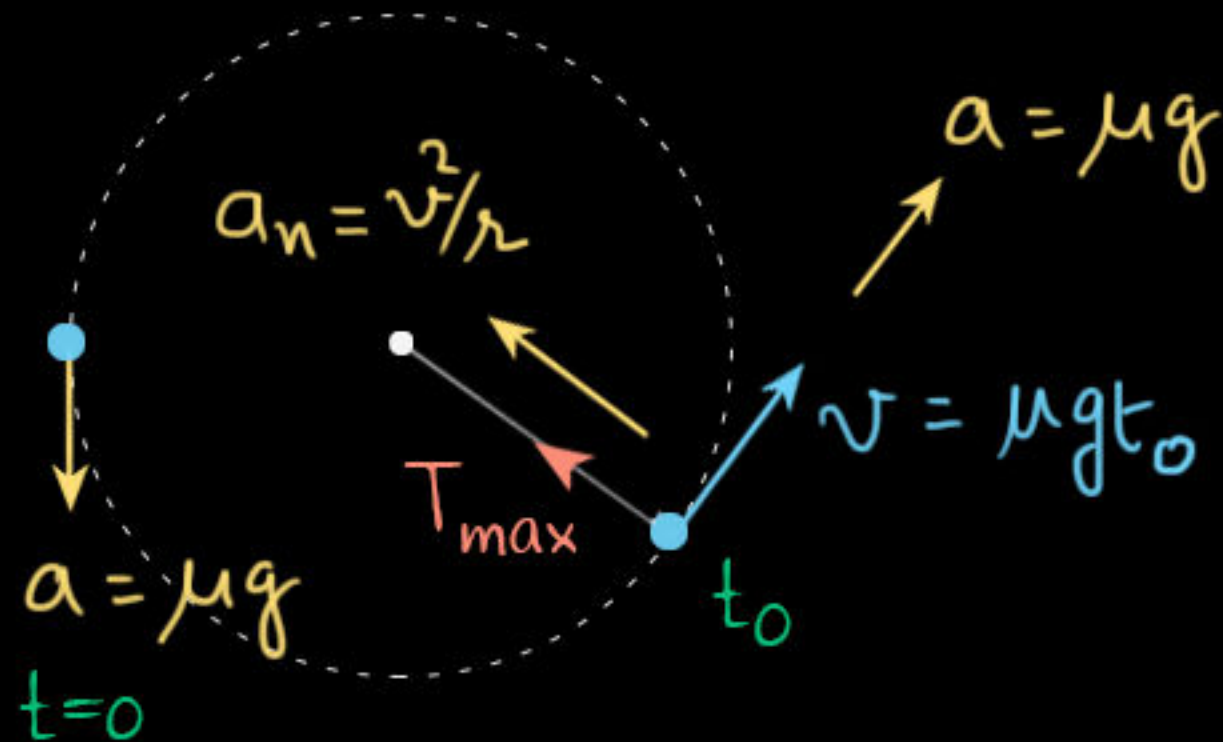
$$\omega_0 = \sqrt{\frac{\mu g (\eta + 1)}{r \sqrt{\mu^2 + 1}}}$$

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56. A disc D of mass $m = 4.0$ kg is tied at one end of a cord of length $l = 1.0$ m, other end of which is fixed at the centre of a large circular platform. The platform is maintained horizontal and rotated about its vertical axis of symmetry. The disc is placed on the platform as shown. How long after the disc is placed, will the cord break? The maximum tensile force, the cord can withstand is $T_{\max} = 100$ N and coefficient of kinetic friction between the disc and the platform is $\mu_k = 0.1$.

Assume that platform is rotating super fast!
i.e, the disc always slips until thread breaks.

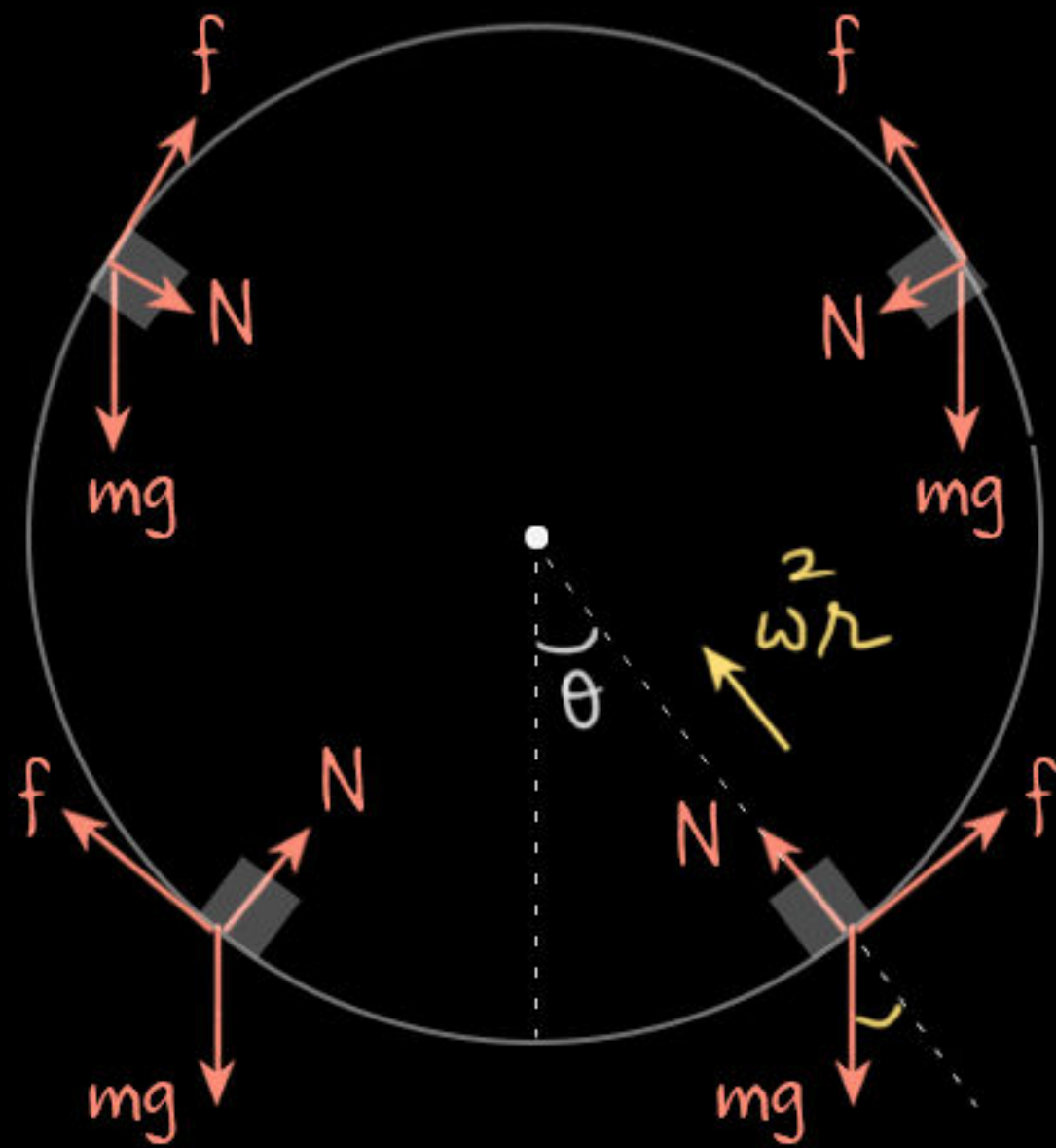


$$T = \frac{mv^2}{r} = \frac{m(\mu g t)^2}{r}$$

$$\Rightarrow T_{\max} = \frac{m(\mu g t_0)^2}{r}$$

$$\Rightarrow t_0 = \frac{1}{\mu g} \sqrt{\frac{T_{\max} r}{m}}$$

Concept: We find direction of friction is always up, by balancing forces, and not by 'how it slips'. Direction of spin doesn't matter



57. Consider a hollow cylinder of radius r fixed in a laboratory with its axis horizontal. A small block is placed inside the cylinder as shown in the figure. Coefficient of friction between the block and the inner surface of the cylinder is μ . Find the angular velocity at which if the cylinder is rotated about its axis, the block does not slide. Acceleration due to gravity is g .

Parallel to surface: $f = mg \sin \theta$ ① To not slide: $f \leq \mu N$ ③

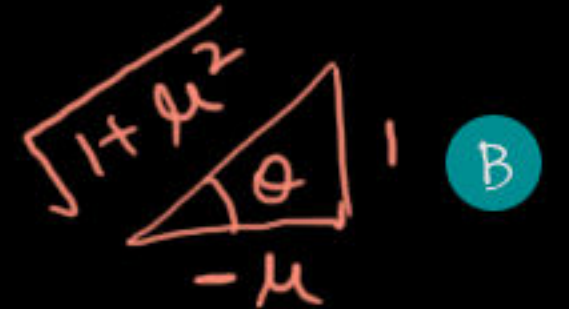
Normal to surface: $N - mg \cos \theta = m \omega^2 r$ ②

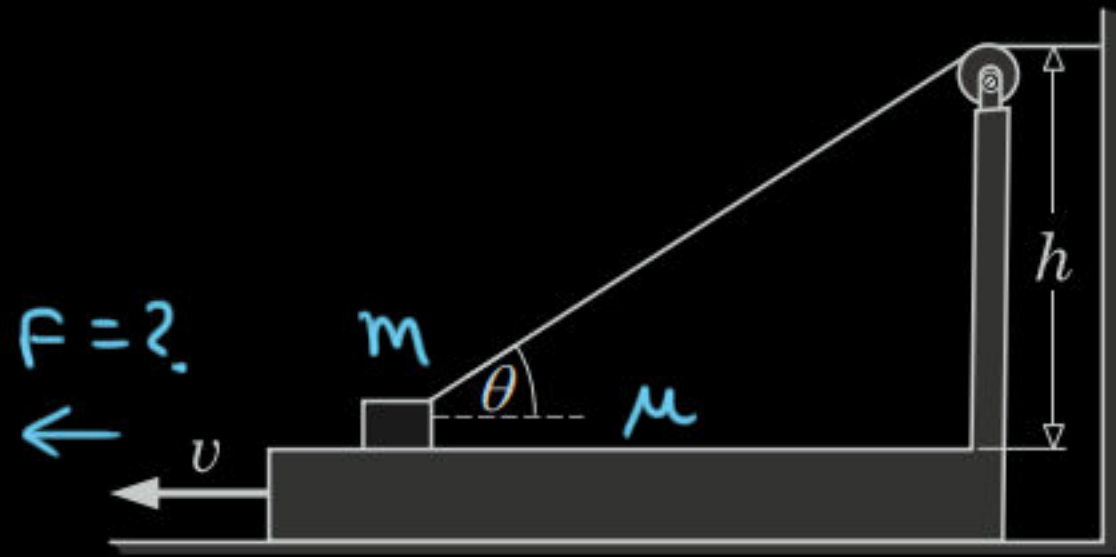
From 1,2,3: $\omega^2 \geq \frac{g (\sin \theta - \mu \cos \theta)}{\mu r}$ A

From A,B: $\omega^2 \geq \frac{g \sqrt{\mu^2 + 1}}{\mu r}$ //

Maximum when $\frac{d(\)}{d\theta} = 0$

$\Rightarrow \tan \theta = -1/\mu$

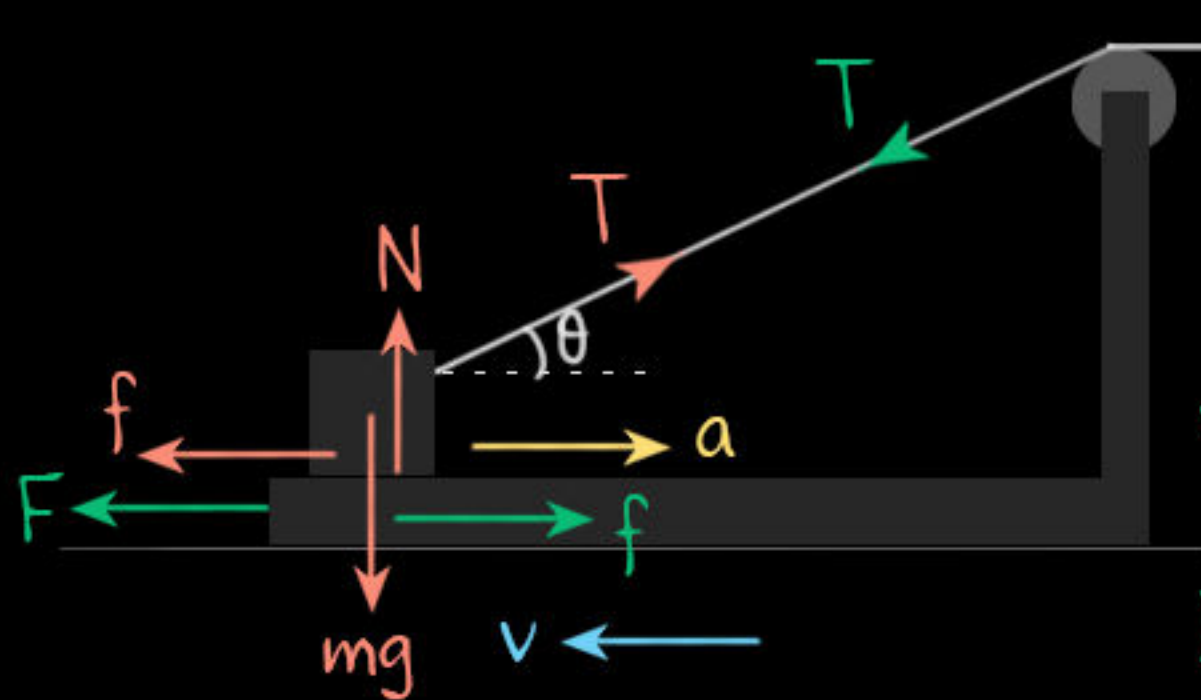




w.r.t ground

Let block's acceleration: a

58. A small block of mass m is placed on a platform that has a rectangular protrusion. The block is connected to one end of a light inextensible cord that passes over an ideal pulley fixed on the corner of the rectangular protrusion and finally attached to the wall. Segment of the cord between the pulley and the wall is horizontal and is at a height h above the top face of the platform as shown in the figure. The platform is on a horizontal frictionless floor. Coefficient of friction between the block and top face of platform is μ . The platform is pulled away from the wall so that it moves with a constant velocity v . Find the force F pulling the platform when segment of the cord between the block and the pulley makes angle θ with the horizontal. Denote acceleration of free fall by g and assume that the block does not leave the platform.

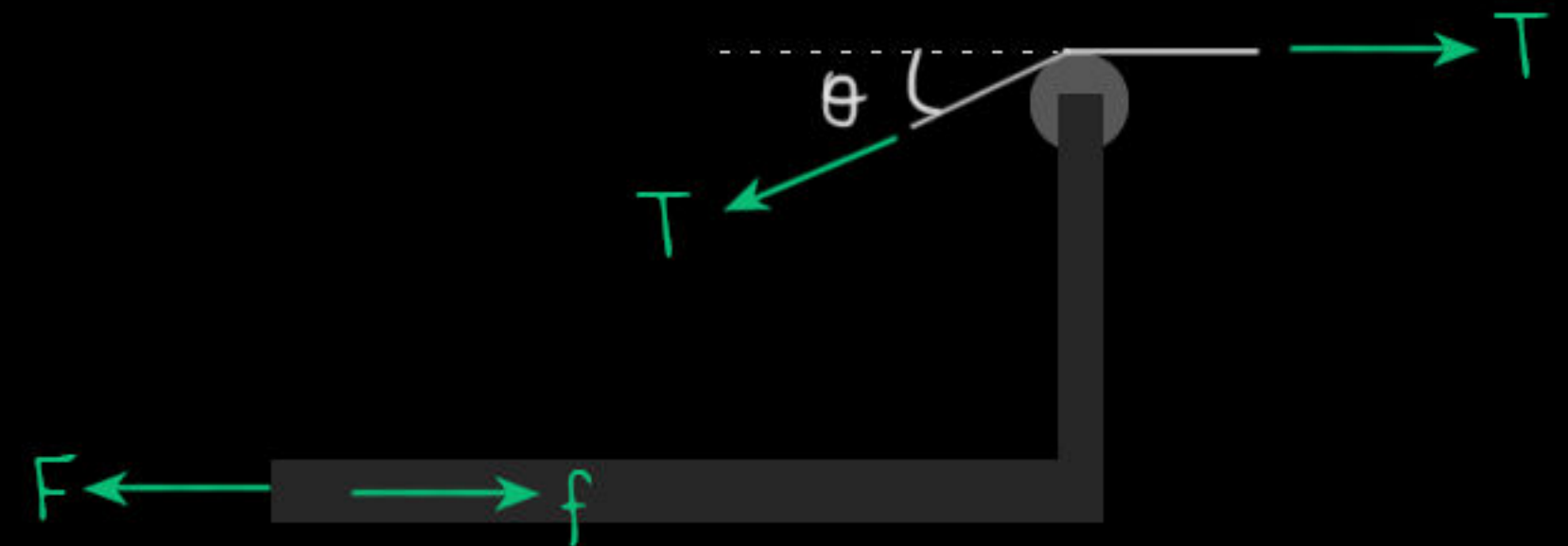


Forces on block:

$$Y: N + T \sin \theta = mg \quad (1)$$

$$X: T \cos \theta - f = ma \quad (2)$$

$$\text{Friction: } f = \mu N \quad (3)$$



∴ Net force on platform is zero in X direction:

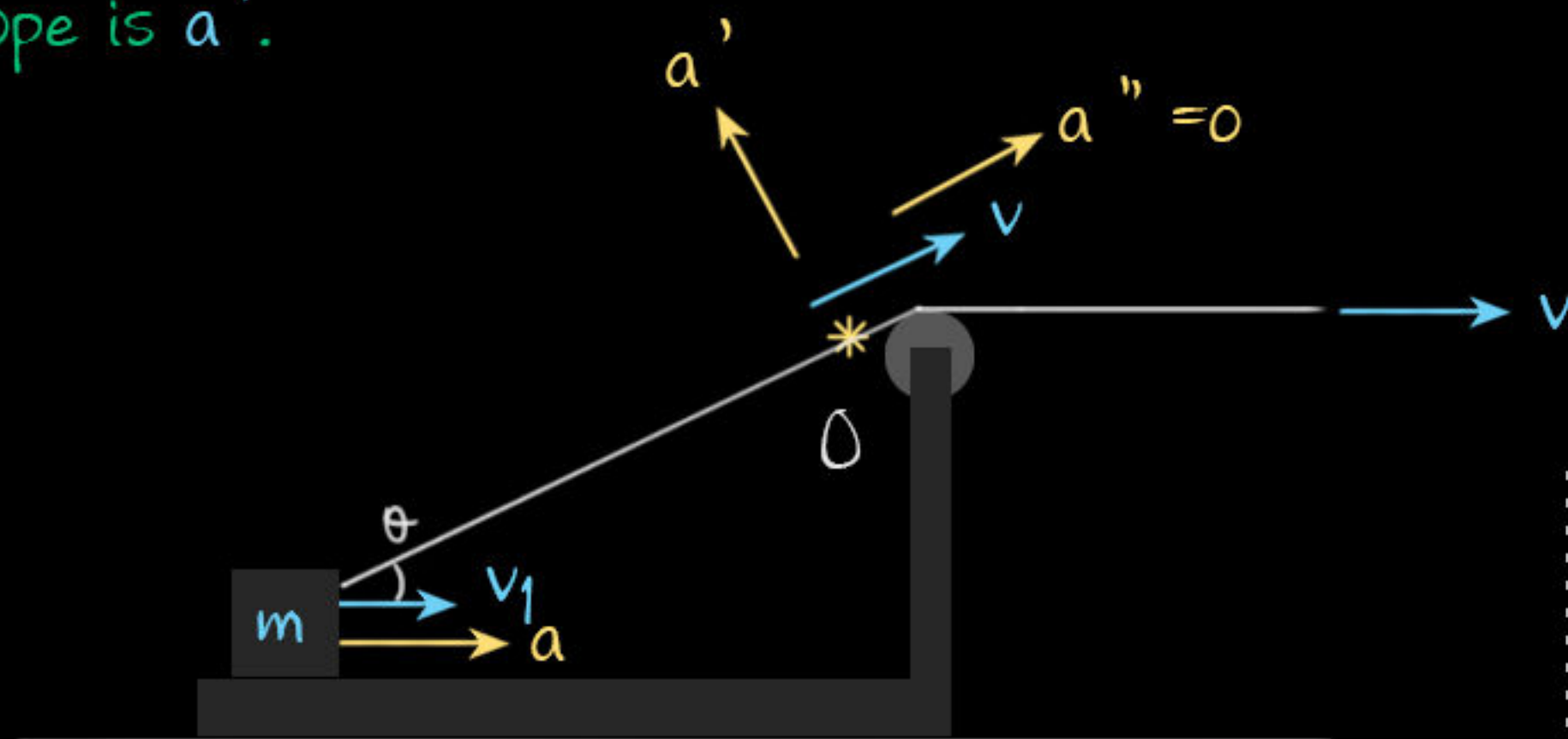
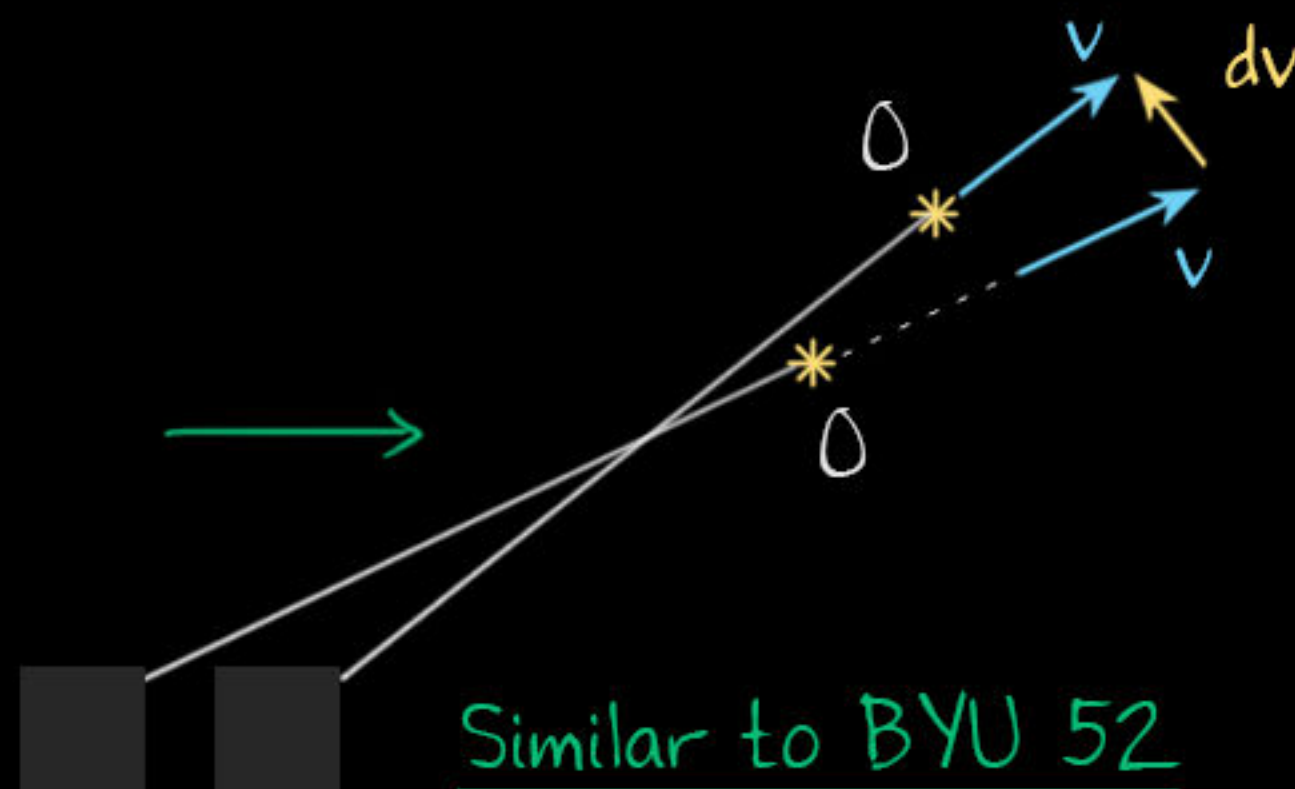
$$F + T \cos \theta = T + f \quad (4)$$

w.r.t platform

Let blocks velocity be v_1

Concept: Acceleration of O along the rope (a'') is zero as its velocity along the rope is constant (v).

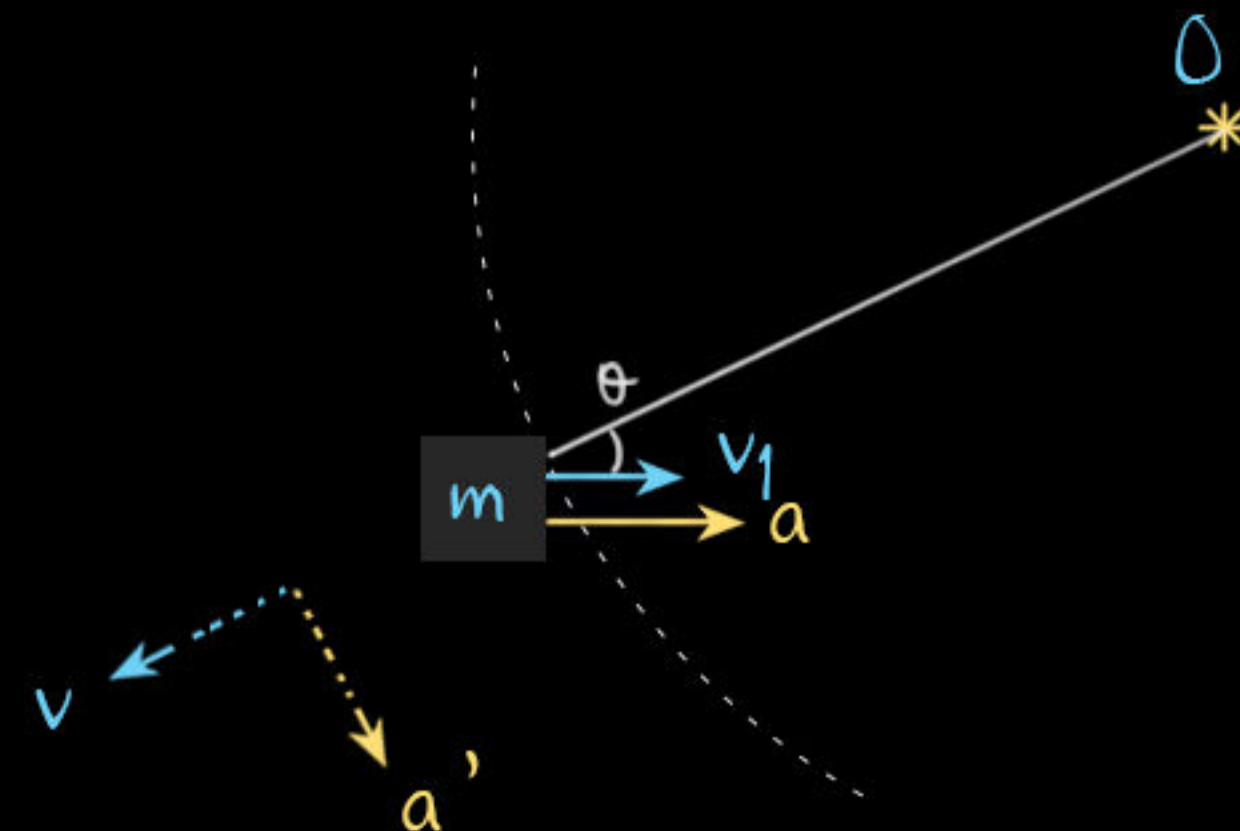
Lets say acceleration of O perpendicular to rope is a' .



Rope is inextensible: $v_1 \cos \theta = v$ 5

Within w.r.t platform, w.r.t top point O on rope

accelaration of block is: $\vec{a} + \vec{a}'$



$$a_c = \frac{v_{\perp}^2}{r} \Rightarrow a \cos \theta = \frac{(v_1 \sin \theta)^2}{\frac{h}{\sin \theta}} \quad 6$$

Variables: $F, f, N, T, a, v,$

Solving 1-6: $F = m \left[\frac{\mu gh + mv^2 \tan^3 \theta}{h (\cos \theta + \mu \sin \theta)} - \frac{v^2 \tan^3 \theta}{h} \right]$

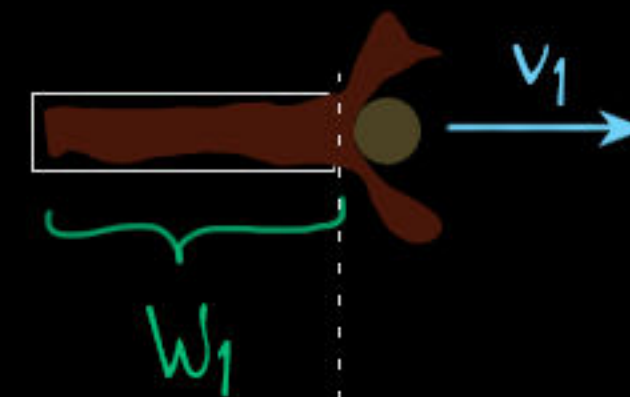
by Ankit Singhvi

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1. Greater is the range, longer should be length of barrel of a gun. What is the reason behind this belief?

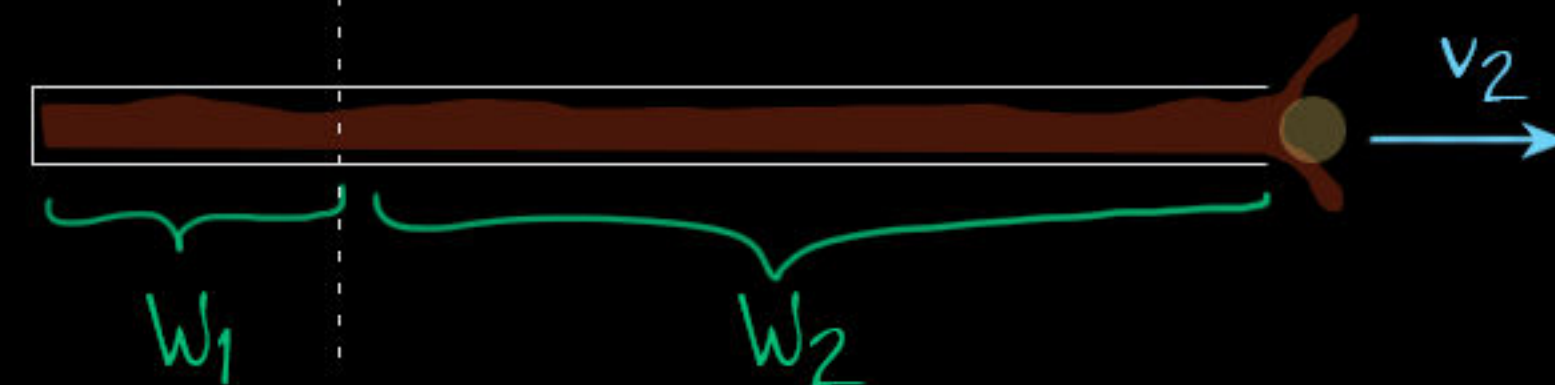
- (a) To allow better cooling of the gun due to increased surface area.
- ✓ (b) To allow the force of the expanding gases to act for a longer distance.
- (c) To increase the force exerted on the shell due to the expanding gases.
- (d) To provide a more favourable ratio of kinetic energy to potential energy.

Let work done by gases in short barrel be $W_1 = \frac{1}{2}mv_1^2$



Then work done by gases in long barrel is $W_1 + W_2 = \frac{1}{2}mv_2^2$

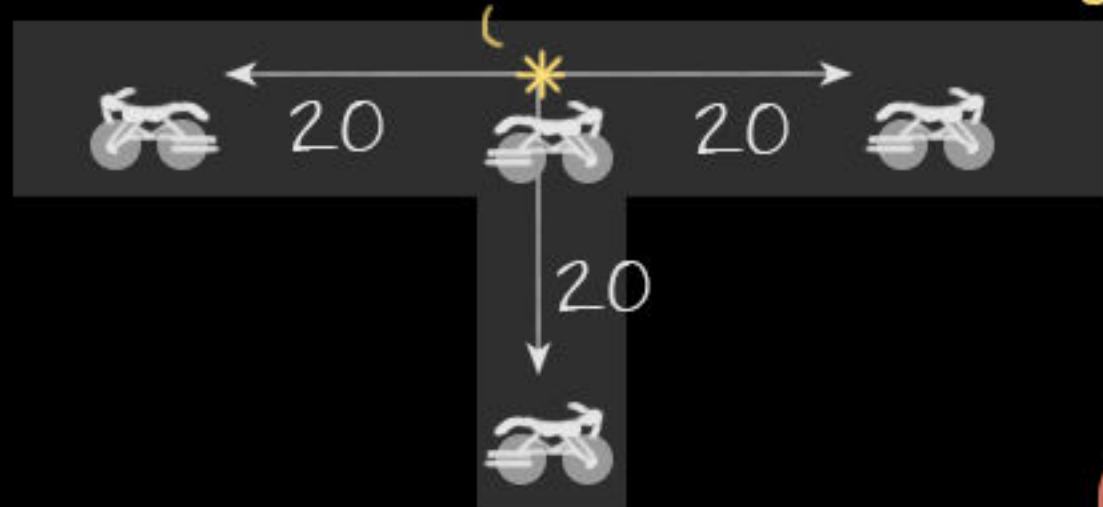
∴ $v_2 > v_1$ and longer barrel will provide longer range



Concept: Work done is frame dependent. As displacement depends on frame, even if force does not.

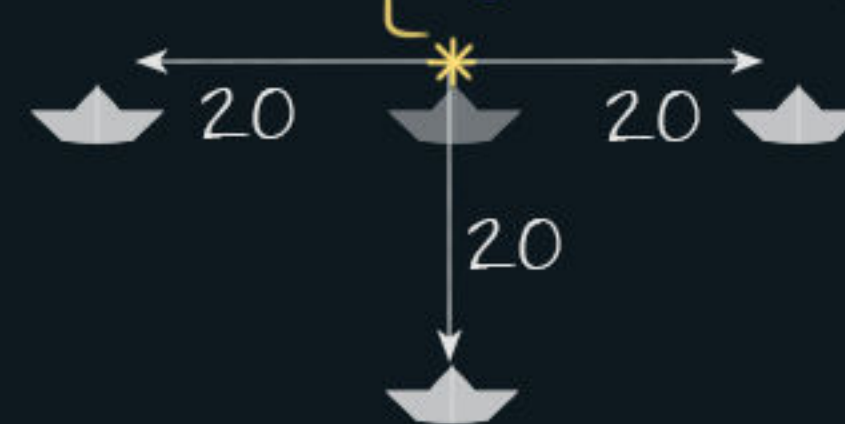
But, Energy expended by a bike, depends on distance it covers w.r.t road. Road is the only reference frame that can be used to calculate 'energy expended'. Similarly for a boat, energy expended depends on distance it covers w.r.t water its floating on.

w.r.t road Junction(stationary)



Energy expended

w.r.t river block(stationary)



2. You are in a paddleboat that is floating alongside a wooden block in a river. You are asked to perform following three experiments.

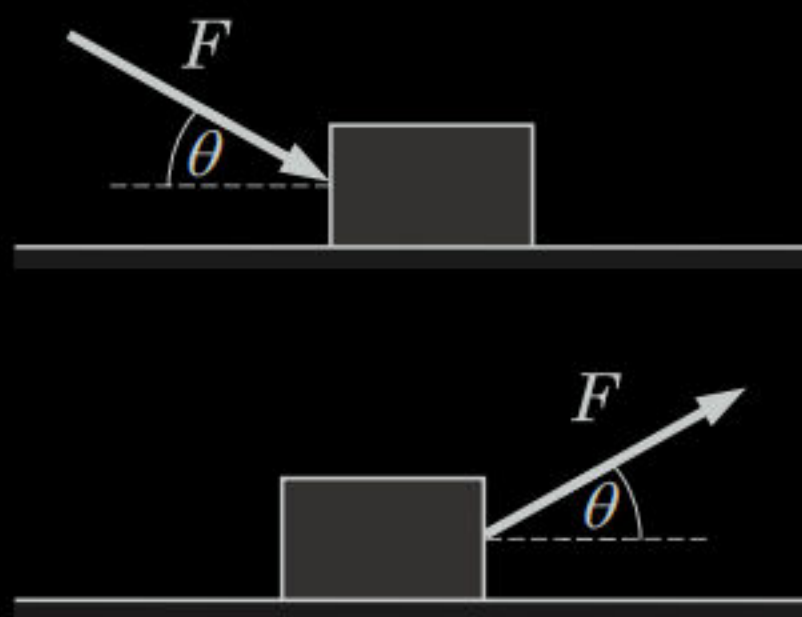
- I. Row your boat 20 m downstream ahead of the wooden block.
- II. Row your boat upstream to go 20 m away from the wooden block.
- III. Row your boat to move along the stream together with the block and simultaneously shift 20 m towards a bank.

In all these experiments, you row your boat at a constant speed relative to water expending energies E_1 , E_2 and E_3 respectively. Which of the following statements is correct?

- (a) $E_1 < E_2 < E_3$
- (b) $E_2 < E_1 < E_3$
- ✓ (c) $E_1 = E_2 = E_3$
- (d) It depends on reference frame you choose.

A bike w.r.t road burns 1 litre petrol per 20 km:

A boat w.r.t river burns 1 litre diesel per 20 km:



3. Two men move identical heavy boxes through equal distances on a horizontal floor (not frictionless) as shown. The first man is pushing, while the second is pulling the box with forces of equal magnitudes.

- I. Both the men deliver equal power.
- ✓ II. Both the men do equal amount of work.
- III. First man does more work than the second does.
- IV. First man delivers more power than the second one.
- ✓ V. Second man delivers more power than the first one.

Let the distance moved be x .

$$W = F \cos \theta x \text{ in both cases.}$$

Power = $\frac{W}{t}$

Equal in both cases

less N
 $\Downarrow$
 less retarding friction
 $\Downarrow$
 Less in case 2, as acceleration is more

∴ More power in case 2.

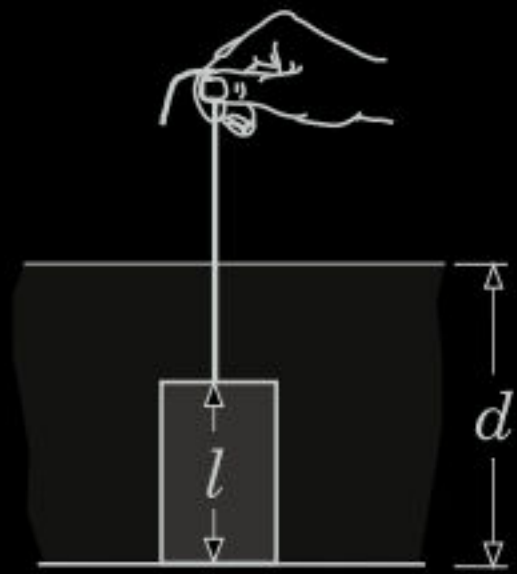


Figure-I

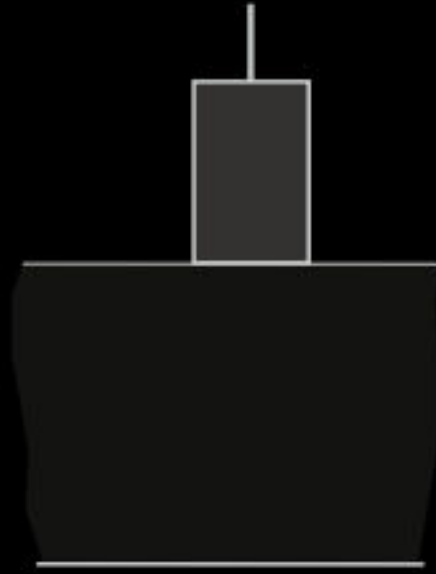


Figure-II

4. A solid right cylinder of length l stands upright at rest on the bottom of a large water reservoir, where depth is d as shown in the figure-I. Density of material of the cylinder is equal to that of water. Now the cylinder is slowly pulled out of water with the help of a thin light inextensible cord as shown in figure-II. Find the work done by the pulling agency.

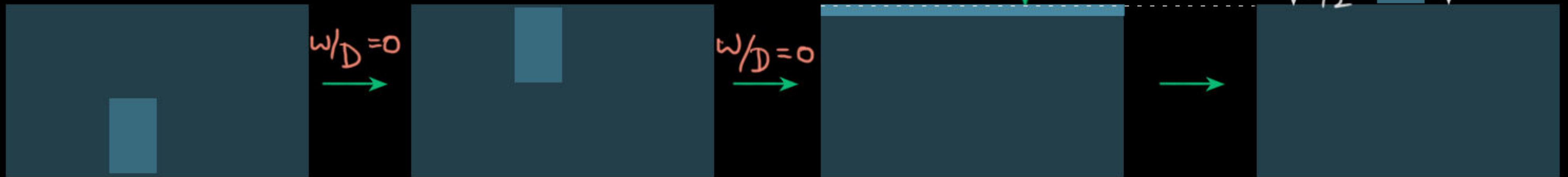
✓(a) $0.5mgl$

(b) mgl

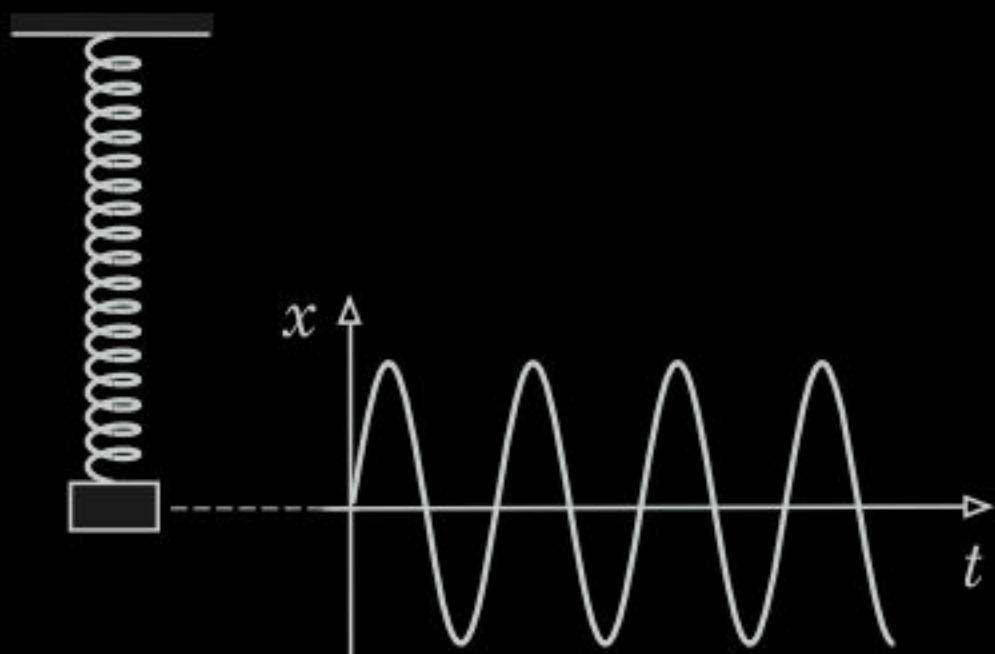
(c) mgd

(d) $mg(0.5l + d)$

Approach: As cylinder's density is same as water. Let's replace it with water.

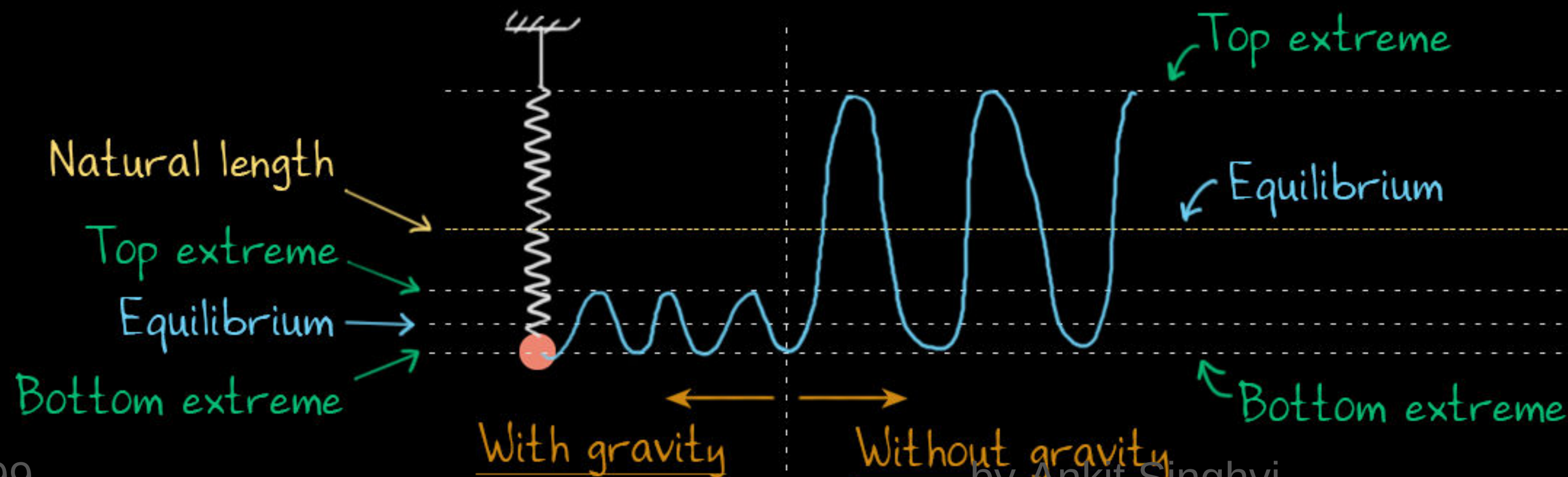
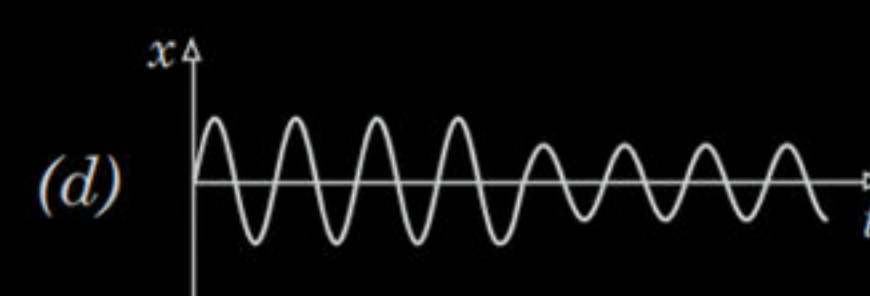
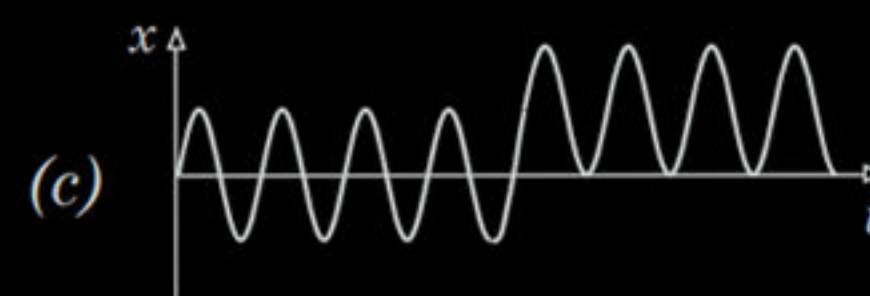
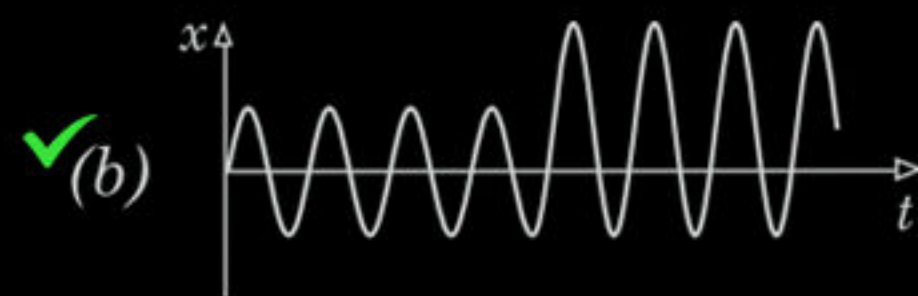
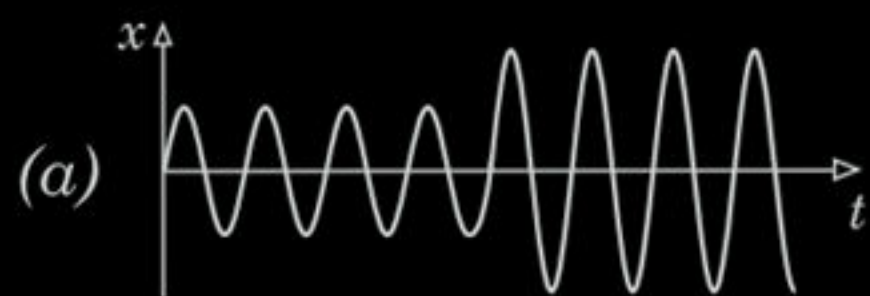


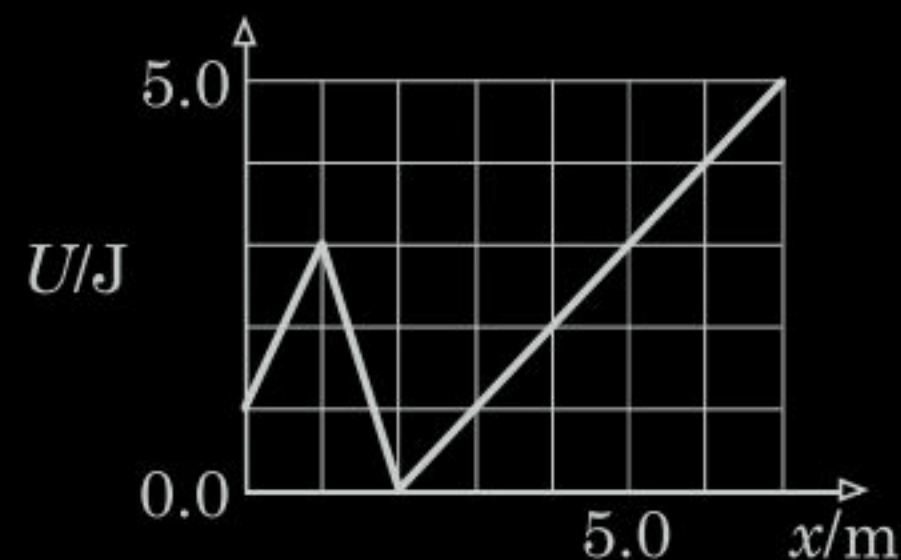
∴ Work done to pull the 'water' cylinder out = $mg \frac{l}{2}$



5. A spring-mass system oscillating in uniform gravity is shown in the figure. If we neglect all dissipative forces, it will keep on oscillating endlessly with constant amplitude and frequency. Accompanying graph shows how displacement x of the block from the equilibrium position varies with time t .

Now at a certain instant when the block reaches its lowest position, gravity is switched off by some unknown mechanism. Which of the following graphs would correctly describe the changes taking place due to switching off the gravity?





6. A potential energy function is shown in the graph. A particle moving in this potential field in the positive x -direction, if possesses 1.0 J of kinetic energy at $x = 1.0$ m, where will it reverse its direction of motion?

(a) $x = 1.0$ m

(b) $x = 2.0$ m

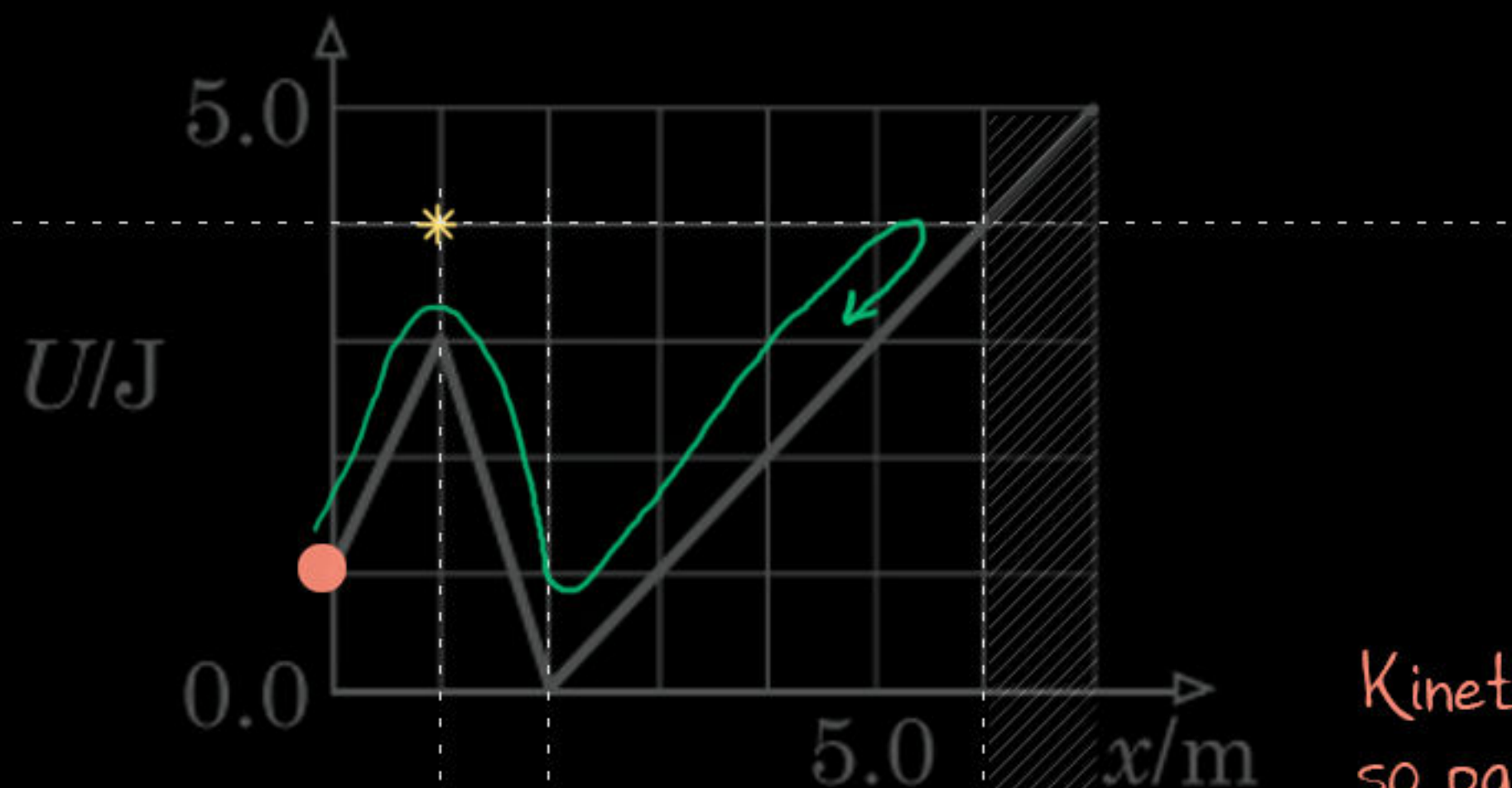
(c) $x = 5.0$ m

✓(d) $x = 6.0$ m

∴ Concept: The fact that $U-x$ can be plotted means it's a conservative Potential energy field.

Total mechanical energy $\rightarrow$
is constant = $3 + 1 = 4$ Joules
(given)

Approach: Think of this graph like a landscape with hills and valleys that the ball is passing through.



Kinetic energy can't be -ve
so particle won't enter this
region

velocity
local minima

velocity
local maxima

velocity zero,
then reversed

by Ankit Singhvi

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Concept: For minimum force F needed to break the cord, ball must come at rest at breaking point.

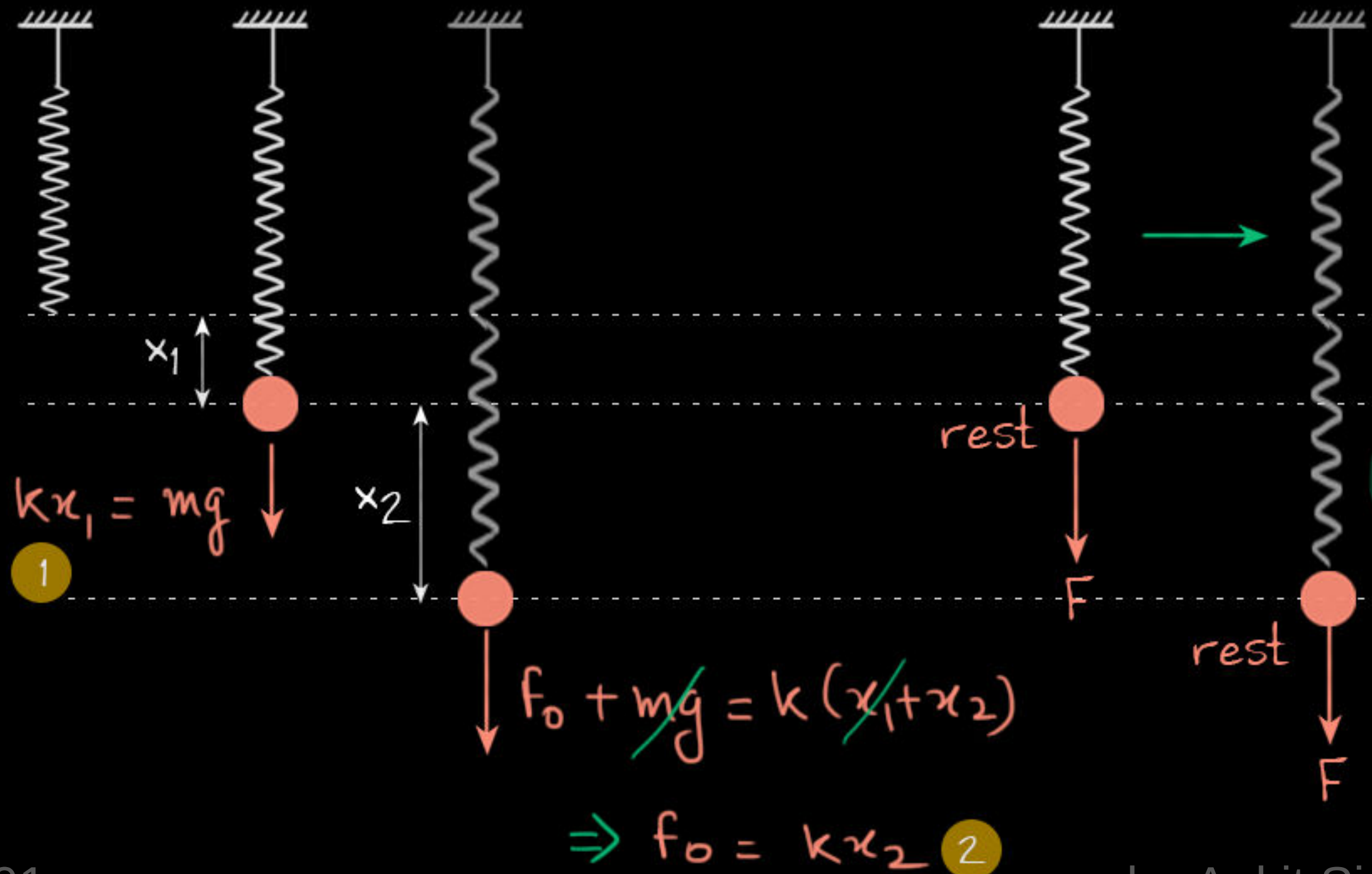
7. A ball of mass m is suspended from the ceiling with the help of an elastic cord. An additional downward force applied on the ball, if increased gradually to a value F_0 , the cord would break. What should be the additional minimum constant force F that will break the cord?

✓ (a) $F = \frac{1}{2} F_0$

(b) $F = \frac{1}{2} F_0 + mg$

(c) $F = \frac{1}{2} (F_0 - mg)$

(d) $F = \frac{1}{2} (F_0 + mg)$



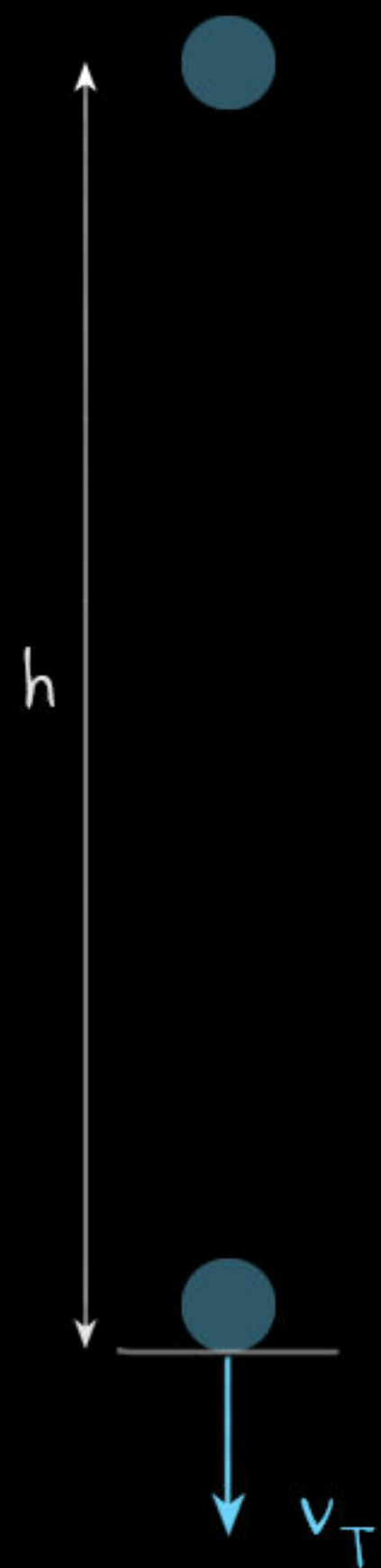
$$W_F + W_g = \Delta PE_{\text{spring}} + \cancel{\Delta KE}$$

$$Fx_2 + mgx_2 = \frac{1}{2}k(x_1 + x_2)^2 - \frac{1}{2}kx_1^2 \quad 3$$

From 1, 2, 3: $F = f_0/2$

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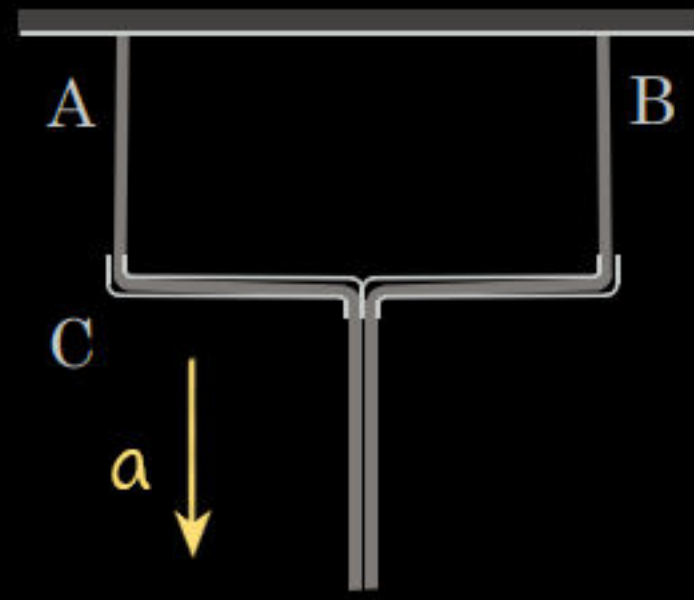
8. A raindrop of mass m at a height h starts from rest and initially accelerates under gravity. Due to viscous drag of air, its motion is resisted and finally it acquires a constant terminal velocity v_T and hits the ground. Neglect the buoyancy; denote acceleration due to gravity by g and assume no wind. The magnitude of average viscous force on the rain drop during the entire downward trip is

- (a) $m \left(g + \frac{v_T^2}{h} \right)$ (b) $m \left(g - \frac{v_T^2}{h} \right)$
 (c) $m \left(g + \frac{v_T^2}{2h} \right)$ ✓ (d) $m \left(g - \frac{v_T^2}{2h} \right)$

$$W_{\text{visc}} + W_g = \Delta KE$$

$$\Rightarrow -F_{av}h + mgh = \frac{1}{2}mv_T^2 - 0$$

$$\Rightarrow F_{av} = \frac{m}{h} \left(gh - \frac{1}{2}v_T^2 \right)$$

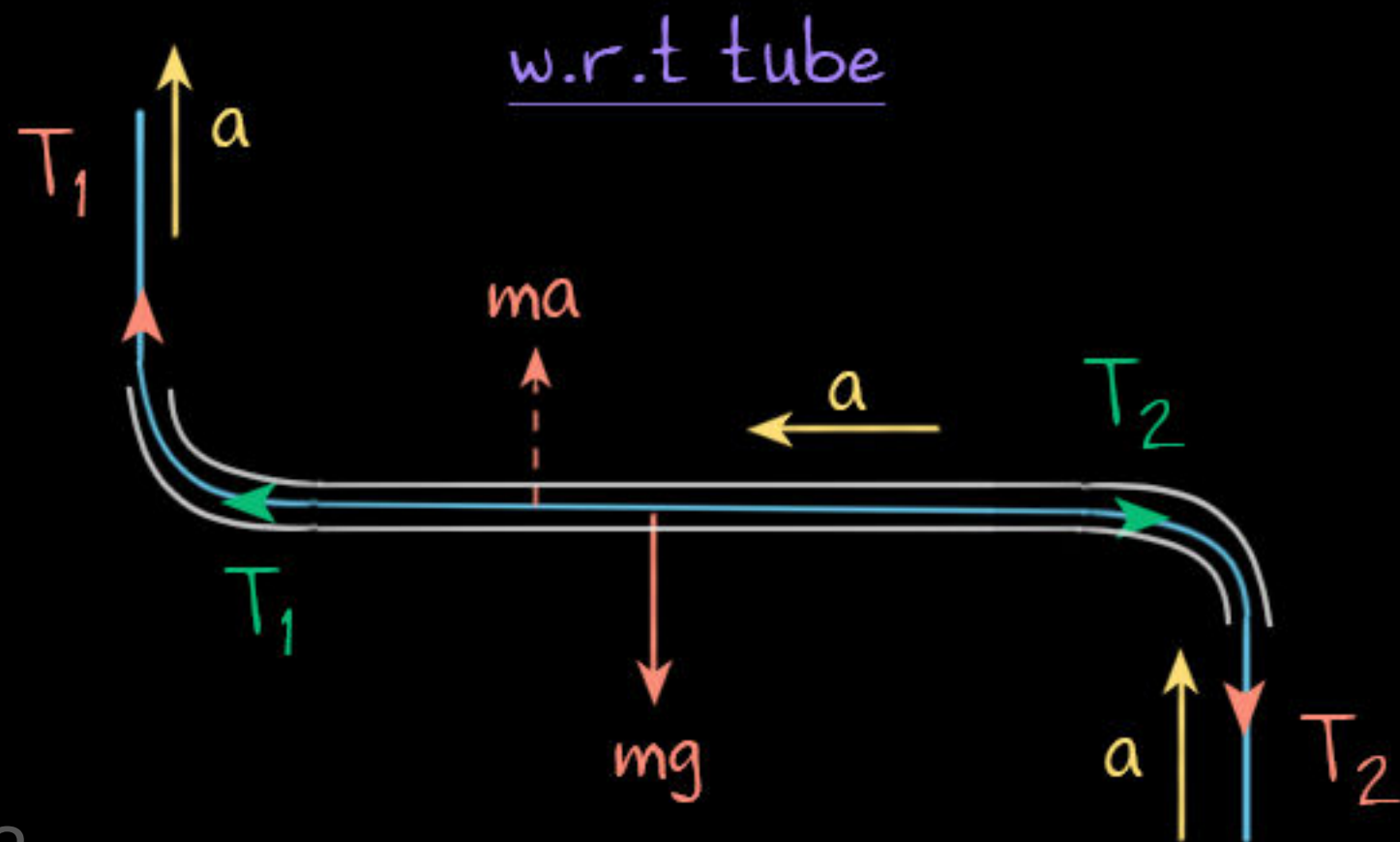


9. Two identical ropes A and B of uniform mass per unit length are suspended from the ceiling and are made to pass through an almost inertia-less metallic tubular structure C as shown in the figure. The structure is made by welding two identical bent tubes. There is no friction between the ropes and the tubes. Initially the structure is held at rest and then released. What will happen?

Approach: w.r.t tube, the trapped rope has only $a_x = a$.

Net normal reaction to tube by rope will be zero (and viceversa), as tube is massless.

- (a) It will not move at all.
- (b) It will move downwards with speed decreasing at the rate $0.5g$.
- ✓(c) It will move downwards with speed increasing at the rate $0.5g$.
- ✓(d) The rope will heat up.

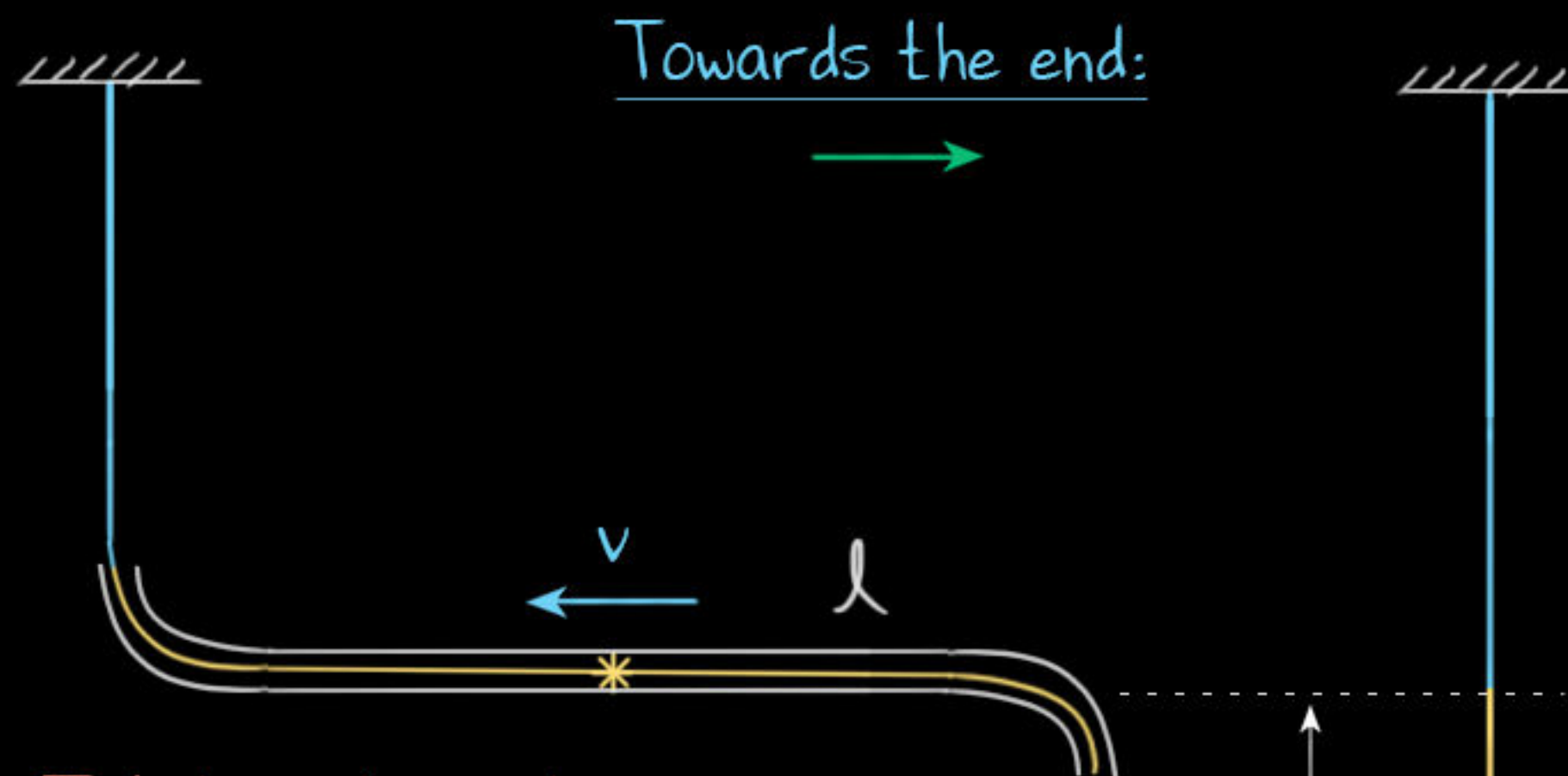


Let the mass of trapped rope be m and acc. of tube be a .

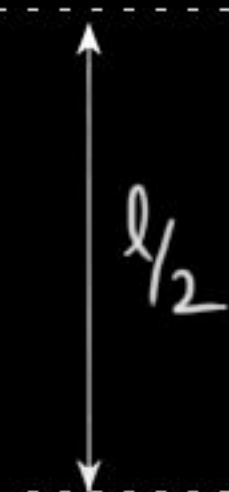
$$F_y \text{ balance on tube: } T_1 + ma = mg + T_2 \quad (1)$$

$$F_x \text{ on rope trapped: } T_1 - T_2 = ma \quad (2)$$

$$\text{From 1,2: } a = g/2 //$$



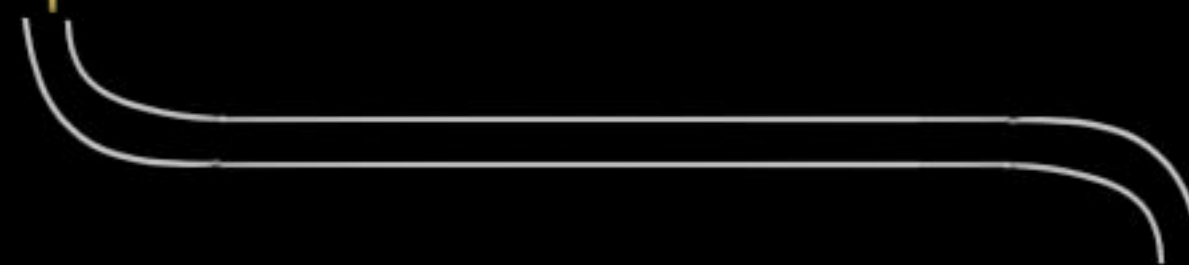
Total mechanical energy
of trapped rope = $\frac{1}{2}mv^2 + \frac{mgl}{2}$



Total mechanical energy
of trapped rope = $0 + 0$

Reference for gravitational energy

$v=0$



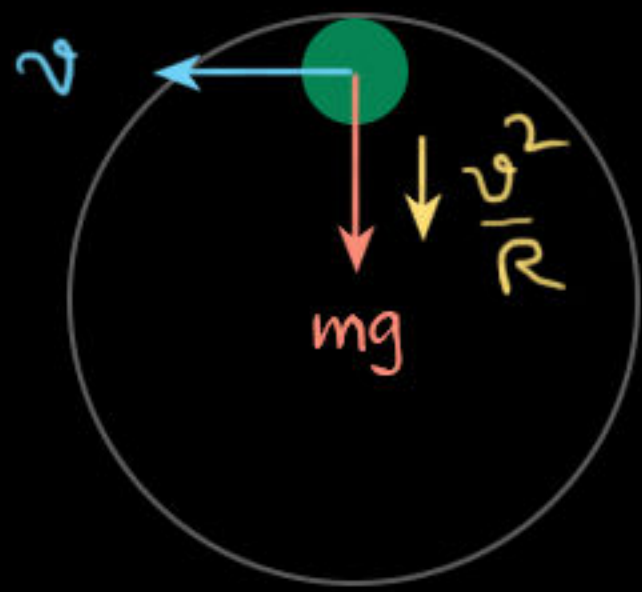
∴ Mechanical Energy lost is converted to heat.

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Concept: For a loop to be completed, Normal reaction at top should just reach zero.

And centripetal force therefore should be provided by mg alone.



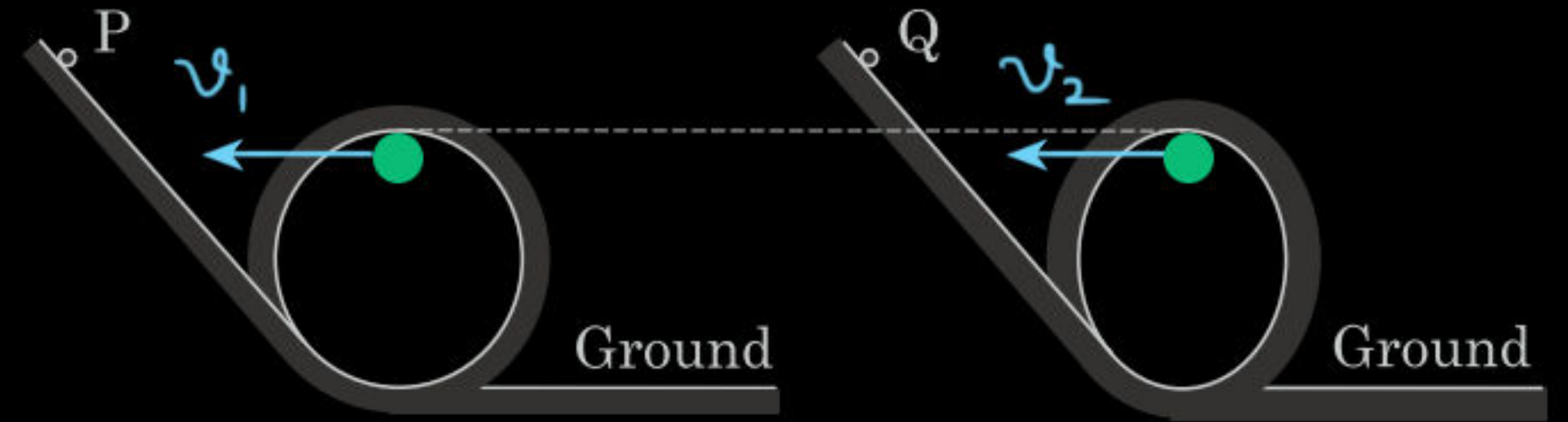
Case 1 $mg = \frac{mv_1^2}{R}$

Case 2 $mg = \frac{mv_2^2}{R_-}$

divide

$$\frac{v_1^2}{v_2^2} = \frac{R}{R_-} \Rightarrow v_1 > v_2 \Rightarrow \{a\}$$

10. A slider is released from a point P on a frictionless track, which bends into a circular loop at its bottom. Height of the point P above the ground is selected such that the slider is just capable of moving round inside the circular loop without breaking contact anywhere. The circular loop is now made elliptical without altering its height. The slider is now released from a point Q as shown in the figure. If the slider again rounds the complete elliptical loop, the minimum height of Q above the ground

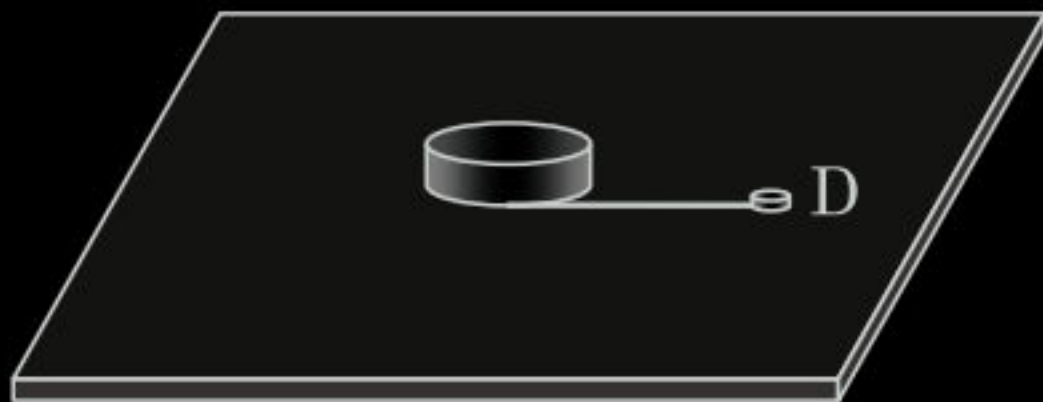


- ✓ (a) should be less than that of P.
- (b) should be equal to that of P.
- (c) should be more than that of P.
- (d) cannot be determined with the given information.

As elliptical radius of curvature on top is less than circular RoC.

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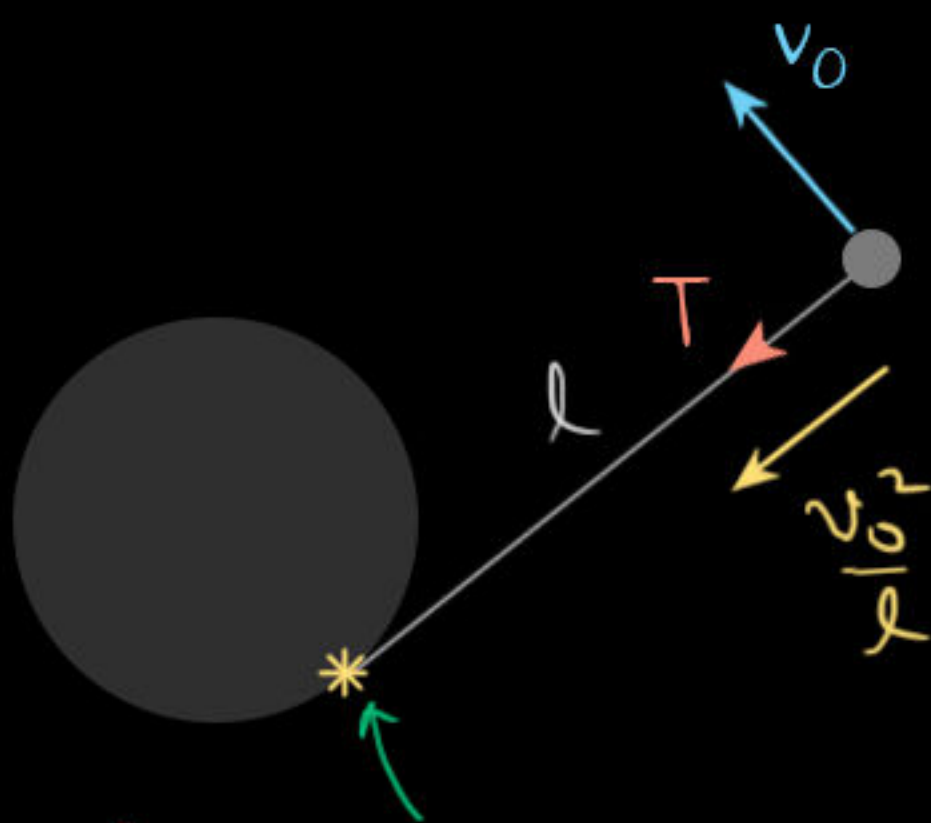
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11. A small disk placed on a frictionless horizontal floor is attached at one end of a light inextensible cord, the other end of which is attached on a vertical cylinder fixed on the floor. The disc is projected with a certain velocity perpendicular to the cord and along the floor. How will the tensile force in the cord and speed of the disc change during the ensuing motion?

- (a) Both will increase.
- (b) Both will remain constant.
- (c) Tensile force will increase and speed will decrease.
- ✓(d) Tensile force will increase and speed will remain constant.

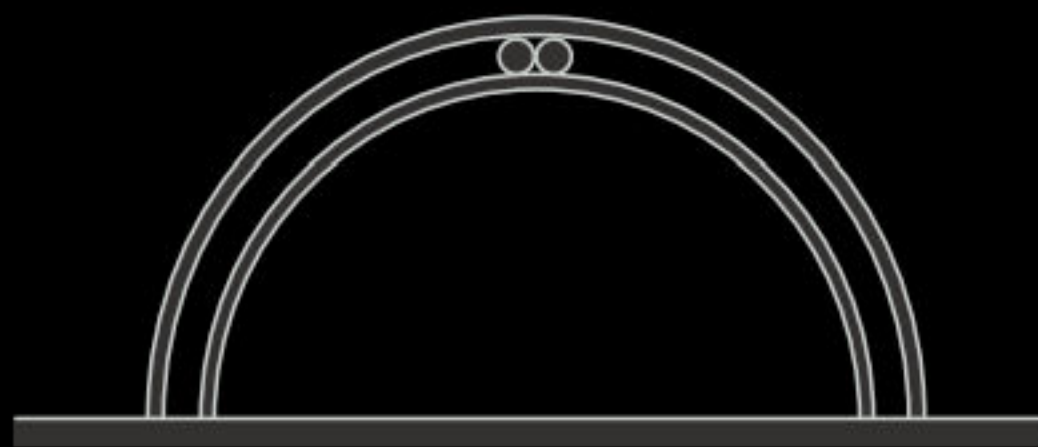
Concept: As T is always perpendicular to velocity, v_0 remains constant.



Instantaneous Axis of Rotation.

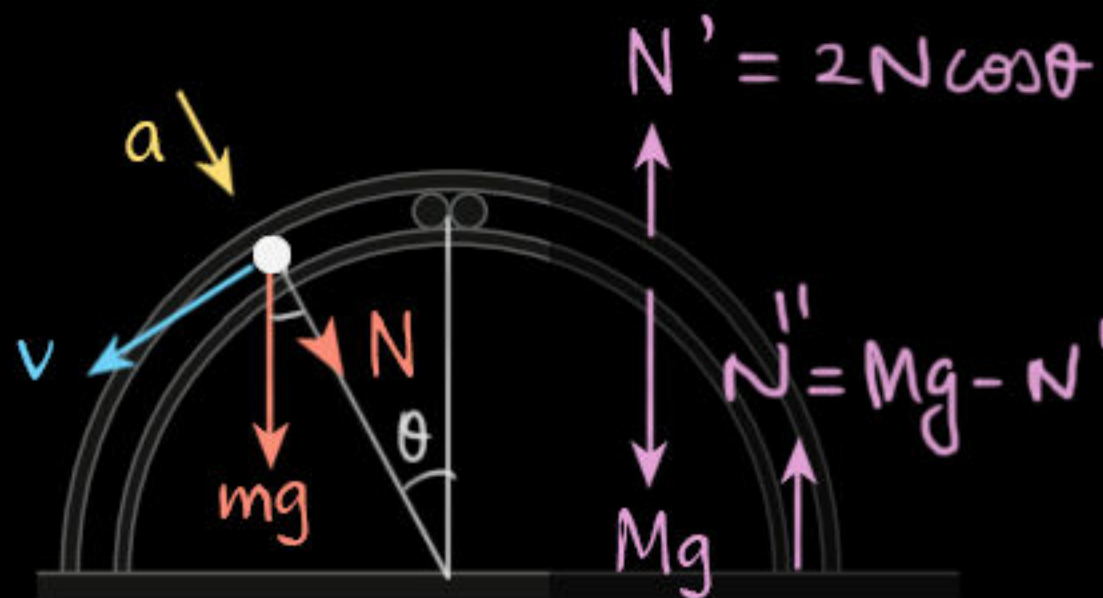
$$\text{Also, } T = \frac{mv_0^2}{l}$$

∴ As l decreases with time, T will increase.



Let N be Normal reaction on ball due to tube towards center.

∴ Due to N , upward force on tube $N' = 2N \cos \theta$
For two balls



Let mass of tube be M

12. A semicircular rigid cylindrical tube is placed on a horizontal floor. Two identical balls that loosely fit inside the tube are **simultaneously released** from the top of the tube. Each ball moves down the tube on either side. Which of the following statements **best describe the total force of normal reaction between the tube and the floor?**

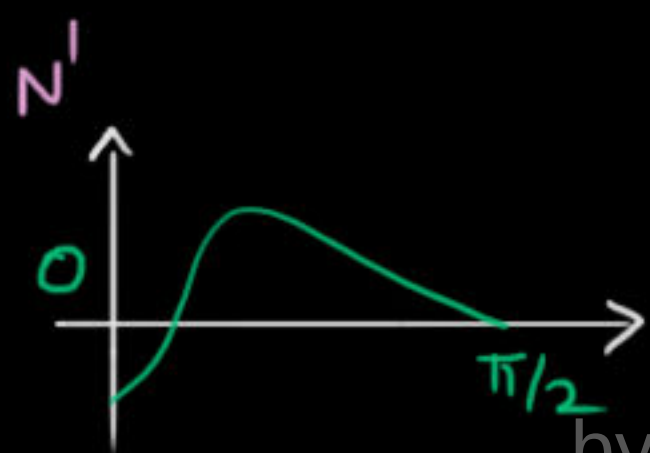
- (a) It decreases continually.
- (b) It increases continually.
- ✓(c) It first decreases, acquires a minimum value then increases.
- ✓(d) The tube may leave the ground.

Energy conservation: $mgR(1 - \cos \theta) = \frac{1}{2}mv^2$ ①

Force normal to surface: $mg \cos \theta + N = \frac{mv^2}{R}$ ②

From 1,2: $N = 2mg - 3mg \cos \theta \rightarrow (0 \rightarrow \pi/2)$

∴ $N' = 2N \cos \theta = 2mg(2 \cos \theta - 3 \cos^2 \theta)$ (Maxima at $\cos \theta = 1/3$)

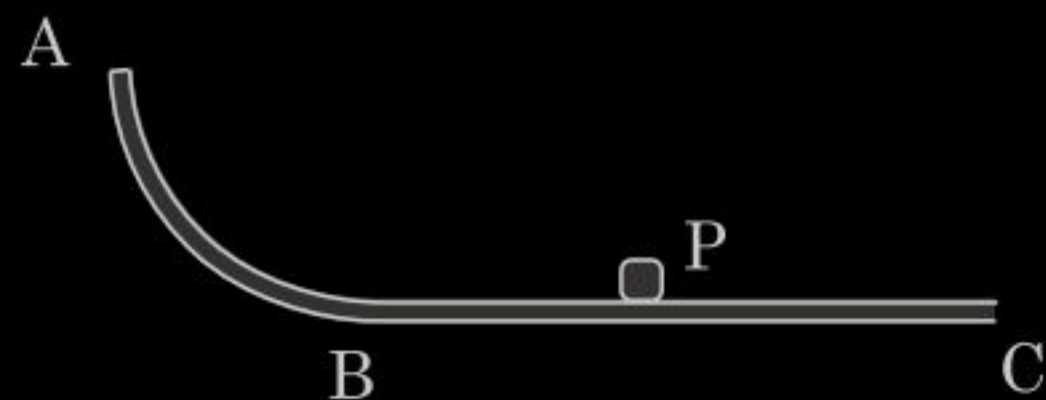


Reaction between ground and tube $N'' = Mg - N'$

Which decreases than increase, if $Mg < N'$ it will lift.

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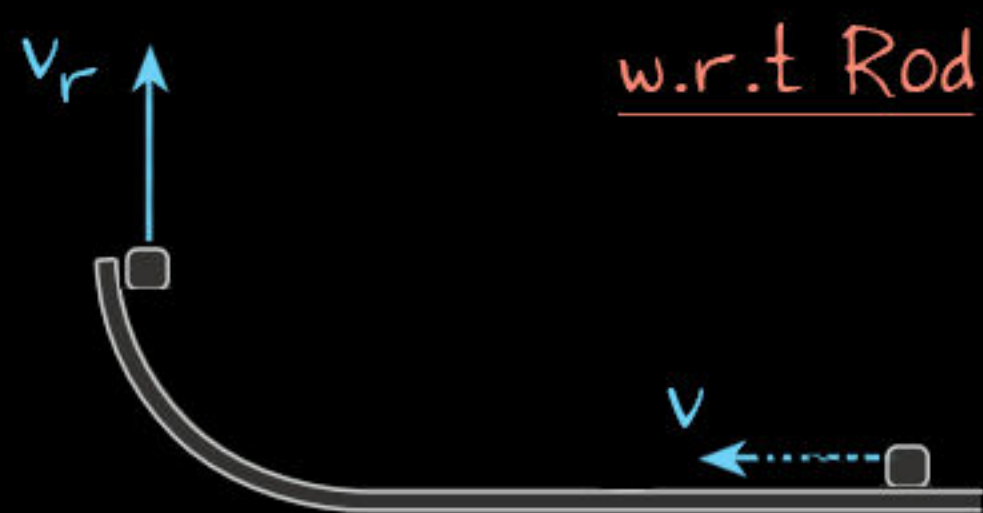


13. A frictionless rod ABC and a particle P of mass m are lying on a frictionless horizontal floor as shown in top view of the situation. The portion AB of the rod is bent to form a quarter of a circle. The rod is suddenly made to move with a constant velocity v towards the right along portion BC. What is the work done by the rod on the particle till it leaves the rod?

Approach: By constrained relation, ball will have same v_x as platform when it leaves it.

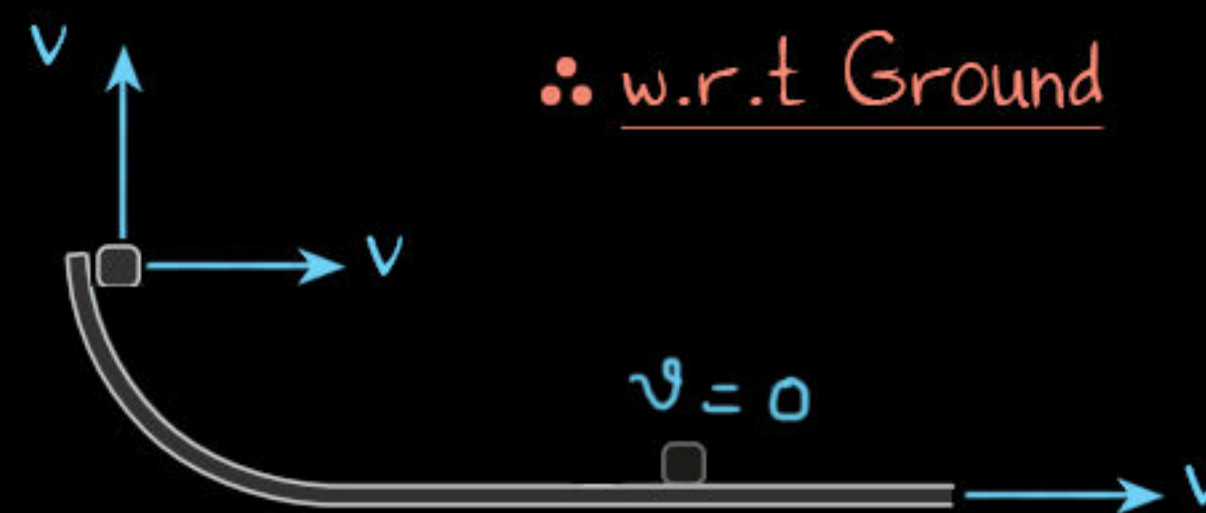
- (a) $0.5mv^2$
(c) $2mv^2$

- ✓ (b) mv^2
(d) None of these.

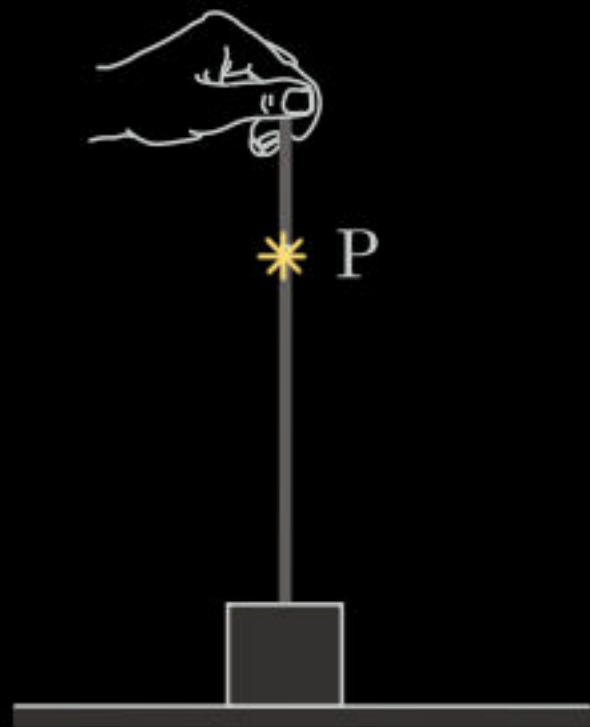


As Normal reaction from rod is always perpendicular to motion, its velocity won't change.

ie. $v_x = v$



$$\begin{aligned} W_{\text{Rod}} &= \Delta KE \\ &= \frac{1}{2}m(v^2 + v^2) - 0 \\ &= mv^2 \end{aligned}$$

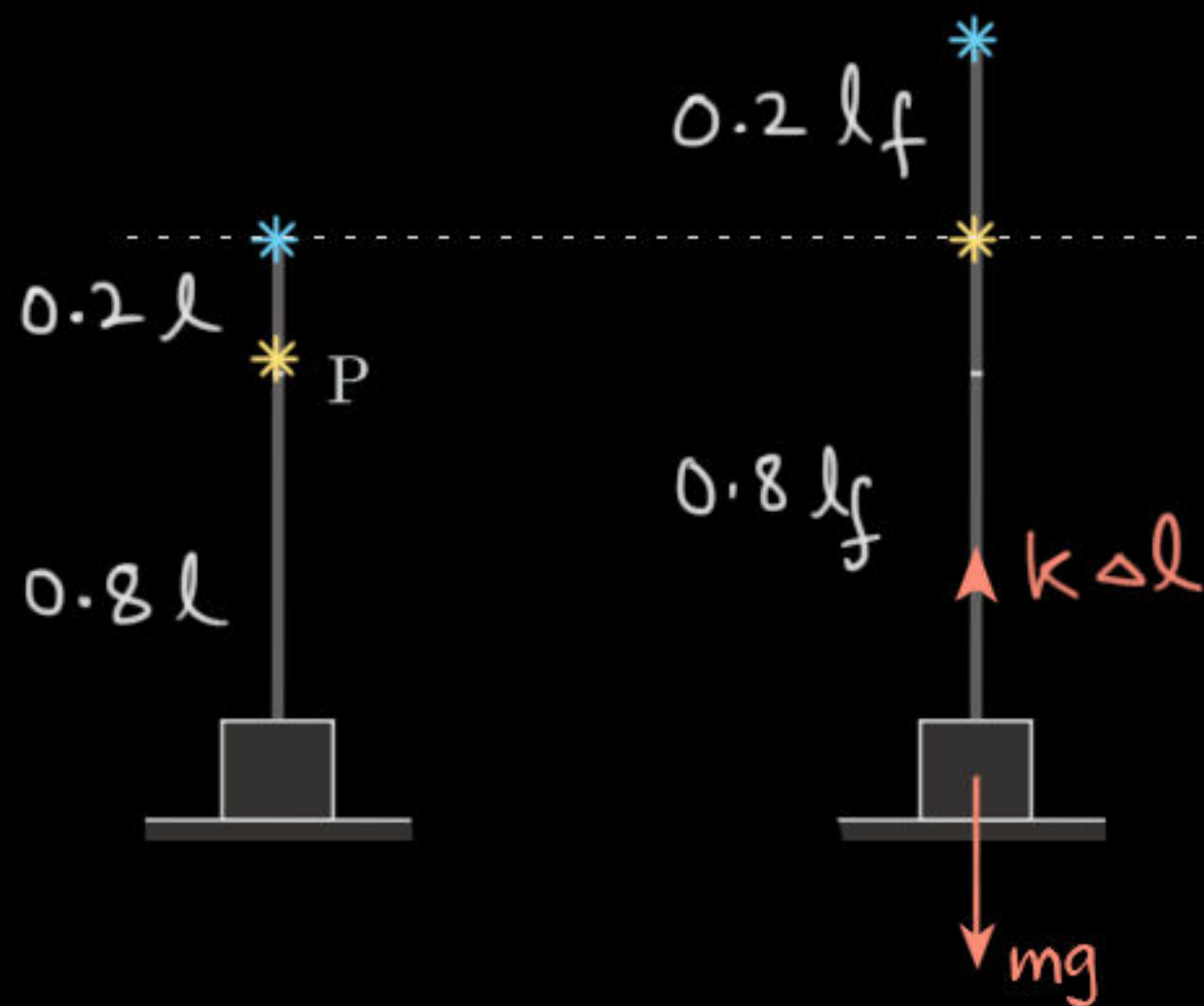


14. One end of an elastic cord is attached to a block of mass m and the other end is held keeping the cord relaxed and vertical. Relaxed length of the cord is l and there is a mark P at a height $0.8l$ from the lower end. If the upper end is slowly raised, the block leaves the ground, when the mark P reaches the point where the upper end of the relaxed cord initially was. Expression for the work done by the force applied by the hand is

(a) $\frac{mgl}{16}$

✓ (b) $\frac{mgl}{8}$

Work done to pull the cord = Energy stored in cord



$$l = 0.8 l_f \quad (1)$$

$$k \Delta l = mg \quad (2)$$

$$= \frac{1}{2} k \Delta l^2$$

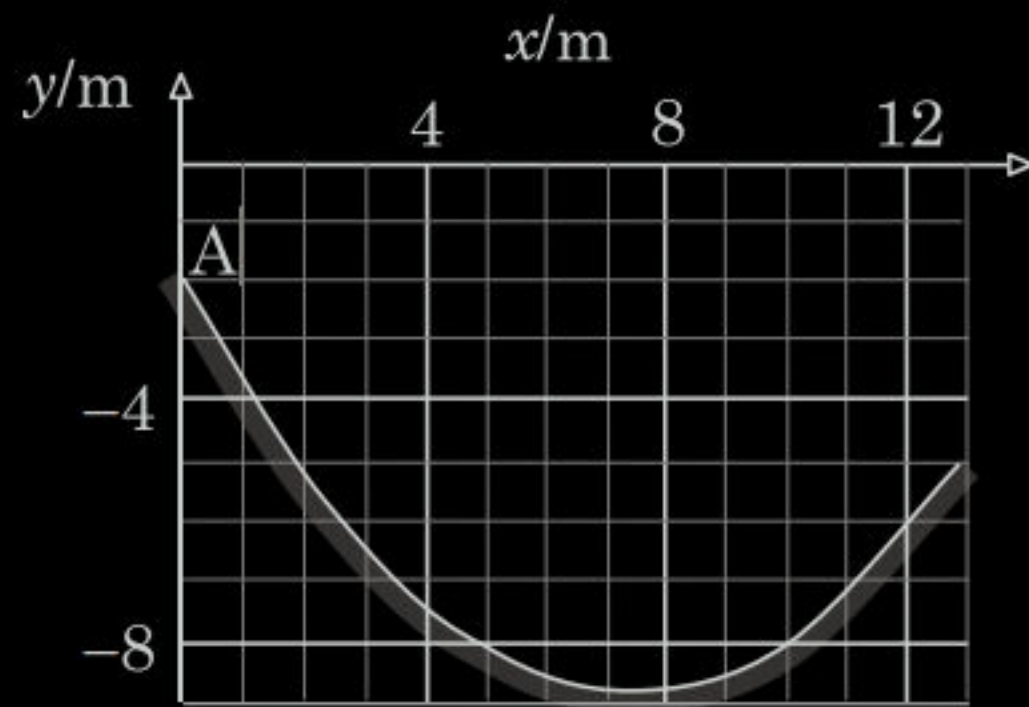
$$= \frac{1}{2} (k \Delta l) \Delta l$$

$$= \frac{1}{2} mg (l_f - l)$$

$$= \frac{1}{2} mg \left(\frac{l}{0.8} - l \right)$$

$$= \frac{mgl}{8} //$$

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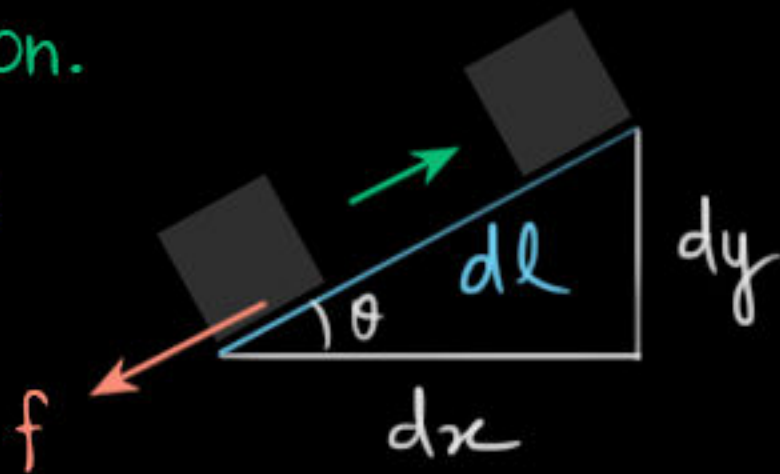
15. A small block starts sliding down from point A on a slide of irregular shape as shown in the figure. Here x and y -axes represent the horizontal and the vertical directions respectively. Coefficient of friction between the slide and the block is 0.6 everywhere. Radius of curvature of the slide everywhere is large enough and speed of the block is small enough so that normal component of acceleration becomes negligible as compared to acceleration of free fall. At which point may the block stop?

- (a) (5, -8)
(c) (12, -6)

- ✓ (b) (10, -8)
(d) Insufficient information

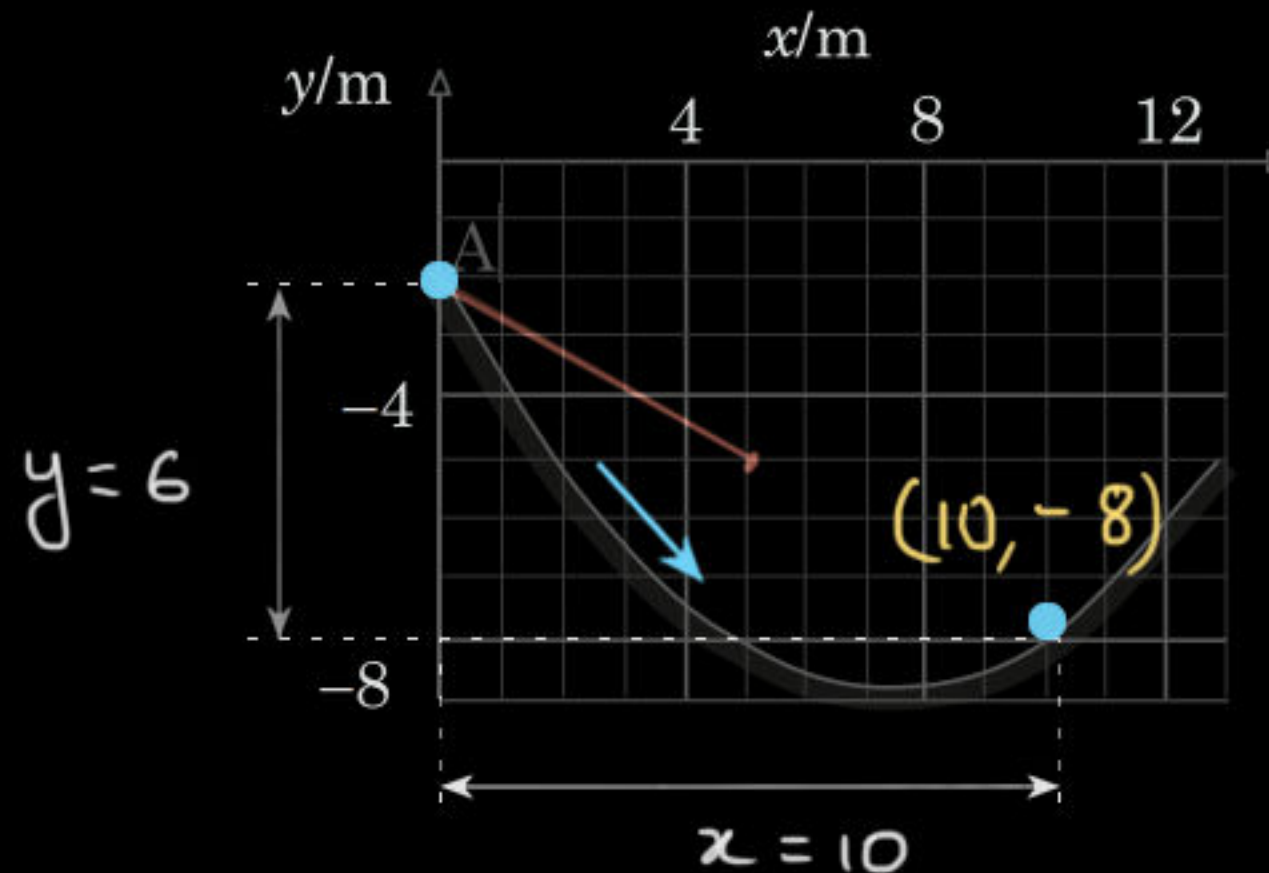
Concept: On a hill/valley path, work done by friction depends only on distance traveled in X direction.

Proof:



$$\begin{aligned} |dw_f| &= f dl \\ &= \mu mg \cos\theta \cdot dl \\ &= \mu mg dx \end{aligned}$$

$$\Rightarrow |W_f| = \mu mg x$$



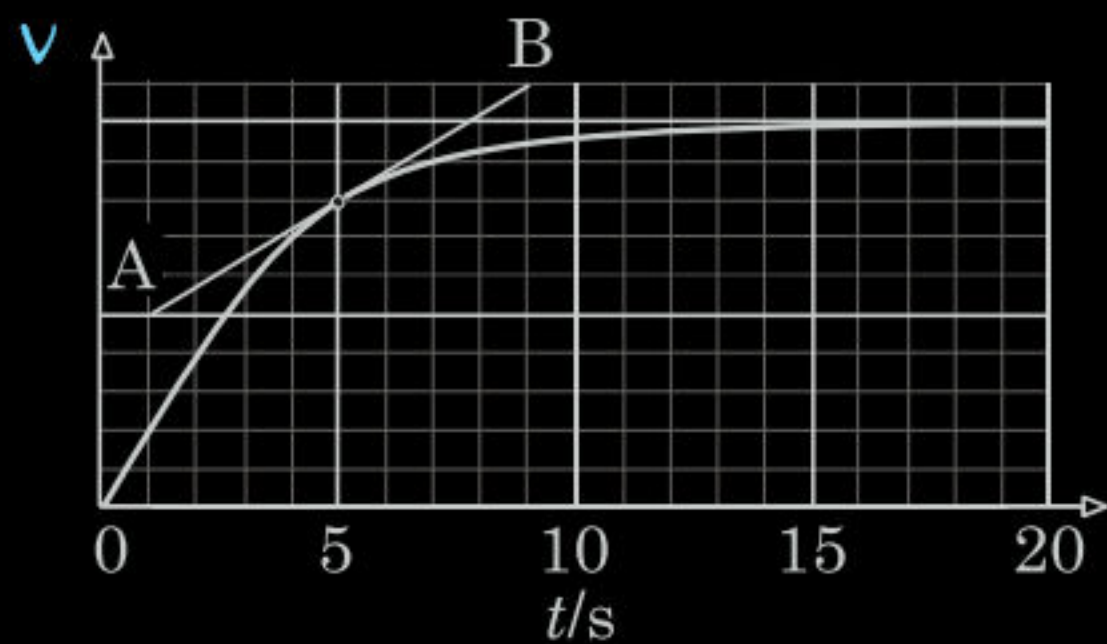
When it will stop:
 $W_g + W_f = \Delta KE \rightarrow 0$

$$\Rightarrow \cancel{m}g y - \mu \cancel{m}g x = 0$$

$$\Rightarrow y = \mu x \quad 0.6$$

$$\Rightarrow 3x = 5y$$

In graph it occurs at (10, -8)



16. A spherical object of mass 60 kg is dropped from a very large height. Its speed at every 1.0 s interval is recorded and is represented in the velocity-time graph, but the experimenter forgot marking velocities. Line AB is a tangent to the graph at an instant $t = 5.0$ s. What is power of air resistance at the instant $t = 5.0$ s. Acceleration due to gravity is 10 m/s^2 .

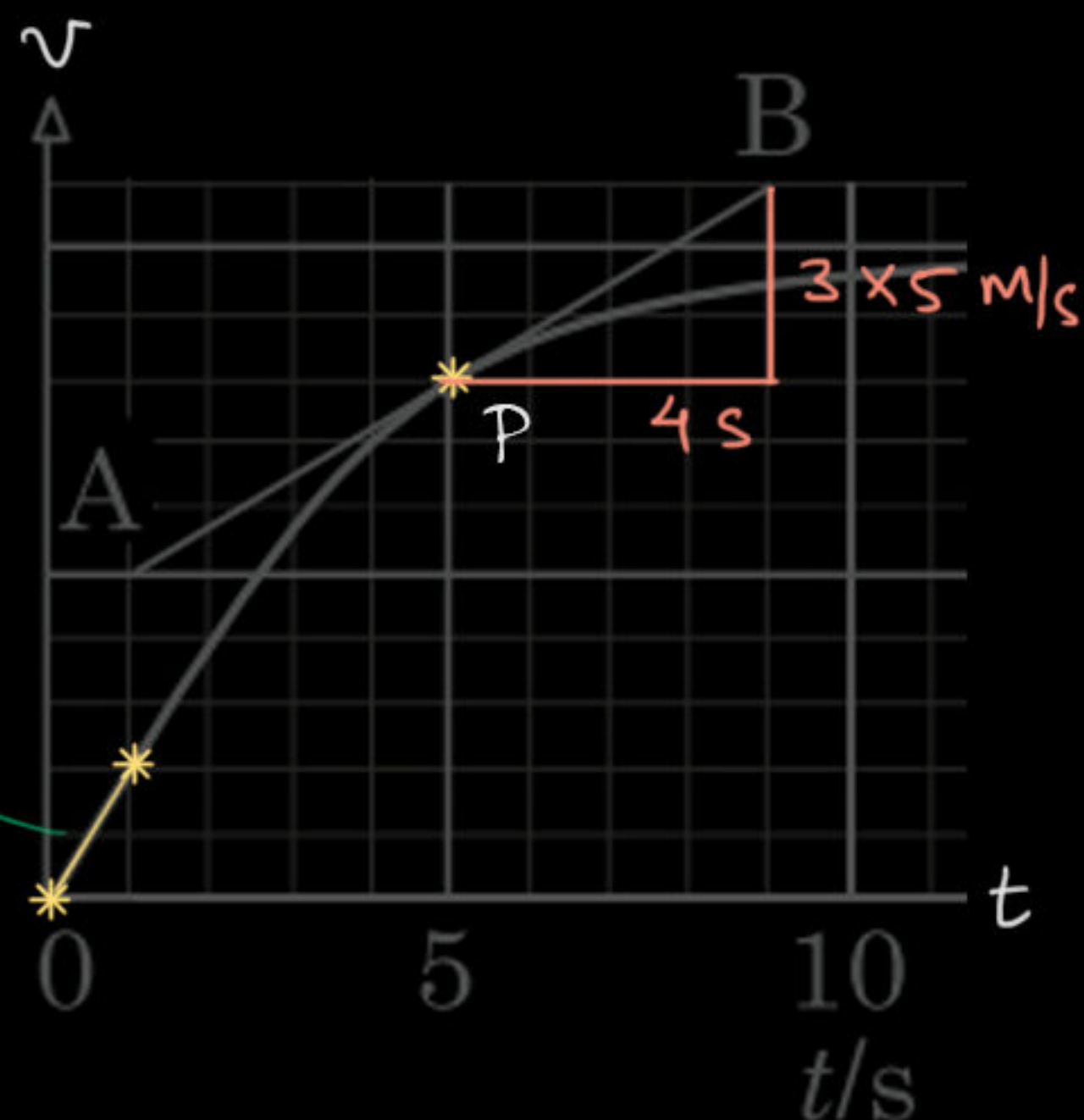
- (a) 0.6 kW
(c) 15.0 kW

- (b) 10.24 kW
(d) 24.0 kW

Concept: Air resistance will be zero initially as velocity will be low.

So first second slope of $v-t$ graph is $g=10$.

Hence one unit on y axis is 5 m/s .



As it falls down Force equation:

$$mg - f = ma$$

$\downarrow$
Slope at P

$$\Rightarrow 600 - f = \frac{15}{4} \cdot \frac{3 \times 5}{4}$$

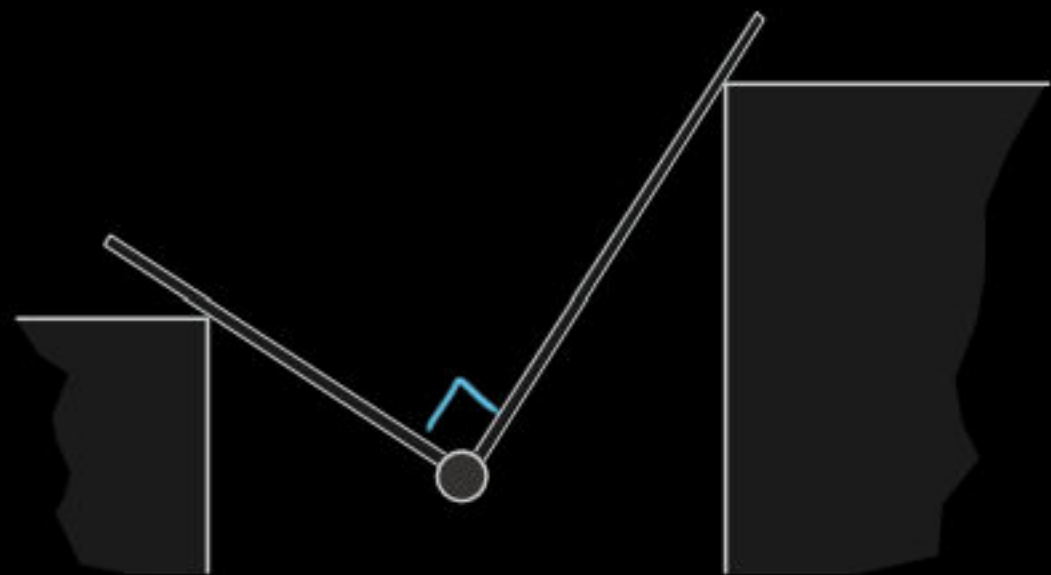
$$\Rightarrow f = 375$$

$$\therefore \text{Power of } f = f v = 375 (5 \times 8)$$

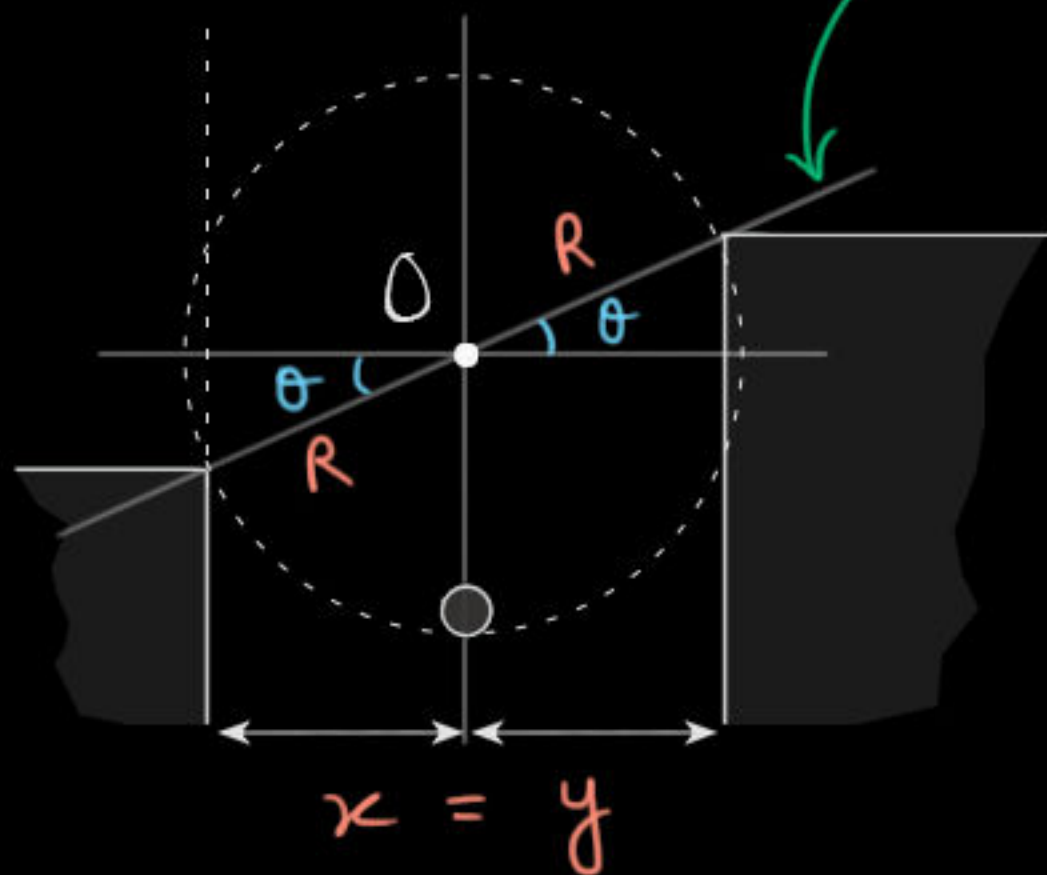
$$= 15 \text{ kW}$$

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Concept: For equilibrium, the ball needs to be at the bottom most point.
As PE of system is then minimum.



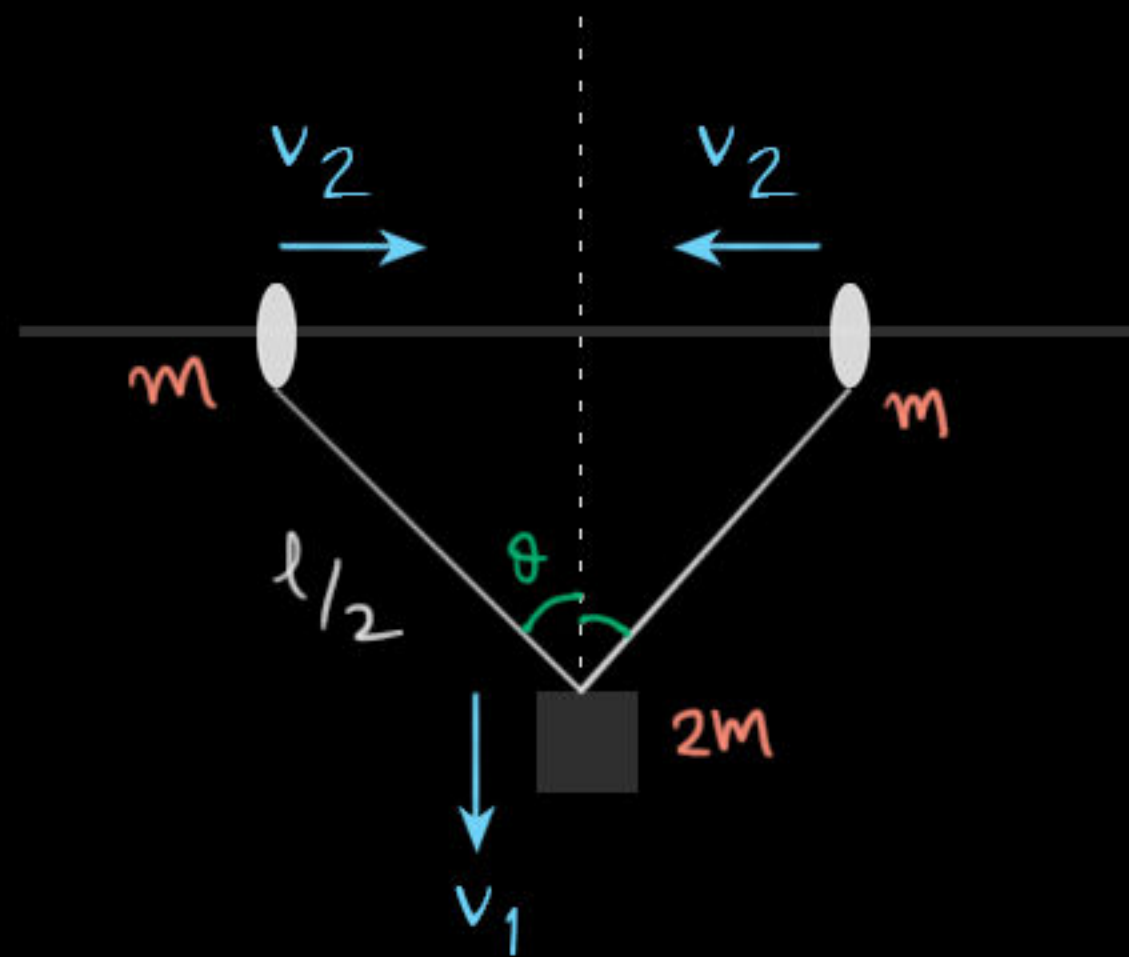
17. A light thin rod is bent into a right angle and a massive bead is affixed at the bend. The structure thus formed is placed on two frictionless platforms of different heights as shown in the figure. When the structure stays in equilibrium, what can you conclude regarding position of the ball relative to the vertical faces of the platforms?

- (a) The ball is closer to the lower platform than the higher one.
- (b) The ball is closer to the higher platform than the lower one.
- ✓ (c) The ball is equidistant from both the platforms irrespective of their heights.
- (d) The ball is equidistant from both the platforms only when both the platforms are of the same height.

This will be the diameter of the circle, as the angle subtended by rods is 90° .

Bottom most point will be below the center O.

$$\therefore x = y = R \cos \theta$$



18. Two identical sleeves A and B each of mass m can slide without friction on a fixed horizontal rod. A load C of mass $2m$ is suspended from the mid-point of a light inextensible cord of length $l = 3\sqrt{3}$ m connecting the sleeves A and B. Initially the sleeves are at a separation l and the load is held motionless. What can you conclude for subsequent motion after the load is released? Acceleration of free fall is 10 m/s^2 .

- (a) Maximum speed of the sleeves is $\sqrt{10} \text{ m/s}$.
 ✓ (b) Maximum speed acquired by the load C is $2\sqrt{5} \text{ m/s}$.
 ✓ (c) The load descends a height 1.5 m to acquire maximum speed.
 ✓ (d) Before the sleeves collide, the net force on the system consisting of the sleeves, the cord and the load becomes zero once.

String is inextensible:

$$\Rightarrow v_1 \cos \theta = v_2 \sin \theta \quad (1)$$

$$Wg = \Delta KE$$

$$\Rightarrow 2mg \frac{l}{2} \cos \theta = \frac{1}{2} 2m v_1^2 + 2 \left(\frac{1}{2} m v_2^2 \right) - 0$$

$$\Rightarrow v_1^2 + v_2^2 = gl \cos \theta \quad (2)$$

From 1,2: $v_1^2 = gl (\cos \theta - \cos^3 \theta)$

$$v_2^2 = gl \cos^3 \theta$$

by Ankit Singhvi

A $gl = \sqrt{30\sqrt{3}}$, B $v_{1\max} @ \cos \theta = 1/\sqrt{3} = 2\sqrt{5}$

C $h = \frac{l}{2} \cos \theta @ \cos \theta = 1/\sqrt{3} \rightarrow 1.5$

D F_x on system is zero always (symmetry).

And @ $v_{1\max} \Rightarrow dv_1/dt = 0 \Rightarrow F_{1y} = 0$

On sleeves F_{2y} is anyway zero.

$\therefore$ Net F_y on system is zero too.

Youtube / Arihant Senior



Immediately after the block is placed on the conveyor

19. A box of mass 1.0 kg moving horizontally with a velocity 10 m/s towards the left, is gently placed on a conveyor belt that is moving horizontally with a velocity 10 m/s towards the right. Coefficient of kinetic friction between the belt and the block is 0.5 . Some physical quantities and their possible value in SI units are mentioned in the given table. Suggest suitable matches.

$$W_f = \Delta KE$$

w.r.t ground.

$$\frac{1}{2}m 10^2 - \frac{1}{2}m 10^2$$

$$= 0 \quad \text{P}$$

w.r.t belt.

$$0 - \frac{1}{2}m 20^2$$

$$= -200 \text{ J} \quad \text{Q}$$

Distance slid in belt frame:

$$v^2 = u^2 - 2as$$

$$\Rightarrow s = \frac{u^2}{2a} = \frac{20^2}{2(10)} = 40 \text{ m} \quad \text{S}$$

Modulus in SI	Physical Quantity
(a) zero	(p) Work of friction on the block with respect to the ground
(b) 20	(q) Work of friction on the block with respect to the belt
(c) 40	*** (r) Work of friction on the belt with respect to the ground
(d) 200	(s) Distance slid by the block on the belt with respect to the belt
	(t) Change in momentum of the block with respect to the ground

$$\Delta p = m(10) - m(-10) = 20 \text{ kg m/s} \quad \text{T}$$

R Concept: Work done by friction on a system (where friction is internal force) in any frame is same. Its value will be the heat generated!.

∴ $W_{\text{friction on system in ground frame}} = W_{\text{friction on system in belt frame}}$

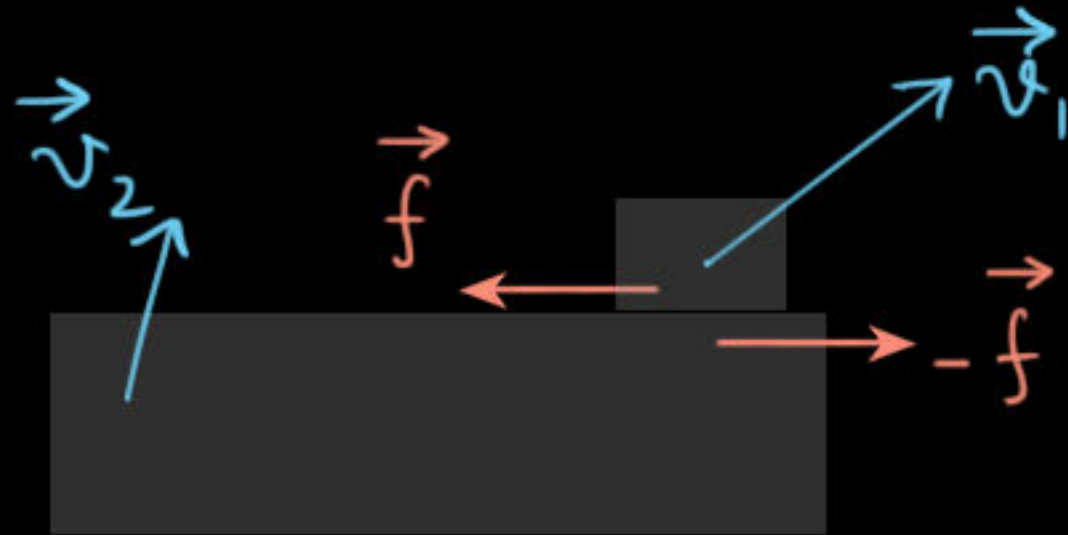
$$\Rightarrow W_{f \text{ on block } | \text{ gnd}} + W_{f \text{ on belt } | \text{ gnd}} = W_{f \text{ on block } | \text{ belt}} + W_{f \text{ on belt } | \text{ belt}}$$

(Calculated in part P) (Calculated in part Q) (As belt w.r.t belt is at rest)

$$\Rightarrow W_{f \text{ on belt } | \text{ gnd}} = -200 //$$

Concept: Work done by friction on a system (where friction is internal force) in any frame is same. Its value will be the heat generated!.

Proof:



Lets say W_{friction} on system in dt time in ground frame is:

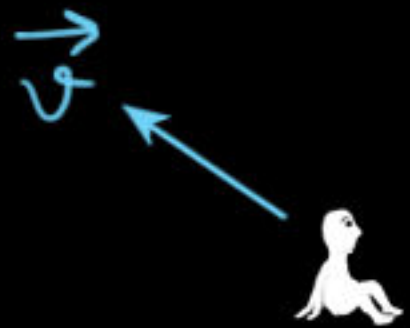
$$dW_f = \vec{f} \cdot (\vec{v}_1 dt) + (-\vec{f}) \cdot (\vec{v}_2 dt)$$

Then W_{friction} on system in a frame moving with $\vec{v}$ is:

$$dW'_f = \vec{f} \cdot [(\vec{v}_1 - \vec{v}) dt] + (-\vec{f}) \cdot [(\vec{v}_2 - \vec{v}) dt] = dW_f$$

$$\Rightarrow \int dW'_f = \int dW_f$$

$$\Rightarrow W_f = W'_f$$



For conservative force:

$$W_{AB} + W_{BC} + W_{CA} = 0$$

A $|W_{CA}| = |2 W_{BC}|$

q ✓

s ✗

(As its certain, not maybe)

C W_{CA} $W_{AB} \uparrow$ $W_{BC} \uparrow$

q	5	-10	5
r	5	-10	5
s	7	-10	3

20. A particle moves from point A to point B then to point C and finally back to A under action of a force. ~~Magnitudes of the~~ work done by the force in these displacements are W_{AB} , W_{BC} and W_{CA} respectively. Suggest suitable match or matches.

Column I

Column II

(a) The force is conservative and $W_{AB} = W_{BC} \neq 0$

(p) W_{CA} may be zero

(b) The force is non-conservative and $W_{AB} = W_{BC} \neq 0$

(q) W_{CA} may be $>$ zero

(c) The force is conservative and $|W_{AB}| > |W_{BC}|$

(r) W_{CA} may be $= W_{BC}$

(d) The force is non-conservative and $|W_{AB}| > |W_{BC}|$

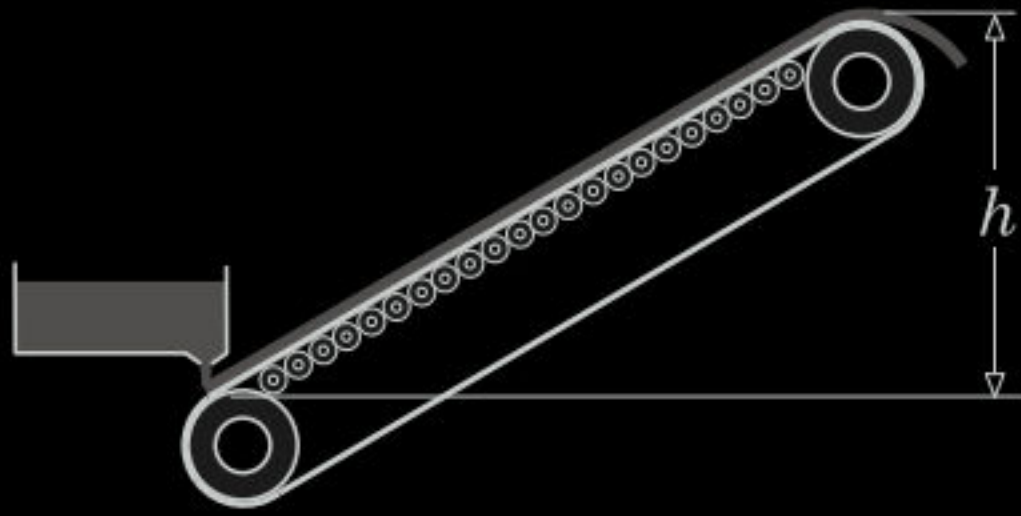
(s) $|W_{CA}|$ may be $> |W_{BC}|$

For Non-conservative force:

W_{AB} , W_{BC} & W_{CA}
can take any random values
independent of each other.

∴ B {p,q,r,s}

D {p,q,r,s}



A conveyor belt collects sand and transports it to a height h as shown in the figure. The sand falls on the belt with negligible speed at constant rate μ (mass per unit time). Friction between the belt and the sand particle is so high that the sand particles stop sliding almost instantaneously after they hit the belt. Acceleration due to gravity is g .

21. What should speed of the belt be for the least possible driving force on the belt applied by the motor?

(a) $\sqrt{0.5gh}$

✓ (b) $\sqrt{gh}$

(c) $\sqrt{2gh}$

(d) A speed however small is possible.

22. What is the power delivered by the motor to the belt, when the motor is applying least possible driving force?

(a) $0.5\mu gh$

(b) μgh

(c) $1.5\mu gh$

✓ (d) $2\mu gh$

23. What is the power dissipated by the sand-belt system, when the motor is applying least possible driving force?

(a) Zero

(b) $0.25\mu gh$

✓ (c) $0.5\mu gh$

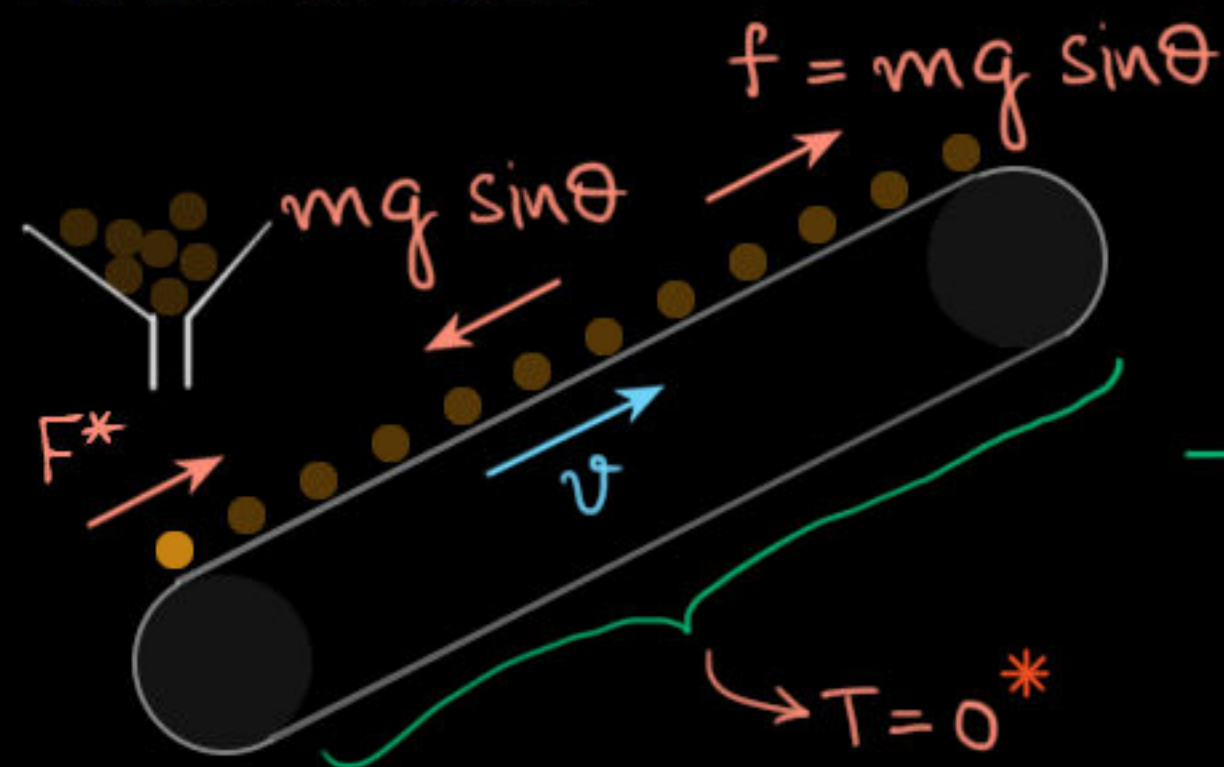
(d) μgh

Least Tension in the belt

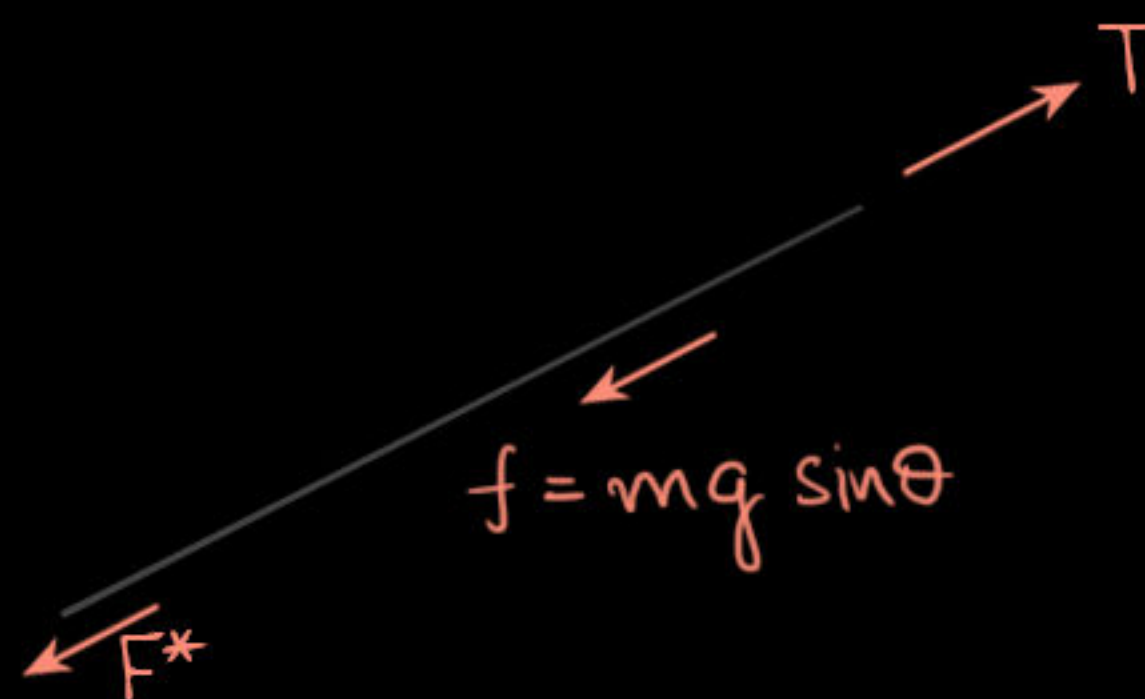
$P = Fv = \text{Tension} \cdot v$

Net rate of work done by friction on sand and belt system.

Forces on sand:



∴ Forces on belt:



∴ Net force on belt is zero

$$\begin{aligned} T &= mg \sin \theta + F^* \\ &= \frac{\mu l}{v} g \sin \theta + \mu v \\ &= \frac{\mu g h}{v} + \mu v \end{aligned}$$

Note: F^* is the frictional force on dm mass of sand. This force accelerates dm to v velocity.

$mg \sin \theta$ is the frictional force on mass m that is being pulled up with constant velocity v .

$$F^* = \frac{dp}{dt} = \frac{dm v - 0}{dt} = \mu v$$

$$m = \mu t_0 = \mu \left(\frac{l}{v} \right)$$

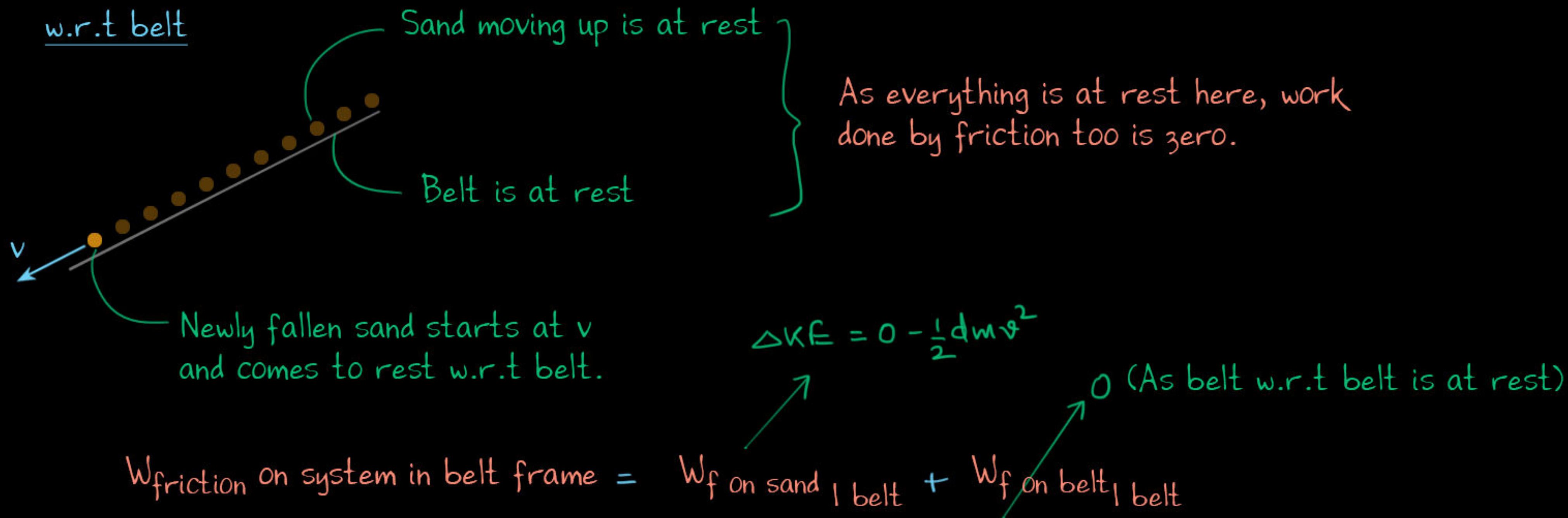
total time sand stays on belt

* $T=0$, Assuming belt is loosely fit

$$21 \quad \frac{dT}{dv} = 0 \Rightarrow T_{\min} @ v = \sqrt{gh} //$$

$$\begin{aligned} 22 \quad P &= T_{\min} v \\ &= \left(\frac{\mu g h}{v} + \mu v \right) v \\ &\quad \downarrow v = \sqrt{gh} \\ &= 2 \mu g h // \end{aligned}$$

23 Power dissipated: Net rate of work done by friction on sand and belt system.



$$\Rightarrow dW_f = -\frac{1}{2} dm v^2$$

$$\Rightarrow \frac{dW_f}{dt} = -\frac{1}{2} \mu v^2 = -\frac{\mu gh}{2}$$

$\therefore$ Power dissipated = $\frac{\mu gh}{2}$

Youtube / Arihant Senior

To show: how T can be zero at bottom (Its just like a cycle chain)
 Analogous replacement of belt by rope, and motor by man.



Note: T is zero at bottom if belt is loosely fit.
 Its possible that belt is tightened before hand.
 Then the tension due to tightning will be same through out. But
 calculations will still be same as we will then deal with excess Tension

by Ankit Singhvi

Youtube / Arihant Senior

1. Efficiency of an engine is measured to be 24% in an experiment running the engine at constant throttle. After the experiment, it was found that 4% of the fuel used in the experiment had actually leaked through a faulty gasket. Find efficiency of the engine after repair of the gasket.

	Diesel	Useful part of it
Experimental:	100 J	24 J
Actual:	96 J	24 J

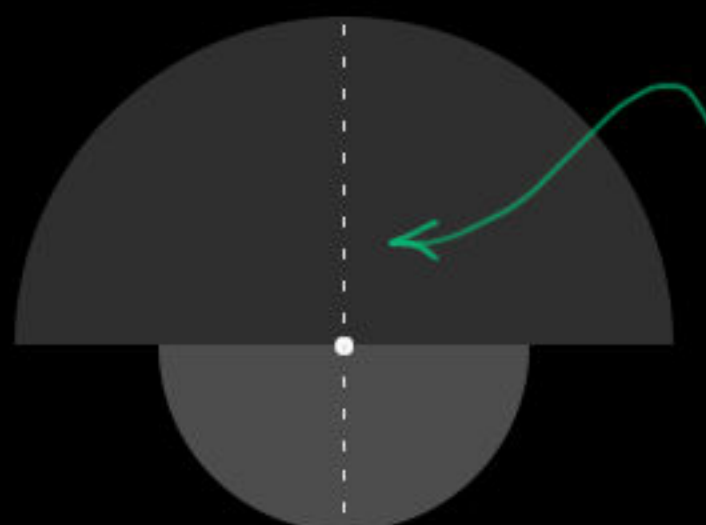
$$\Rightarrow \eta = \frac{24}{96} \times 100 = 25\%$$



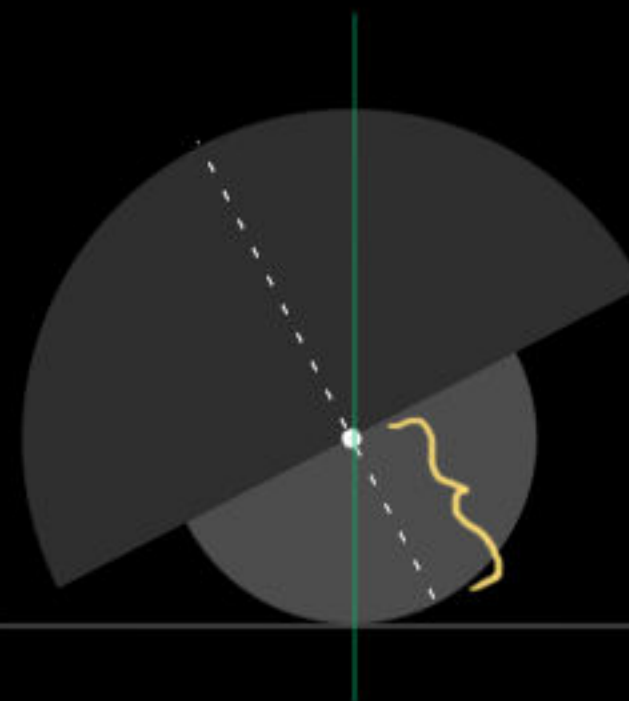
2. Two uniform hemispheres A and B are concentrically fixed to each other and the composite body thus formed is placed on a horizontal floor as shown. Masses of the hemisphere A and B are m_A and m_B and their radii are r_A and r_B respectively. Find conditions for stable, unstable and neutral equilibria of the composite body in the situation shown.

Concept: CoM of solid hemisphere lies at $3R/8$.

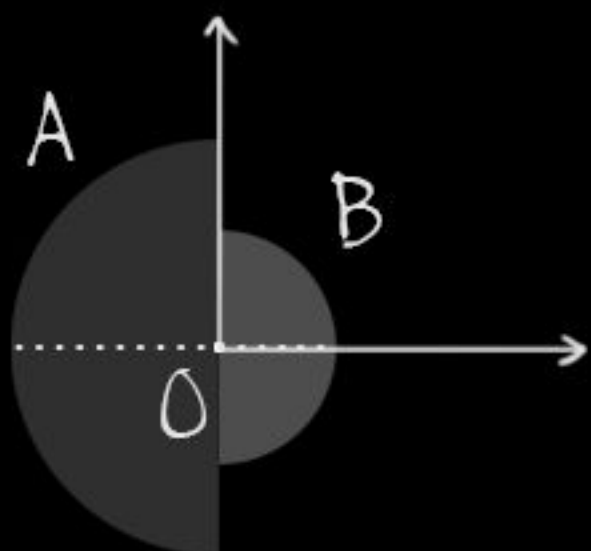
When tilted, restoring torque should be opposite the tilt for stable equilibrium.



By symmetry, CoM will lie on this line

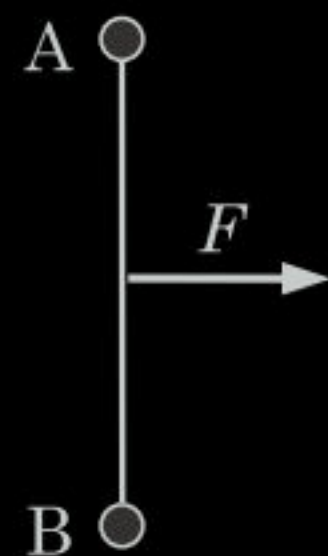


∴ For stable equilibrium CoM must lie inside lower hemisphere.

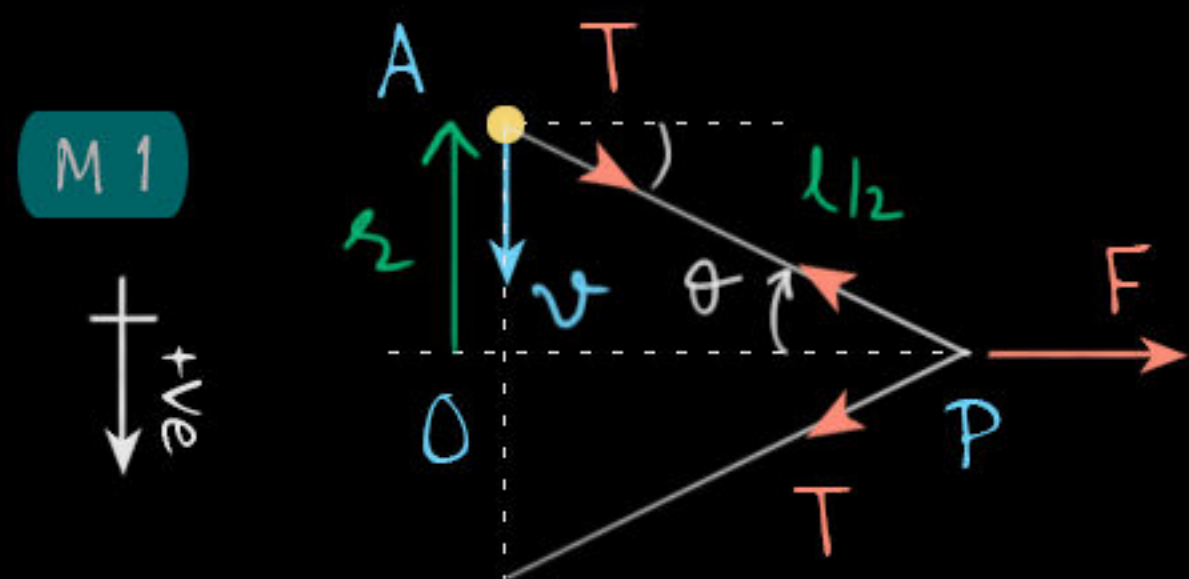


∴ For stable eq.

$$x_{cm} = \frac{m_B r_B - m_A r_A}{m_B + m_A} > 0 \Rightarrow m_B r_B > m_A r_A //$$



3. Two small identical discs each of mass $m = 1.0$ kg placed on a frictionless horizontal floor are connected by a light inextensible cord of length $l = 1.0$ m. Now the midpoint of the cord is pulled perpendicular to the line joining centres of the discs by a constant force $F = 2.0$ N. What is velocity of approach of the discs, when they are about to collide?



On P: $F = 2T \cos \theta$ ①

On disc: $T \sin \theta = m \frac{v dv}{-dr}$ ②

In triangle AOP: $\frac{l}{2} \sin \theta = r$
 $\Rightarrow \frac{l}{2} \cos \theta d\theta = dr$ ③

From 1,2,3:

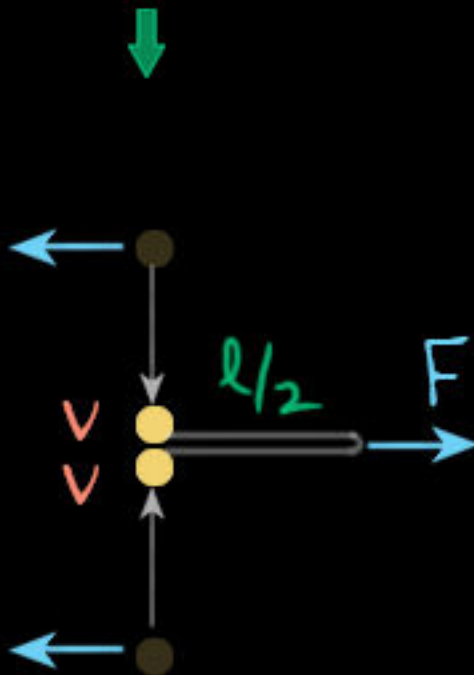
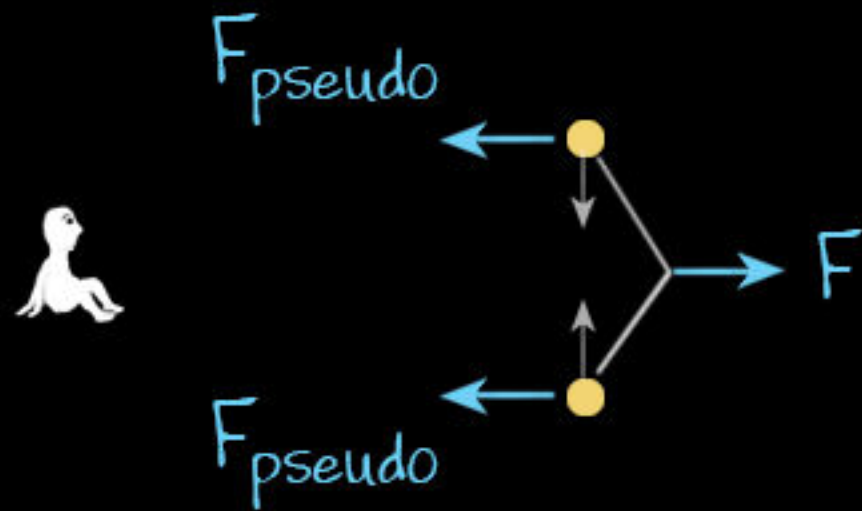
$$m v dv = \frac{F}{2 \cos \theta} \sin \theta \left(-\frac{l}{2} \cos \theta d\theta \right)$$

$$\Rightarrow \int_0^v m v dv = -\frac{Fl}{4} \int_{\pi/2}^0 \sin \theta d\theta$$

$$\Rightarrow \frac{mv^2}{2} = \frac{Fl}{4}$$

$$\Rightarrow v = \sqrt{\frac{Fl}{2m}}$$

w.r.t CoM



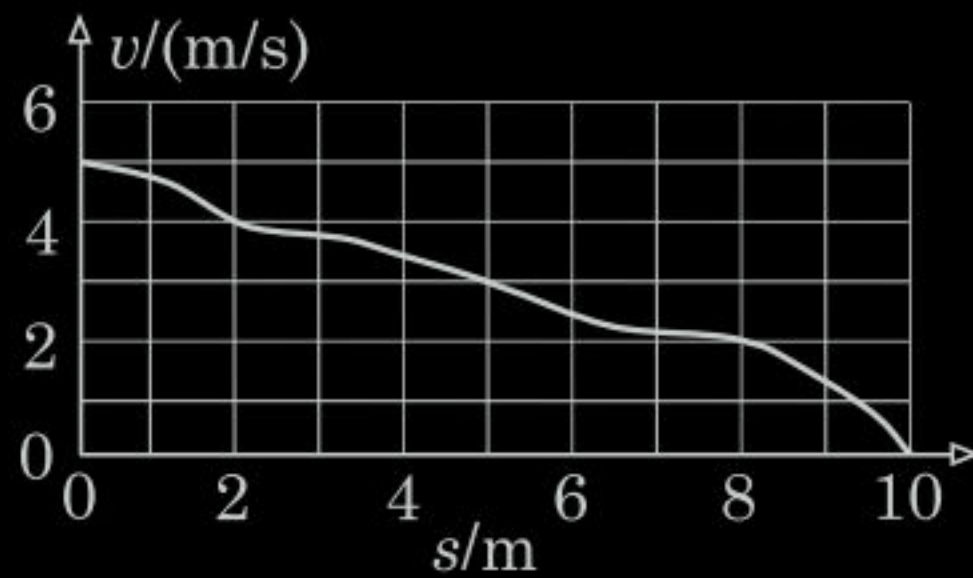
On the system: $W_F + W_{pseudo} + W_T = \Delta KE$

0 (as displacement in X direction is 0)

0 (as massless rope cant do work)

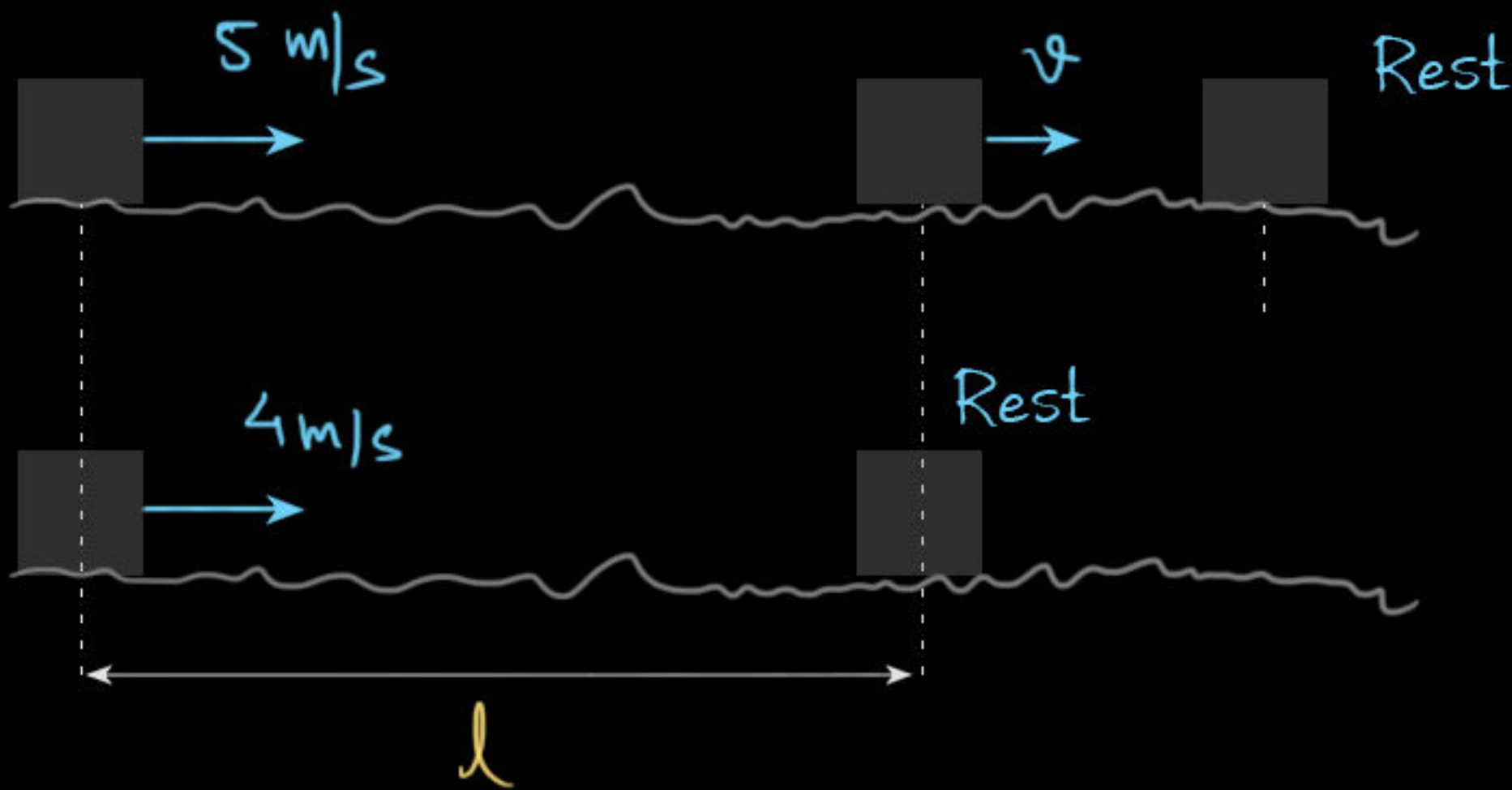
$$F \cdot \frac{l}{2} = 2 \left(\frac{1}{2} m v^2 \right)$$

$$\Rightarrow v = \sqrt{\frac{Fl}{2m}}$$



4. On a horizontal surface of non-uniform texture, friction varies from point to point. On this surface a small disc projected with speed 5 m/s covers 10 m along a straight path before it stops. Variation in speed v of the disc with distance s covered is shown in the figure. If the disc is projected with speed 4 m/s from the same initial point in the same direction, how much total distance will it cover?

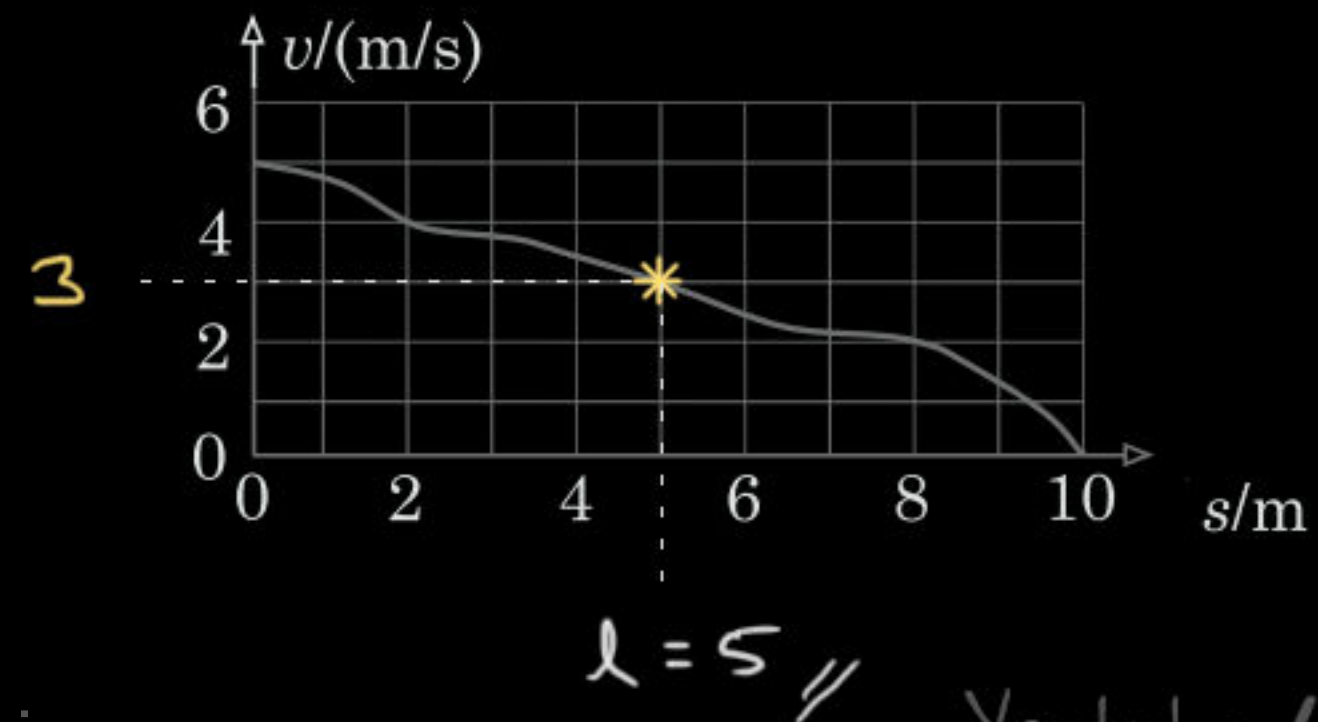
Lets say the disc's velocity in first attempt at the position where it stops in second attempt be v .



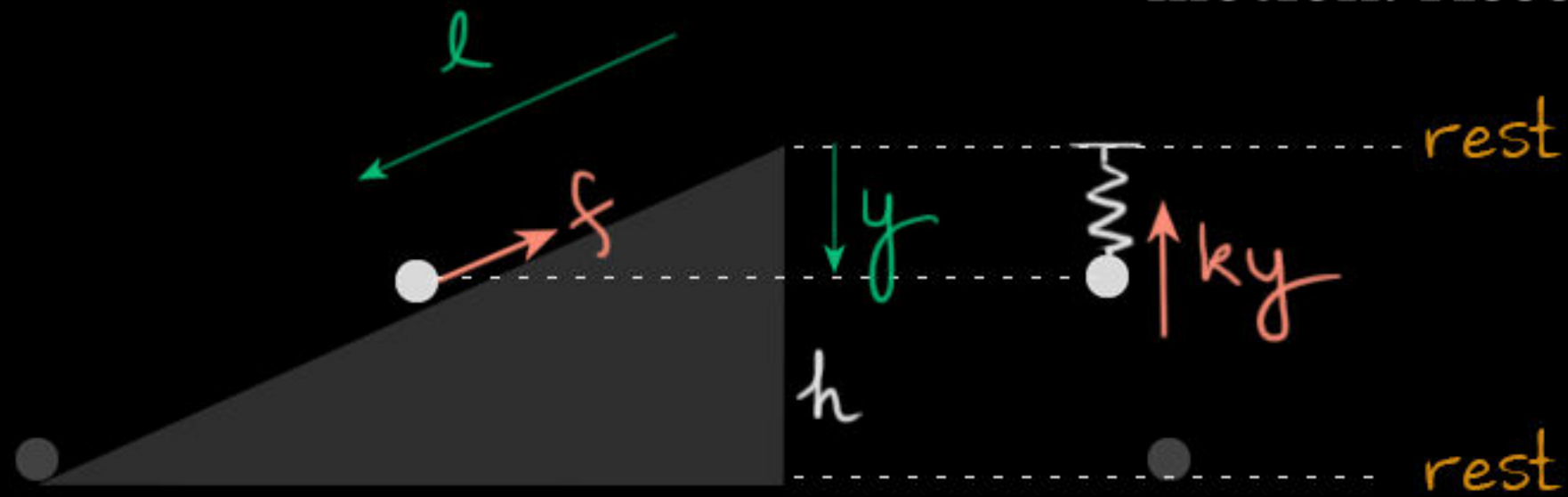
Energy loss (due to friction) is equal for the distance l in both cases.

$$\therefore \frac{1}{2}mv^2 - \frac{1}{2}mv^2 = \frac{1}{2}m4^2$$

$$\Rightarrow v = 3 \text{ m/s}$$



5. A small block when released on an inclined plane, it first slides down and then stops after sliding down a height h . This strange behavior is due to the coefficient of friction that is here proportional to the distance slid by the block. Find the maximum speed of the block during this motion. Acceleration due to gravity is g .



Given $\mu \propto l$ We know $f \propto \mu$

$$\Rightarrow f \propto l \quad \& \quad l \propto y$$

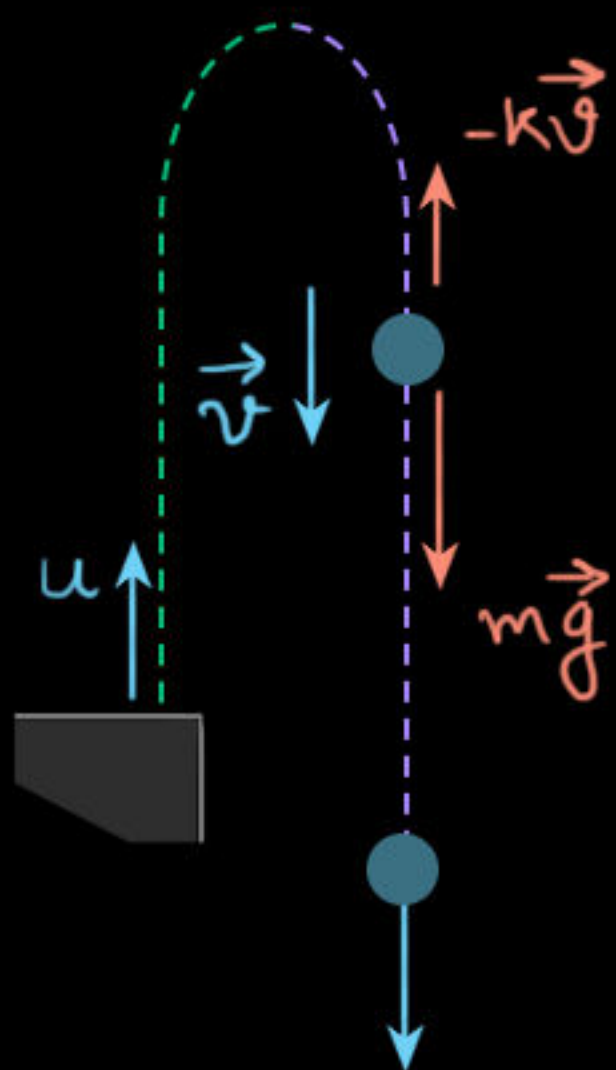
$\therefore$ Lets replace f as: $f = ky$

$\therefore$ For equivalent SHM:

At equilibrium: $mg = k \cdot h/2$
(at $h/2$)

$$\Rightarrow \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2g}{h}}$$

$$\therefore v_{\max} = \omega A = \sqrt{\frac{2g}{h}} \cdot \frac{h}{2} = \sqrt{\frac{gh}{2}}$$



6. An experimenter throws a ball of mass $m = 1.0 \text{ kg}$ vertically upwards with a velocity $u = 4.0 \text{ m/s}$ from the top of a high tower. During the flight of the ball, modulus of the force of air resistance on the ball is given by equation $F = kv$, here $k = 0.41 \text{ kg/s}$ and v is the speed of the ball. The tower is so high that the ball achieves a constant speed before striking the ground. Find velocity of the ball, when its kinetic energy changes most rapidly. Acceleration due to gravity is $g = 10 \text{ m/s}^2$.

We need v
at which this is maximum

$$\text{Net Power} = (\text{Net } \vec{F}) \cdot \vec{v} = \frac{dW_{\text{net}}}{dt} = \frac{dKE}{dt}$$

Required: At what velocity $d(KE)/dt$ is maximum.

i.e, at what velocity, Power is maximum?

$$\begin{aligned} \text{Power} &= \vec{F} \cdot \vec{v} = (m\vec{g} - k\vec{v}) \cdot \vec{v} \\ &= m(\vec{g} \cdot \vec{v}) - kv^2 \end{aligned}$$

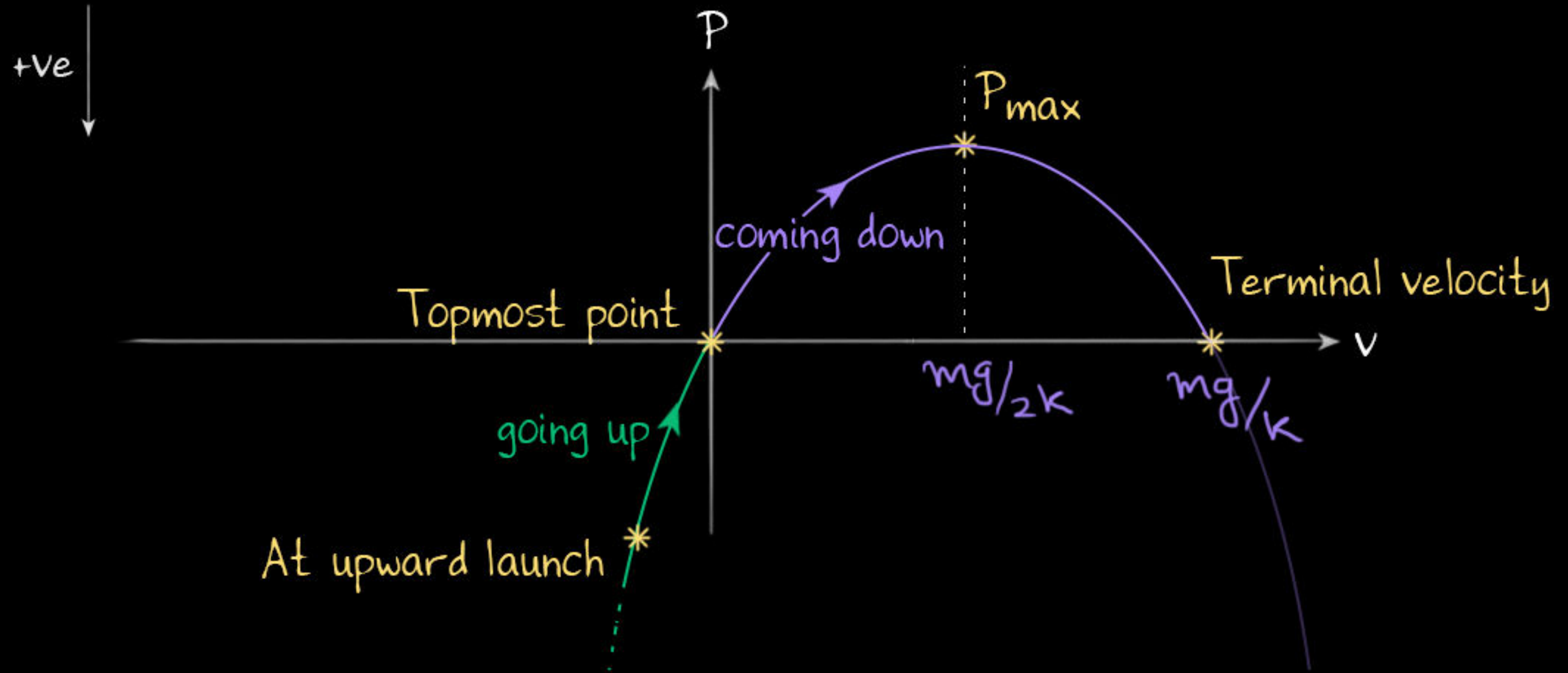
+ve
↓

Taking downward direction of velocity as +ve and upwards as -ve.

$$\text{Power} = mgv - kv^2$$

A inverted parabola

$$P = mgv - kv^2$$



∴ $|P|$ is maximum, either when coming down with $v = mg/2k$ or at upward launch.

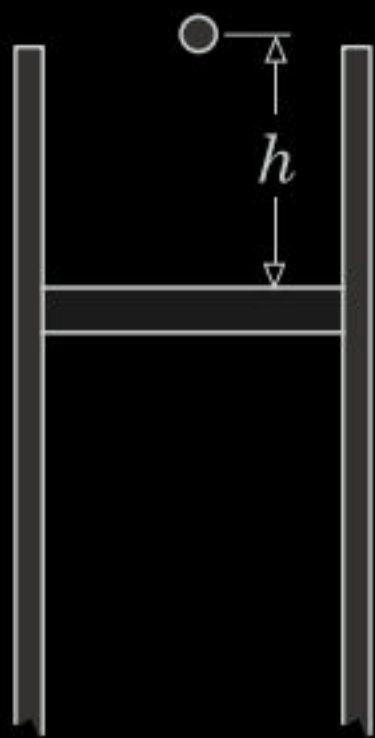
$$\text{@ } mg/2k, P = mg\left(\frac{mg}{2k}\right) - k\left(\frac{mg}{2k}\right)^2 = \frac{m^2g^2}{4k} = 61 \text{ W} \checkmark \text{ (more magnitude of } P\text{)}$$

$$\text{@ launch } v = -4 \text{ m/s}, P = mg(-4) - k(-4)^2 = -47 \text{ W}$$

∴ Rate of change of KE is maximum when, $v = mg/2k$ going down //

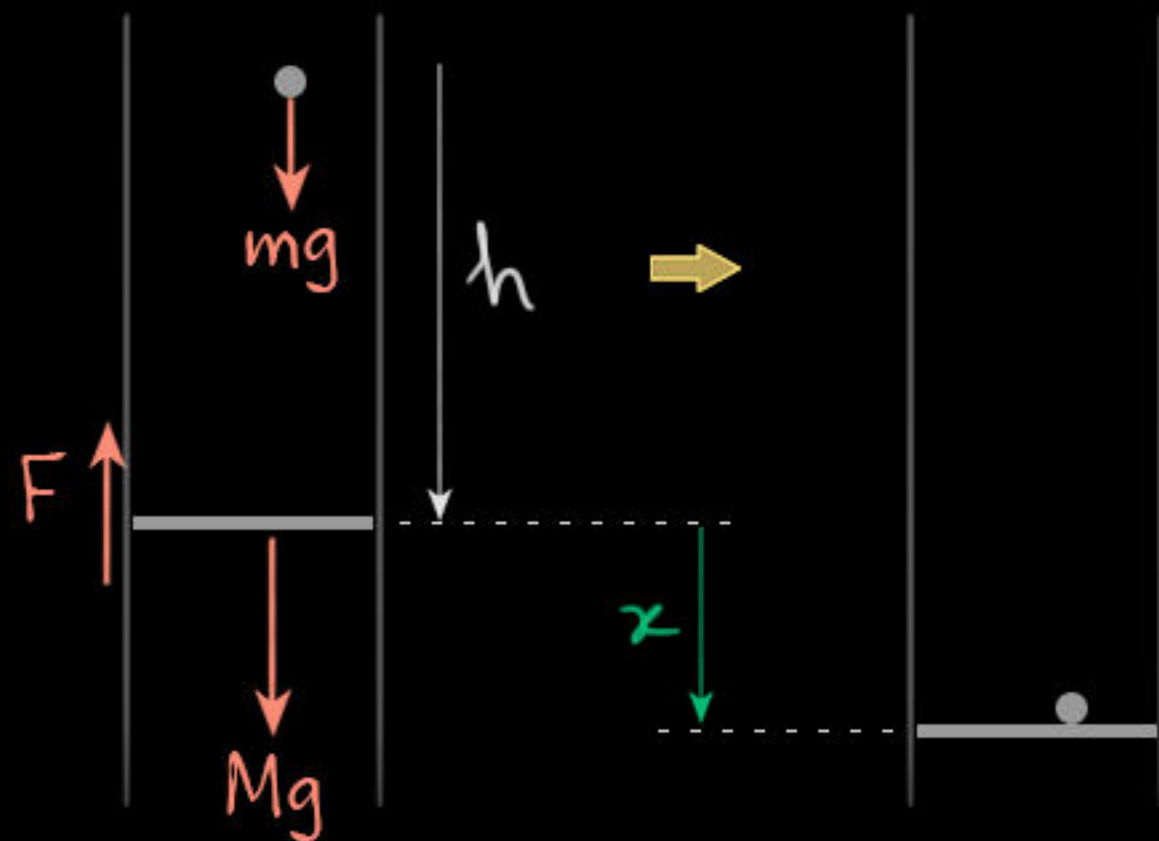
by Ankit Singhvi

Youtube / Arihant Senior



7. A piston of mass M with a ball of mass m rests inside a fixed long vertical tube due to frictional forces acting between the periphery of the piston and the inner surface of the tube. The ball is raised to height h above the piston and then released. All the collisions of the ball with the piston are perfectly elastic. Net frictional force on the piston has a constant value F that is more than the total weight of the ball and the piston. How far will the piston eventually move down in the tube? Acceleration due to gravity is g and air resistance is negligible.

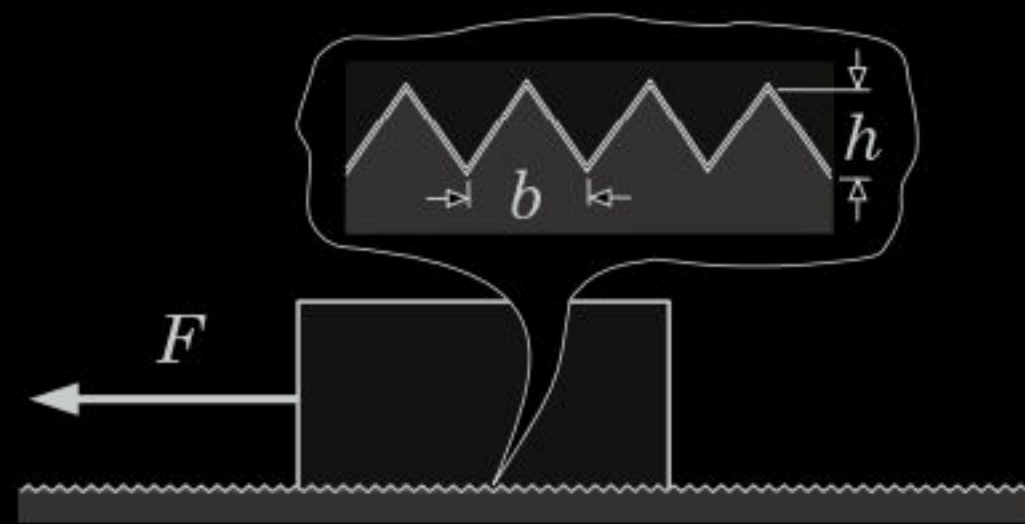
Eventually ball and piston both will be at rest.



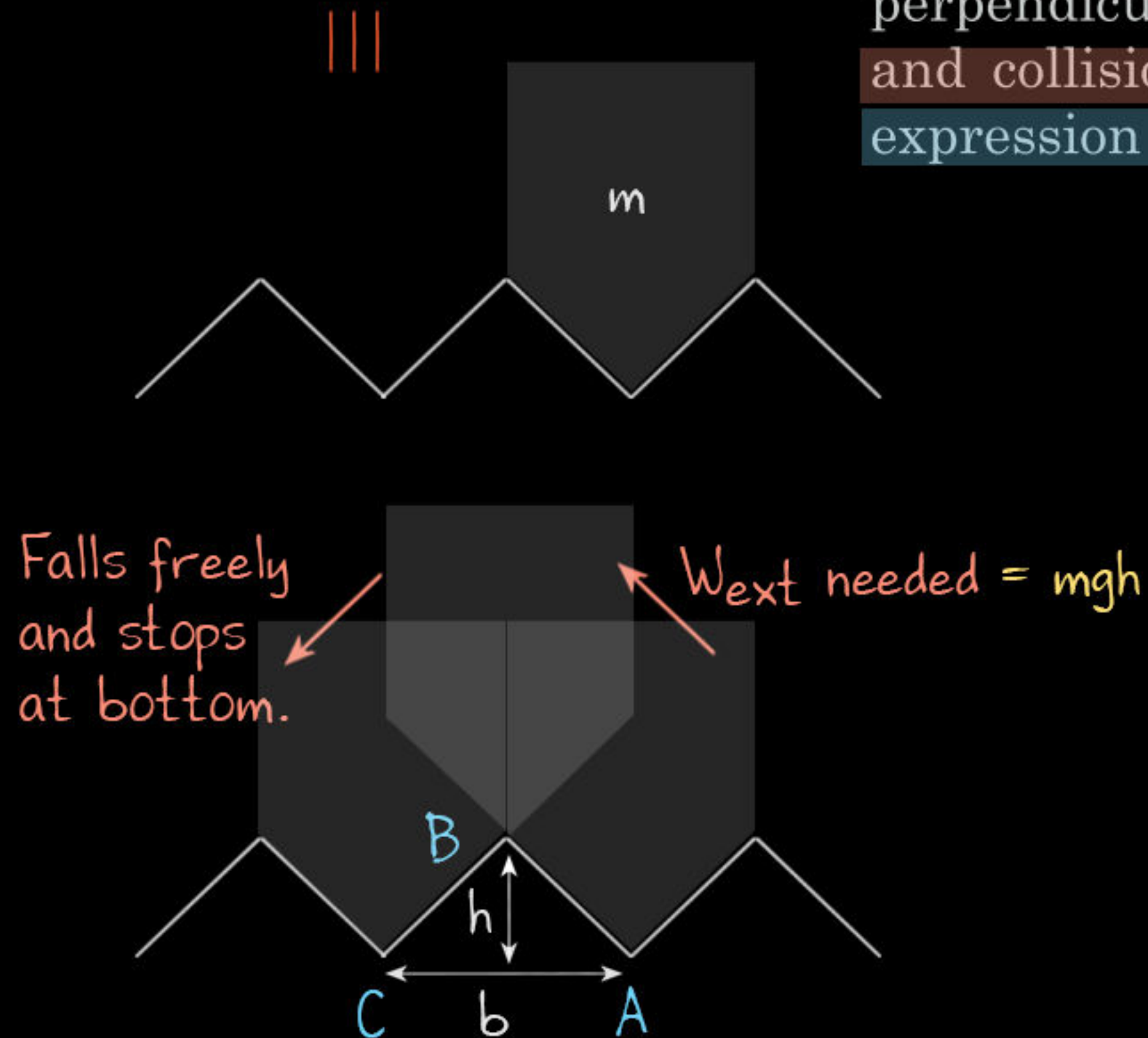
$$W_g + W_f = \cancel{\Delta KE}$$

$$\Rightarrow [mg(h+x) + Mg x] - Fx = 0$$

$$\Rightarrow x = \frac{mgh}{F - (m+M)g} //$$



8. In a very simple experiment suggested to understand contribution of surface irregularities on the force of dry friction, a portion of a floor in a laboratory and bottom of a block are knurled by cutting V-shape parallel grooves of very small dimensions (height h and width b). The block is placed on the floor so that their grooves exactly fit into each other as shown in the figure and then the block is pulled with a constant velocity perpendicular to the grooves. Assuming faces of the grooves frictionless and collisions between them completely inelastic, suggest a suitable expression for an effective coefficient of kinetic friction.



∴ As it goes from A to C:

$$W_{\text{ext}} + W_g + W_f = \Delta K$$

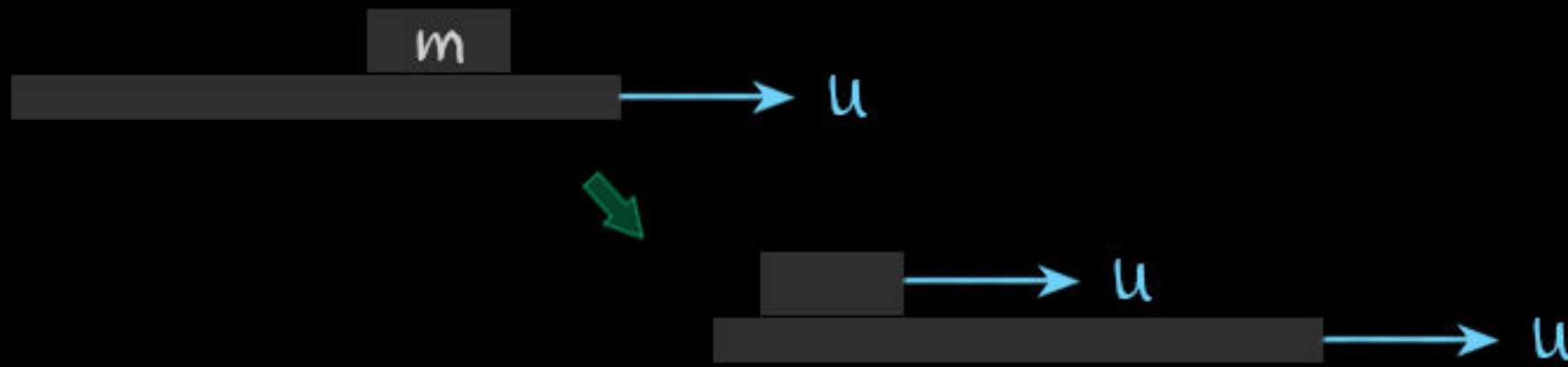
$$mgh + 0 - \mu_{\text{eff}} mgb = 0$$

$$\Rightarrow \mu_{\text{eff}} = \frac{h}{b}$$

Concept: Total W_f , i.e. frictional losses are independent of reference frame.

9. A plank is being pushed on a frictionless horizontal floor with a constant velocity $u = 1.5 \text{ m/s}$. When a block is gently placed on the plank, it slides for a while, acquires velocity of the plank and then moves together with the plank. If the agency pushing the plank has to do a work $W = 11.25 \text{ J}$ during the sliding of the block, find mass of the block.

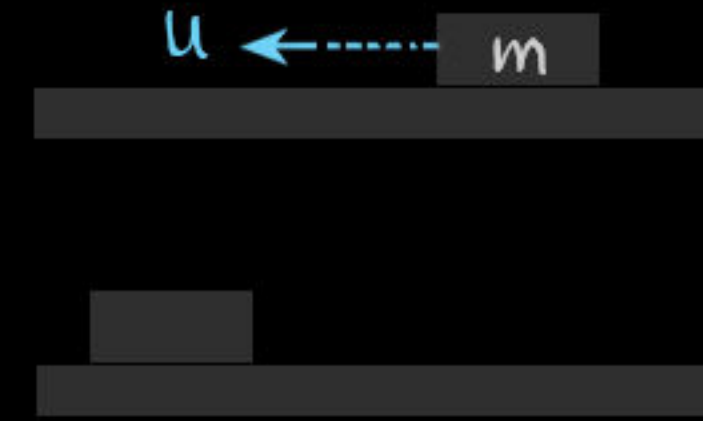
w.r.t ground



$$W_{\text{ext}} + W_f = \Delta KE$$

$$\Rightarrow W + W_f = \frac{1}{2}mu^2 \quad (1)$$

w.r.t plank



$$W_f = \Delta KE$$

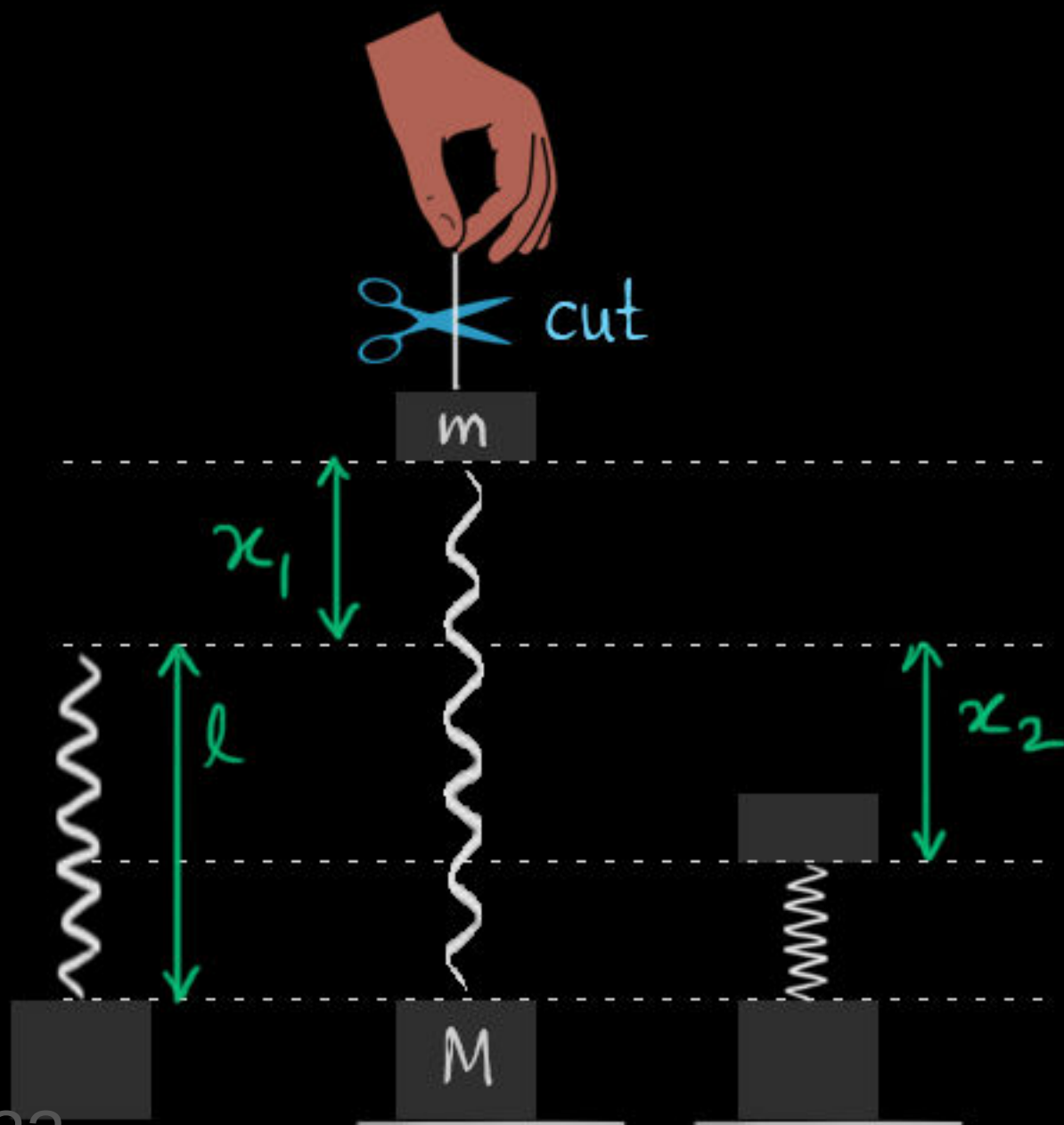
$$\Rightarrow W_f = -\frac{1}{2}mu^2 \quad (2)$$

From 1, 2: $m = \frac{2W}{u^2}$

10. A boy is holding one end of an inextensible cord supporting a block of mass m at its lower end. From this block, another block of mass M is suspended with the help of a light spring of stiffness k . The boy brings the assembly over a table and lowers it until the lower block touches the tabletop and then releases the cord. How far will the upper block move before coming to an instantaneous rest first time?

Let the natural length be l .

Initial extension be x_1 and final compression be x_2



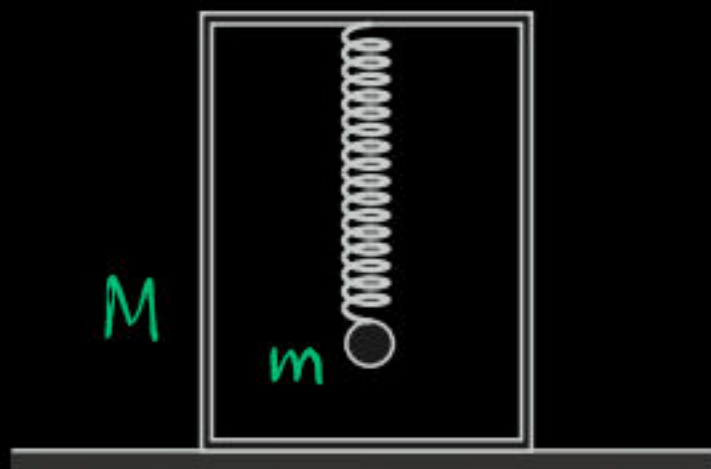
$$W_g = \Delta PE_{\text{spring}} + \cancel{\Delta KE}$$

$$\Rightarrow mg(x_1 + x_2) = \frac{1}{2}k(x_2^2 - x_1^2)$$

$$\Rightarrow mg = \frac{1}{2}k(x_2 - x_1)$$

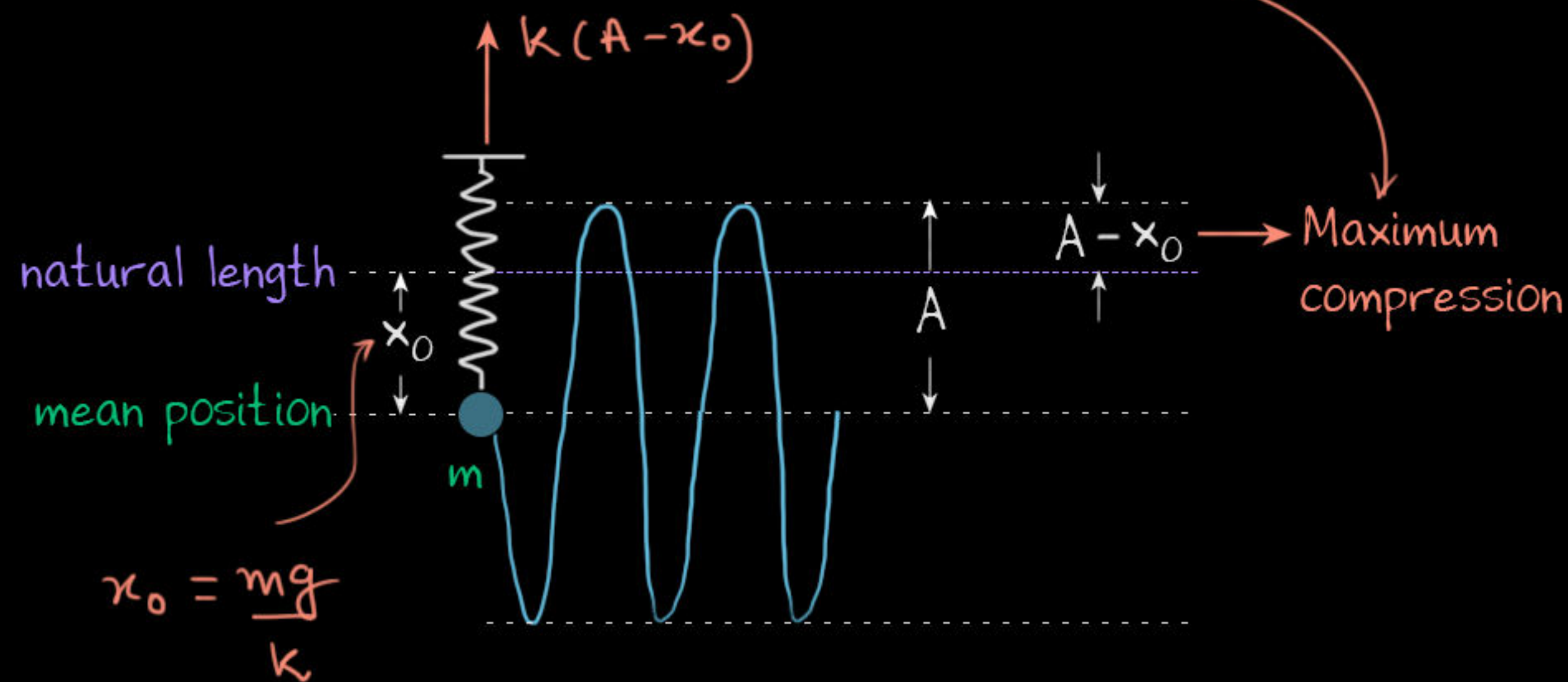
$$\Rightarrow \frac{2mg}{k} = \underbrace{x_2 + x_1}_{\text{required}} - 2x_1 \quad \leftarrow kx_1 = Mg$$

$$\Rightarrow x_1 + x_2 = \frac{2g(M+m)}{k}$$



11. A ball of mass m is suspended in a box of mass M with the help of a light spring of force constant k and the setup is placed on a horizontal floor as shown in the figure. With what maximum amplitude can the ball oscillate up and down so that the box remains standstill? Acceleration of free fall is g .

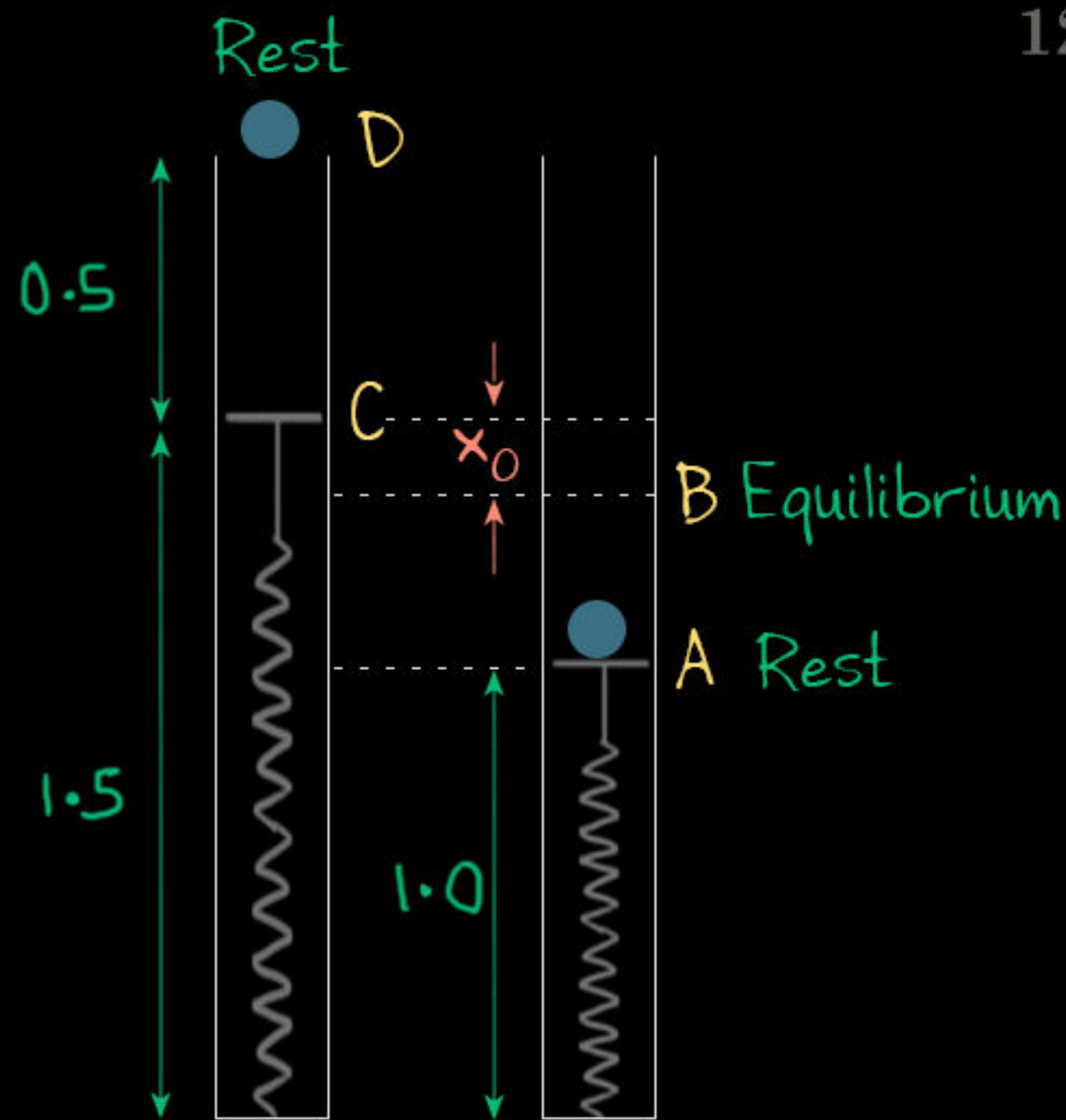
When spring is compressed fully, it must exert Mg on roof to lift the box.



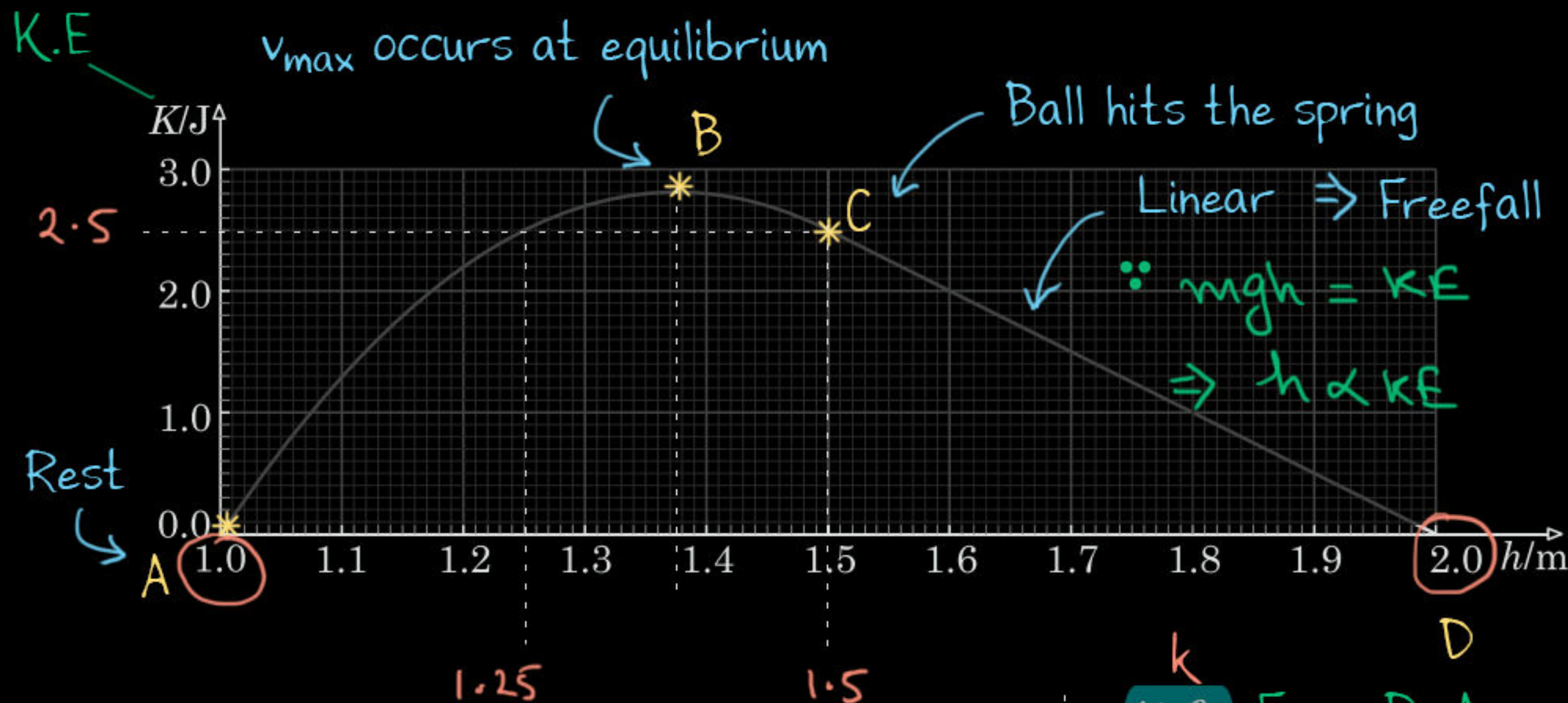
$$\therefore k(A - x_0) = Mg$$

$$\Rightarrow A = (M + m) \frac{g}{k}$$

Youtube / Arihant Senior



12. A spring is inserted in a 2.0 m long transparent pipe. Ball is dropped. Determine length of the relaxed spring, stiffness of the spring and mass of the ball.



In free fall D-C

$$mg(0.5) = KE_C = 2.5$$

$$\Rightarrow m = 0.5 \text{ kg}$$

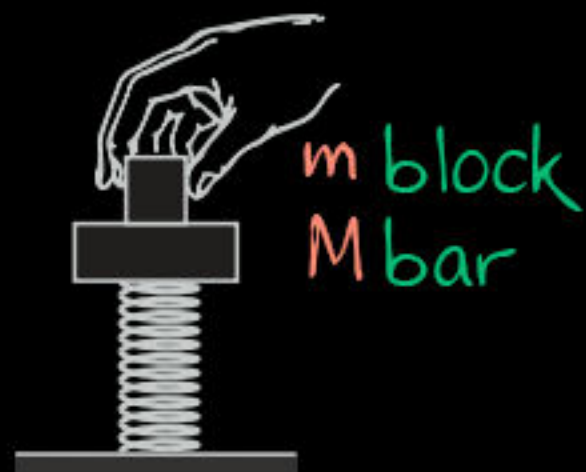
$$\text{Relaxed spring} = 1.5 \text{ m}$$

$$\begin{aligned} \text{M1 } mg &= kx_0 \\ CB &= \frac{1.5 - 1.25}{2} = \frac{0.25}{2} \\ \Rightarrow k &= \frac{mg}{x_0} = \frac{0.5(10)}{0.25/2} = 40 \text{ N/m} \end{aligned}$$

$$\begin{aligned} \text{M2 } \text{From D-A} \\ mgh &= \frac{1}{2}kx^2 \\ &= \frac{1}{2}k(1.5 - 1)^2 \\ \Rightarrow k &= 40 \text{ N/m} \end{aligned}$$

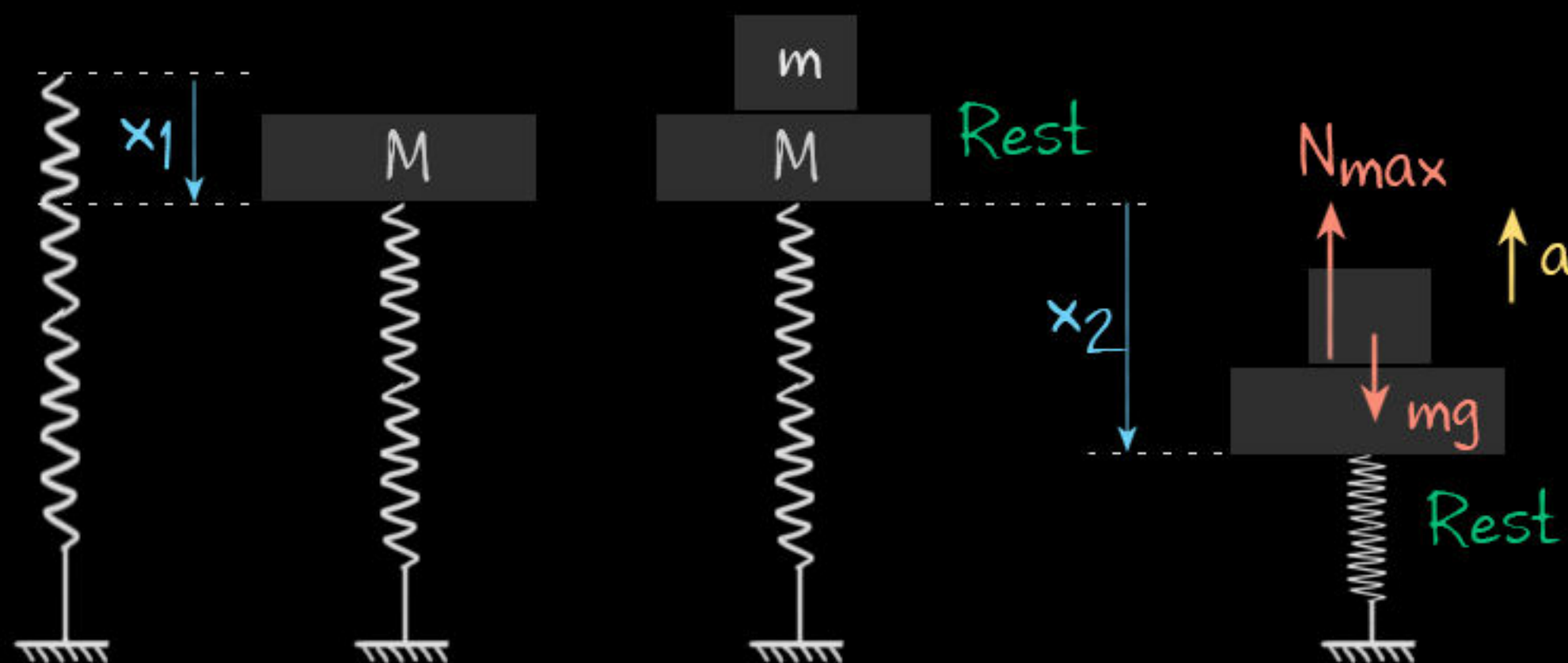
by Ankit Singhvi

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13. A bar of mass M rests in equilibrium on a vertical spring, lower end of which is affixed on the ground. Now a block of mass m is held on the bar without exerting any force on it and then the block is released. How much maximum force the bar will apply on the block during the subsequent motion? Acceleration of free fall is g .

Concept: Only upward force on m is Normal reaction. So $N_{\max}$ is when upward acceleration is maximum. This will occur when spring compression is maximum at bottom.



Natural length

$$x_1 = \frac{Mg}{k}$$

$$N_{\max} - mg = ma \quad (1)$$

$$W_g = \Delta PE_{\text{spring}} + \Delta KE$$

$$(m+M)gx_2 = \frac{1}{2}k(x_1+x_2)^2 - \frac{1}{2}kx_1^2$$

$x_1 = \frac{Mg}{k}$

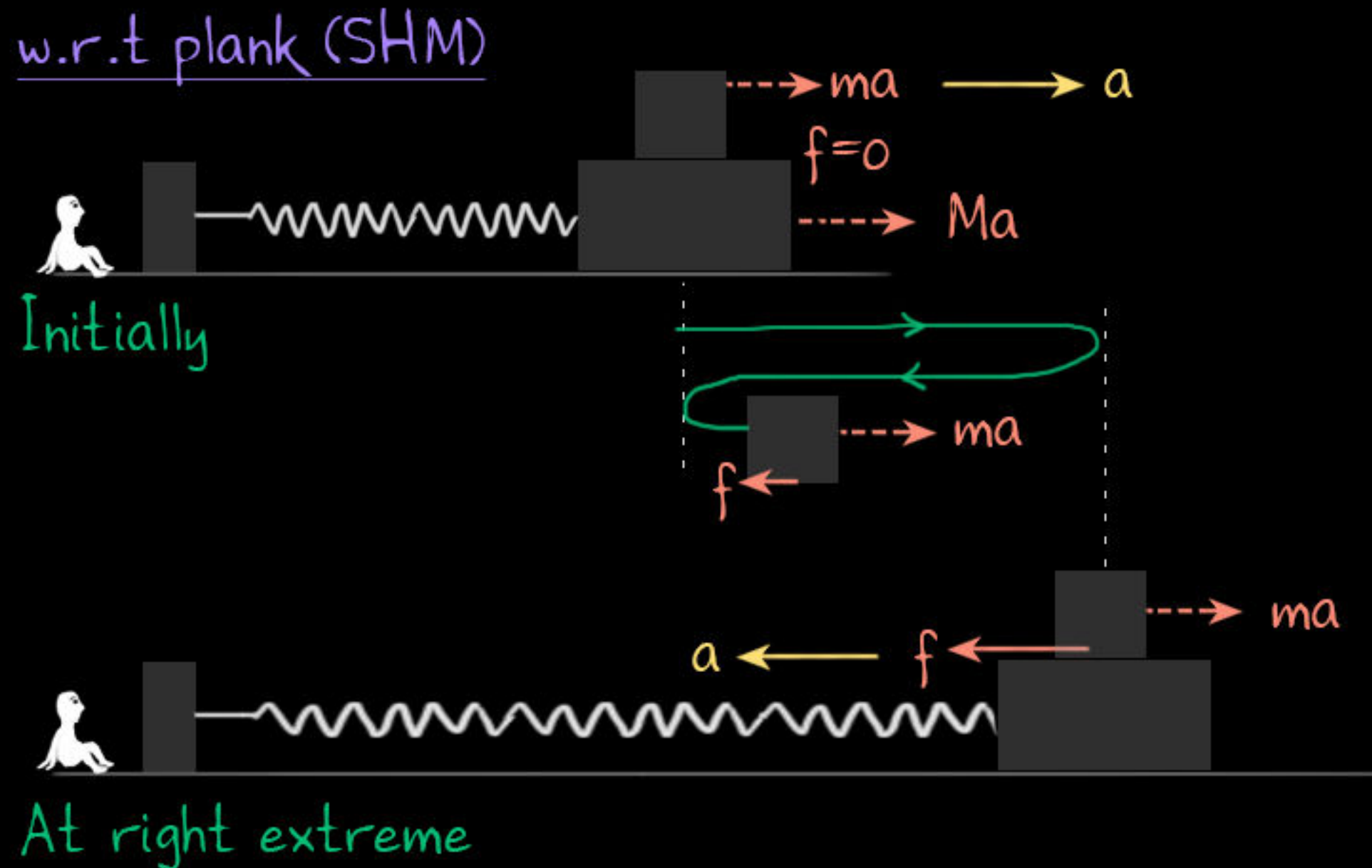
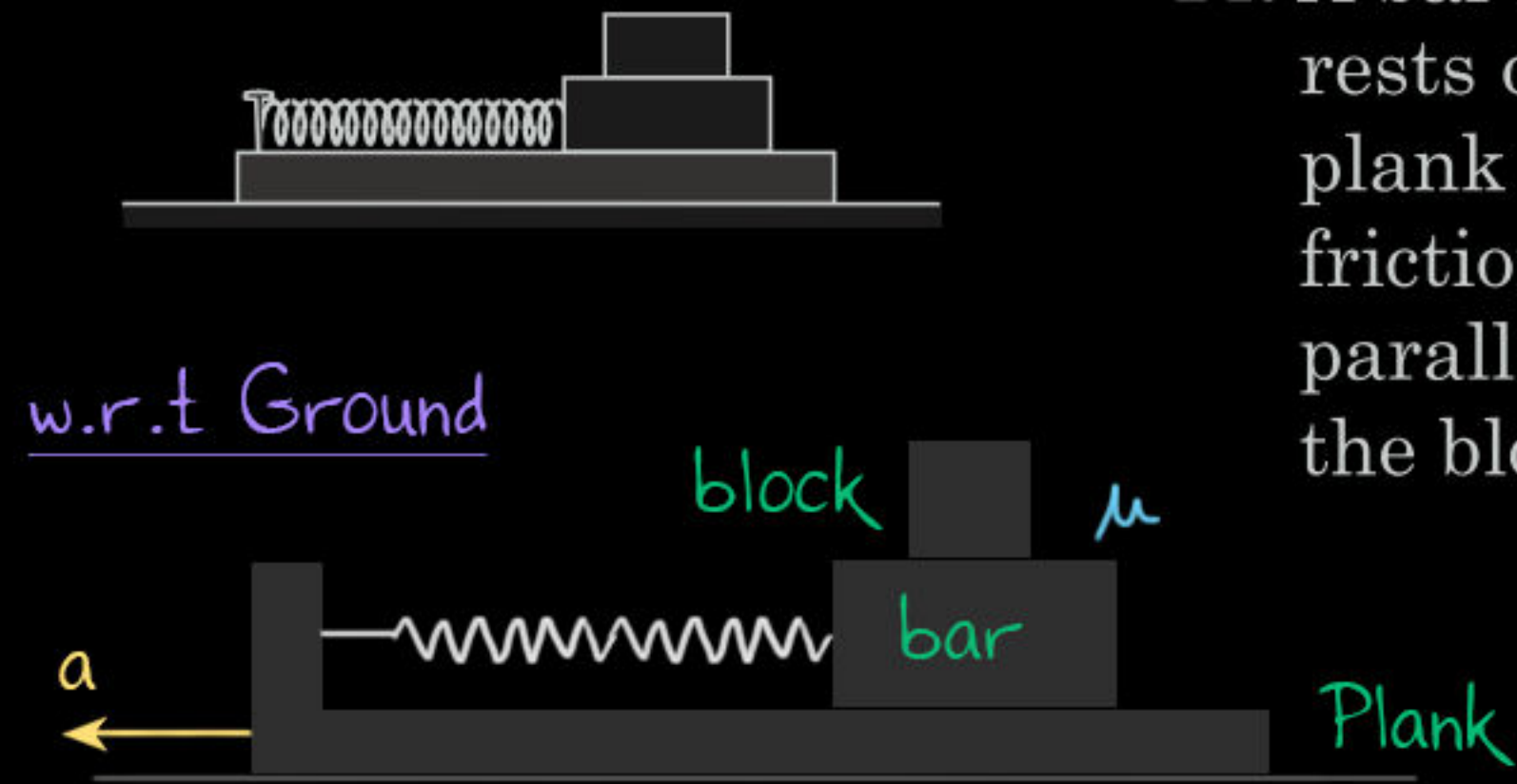
$$\Rightarrow x_2 = \frac{2mg}{k} \quad (2)$$

Force on $(m+M)$:

$$k(x_1+x_2) - (m+M)g = (m+M)a \quad (3)$$

$$\text{From 1,2,3: } N_{\max} = \frac{mg(2m+M)}{(m+M)}$$

14. A bar supporting a block on its top is placed on a frictionless plank that rests on a horizontal floor. The bar is attached to a nail driven into the plank with the help of a spring as shown in the figure. Coefficient of friction between the bar and the block is μ . What maximum acceleration parallel to the spring can the plank be given preventing sliding between the block and the bar? Acceleration due to gravity is g .



As acceleration goes from $+a$ to $-a$, friction will go from zero to extreme.

At right extreme: $f - ma = ma$

$\therefore$ For no slipping: $f = 2ma < \mu mg$

$\Rightarrow a < \frac{\mu g}{2}$
 Youtube / Arian Senior

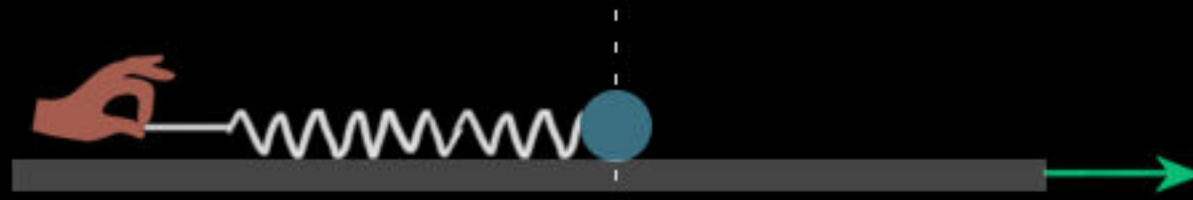
by Ankit Singhvi

15. A block of mass $m = 1.0$ kg placed on a floor is connected at one end of a long spring of stiffness $k = 100$ N/m. Coefficient of kinetic friction between the floor and the block is $\mu_k = 0.47$. The free end of the spring is pulled gradually away from the block until the block begins to slide. If the block stops after sliding a distance $s = 0.6$ cm in one stroke, find coefficient of static friction between the block and the floor.

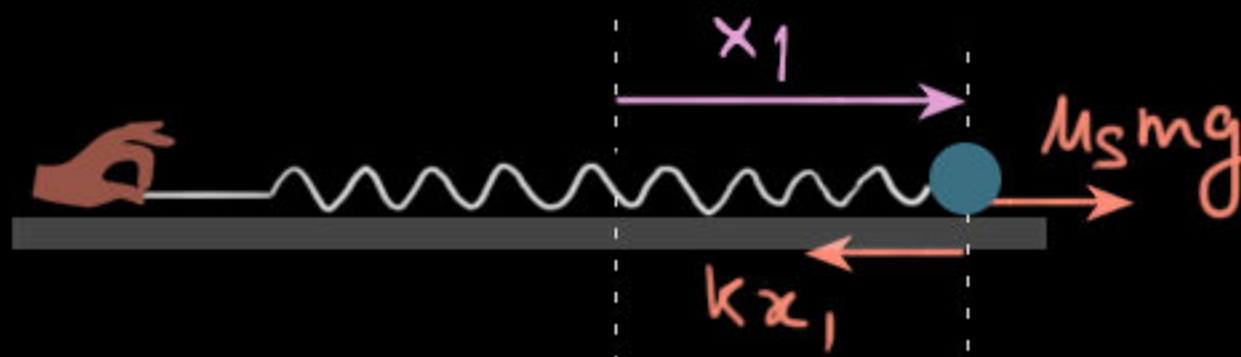
w.r.t hand: Floor moves away gradually

I am taking it $\times 2$

Natural length:



Sliding begins



At extreme x_1 : $kx_1 = \mu_s mg$ (1)

Sliding ends

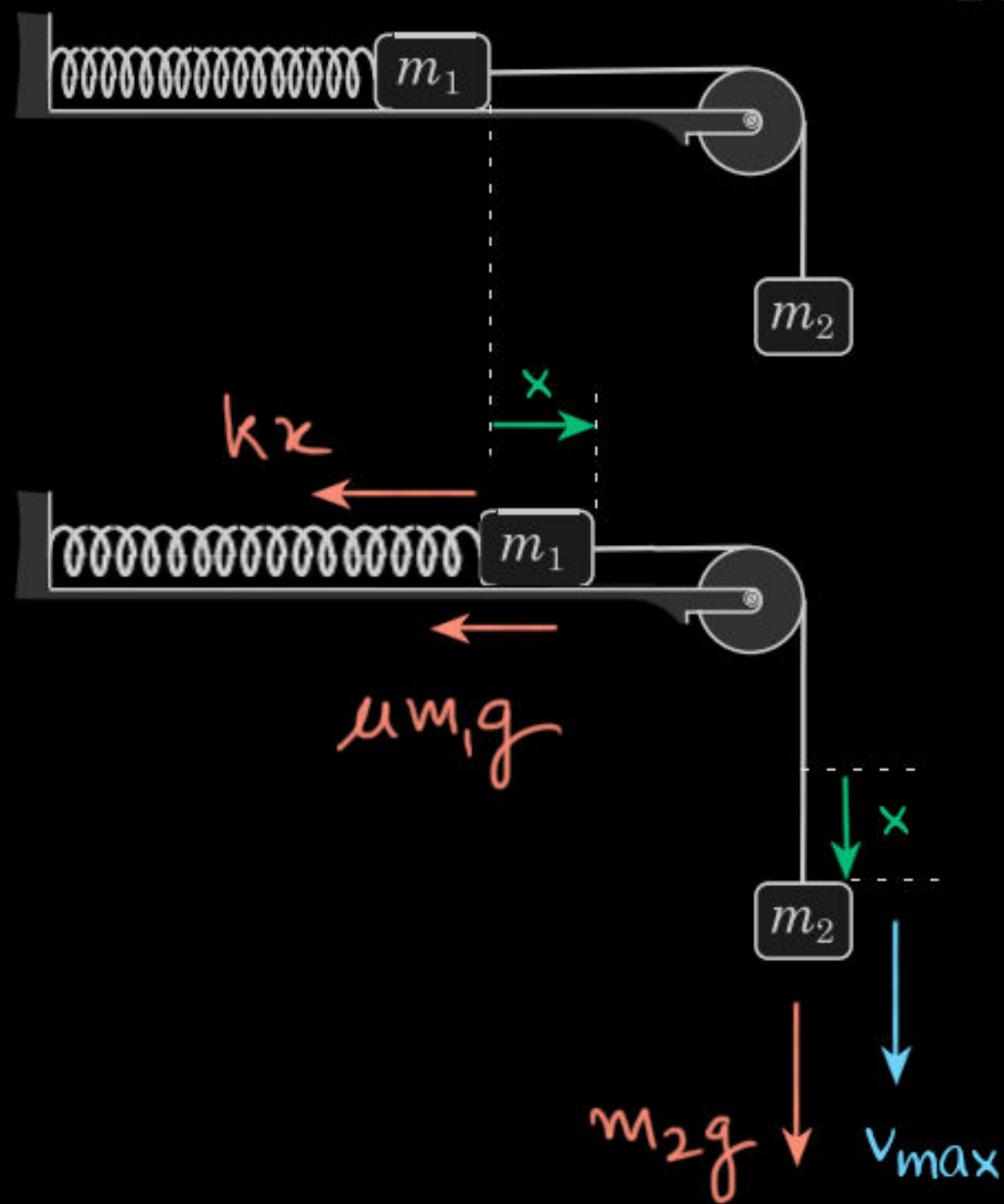


During x_2 : $W_f = \Delta PE + \Delta KE$
 $\Rightarrow -\mu_k mg x_2 = \frac{1}{2} k (x_1 - x_2)^2 - \frac{1}{2} k x_1^2$ (2)

From 1,2: $\mu_s = \mu_k + \frac{kx_2}{2mg}$

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16. In the arrangement shown, the blocks have masses m_1 and m_2 , the spring constant is k and coefficient of friction is μ . Initially the lower block is held at rest keeping the spring relaxed and the vertical and horizontal segments of the cord straight. Find maximum possible speeds of the blocks after the lower block is released.



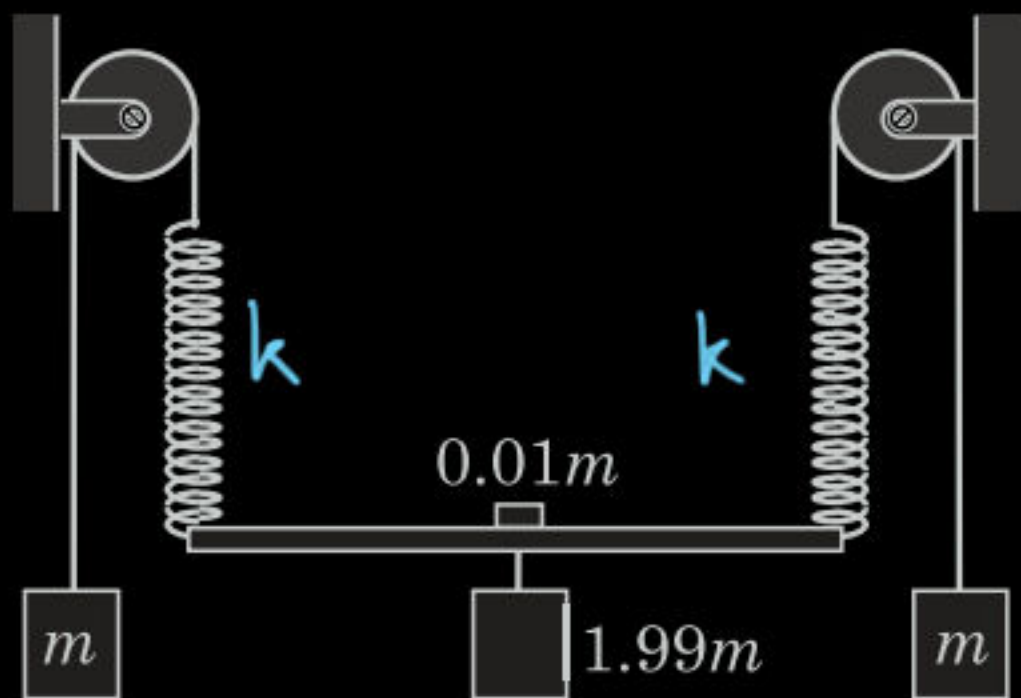
Concept: At maximum speed, acceleration will be zero. That is net force on both blocks will be zero.

$$\Rightarrow m_2 g - kx - \mu m_1 g = 0 \quad (1)$$

$$\Rightarrow \underbrace{m_2 g x - \mu m_1 g x}_{\substack{kx^2 \\ \text{(from 1)}}} = \frac{1}{2} kx^2 + \frac{1}{2} (m_1 + m_2) v_{\max}^2 \quad (2)$$

$$\text{From 1,2: } v_{\max} = \frac{(m_2 - \mu m_1) g}{\sqrt{k(m_1 + m_2)}}$$

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17. In the setup shown, an almost inertia-less bar is suspended horizontally with the help of two identical springs each of stiffness k , two light inextensible cords that pass over fixed ideal pulleys and two counterweights each of mass m . A small disc of mass $0.01m$ is placed at the midpoint of the bar. A block of mass $1.99m$ is suspended from the disc with the help of a light cord that passes through a hole in the bar. If this cord is cut, up to what maximum height will the disc jump?

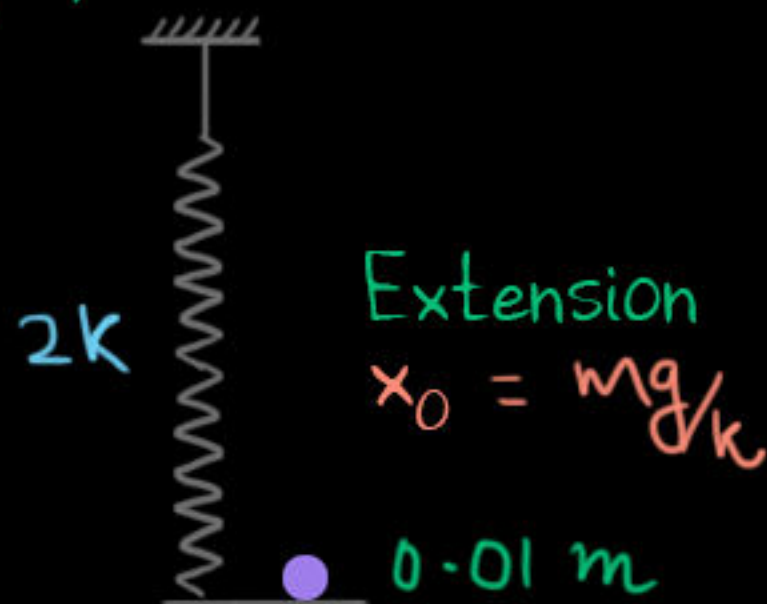
Mass is miniscule. So spring will quickly reach its natural length, launch the mass and stop.

Let the initial extension be x_0

$$2kx_0 = 2mg$$

|||

Simplified:



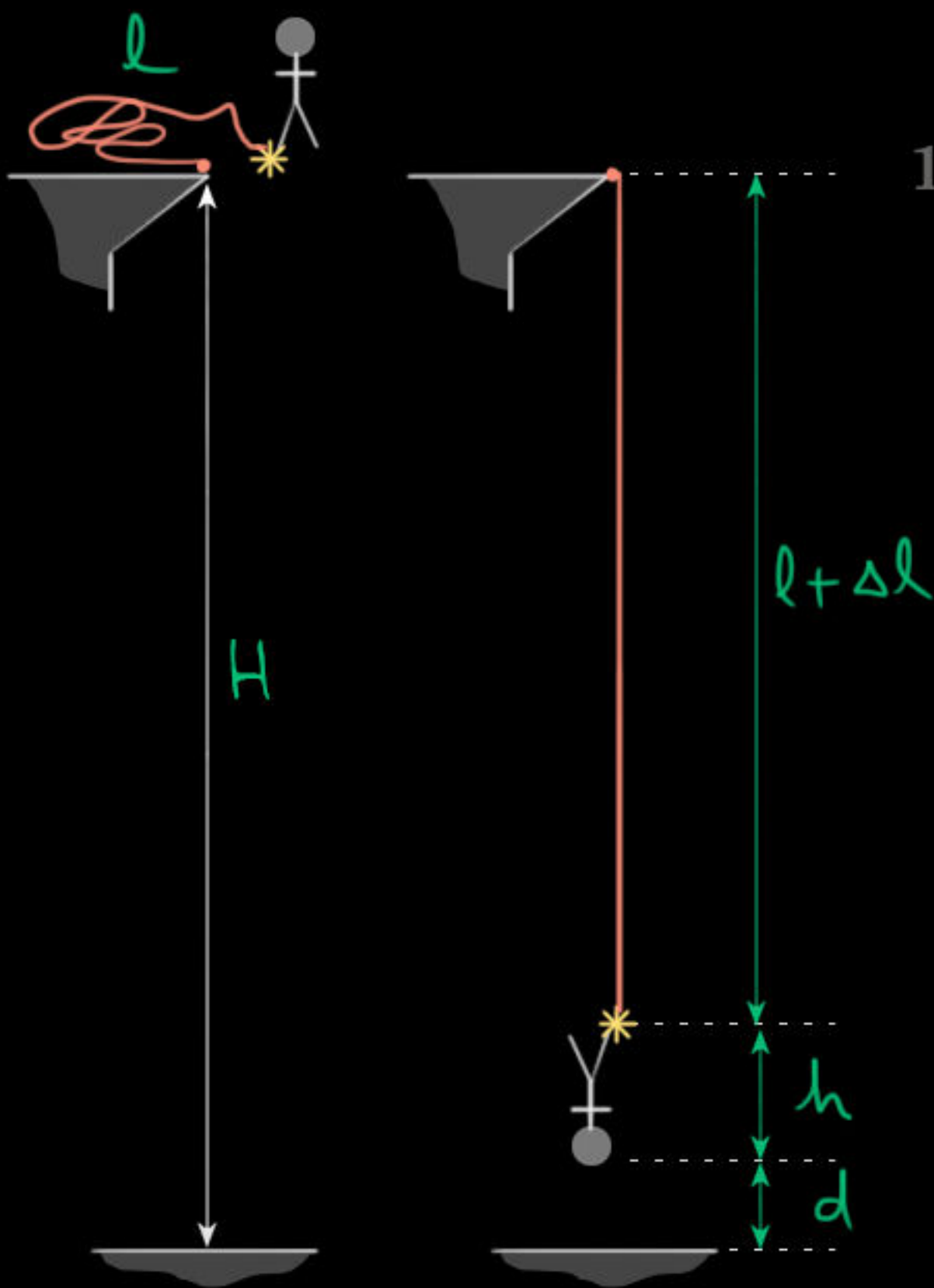
To what height will it jump?

$$W_g = \Delta PE_{spring} + \Delta KE$$

$$-(0.01)mgh = 0 - \frac{1}{2}(2k)x_0^2$$

$$\Rightarrow h = \frac{\frac{1}{2}k \left(\frac{m^2 g^2}{k^2} \right)}{(0.01)mg}$$

$$= \frac{100mg}{2k}$$



$$H = l + \Delta l + h + d \quad (1)$$

Assume the bungee jumper's CoM lies in his feet! (To get the right answer given in the book)

18. In bungee jumping, a performer ties one end of an elastic cord with his feet and the other end to a fixed support on a very high place and then jumps off. A bungee jumper of height $h = 1.5$ m and mass $m = 66$ kg has to jump from a tower of height $H = 62.5$ m. The bungee cord available is $l_0 = 30$ m long and has a force constant $k = 33$ N/m. If the minimum safe clearance from the ground is $d = 1.0$ m, what maximum length of this cord can the bungee jumper use?

Spring constant for usable shortened rope be $k' \Rightarrow k l_0 = k' l$

$$\Rightarrow k' = \frac{k l_0}{l} \quad (2)$$

$$W_g = \Delta PE_{\text{cord}} + \Delta KE$$

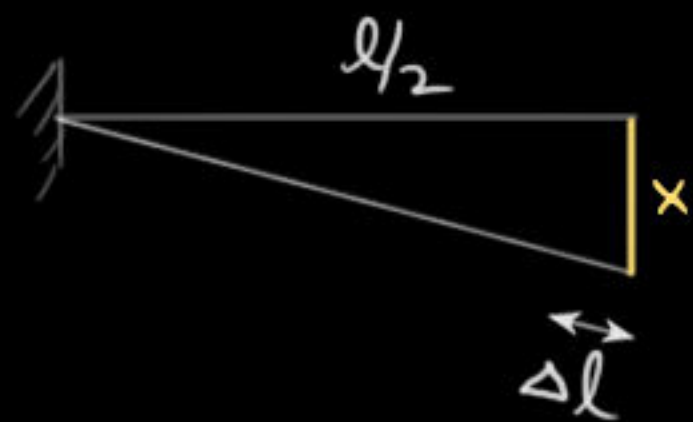
$$\Rightarrow mg(H - h - d) = \frac{1}{2} k' \Delta l^2 \quad (3)$$

From 1, 2, 3:

$$80l = (60 - l)^2$$

$$\Rightarrow l = 20 \text{ m}$$

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Approach: Find $\Delta l > k > h$

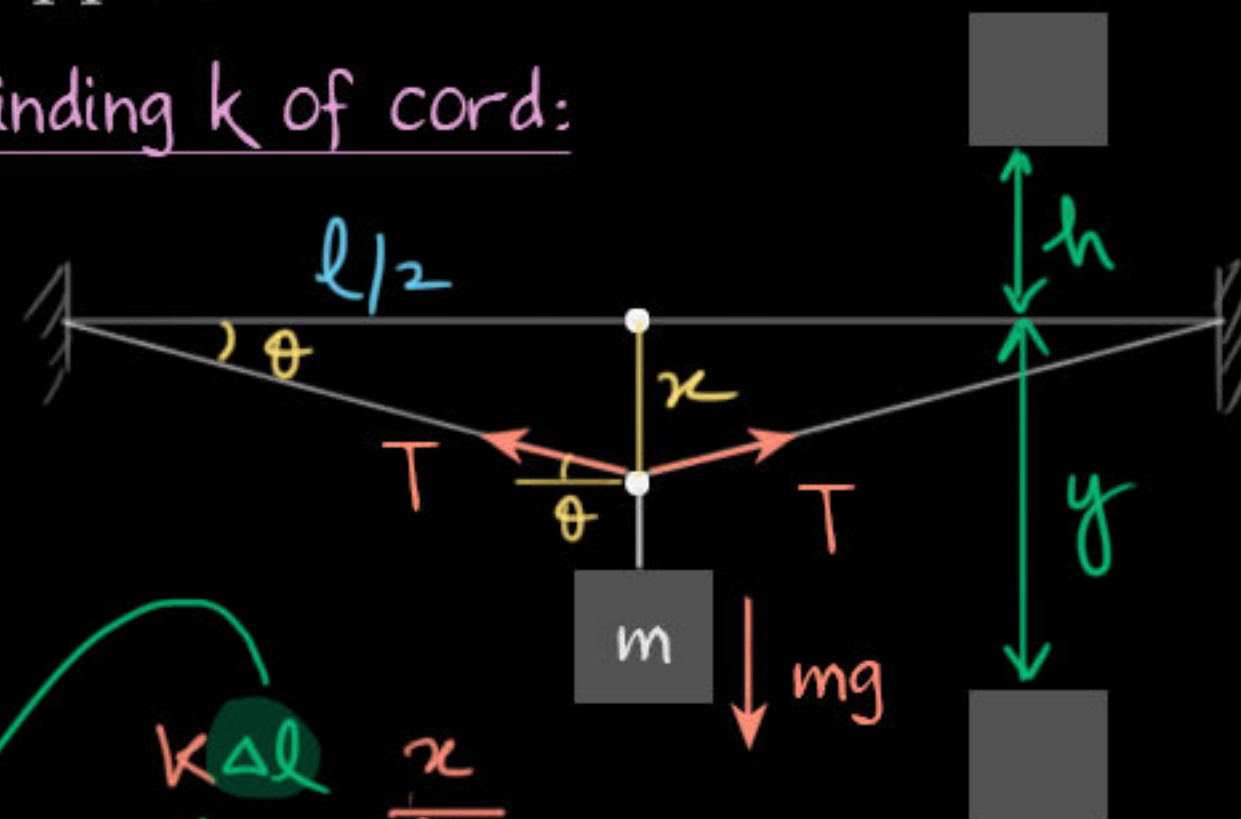
Extension in half wire:

$$\begin{aligned}\Delta l &= \sqrt{\left(\frac{l}{2}\right)^2 + x^2} - \frac{l}{2} \\ &= \frac{l}{2} \left(1 + \left(\frac{2x}{l}\right)^2\right)^{1/2} - \frac{l}{2} \\ &\approx \frac{l}{2} \left(1 + \frac{1}{2} \frac{4x^2}{l^2}\right) - \frac{l}{2} \\ &= \frac{l}{2} + \frac{x^2}{l} - \frac{l}{2} \\ &= \frac{x^2}{l}\end{aligned}$$

Assume that initially taut cord is at natural length.

19. An elastic light cord of length 10 m is taut between two nails that are in the same level. A load suspended at the midpoint of the cord, if allowed to move gradually, stops after descending a height $x = 1$ cm. If the same load is dropped from a height above the midpoint of the cord, the load on striking the midpoint sticks there and then descends a maximum height $y = 2$ cm. Assuming collision of the load with the cord to be lossless, estimate the height above the cord from where the load was dropped.

Finding k of cord:



$$2T \sin \theta = mg$$

$$\Rightarrow k = \frac{mg l^2}{4x^3}$$

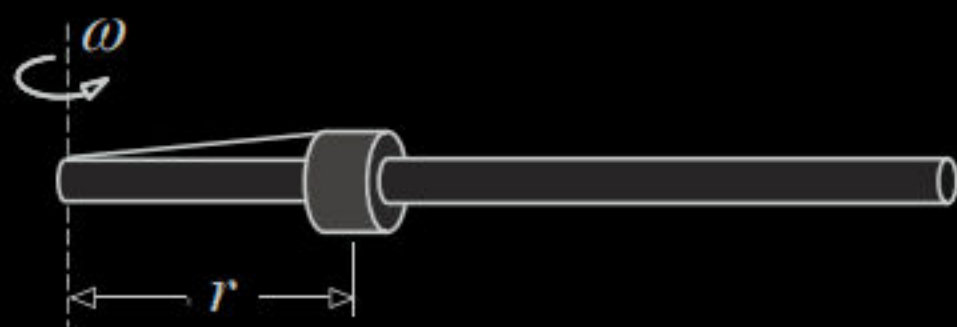
When mass is dropped:

$$W_g = 2 \left[\frac{1}{2} k (\Delta l_2)^2 \right]$$

$$\Rightarrow mg(h + y) = k \left[\frac{y^2}{l} \right]^2$$

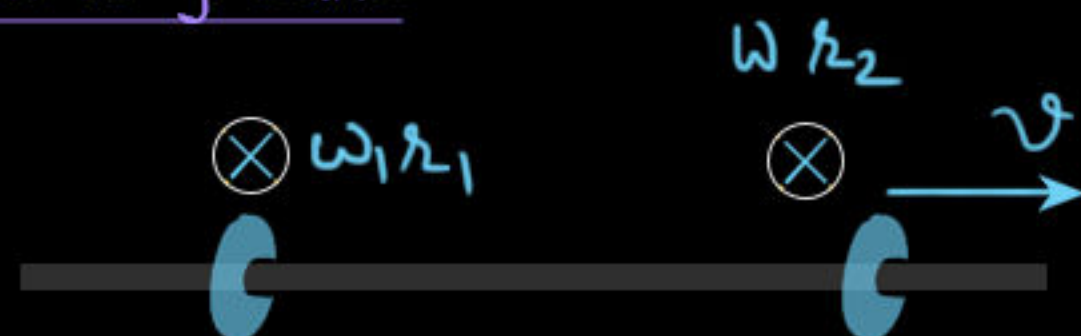
$$\frac{mg l^2}{4x^3}$$

$$\Rightarrow h = \frac{y^4}{4x^3} - y$$

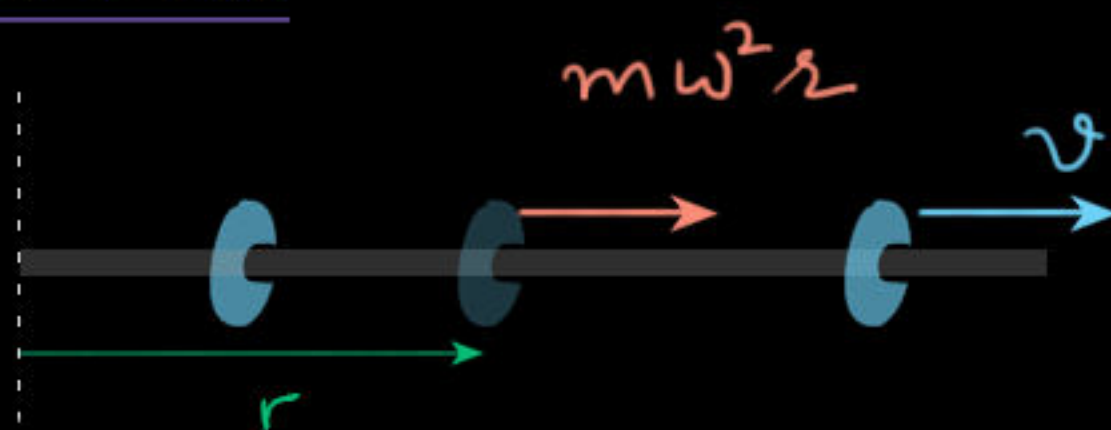


20. A horizontal frictionless thin rod wearing a sleeve of mass m is being rotated at a constant angular velocity ω about a stationary vertical axis through one of its ends. The sleeve is held stationary with respect to the rod at a distance $r = r_1$ from the axis with the help of a light cord as shown in the figure. At some instant of time, the cord is cut. Find work done by the external agency in maintaining the angular velocity of the rod constant until the sleeve slides to a distance $r = r_2$.

w.r.t ground



w.r.t rod



$$\begin{aligned}
 W_{\text{pseudo}} &= \Delta KE_{\text{along rod}} \\
 \Rightarrow m\omega^2 \int_{r_1}^{r_2} r dr &= \frac{1}{2}mv^2 \\
 \Rightarrow \cancel{m}\omega^2 \frac{(r_2^2 - r_1^2)}{2} &= \frac{1}{2}\cancel{m}v^2 \\
 \Rightarrow v^2 &= \omega^2(r_2^2 - r_1^2) \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 W_{\text{ext}} &= \Delta KE \\
 &= \frac{1}{2}m(v^2 + \omega^2 r_2^2) - \frac{1}{2}m(\omega^2 r_1^2) \\
 &\quad \downarrow \text{From 1} \\
 W_{\text{ext}} &= m\omega^2(r_2^2 - r_1^2) //
 \end{aligned}$$

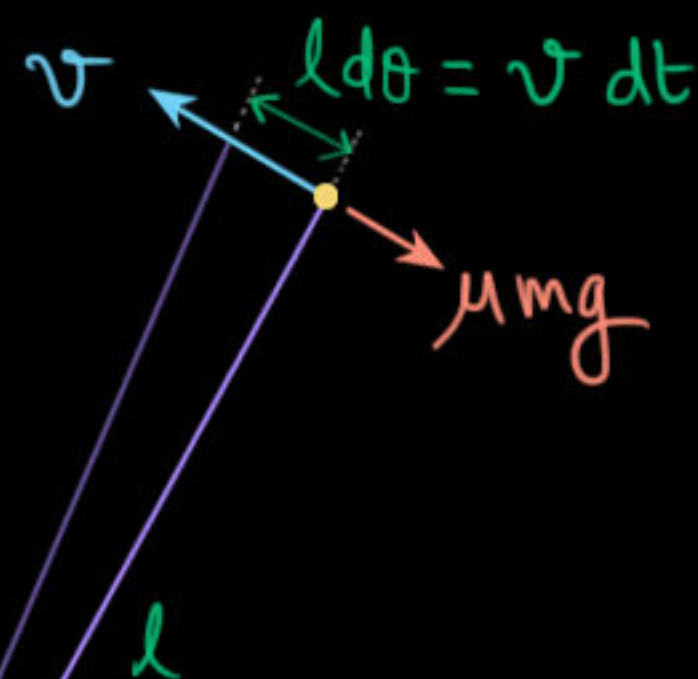
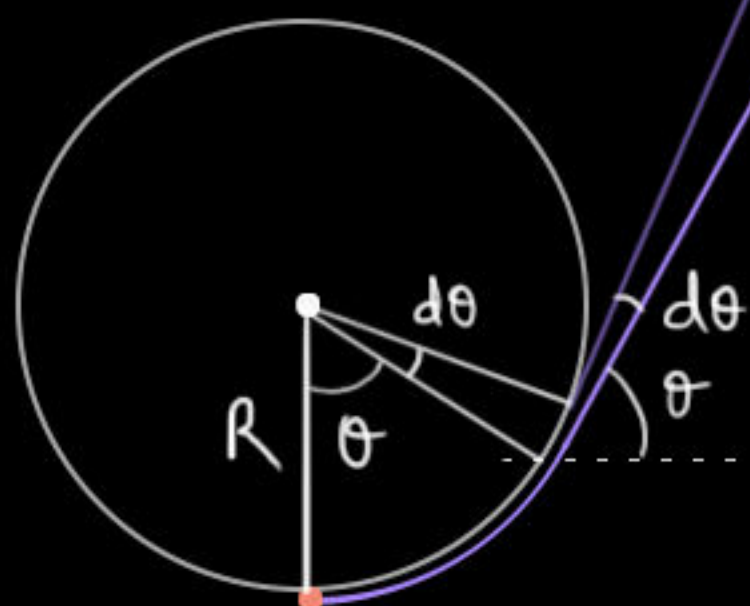


21. A disc placed on a large horizontal floor is connected from a vertical cylinder of radius r fixed on the floor with the help of a light inextensible cord of length l_0 as shown in the figure. Coefficient of friction between the disc and the floor is μ . The disc is given a velocity u parallel to the floor and perpendicular to the cord. How long will the disc slide on the floor before it hits the cylinder?

Time if thread is very long, so it stops before hitting the cylinder

$$\therefore u = \mu g t_1$$

$$\Rightarrow t_1 = \frac{u}{\mu g}$$



Time if thread is small, and it hits the cylinder

$$l d\theta = v dt$$

$$R d\theta = -dl$$

$$(u - \mu g t)$$

$$\text{Divide: } -l dl = R (u - \mu g t) dt$$

$$\Rightarrow \int_{l_0}^0 l dl = R \int_0^t (u - \mu g t) dt$$

($D > 0$, real root, and $t_2 < t_1$, $\therefore$ hit cylinder)

$$\text{Solving: } t_2 = \frac{u}{\mu g} \left(1 \pm \sqrt{1 - \frac{l_0^2 \mu g}{u^2 R}} \right) \text{ if } u \geq l_0 \sqrt{\frac{\mu g}{R}}$$

otherwise, doesn't hit

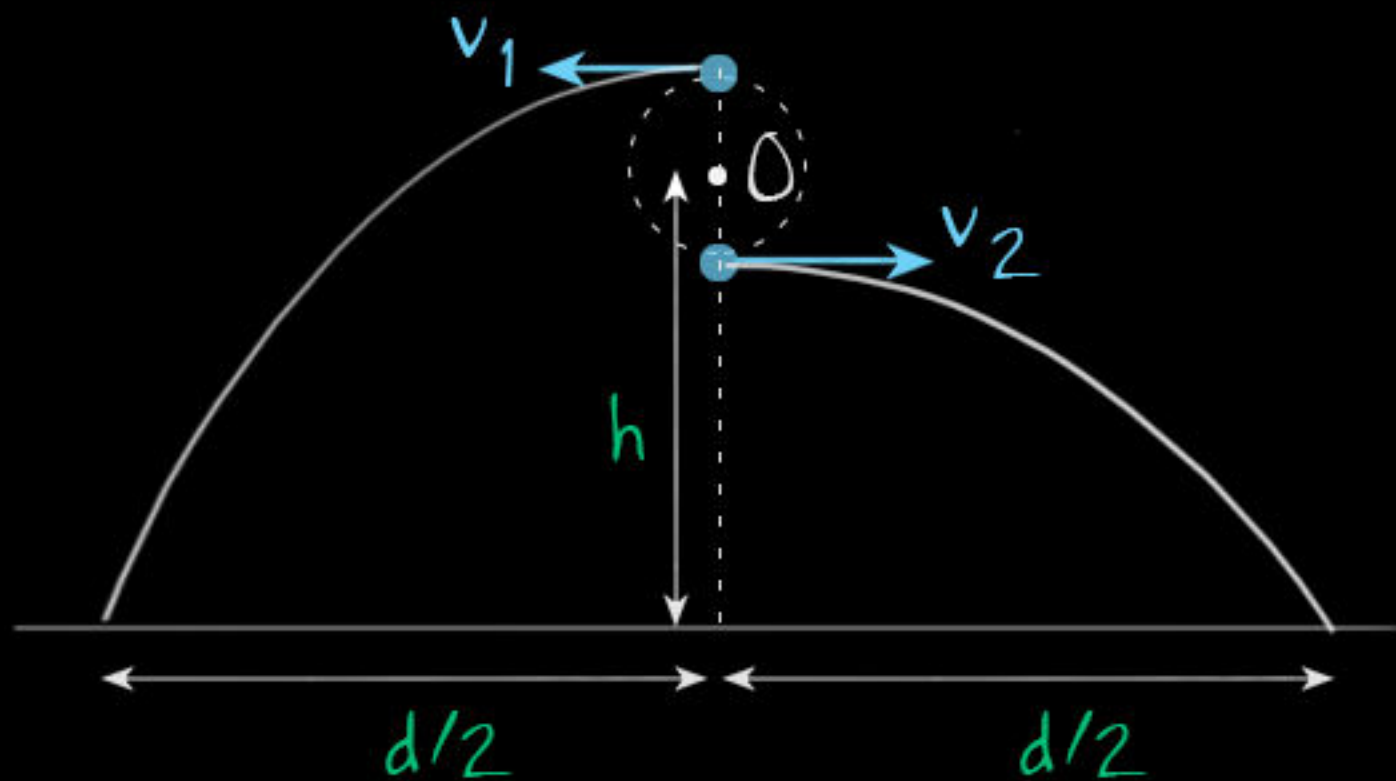
Rejecting +ve root, as t can NOT exceed t_1

$$t = t_1 \text{ if } u < l_0 \sqrt{\frac{\mu g}{R}}$$

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22. A marble is being whirled on a circular path in a vertical plane with the help of a light inextensible cord of length $l = 50$ cm around a fixed center O. If the cord is cut when the ball is at the bottom of the circular path, the ball hits the horizontal ground at point A and if the cord is cut when the ball is at top of the circular path, the ball hits the ground at point B a distance $d = 4.8$ m away from the point A. If the points A and B are equidistant from the centre O, find height of the centre O above the ground. Ignore effects of air resistance.

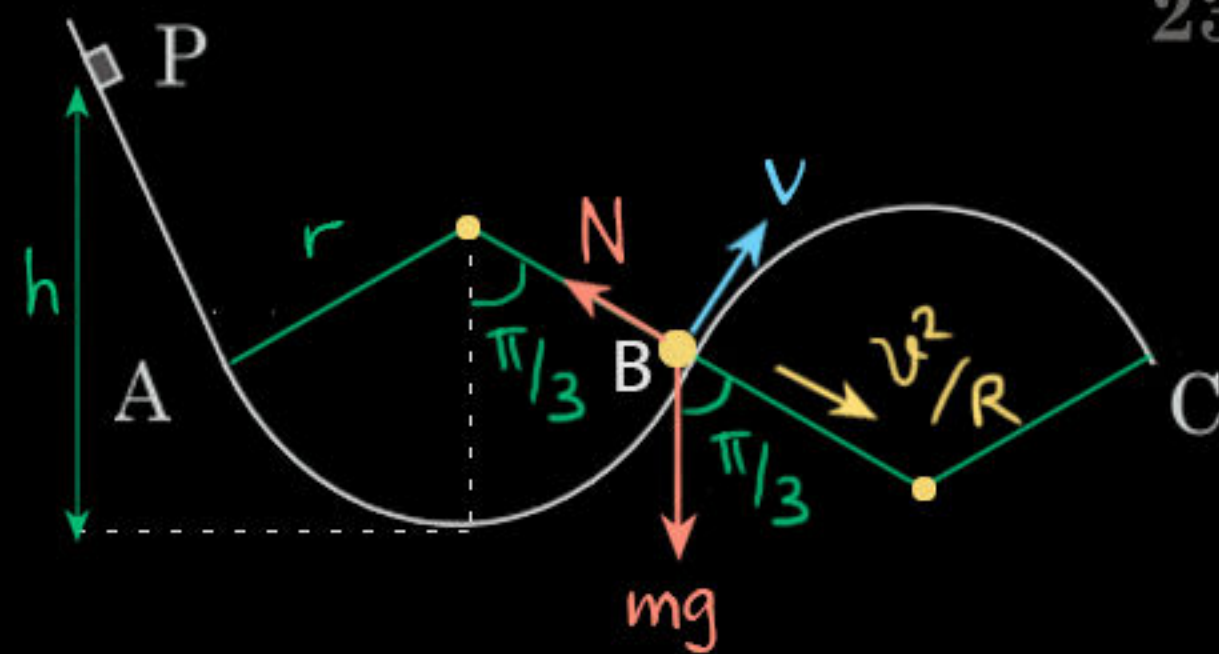


$$\text{Range} = \frac{d}{2} = v_1 \sqrt{\frac{2(h+R)}{g}} = v_2 \sqrt{\frac{2(h-R)}{g}}$$

$$\text{Energy: } mg2R = \frac{1}{2}m(v_2^2 - v_1^2)$$

Eliminating v_1 and v_2

$$h = \sqrt{\frac{d^2}{16} + R^2} //$$



23. A point mass can slide in a vertical plane on a **frictionless** curvilinear track as shown in the figure. Sections AB and BC of the track are **circular arcs each of angular span 120°** and radius **r** . **From what minimum height** above the bottom of section AB should a small block P be released **so that it certainly loses contact** with the track?

Concept: Up to B it will stick, as centripetal acceleration exists and its due to normal reaction.

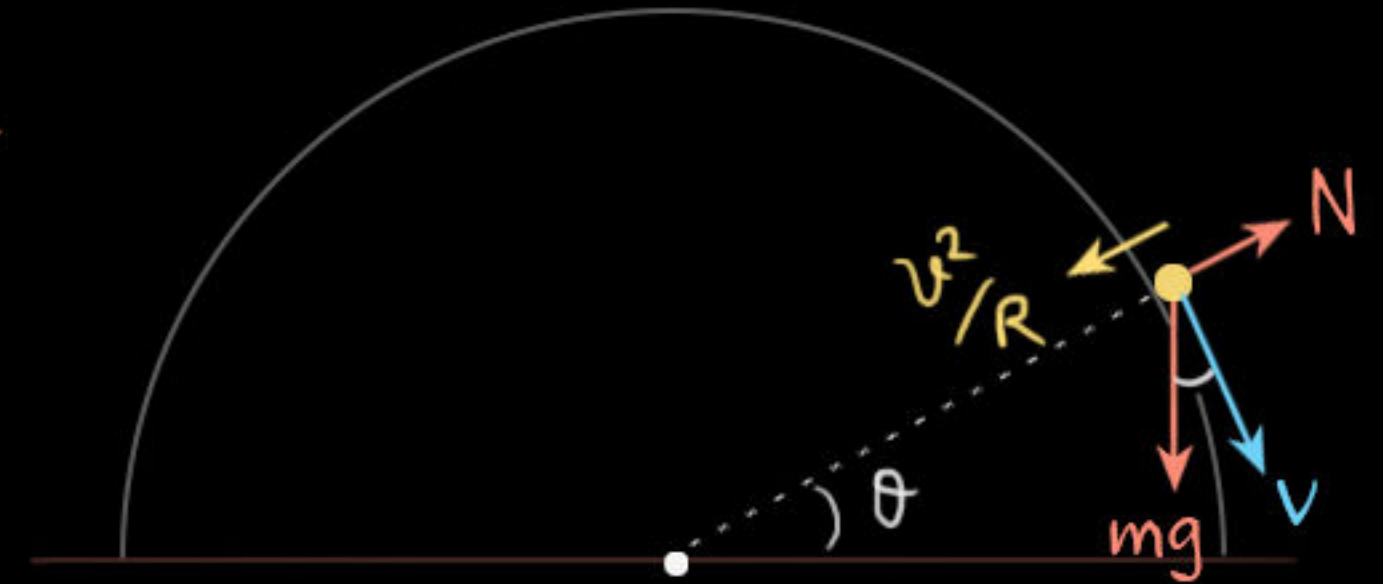
After that, its chance of losing contact is maximum at just after B. If it doesn't lose contact there, it will complete the whole path.

Force: $mg \cos \pi/3 - N = \frac{mv^2}{R}$

Energy: $mg \left[h - r(1 - \cos \pi/3) \right] = \frac{1}{2}mv^2$

$\Rightarrow h = 3R/4 //$

Proof

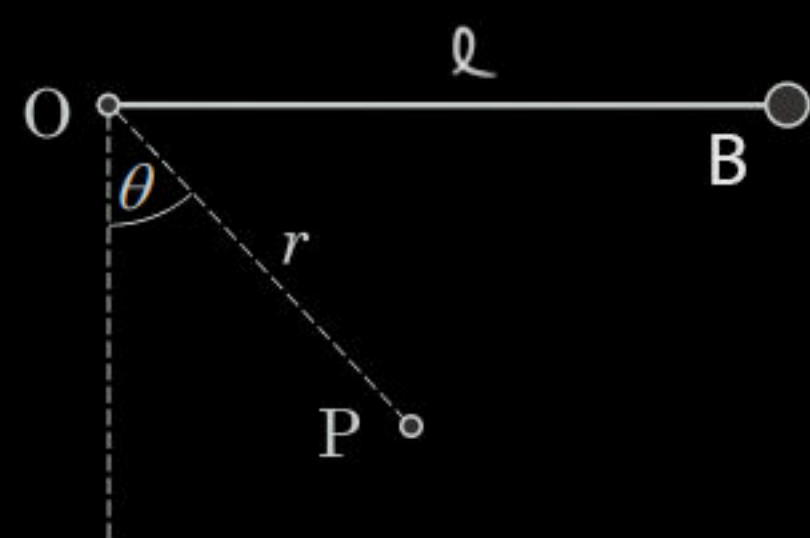


$$mg \sin \theta - N = \frac{mv^2}{R}$$

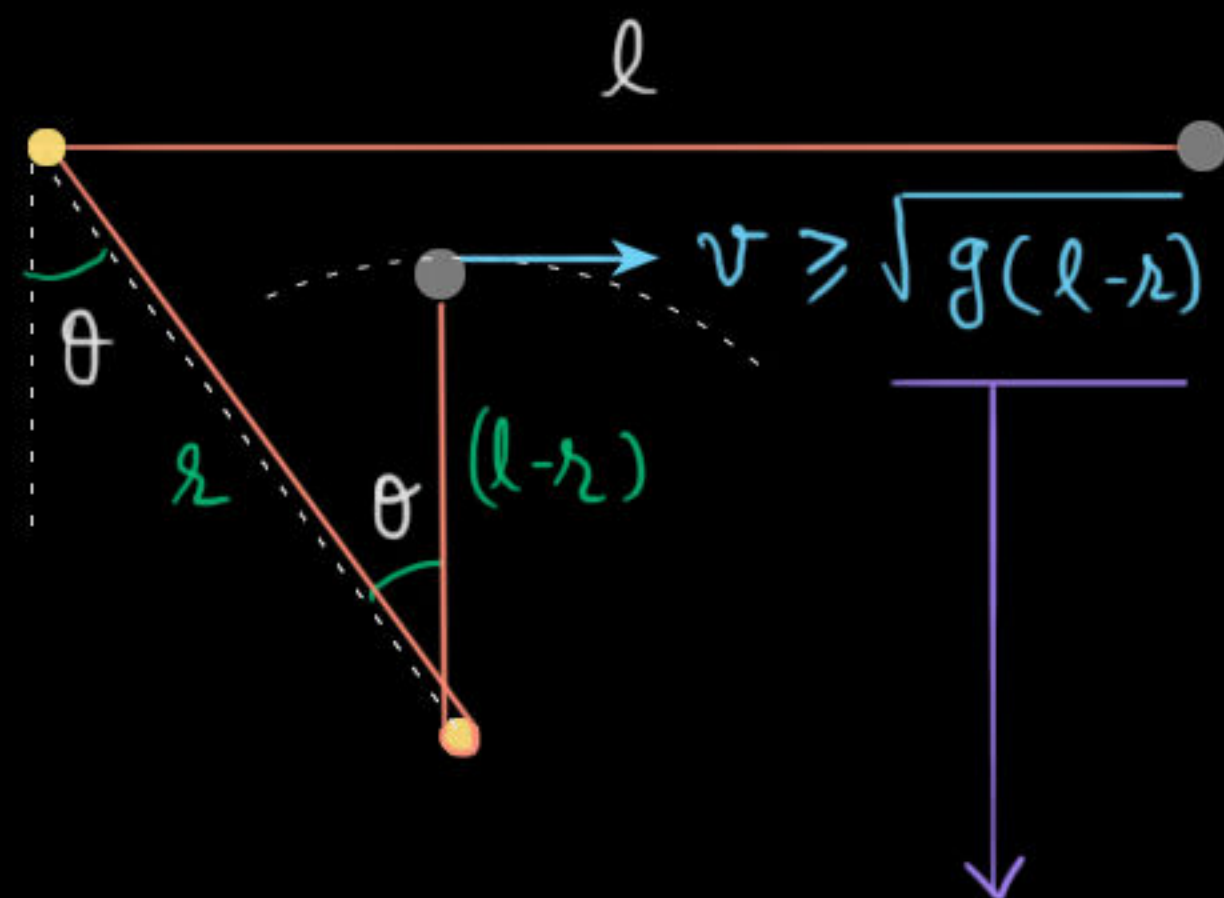
$$\Rightarrow N = mg \sin \theta - \frac{mv^2}{R}$$

$\therefore$ As $\theta \downarrow$, $\sin \theta \downarrow$ & $v \uparrow \Rightarrow N \downarrow$

Note: particle moving up or down is irrelevant as equations will remain the same



24. A small ball B is suspended from a fixed nail O with the help of a light inextensible cord of length l . There is another nail P at a distance r from O. The ball is pulled aside stretching the cord horizontal and released. In the ensuing motion, the cord clings to the nail P and the ball begins to turn around nail P. Find range of values of r , so that the ball makes a complete circular turn around the nail P. Ignore air resistance and radii of the nails.



Using a known result.

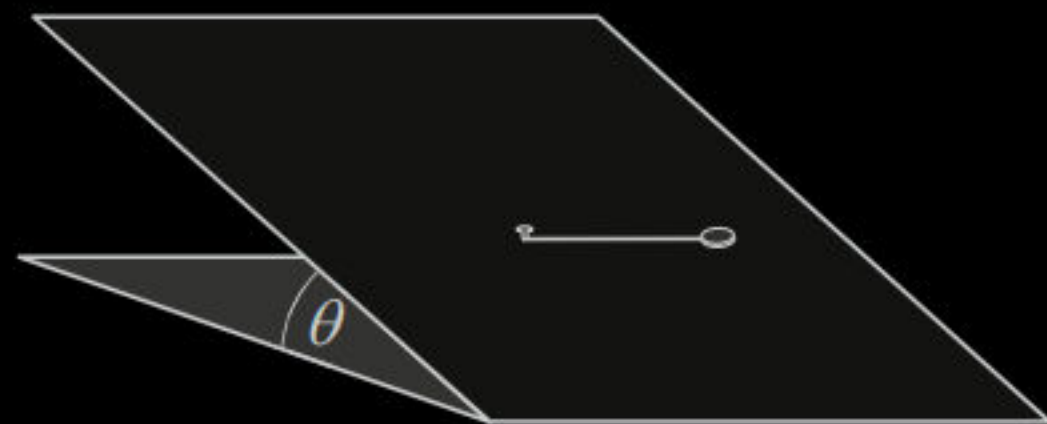
A pendulum bob must have $\sqrt{5gl}$ at bottom and hence $\sqrt{gl}$ at top to complete the circle.

$$W_g = \frac{1}{2}mv^2 \geq \frac{1}{2}mg(l-r)$$

$$\Rightarrow mg[r \cos \theta - (l-r)] \geq \frac{1}{2}mg(l-r)$$

$$\Rightarrow r \geq \frac{3l}{2\cos \theta + 3}$$

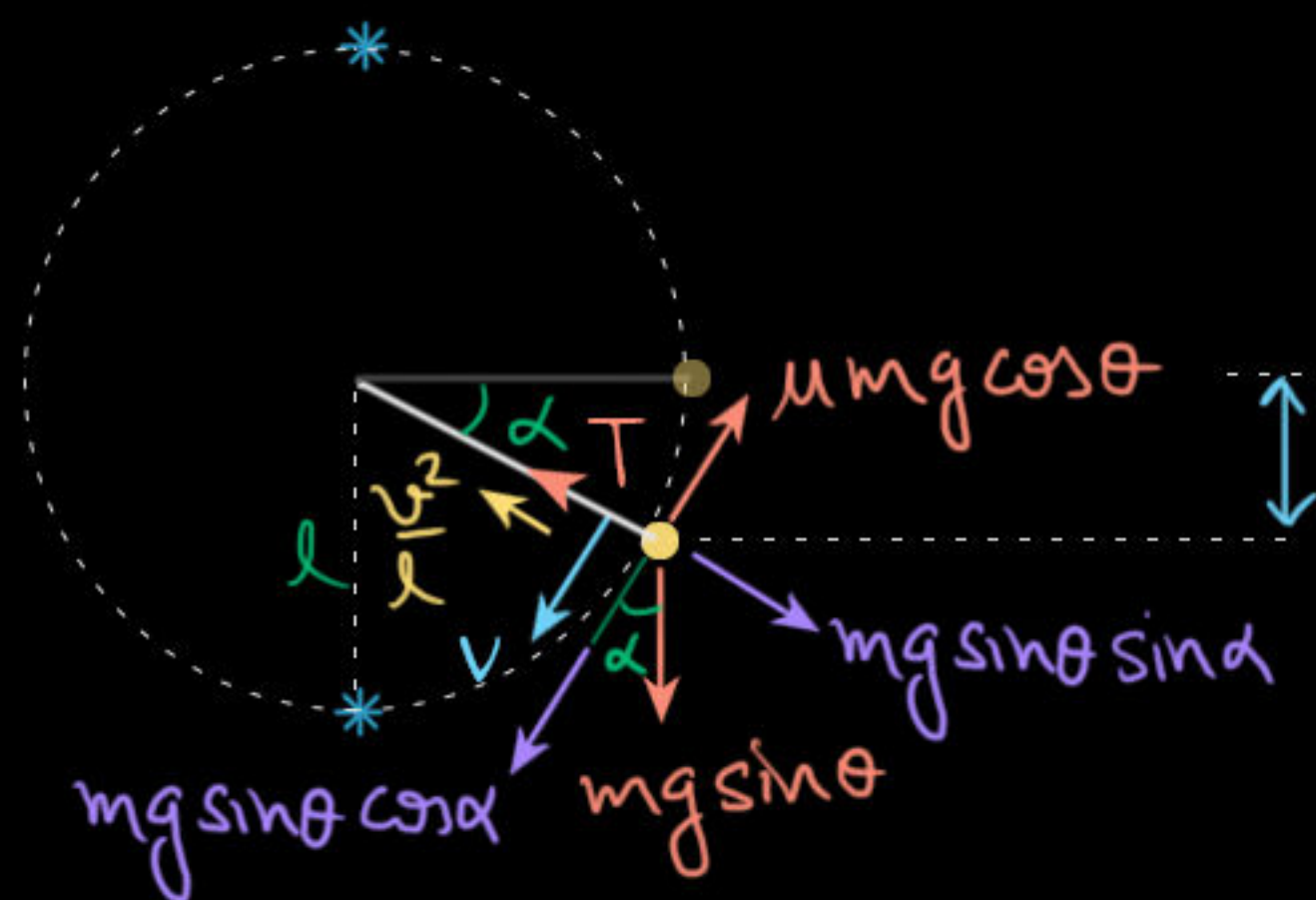
And of course r should be less than l for clinging !



25. A small disc of mass m placed on a plane of inclination $\theta = \cos^{-1}(0.8)$ with the horizontal is connected from a nail driven into the plane with the help of a light inextensible cord of length $l = 1.0$ m. Coefficient of friction between the plane and the disc is $\mu = 3/8$. The disc is held on the plane stretching the cord horizontal as shown in the figure and released. Find maximum speed of the disc and angular displacement where tensile force in the cord becomes maximum during subsequent motion

top of plane view (NOT top view)

Lets say it turns by α



Speed is maximum, when tangential acceleration is zero.

$$\Rightarrow \cancel{m}g \sin \theta \cos \alpha = \mu \cancel{m}g \cos \theta \Rightarrow \cos \alpha = 1/2$$

side view



$$W_g + W_f = \Delta KE$$

$$\Rightarrow \cancel{m}g l \sin \alpha \sin \theta - \mu \cancel{m}g \cos \theta l \alpha = \frac{1}{2} \cancel{m} v^2 \quad (1)$$

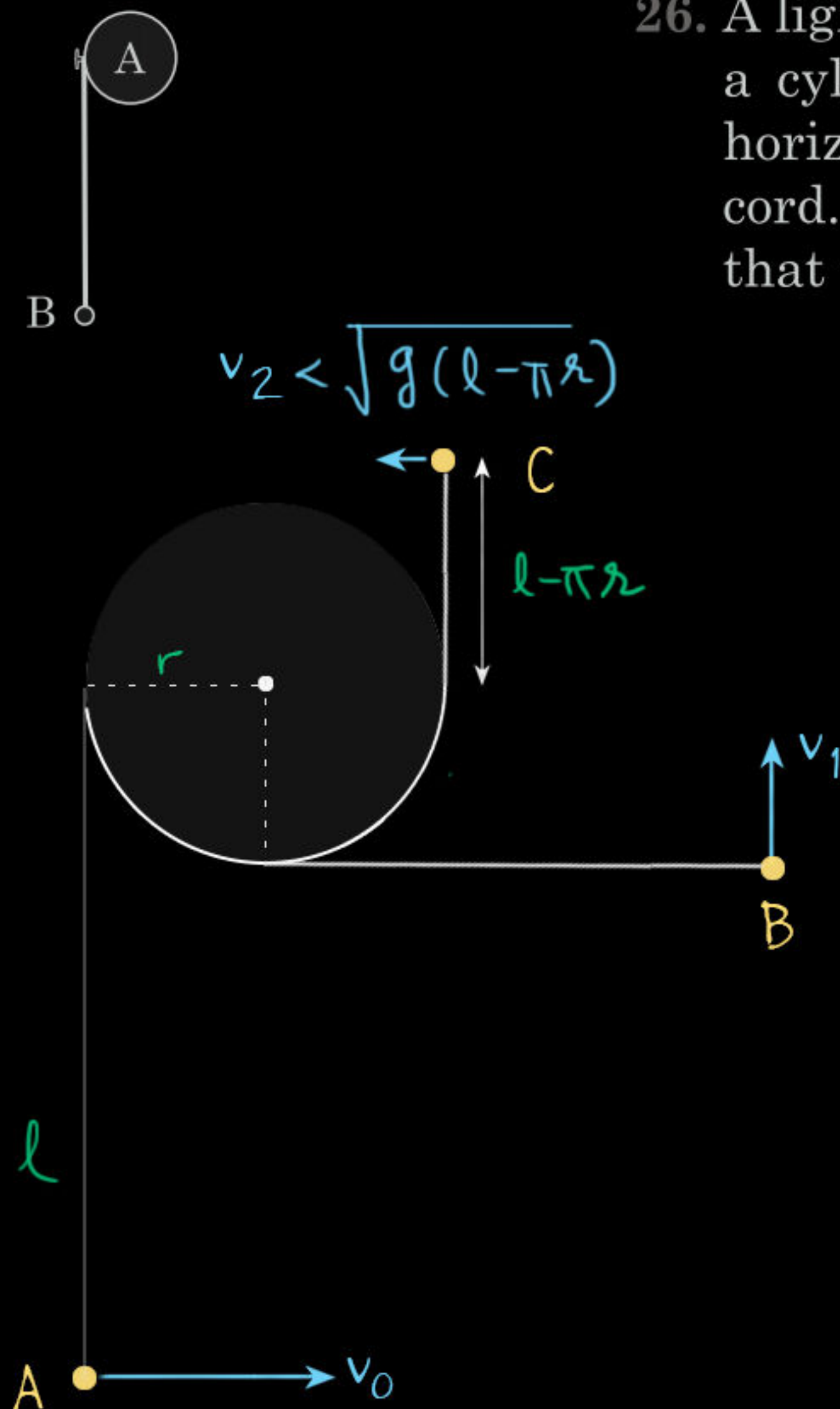
$$\cos \alpha = 1/2, \alpha = \pi/3$$

$$v_{\max} = 2 [3\sqrt{3} - \pi]$$

$$T - mg \sin \theta \sin \alpha = \frac{m v^2}{l} \quad (2)$$

Eliminating v from 1,2 and differentiating w.r.t α to get maximum value of T :

$$2 [\cos \alpha \sin \theta - \mu \cos \theta] + \sin \theta \cos \alpha = 0 \Rightarrow \cos \alpha = 1/3$$



26. A light cord of length l is attached to a nail driven into curved surface of a cylinder A of radius r ($l > \pi r$). The cylinder is fixed with its axis horizontal. A small ball B of mass m is hanging at the lower end of the cord. How much horizontal velocity must be imparted to the ball B so that the cord will certainly slack during the subsequent motion?

It will be taut if it stops before B. As centripetal acceleration will exist and it will be because of Tension. We must make it cross B at least to make it slack.

$$\therefore \frac{1}{2} m v_0^2 > m g (l - r) \text{ for positive } v_1. \quad (1)$$

If it crosses B, then for it to become slack, it must never reach C.

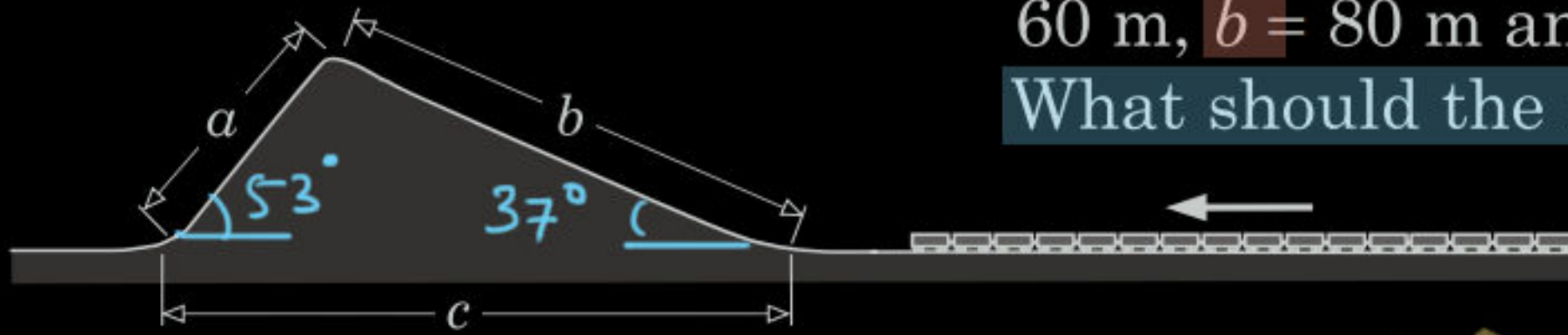
$$\therefore \frac{1}{2} m (v_0^2 - \overbrace{g(l - \pi r)}^{v_2^2}) < m g [l + (l - \pi r)]$$

$$\Rightarrow v_0^2 < 5gl - 3g\pi r \quad (2)$$

$$\text{From 1, 2: } \sqrt{2g(l - r)} < v_0 < \sqrt{g(5l - 3\pi r)}$$

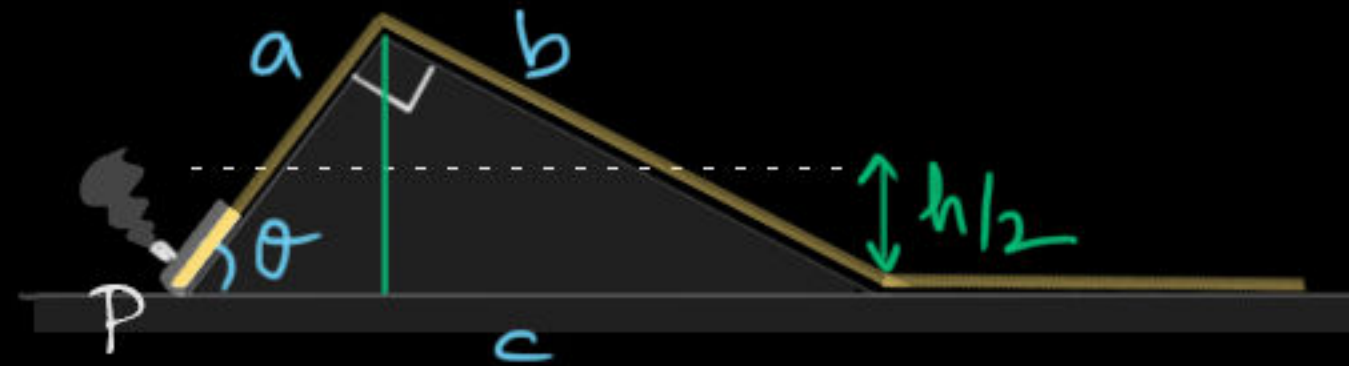
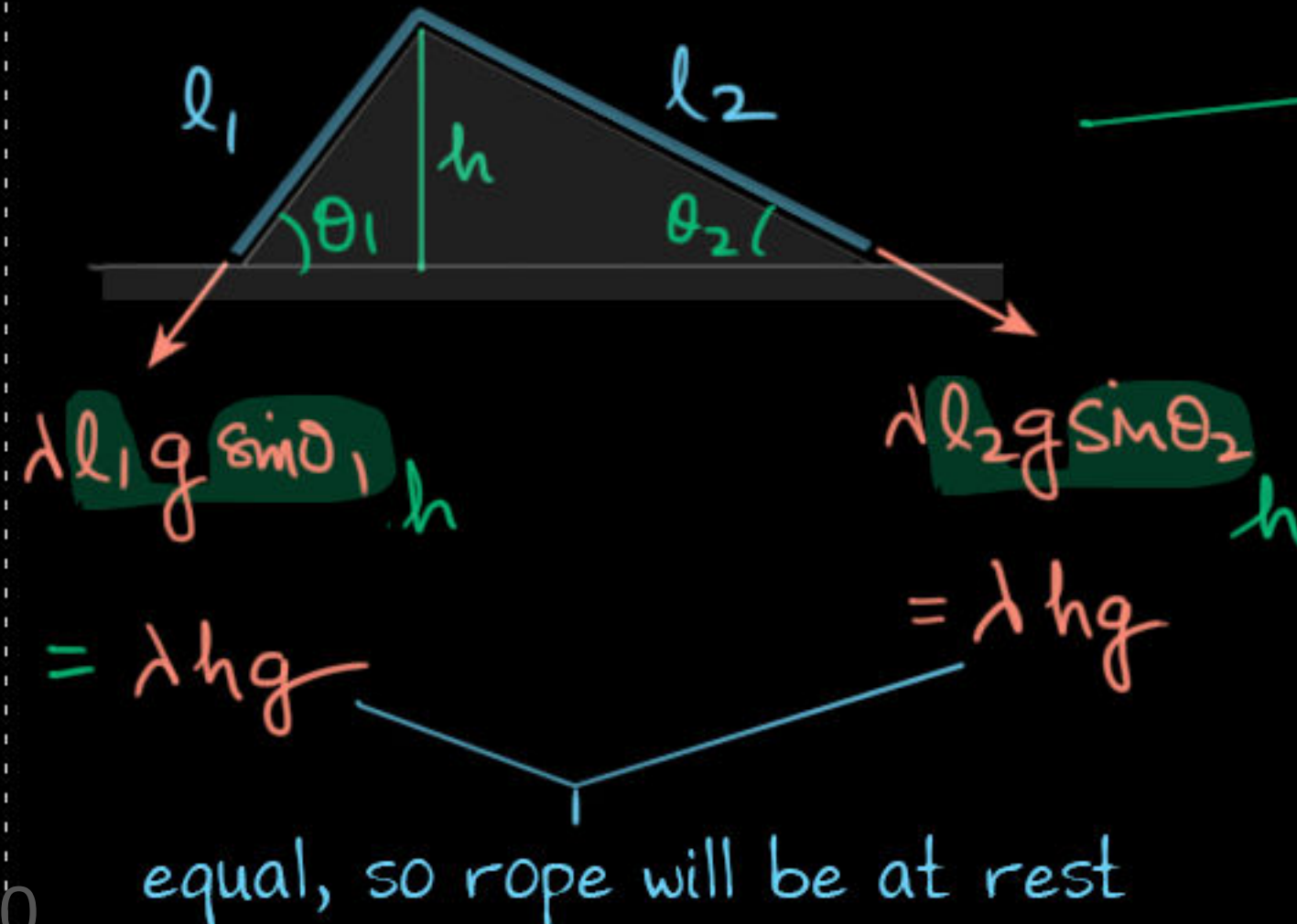
27. A prototype train of length $l = 168$ m is running at a constant speed v with its engine off on horizontal portion of a track that has a hilly portion as shown in the figure. Dimensions of the hilly portion are $a = 60$ m, $b = 80$ m and $c = 100$ m respectively.

What should the minimum speed of the train be to pass over the hill?



Analogy: Lets say a rope is lying down on a random wedge as shown.

Linear mass density be λ



Here, train is longer than $a+b$.

∴ If train stops anywhere before P, train will roll back.

So once the engine reaches P, train will cross over easily.

As potential energy doesn't change anymore (It's $m_{ab}gh/2$).

∴ From flat terrain to when engine reaches P:

$$\Delta KE + \Delta PE = 0$$

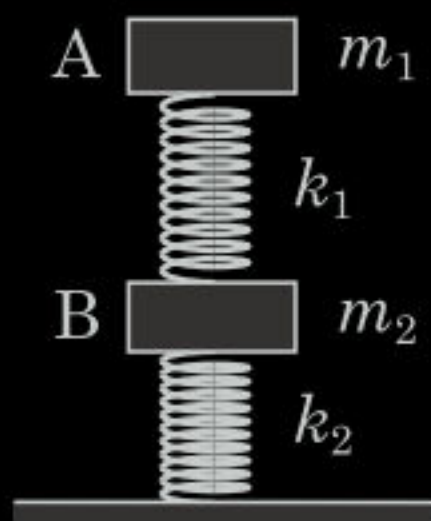
$$\Rightarrow -\frac{1}{2}mv^2 + m_{ab}gh/2 = 0$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{a+b}{l} m g \left(\frac{a \sin \theta}{2} \right)$$

$$v > \sqrt{\frac{g(a+b)ab}{lc}}$$

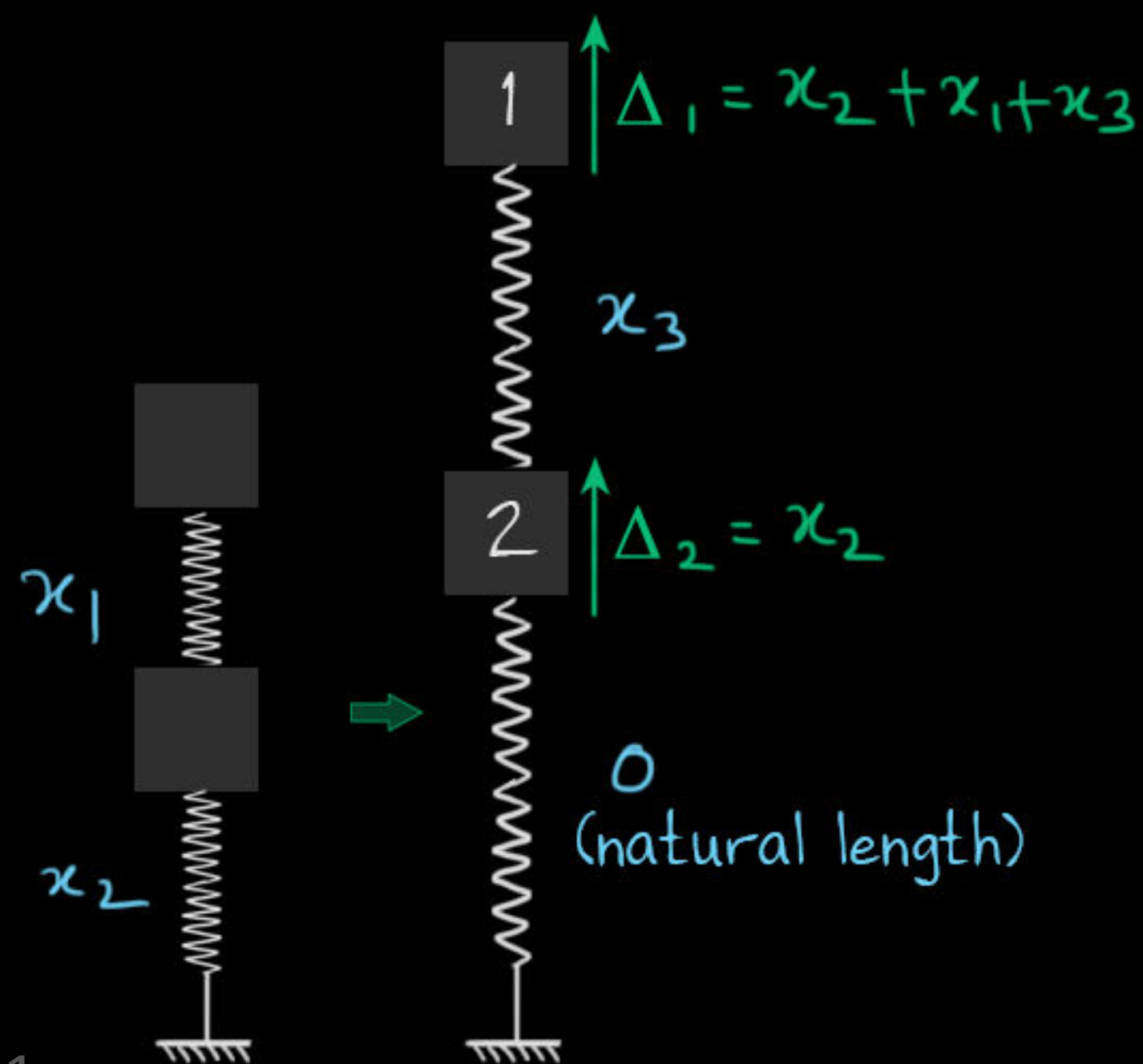
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28. A system of blocks A and B of masses m_1 and m_2 connected by a spring of force constant k_1 is placed on a spring of force constant k_2 , which rests on the ground. Both the springs are vertical and the whole system is in equilibrium. Find increment in gravitational potential energy of the system in lifting the block A upwards until the lower spring becomes relaxed.

Let the initial compressions be x_1 and x_2 . And final extensions be x_3 and 0.



$$\text{Compression } x_1 = \frac{m_1 g}{k_1}$$

$$\text{Compression } x_2 = \frac{(m_1 + m_2) g}{k_2}$$

$$\text{Elongation: } x_3 = \frac{m_2 g}{k_1}$$

$$\begin{aligned} \Delta PE &= m_1 g \Delta_1 + m_2 g \Delta_2 \\ &= g^2 \left[m_1 \left(\frac{m_1 + m_2}{k_2} + \frac{m_1}{k_1} + \frac{m_2}{k_1} \right) + \frac{m_2 (m_1 + m_2)}{k_2} \right] \\ &= g^2 (m_1 + m_2) \left[\frac{(k_1 + k_2) m_1 + k_1 m_2}{k_1 k_2} \right] \end{aligned}$$

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$$\vec{F} = -\frac{\partial U}{\partial x} \hat{i} - \frac{\partial U}{\partial y} \hat{j}$$

29. When a particle passes a point (x, y) in a force field created by stationary sources, potential energy of the system associated with the force field is given by equation $U = kx / (x^2 + y^2)^{3/2}$. If at a position $\vec{r} = r_0(\hat{i} + \hat{j})$, the particle is observed moving perpendicular to $\vec{r}$, find the tangential and normal components of acceleration of the particle at this point.

$$F_x = -\frac{\partial U}{\partial x} \hat{i} = \frac{k(2x^2 - y^2)}{(x^2 + y^2)^{5/2}} \hat{i}$$

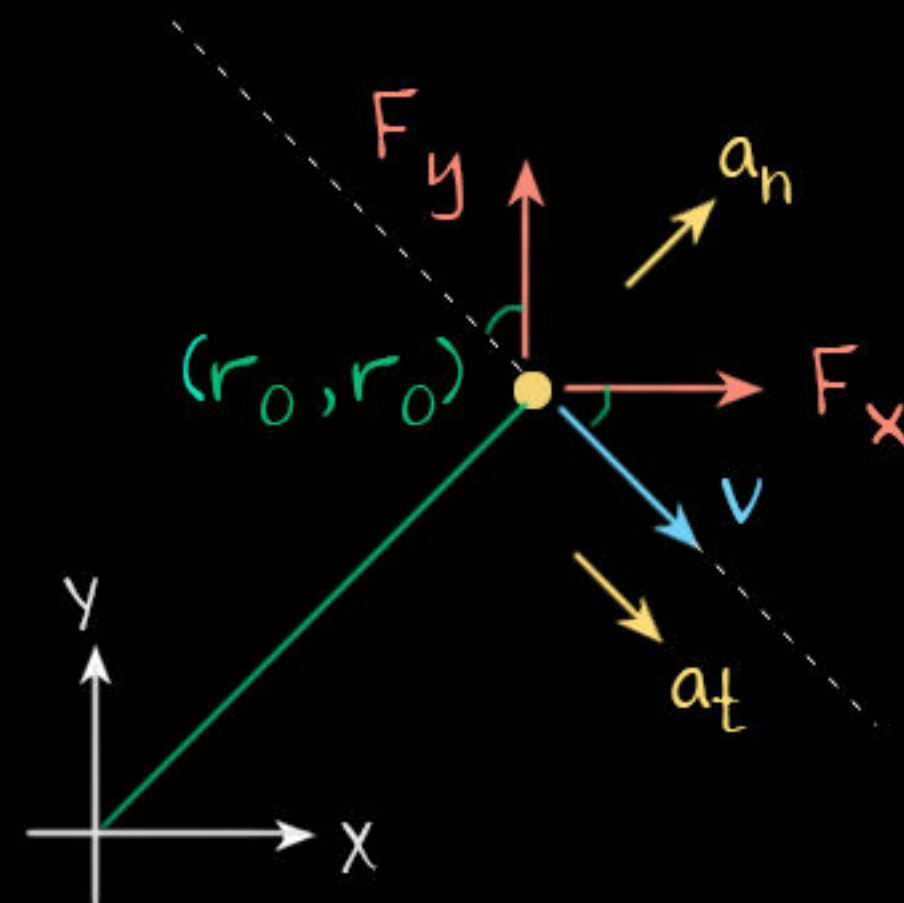
↓ (r_0, r_0)

$$\frac{k}{4\sqrt{2}r_0^3} \hat{i}$$

$$F_y = -\frac{\partial U}{\partial y} \hat{j} = \frac{3kxy}{(x^2 + y^2)^{5/2}} \hat{j}$$

↓ (r_0, r_0)

$$\frac{3k}{4\sqrt{2}r_0^3} \hat{j}$$

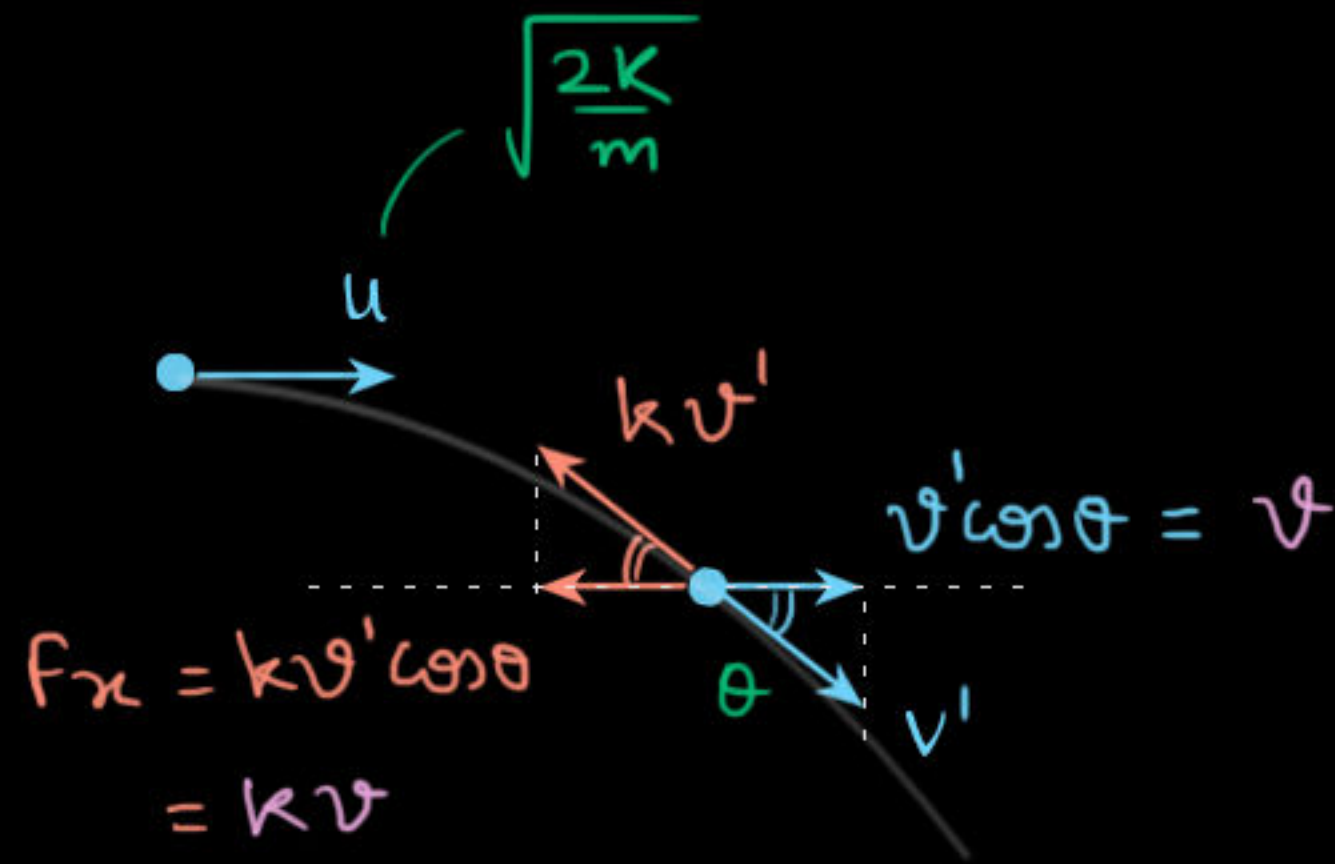


$$ma_t = (F_x \cos \frac{\pi}{4} - F_y \sin \frac{\pi}{4}) \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}} \right) = \frac{-k}{4\sqrt{2}r_0^3} (\hat{i} - \hat{j}) //$$

$$ma_n = (F_x \sin \frac{\pi}{4} + F_y \cos \frac{\pi}{4}) \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} \right) = \frac{k}{2\sqrt{2}r_0^3} (\hat{i} + \hat{j}) //$$

Approach: We shall do everything in horizontal direction.

30. A ball of mass m projected horizontally from the top of a tower with kinetic energy K flies in the air for a time interval τ before it hits the horizontal ground. If force of air drag is proportional to the speed, and the proportionality constant is k , find horizontal range of the ball.



$$F_x = ma_x$$

$$-kv' = m \frac{dv'}{dx}$$

$$-kv = m \frac{dv}{dt}$$

$$\Rightarrow \int_0^x -k dx = m \int_u^v dv$$

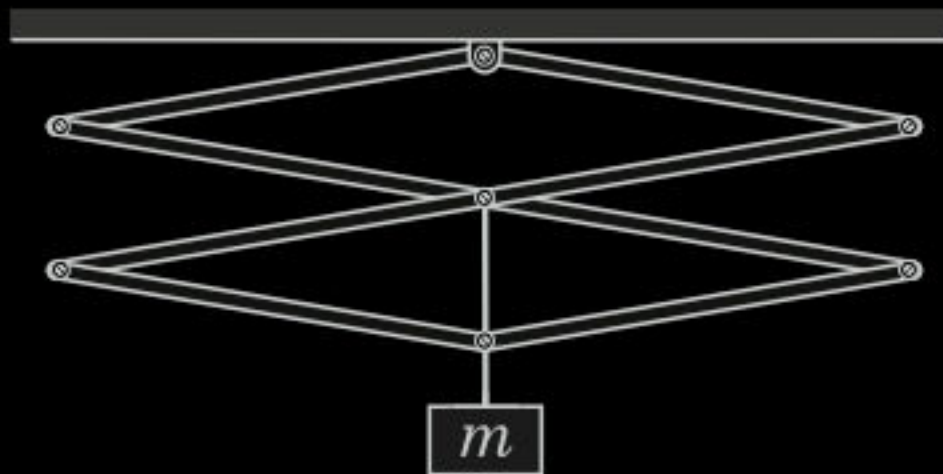
$$\Rightarrow -kx = m(v - u) \quad (1)$$

$$\Rightarrow \int_u^v \frac{dv}{v} = -\frac{k}{m} \int_0^z dt$$

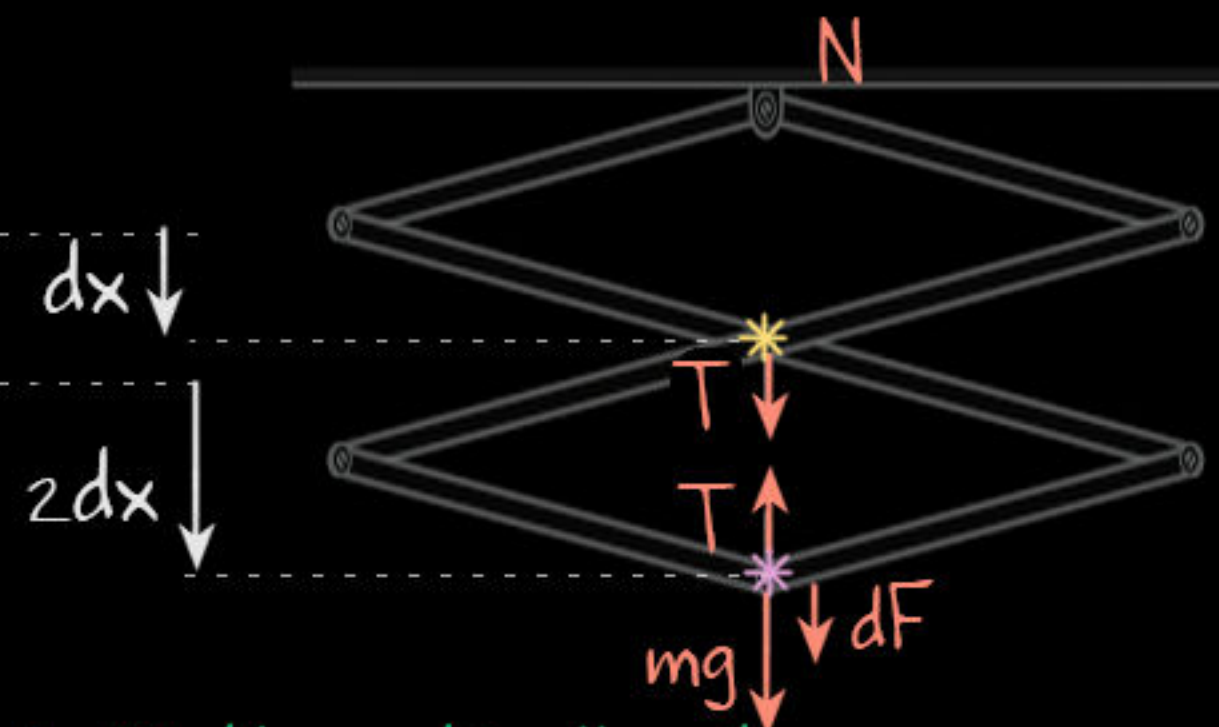
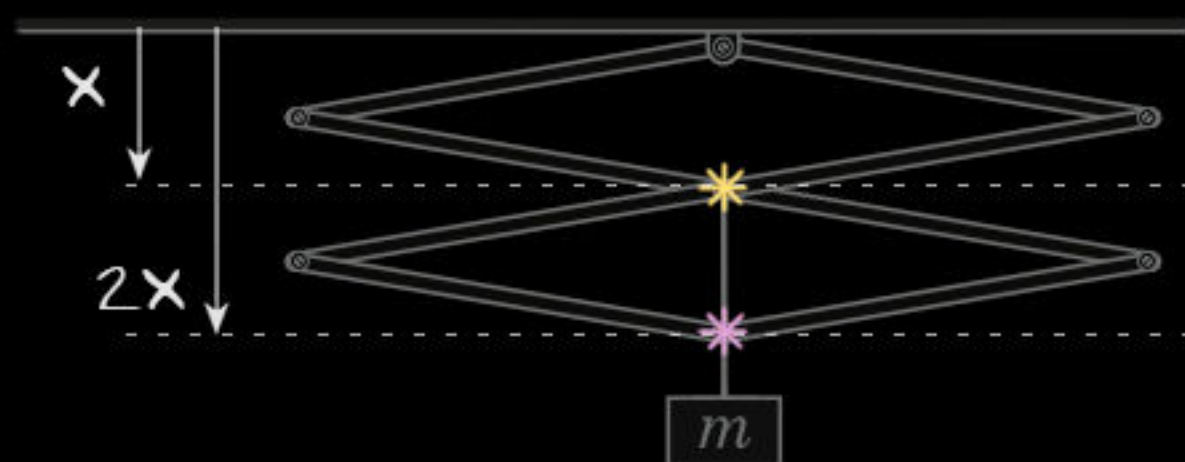
$$\Rightarrow v = u e^{-kz/m} \quad (2)$$

Eliminating v from 1,2

$$x = \frac{mu}{k} (1 - e^{-kz/m})$$



31. Two long and four half as long light rods are hinged at their ends to form a pantograph consisting of two identical rhombi. The pantograph is suspended from the ceiling, the lowest hinge of the pantograph is connected to the hinge above it by a light inextensible cord and a load of mass m is suspended from the lowest hinge as shown in the figure. Find the tensile force developed in the cord connecting the two hinges.



M 1 Energy

Replace the cord with elastic cord. Mathematically, its same. We pull m down a little bit further with negligible dF .

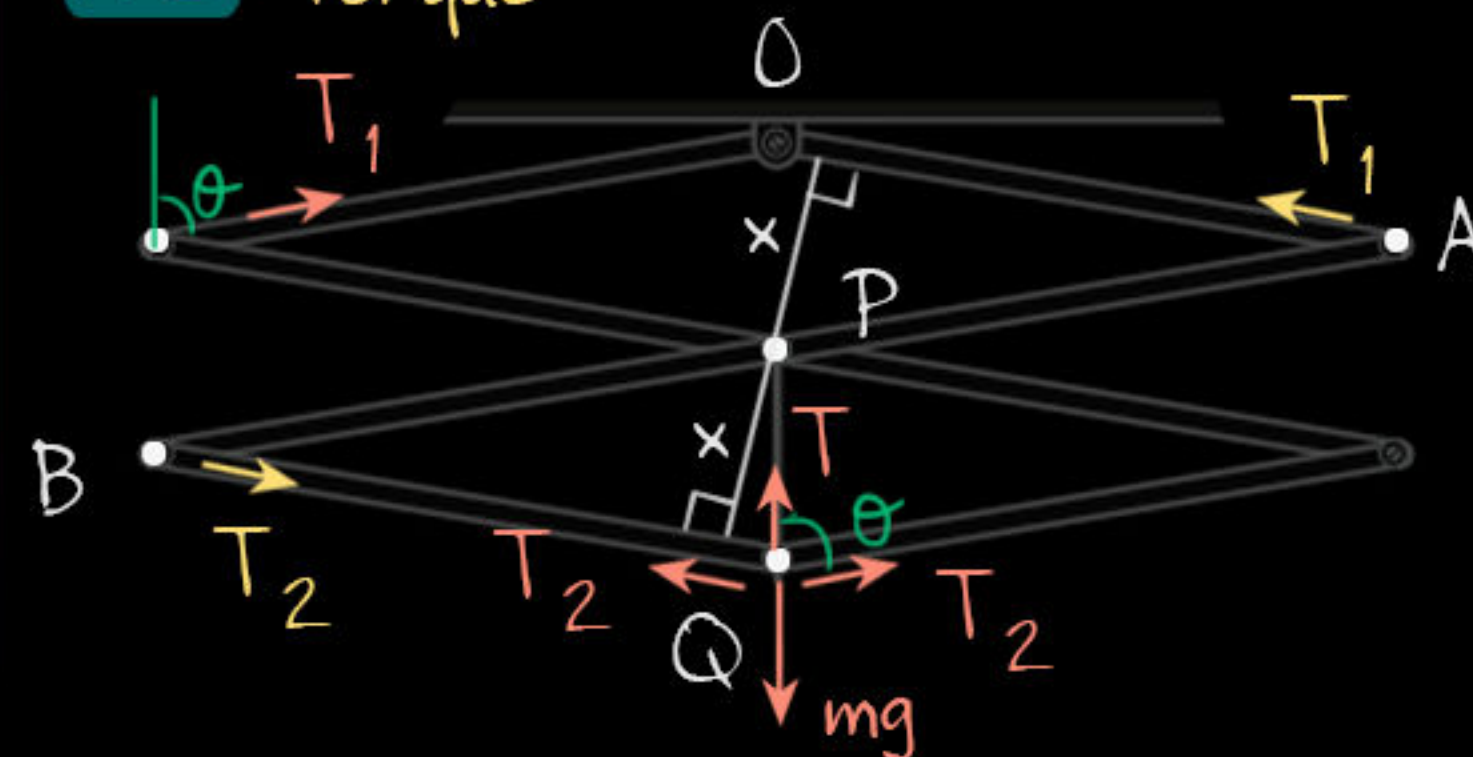
WE theorem on massless system of rods:

$$W_{df} + W_N + W_T + W_T + W_g = \Delta E$$

$$\Rightarrow dF \cdot 2dx + Tdx - T(2dx) + mg(2dx) = 0$$

$$\Rightarrow T = 2mg //$$

M 2 Torque



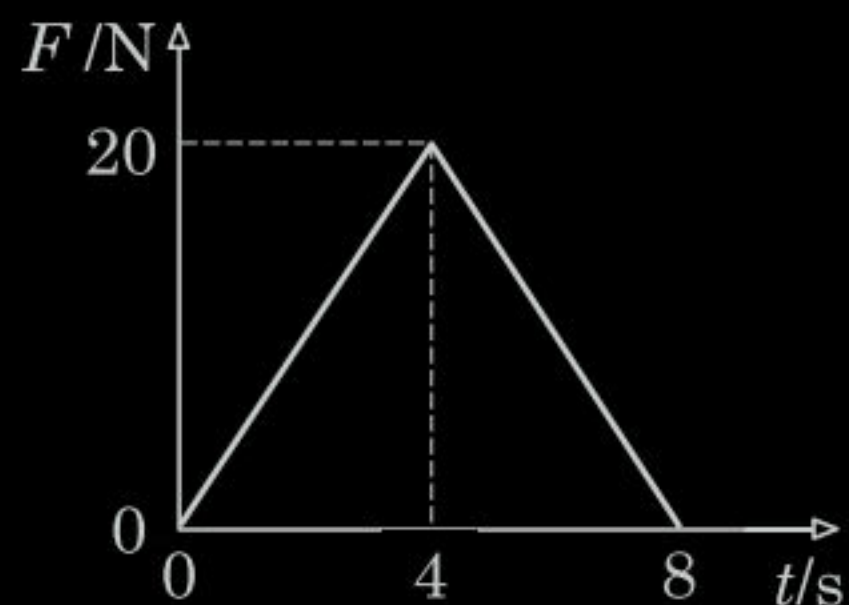
$$F_y: 2T_1 \cos \theta = mg \quad (1)$$

$$F_Q: 2T_2 \cos \theta + T = mg \quad (2)$$

$$\tau_P \text{ on rod AB: } T_1 x + T_2 x = 0 \quad (3)$$

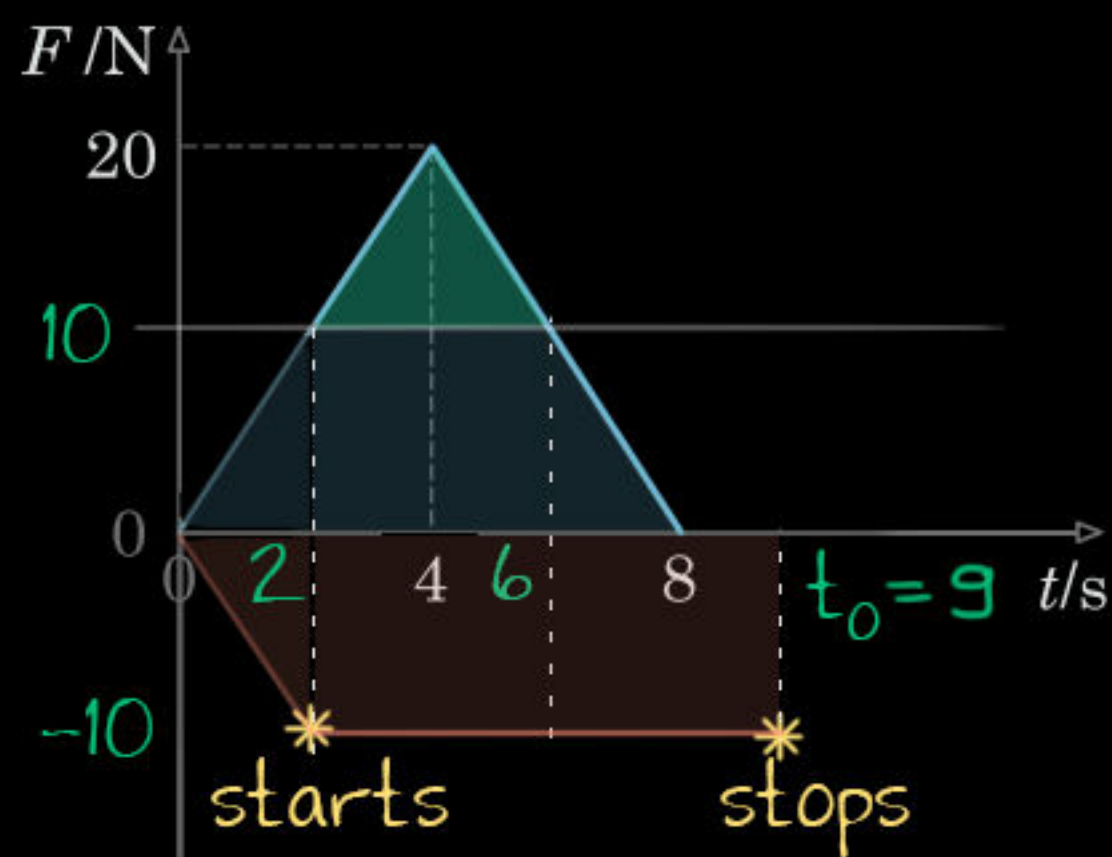
$$\text{From 1,2,3: } T = 2mg //$$

by Ankit Singhvi



1. A box of mass **4 kg** placed on a horizontal floor, is acted upon by a horizontal force **F** that varies with time **t** as shown in the given graph. If **coefficients of static and kinetic frictions are 0.25**, on which of the following **conclusions** can you arrive?

- ✓(a) The box **starts moving** at the instant $t = 2.0$ s.
- ✓(b) The **maximum velocity** acquired by the box is 5.0 m/s.
- ✓(c) The box **stops** at the instant $t = 9.0$ s.
- ✓(d) Modulus of **average power** of the force **F** is more than that of the **frictional force**.



A Box moves when: $F \geq \mu mg = (0.25)(4)(10) = 10 \text{ N @ } 2 \text{ sec}$

B @ Maximum velocity, acceleration and hence Force is zero. So @ 6 sec.

$J = \text{Area} = mv \Rightarrow \triangle \frac{1}{2}(6-2)(20-10) = 4 v_{\max} \Rightarrow v_{\max} = 5 \text{ m/s}$

C Lets say box stops at t_0 sec. Its $\Delta p = 0$

∴ Box will stop when J_{net} during above period is zero.

So both areas must be equal. @ $t_0 = 9 \text{ sec}$

D $W_{\text{net}} = W_F + W_f = \Delta KE = 0$

Lets say its W each

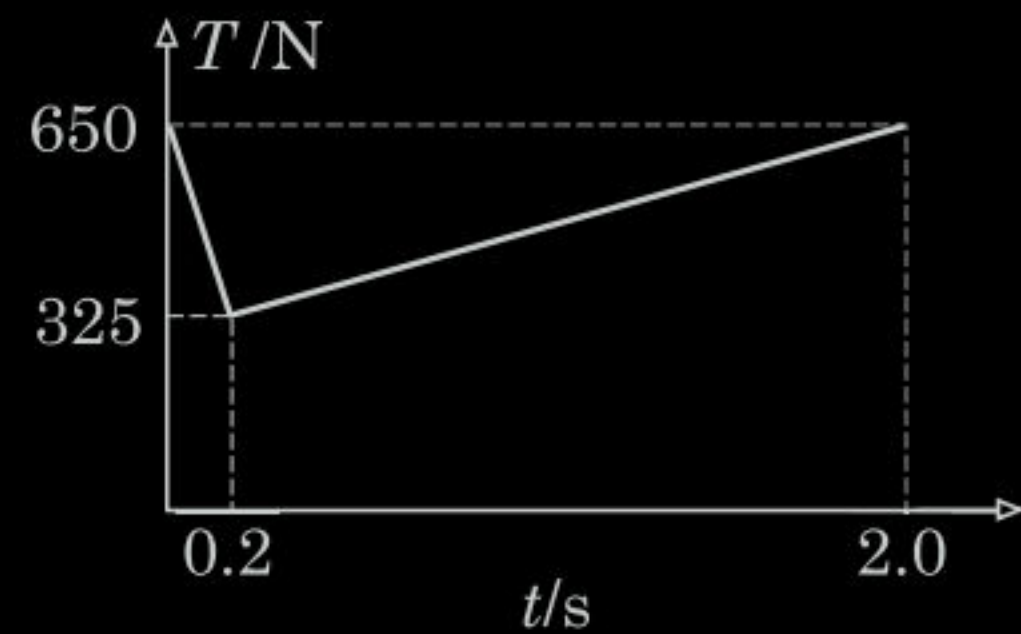
$$P_{\text{av}|F} = \frac{W}{8}$$

$$P_{\text{av}|f} = \frac{W}{9}$$

$$\Rightarrow P_{\text{av}|F} > P_{\text{av}|f}$$

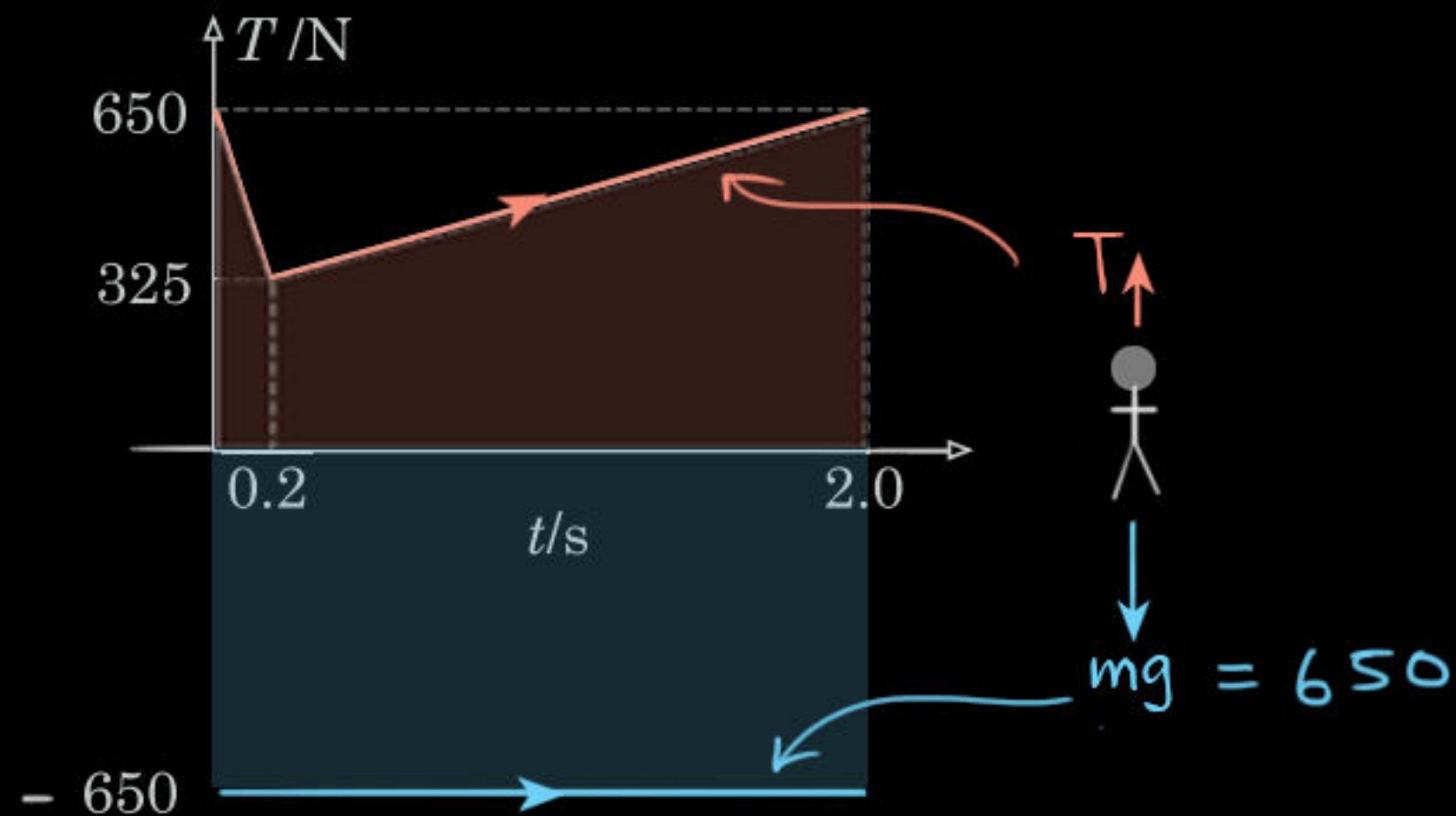
by Ankit Singhvi

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2. A mountain climber is sliding down a vertical rope. Her total mass, including equipment, is 65 kg. By adjusting frictional force of the rope, she controls the force that the rope exerts on her. For a 2.0 s interval, this force T is shown as a function of time t in the given graph. What is change in her speed in this interval? Assume acceleration of free fall to be 10 m/s^2 and neglect air resistance.

✓(a) 5 m/s



$$\Delta p = \int F dt$$

$$\Rightarrow m(v_f - v_i) = \int F_{\text{net}} dt$$

$$\Rightarrow m \Delta v = \int (650 - T) dt$$

$$= \int 650 dt - \int T dt$$

$$= \text{[Blue Rectangle]} - \text{[Brown Area Under T Curve]}$$

$$= \text{[Green Triangle]}$$

$$\Rightarrow \Delta v = \frac{1}{2} \frac{(650 - 325)(2)}{65} = 5 \text{ m/s}$$

by Ankit Singhvi

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3. A boy of mass 50 kg is standing at one end of a 9.0 m long boat of mass 100 kg that is floating motionless on a calm lake. The boy walks to the other end of the boat and stops there. The boat also moves and finally stops due to water resistance. If force of water resistance is proportional to velocity of the boat relative to the water. What is the magnitude of the net displacement s of the boat?

✓ (a) $s = 0.0$ m
(c) $s < 3.0$ m

(b) $s = 3.0$ m
(d) $s > 3.0$ m

For the boat man system:

$$\int f dt = \Delta p \quad \begin{matrix} \nearrow 0 \text{ (As both man and boat ultimately stop)} \\ \searrow k v_b \text{ (Viscous force on the bottom of boat)} \end{matrix}$$

$$\Rightarrow \int k v_b dt = 0$$

$$\Rightarrow k \int v_b dt = 0$$

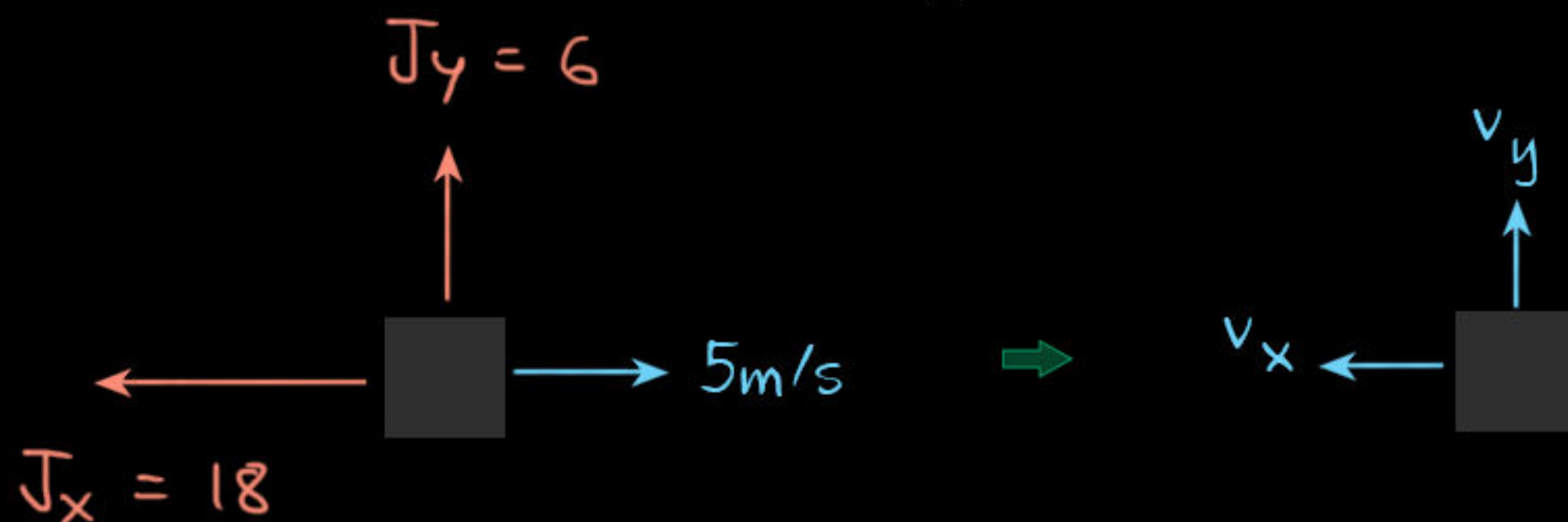
$$\Rightarrow \Delta s_b = 0 //$$

Lets show the information in vector form.

4. A block of mass 2.0 kg is sliding on a frictionless horizontal floor with a velocity of 5.0 m/s due east. It is subjected to a variable net force for 10 seconds resulting in an impulse of 6.0 N·s in the north and 18.0 N·s in the west. What is the total work done by this force?

✓ (a) 0.0 J
(c) 100 J

(b) 90 J
(d) 200 J



Towards West:

$$J_x = m(v_x - (-5))$$

$$\Rightarrow 18 = 2(v_x + 5)$$

$$\Rightarrow v_x = 4 \text{ m/s}$$

Towards North:

$$J_y = m(v_y - 0)$$

$$\Rightarrow 6 = 2v_y$$

$$\Rightarrow v_y = 3 \text{ m/s}$$

$$W_{\text{net}} = \Delta KE$$

$$= \frac{1}{2}m(5^2 - 5^2)$$

$$= 0 //$$

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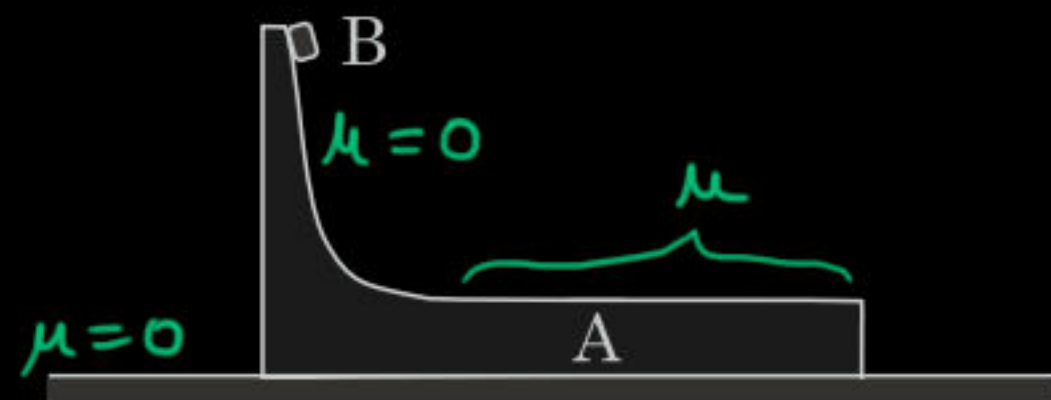
$$KE = \frac{p^2}{2m}$$

5. An astronaut leaves his spaceship for some experiment and floats freely in space at rest relative to his spaceship. His friend in the spaceship ignites a rocket installed on the spaceship for a very short duration. Which of the following observations can you make relative to a reference frame moving together with the astronaut outside the spaceship?
- ✓(a) The spaceship has lesser kinetic energy than the ejected gases.
 - (b) The spaceship and the ejected gases have equal kinetic energies.
 - (c) The spaceship has greater kinetic energy than the ejected gases.
 - ✓(d) Magnitudes of momenta of spaceship and ejected gases are equal.

w.r.t astronaut, initially ship was at rest.

After ignition, ship moves but for ship gas system momentum is conserved. Let each of them have momentum p in opposite direction.

$$\begin{array}{l}
 KE_{\text{rocket}} = \frac{p^2}{2M} \\
 KE_{\text{gases}} = \frac{p^2}{2m}
 \end{array}
 \begin{array}{c}
 \nearrow \\
 \searrow
 \end{array}
 \begin{array}{c}
 \because M > m \\
 \longrightarrow
 \end{array}
 KE_{\text{rocket}} < KE_{\text{gases}}$$



Concept: As there is no external force, final momentum must be zero too.

So both A and B will be at rest when B stops sliding.

6. Top of a large block A of mass 4 kg consists of a frictionless slope of height 40 cm which smoothly merges into a horizontal flat portion. The block rests on a frictionless horizontal floor. When a small block B of mass 1.0 kg is released on the top of the slope, it slides down the slope and then stops after sliding a distance 1.0 m on the horizontal portion of the top of the block A. Find coefficient of friction between the block B and horizontal portion of the top of the block A.

(a) 0.2

(b) 0.3

✓(c) 0.4

(d) 0.5

For block-wedge system:

$$W_g + W_f|_{w.r.t \text{ gnd}} = \cancel{\Delta K E}$$

$$\rightarrow = W_f|_{w.r.t A} = -\mu m_B g x$$

{WPE, MCQ 19,20}

$$\Rightarrow \cancel{m_B g h} - \mu \cancel{m_B g} x = 0$$

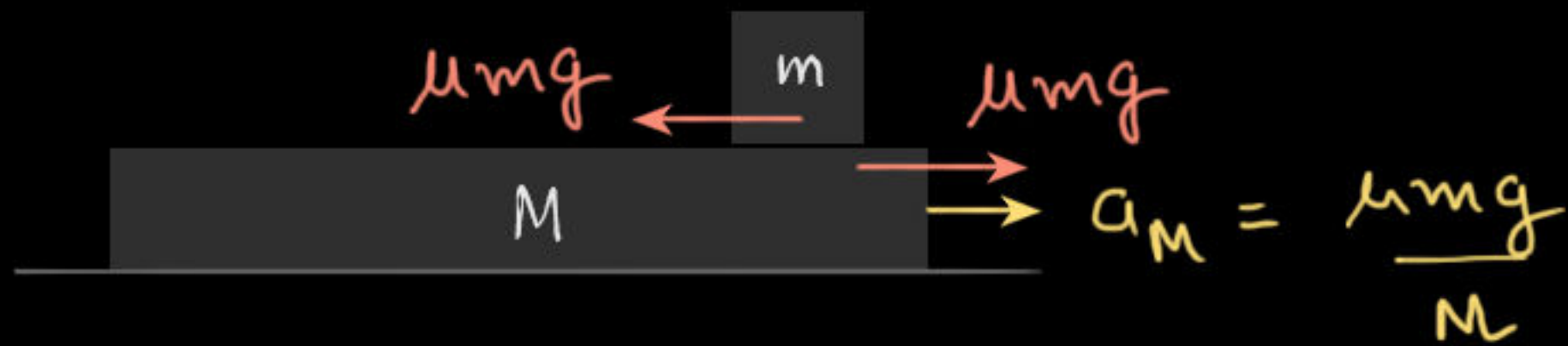
$$\Rightarrow \mu = \frac{h}{x} = 0.4 //$$



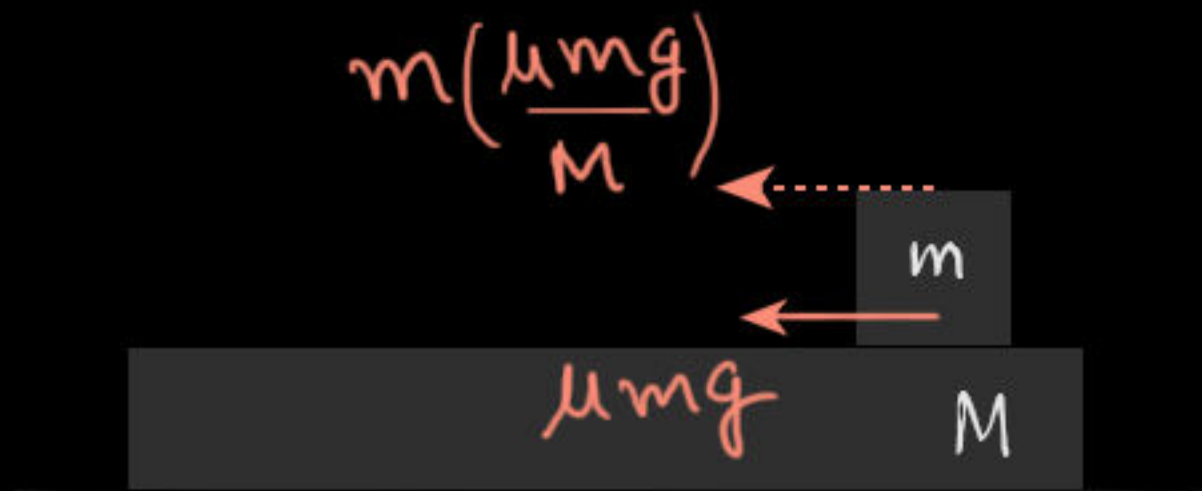
7. A small block of mass m is placed on a plank of mass M , which rests on a frictionless horizontal floor. Coefficient of friction between the block and the plank is μ . The block is given a sharp impulse, so that it starts sliding on the plank with an initial velocity u . Due to friction between the block and the plank, the block slows down and finally stops sliding on the plank. What average thermal power is dissipated in this process?

While there is sliding:

w.r.t ground



w.r.t M



Lets say block slides x , w.r.t M . $\checkmark(a) \frac{\mu m g u}{2}$

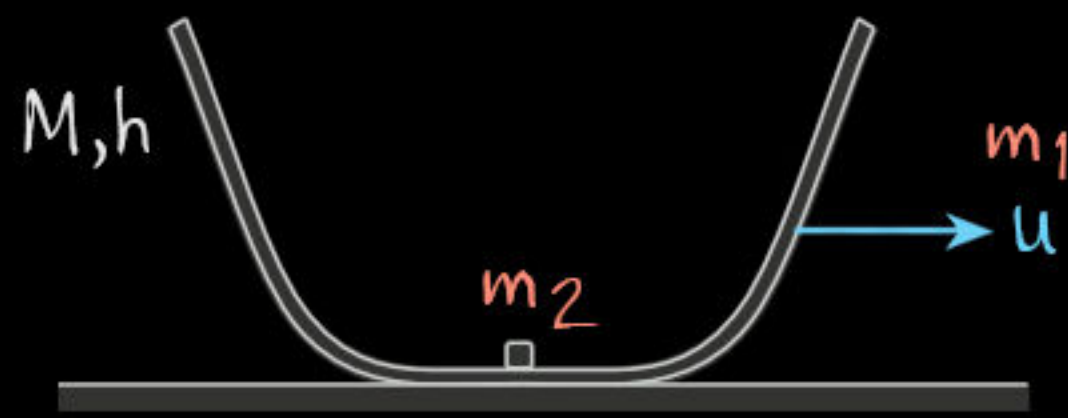
w.r.t M

$$P_{\text{av dissipated}} = \frac{W_f}{t} = \frac{(\mu m g) x}{t} \quad (1)$$

∴ uniform acceleration

$$\left(\frac{u + 0}{2} \right) t = x \Rightarrow \frac{x}{t} = \frac{u}{2} \quad (2)$$

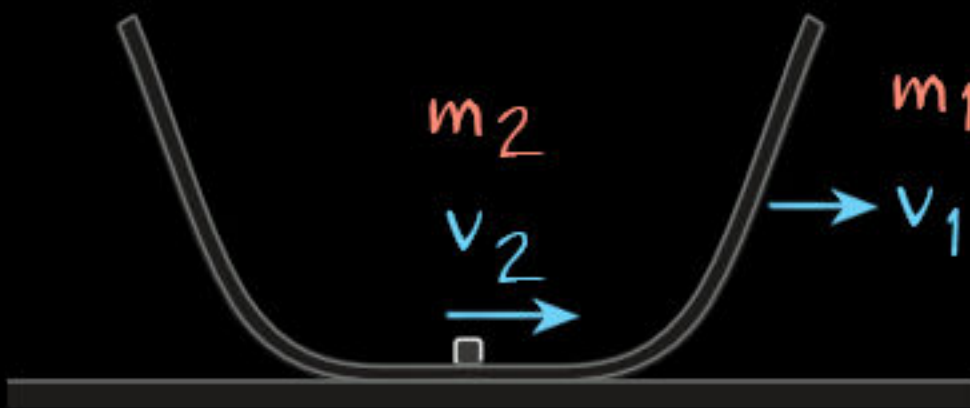
$$\text{From 1,2: } P_{\text{av dissipated}} = \frac{\mu m g u}{2}$$



A Initially



B When 2 is at peak



C When v_1 is minimum.

8. A small block rests on flat bottom of a deep bowl that is placed on a horizontal floor. There is **no friction** between the block and the bowl as well as between the bowl and the floor. **Total mass** of the bowl and the block is M . If the bowl is given a horizontal velocity by a **sharp hit**, the **block rises to a maximum height h** sliding on the walls of the bowl. Which of the following statements are **correct**?

- ✓ (a) **Maximum kinetic energy of the bowl is Mgh .**
- ✓ (b) If the bowl is lighter than the block, **minimum kinetic energy of the bowl is zero.**
- ✓ (c) To predict minimum kinetic energy of the bowl, we must know ratio of the masses.
- (d) To predict maximum as well as minimum kinetic energy of the bowl, we must know ratio of the masses.

$$KE_{\text{max of bowl}} : \frac{1}{2} m_1 u^2$$

Between A and B:

$$m_1 u = M v$$

$$\frac{1}{2} m_1 u^2 = \frac{1}{2} M v^2 + m_2 g h$$

$$\text{Eliminating } v: \frac{1}{2} m_1 u^2 = Mgh //$$

by Ankit Singhvi

Between A and C:

$$m_1 u = m_1 v_1 + m_2 v_2$$

$$\frac{1}{2} m_1 u^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Putting $m_2/m_1 = k$ & eliminating v_2

$$v_1 = \left(\frac{1-k}{1+k} \right) u = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u$$

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Let mass of sledge be M .
And snow fall rate $dm/dt = r$.

Case I Thrown perpendicular
w.r.t sledge

fallen $dm \bullet v=0$
 $M \longrightarrow v$
 thrown $dm \bullet \longrightarrow v$

$$dp = dm(v-0) = dm v$$

$$F_{\text{sledge}} = -F_{\text{snow}} = -\frac{dp}{dt} = -v \frac{dm}{dt} = -vr$$

$$\therefore a_{\text{sledge}} = -\frac{vr}{M} //$$

9. A sledge on which you are riding is given an initial push to slide on a straight level frictionless track. Snow is falling vertically on the sledge in the frame of the track. You have to adopt any one of the following three strategies to keep your sledge moving fastest.

I: You sweep the snow off the sledge so that it leaves the sledge perpendicular to the track, as seen by you in the frame of the sledge.

II: You sweep the snow off the sledge so that it leaves the sledge perpendicular to the track, as seen by someone in the frame of the track. continually

III: You do nothing

Order the strategies from the best to the worst.

✓ (a) II, III and I

Case II Thrown perpendicular
w.r.t ground

fallen $\bullet v=0$
 $\longrightarrow v$
 thrown $\bullet v=0$

$$F_{\text{sledge}} = -F_{\text{snow}} = \frac{dp}{dt} = 0$$

$$\therefore a_{\text{sledge}} = 0 //$$

Case III Snow gets accumulated

fallen $dm \bullet v=0$
 $M+ \longrightarrow v$

$$dp = dm(v-0) = dm v$$

$$F_{\text{sledge}} = -F_{\text{snow}} = -\frac{dp}{dt} = -vr$$

$$\therefore a_{\text{sledge}} = -\frac{vr}{M+}$$

$\therefore \text{II} > \text{III} > \text{I}$
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10. A particle of mass m moving with velocity v makes a head-on elastic collision with a stationary particle of mass $2m$. Kinetic energy lost by the lighter particle during period of deformation is

(a) $\frac{1}{2}mv^2$

(b) $\frac{1}{18}mv^2$

✓ (c) $\frac{4}{9}mv^2$

(d) $\frac{8}{9}mv^2$

i.e., from zero to maximum deformation
(After that, its period of reformation)

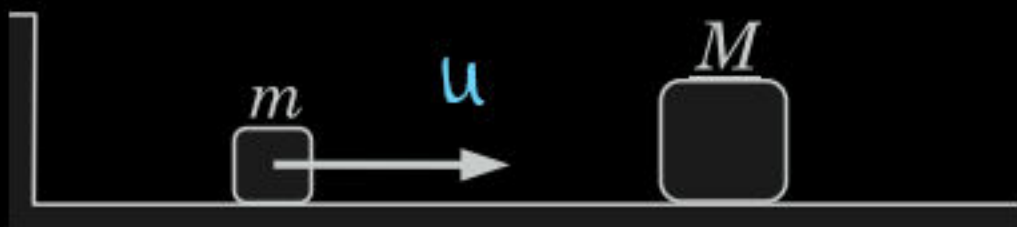


$$p_i = p_f$$

$$mv = 3mv'$$

$$\Rightarrow v' = v/3$$

$$\begin{aligned} \therefore \text{Loss of KE of } m \text{ is: } & \frac{1}{2}m(v^2 - v'^2) \\ &= \frac{1}{2}m\left(v^2 - \frac{v^2}{9}\right) \\ &= \frac{4mv^2}{9} // \end{aligned}$$



11. A block of mass m is given a velocity towards another block of mass M kept at rest on a frictionless horizontal floor some distance apart from a wall as shown in the figure. If all the collisions are elastic, under which of the following conditions, will the blocks collide only once?

Lets say initial velocity is u and later be v_1 and v_2 as shown.



(a) $m \leq M$

(c) $M \geq 3m$

✓ (b) $M \leq 3m$

(d) $M \leq 3m \leq 3M$

For single collision: $v_2 \geq v_1$ ①

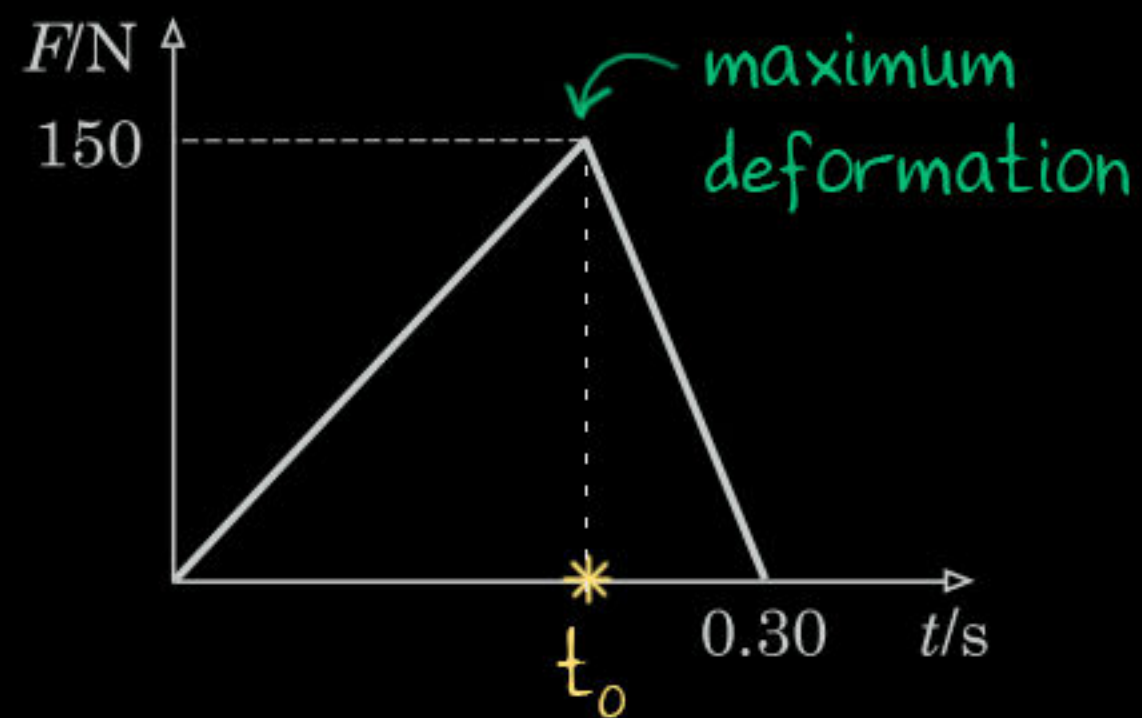
$p_i = p_f$: $m u = M v_2 - m v_1$ ②

$v_{\text{approach}} = v_{\text{separation}}$ $u = v_1 + v_2$ ③

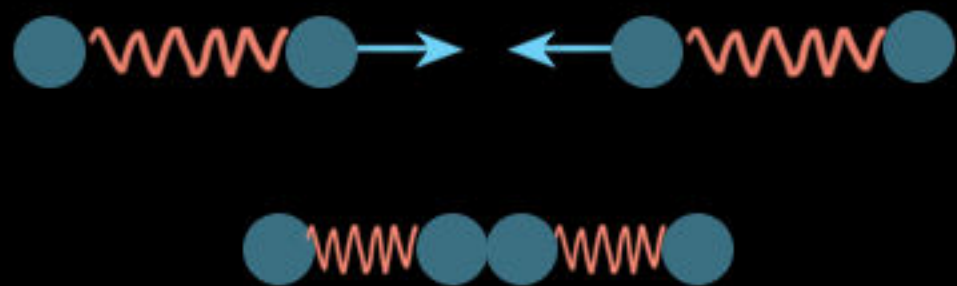
From 1, 2 and 3:

$$\frac{2m}{M-m} \geq 1$$

$$\Rightarrow M \leq 3m$$



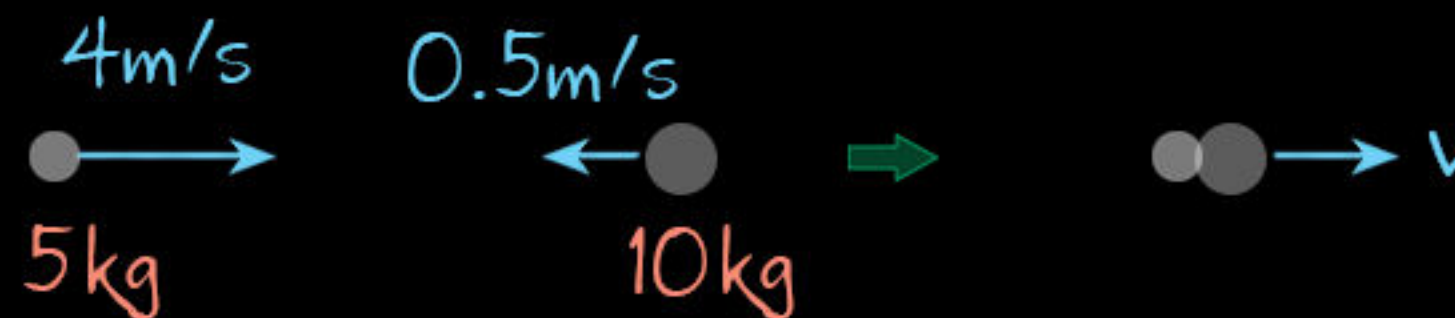
Concept: At maximum deformation, internal force is maximum. Just like two springs colliding.



12. Two bodies A and B of masses 5.00 kg and 10.0 kg moving in free space in opposite directions with velocities 4.00 m/s and 0.50 m/s respectively undergo a head-on collision. The force F of their mutual interaction varies with time t according to the given graph. What can you conclude from the given information?

- ✓(a) Period of deformation is 0.20 s.
- ✓(b) Coefficient of restitution is 0.5.
- (c) Body A will move with velocity 0.5 m/s in the original direction.
- ✓(d) Body B will move with velocity 1.75 m/s in reverse direction.

Upto maximum deformation



$$5(4) - 10(0.5) = (5+10)v \Rightarrow v = 1 \text{ m/s}$$

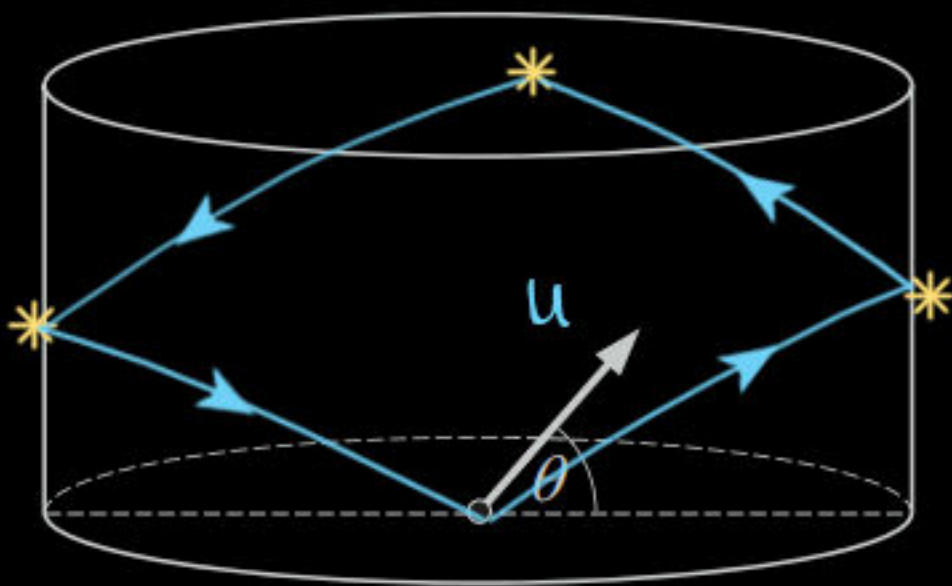
$$\begin{aligned} \therefore \text{J imparted at maximum deformation} &= 5 \text{ kg}(4 - 1) \\ &= 15 \text{ Ns} = \frac{1}{2} (150) t_0 \text{ (From graph)} \\ \Rightarrow t_0 &= 0.2 \text{ s} \end{aligned}$$

$$\text{Total } J \text{ imparted} = \text{Total area under } F-t = \frac{1}{2}(0.3)150 = 22.5 \text{ Ns}$$



- $5(4) - 22.5 = 5v_1 \Rightarrow v_1 = -0.5 //$
- $-10(0.5) + 22.5 = 10v_2 \Rightarrow v_2 = 1.75 //$

$$\text{Coefficient of restitution} = \frac{v_2 - v_1}{4 + 0.5} = \frac{1.75 - (-0.5)}{4.5} = \frac{2.25}{4.5} = 0.5 //$$



13. A particle projected at an angle θ with the horizontal from the centre of the floor of a cylindrical room of radius r returns to the point of projection after three elastic collisions with the walls and the ceiling. If the particle remains in air for time T , find the speed of projection.

(a) $\frac{3r}{T \cos \theta}$

✓ (b) $\frac{4r}{T \cos \theta}$

(c) $2\sqrt{\frac{gr}{\sin 2\theta}}$

✓ (d) $> 2\sqrt{\frac{gr}{\sin 2\theta}}$

Approach: Cylindrical room is irrelevant. Path will be same in a normal cuboidal room. It will travel in the vertical plane.

$$u \cos \theta T = r + 2r + r$$

$$\Rightarrow u = \frac{4r}{T \cos \theta}$$

As it strikes the roof, it shortens the travel time
 $\therefore T$ must be less than that of a normal projectile.

$$\Rightarrow \frac{4r}{u \cos \theta} < \frac{2u \sin \theta}{g}$$

$$\Rightarrow u > 2\sqrt{\frac{gr}{\sin 2\theta}}$$

14. Travelling a distance on a bicycle is less toiling than running the same distance in equal time. Which of the following factor is responsible for this well-known fact?

- ✓ (a) Gravity
- (b) Friction force
- (c) Mechanical advantage of the bicycle
- (d) All the above

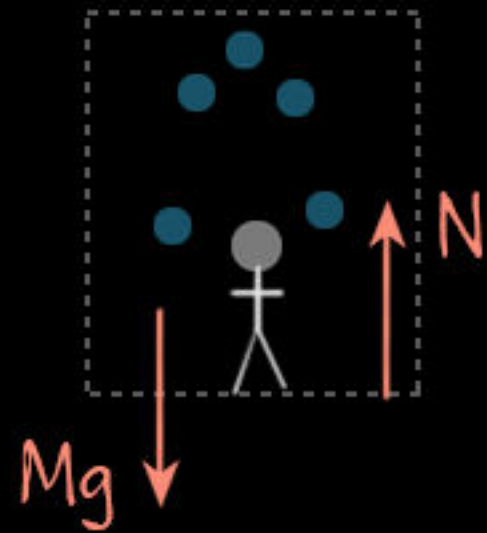
While walking our center of mass keeps bobbing up and down.

Lets say it shifts 1 cm each step for a 100 kg man.

So every step, amount of work is same as lifting a 1kg mass by 100cm.

Let normal reaction at feet of juggler be N . Its variable.

Mass of (juggler + balls) be M .



15. An expert juggler, carrying five identical balls, has to cross a swing bridge, maximum safe load rating of which is 50 kgf. The juggler weighs 47 kgf and each ball weighs 2 kgf. He believes that he can cross the bridge safely by juggling the balls in such a way that he never has more than one ball in his hands. His skill enables him to juggle smoothly without any jerk. What can you say about his belief?

- (a) Correct, as total load on the bridge never exceeds 49 kgf.
- (b) Correct, as total load on the bridge slightly exceeds 49 kgf.
- ✓ (c) Incorrect, as average load on the bridge is $47 \text{ kgf} + 5 \times 2 \text{ kgf} = 57 \text{ kgf}$.
- (d) I am indecisive, as more information is needed.

In a long period Δt : $\int F dt = \Delta p$

$$\Rightarrow \int_0^{\Delta t} (N - Mg) dt = \Delta p_{\text{juggler + balls}} = 0$$

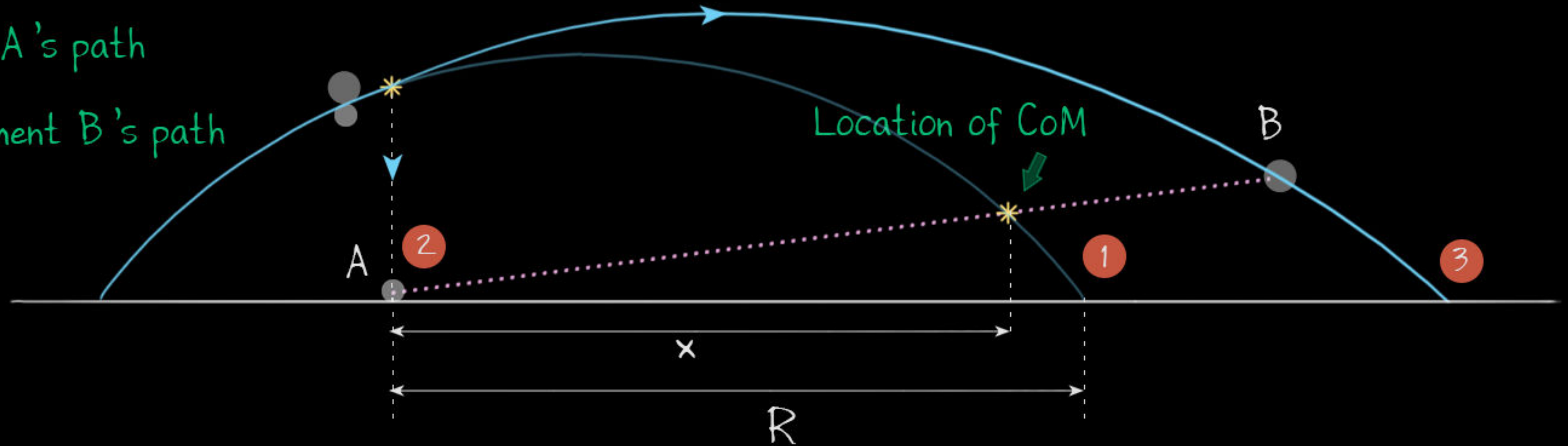
$$\Rightarrow \int_0^{\Delta t} N dt = Mg \Delta t$$

$$\begin{aligned} \therefore N_{av} &= \frac{\int_0^{\Delta t} N dt}{\Delta t} = Mg \\ &= (47 + 5 \times 2) \text{ kgf} // \end{aligned}$$

Concept: – CoM will continue to traverse the original path.

As A's vertical velocity upwards is reduced, B's will increase.

- 1 Original path
- 2 Free falling A's path
- 3 Other fragment B's path

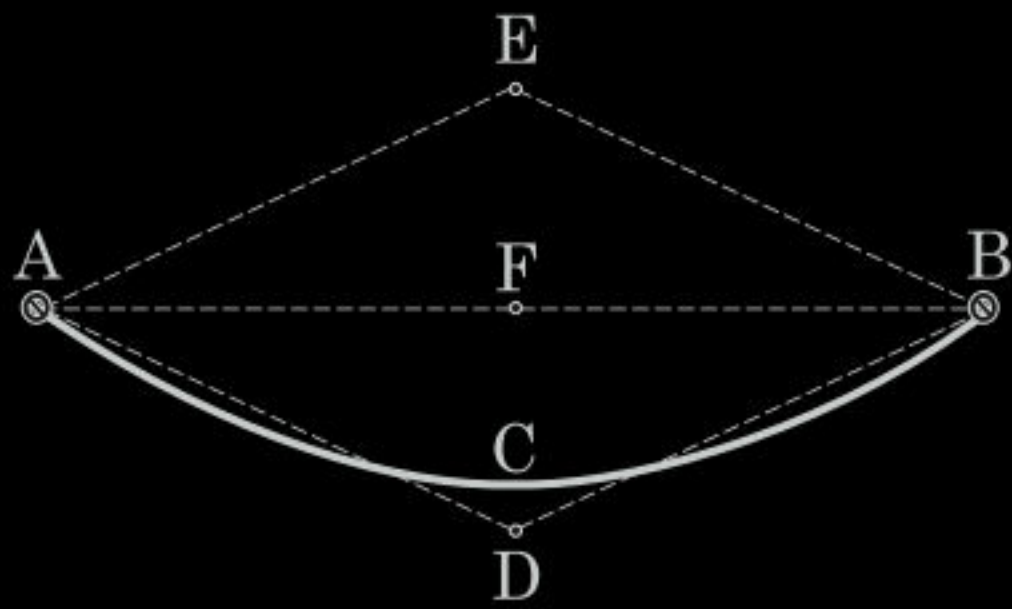


As time depends on v_y : $T_2 < T_1 < T_3$

So when A hits the ground, B will be still up in air, and $x < R$.

16. A shell fired from the ground explodes into two fragments A and B somewhere before the highest point of its trajectory. The fragment A immediately comes to an instantaneous rest and then falls. Knowing that horizontal range of the shell would have been R , if it were not exploded, predict location of mass centre of the two fragments relative to the location of firing, when the fragment A hits the ground.

✓(c) In the air at a horizontal distance that is less than R .



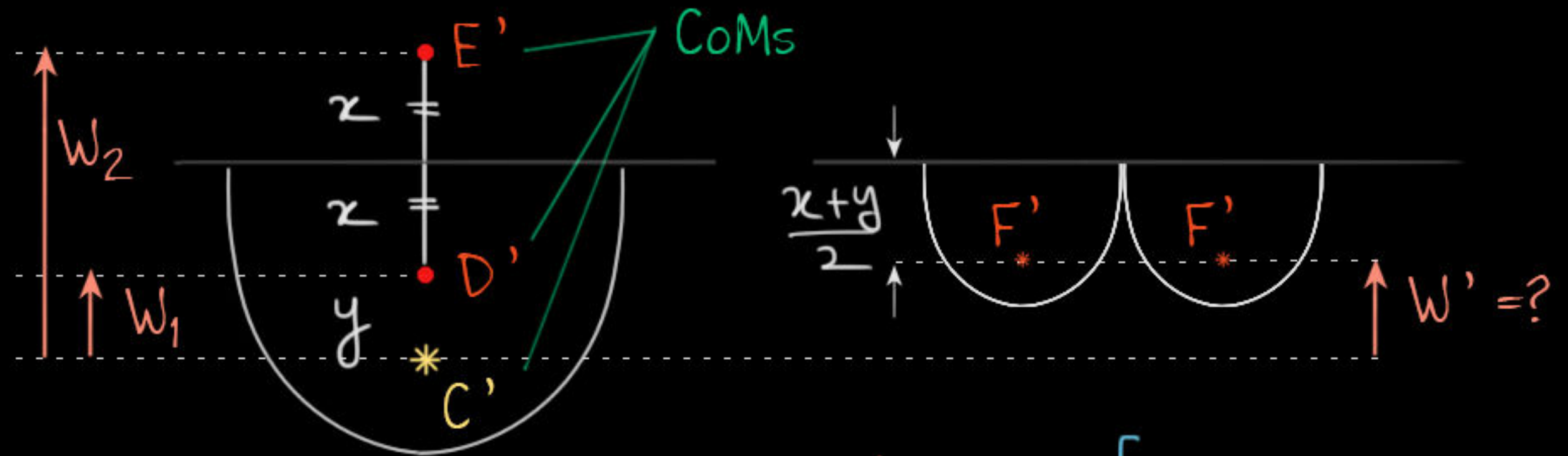
Concepts:

- External work is done to raise the CoM of rope. Because rope is inextensible and can't store energy.
- Shape of rope AB at equilibrium is similar to shape of rope AF and FB at equilibrium. That is:



17. A uniform rope suspended between two level nails A and B assume a curved shape ABC as shown in the figure. The midpoint C of the rope is pulled vertically down to a position D to make a V shape and released. When the rope again acquires the curved shape ACB, the midpoint is pulled vertically up to a position E to make an inverted V shape. If in these two pulling processes, total work done by the pulling agency is W , how much work will be done by the pulling agency in lifting vertically the midpoint from equilibrium to a position F in level with the nails?

- ✓(a) $W/4$ (b) $W/3$ (c) $W/2$ (d) None of these



$$W_1 = mgy$$

$$W_2 = mg(2x+y)$$

$$W_1 + W_2 = 2mg(x+y) = W$$

$$W' = mg \left[(x+y) - \left(\frac{x+y}{2} \right) \right]$$

$$= \frac{mg(x+y)}{2} = \frac{W/2}{2}$$

$$= W/4$$

Youtube / Arihant Senior

by Ankit Singhvi

18. Total mass of a system consisting of several particles is 10 kg. To apply Newton's laws of motion in **centroidal frame**, you have to assume a pseudo force of $(4\hat{i} - 2\hat{j})$ N acting on a particle of mass 2 kg of the system. What is the net external force acting on the whole system?

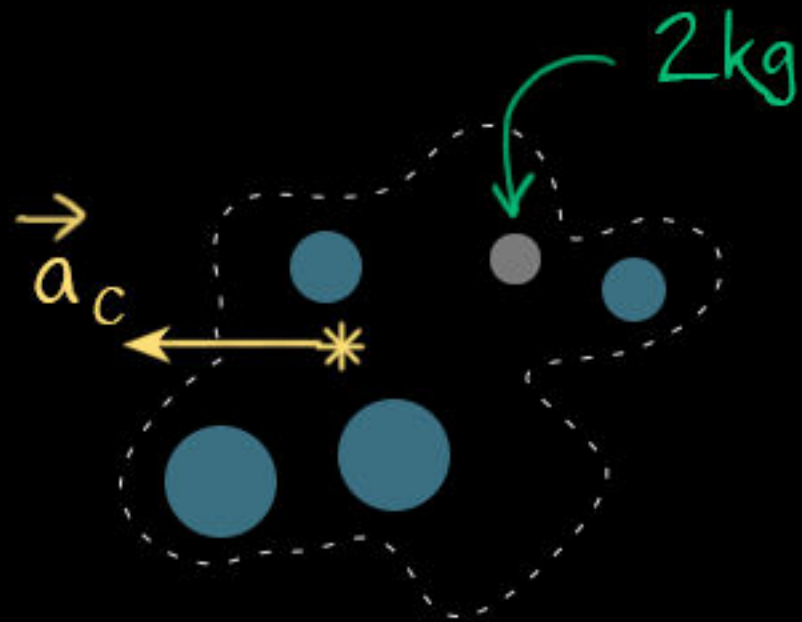
(a) $(10\hat{i} - 20\hat{j})$ N

(b) $(-10\hat{i} + 20\hat{j})$ N

(c) $(20\hat{i} - 10\hat{j})$ N

✓(d) $(-20\hat{i} + 10\hat{j})$ N

CoM frame



$$\begin{aligned} \text{2kg} \quad \vec{F}_{\text{psuedo}} &= 2(-\vec{a}_c) = 4\hat{i} - 2\hat{j} \\ \Rightarrow \vec{a}_c &= -2\hat{i} + \hat{j} \end{aligned}$$

$$\begin{aligned} \therefore \text{For 10kg system } F_{\text{net}} &= 10 a_c \\ &= -20\hat{i} + 10\hat{j} \end{aligned}$$

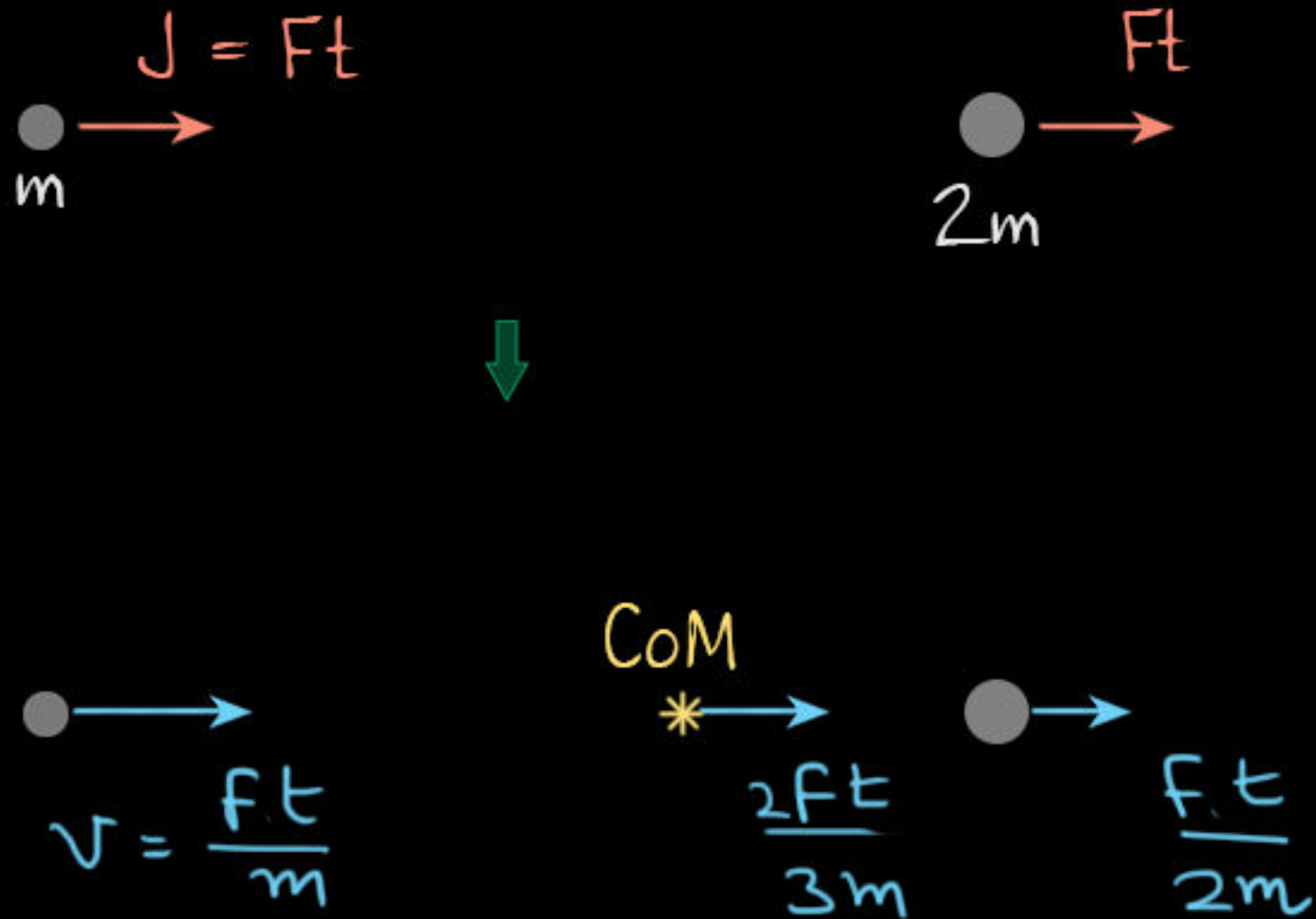
19. Consider two particles A and B of masses m and $2m$ at rest in an inertial frame. Each of them are acted upon by net forces of equal magnitudes F in the positive x -direction for equal amounts of time t . Momenta of the particles A and B in centre of mass frame respectively are

(a) $\frac{2Ft}{3}\hat{i}$ and $-\frac{2Ft}{3}\hat{i}$

(b) $-\frac{2Ft}{3}\hat{i}$ and $\frac{2Ft}{3}\hat{i}$

✓(c) $\frac{Ft}{3}\hat{i}$ and $-\frac{Ft}{3}\hat{i}$

(d) $-\frac{Ft}{3}\hat{i}$ and $\frac{Ft}{3}\hat{i}$

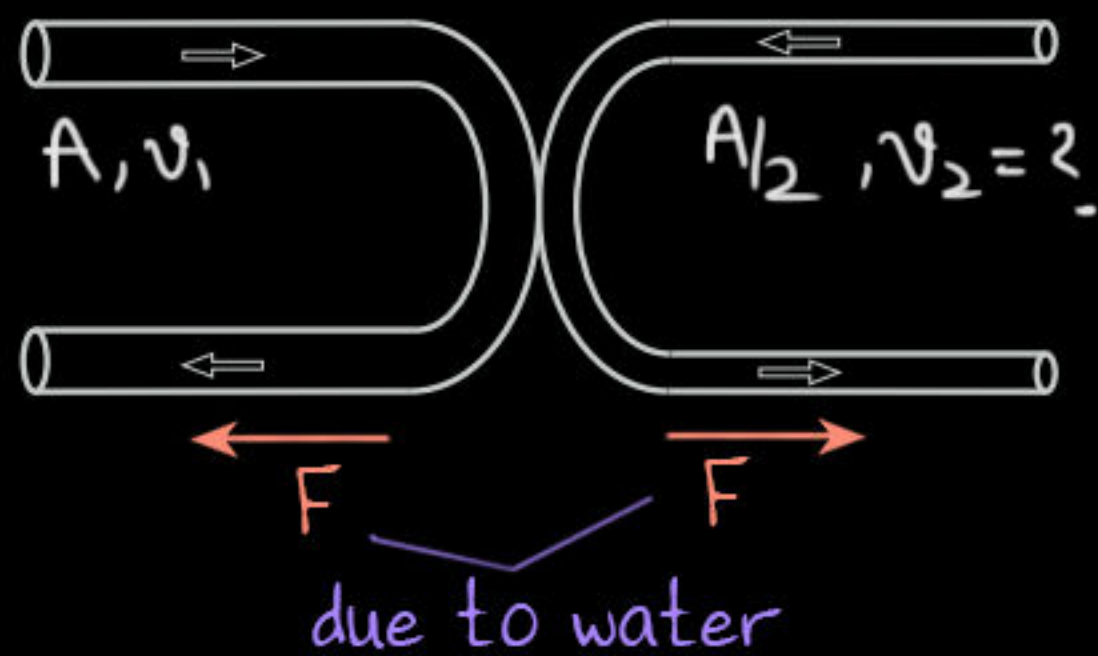


∴ w.r.t CoM

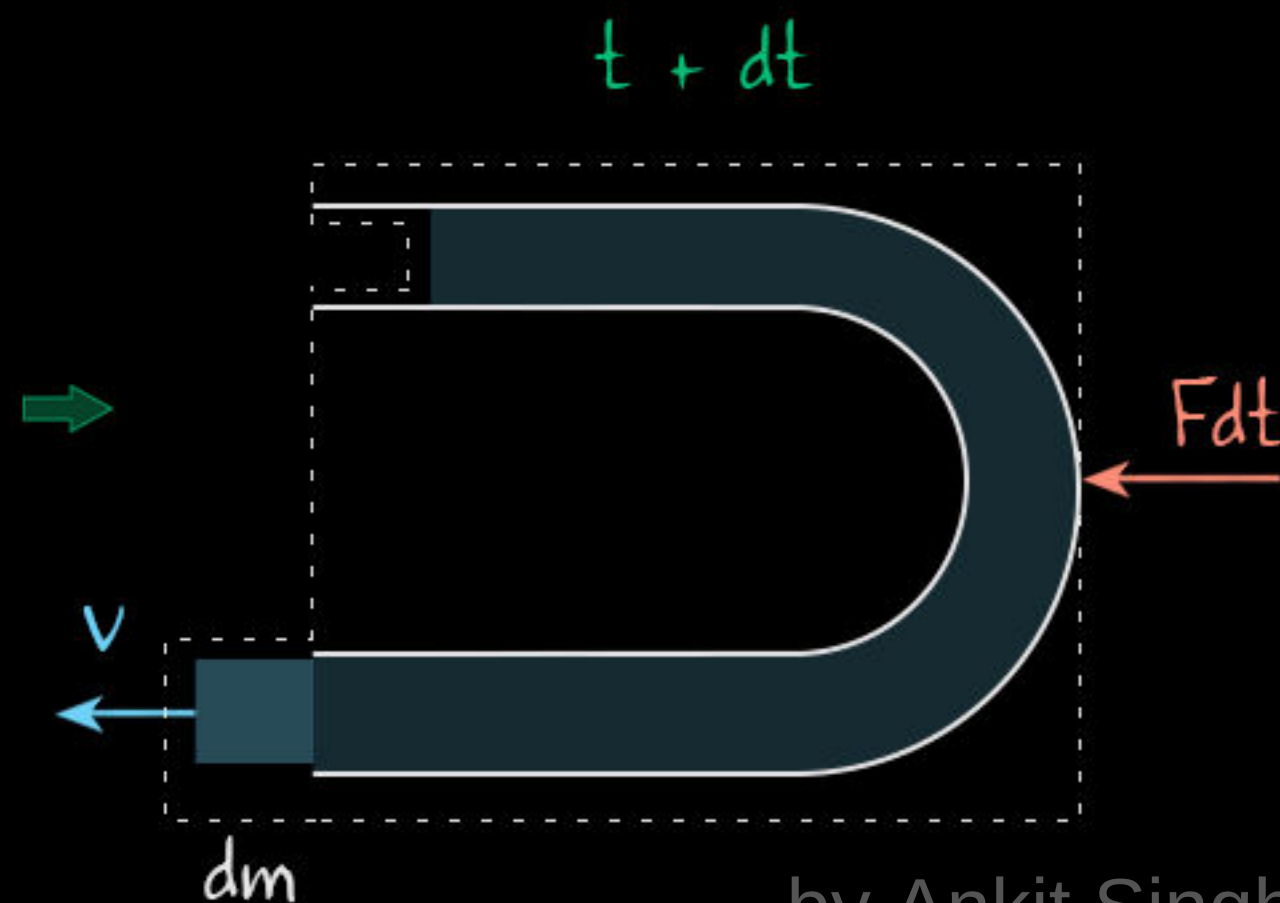
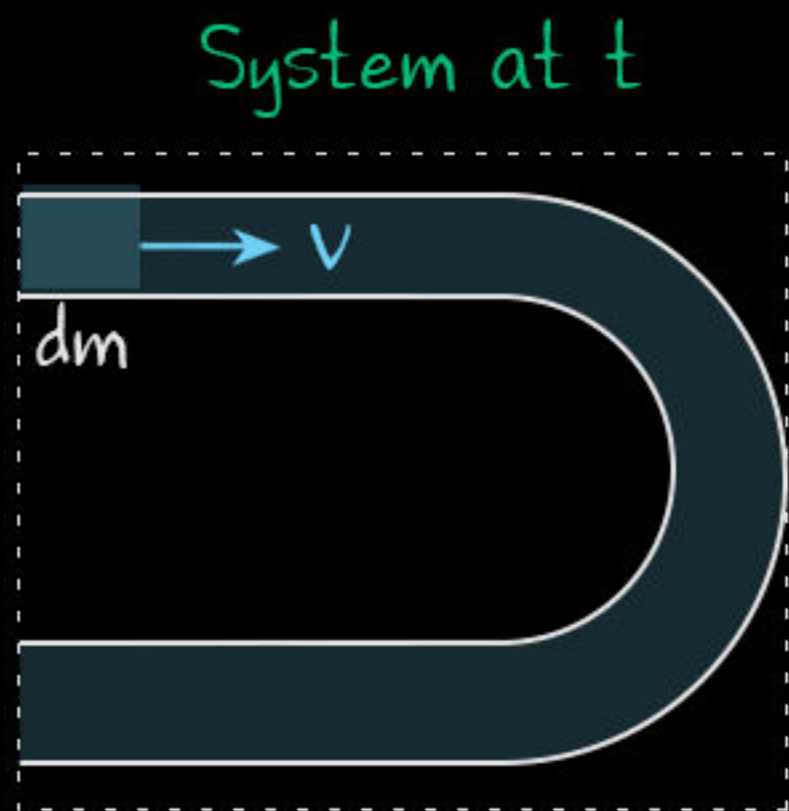
$$\bullet \rightarrow \frac{Ft}{3m} \Rightarrow p_1 = m \left(\frac{Ft}{3m} \right) = \frac{Ft}{3} \hat{i} //$$

$$p_2 = -p_1 = -\frac{Ft}{3} \hat{i} //$$

Youtube / Arihant Senior



Approach: In flowing liquid, we must define the system first and then apply rate of change of momentum of that system to get the force.



20. Water is flowing through two rigidly connected coplanar U tubes. In the left tube of cross-sectional area A , speed of water flow is u and in the right tube of cross-sectional area $A/2$ speed water flow is unknown. If the net force on the tube assembly due to water flow is zero, what must be the speed of the water flow in right tube? Neglect effect of gravity.

(a) $\frac{u}{2}$
 ✓ (c) $u\sqrt{2}$

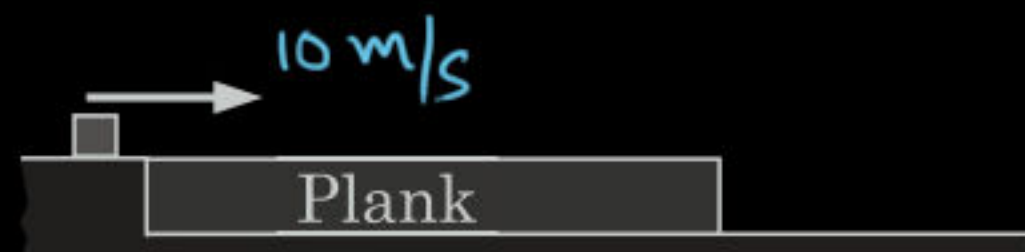
(b) $\frac{u}{\sqrt{2}}$
 (d) $2u$

General derivation

$$\begin{aligned}
 Fdt &= dm(2v) \\
 \Rightarrow F &= 2v \frac{dm}{dt} \\
 &= 2v(\rho A v) \\
 &= 2\rho A v^2 //
 \end{aligned}$$

Sol:

$$\begin{aligned}
 F &= F \\
 \Rightarrow \cancel{A_1} v_1^2 &= \cancel{A_2} v_2^2 \\
 \Rightarrow v_2 &= \sqrt{2} v_1 //
 \end{aligned}$$

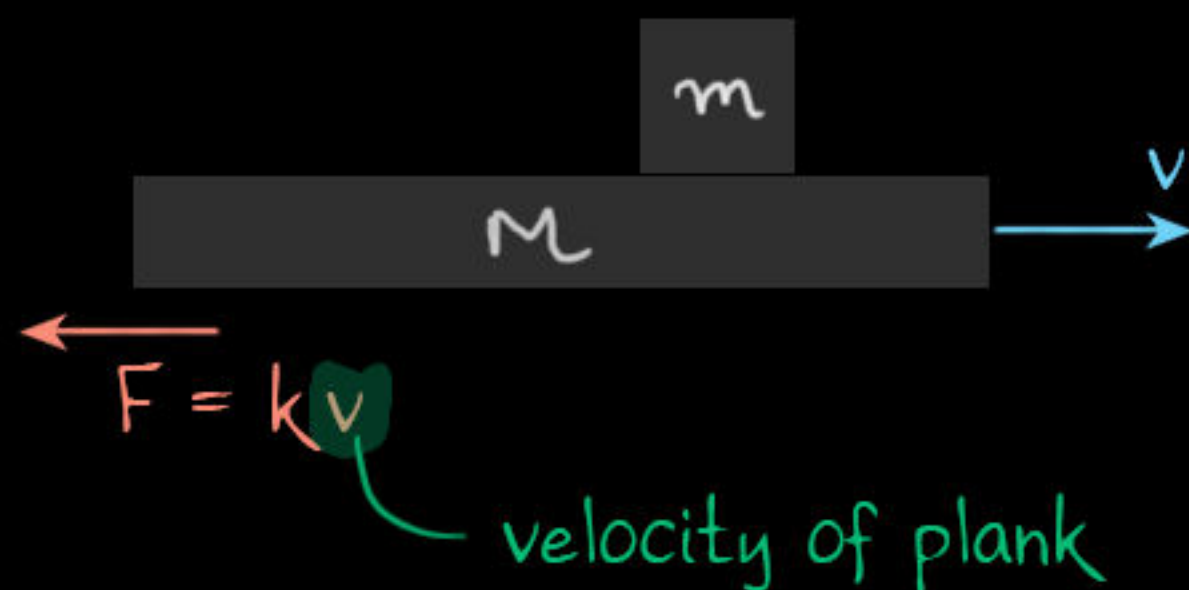


A plank of mass 1.5 kg is placed on a horizontal floor that is lubricated with oil. Top of the plank is in level with a platform as shown in the figure. Force of viscous drag on the plank due to layer of lubricating oil, when the plank slides is given by equation $\vec{f} = -k\vec{v}$, where $\vec{f}$ is in N, $\vec{v}$ is velocity of the plank in m/s and $k = 2.0 \text{ kg/s}$. A small block of mass 0.5 kg lands from the platform on the plank with a velocity 10 m/s and after sliding some distance on the plank, the block stops on the plank.

21. How far does the plank slide on the floor?

- (a) 1.0 m (b) 1.5 m
 ✓ (c) 2.5 m (d) 3.0 m

On block plank system:



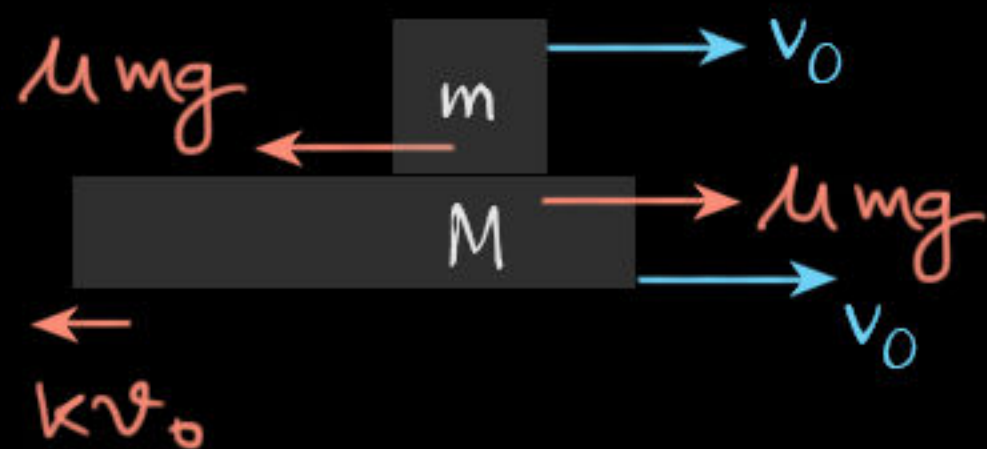
$$F dt = \Delta p$$

$$\Rightarrow -kv dt = 0 - mu$$

$$\Rightarrow -k \int v dt = -mu$$

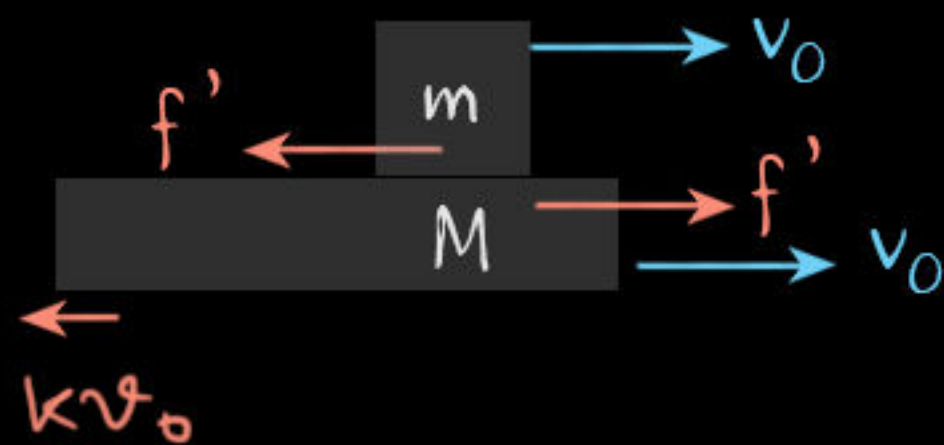
$$\therefore S_{\text{plank}} = \frac{mu}{k} = \frac{0.5(10)}{2} = 2.5 \text{ m}$$

Just before sliding stops:



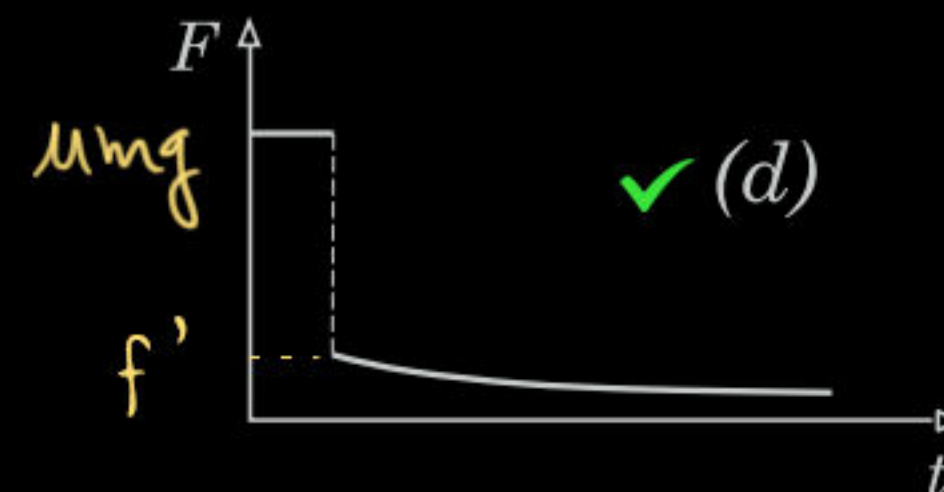
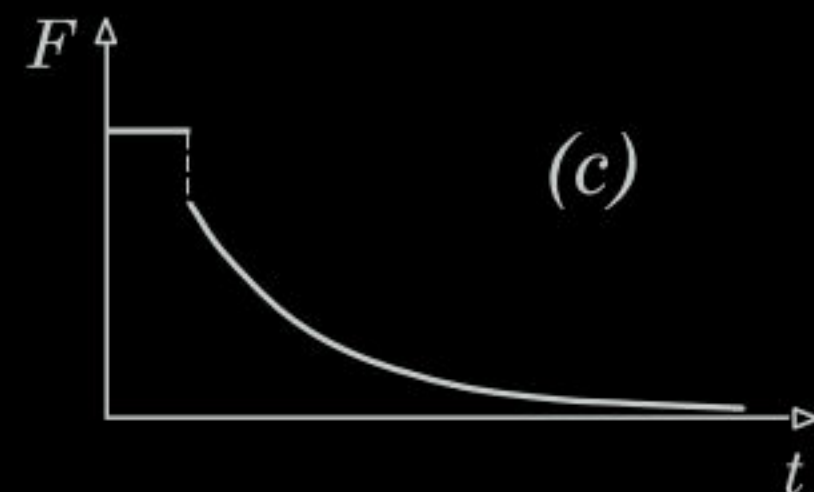
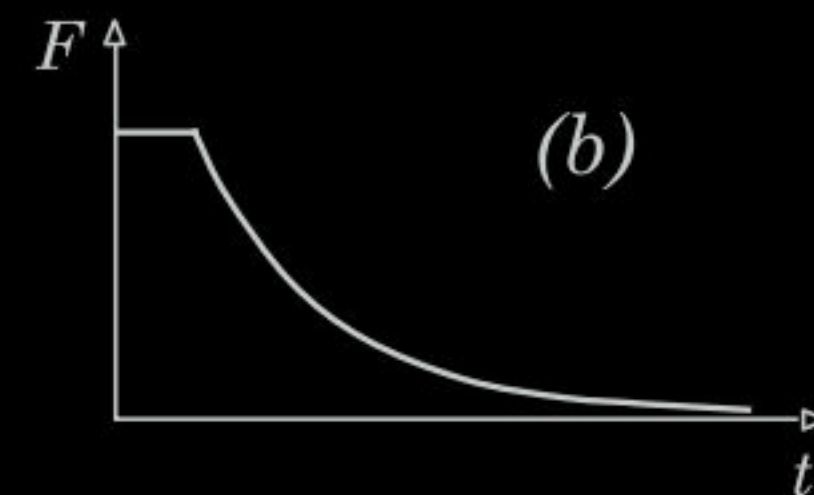
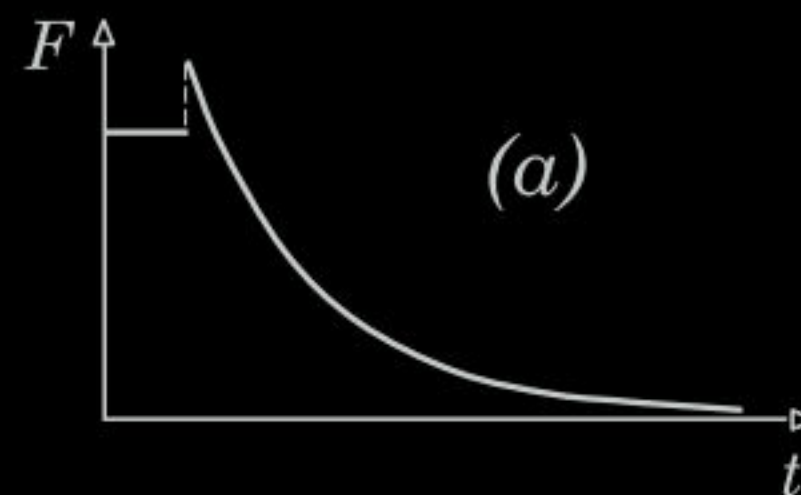
$$\mu mg \geq kv_0 \quad (1)$$

Just after sliding stops:



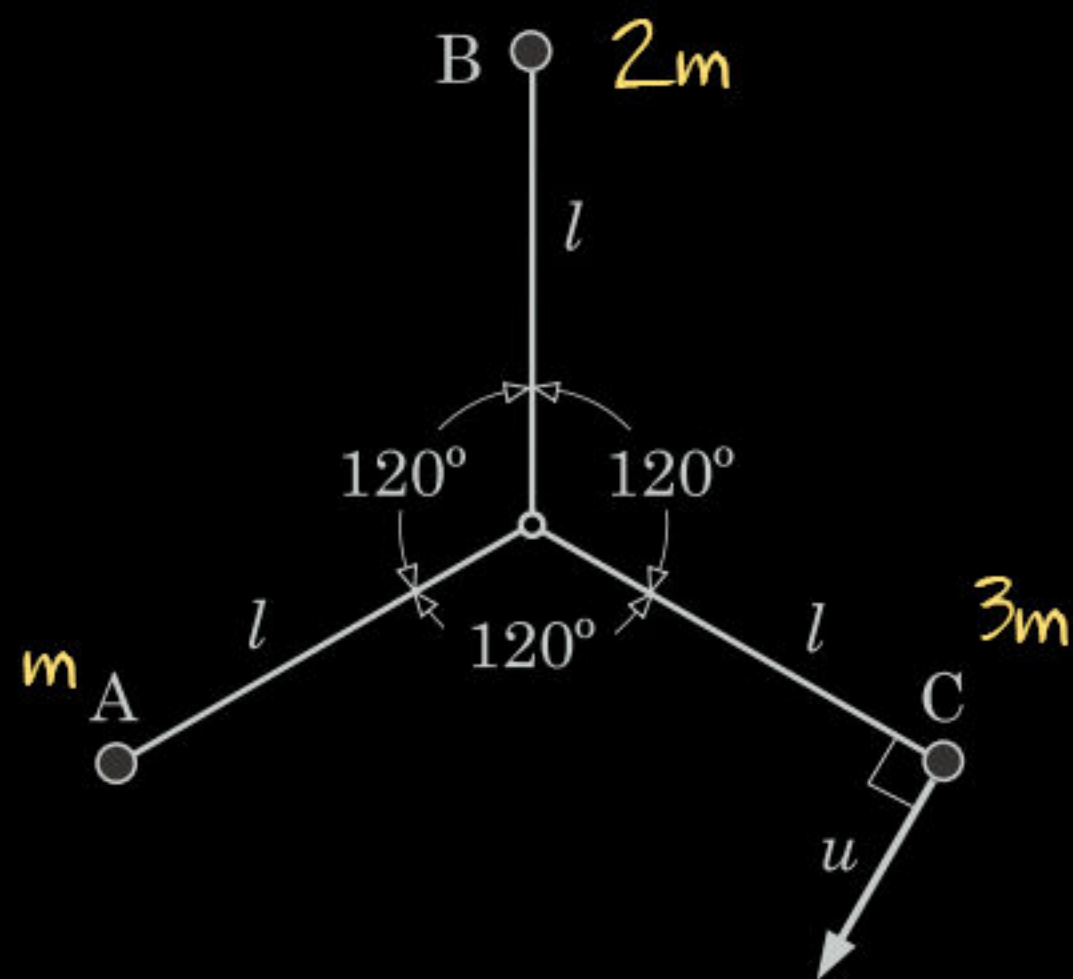
$$a = \frac{f'}{m} = \frac{kv_0 - f'}{M} \quad (2)$$

22. Which of the following graph best describes relation between magnitude of frictional force F between the block and the plank with time t ?



From 1,2: $\frac{f'}{m} \leq \frac{\mu mg - f'}{M}$

$$\Rightarrow f' \leq (\mu mg) \frac{m}{m+M} \Rightarrow f' \leq \frac{\mu mg}{4}$$

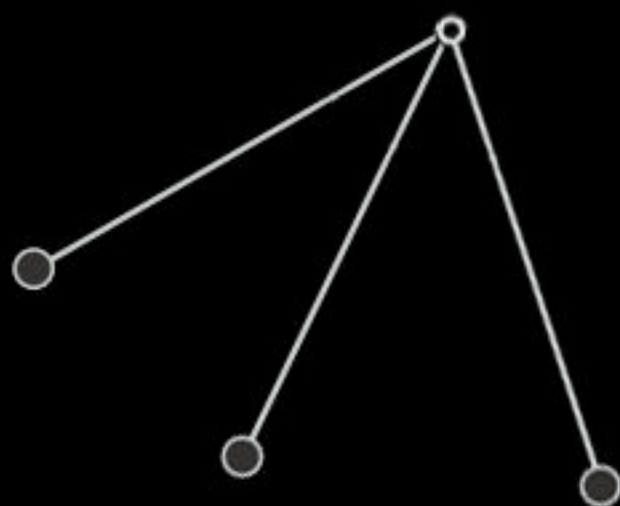


Ends of three light identical rods each of length l are connected to a light pivot that enables them to rotate in any direction. At the other ends of the rods, three particles A, B and C of masses m , $2m$ and $3m$ respectively are affixed. Initially, the rods are coplanar, angle between any two adjacent rod is 120° and the particles are at rest. Now the particle C is given a velocity u perpendicular to the rod connected to it and in the plane of the rods as shown in the figure. Ignore gravitational interaction between the particles.

23. Denoting acceleration vectors of the particles A, B and C by $\vec{a}_A$, $\vec{a}_B$ and $\vec{a}_C$ respectively, which of the following conclusion can you make?

✓ (c) $\vec{a}_A + 2\vec{a}_B + 3\vec{a}_C = \vec{0}$

⇒ System can be rearranged like this:



$F_{\text{net on system}} = 0$

⇒ $m\vec{a}_1 + 2m\vec{a}_2 + 3m\vec{a}_3 = 0$

⇒ $\vec{a}_1 + 2\vec{a}_2 + 3\vec{a}_3 = 0$

24. What are moduli of acceleration vectors of the particles A, B and C immediately after the particle C is given velocity u ?

1 F_{net} on light joint = 0

∴ All tensions are equal (by Lami's theorem). Let's say it's T .

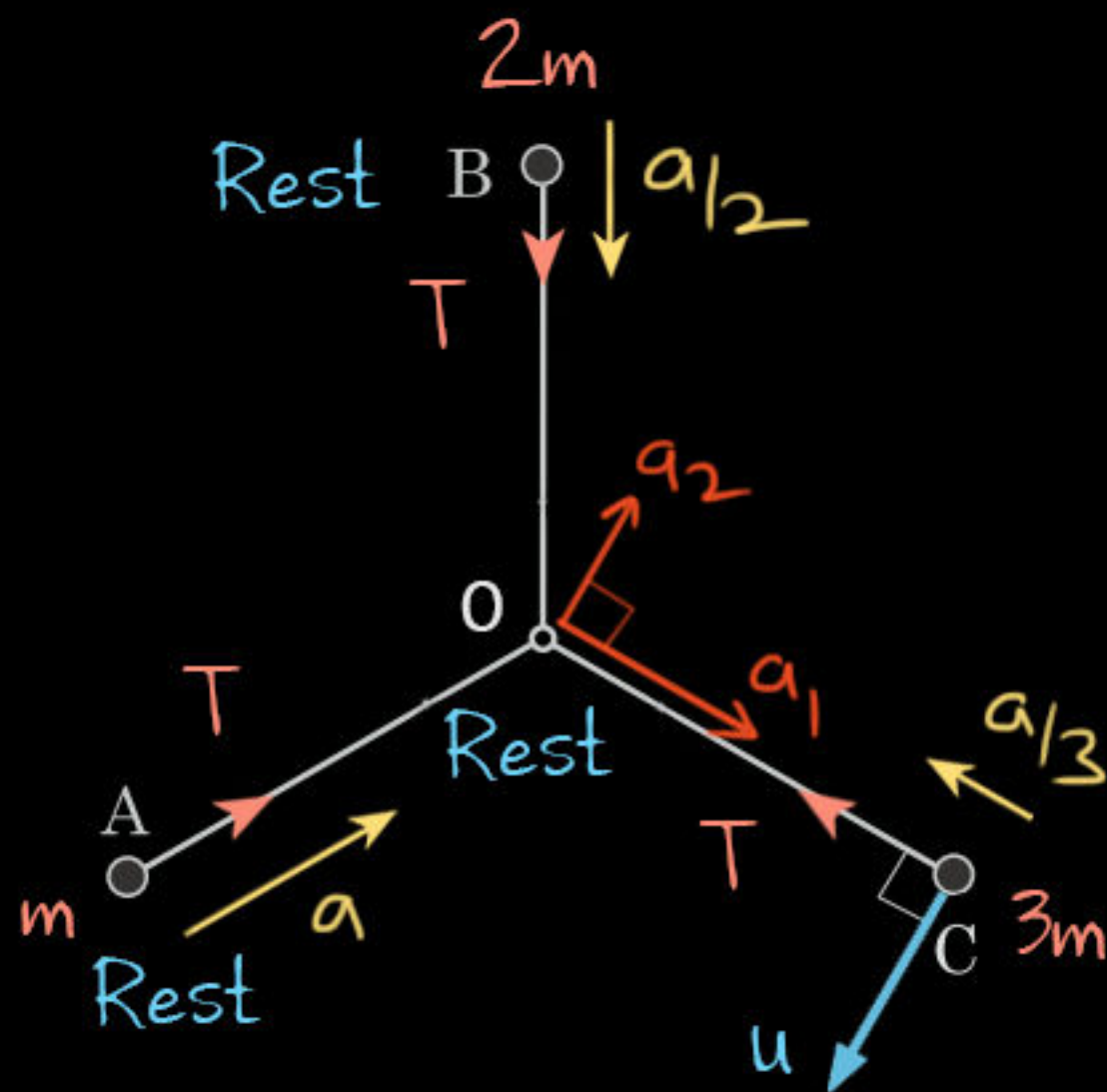
2 Each rod can apply force only along its length. As rods can freely rotate about pivot that is at rest immediately afterwards. A and B are at rest too btw.

∴ Accelerations of A, B, C are $\frac{T}{m}$, $\frac{T}{2m}$, $\frac{T}{3m} = a$, $a/2$, $a/3$

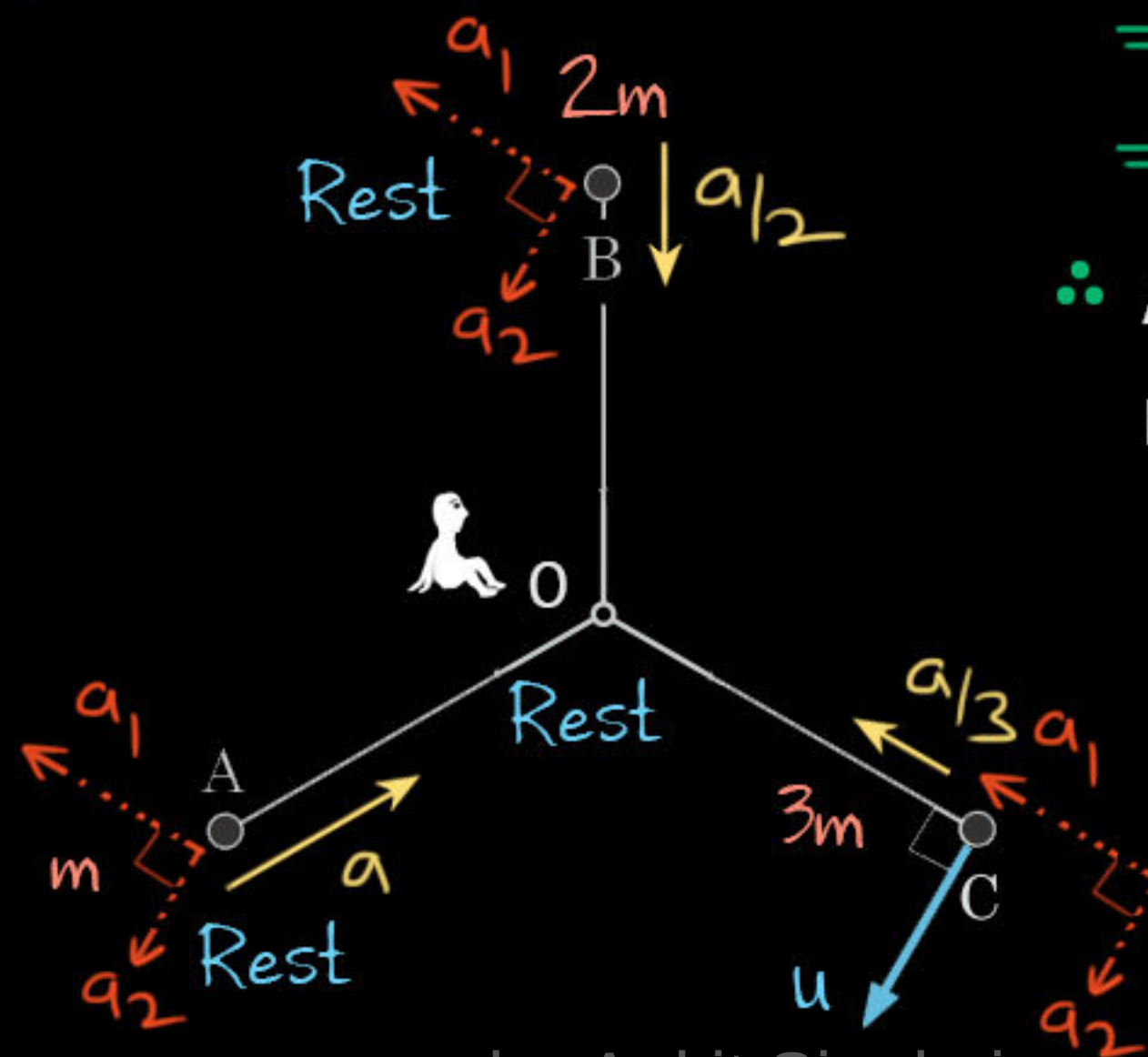
3 Let O's acceleration be a_1 and a_2 as shown.

✓ (c) $\frac{6u^2}{11l}$, $\frac{3u^2}{11l}$ and $\frac{2u^2}{11l}$

w.r.t ground



w.r.t O



Acceleration of A and B along the rod = acceleration of pivot along the rod = zero. (∴ ω of these rods is zero)

∴ A: $\frac{a}{2} + a_2 \cos 30^\circ = a \cos 60^\circ$ (1)

B: $a = a_2 \cos 30^\circ + a_1 \cos 60^\circ$ (2)

Acceleration of C along the rod = $\frac{u^2}{l}$

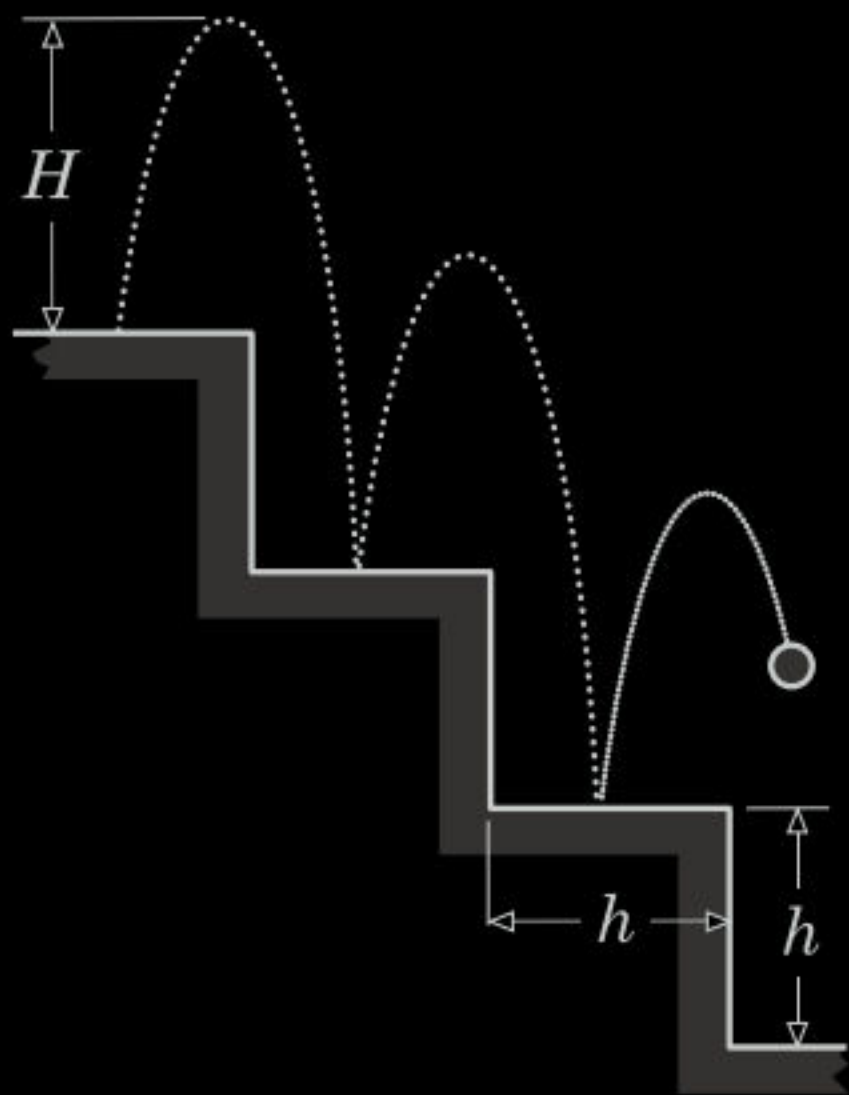
C: $a_1 + \frac{a}{3} = \frac{u^2}{l}$ (3)

From 1, 2, 3:

a , $a/2$, $a/3$
 $\frac{6u^2}{11l}$, $\frac{3u^2}{11l}$, $\frac{2u^2}{11l}$

by Ankit Singhvi

Youtube / Arvind senior



A marble is bouncing down a stairs in a regular manner, hitting each step at identical points and then rising the **same height** H above each step. Height and depth of a stair each are equal to h as shown in the figure. Coefficient of restitution for each bounce is e .

25. **Air time** between two successive bounces is

26. **Horizontal component of velocity** of the marble is

27. The height H attained by the marble after a bounce is

✓ 25: $\sqrt{\frac{2h(1+e)}{g(1-e)}}$ ✓ 26: $\sqrt{\frac{gh(1-e)}{2(1+e)}}$ ✓ 27: $\frac{e^2h}{1-e^2}$

$$v_1 = \sqrt{2gH}$$

$$v_2 = \sqrt{2g(h+H)}$$

dividing:

$$\frac{v_1}{v_2} = e = \sqrt{\frac{H}{h+H}}$$

$$\Rightarrow H = \frac{e^2h}{1-e^2} \quad \text{①}$$

$$T = T_{\text{up}} + T_{\text{down}} \\ = \sqrt{\frac{2H}{g}} + \sqrt{\frac{2(h+H)}{g}}$$

↓ From 1

$$T = \sqrt{\frac{2h(1+e)}{g(1-e)}} \quad \text{②}$$

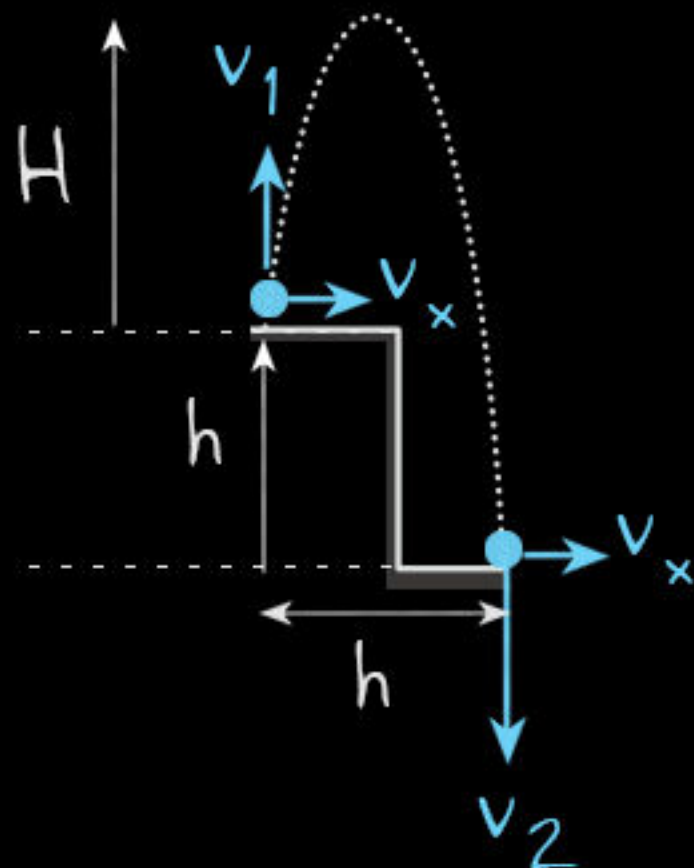
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$$v_x T = h \\ v_x = \frac{h}{T}$$

↓ From 2

$$v_x = \sqrt{\frac{gh(1-e)}{2(1+e)}}$$

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Everything we will talk about will be in given reference frame.

$$\vec{p}_{\text{system}} = 10 (\vec{v}_1 + \vec{v}_2 + \vec{v}_3) \\ = 30 (2\hat{i} - \hat{j})$$

∴ Shell was traveling with

$$\vec{v}_0 = 2\hat{i} - \hat{j} \text{ before explosion.}$$

$$\therefore E_{\text{released}} = KE_f - KE_i$$

$$= \frac{1}{2} 10 (v_1^2 + v_2^2 + v_3^2) - \frac{1}{2} 30 v_0^2$$

$$= 335 - 75 = 260 \text{ J} //$$

A 30 kg shell in free space explodes into three identical fragments. After the explosion, the fragments rush apart. Forces of mutual interaction between the fragments are long range attractive conservative forces. Immediately after the explosion, velocities of the fragments relative to an inertial frame are $(3\hat{i} + 2\hat{j})$ m/s, $-2\hat{i}$ m/s and $(5\hat{i} - 5\hat{j})$ m/s.

28. If entire energy released in the explosion is converted into kinetic energy of the fragments, how much energy has been released in the explosion?

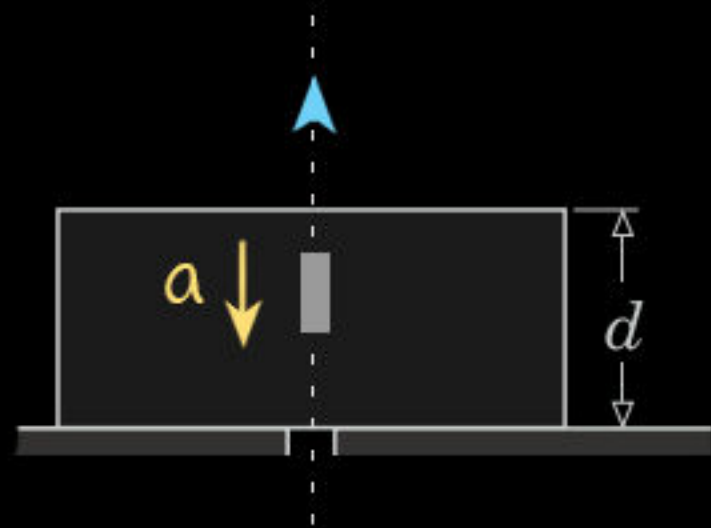
$$(a) 260 \text{ J} \quad (c) 335 \text{ J}$$

29. After sufficiently long time, when the forces of mutual interactions cease to act, what can be minimum total kinetic energy of all the fragments?

$$(c) 75 \text{ J}$$

$$\text{Energy retained} = KE_{\text{cm}} = KE_i$$

$$= \frac{1}{2} 30 v_0^2 = 75 \text{ J} //$$



For bullet:

$$v^2 = u^2 - 2ad$$

$$\Rightarrow a = \frac{u^2 - v^2}{2d}$$

Note: We are neglecting the effect of gravity on bullet, as its contribution in a is negligible compared to friction.

1. A wooden bar of thickness $d = 20$ cm is symmetrically placed on a hole made in a horizontal tabletop. A bullet of mass $m = 20$ g shot vertically up through the hole, pierces the wooden bar. Within the bar, speed of the bullet reduces uniformly from $u = 100$ m/s at the bottom to $v = 80$ m/s at the top. What should the minimum mass of the bar be so that it will not leave the tabletop? Acceleration of free fall is $g = 10$ m/s².

with time

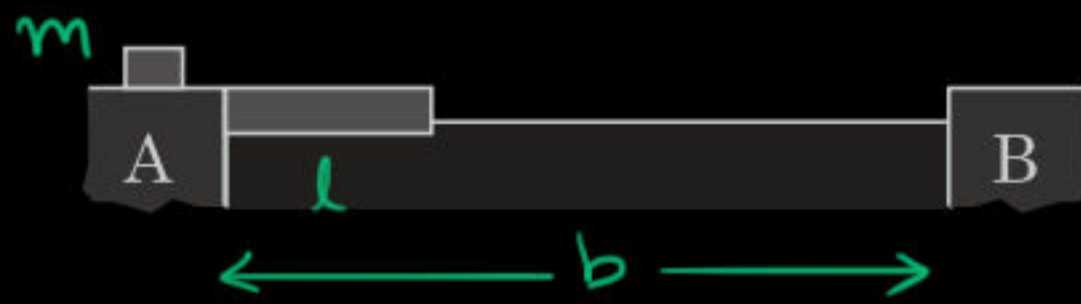
uniform acceleration



Force on bullet by bar = ma = Force on bar by bullet

∴ For bar to not lift: $Mg > ma$

$$\Rightarrow M > \frac{m}{g} \left(\frac{u^2 - v^2}{2d} \right)$$



Concept: Plank must stop at B for minimum speed of block.

∴ Displacement of plank: $S_p = b - l$

Let 'u' be the minimum velocity of block required.

2. A plank of length $l = 2.0$ m rests on a calm water close to a frictionless platform A that is a distance $b = 17$ m from another platform B as shown in the figure. A block of mass $m = 25$ kg sliding on the platform A lands on the plank and stays on it. Force of water resistance on the plank is given by equation $\vec{F} = -k\vec{v}$, where $\vec{F}$ in N is the force of water resistance, $\vec{v}$ in m/s is velocity of the plank and $k = 15$ N·s/m. Calculate the minimum speed of the block on the platform A so that the plank reaches the platform B.

Similar: MCQ 21,22

For system: $\int F dt = \Delta p$

$$\Rightarrow \int -k v_p dt = 0 - mu$$

$$\Rightarrow -k \int v_p dt = -mu$$

$$k S_p = mu$$

$$k(b-l) = mu$$

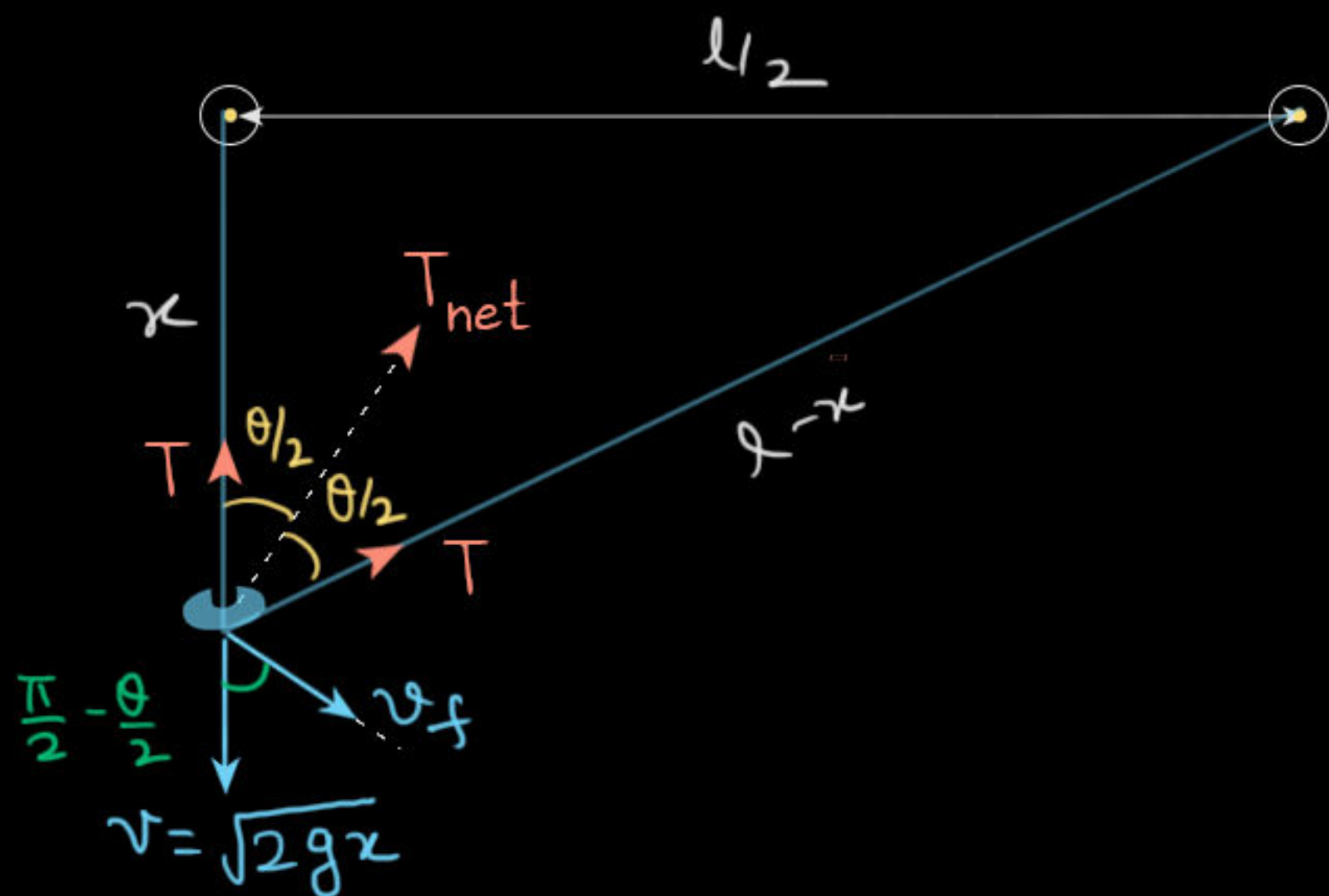
$$\therefore u = \frac{k(b-l)}{m}$$

Concept: Until taut, the massless cord can't exert any force on bead.
So bead will freefall until cord becomes taut.

3. A thin light inextensible frictionless cord of length l wearing a small bead is tied between two nails that are in the same level a distance $0.5l$ apart. Initially the bead is held close to a nail and released. Find speed of the bead immediately after the cord becomes taut.



Just after being taut



$$x^2 + \left(\frac{l}{2}\right)^2 = (l-x)^2 \Rightarrow x = \frac{3l}{8}$$

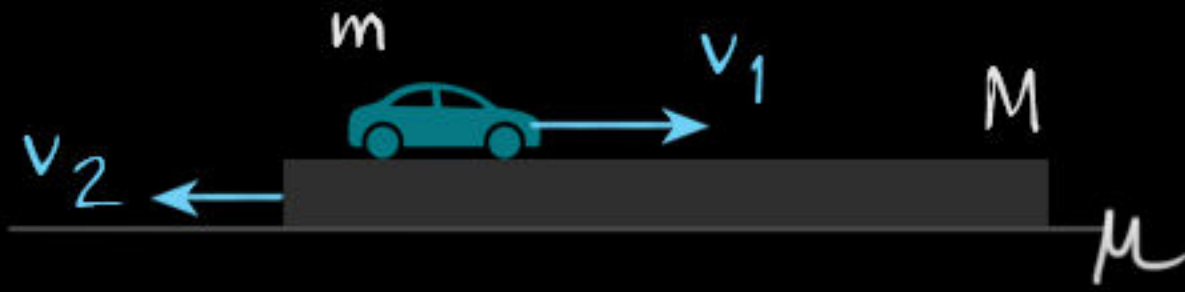
$$\Rightarrow \tan \theta = \frac{4/2}{3l/8} = \frac{4}{3} \Rightarrow \theta = 53^\circ$$

Concept: T_{net} will cause the jerk and stop the bead in its direction.

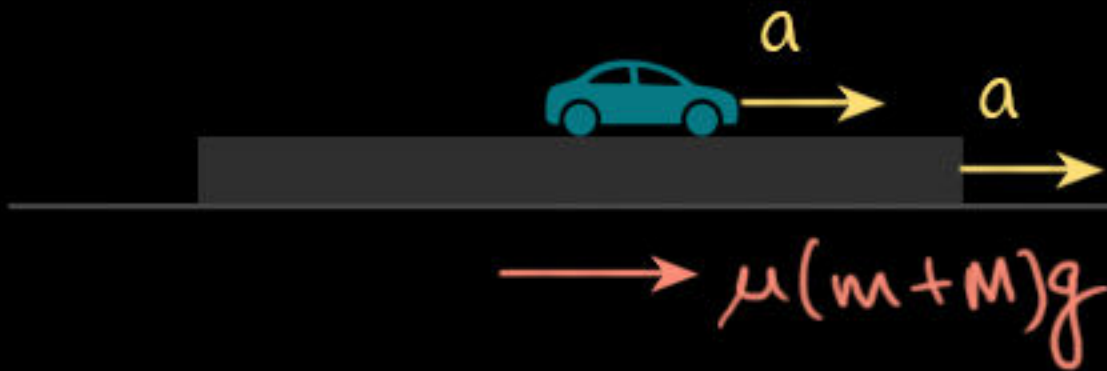
Perpendicular to T_{net} the velocity will remain the same.

$$\begin{aligned} \therefore v_f &= \sqrt{2gx} \cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) \\ &= \sqrt{2g \cdot \frac{3l}{8}} \sin\left(\frac{53}{2}\right) = \sqrt{\frac{3gl}{20}} \end{aligned}$$

Concept: At $t=0$, the car will jerk start with v w.r.t plank.



$0 < t < t_0$ (Until plank stops):
Since relative velocity v is constant, relative acceleration must be zero too.
So, both will have same a .



$t > t_0$. Once plank stops, car will continue to move with v .



4. A remote controlled car of mass m can move on a long plank of mass M that rests on a horizontal floor. The mechanism of the car is so designed that it always moves on the plank with a constant velocity v with respect to the plank. Coefficient of friction between the plank and the floor is μ and acceleration of free fall is g . How will velocity of the car with respect to the floor vary with time t after it is switched on?

At $t=0$: $mv_1 = Mv_2$
Given: $v_1 + v_2 = v$

$$v_1 = \frac{Mv}{m+M}, \quad v_2 = \frac{mv}{m+M}$$

$0 < t < t_0$ $a = \frac{\mu(m+M)g}{m+M} = \mu g$

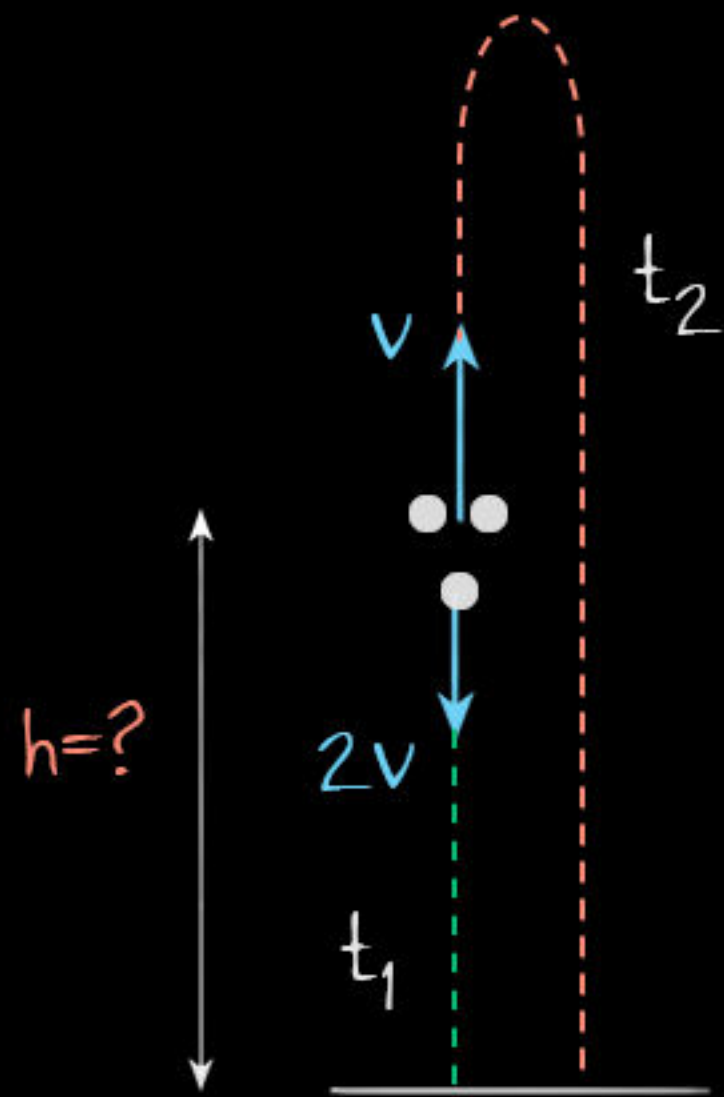
Plank: $0 = -v_2 + \mu g t_0$

$$\Rightarrow t_0 = \frac{v_2}{\mu g} = \frac{mv}{(m+M)\mu g}$$

$$\begin{aligned} \therefore v_{car} &= v_1 + \mu g t \\ &= \frac{Mv}{m+M} + \mu g t \end{aligned}$$

$t > t_0$ $v_{car} = v$
by Ankit Singhvi

Concept: Lets bother with vertical velocity components of particles only as both height and time are unaffected by horizontal components.



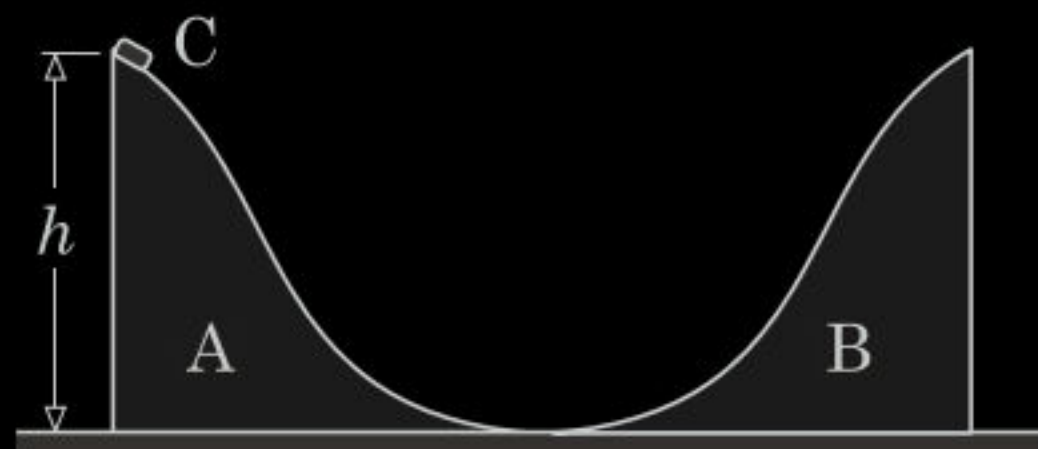
5. A shell projected vertically upward explodes into three identical fragments at the top of its flight. After the explosion, one fragment moves vertically downwards and hit the ground in time t_1 , while the other two hit the ground in time t_2 . Denoting acceleration due to gravity by g , find the height above the ground where the shell exploded.

$$h = vt_1 + \frac{1}{2}gt_1^2$$

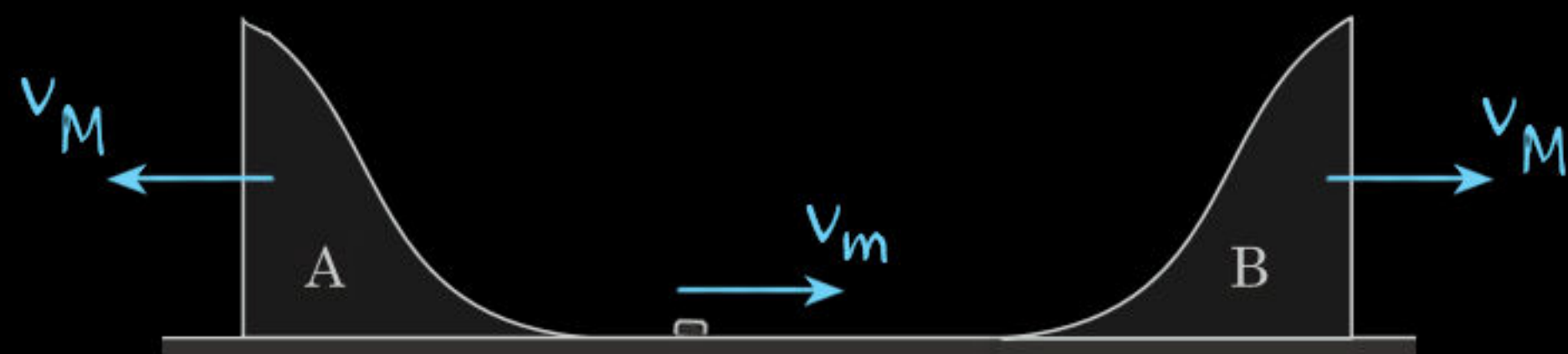
$$h = -vt_2 + \frac{1}{2}gt_2^2$$

Eliminating v :

$$h = \frac{gt_1t_2}{2} \frac{(t_1 + 2t_2)}{t_2 + 2t_1} //$$

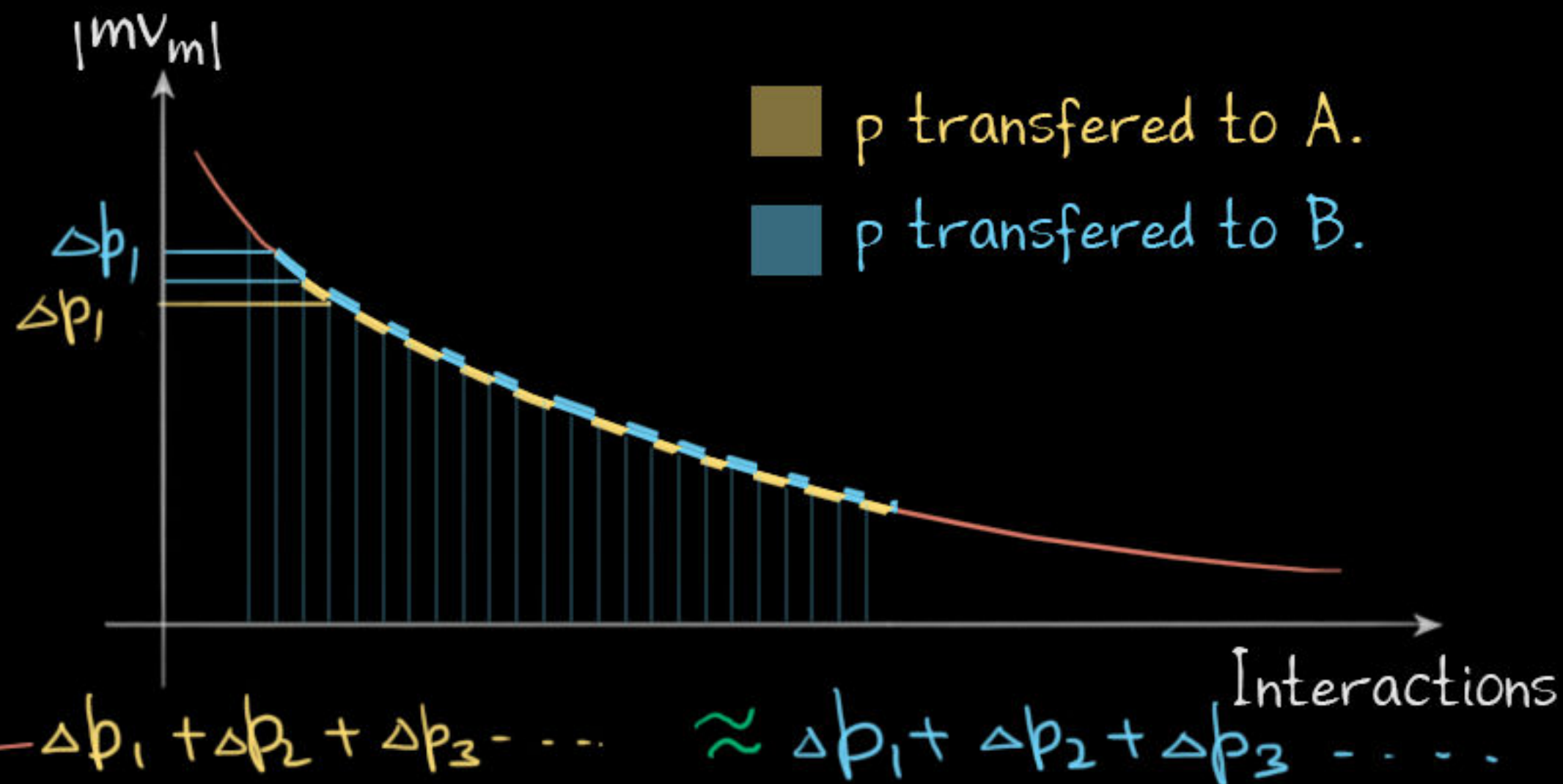


6. Two identical frictionless slides A and B each of height h and mass M are placed on a horizontal frictionless floor. A small disc C of mass $m \ll M$ is released on the top of the slide A. Find expressions for the speeds eventually acquired by both the slides when interaction of the slides with the bead ceases.



Concept: As $m \ll M$, there is little gain in velocity by slides everytime m interacts with any of the slides.

So at any given time: $v_A \approx v_B$



Interaction eventually ceases, when: $v_m \approx v_M$

Let the common velocity be v .

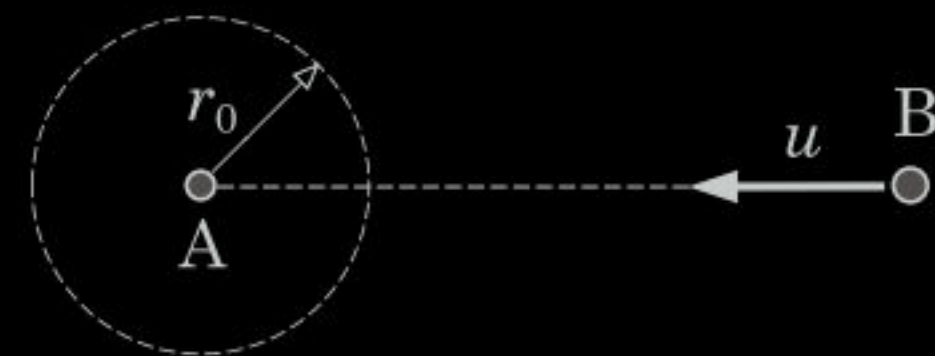
$$\therefore \text{Eventually: } mgh \approx \frac{1}{2} (m + 2M) v^2$$

$$\Rightarrow v \approx \sqrt{\frac{2mgh}{m + 2M}} \approx \sqrt{\frac{mgh}{M}}$$

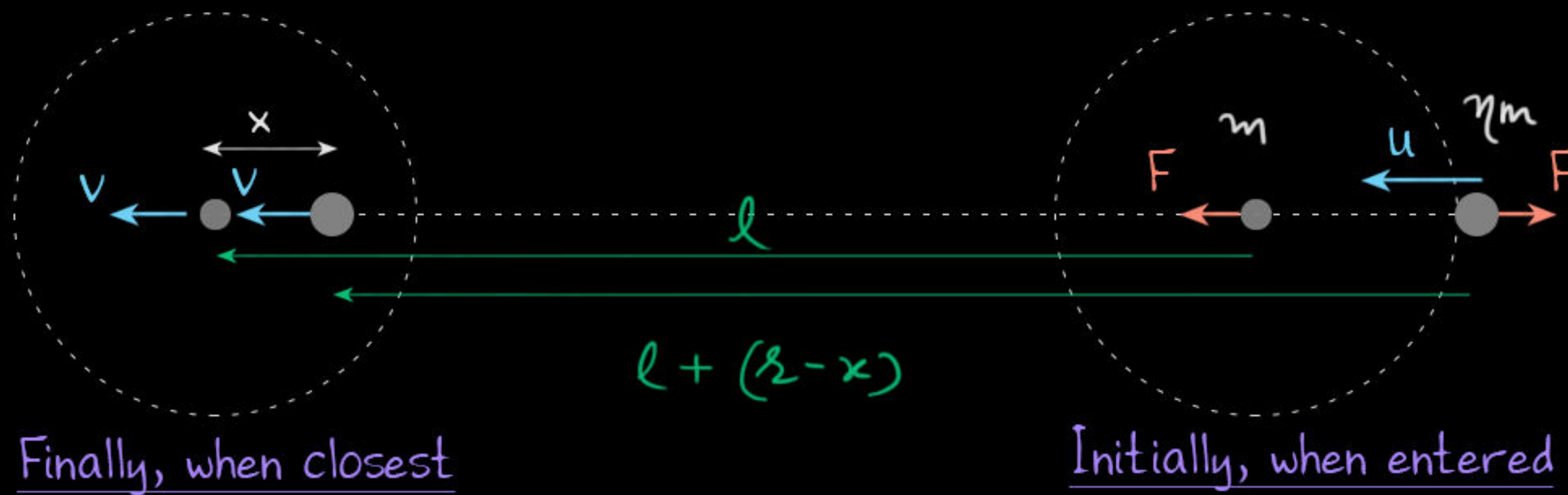
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7. Two particles A and B repel each other by a force $F = 6.0 \text{ N}$ when distance between them is less than $r_0 = 1.0 \text{ m}$ and apply no force on each other when distance between them is greater than r_0 . Mass of particle A is $m = 1.0 \text{ kg}$ and mass of particle B is $\eta = 3$ times the mass of A. If the particle B is projected with velocity $u = 2.0 \text{ m/s}$ towards the particle A kept at rest in free space, find the minimum distance between them.



Concept: They will be closest, when their velocities become equal. After that the gap will increase again.



$$P: \eta m u = (\eta m + m) v \quad (1)$$

$$E: W_F = \Delta KE$$

$$\Rightarrow f(l) + (-f)(l + (l - x)) = \frac{1}{2}(\eta m + m)v^2 - \frac{1}{2}\eta m u^2 \quad (2)$$

From 1,2:

$$x = l - \frac{\eta m u^2}{2f(\eta + 1)}$$

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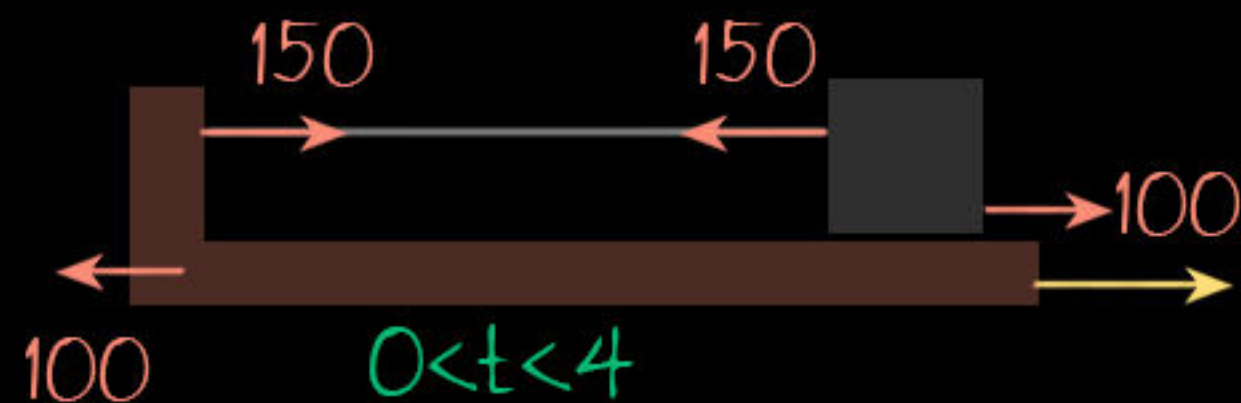
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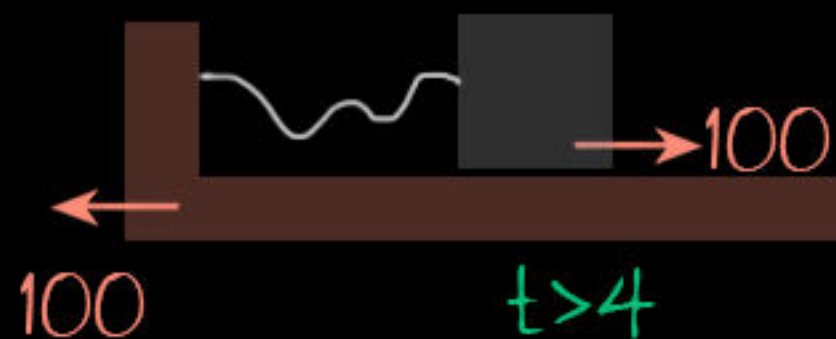
8. A barge stays motionless on a lake. An electric motor affixed on the deck can pull a block with the help of a light inextensible string as shown in the figure. Coefficient of friction between the block and the deck is 0.2 and water offers no resistance to motion of the barge. Mass of the barge including the motor is 200 kg and mass of the block is 50 kg. When the motor is switched on, it pulls the block by a constant force of 150 N. If the motor is switched off after 4 s, determine displacement of the block on the deck and draw velocity-time ($v-t$) graph for the barge. Acceleration due to gravity is 10 m/s^2 .

Concept: As the net force on system is zero, both will start and stop at the same time.

Approach: As it's asking $v-t$ as well, let's do that and from that area we will find barge and then block's displacement.



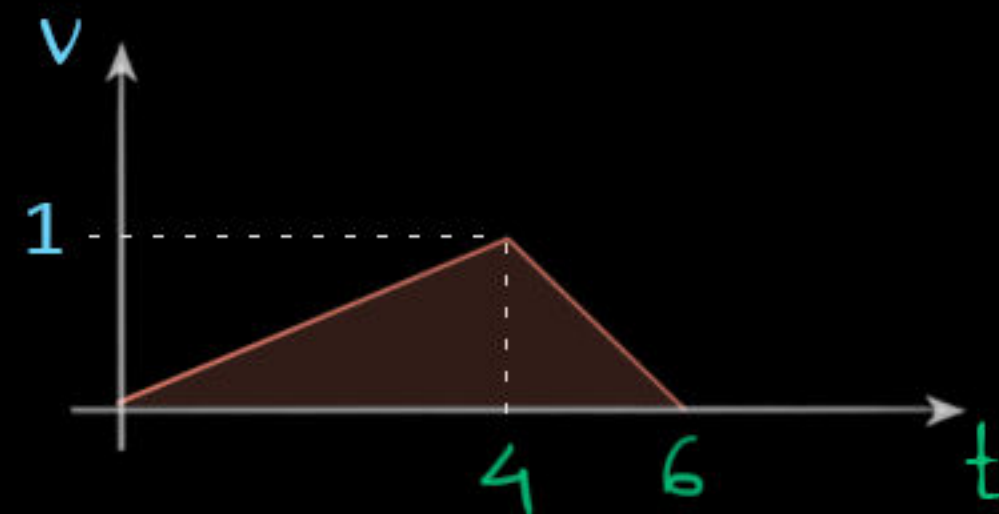
$$a_{\text{barge}} = \frac{50}{200} = \frac{1}{4}$$



$$a_{\text{barge}} = -\frac{100}{200} = -\frac{1}{2}$$

$$\mu mg = (0.2) 50 \cdot (10) = 100 \text{ N}$$

$$v_{t=4} = at = \frac{1}{4}(4) = 1$$

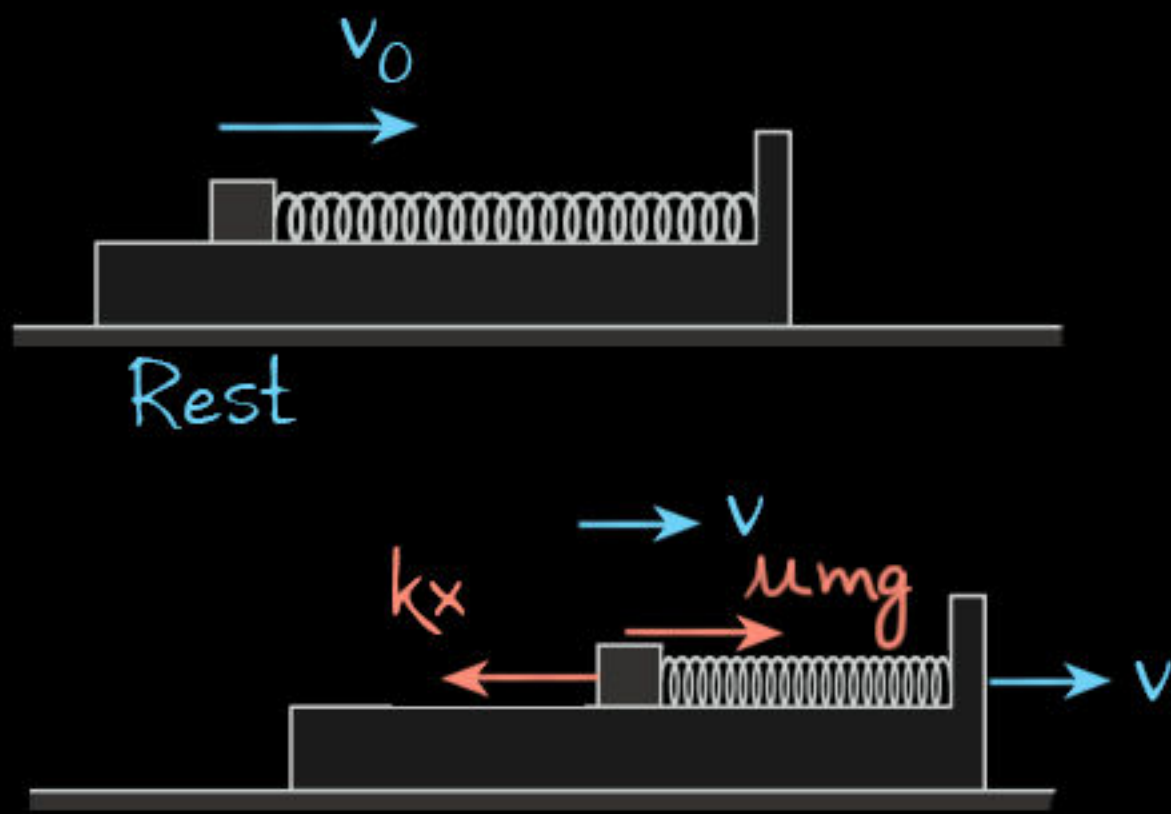


Disp. of barge: $x_1 = \frac{1}{2} \cdot 6(1) = 3$

∴ CoM is at rest: $x_1(200) = x_2(50)$

$$\Rightarrow x_2 = 12$$

∴ w.r.t barge block moves: $x_1 + x_2 = 15 \text{ m}$



9. A block of mass $m = 1.0$ kg is placed on a plank of mass $M = 15$ kg that rests on a frictionless floor. The block is connected to an obstruction on the plank with the help of a light spring of stiffness $k = 500$ N/m as shown in the figure. Coefficient of friction between the plank and the block is $\mu = 0.5$. Initially when the spring is relaxed, the block is given a velocity v_0 towards the obstruction. If the block stops on the plank before reversing its direction of motion relative to the plank, find the velocity v_0 . Acceleration due to gravity is $g = 10$ m/s².

Force: $\mu mg \leq kx$ ①

Momentum: $mv_0 = (m+M)v$ ②

Energy: $W_f = \Delta KE + \Delta PE_{\text{spring}}$

$\Rightarrow -\mu mgx = \left[\frac{1}{2}(m+M)v^2 - \frac{1}{2}mv_0^2 \right] + \frac{1}{2}kx^2$ ③

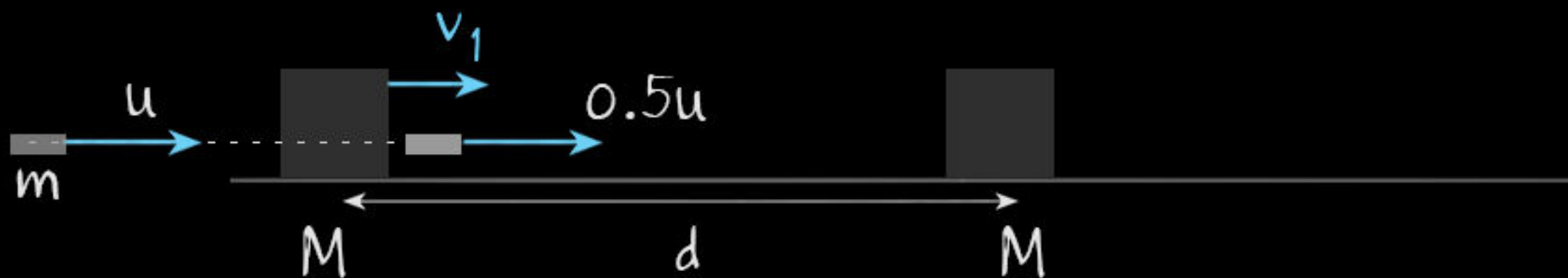
From 1,2,3:

$v_0 \geq \mu g \sqrt{\frac{3m(m+M)}{kM}}$

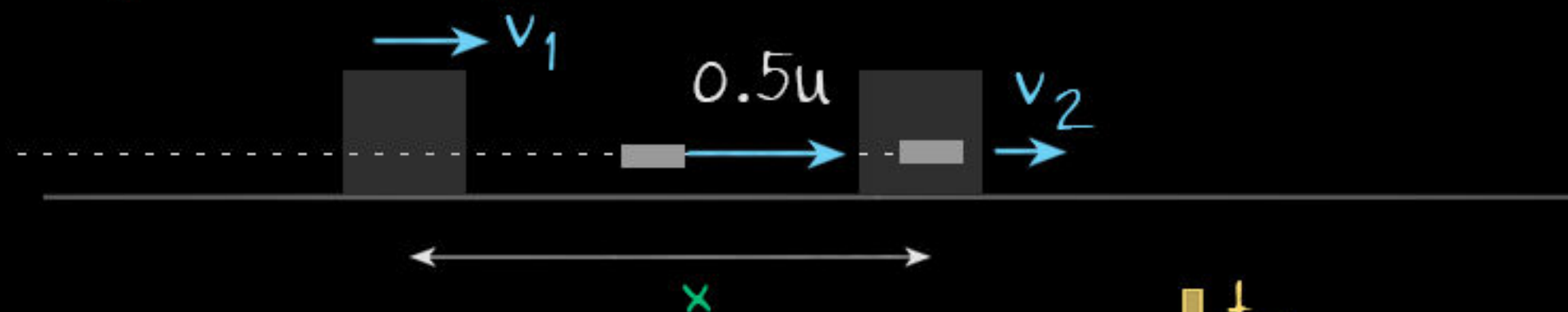
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10. Two cubes each of mass $M = 240$ g are placed a distance $d = 90$ cm apart on a frictionless horizontal floor. A bullet of mass $m = 12$ g hits the first cube moving horizontally with a speed $u = 180$ m/s, emerges with a speed $0.5u$, hits the second cube and gets embedded there. How long after the bullet emerges from the first cube, will the cubes collide? Neglect time of interaction of the bullet with the first cube.

Bullet strikes 1st block



Bullet strikes 2nd block



blocks collide



$$mu = Mv_1 + m(0.5u) \quad (1)$$

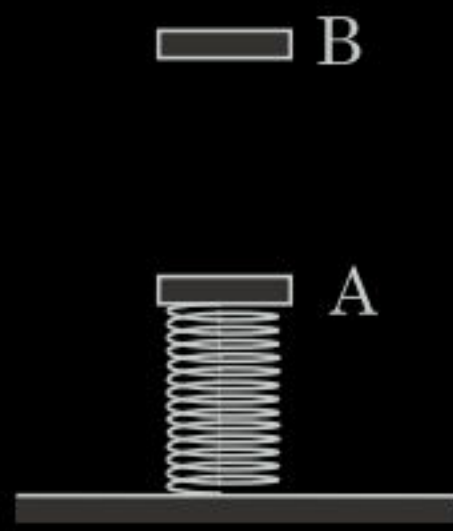
$$t_1 = \frac{d}{0.5u} = \frac{d-x}{v_1} \quad (2 \text{ and } 3)$$

$$m(0.5u) = (m+M)v_2 \quad (4)$$

$$t_2 = \frac{x}{v_1 - v_2} \quad (5)$$

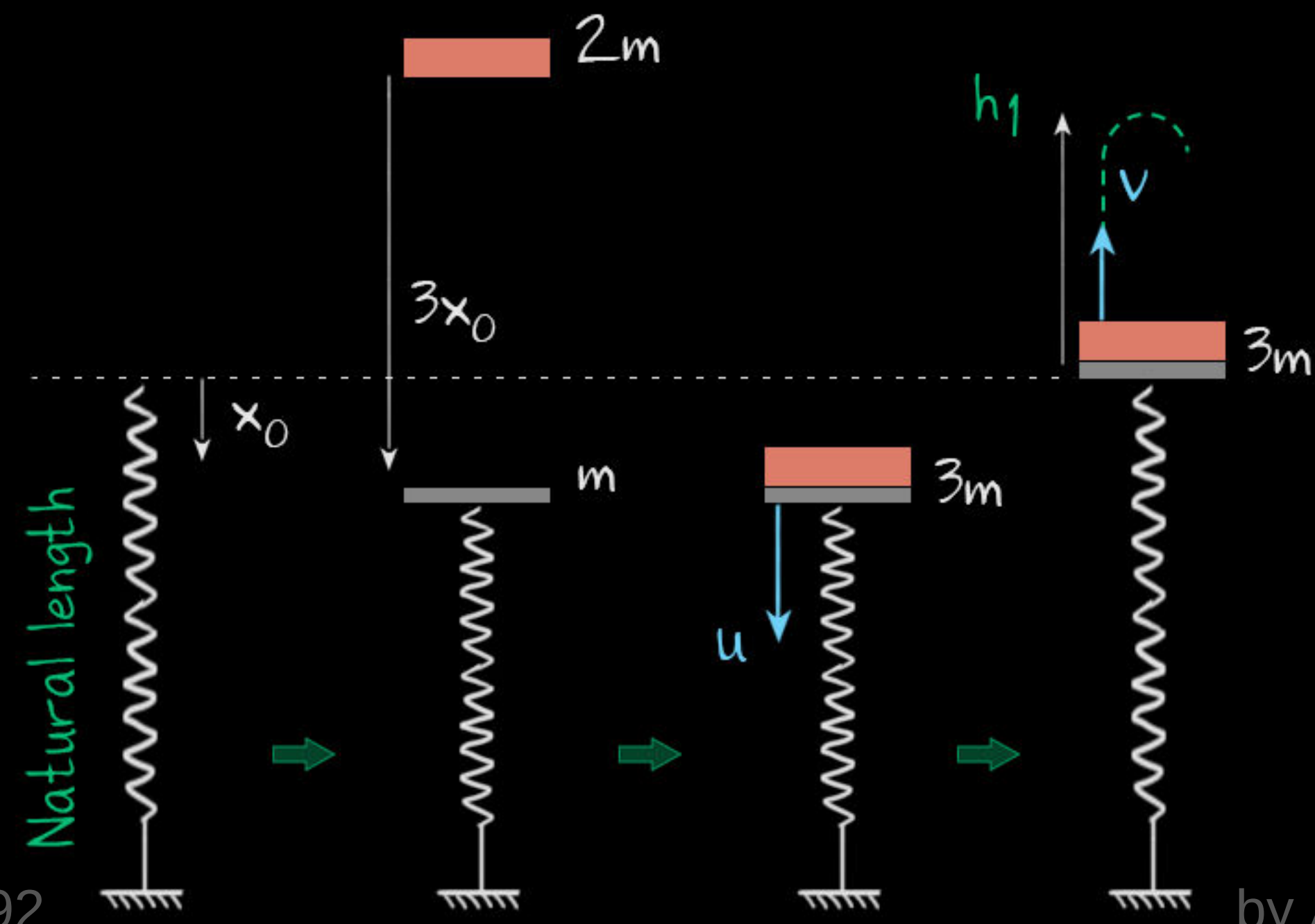
$$\text{From 1,2,3,4,5: } t_1 + t_2 = \frac{2dM^2}{um^2}$$

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Concept: On rebound, B will loose contact when spring reaches its natural length.

11. A thin plate A of mass m is affixed on the upper end of a spring, lower end of which is affixed on the ground. In equilibrium, the spring is compressed by an amount x_0 . Another thin plate B of mass $2m$ dropped from a height $3x_0$ above plate A hits the plate A, moves downwards together with the plate A and after reaching a lowest position, both the plates rebound upwards. What maximum height above the initial position of the plate A will the plate B rise?



Force: $mg = kx_0$ ①

Momentum: $2m\sqrt{2g(3x_0)} = 3m u$ ②

Energy: $W_g = \Delta KE + \Delta PE_{sp}$

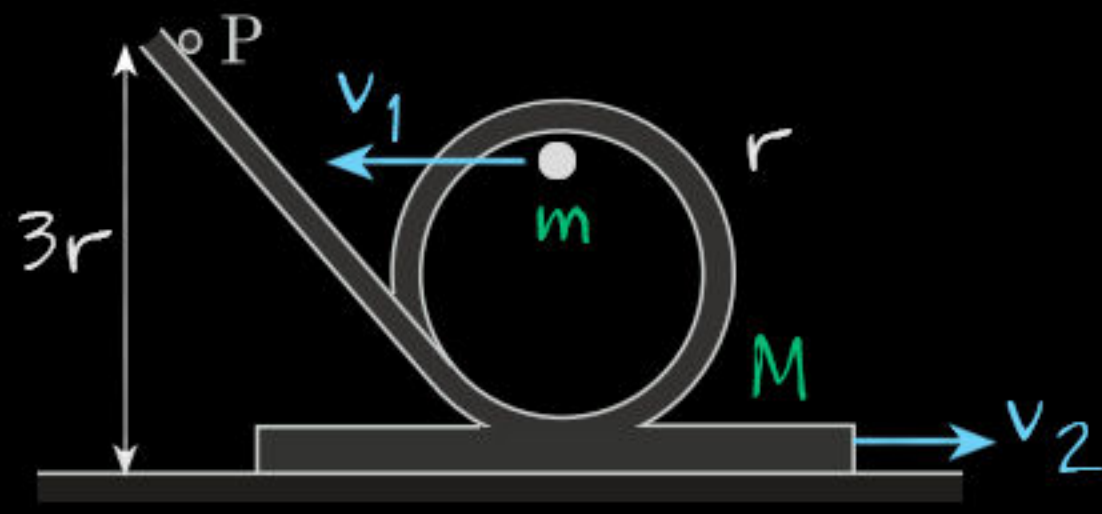
$\Rightarrow -3mgx_0 = \frac{1}{2} 3m (v^2 - u^2) + (0 - \frac{1}{2} kx_0^2)$ ③

$h_1 = \frac{v^2}{2g}$ ④

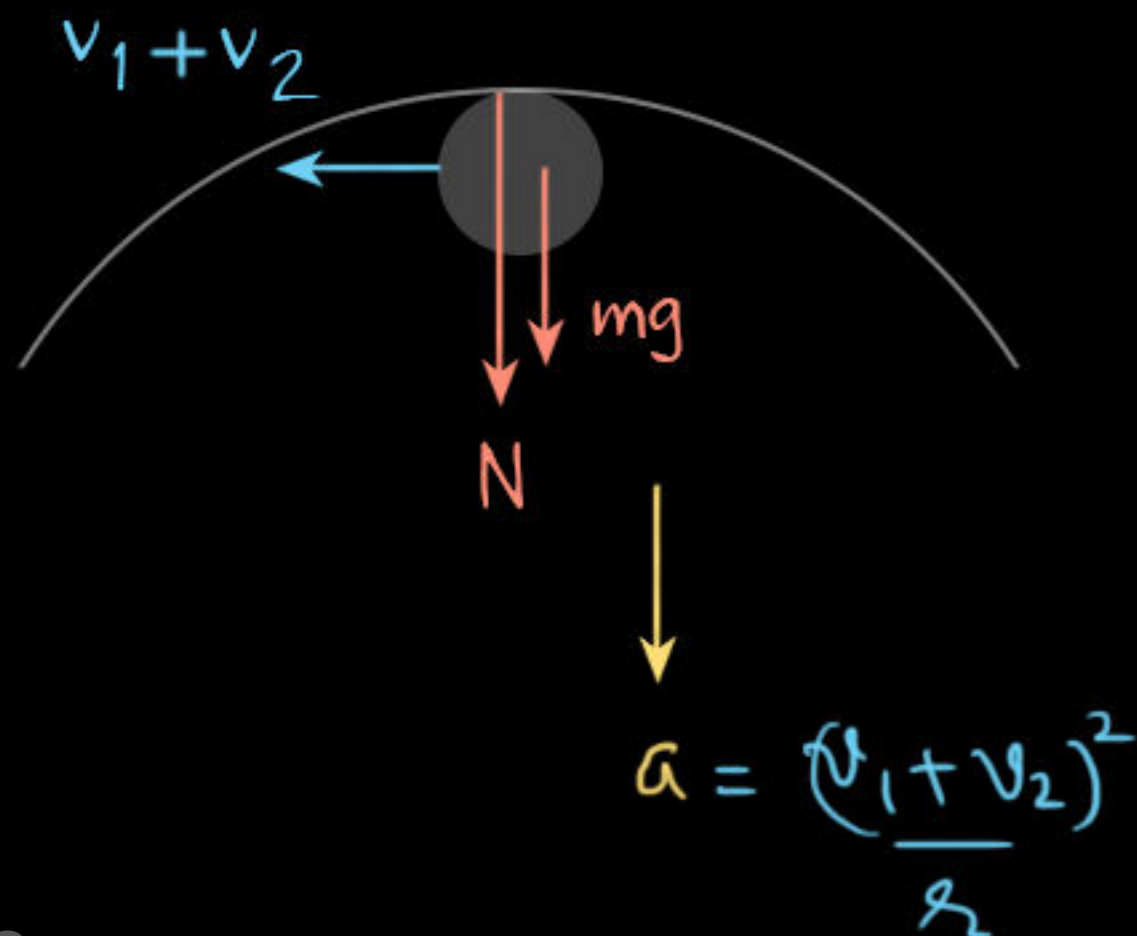
From 1,2,3,4:

$h_1 = \frac{x_0}{2} \Rightarrow h_{net} = x_0 + h_1 = \frac{3x_0}{2}$

w.r.t ground:



w.r.t loop: RoC is r



12. A frictionless inclined plane terminates smoothly into a frictionless vertical circular loop mounted on a plank that can slide on a frictionless horizontal floor. This assembly has a total mass of **0.8 kg**. A small ball P of mass **0.2 kg** is released on the inclined plane from a **height that is triple of the radius** of the circular loop. Find force of normal reaction between the ball and the circular loop, **when the ball is at its topmost position in the circular loop.**

Momentum: $m v_1 = M v_2$ ①

Energy: $W_g = \Delta KE$

$\Rightarrow m g r = \frac{1}{2} m v_1^2 + \frac{1}{2} M v_2^2$ ②

F_y: $N + mg = \frac{m (v_1 + v_2)^2}{r}$ ③

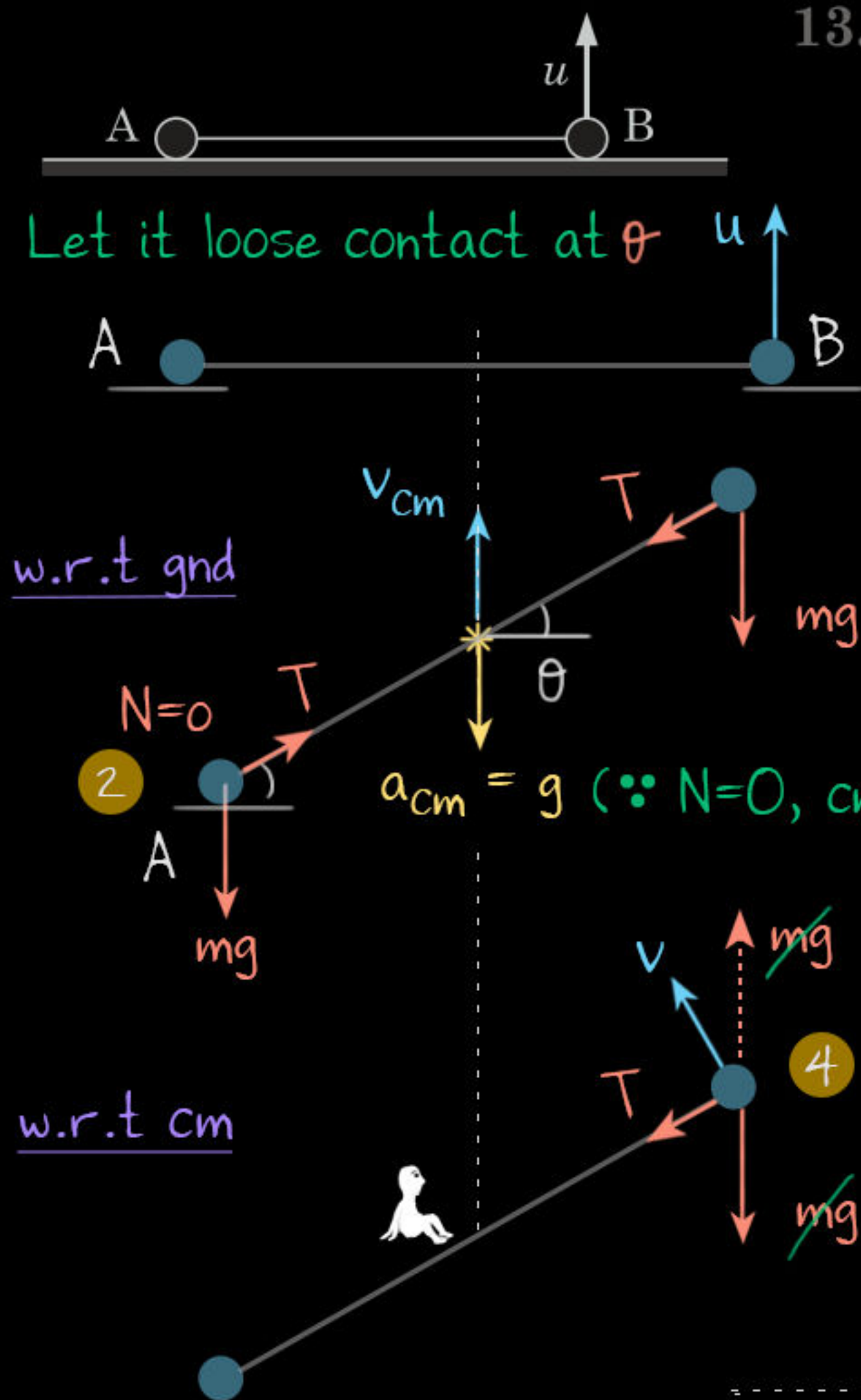
From 1,2,3:

$N = 3 N$

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13. Two identical small balls A and B each of mass m connected by a light inextensible cord of length l are placed on a frictionless horizontal floor. With what velocity u must the ball B be projected vertically upwards so that the ball A leaves the floor? Acceleration of free fall is g .



Energy: $W_g = \Delta KE = KE_f - KE_i \rightarrow [KE_{wrt\ cm} + KE_{cm}]$

$$\Rightarrow -2mg \frac{l}{2} \sin \theta = 2 \left(\frac{1}{2} m v^2 \right) + \frac{1}{2} (2m) v_{cm}^2 - \frac{1}{2} m u^2$$

$$\Rightarrow u^2 = 2v^2 + 2v_{cm}^2 + 2gl \sin \theta \quad (1)$$

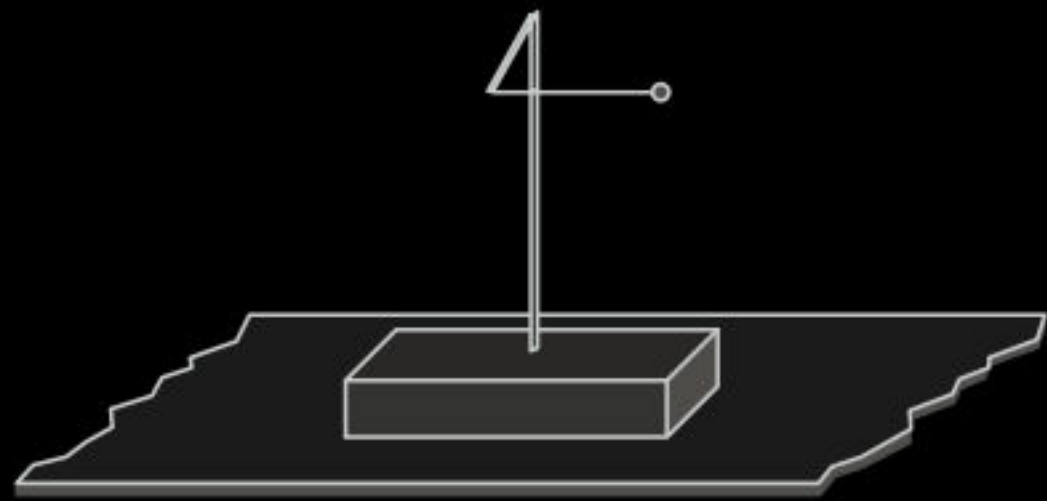
On A, F_y : $T \sin \theta = mg \quad (2)$

On A, $v_y = 0$: $\vec{v}_{A/cm} + \vec{v}_{cm} = 0$ (In Y direction)

$$\Rightarrow v \cos \theta - v_{cm} = 0 \quad (3)$$

On B, along thread: $T = \frac{mv^2}{l/2} \quad (4)$

From 1,2,3,4: $u^2 = gl \left(\frac{2}{\sin \theta} + \sin \theta \right) \xrightarrow{\text{min @ } \sin \theta = 1} \Rightarrow u \geq \sqrt{3gl}$



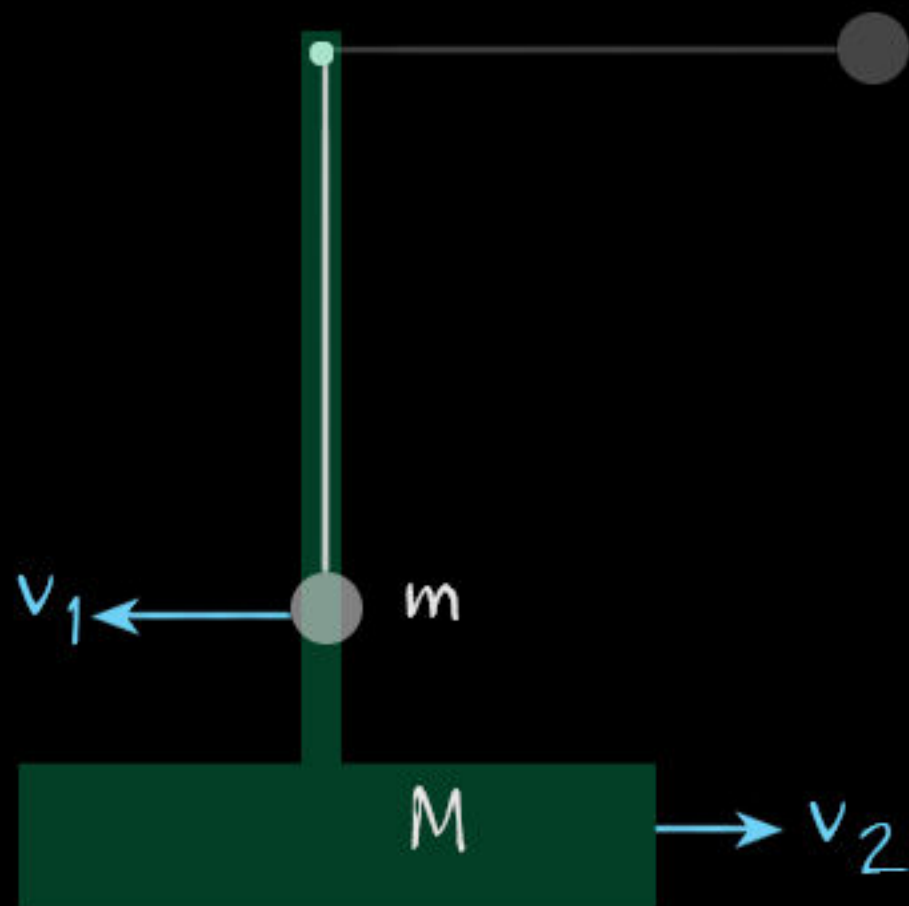
14. A block of mass M can slide without friction on a horizontal floor. A small ball of mass m is suspended by a light inextensible cord of length l from a hanger fixed at the top of a vertical pole mounted on the block so that the ball can swing freely in a vertical plane. The ball is released from a horizontal position as shown. Find the maximum tensile force in the string during the subsequent motion.

Concept: Tension is maximum at bottom, as T is opposite mg and tangential velocity: $v_1 + v_2$ is maximum too.

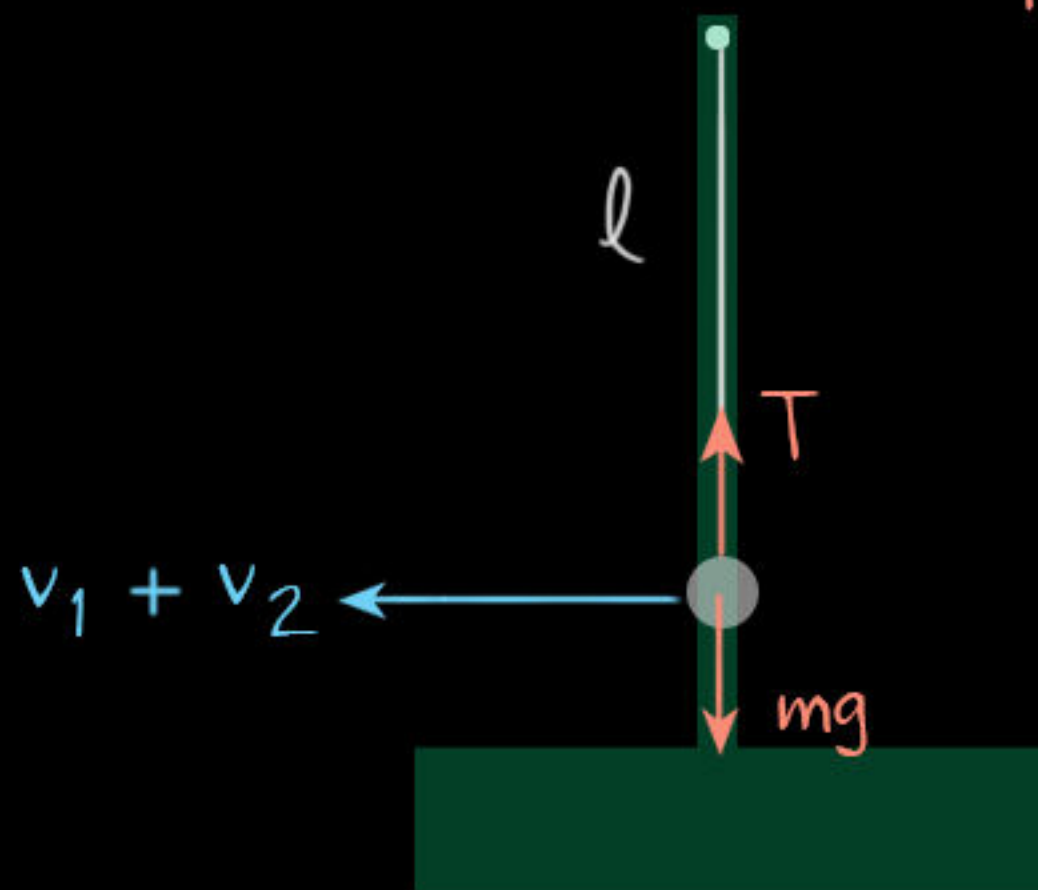
$$\text{Energy: } mgl = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$$

$$\text{Momentum: } mv_1 = Mv_2$$

$$\text{Force: } T - mg = \frac{m(v_1 + v_2)^2}{l}$$



w.r.t gnd



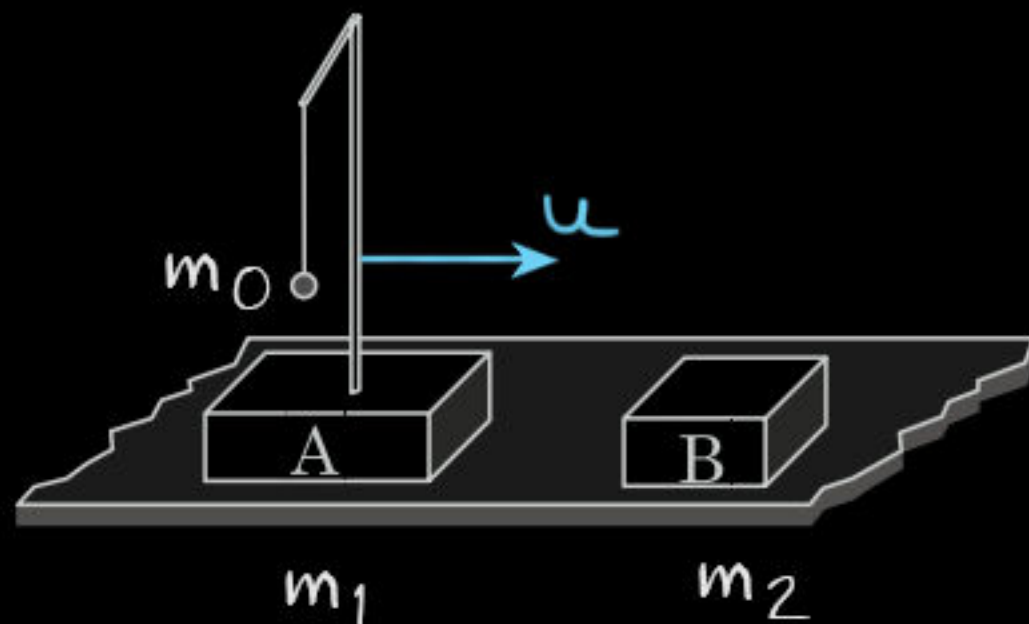
w.r.t M

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From 1,2,3:

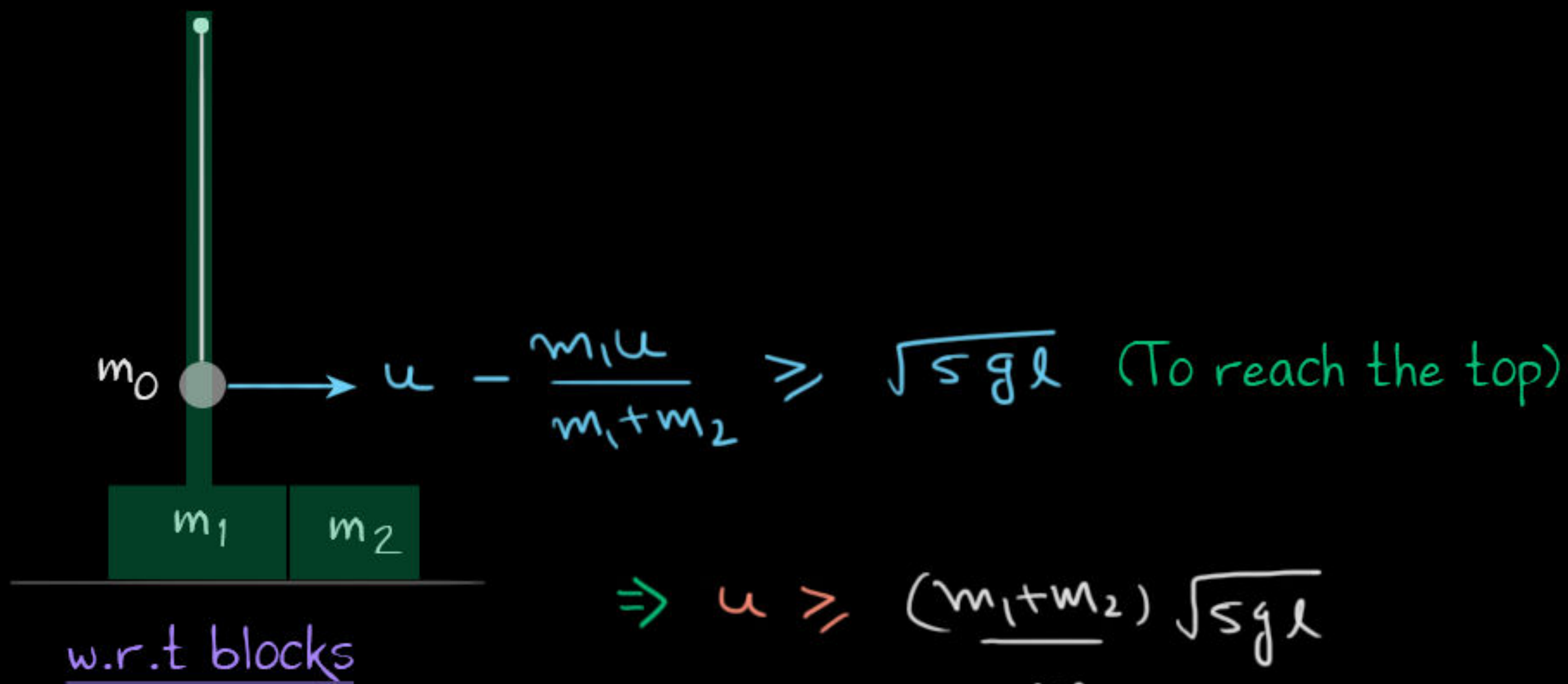
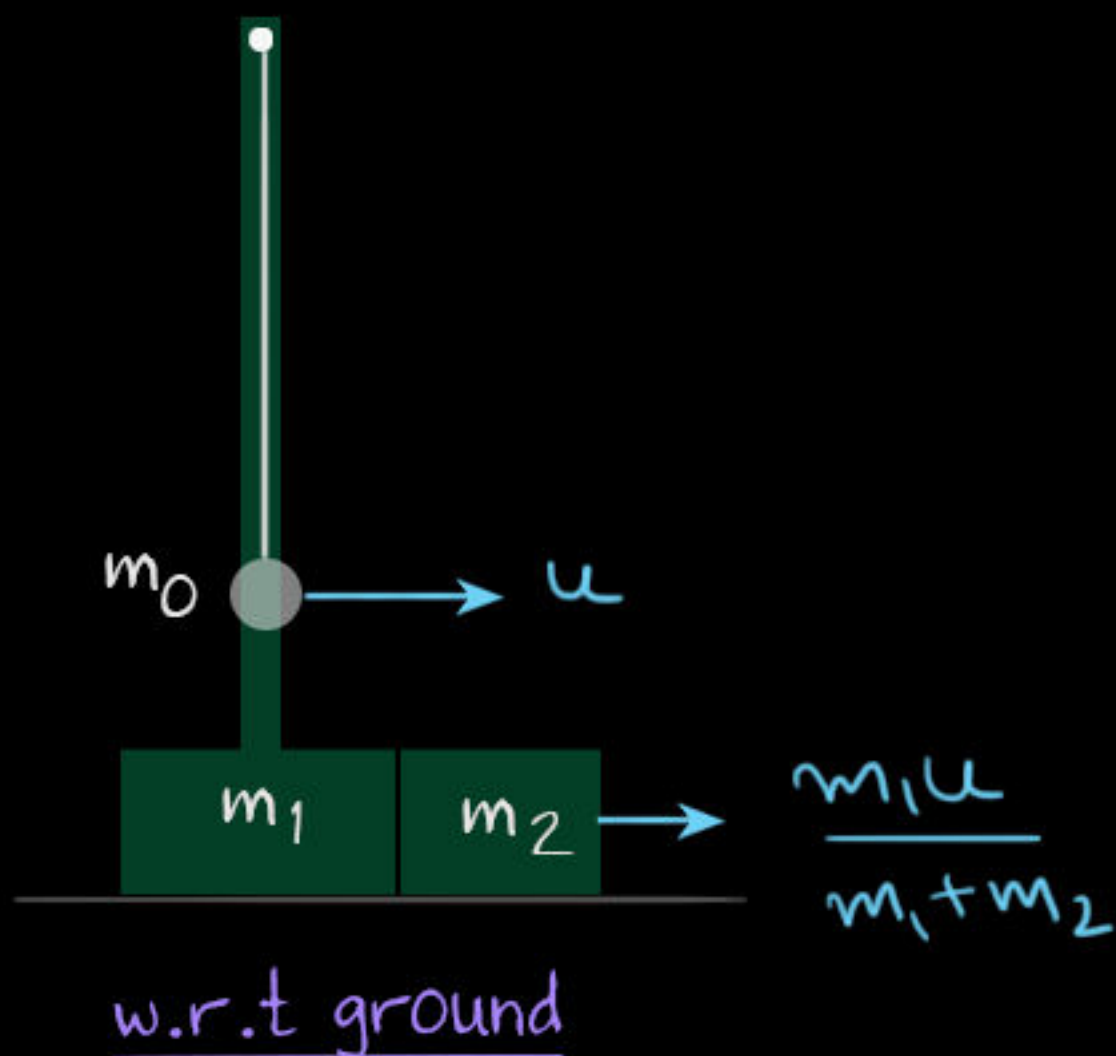
$$T = mg \left(3 + \frac{2m}{M} \right)$$

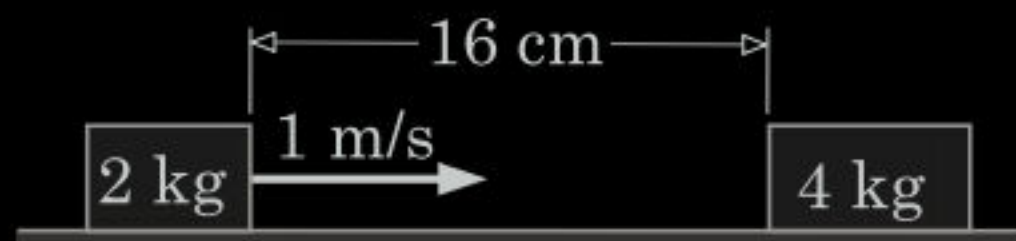
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15. A small ball of mass m_0 is suspended from a thread of length l attached at the top of a pole, which is mounted on a block A of mass m_1 . The block A and the ball both are moving with a velocity u when the block A strikes another block B of mass m_2 and sticks to it. If the ground is frictionless, find the minimum velocity u so that the ball can move on a complete circular path. Assume that the blocks A and B are much heavier than the ball.

Just after collision:





16. A block of mass 2 kg is sliding on a horizontal floor towards another block of mass 4 kg placed at rest on the floor. When the blocks are a distance 16 m apart, velocity of the first block is 1 m/s as shown in the figure. Coefficient of friction between the floor and the blocks is 0.2 and the collision is perfectly elastic. Find separation between the blocks, when they come to rest. Acceleration due to gravity is 10 m/s^2 .

Just before collision:



$$v^2 = u^2 + 2as$$

$$\Rightarrow v^2 = 1 - 2(0.2g) \frac{16}{100} \Rightarrow v = 0.6$$

Just after collision:



$$v_1 + v_2 = 0.6$$

$$2(0.6) = 4v_2 - 2v_1$$

$$v_1 = 0.2$$

$$v_2 = 0.4$$

$$\text{Total separation, } x = x_1 + x_2 = \frac{v_1^2}{2\mu g} + \frac{v_2^2}{2\mu g} = 0.05 = 5 \text{ cm} //$$

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Approach:

Use v_{cm} for finding u .

Diagram for finding l .

Let the CoM move x_{cm} in z time.

As it moves with constant velocity:

$$\therefore v_{cm} = v_{cm}$$

$$\frac{u}{2} = \frac{x_{cm}}{z}$$

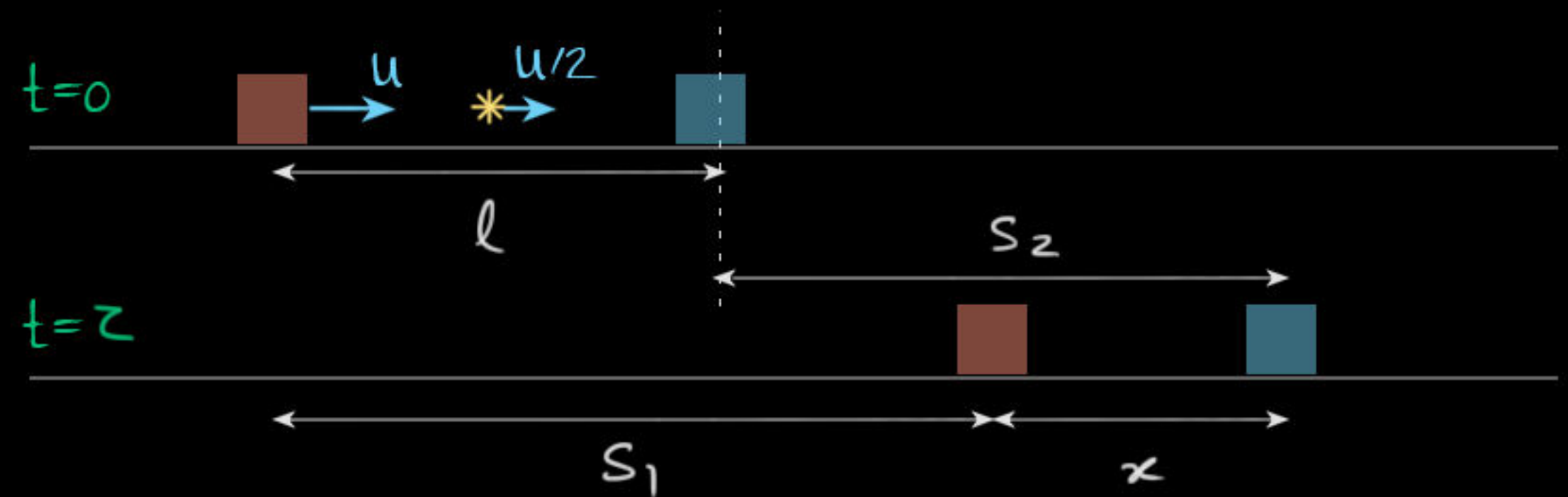
$$\Rightarrow \frac{u}{2} = \left(\frac{ms_1 + ms_2}{2m} \right) \frac{1}{z}$$

$$\Rightarrow u = \frac{s_1 + s_2}{z}$$

17. A small disc sliding on a frictionless horizontal floor with an unknown velocity collides head on with another identical disc kept at rest on the floor. If during a time interval τ , the first and the second discs cover distances s_1 and s_2 respectively, find possible values of initial velocity u of the first disc and initial distance l between the discs.

Collision can be elastic or inelastic or inbetween

At z , let the distance between blocks be x



$$l + s_2 = s_1 + x$$

$$\Rightarrow l = s_1 - s_2 + x$$

$$\Rightarrow s_1 - s_2 \leq l \leq s_1$$

$$0 \leq x \leq s_2$$

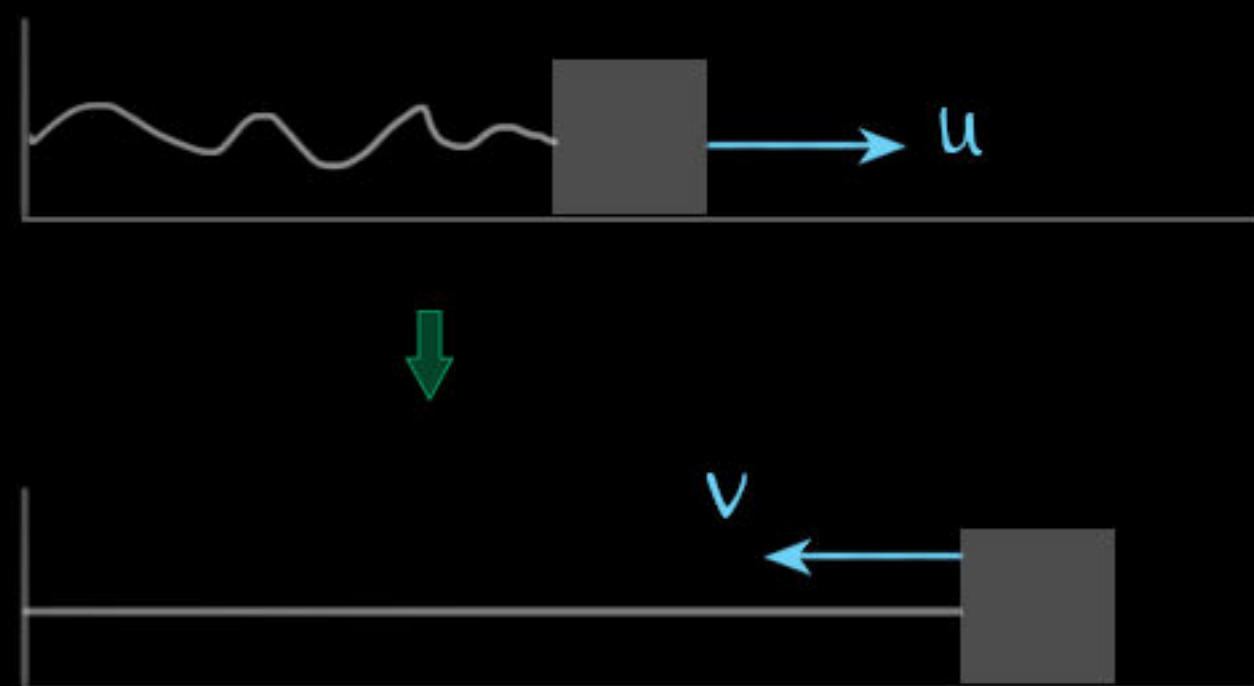
For elastic collision

For inelastic collision



18. Two blocks A and B of masses m_A and m_B placed on a frictionless horizontal floor are connected by a light and almost inextensible cord. The block A is given a velocity u away from the block B. If coefficient of restitution defined for the cord has a value e , find velocity with which block B begins to move.

A general case when block is tied to the wall:



Between two ends of the cord:

$$e = \frac{v_{\text{separation}}}{v_{\text{approach}}} = \frac{-v}{-u}$$

$$\Rightarrow v = eu$$

Sol:



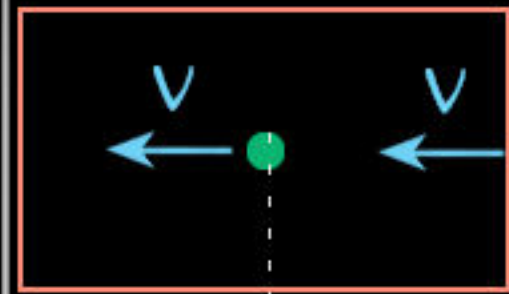
$$e = \frac{v_{\text{separation}}}{v_{\text{approach}}} = \frac{v_1 - v_2}{-u} \quad (1)$$

$$m_A u = m_A v_1 + m_B v_2 \quad (2)$$

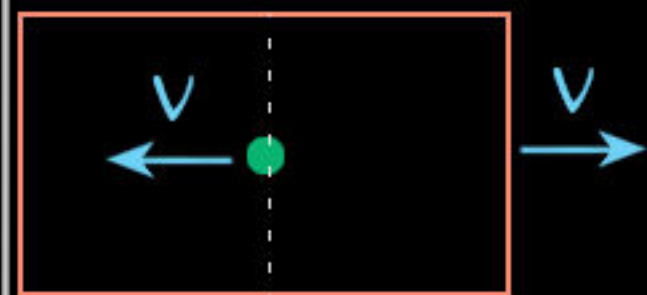
$$\text{From 1,2: } v_2 = \frac{m_A e u}{m_A + m_B}$$

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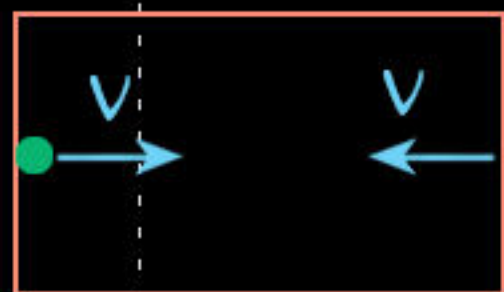
Initially just before collision



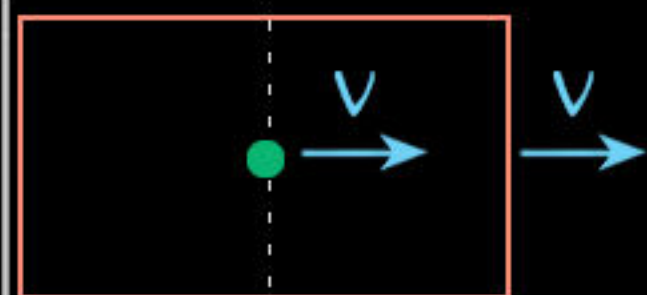
After 1st collision with wall



After collision between block and ball



After collision with wall again

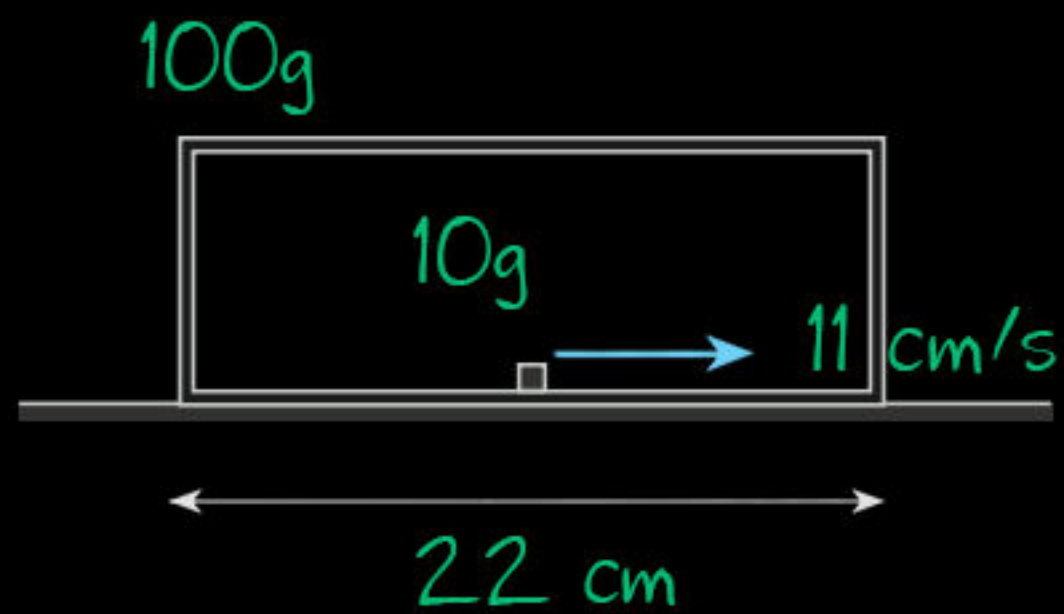


B

19. In gravity free space, a small ball is placed at the centre of a box of the same mass. The ball and the box both are moving with a velocity v towards a wall. Assume all collisions to be perfectly elastic.

- (a) How many collisions are possible?
- (b) Find velocities of the box and the ball and position of the ball relative to the box after all possible collisions.

A $\therefore$ three collisions in total.



20. A box of length 22 cm and mass 100 g rests on a horizontal frictionless floor. A small bead of mass 10 g is placed at the centre of the bottom of the box. There is no friction between the bead and the bottom of the box. The bead is projected with a velocity 11 cm/s parallel to the length of the box. If all the collisions are perfectly elastic, find displacement of the box in a time interval 60 s after the bead was projected.

Sol: w.r.t box ball will just go to and fro with v

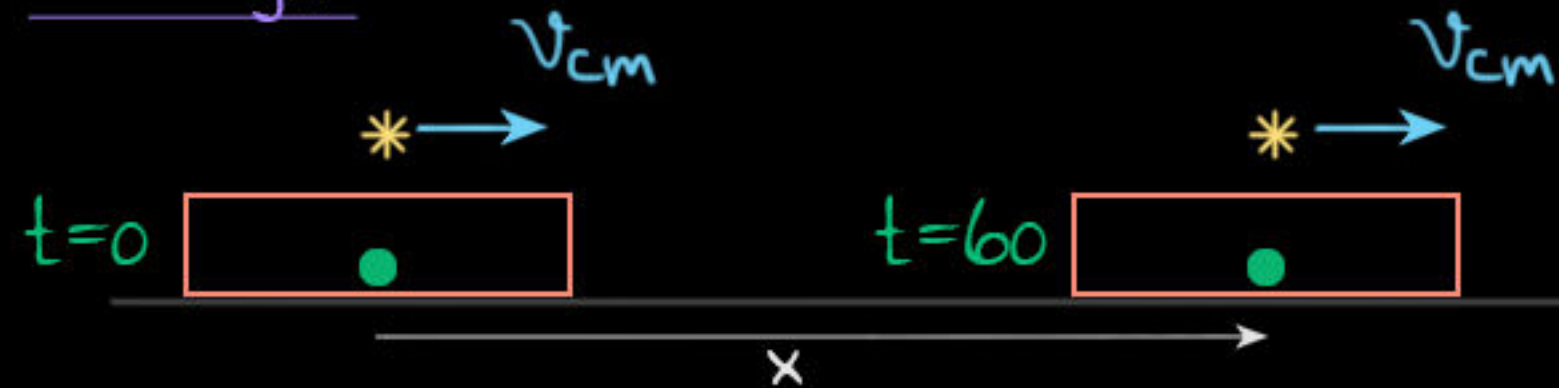
At $t=0$ and after every 4 seconds:



$\therefore$ At $t=60$:



w.r.t gnd



$$\therefore x = \text{displacement of CoM} = v_{cm} \times 60 = \left[\frac{11 \times 10}{(10 + 100)} \right] \cdot 60 = 60 //$$

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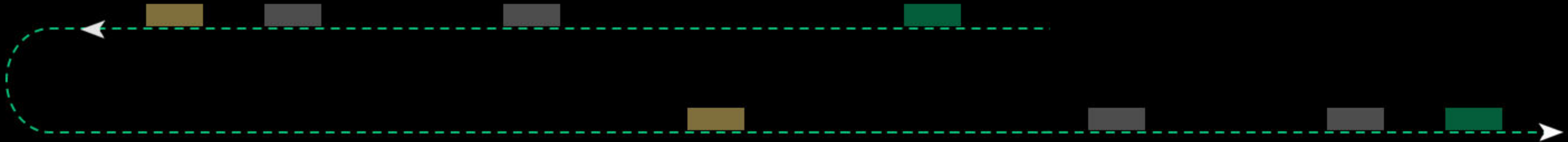
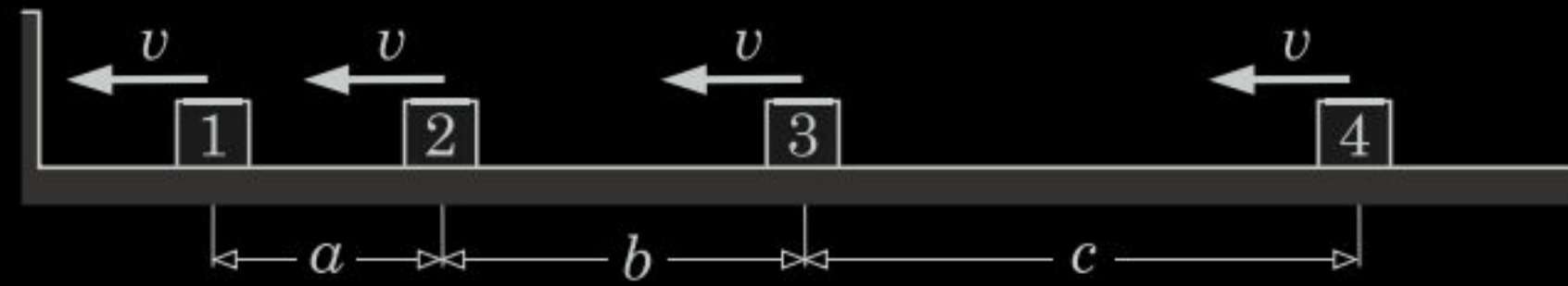
by Ankit Singhvi

Concept: e is a constant independent of reference frames.

- $\therefore$ Elastic collisions are elastic w.r.t any frame.

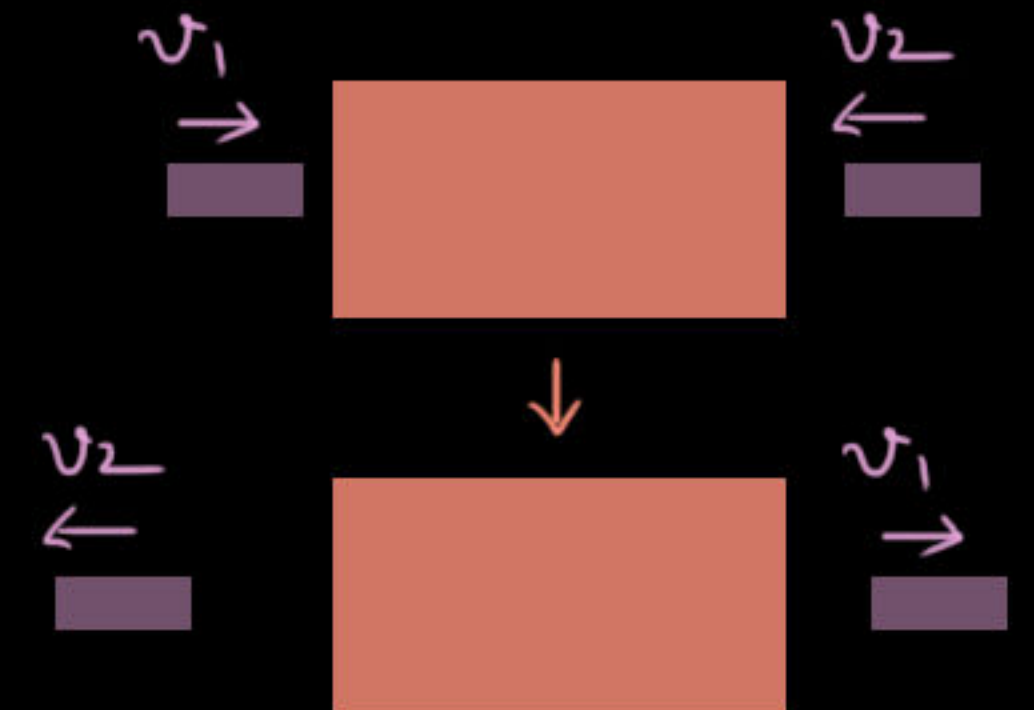
Concept: In an elastic head-on collision of equal masses, velocities get exchanged.

21. Four identical small blocks marked 1, 2, 3 and 4 with separations a , b and c as shown in the figure are moving on a frictionless floor in a line all with a velocity v towards a wall. If all the collisions are perfectly elastic, with what velocities and separations will the blocks move after all possible collisions?



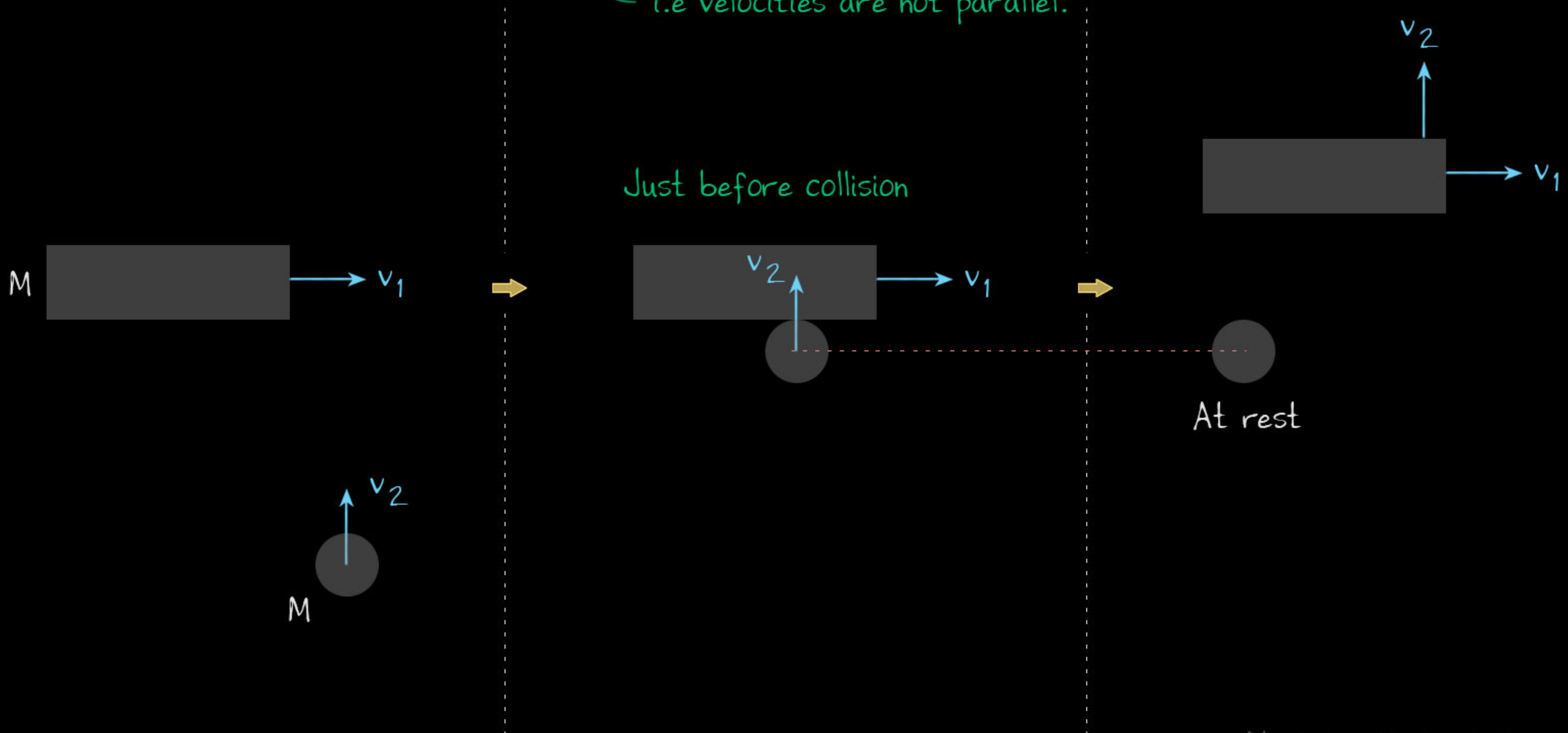
It will appear that the 'train' took a U-turn from wall.

But, in reality, only the velocities are exchanged, not the blocks. So the block closest to the wall will remain closest to the wall.



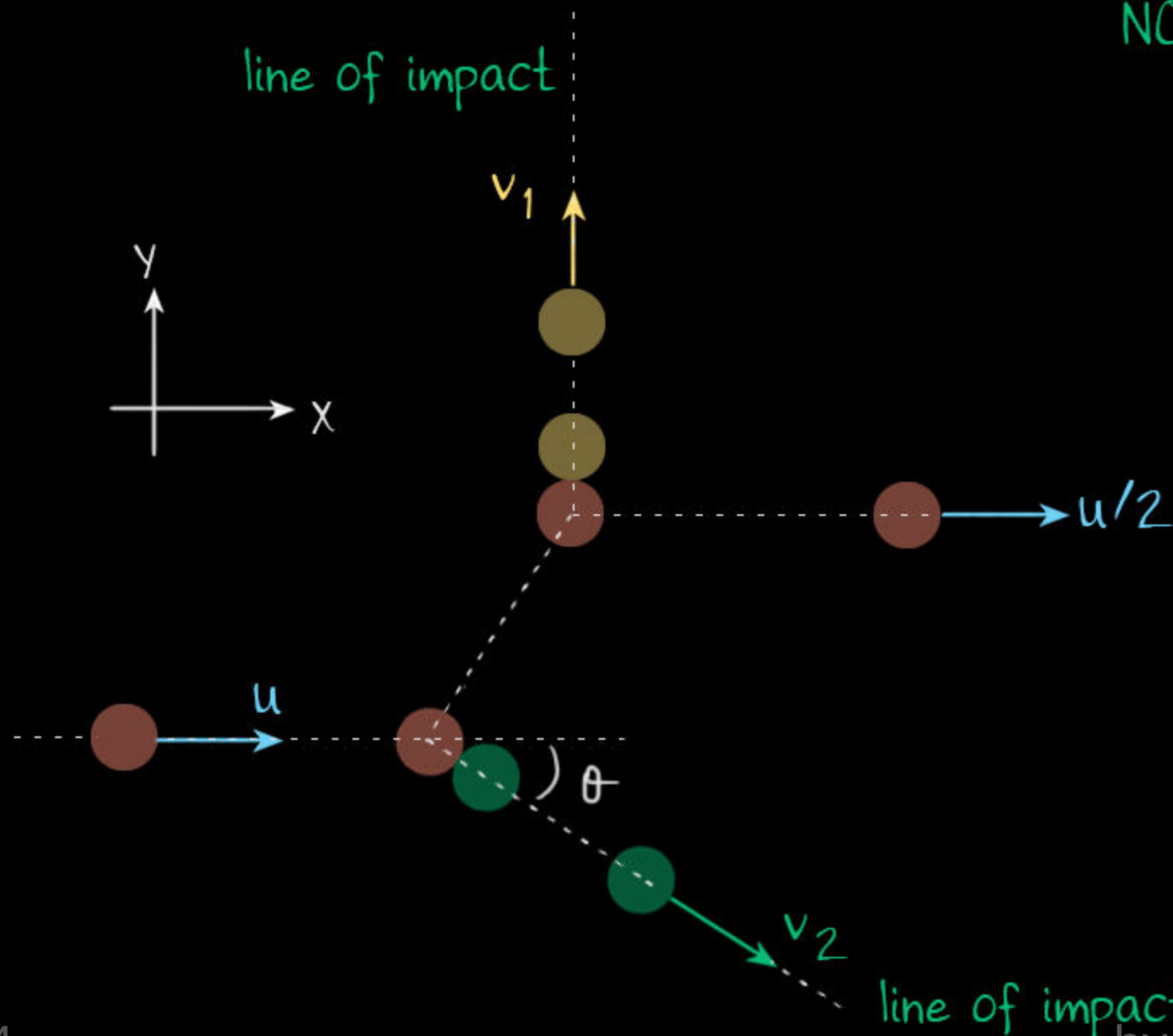
22. Is it possible that after an elastic oblique collision between two identical uniform balls, one of them will stop? If yes, find necessary conditions.

i.e velocities are not parallel.



23. Three identical small discs rest on a frictionless horizontal floor. One of them is projected along the floor to successively collide elastically with the other two and thereafter to move in its original direction with half of its initial speed. Assume the floor in the x - y plane and initial velocity of the first disc u in the positive x -direction, find velocities of the other two discs after the collisions.

One after another,
NOT simultaneously.



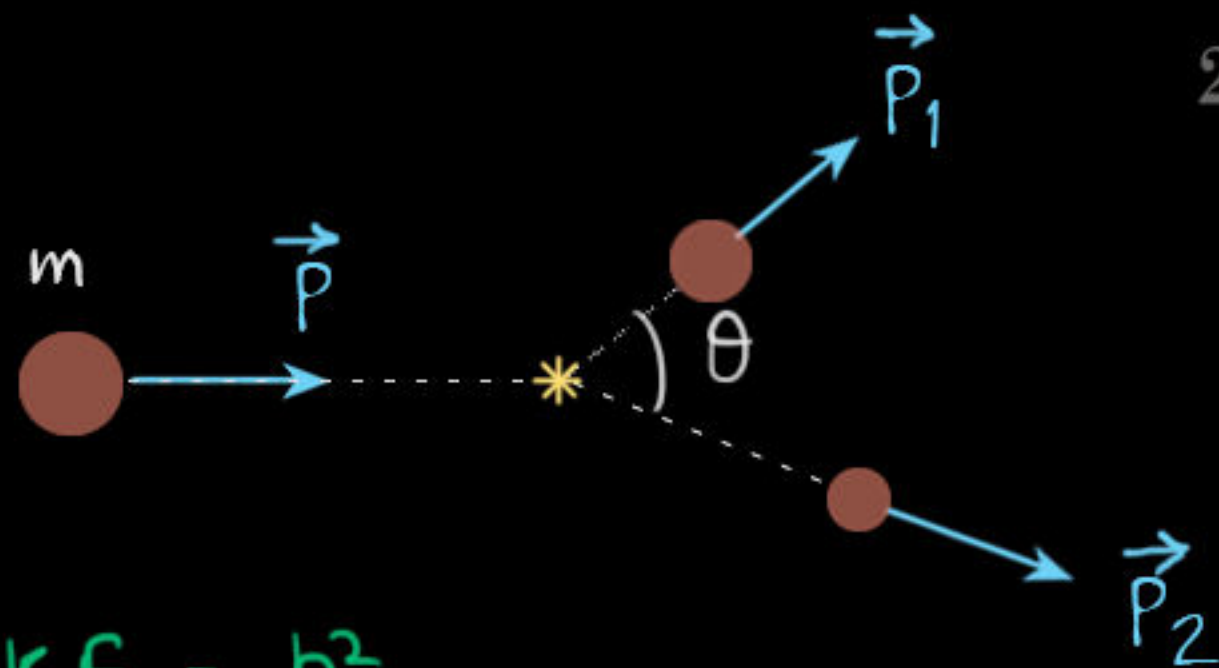
$$P_y: v_1 = v_2 \sin \theta \quad (1)$$

$$P_x: u = \frac{u}{2} + v_2 \cos \theta \quad (2)$$

$$\text{Energy: } u^2 = \left(\frac{u}{2}\right)^2 + v_1^2 + v_2^2 \quad (3)$$

From 1, 2, 3:

$$v_1 = u/2, v_2 = u/\sqrt{2}, \theta = \frac{\pi}{4}$$



24. A shell of mass $m = 100$ kg explodes at some point on its trajectory into two fragments that fly with momenta $p_1 = 36 \times 10^4$ kg·m/s and $p_2 = 24 \times 10^4$ kg·m/s making angle $\theta = 60^\circ$ from each other. Determine ratio of masses of the fragments for which kinetic energy released in the explosion will be minimal. How much is this kinetic energy?

$$KE = \frac{p^2}{2m}$$

$$\vec{p} = \vec{p}_1 + \vec{p}_2 \Rightarrow p^2 = p_1^2 + p_2^2 + 2p_1p_2 \cos \theta \quad (1)$$

$$KE_{\text{explosion}} + \frac{p^2}{2m} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} \quad (2)$$

$$\Rightarrow KE_{\text{exp}} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} - \frac{(p_1^2 + p_2^2 + 2p_1p_2 \cos \theta)}{2m}$$

$$m = m_1 + m_2$$

$$\&$$

$$m_1/m_2 = t$$

$$=$$

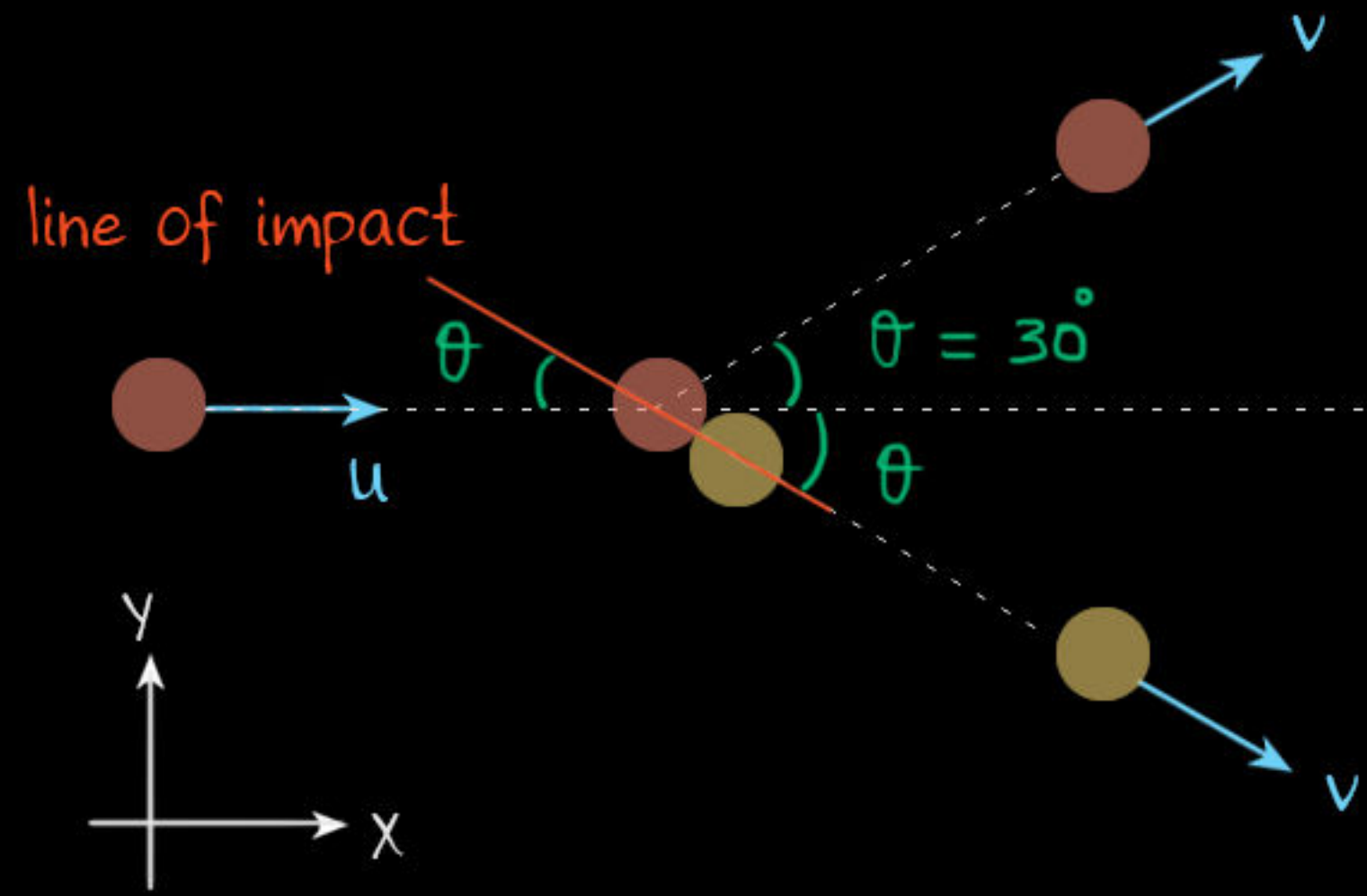
$$\frac{1}{2m} \left[p_1^2 \left(1 + \frac{1}{t} \right) + p_2^2 (t + 1) - (p_1^2 + p_2^2 + 2p_1p_2 \cos \theta) \right] \quad (3)$$

$$\text{minimum @ } t = \frac{m_1}{m_2} = p_1/p_2 //$$

Putting in (3)

$$\Rightarrow \text{Min } KE_{\text{exp}} = \frac{p_1p_2(1 - \cos \theta)}{m} //$$

25. A ball moving with a uniform velocity in free space collides with another stationary identical ball and thereafter each of them move at angle 30° from the initial direction of motion of the first ball. Find fraction of the initial kinetic energy lost in the collision and coefficient of restitution?



$$P_y: v = v$$

$$P_x: mu = 2(mv \cos \theta)$$

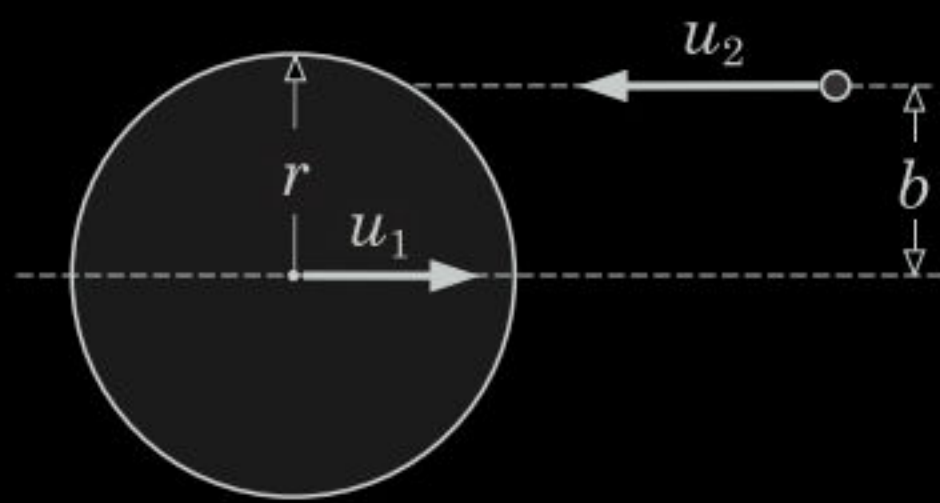
$$\Rightarrow v = u/\sqrt{3}$$

∴ Along line of impact:

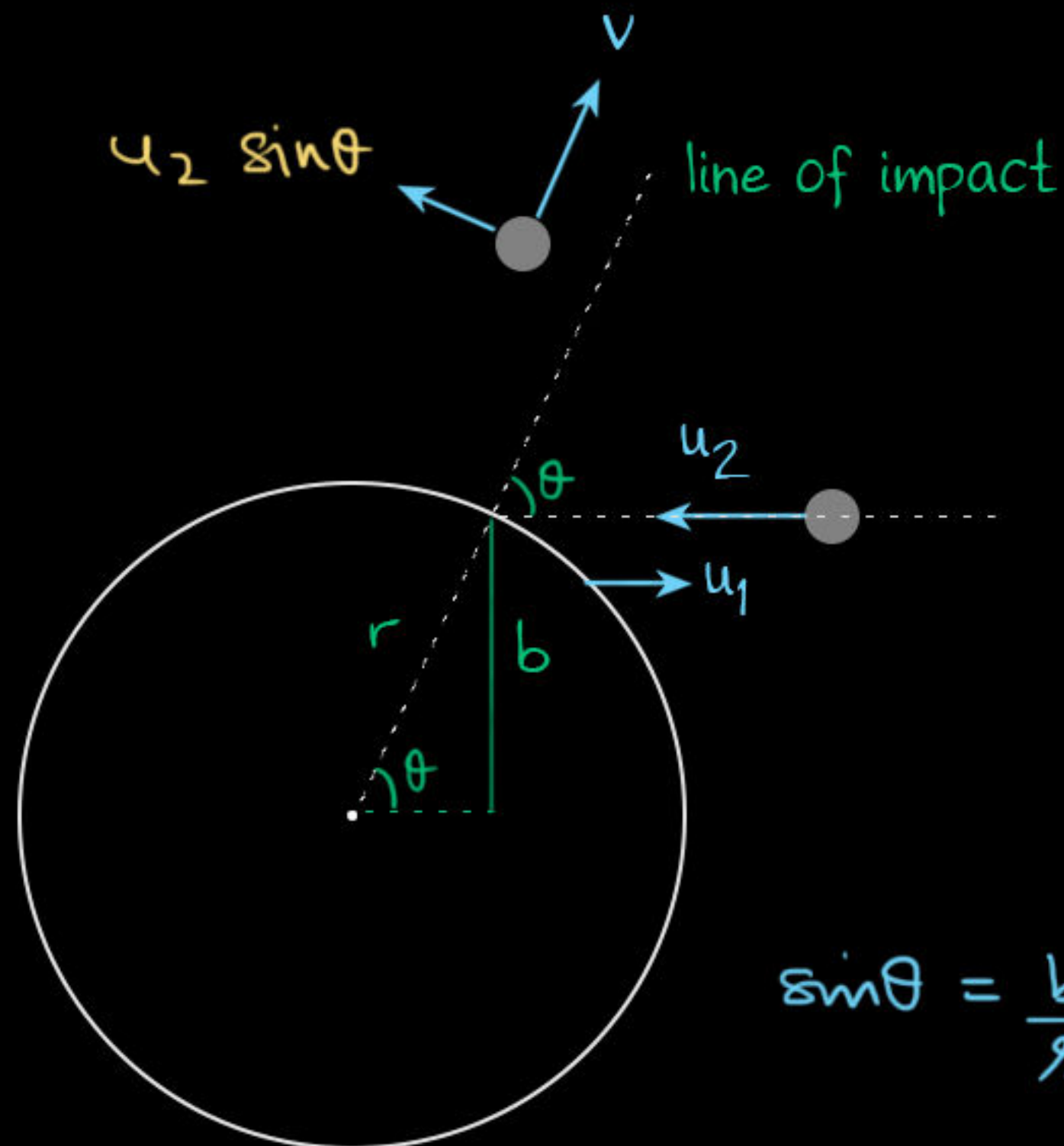
$$e = \frac{v - v \cos 2\theta}{u \cos \theta} = \frac{\frac{u}{\sqrt{3}} \left(1 - \frac{1}{2}\right)}{u \cdot \frac{\sqrt{3}}{2}} = \frac{1}{3} //$$

$$\text{Fraction of initial KE lost} = \frac{KE_i - KE_f}{KE_i} = 1 - \frac{KE_f}{KE_i}$$

$$= 1 - \frac{2 \left(\frac{1}{2} m \frac{u^2}{3} \right)}{\frac{1}{2} m u^2} = 1 - \frac{2}{3} = \frac{1}{3} //$$



26. A massive frictionless cylinder of radius r , moving with a constant velocity u_1 collides elastically with a small light ball moving with a velocity u_2 towards the cylinder perpendicular to its axis as shown in the figure. The distance between the lines of motion (impact parameter) of the bodies is b . Find speed of the ball after the collision. Assume gravity free conditions.



$$\sin \theta = \frac{b}{r} \quad (1)$$

Along line of impact: $v_{\text{approach}} = v_{\text{separation}}$

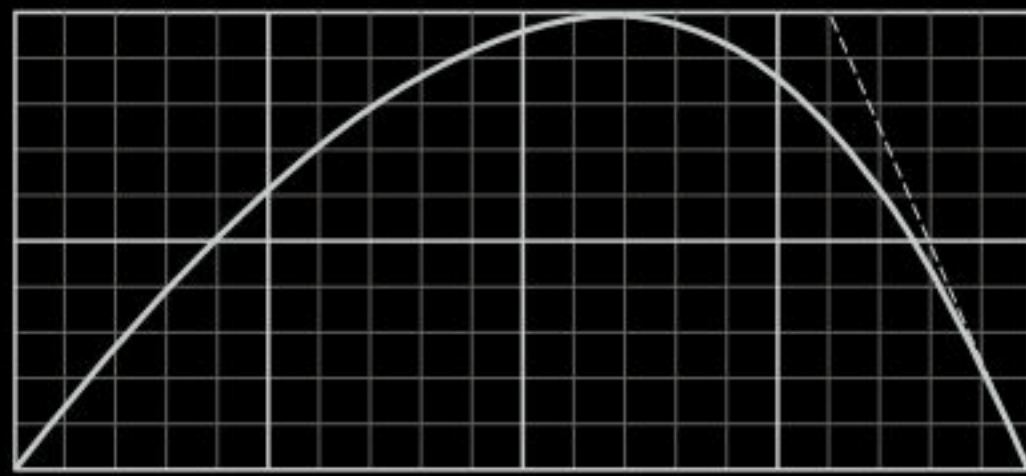
$$u_1 \cos \theta + u_2 \cos \theta = v - u_1 \cos \theta \quad (2)$$

$$\therefore v_{\text{ball}} = \sqrt{(u_2 \sin \theta)^2 + v^2}$$

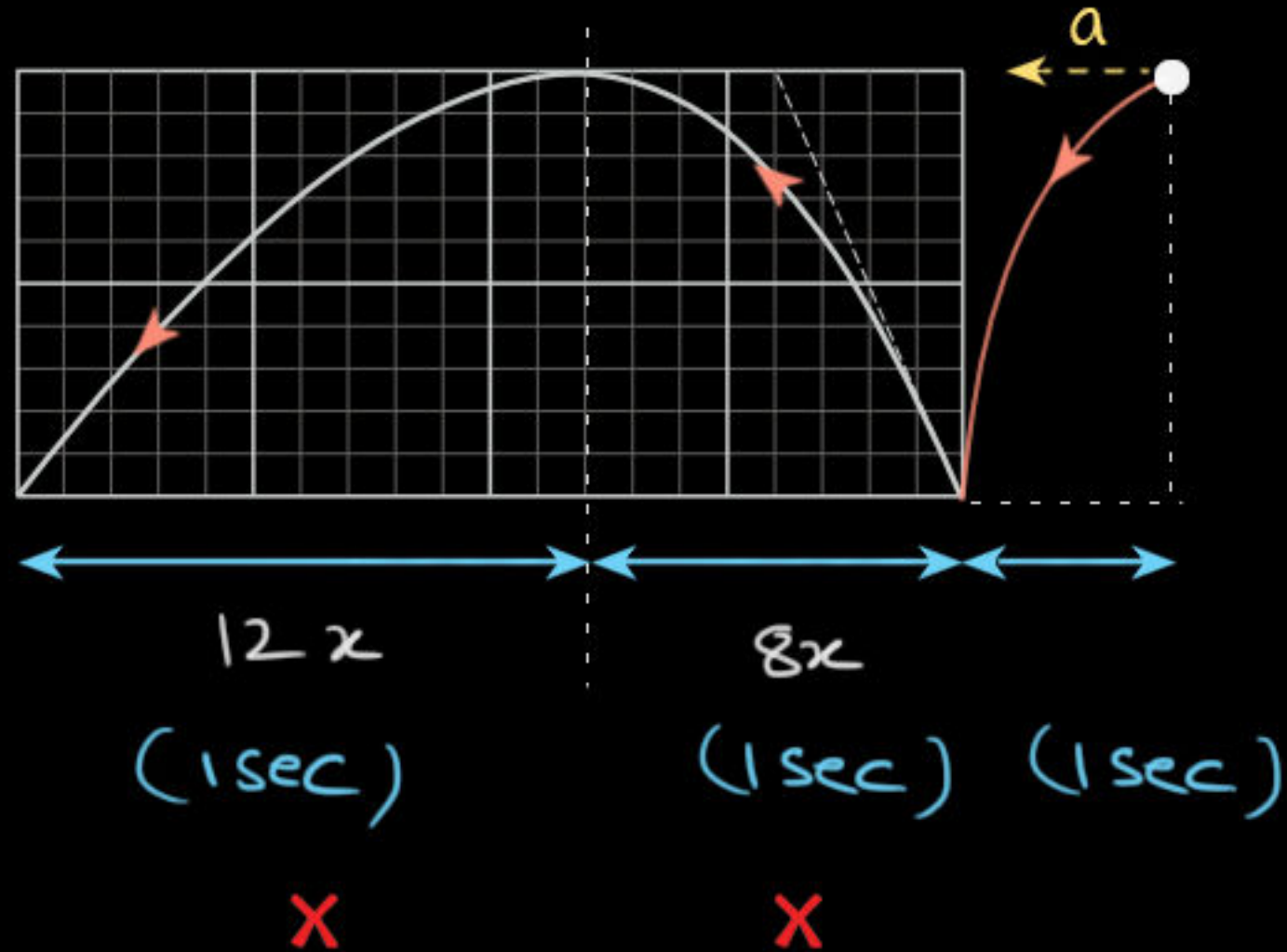
From 1,2

$$= \sqrt{u_2^2 + 4u_1(u_1 + u_2) \left(1 - \frac{b^2}{r^2}\right)}$$

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Its a trajectory graph.
Trajectory w.r.t car. 



27. A ball is dropped from a certain height on a straight horizontal road and its motion is recorded from a car that starts moving with a uniform acceleration when the ball is dropped. Path of the ball relative to the car between the first and the second bounce is shown in the graph. The dashed line shown is a tangent to the path. If collision of the ball with the road is perfectly elastic and duration between the first and the second bounce is 2.0 s, find displacement of the car between the first and the second bounce of the ball. Acceleration due to gravity is 10 m/s^2 .



We know, for a body starting at uniform acceleration, distances in 1st, 2nd and 3rd second are in the ratio of: $1 : 3 : 5$

But in graph the second ratio is $8x:12x$ ie $2:3$ and not required $3:5$.
So given graph is wrong in my opinion.

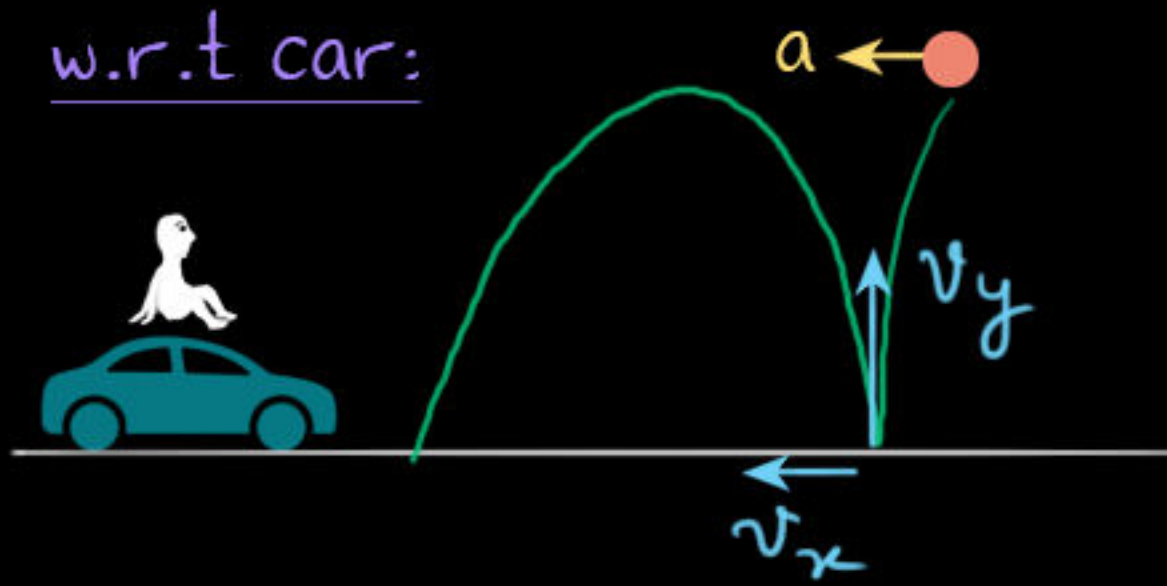
However it can be solved if we dont consider the full graph.

Ref: Kinematics, BYU 20

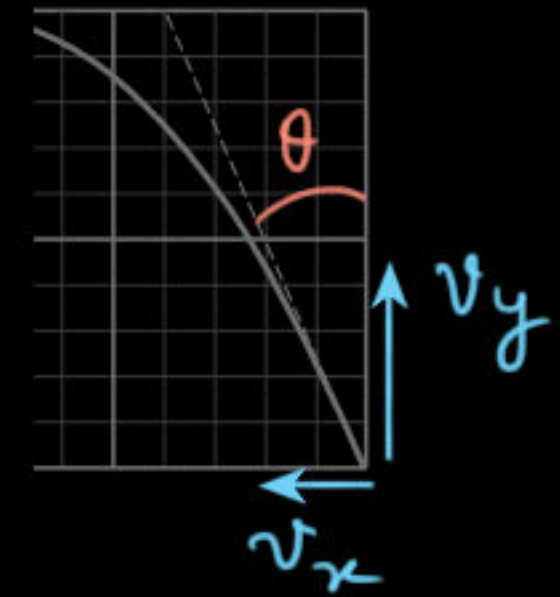
Sol: w.r.t ground:



w.r.t car:



w.r.t car:



As dropped for 1 second: $v_y = u + gt = 0 + 10 \times 1 = 10 \text{ m/s}$

At first bounce, slope of trajectory: $\frac{v_x}{v_y} = \tan \theta = \frac{4}{10} \Rightarrow v_x = 4 \text{ m/s}$ ①

Also: $v_x = u + at = 0 + a \times 1 \text{ sec}$
 $\Rightarrow 4 = a$ ②

Along the road: $S_{\text{ball}|_{\text{car}}} = S_{\text{ball}} - S_{\text{car}}$

$$\Rightarrow |S_{\text{car}}| = |S_{\text{ball}|_{\text{car}}}| = v_x(2) + \frac{1}{2}a(2)^2 \xrightarrow{\text{From 1,2:}} = 16 \text{ m}$$

by Ankit Singhvi

Youtube / Arihant Senior

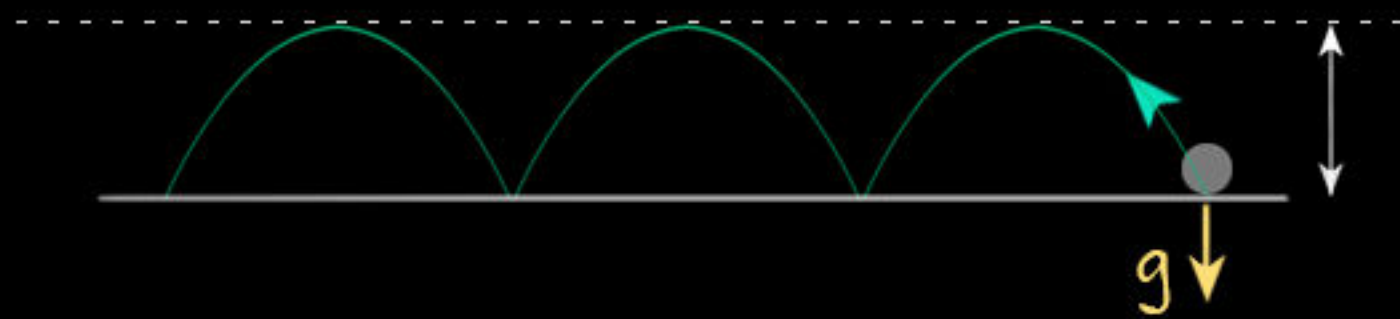
28. A ball projected horizontally from a large inclined plane, collides successively with the plane. Coefficient of friction between the plane and the ball during these collisions is μ . What should be inclination of the plane and coefficient of restitution for the collisions so that between every two successive bounces the ball follows identical trajectories?

Sol:

Given identical trajectories:

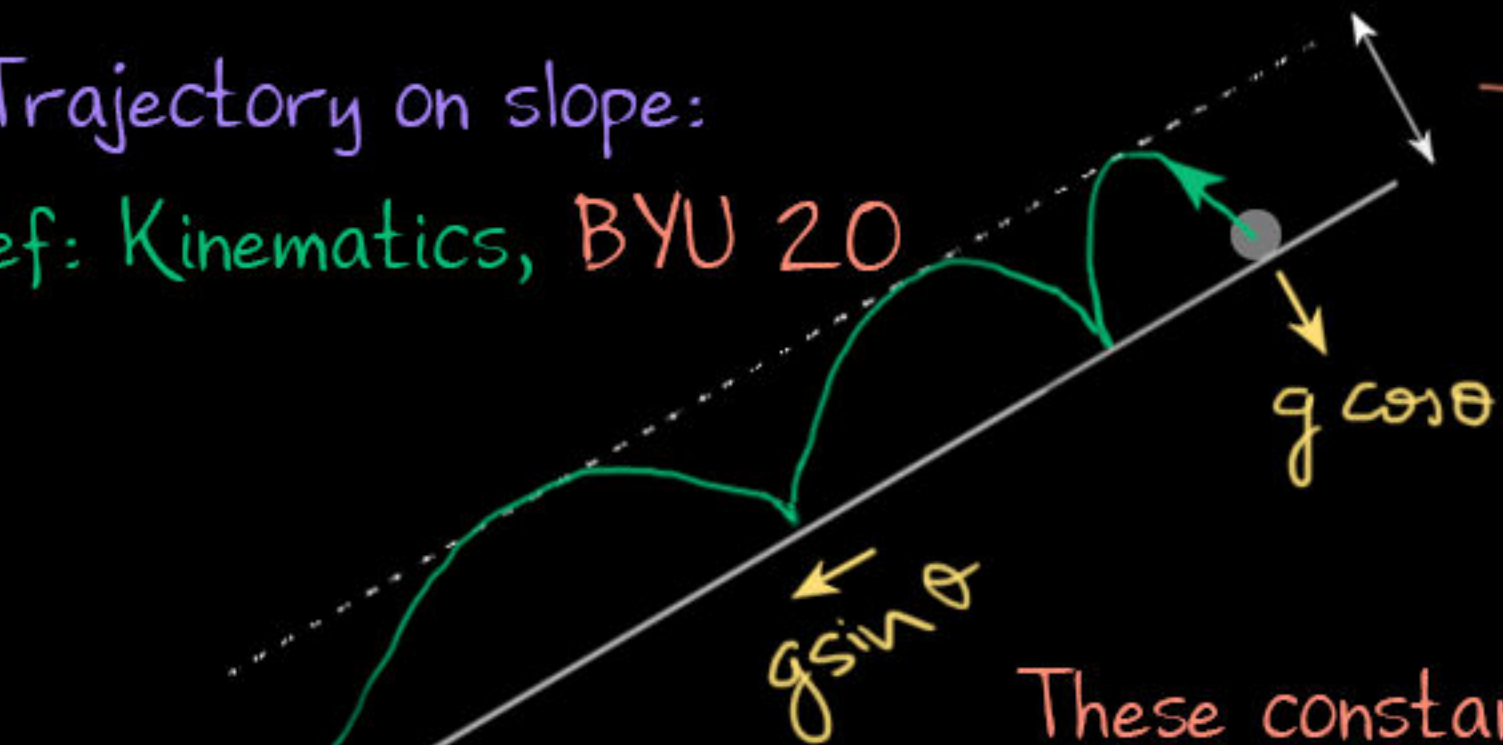
In elastic collisions on frictionless surface:

Trajectory on horizontal plane:

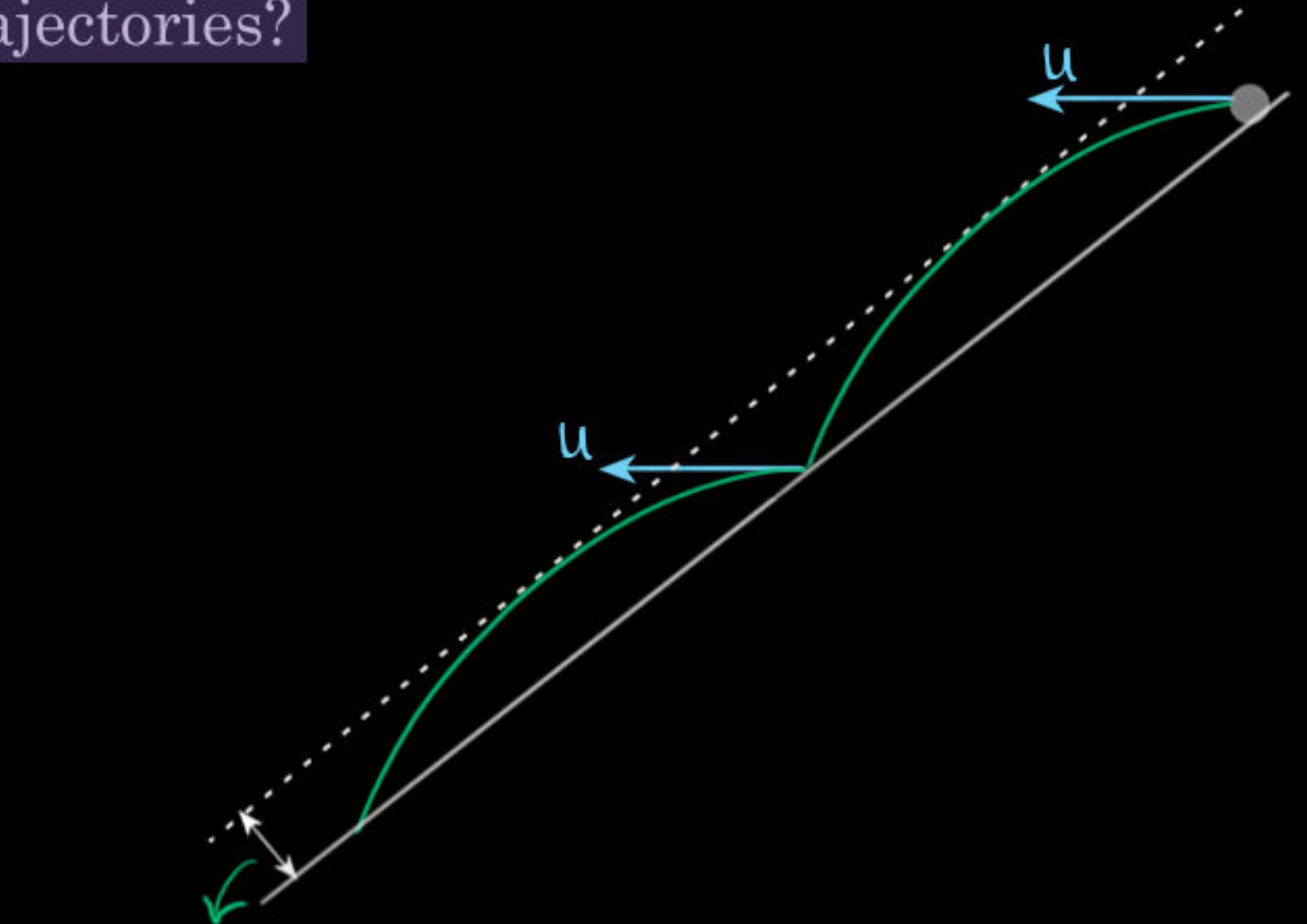


Trajectory on slope:

Ref: Kinematics, BYU 20



These constant heights mean elastic collision.

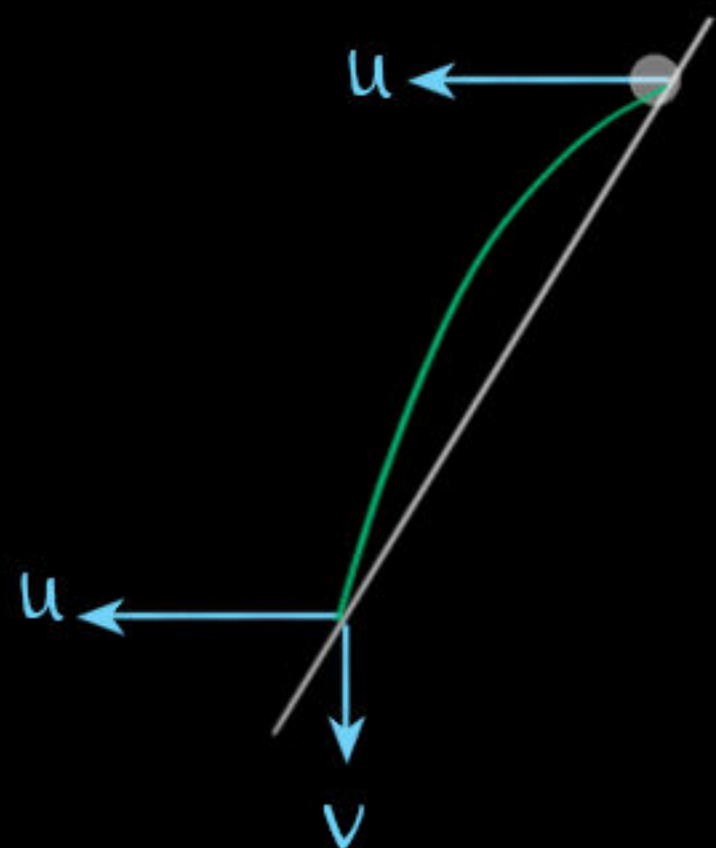


Constant height

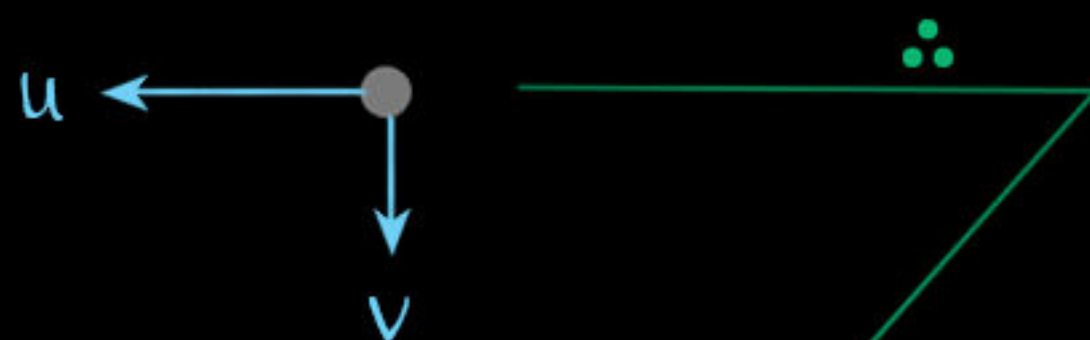
Elastic collision

$$e=1$$

Sol:



Just before collision



Just after collision



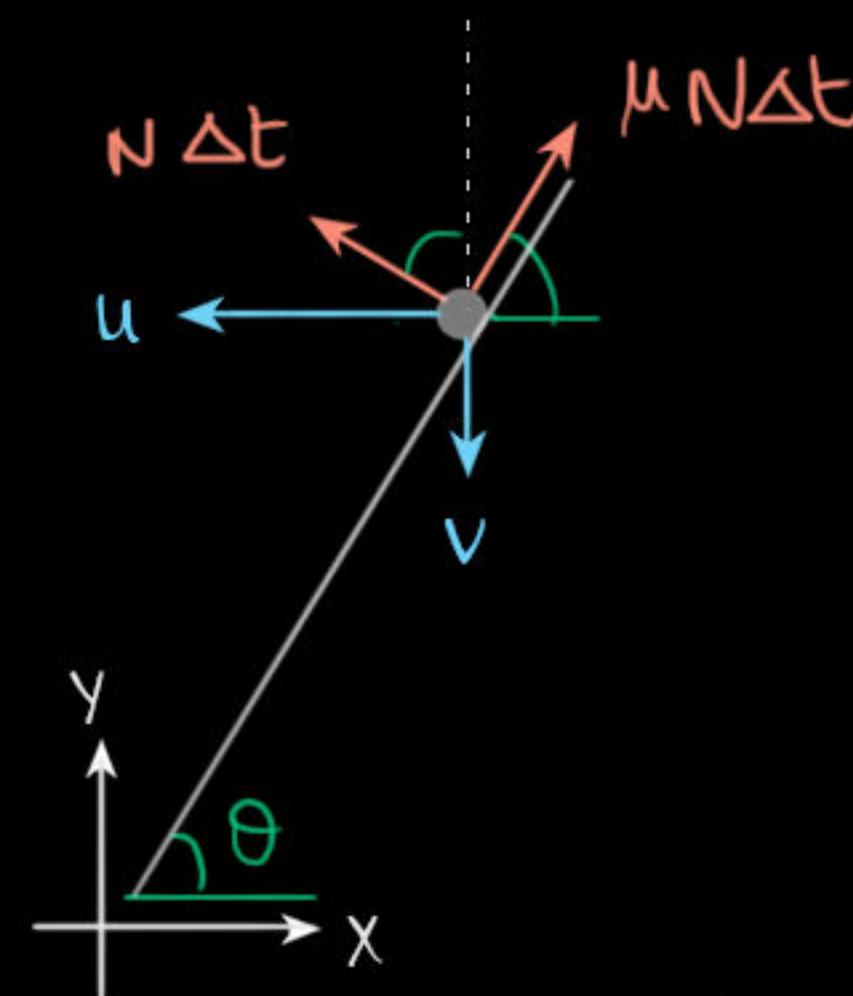
In horizontal direction
net impulse is zero.

$$\Rightarrow P_x: \cancel{N \Delta t} \sin \theta = \mu \cancel{N \Delta t} \cos \theta$$

$$\Rightarrow \theta = \tan^{-1} \mu$$

$$\text{Also, } P_y: N \Delta t \cos \theta + \mu N \Delta t \sin \theta = mv$$

(A good equation, but not useful in this problem!)

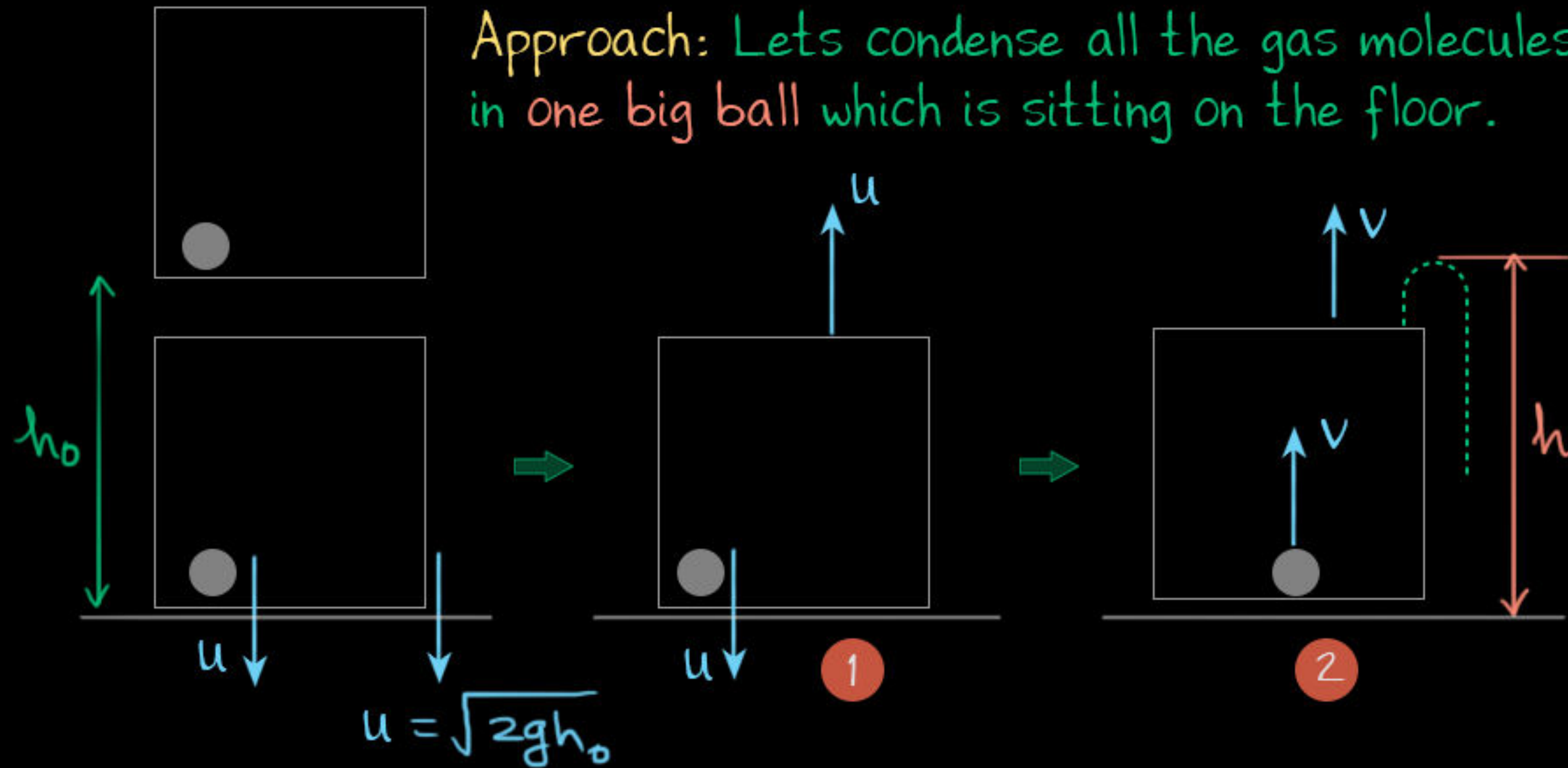


29. A container of mass $M = 10.0$ kg, containing $m = 1.0$ kg of an ideal gas is released without initial velocity from a height $h_0 = 12.1$ m. If collision of the container with the ground is perfectly elastic, how high above the ground will the container bounce? Neglect air resistance and assume that all the internal fluctuations in the gas damp out almost in no time. Acceleration of free fall is $g = 10$ m/s².

Pretty huge assumption

⇒ All collisions of gas molecules are inelastic.

Approach: Lets condense all the gas molecules in one big ball which is sitting on the floor.



Just before collision
between box and floor

Just after, but
ball is yet to collide

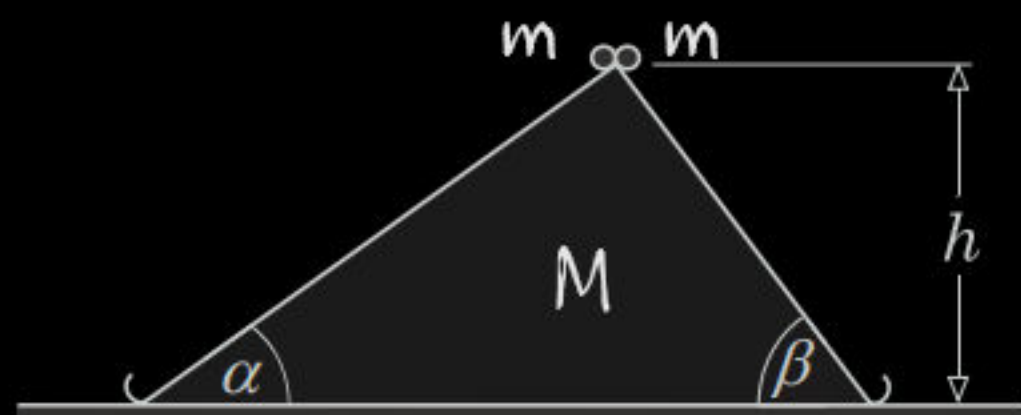
Just after collision
between box and ball

p conservation between 1 and 2:

$$Mu - mu = (M+m)v$$

$$\Rightarrow v = \frac{(M-m)u}{(M+m)} \rightarrow \sqrt{2gh_0}$$

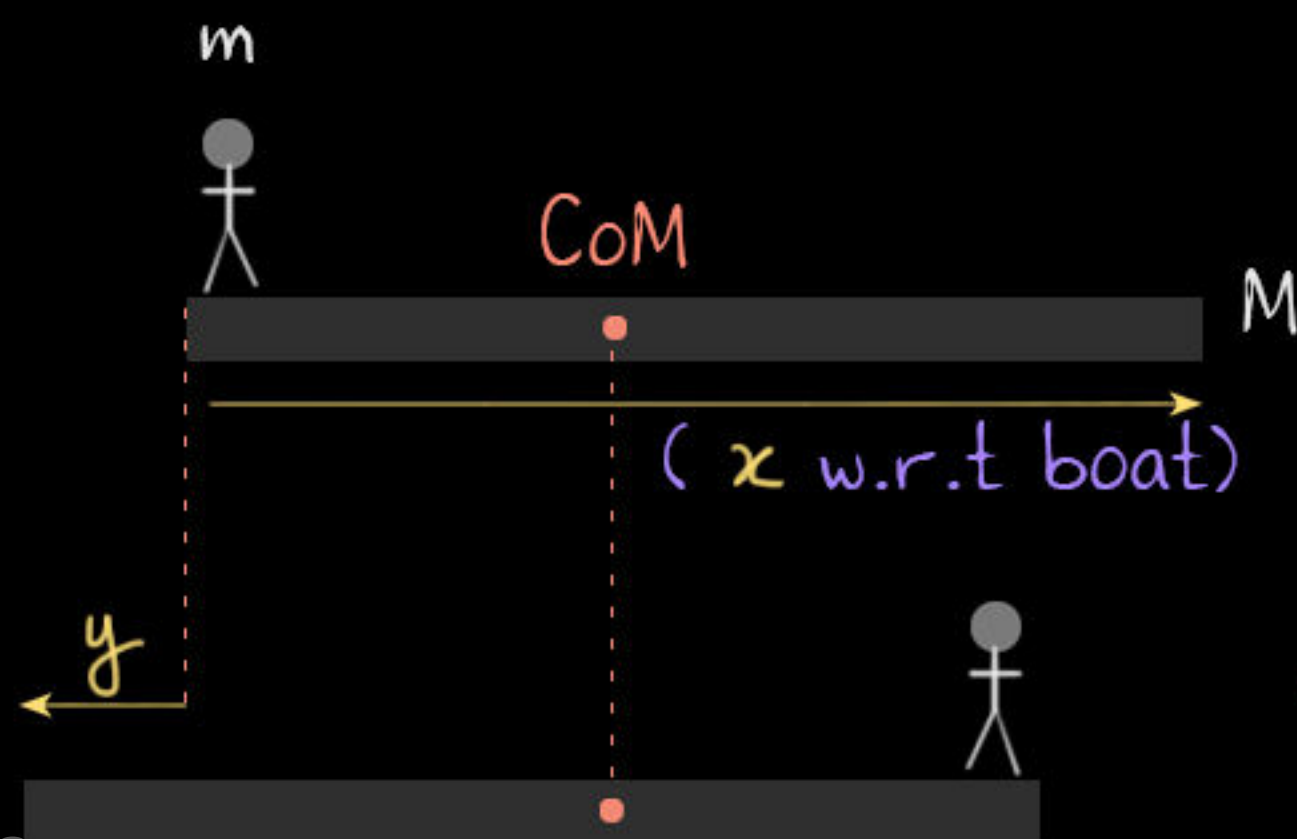
$$\therefore h = \frac{v^2}{2g} = \left[\frac{M-m}{M+m} \right]^2 h_0 //$$



30. Two identical small balls each of mass m are held on the top of a triangular wedge of mass M and height h that is at rest on a frictionless horizontal floor. When the balls are released, they roll down the inclined faces of the wedge and at the bottom get stuck in special traps. How far will the wedge shift during movement of the ball relative to the wedge?

Approach: As there is no horizontal force on system. CoM will remain at rest.

General man boat problem: If Man of mass m moves x w.r.t boat of mass M . How much boat moves w.r.t water?

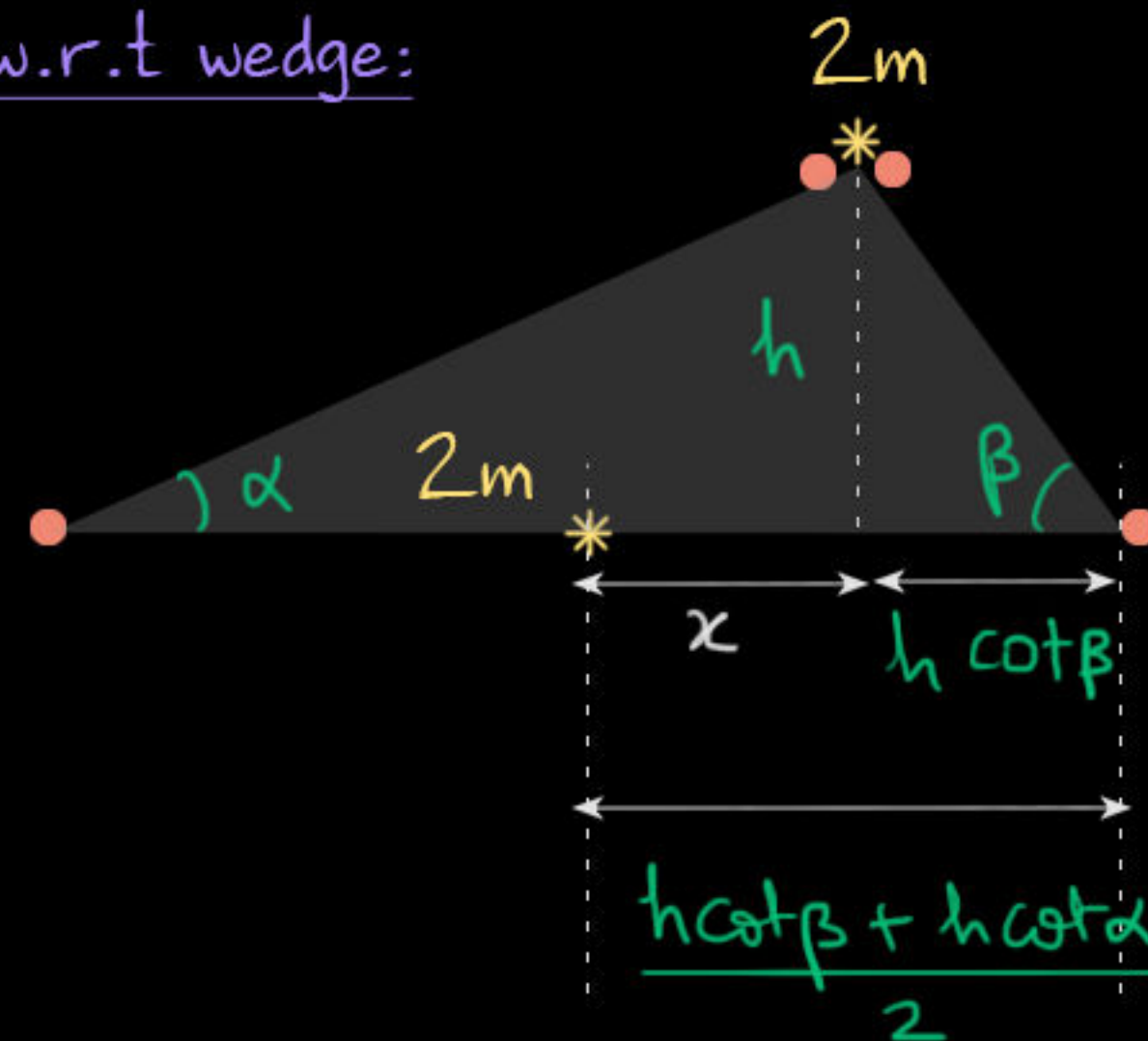


Location of CoM of boat is not needed

$$My = m(x - y)$$

$$\Rightarrow y = \frac{mx}{m+M}$$

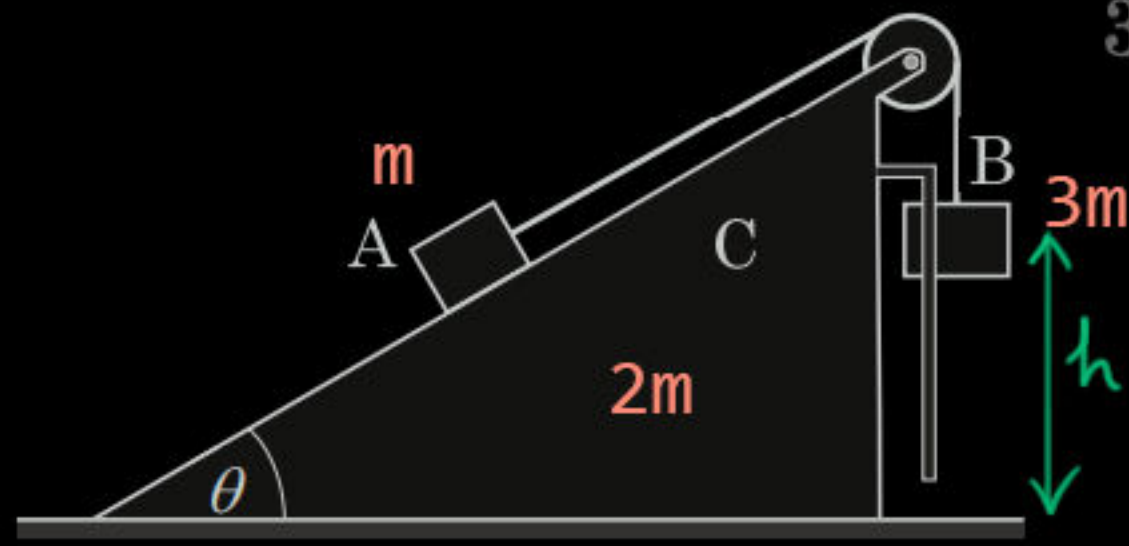
Sol: w.r.t wedge:



$$\therefore y = \frac{2mx}{2m+M} = \frac{mh}{2m+M} (\cot \alpha - \cot \beta)$$

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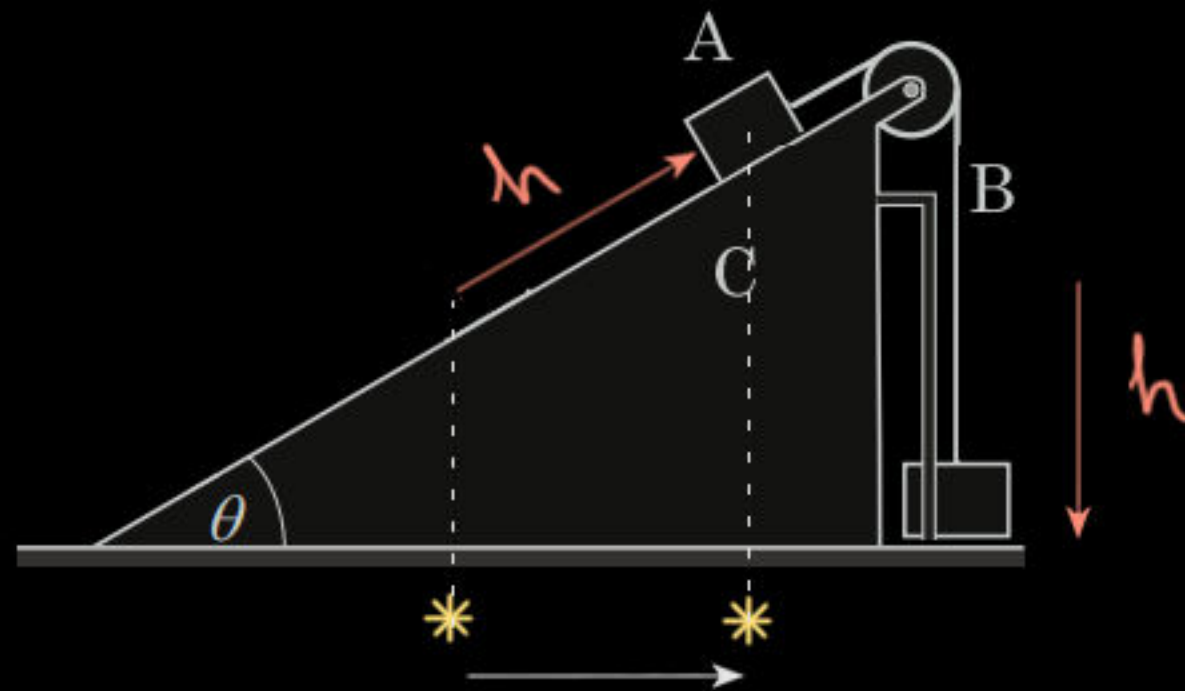
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31. In the setup shown, the block B can slide on a vertical guide rigidly fixed to the wedge C. Masses of the block A, the wedge C and the block B are m , $2m$ and $3m$ and cosine of angle of inclination θ of the inclined face of the wedge with the horizontal is $2/3$. The pulley and the thread are ideal and there is no friction between any of the surfaces in contact. The block B is released from a height 27 cm above the floor. Find displacement of the wedge when the block B touches the floor.

Approach: Similar to BYU 30

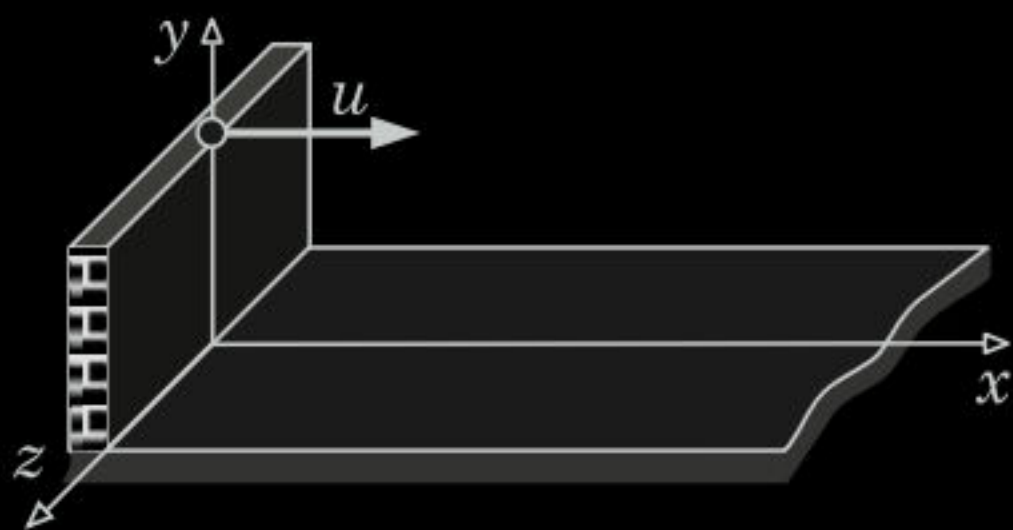
w.r.t wedge: Only horizontal movement is of A



Horizontally, the CoM is at rest.

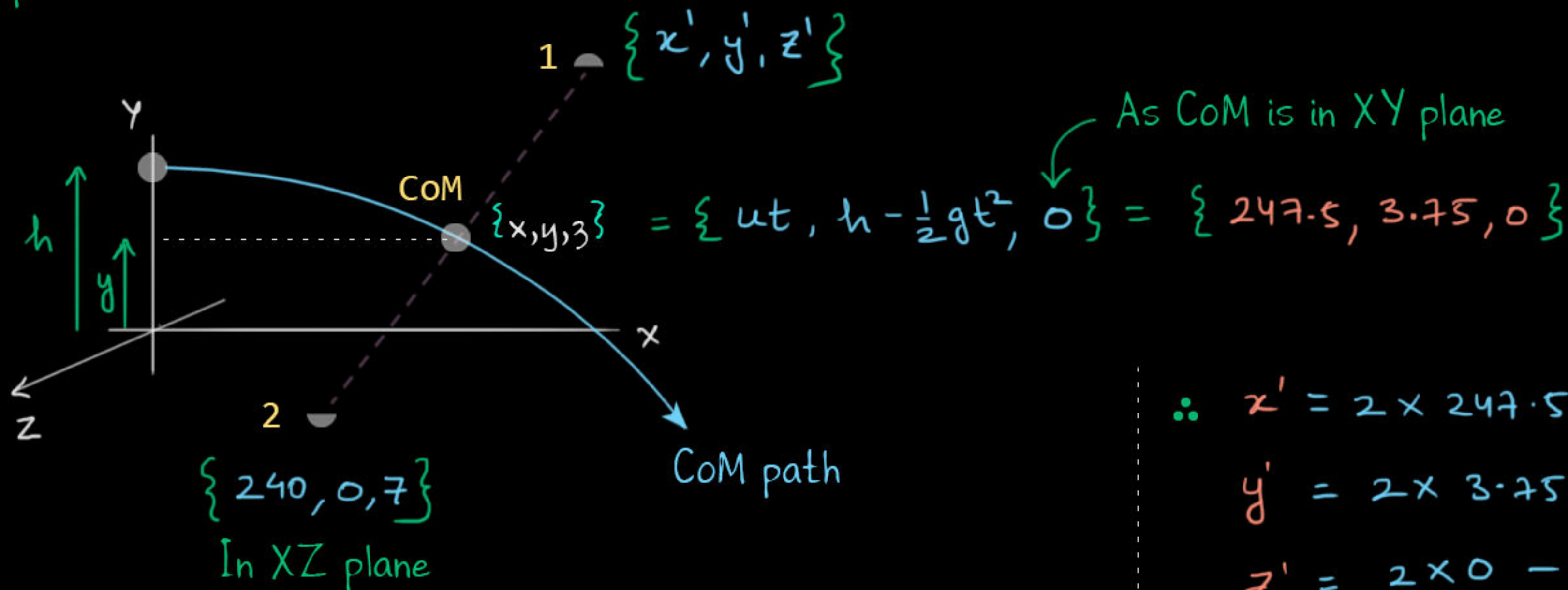
$\therefore$

$$\begin{aligned}
 y &= \frac{m_A x}{m_A + m_B + m_C} \\
 &= \frac{m \left(27 \cdot \frac{2}{3} \right)}{m + 3m + 2m} \\
 &= 3\text{ cm} //
 \end{aligned}$$



32. A shell projected horizontally with a velocity of 165 m/s from the top of a 15 m high wall explodes into two identical fragments in the air. One of the fragments strikes the ground 1.5 s after the shell was projected at a distance of 240 m from the foot of the wall and 7 m to the right of the x-axis. Determine position of the other fragment at this instant. Neglect air resistance and assume acceleration of free fall to be 10 m/s^2 .

Approach: CoM will lie at the center of both identical particles 1 and 2.



$$\therefore x' = 2 \times 247.5 - 240 = 255 //$$

$$y' = 2 \times 3.75 - 0 = 7.5 //$$

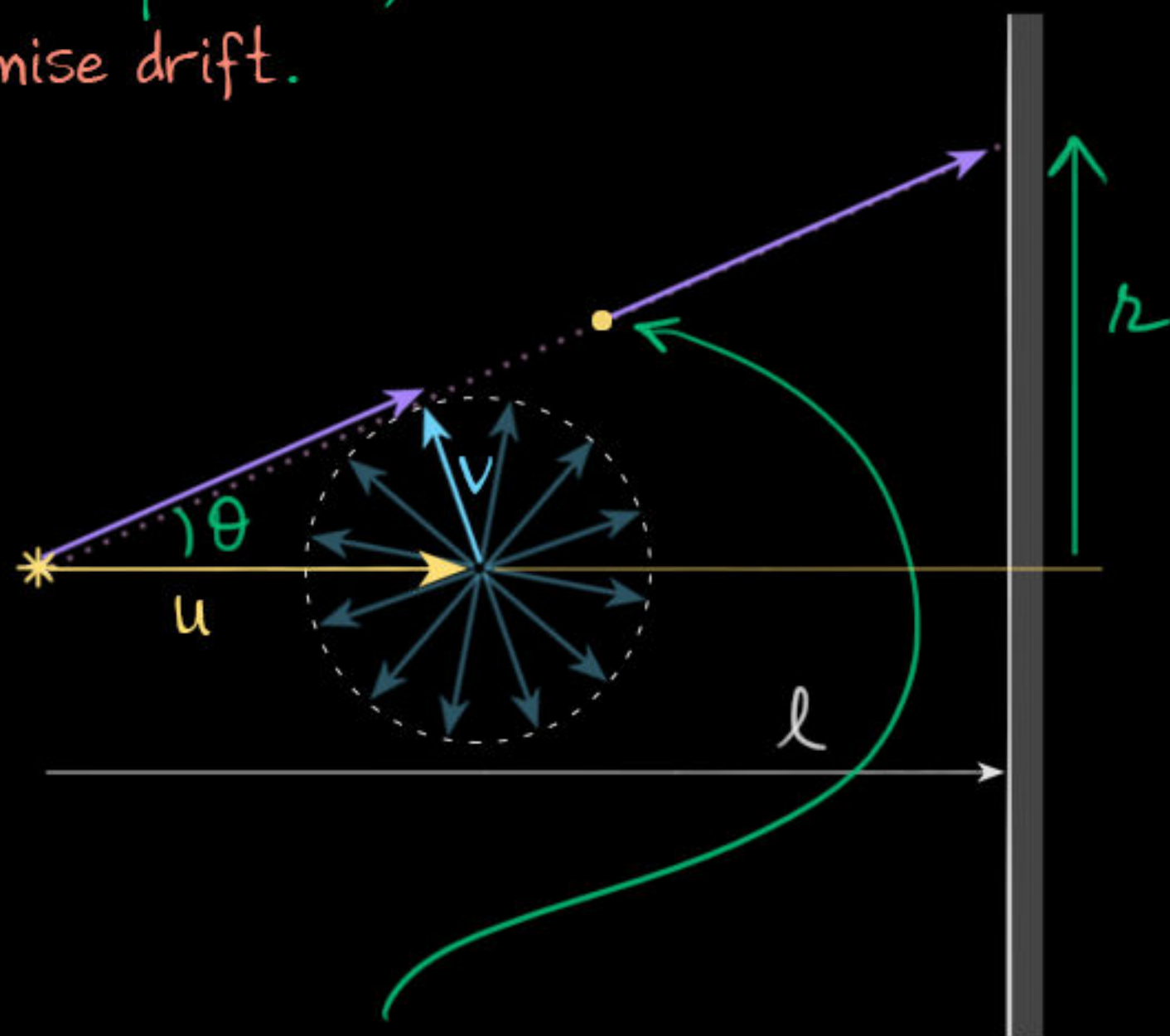
$$z' = 2 \times 0 - 7 = -7 //$$

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$$v_{CoM} \xrightarrow{u}$$

$$v_{fragments|CoM} \xrightarrow{v}$$

Approach: Similar to river (u) boat (v) problem, where we minimise drift.



This fragment will hit the plane farthest

33. In free space, a shell is flying with a speed u towards a large stationary plane along its normal. When the shell is at a distance l from the plane, it explodes into an infinitely large number of fragments that fly apart in all directions such that speed of each fragment relative to the mass centre is v . Find area on the plane in which all the fragments strike.

$$\sin \theta = \frac{v}{u} \quad (1)$$

$$h = l \tan \theta \quad (2)$$

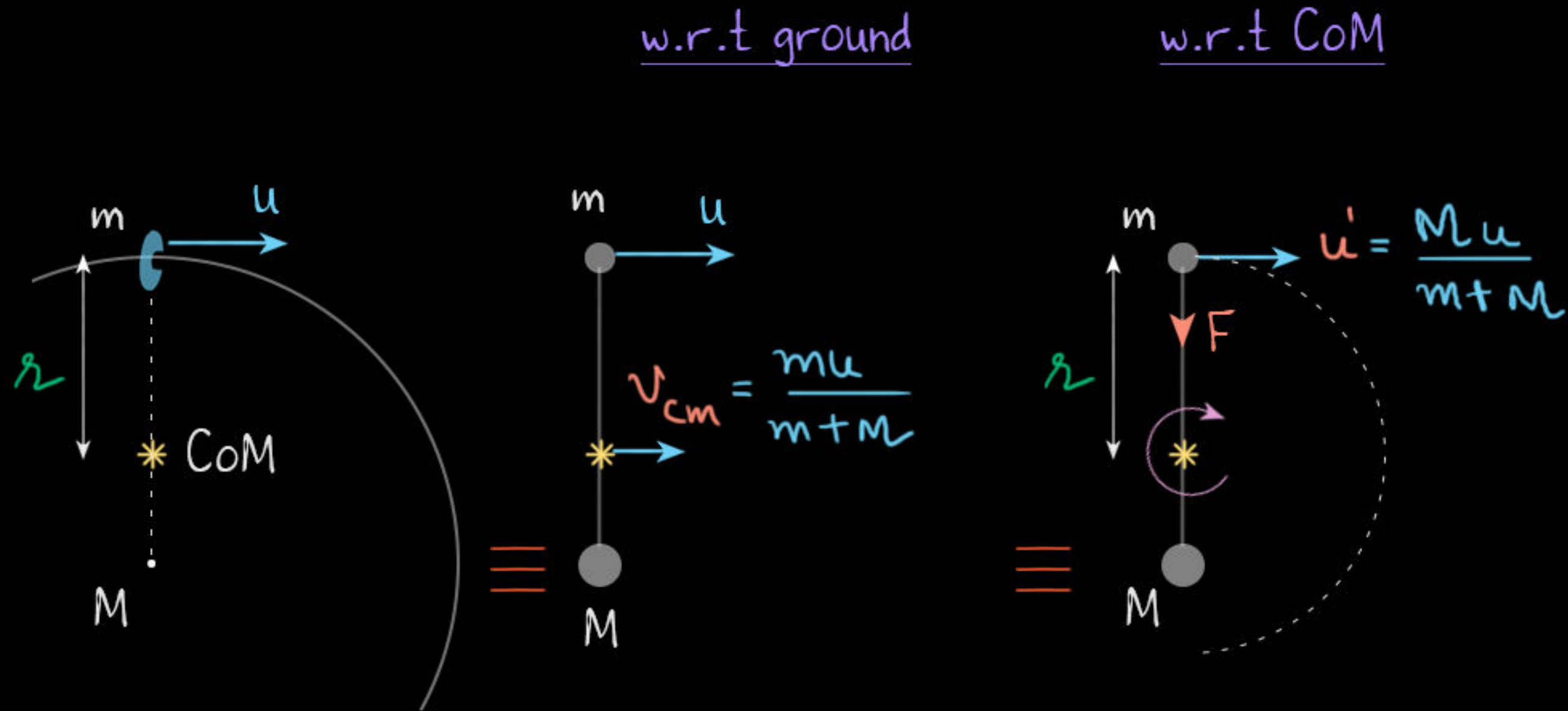
From 1,2:

$$h = l \frac{v}{\sqrt{u^2 - v^2}}$$

Note: If $v > u$, complete area of plane will be struck

Approach: m and M will rotate about their CoM with constant angular velocity. As net force on system is zero.

34. A small bead of mass $m = 4.0$ kg is threaded on a homogeneous frictionless ring of mass $M = 6.0$ kg and radius $R = 0.5$ m. Initially the system rests in free space. Now the bead is struck to impart it a velocity $u = 5.0$ m/s along the tangent to the ring. Find magnitude of force of interaction between them and the minimum kinetic energy of the bead in subsequent motion.



$$F = \frac{m u'^2}{R} = \frac{m}{\left(\frac{MR}{m+M}\right)^2} \cdot \frac{MR}{m+M}$$

$$= \frac{m}{\left(\frac{MR}{m+M}\right)^2} = \frac{mMu^2}{(m+M)R}$$

For bead: $v_{min} = u' - v_{cm}$

$$= \frac{Mu}{m+M} - \frac{mu}{m+M}$$

$$= \left(\frac{M-m}{m+M}\right)u$$

Approach: Kinetic Energy of two particle system w.r.t CoM is given by:

$$KE_{cm} = \frac{1}{2} \mu v_{rel}^2$$

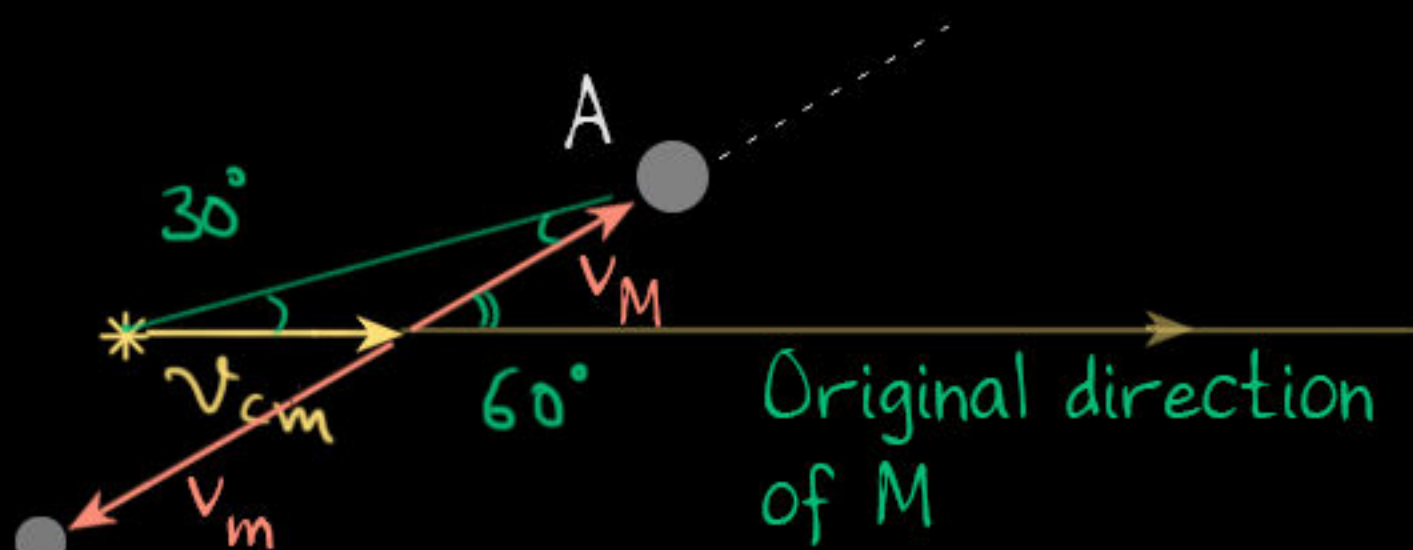
w.r.t. ground before collision:



w.r.t. CoM after collision: Let v_M and v_m be w.r.t CoM

$\therefore (\vec{v}_{cm} + \vec{v}_M)$ is w.r.t ground.

Expressing everything in vector form:



Original direction of M

35. Two balls of unequal masses, moving in opposite directions with equal speeds collide elastically. Thereafter, the heavier particle is observed deviated from its original direction of motion by an angle $\alpha = 30^\circ$ in the laboratory frame and by an angle $\beta = 60^\circ$ in the centre of mass frame. How many times of the mass of lighter ball is the mass of heavier ball?

$$v_{cm} = \frac{M-m}{M+m} \cdot u \quad (2)$$

KE before collision = KE after collision

$$\therefore \frac{1}{2} \mu (2u)^2 = \frac{1}{2} \mu (v_m + v_M)^2$$

$$\Rightarrow 2u = v_m + v_M \quad (3)$$

$$\text{Momentum: } Mv_M = mv_m \quad (4)$$

$$\text{From 1,2,3,4: } \frac{M}{m} = 3 //$$

$$\begin{aligned} \angle A + 30^\circ &= 60^\circ \\ \Rightarrow \angle A &= 30^\circ \\ \Rightarrow v_{cm} &= v_M \quad (1) \\ (\because \text{isosceles triangle}) \end{aligned}$$

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Proof of Kinetic Energy of two particle system w.r.t CoM is given by:

$$KE_{cm} = \frac{1}{2} \mu v_{rel}^2$$



We need to prove:

$$\frac{1}{2} M v_1^2 + \frac{1}{2} m v_2^2 = \frac{1}{2} \frac{mM}{m+M} (v_1 + v_2)^2$$

Multiply and divide LHS by $(m + M)$

$$= \frac{1}{2(m+M)} \left[mM v_1^2 + M^2 v_1^2 + m^2 v_2^2 + mM v_2^2 \right]$$

$$= \frac{1}{2(m+M)} \left[mM v_1^2 + mM v_2^2 + \cancel{(M v_1 - m v_2)^2} + 2mM v_1 v_2 \right]$$

$$= \frac{1}{2(m+M)} mM (v_1 + v_2)^2 = RHS \quad \text{Hence proved!}$$

Proof of

$$p_{1cm} = p_{2cm} = \mu v_{rel}$$

w.r.t. CoM:



We need to prove: $Mv_1 = \frac{mM}{m+M}(v_1 + v_2)$

Cross Multiply

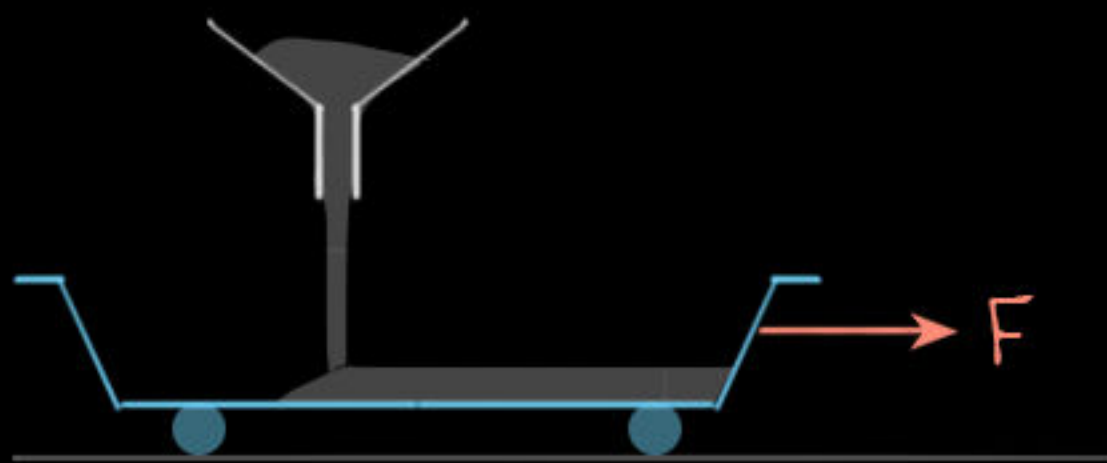
$$\cancel{M}mv_1 + M^2v_1 = \cancel{m}Mv_1 + \cancel{m}Mv_2$$

$$\Rightarrow Mv_1 = mv_2$$

Which is true. Hence proved!

Approach: Net force on system is F .

System: Train (m_0) + coal already on it (rt) + (dm) that falls and comes to rest.



36. When an empty freight train of mass m_0 starts, loading of coal in the train begins at a constant rate r from a stationary hopper. If the track is horizontal and engine pull F is a constant, deduce expression for speed of the train as a function of time t . Neglect all resistive forces.

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m \frac{dv}{dt} + v \frac{dm}{dt}$$

Given constant $\leftarrow$

$$\Rightarrow F = (m_0 + rt) \frac{dv}{dt} + v r$$

$$\Rightarrow F = (m_0 + rt) \frac{dv}{dt} + v r$$

$$\Rightarrow \int_0^v \frac{dv}{F - vr} = \int_0^t \frac{dt}{m_0 + rt}$$

$$\Rightarrow v = \frac{Ft}{m_0 + rt}$$

on next slide

Solving equation:

$$\int_0^v \frac{dv}{F - vR} = \int_0^t \frac{dt}{m_0 + Rt}$$

$$\Rightarrow \frac{-1}{\cancel{R}} \ln\left(\frac{F - vR}{F}\right) = \frac{1}{\cancel{R}} \ln\left(\frac{m_0 + Rt}{m_0}\right)$$

$$\Rightarrow \frac{F}{F - vR} = \frac{m_0 + Rt}{m_0}$$

$$\Rightarrow v = \frac{Ft}{m_0 + Rt} //$$

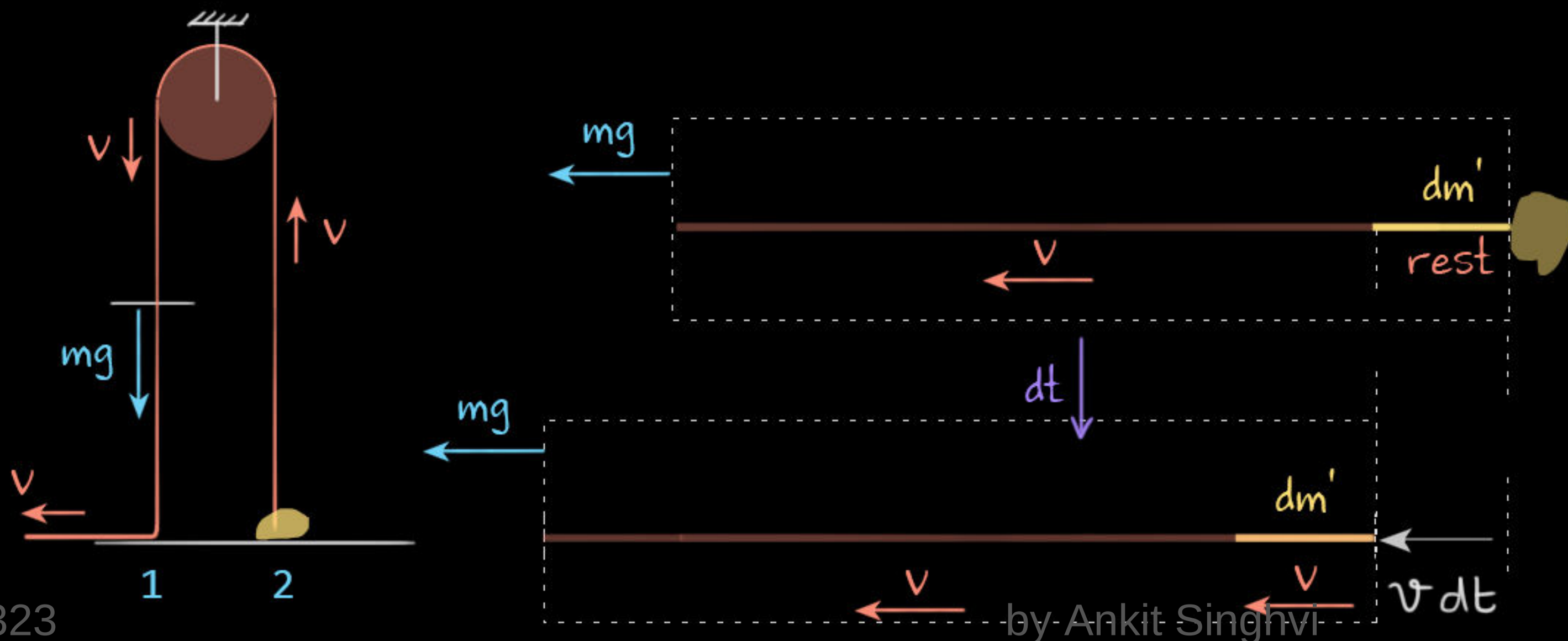
$$F \text{ on bug by rope} \\ = F \text{ on rope by bug} \\ = mg$$



37. A uniform thread of linear mass density $\lambda = 2.0 \text{ g/m}$ passes through a fixed ideal pulley. At its ends, the thread is stacked into two heaps that do not interfere motion of the thread. A beetle of mass $m = 80 \text{ g}$ tries to manage itself at a constant height on the thread by adjusting its speed relative to the thread. At some steady speed of the thread, the beetle succeeds in doing so. Find this steady speed of the thread. Acceleration due to gravity is $g = 10 \text{ m/s}^2$.

Concept: Rope coming to rest at 1 does nothing to hanging part. So we can ignore it. That is assume that it keeps going with same velocity v on a frictionless floor.

Approach: straighten the rope, define the system and write $F = dp/dt$

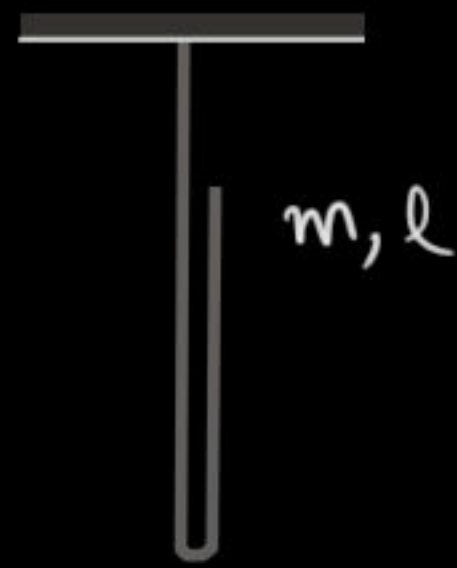


$$F = \frac{dp}{dt}$$

$$\Rightarrow mg = \frac{dm'(\overset{\lambda v dt}{v} - 0)}{dt}$$

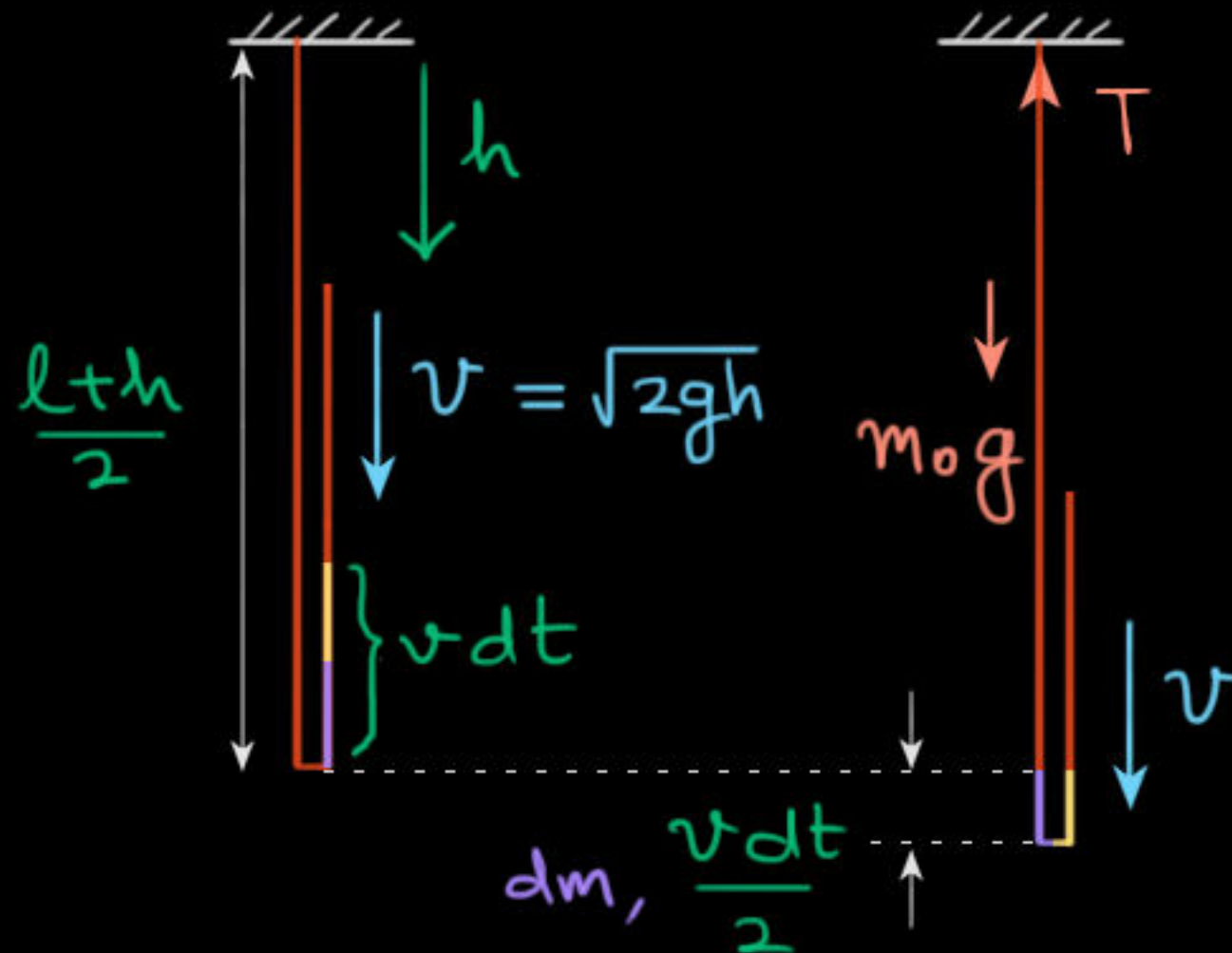
$$\Rightarrow mg = \lambda v^2$$

$$\Rightarrow v = \sqrt{\frac{mg}{\lambda}}$$



38. Ends of a thin uniform inextensible rope of mass $m = 1.0$ kg and length $l = 2.0$ m are fastened to the ceiling close to each other. If one of the ends is released and allowed to fall freely as shown in the figure, find the largest force that the fastening at the other end must be capable of bearing so that rope remains fastened there. Acceleration of free fall is $g = 10$ m/s².

Concept: Tension in Rope on right is zero and it will free fall.
If x amount falls from right, $x/2$ gets added to left.



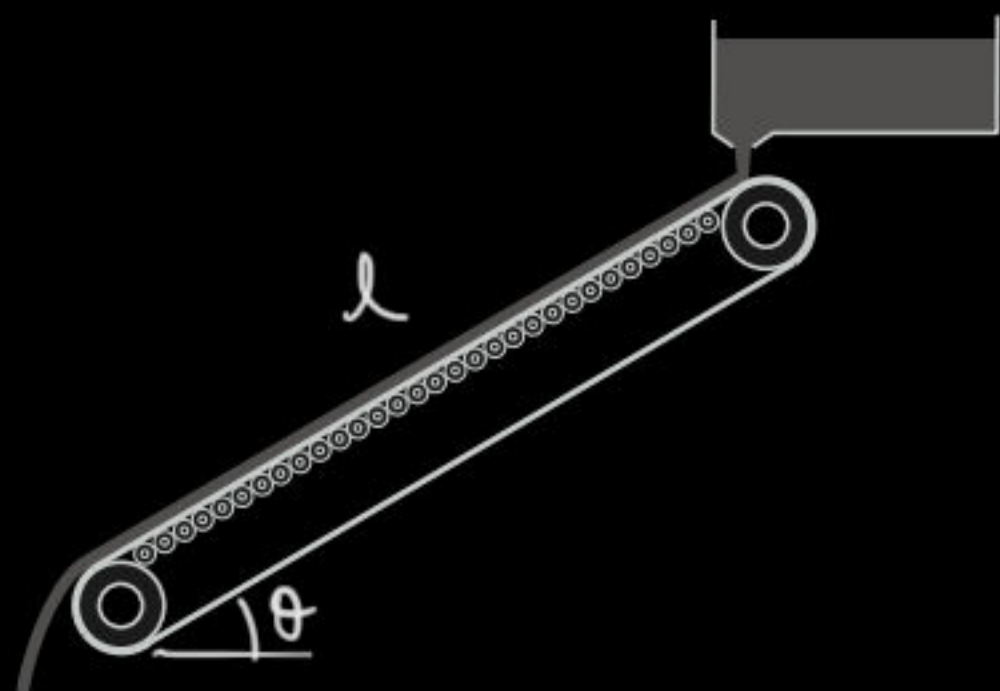
System On system: $F_{net} = \frac{dp}{dt}$

$$\Rightarrow T - m_0 g = \frac{dm \left[0 - (-v) \right]}{dt}$$

Labels in the diagram: $\lambda \left(\frac{l+h}{2} \right)$ points to m_0 ; $\lambda \left(\frac{v dt}{2} \right)$ points to dm ; $\sqrt{2gh}$ points to v .

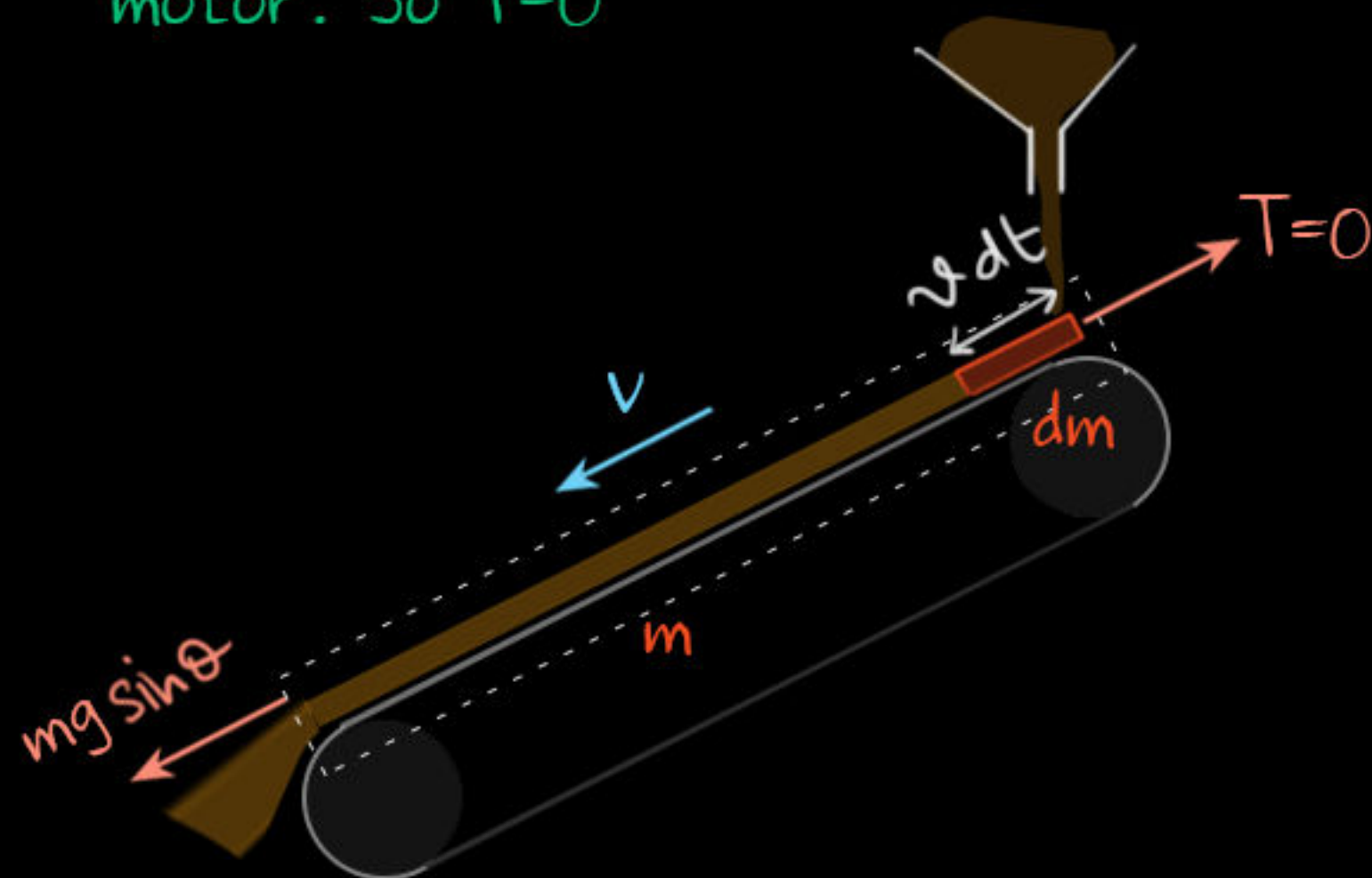
$$\Rightarrow T = \lambda \left(\frac{l+h}{2} \right) g + \lambda g h$$

$$\therefore @ h \rightarrow l, T_{max} = 2\lambda g l //$$



Concept: There is no motor. So $T=0$

39. A conveyor belt is inclined at an angle θ with the horizontal. Length of a straight portion of the belt is l , resistive forces in the belt mechanism are negligible and radii of the end cylinders are negligible as compared to the length l . Sand falls on the belt with a negligible speed at a constant rate at the top and leaves at the bottom. Friction between the belt and the sand is high enough to stop sliding of sand particles almost in no time. Inertia of the belt mechanism is negligible as compared to that of the total sand on it. Find the steady speed acquired by the belt. Acceleration due to gravity is g .



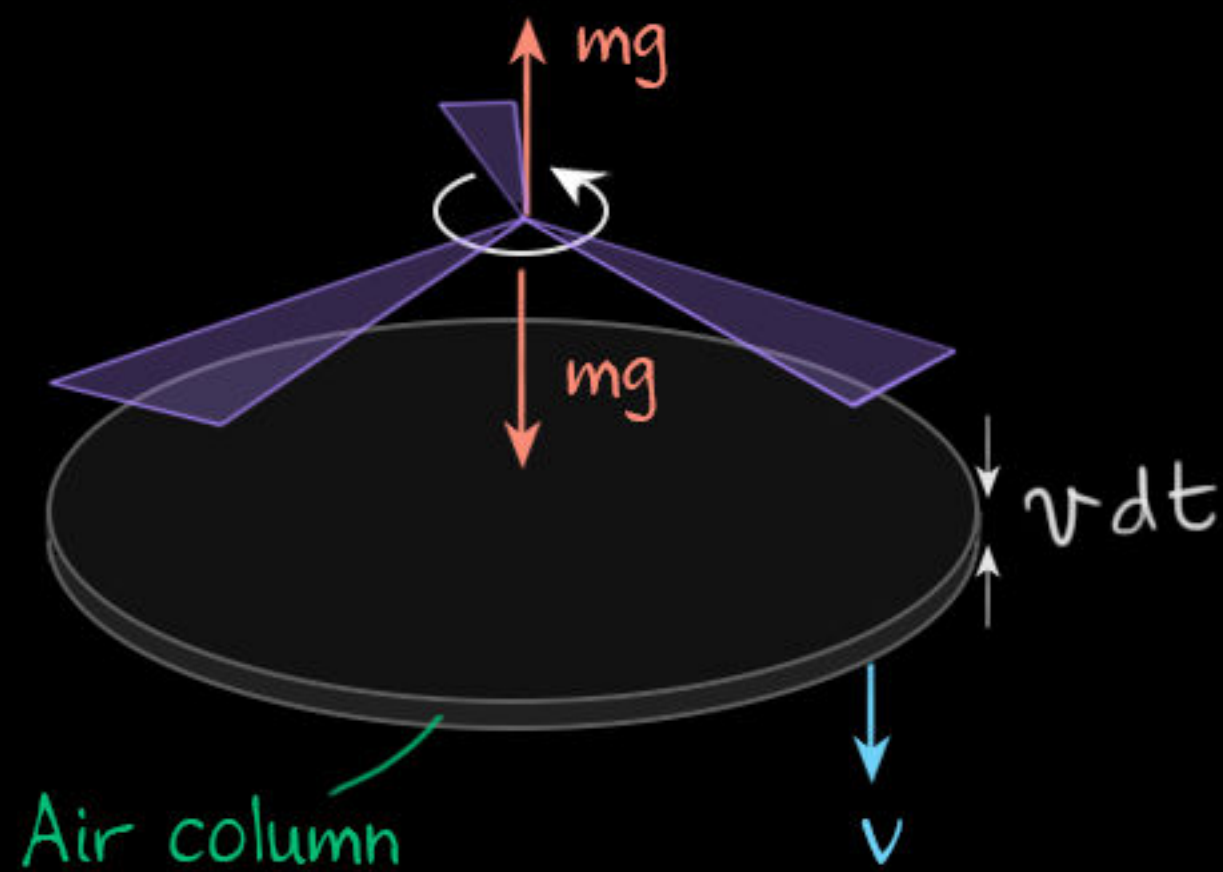
$$F = \frac{dp}{dt}$$

$$\cancel{mg \sin \theta} = \frac{dm(v-0)}{dt} = \frac{\lambda v dt v}{dt} = \frac{mv^2}{l}$$

$$\Rightarrow v = \sqrt{lg \sin \theta}$$

Lets say the air is pushed down from rest to v by the fan.

As a result fan gets a lift of mg for minimum power needed.



40. Fan of a remote operated model helicopter of mass m has diameter d . Denoting density of air by ρ and acceleration of free fall by g , deduce expression for the minimum power required for take-off.

∴ In dt time, momentum gained by air is:

$$\delta \Delta V = \delta \pi \frac{d^2}{4} \cdot v dt \quad (1)$$

$$dp = dm v$$

$$\Rightarrow dp = \left[\frac{\delta \pi d^2 v dt}{4} \right] \cdot v$$

$$\therefore F = mg = \frac{dp}{dt} = \frac{\delta \pi d^2 v^2}{4} \Rightarrow v = \sqrt{\frac{4mg}{\delta \pi d^2}} \quad (2)$$

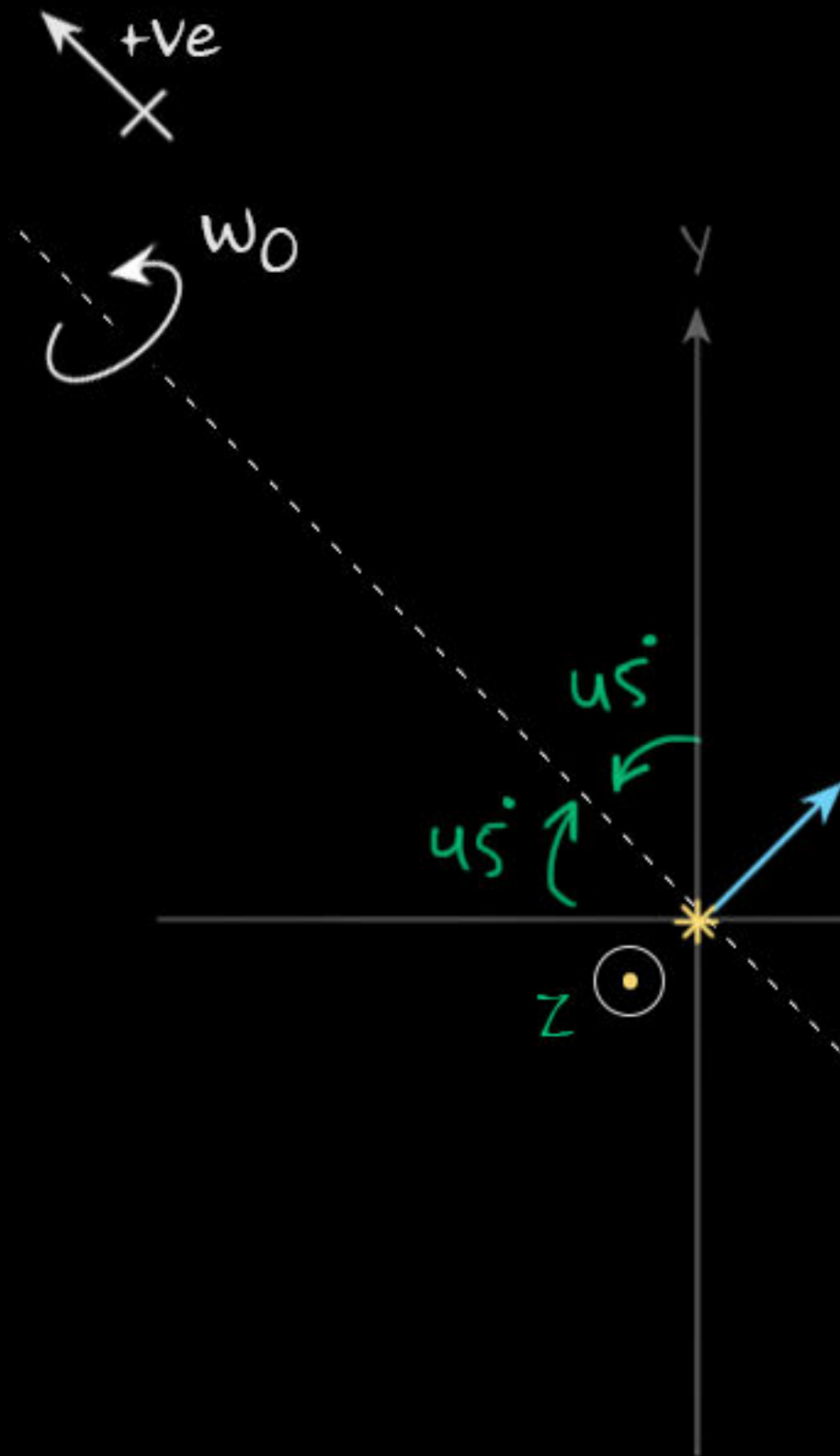
$$\text{Power} = \frac{dKE}{dt} = \frac{\left(\frac{1}{2} dm v^2 - 0 \right)}{dt} = \frac{1}{2} \frac{\delta \pi d^2}{4} \cdot \sqrt{\frac{4mg}{\delta \pi d^2}} \cdot dt \cdot \frac{4mg}{\delta \pi d^2 dt} = \frac{(mg)^{3/2}}{d \sqrt{\delta \pi}} //$$

A wrong approach to calculate power:

$$\text{Power} = F \cdot v = mgv = \frac{2(mg)^{3/2}}{d\sqrt{8\pi}} \quad \times$$

As v is not constant. Air is accelerated from 0 to v .

M 1 Visually



1. An object is rotating at an angular velocity ω_0 about an axis that passes from the origin and is directed in the x - y plane at 45° from the y -axis as well as from the negative x -axis. What is velocity of a point on the body located on the positive z -axis at a distance z from the origin?

(a) $\frac{\omega_0 z}{\sqrt{2}}(\hat{i} + \hat{j} + \hat{k})$

(b) $\frac{\omega_0 z}{\sqrt{2}}(-\hat{i} + \hat{j} + \hat{k})$

✓ (c) $\frac{\omega_0 z}{\sqrt{2}}(\hat{i} + \hat{j})$

(d) $\frac{\omega_0 z}{\sqrt{2}}(-\hat{i} + \hat{j})$

$$v = \omega_0 z = \omega_0 z \left(\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}} \right)$$

M 2 By equation ✓

$$\vec{\omega} = \omega_0 \left(-\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}} \right)$$

$$\vec{r} = z \hat{k}$$

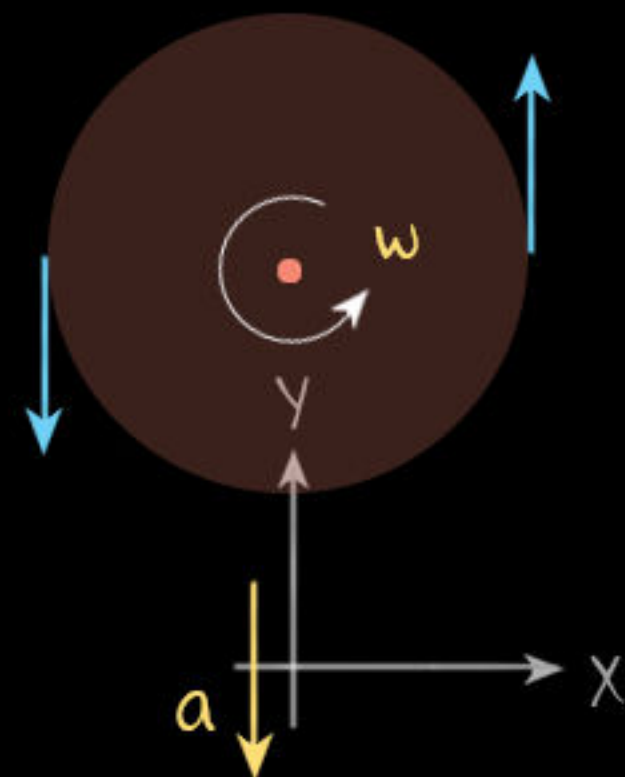
$$v = \vec{\omega} \times \vec{r} = \omega_0 z \left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{i}}{\sqrt{2}} \right)$$

Concept: ω of a rigid body is frame independent.

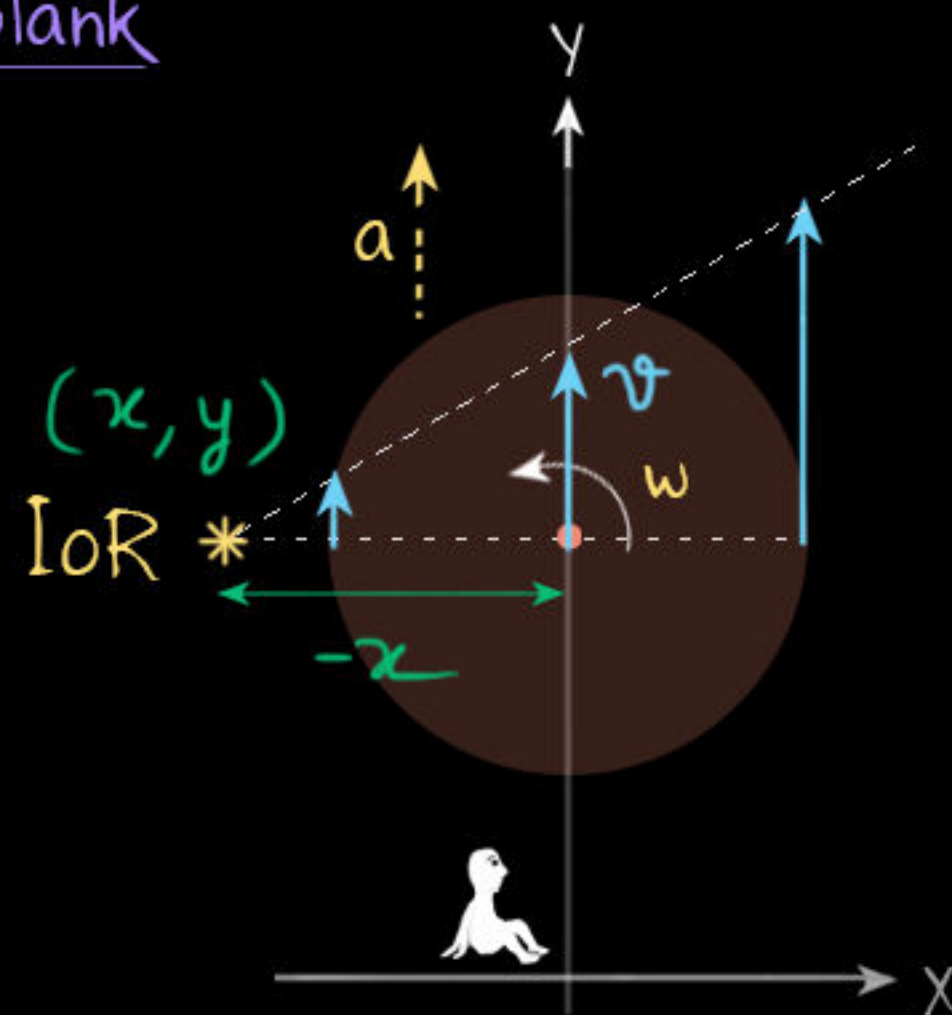
lets say

$$\vec{\omega} = \omega \hat{k}$$

w.r.t ground



w.r.t plank



2. A disc placed on a frictionless horizontal plank is rotating with a constant angular velocity ω about its central vertical axis. Now the plank is made to move with a constant acceleration a on a straight path. If you assume initial location of the centre of disc as origin, direction of motion of the plank in negative y -axis of the coordinate system attached with the plank, which of the following equations represent trajectory of instantaneous centre of rotation of the disc.

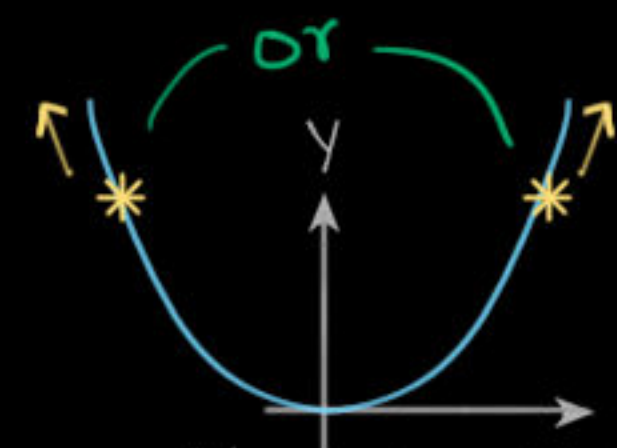
✓(a) If $\vec{\omega} = \omega \hat{k}$, then $y = \frac{\omega^2 x^2}{2a}$ and $x \leq 0$.

✓(b) If $\vec{\omega} = -\omega \hat{k}$, then $y = \frac{\omega^2 x^2}{2a}$ and $x \geq 0$.

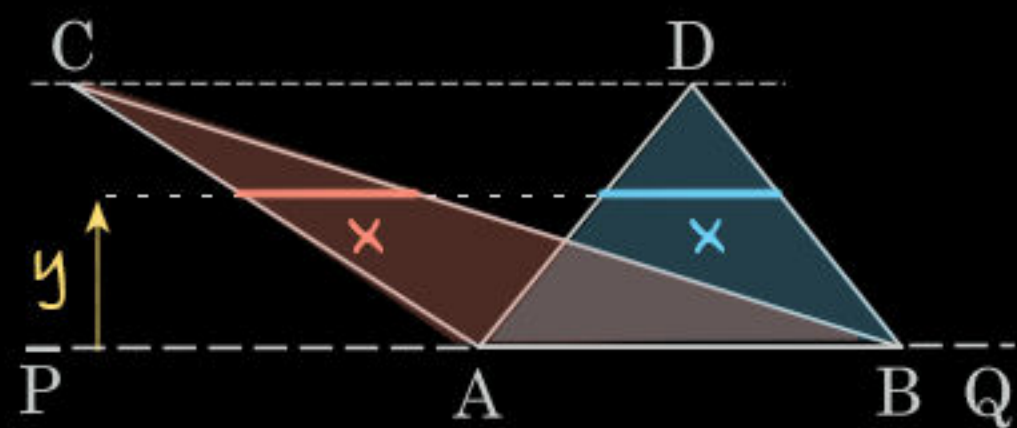
$$v = \omega(-x) = at \quad (1)$$

$$y = \frac{1}{2} at^2 \quad (2)$$

Eliminating t : $y = \frac{\omega^2 x^2}{2a}$



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3. Two uniform triangular plates ABC and ABD of the same thickness, height and made of the same material are shown in the figure. If moment of inertias of these plates about the axis PQ are I_C and I_D , which of the following statements is true?

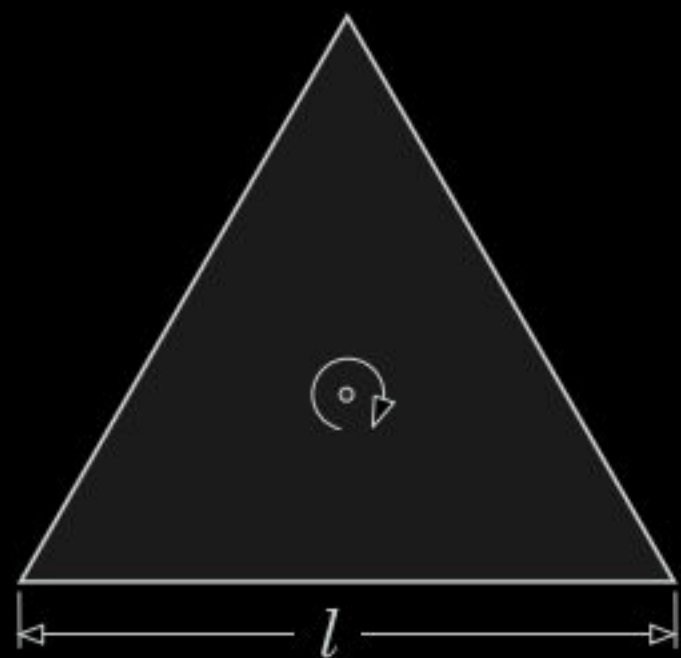
- (a) $I_C > I_D$ (b) $I_C < I_D$
 ✓ (c) $I_C = I_D$ (d) More information needed.

At same distance y , Length of a small strip is same (x) for both triangles.

$$\therefore dI_1 = dI_2$$

$$\Rightarrow \int dI_1 = \int dI_2$$

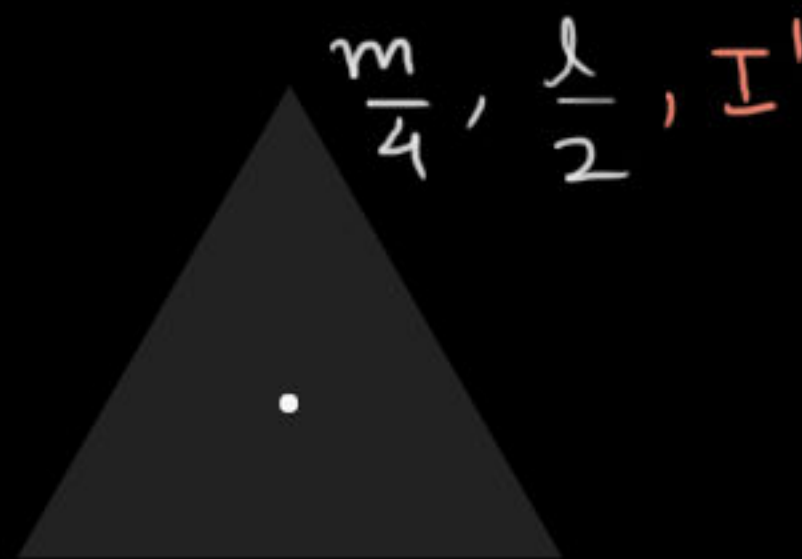
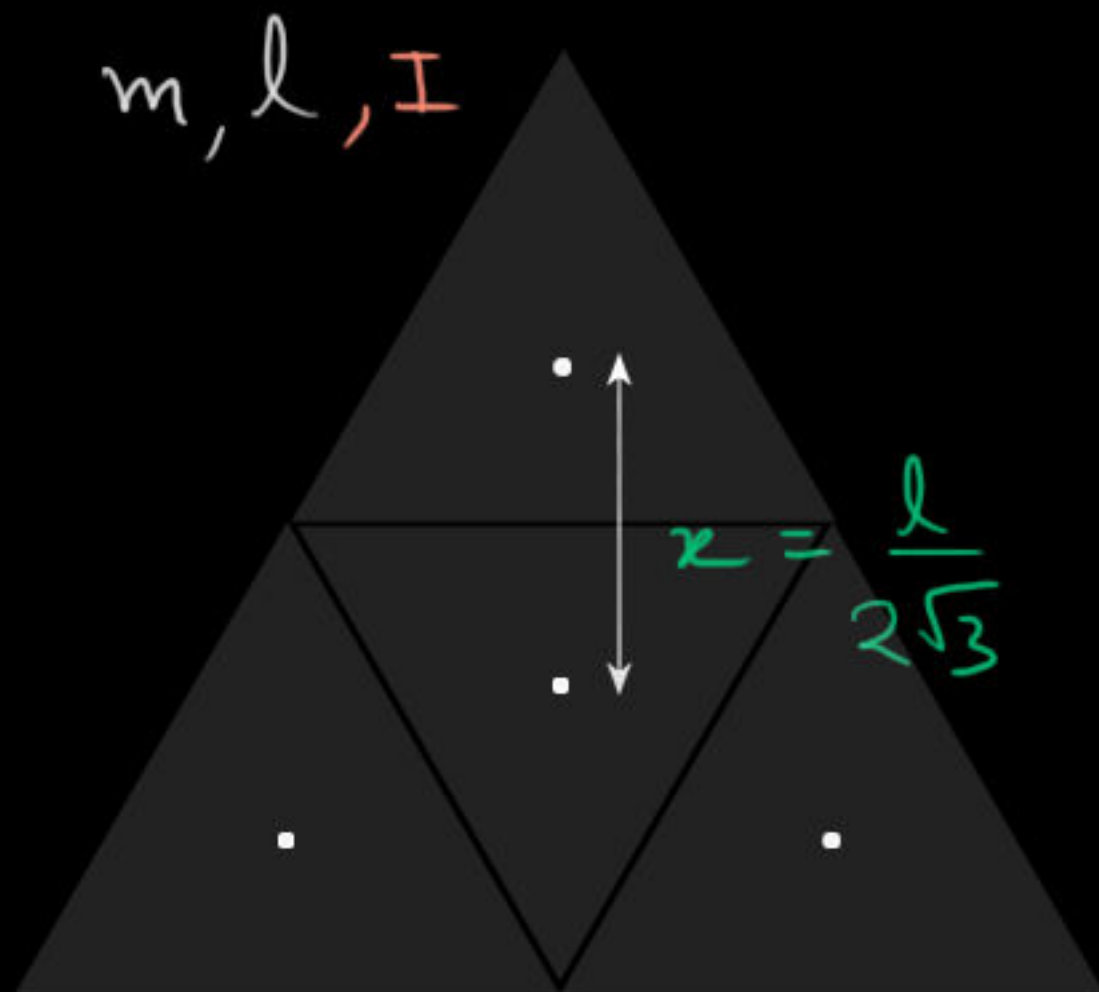
$$\Rightarrow I_1 = I_2$$



4. A uniform equilateral triangular lamina of side l has a mass m . Its moment of inertia about the axis through the centroid and perpendicular to its plane is

- (a) $\frac{1}{3}ml^2$ (b) $\frac{1}{6}ml^2$
 ✓ (c) $\frac{1}{12}ml^2$ (d) None of these

Approach: Scaling



Let required $I = k ml^2 \Rightarrow I' = k \left(\frac{m}{4}\right) \left(\frac{l}{2}\right)^2 = \frac{k ml^2}{16}$

$$I = I' + 3 \left(I' + \frac{m}{4} x^2 \right)$$

$$\Rightarrow k ml^2 = 4 \frac{k ml^2}{16} + \frac{3}{4} m \frac{l^2}{4 \cdot 3}$$

$$\Rightarrow \frac{3k}{4} = \frac{1}{16}$$

$$\Rightarrow k = \frac{1}{12}$$

$$\therefore I = k ml^2 = \frac{ml^2}{12}$$

Concept: When $m_p = 0$, beam will tilt about 1. So we need a minimum m_p to avoid that.

At that minimum m_p , rod will just balance about 1 and therefore $T_2 = 0$.

Now we increase m_p . As we do so, T_1 will decrease and T_2 will increase. So at maximum m_p string 2 will break once $T_2 = 1200\text{ N}$

5. A light beam suspended with the help of two ropes supports two loads as shown in the figure on the next page. The load at the left end is of 75 kg and the load P at the right end is unknown. The maximum tension that either of the rope can bear is 1200 N. In an experiment, mass of the load P is gradually increased from zero. If the rod remains horizontal in equilibrium, which of the following conditions must be satisfied?



- ✓ (a) Minimum mass of the load P can be 12.5 kg.
- ✓ (b) Maximum mass of the load P can be 102.5 kg.
- ✓ (c) When mass of the load P is minimum, tension in one of the ropes vanishes.
- (d) When mass of the load P is maximum, tension in one of the ropes vanishes.

$$\therefore T_1 = 0, T_2 = 0$$

$$\Rightarrow 75g(0.5) = m_{p|_{\min}}g(3)$$

$$\Rightarrow m_{p|_{\min}} = 12.5 \text{ kg} //$$

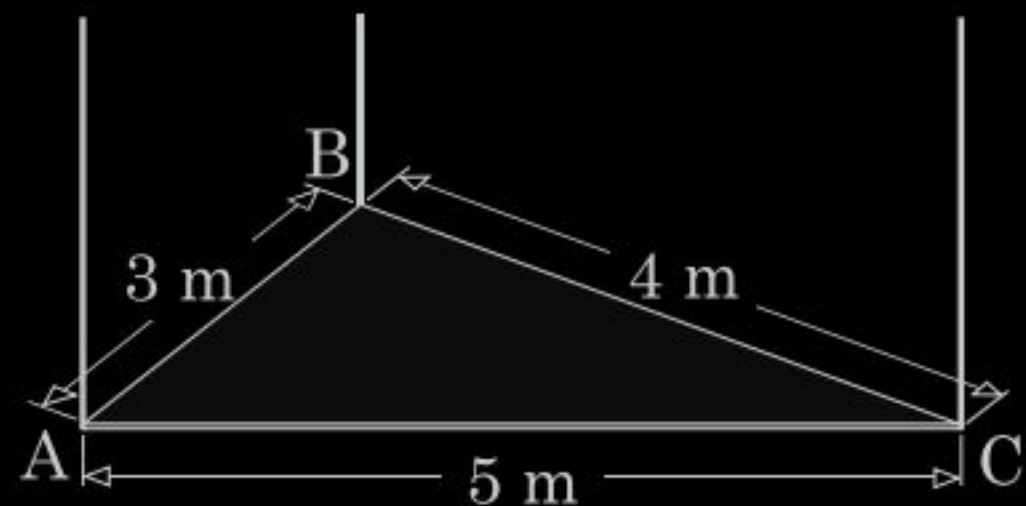
$$T_1 = 0, T_2 = 1200 \text{ N}$$

$$\Rightarrow 75g(0.5) + (1200)(2.25) = m_{p|_{\max}}g(3)$$

$$\Rightarrow m_{p|_{\max}} = 102.5 \text{ kg} //$$

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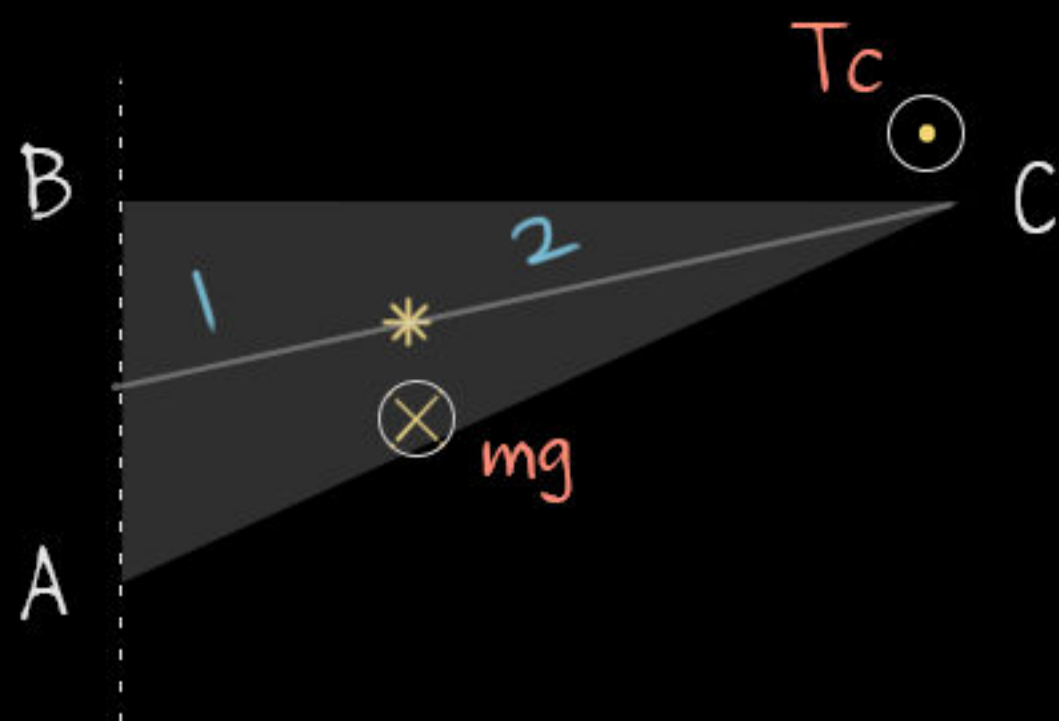
6. A uniform right-angled triangular lamina is suspended from the ceiling with the help of three light strings. The plane of the lamina is horizontal and the strings are vertical as shown in the figure. In which of the following options, tensile forces T_A , T_B and T_C developed in the strings connected to the vertices A, B and C respectively of the lamina are arranged in correct order of their magnitudes?

(a) $T_A > T_B > T_C$

(b) $T_B > T_A > T_C$

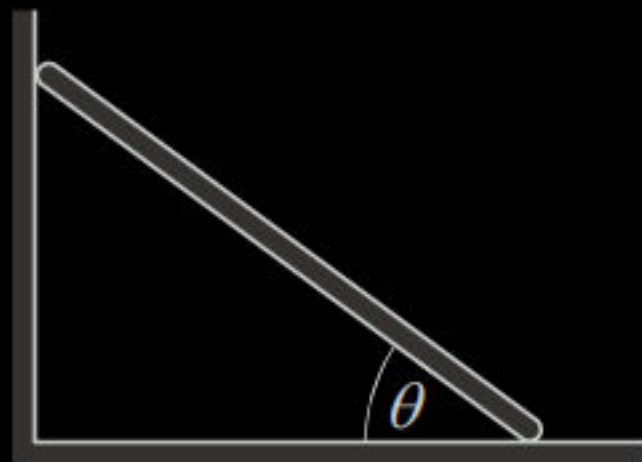
(c) $T_C > T_A > T_B$

✓ (d) $T_A = T_B = T_C$



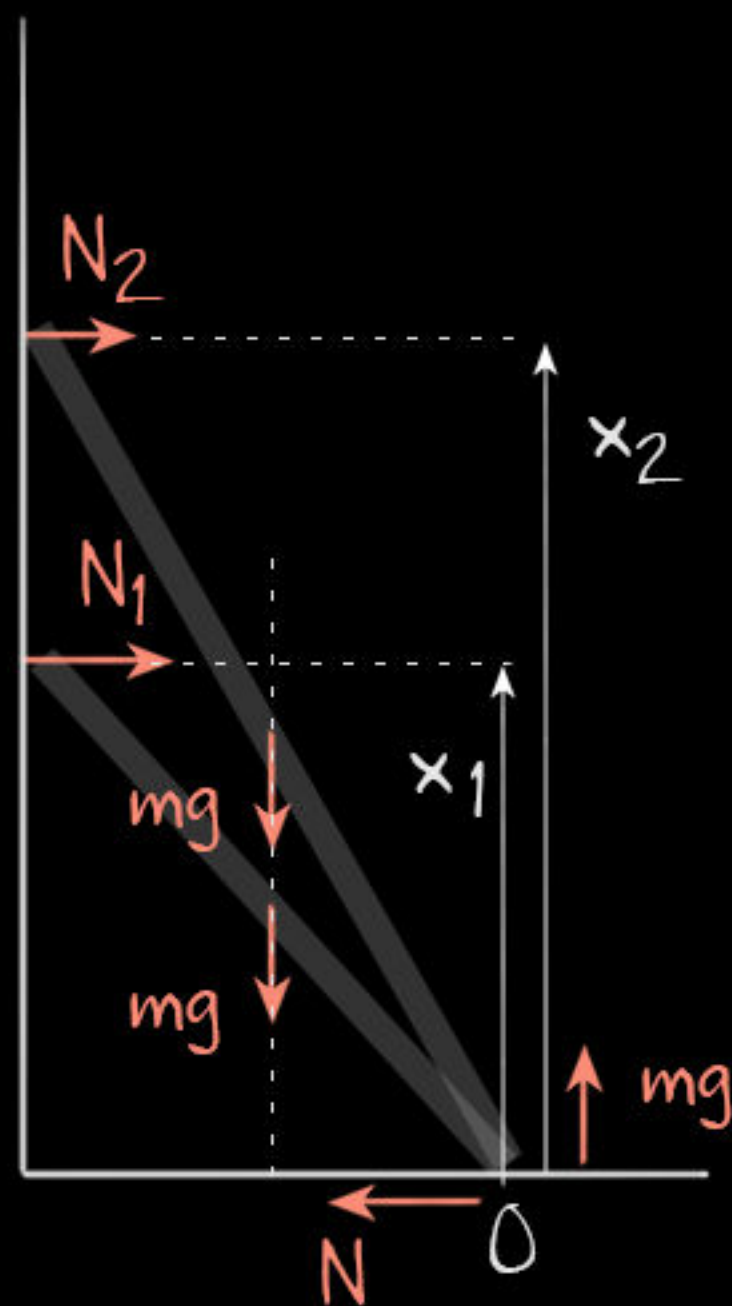
$$\begin{aligned} \sum \tau_{AB} &\Rightarrow mg \cancel{x} = T_C (3\cancel{x}) \\ &\Rightarrow T_C = \frac{mg}{3} \end{aligned}$$

$$\text{Similarly: } T_A = T_B = \frac{mg}{3}$$



7. A uniform steel rod leans against a **frictionless wall** in static **equilibrium**. Frictional force between the lower end and the floor is less than its limiting value by a finite amount. **The rod is supplied some amount of heat so that it expands.** Assume that the coefficient of friction does not change on heating. What can you **conclude regarding contact forces from the wall and from the floor?**

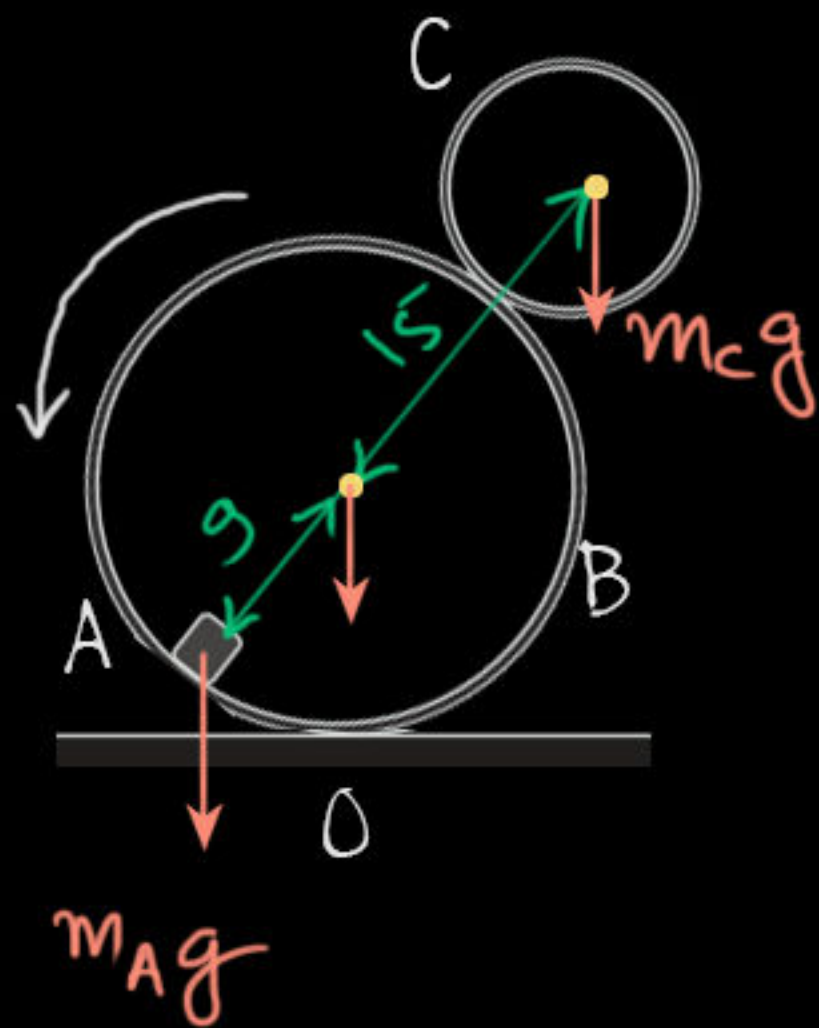
- (a) From the wall it decreases and from the floor it increases.
- (b) From the wall it increases and from the floor it decreases.
- (c) From the wall as well as from the floor it increases.
- ✓ (d) From the wall as well as from the floor it decreases.



In both cases O doesn't move, and rod is in equilibrium.
Also, torque due to mg is same.

$$N_1 x_1 = \tau_{mg} = N_2 x_2$$

$$\therefore N_1 > N_2$$



8. A toy "roly-poly" consists of two rigidly attached hollow uniform plastic balls. The upper and the lower balls are of radii 6 cm and 9 cm respectively. A small load of mass 250 g is glued at the bottom of the lower ball. This toy has a property that if it is tilted to one side by any angle and then set free, it oscillates and finally settles in vertical orientation. Which of the following statements are correct?

- ✓ (a) More is the mass of the load, faster is the response.
- ✓ (b) More is the mass of the lower ball, more sluggish is the response.
- (c) Mass of the upper ball should be less than 150 g and that of the lower ball should be more than 150 g.
- ✓ (d) Mass of the upper ball should be less than 150 g and that of the lower ball may have any value.

D To restore roly-poly: τ_0 must be anticlockwise

$$\Rightarrow m_A g(9) > m_C g(15)$$

250

$$\Rightarrow m_C < 150$$

B $\tau_0 = I_0 \alpha$

$$\Rightarrow \alpha = \frac{\tau_0}{I_0} = \frac{m_A c_1 - m_C c_2}{m_A c_3 + m_B c_4 + m_C c_5} \quad 1$$

A From 1: $\alpha = 0$ at certain value of m_A

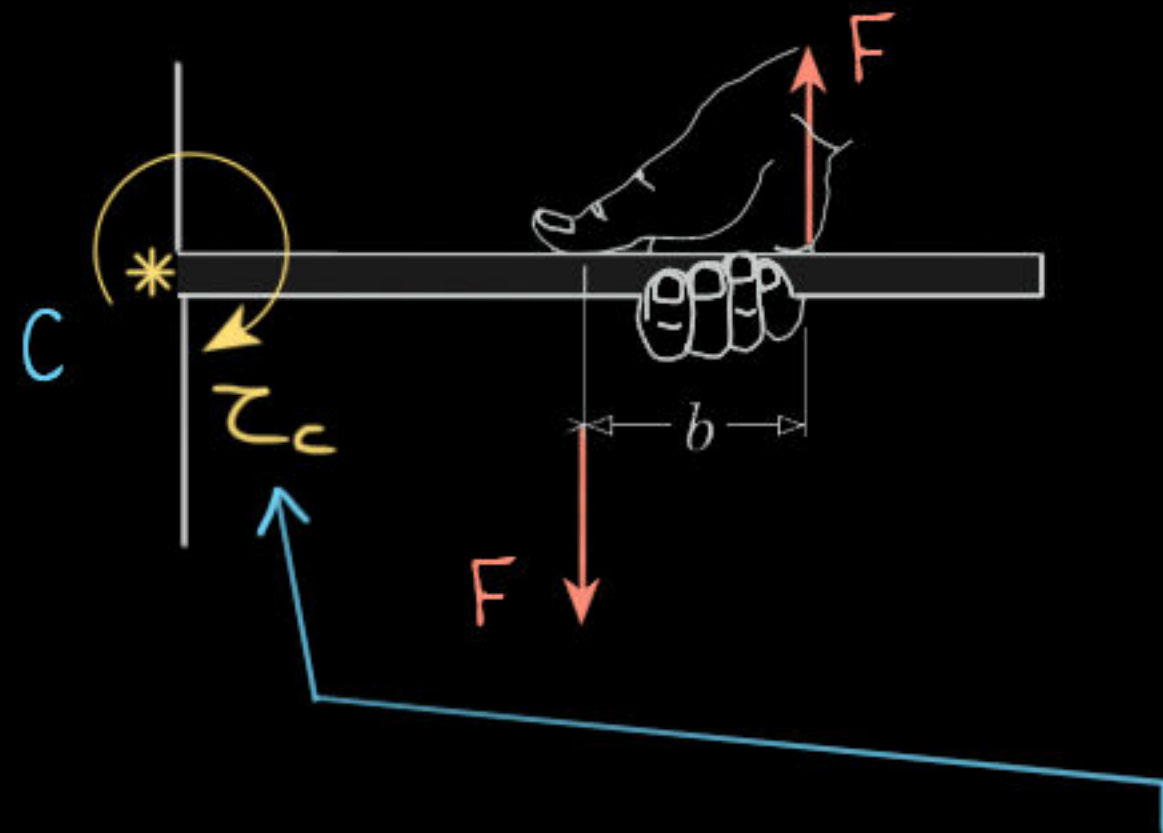
∴ If we increase m_A further, α will increase.

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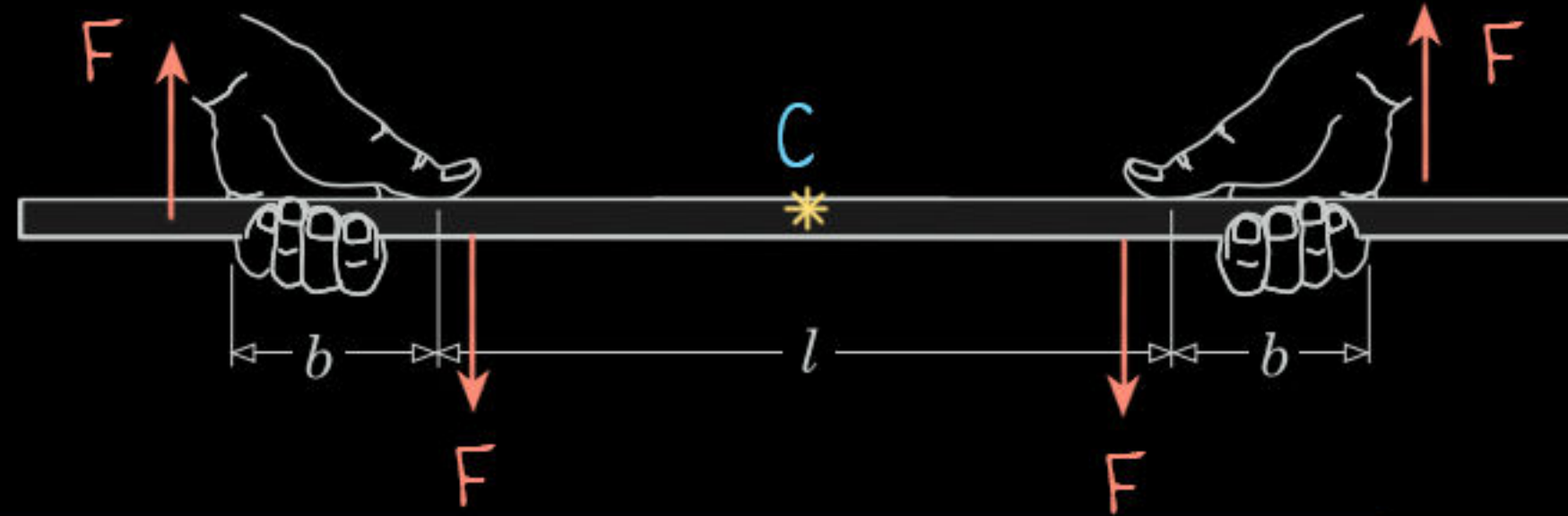
Concept: As your hand is at translational equilibrium, you must be exerting couple force, exerting a torque.

Concept: Until it breaks, bar is at equilibrium. So given situation is same as shown below (as reaction forces on C are same in both cases)



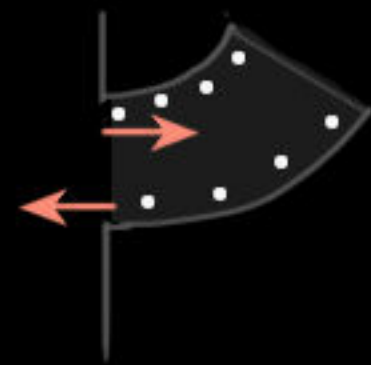
Concept: It will break when contact torque due to reaction forces can no longer balance couple Force torque $F \times b$.

9. It is easier to break a wooden bar by bending it as compared to pulling it. To break a wooden bar a man holds the bar in his fists and then tries to twist his fists as shown in the figure, where b is width of his fists and l is a characteristic distance between places where he holds the bar.



To break the bar most easily, the best strategy is to

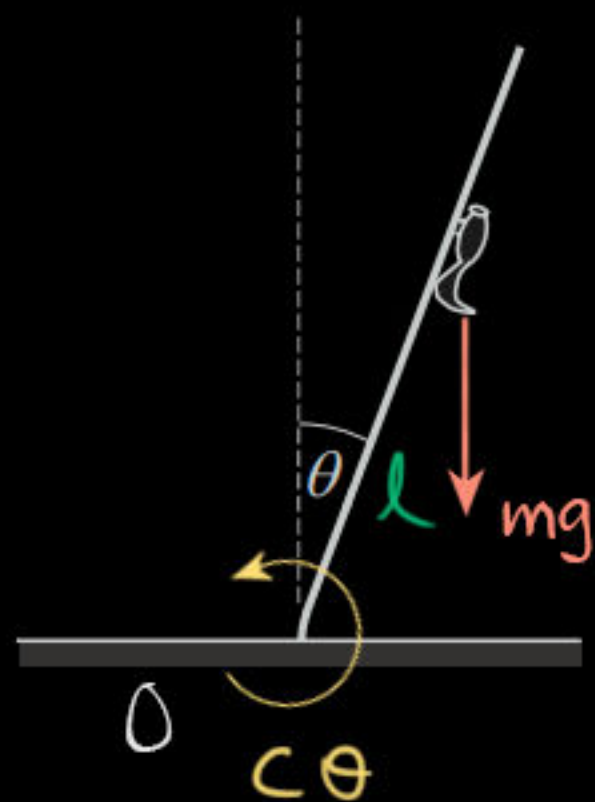
- (a) increase l and keep b unchanged.
- ✓ (b) increase b and **keep l unchanged.** → actually, l doesn't matter
- (c) increase b and l both.
- (d) It is the strength of the bar, which matters. The quantities b and l have no effect.



∴ For rod to not break, and be in equilibrium

$$\tau_c = Fb < \tau_{\text{break}}$$

∴ For rod to break easily, maximize b .



10. A squirrel of mass m climbs slowly on a thin straight vertical rod of length L . Mass of the rod is negligible as compared to that of the squirrel and size of the squirrel is negligible as compared to the length l that it climbs on the rod. Due to weight of the squirrel, the rod bends at its lower end through an angle θ and due to elasticity of the material of the rod a restoring torque $C\theta$ is developed. If $C = 2mgL$, find the maximum length the squirrel can climb on the rod.

- (a) $L/2$
 ✓ (c) L

- (b) $3L/4$
 (d) $2L$

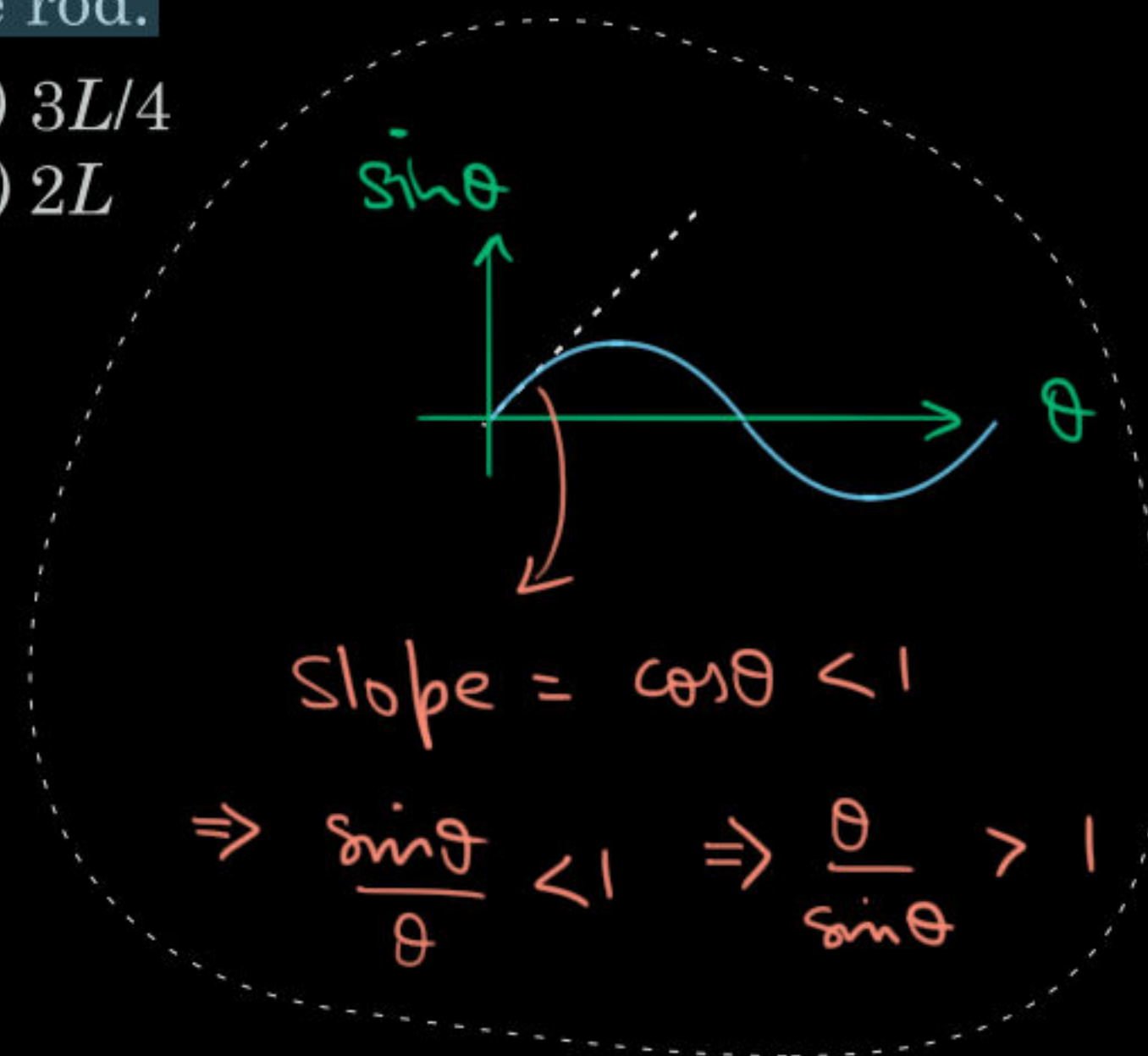
If the squirrel can not be supported anymore:

$$\Rightarrow mgl \sin \theta > \overset{2mgL}{C\theta}$$

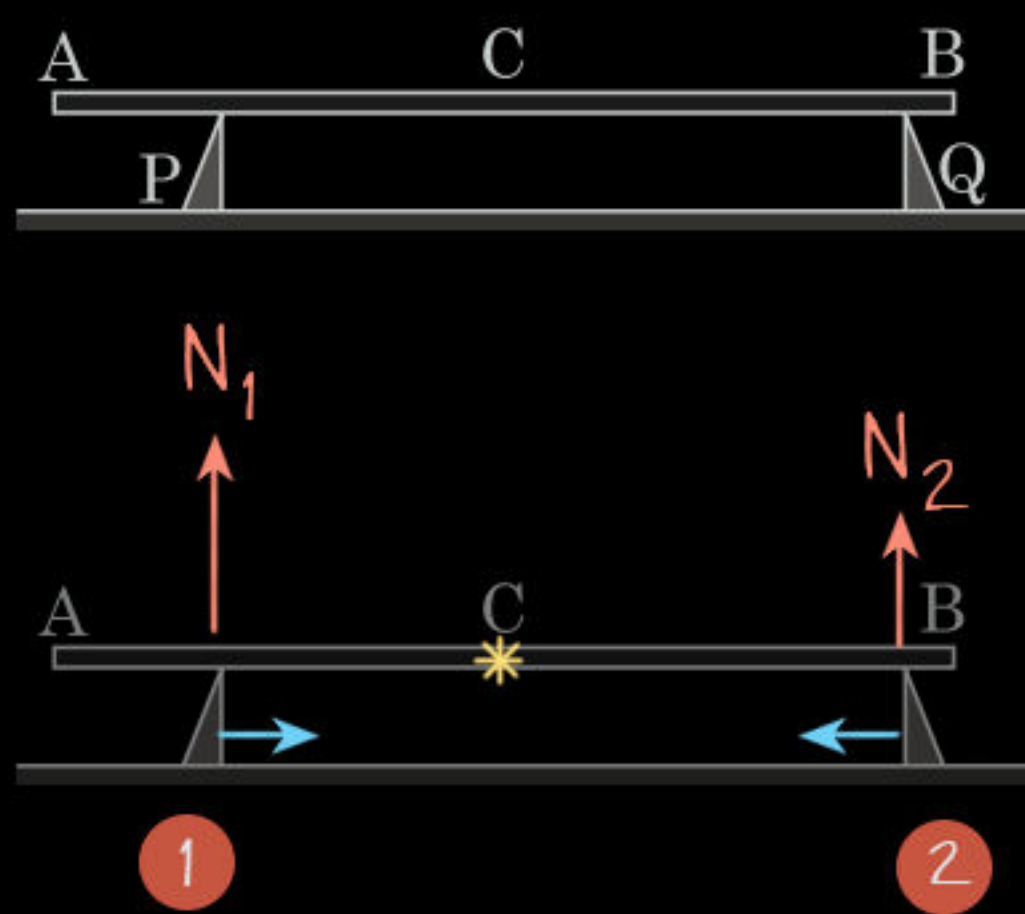
$$\Rightarrow \cancel{mgl} \sin \theta > 2\cancel{mgL} \theta$$

$$\Rightarrow l > \frac{2L\theta}{\sin \theta}$$

$$\frac{\theta}{\sin \theta} > 1 \quad \text{proof} \rightarrow$$



$\Rightarrow l > 2L$ But maximum length of rod is only L .
 So it can climb its maximum length L .



11. A uniform bar AB is placed asymmetrically on two identical wedges P and Q that can slide on a frictionless horizontal tabletop as shown in the figure. Centre of the bar is C. Coefficient of static friction between the bar and the wedges is more than that of kinetic friction. If the wedges are shifted slowly with equal speeds on the tabletop towards each other, what will you observe?

- ✓(a) The wedges meet below the centre C.
- ✓(d) At a time, only one wedge that is farther away from the centre C slides relative to the bar.

Concept 1: $N_1 > N_2$

Concept 2:

As the wedges are moved closer, slipping has to occur atleast at top of one wedge.

As N_2 is lesser, static friction at 2 is lesser. So here slipping definitely occurs

Concept 3: acceleration of bar is nearly zero. $\therefore f \approx f$

Concept 4:

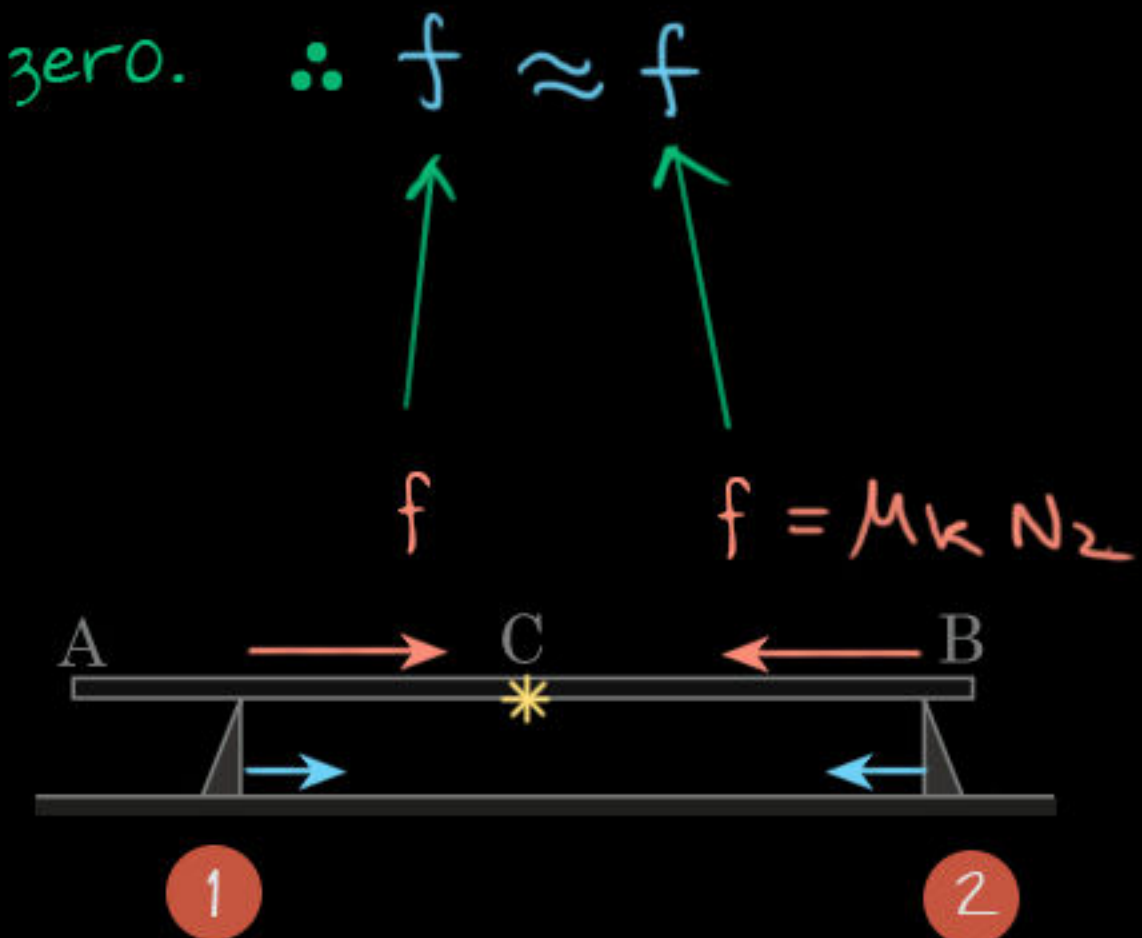
$$f = \mu_k N_2 < \mu_s N_1$$

$$\text{as } \mu_k < \mu_s \text{ \& } N_2 < N_1$$

$\Rightarrow$ no slipping at 1

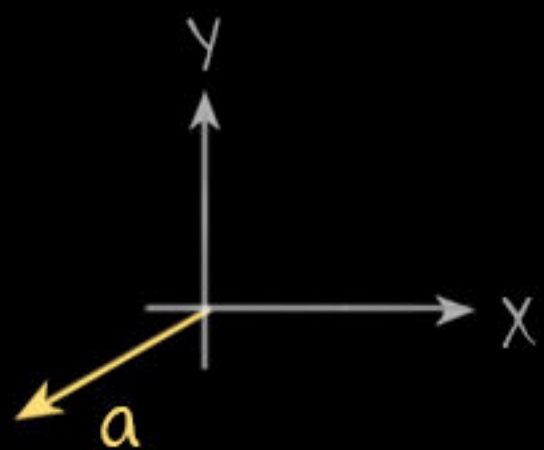
Until C is further to 1 than 2. Then, N_1 will be smaller than N_2 . Then 1 will slip and 2 wont and so on until both wedges meet at C.

by Ankit Singhvi



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Let the acceleration of non-inertial, non-rotating frame be a .

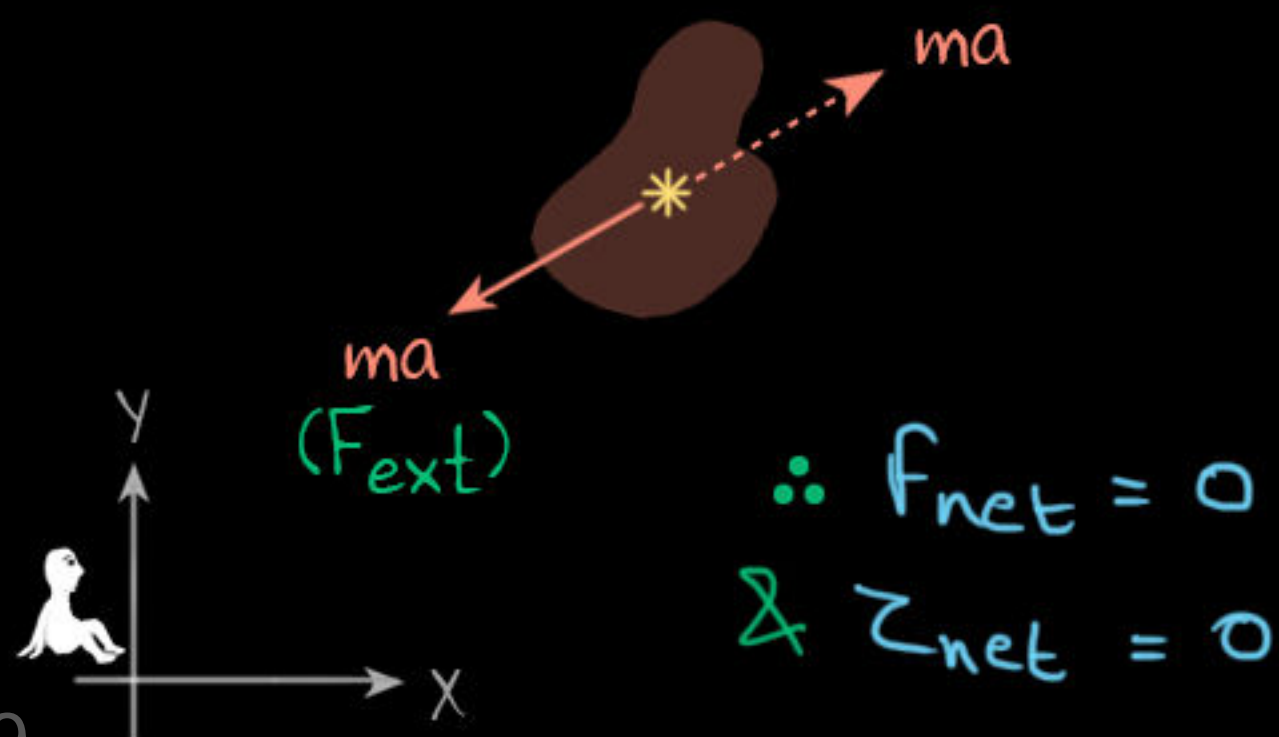


Approach: Similar to a box at rest inside an accelerating elevator.

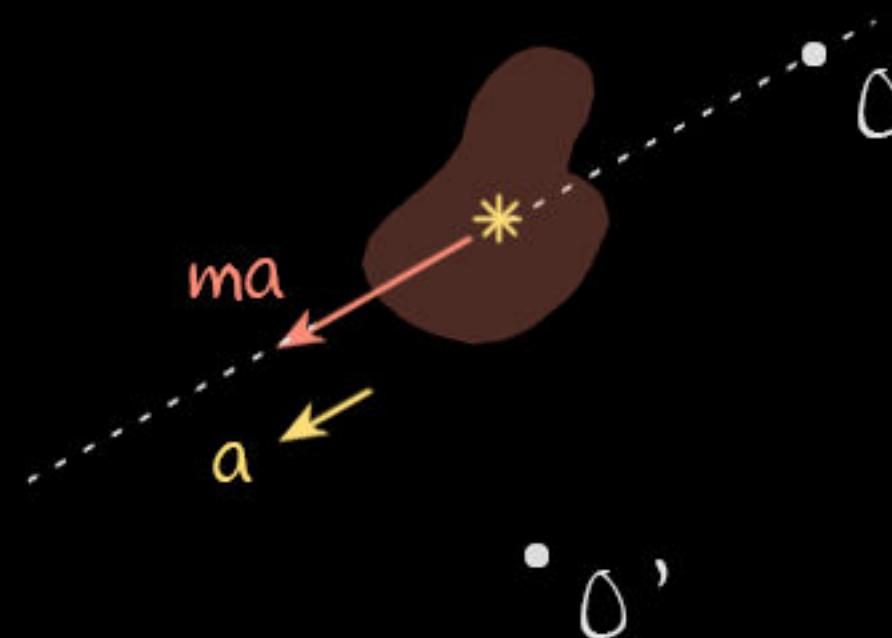
12. A rigid body is observed in equilibrium in a particular non-rotating non-inertial frame. What can you conclude, if the body is observed from an inertial frame?

- ✓ (a) The body is in rotational equilibrium but not in translational equilibrium.
- ✓ (b) Net torque of all the forces on the body about its mass centre is a null vector.
- ✓ (c) Net torque of all the forces on the body about any point that is collinear with line of the acceleration of the mass centre is a null vector.
- (d) Net torque of all the forces on the body about all the points of a line that is parallel to the line of the acceleration of the mass centre is a null vector.

w.r.t non-inertial, non-rotating frame



w.r.t inertial frame

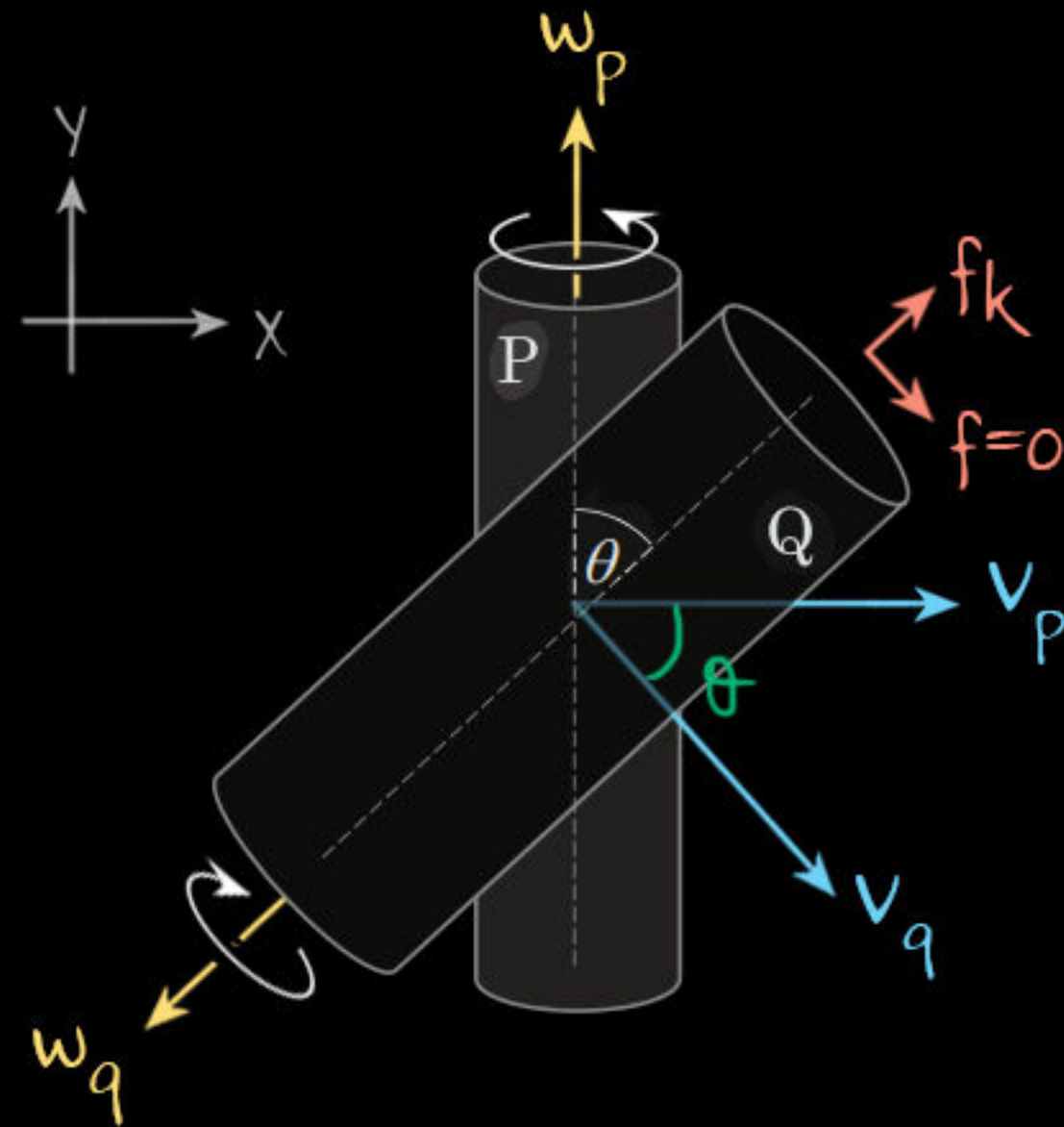


$$\therefore a \neq 0$$

$$\tau_O = 0$$

$$\tau_{O'} \neq 0$$

Let the velocities at point of contact be v_p and v_q at steady state



Concept: At steady state, torque on Q about its axis is zero. So, no slipping will occur perpendicular to Q's axis.

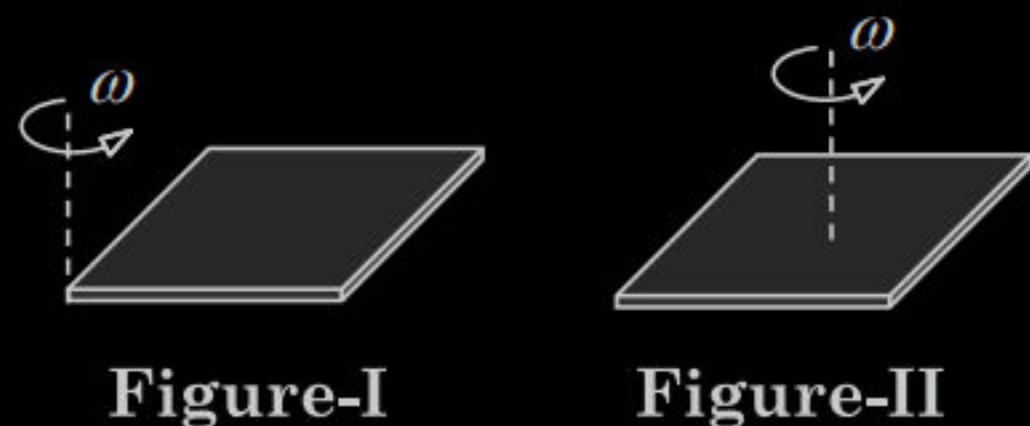
13. A cylinder P of radius r_P is being rotated at a constant angular velocity $\omega_P \hat{j}$ with the help of a motor about its axis that is fixed. Another cylinder Q of radius r_Q free to rotate about its axis that is also fixed is touched with and pressed on P making an angle θ between their axes. Soon after the cylinders are pressed against each other, a steady state is reached and the cylinder Q acquires a constant angular velocity. What can you conclude when the steady state is reached?

- (a) Angular velocity of cylinder Q is $\vec{\omega}_Q = -\frac{\omega_P r_P}{r_Q \cos \theta} (\sin \theta \hat{i} + \cos \theta \hat{j})$.
- ✓ (b) Angular velocity of cylinder Q is $\vec{\omega}_Q = -\frac{\omega_P r_P \cos \theta}{r_Q} (\sin \theta \hat{i} + \cos \theta \hat{j})$.
- ✓ (c) Unit vector of frictional force on cylinder Q is $(\sin \theta \hat{i} + \cos \theta \hat{j})$.
- (d) Frictional force on each cylinder becomes vanishingly small.

$$\Rightarrow v_P \cos \theta = v_Q$$

$$\Rightarrow \omega_P r_P \cos \theta = \omega_Q r_Q$$

$$\Rightarrow \omega_Q = \frac{\omega_P r_P \cos \theta}{r_Q} \hat{\omega}_Q \quad \text{where } \hat{\omega}_Q = -(\sin \theta \hat{i} + \cos \theta \hat{j})$$

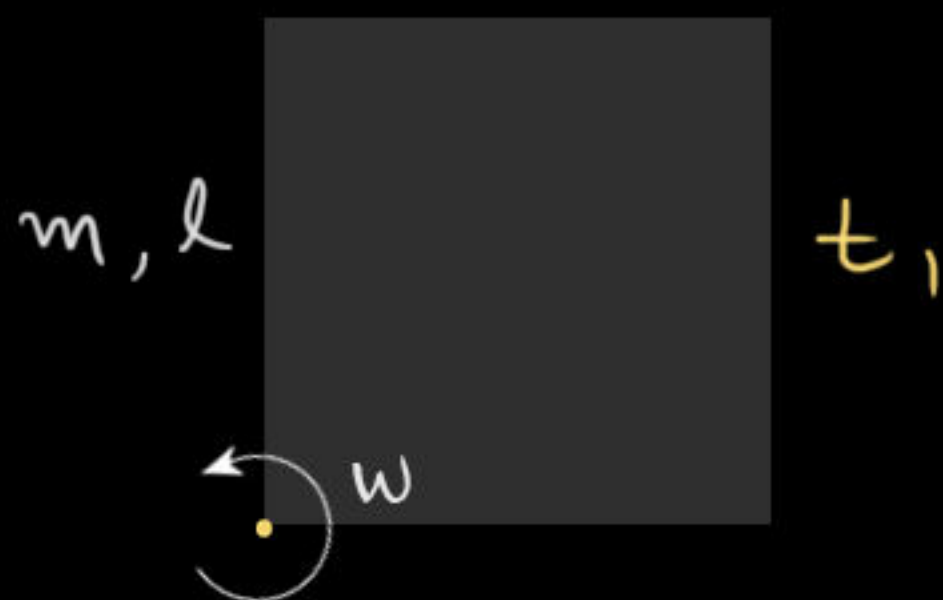


14. A uniform square plate is placed on a horizontal floor. When it is given an angular velocity ω about a vertical axis through one of its corners as shown in figure-I, it takes time t_1 to come to a complete stop. Now the same square plate is given the same angular velocity to rotate about another vertical axis through its centre as shown in figure-II. How long will it take to come to a complete stop now?

Approach: Scaling

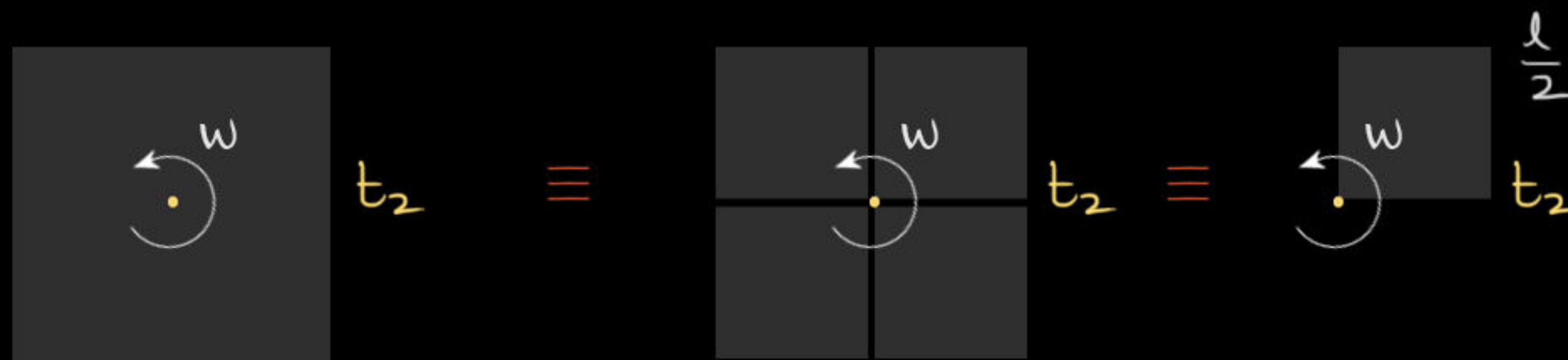
Torque is constant so:

$$\omega = \alpha t \quad (1)$$



$$\begin{aligned} \tau &= I \alpha \\ \cancel{k_1} m \cancel{l} &= \cancel{k_2} m \cancel{l}^2 \alpha \\ \Rightarrow \alpha &\propto \frac{1}{l} \quad (2) \end{aligned}$$

- ✓ (a) $t_1/2$ (b) $2t_1/3$
(c) $3t_1/2$ (d) $2t_1$



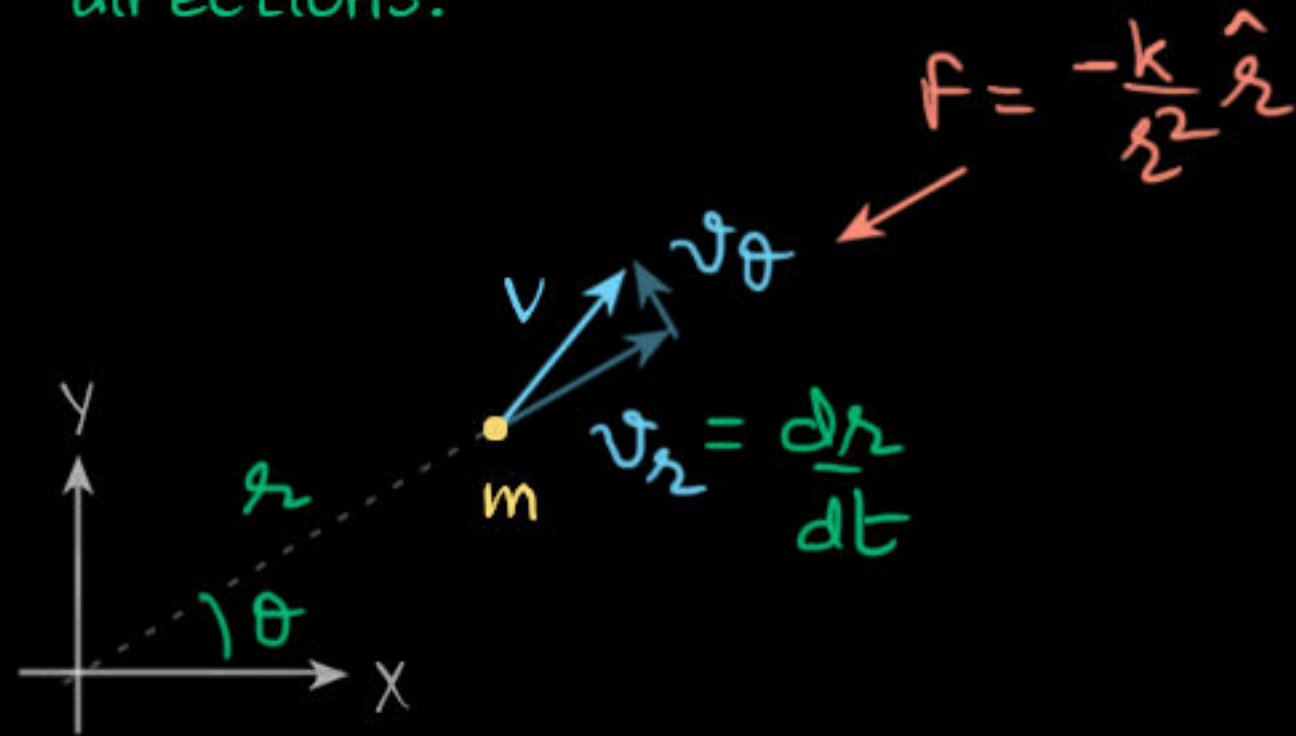
From 1 and 2: $\frac{t}{l} = \text{constant}$

$$\therefore \frac{t_1}{l_1} = \frac{t_2}{l_2} \Rightarrow \frac{t_1}{l} = \frac{t_2}{l/2} \Rightarrow t_2 = \frac{t_1}{2}$$

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Lets break velocity v in radial and transverse directions.



15. A particle of mass m at a distance r from a fixed point is attracted towards the fixed point by a force given by equation $F = k/r^2$, where k is a positive constant. If L be its angular momentum with respect to the fixed point, which of the following equations is correct?

(a) $\Delta \left(\frac{L^2}{2mr^2} - \frac{k}{r} \right) = 0$

(b) $\Delta \left(\frac{1}{2}m \left(\frac{dr}{dt} \right)^2 + \frac{k}{r} + \frac{L^2}{2mr^2} \right) = 0$

(c) $\Delta \left(\frac{1}{2}m \left(\frac{dr}{dt} \right)^2 - \frac{k}{r} \right) = 0$

✓ (d) $\Delta \left(\frac{1}{2}m \left(\frac{dr}{dt} \right)^2 - \frac{k}{r} + \frac{L^2}{2mr^2} \right) = 0$

Total Energy = $KE + U = \text{constant}$

$$= \frac{1}{2}mv^2$$

$$= \frac{1}{2}m \left[v_\theta^2 + \left(\frac{dr}{dt} \right)^2 \right]$$

$\downarrow$
 L/mr

$$= \frac{L^2}{2mr^2} + \frac{1}{2}m \left(\frac{dr}{dt} \right)^2$$

$$= - \int_{\infty}^r \vec{F} \cdot d\vec{r}$$

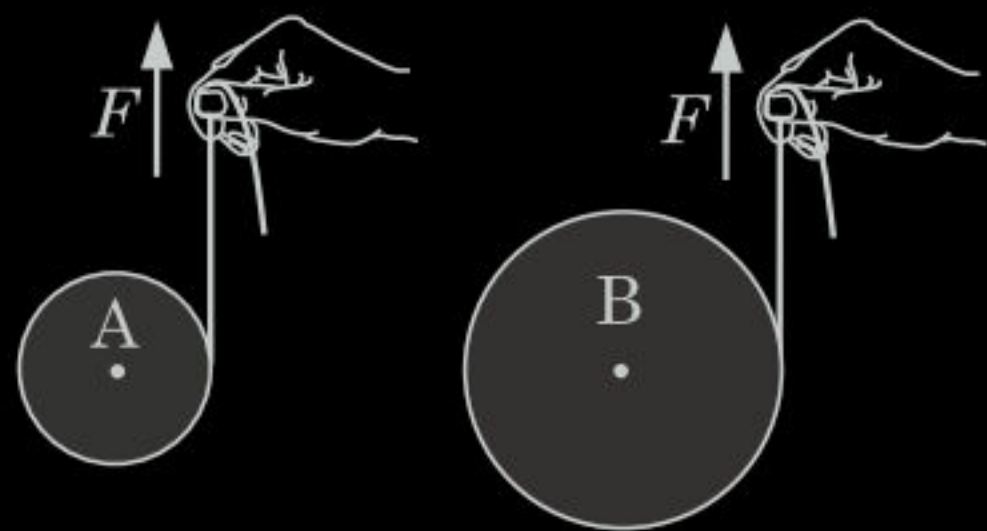
$$= \int_{\infty}^r \frac{k}{r^2} dr$$

$$= -\frac{k}{r}$$

Concept: A central force is conservative in nature
So we can conserve energy

$$\Rightarrow \Delta (KE + U) = 0$$

$$\Rightarrow \Delta \left(\frac{L^2}{2mr^2} + \frac{1}{2}m \left(\frac{dr}{dt} \right)^2 - \frac{k}{r} \right) = 0 //$$



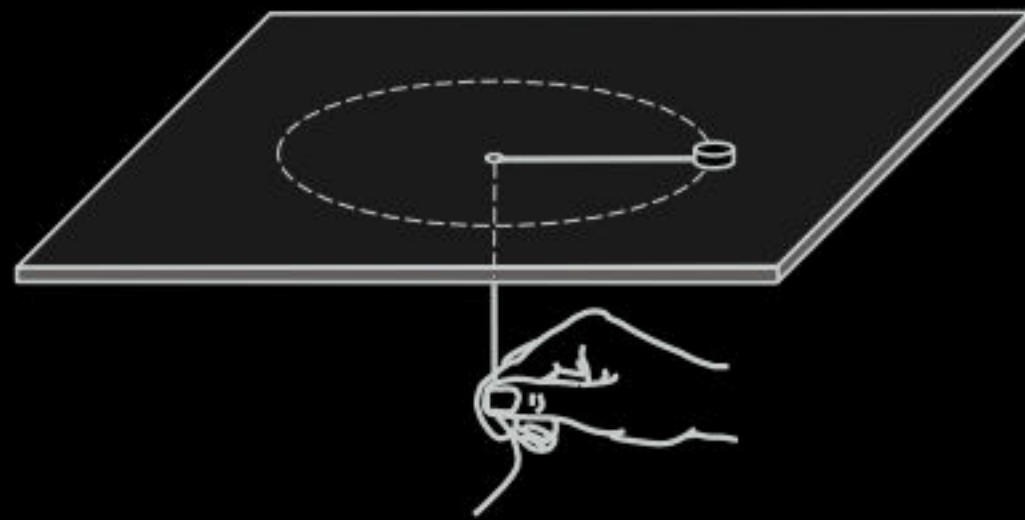
16. Two uniform discs A and B of equal masses and different radii r_A and r_B are mounted on fixed horizontal axles passing through their centres. On each of the discs, a light inextensible long cord is wound and the discs are held motionless. If free ends of both the cords are pulled with equal forces for equal amounts of time, lengths of the cord unwound from the discs l_A and l_B bear the relation

- (a) $l_A > l_B$ (b) $l_A < l_B$
 ✓ (c) $l_A = l_B$ (d) More information needed.

$$\tau = I\alpha$$

$$\Rightarrow F\cancel{r} = m\frac{\cancel{r}^2}{2}\alpha$$

$$\therefore a_{\text{thread}} = \alpha\cancel{r} = \frac{2F}{m} = \text{same for both}$$

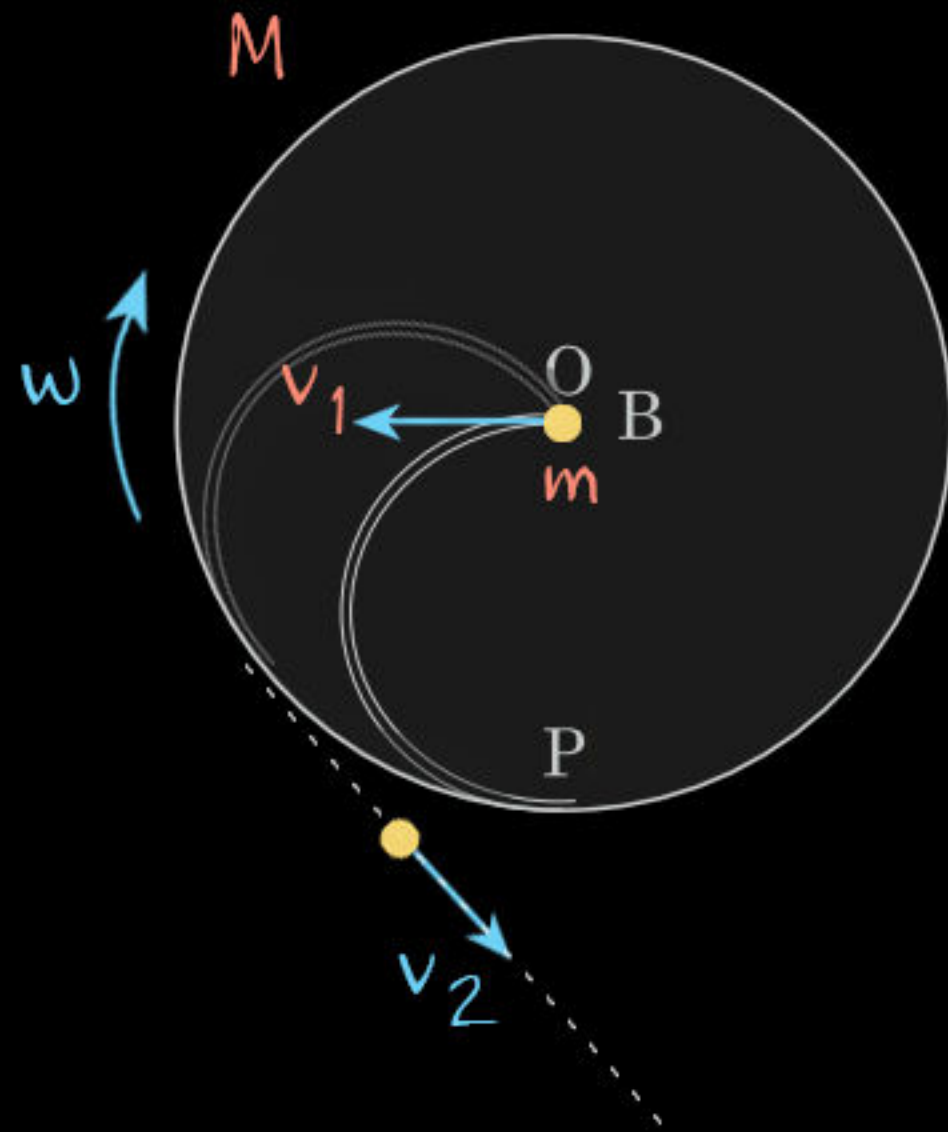


17. A small disc is attached at one end of a light inextensible string that passes through a hole in a frictionless horizontal tabletop. Initially the disc moves on a circle of radius R with kinetic energy K_0 . The other end of the string is slowly pulled so that the disc finally moves on a circle of radius R/η . What is the work W done by the pulling agency?

- (a) $W = 0$ (b) $W = \eta^2 K_0$
 ✓ (c) $W = (\eta^2 - 1) K_0$ (d) $W = (\eta - 1) K_0$

L: $\cancel{mv_1 R} = \cancel{mv_2} \frac{\cancel{R}}{\eta} \Rightarrow v_2 = \eta v_1$

E: $W_{\text{ext}} = \Delta KE$
 $= \frac{1}{2} m (v_2^2 - v_1^2)$
 $= \frac{1}{2} m v_1^2 (\eta^2 - 1)$
 $= K_0 (\eta^2 - 1)$
 $\equiv$



18. A horizontal disc of mass 1.0 kg mounted on a fixed vertical axle through its centre can rotate without friction. On the disc a semicircular groove OP is cut. A small ball B of mass 1.5 kg when pushed on the disc towards the centre O, the ball enters the groove with a speed 2.0 m/s and moves through the groove without friction. With what speed will the ball leave the disc? Neglect loss of mass due to cutting the groove.

- ✓ (a) 1.0 m/s
(c) 3.0 m/s

- (b) 1.5 m/s
(d) Insufficient information

L: $m v_2 R = \left(\frac{MR^2}{2} \right) \omega$

E: $\frac{1}{2} m v_1^2 = \frac{1}{2} m v_2^2 + \frac{1}{2} \left(\frac{MR^2}{2} \right) \omega^2$

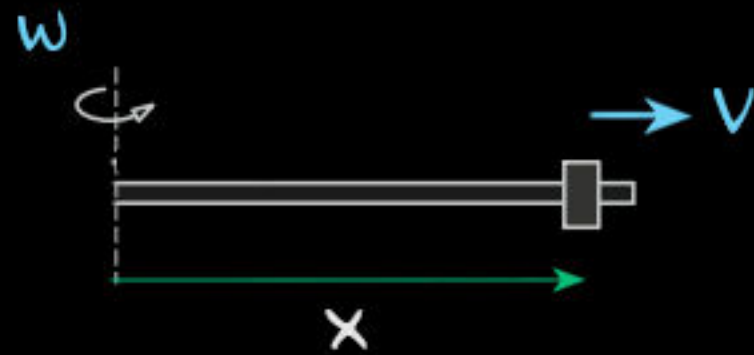
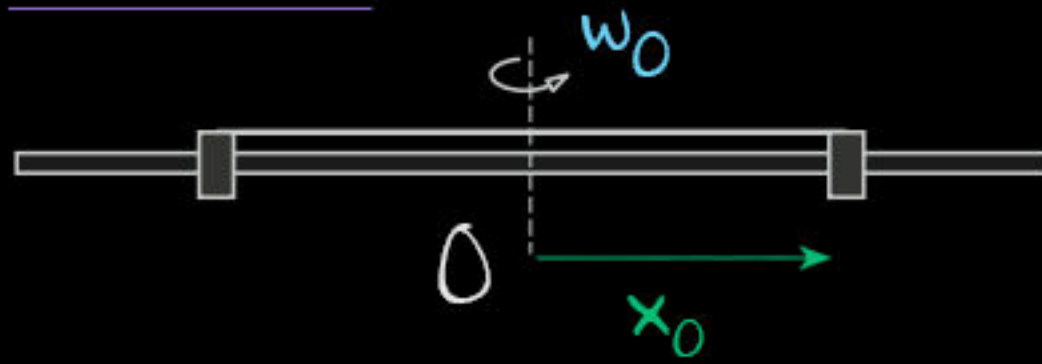
$\therefore$

$$v_2^2 = \frac{M v_1^2}{M + 2m} = \frac{2^2}{4} = 1$$

$$\Rightarrow v_2 = 1 \text{ m/s}$$

Method 1

w.r.t rod:



$$L: \omega_0 x_0^2 = \omega x^2$$

$$E: (\omega_0 x_0)^2 = (\omega x)^2 + v^2$$

$$\therefore v = \omega_0 x_0 \sqrt{1 - \frac{x_0^2}{x^2}}$$

19. Two identical small beads A and B threaded on an almost inertia-less horizontal rod of length $2l$ are connected by a light inextensible cord of length $1.2l$. Initially the beads are adjusted equidistant from the centre of the rod and the rod is rotating about its central vertical axis at an angular velocity $1/3 \text{ rad/s}$. At some point of time, the cord is cut and the beads begin to slide on the rod without friction. Neglect effects of gravity. Which of the following statements are correct?

- ✓(b) Path of a bead in an inertial frame is a straight line.
- ✓(c) To leave the rod, the beads take 4 s after the cord is cut.

$$\Rightarrow \omega_0 x_0 \int_0^t dt = \int_{x_0}^l \frac{x dx}{\sqrt{x^2 - x_0^2}}$$

$$\Rightarrow \omega_0 x_0 t = \sqrt{x^2 - x_0^2} \Big|_{x_0}^l$$

$$\downarrow x_0 = 0.6l, \omega_0 = 1/3$$

$$\frac{1}{3} (0.6l) t = 0.8l \Rightarrow t = 4$$

(b) Path of a bead in an inertial frame is a straight line.

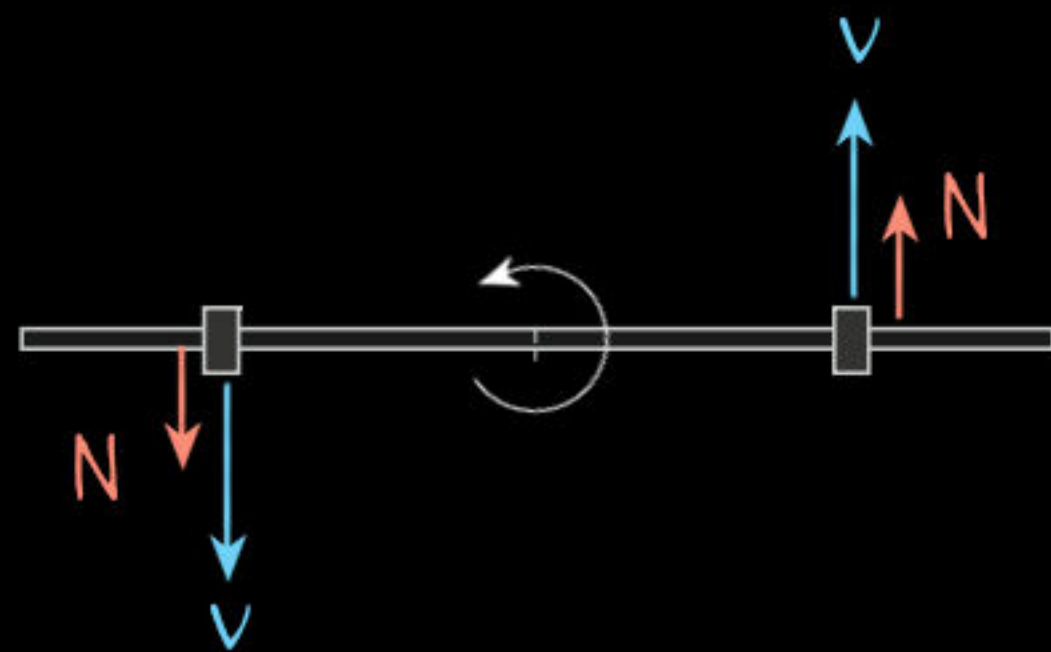
Concept: On a massless object (rod in this case), net force or torque is always zero.

Concept: As the rod is frictionless, there is no force along the rod. Therefore, only possible force can be Normal reaction.

Sol:

Once the thread is cut, let the Normal reaction on rod be N .

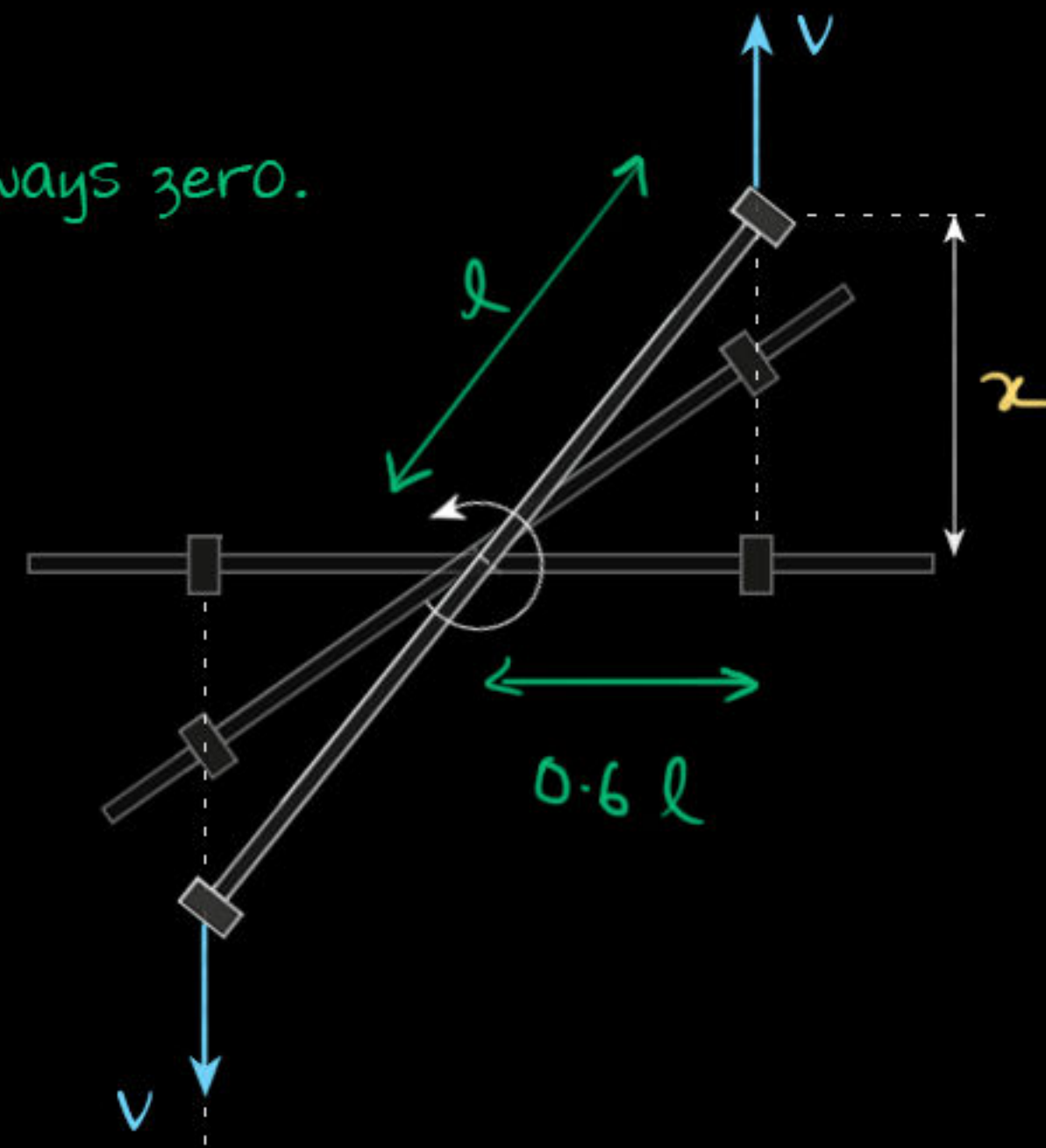
By symmetry, both reactions will be in same sense, let's say counter clockwise..



This would mean, there will be a finite torque on massless rod. Which is not possible.

$$\therefore N = 0$$

- Without any external force, beads will travel in straight line with same velocity.

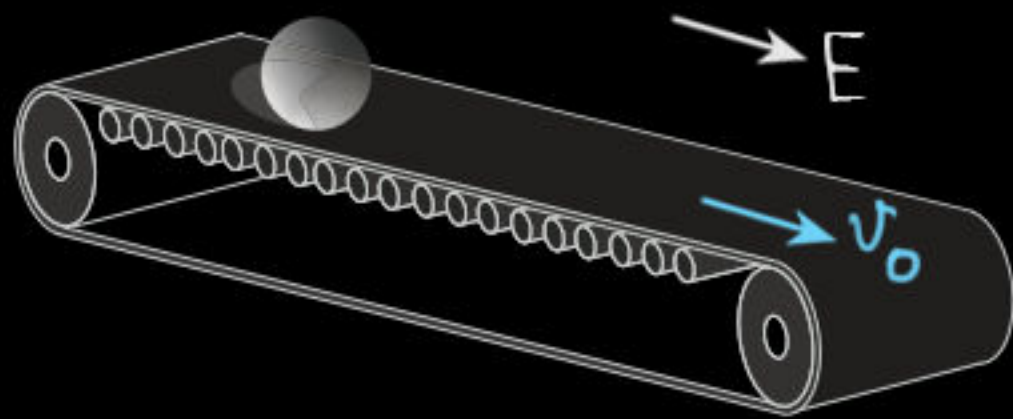


Method 2

$$t = \frac{x}{v} = \frac{\sqrt{l^2 - (0.6l)^2}}{\omega_0 (0.6l)} = \frac{0.8l}{\frac{1}{3} 0.6l} = 4 \text{ sec}$$

Note: Rod is basically useless. It never exerted any force on beads.

Initially too, external force on a bead was due to thread tension alone.



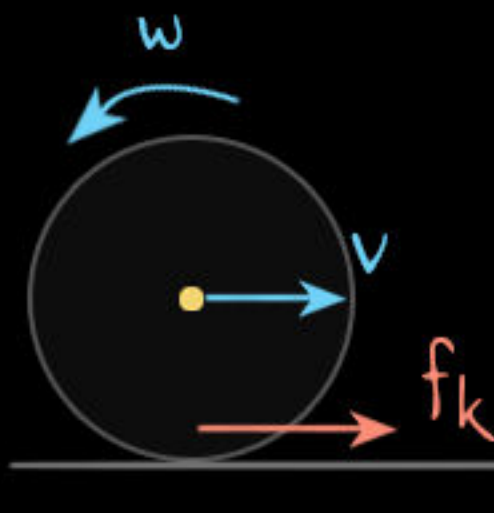
A typical conveyor belt used to transport luggage is shown in the figure. Strip of the conveyor belt is moving with a constant velocity v_0 towards the east. A stationary homogeneous sphere of radius R and mass M is gently placed on the conveyor belt. Assume coefficient of static and kinetic friction to be μ_s and μ_k respectively.

20. Which of the following statements are correct?

- (a) It will appear rotating clockwise, if seen facing due north.
- ✓ (b) It will appear rotating anticlockwise, if seen facing due north.
- (c) Its linear speed relative to the ground as well as relative to the belt decreases.
- ✓ (d) Its linear speed relative to the ground increases and relative to the belt decreases.

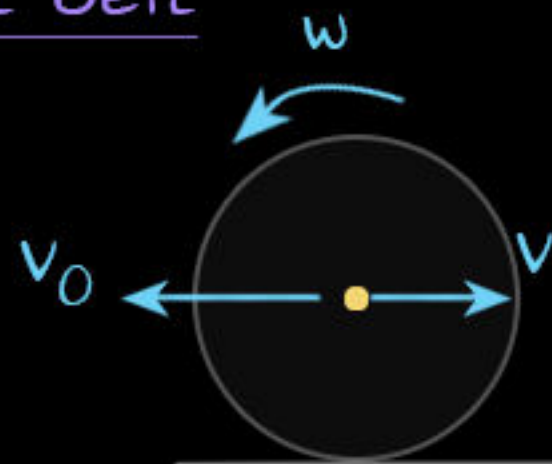
At time t
w.r.t ground

North
⊗



Velocity increases from 0 to v .

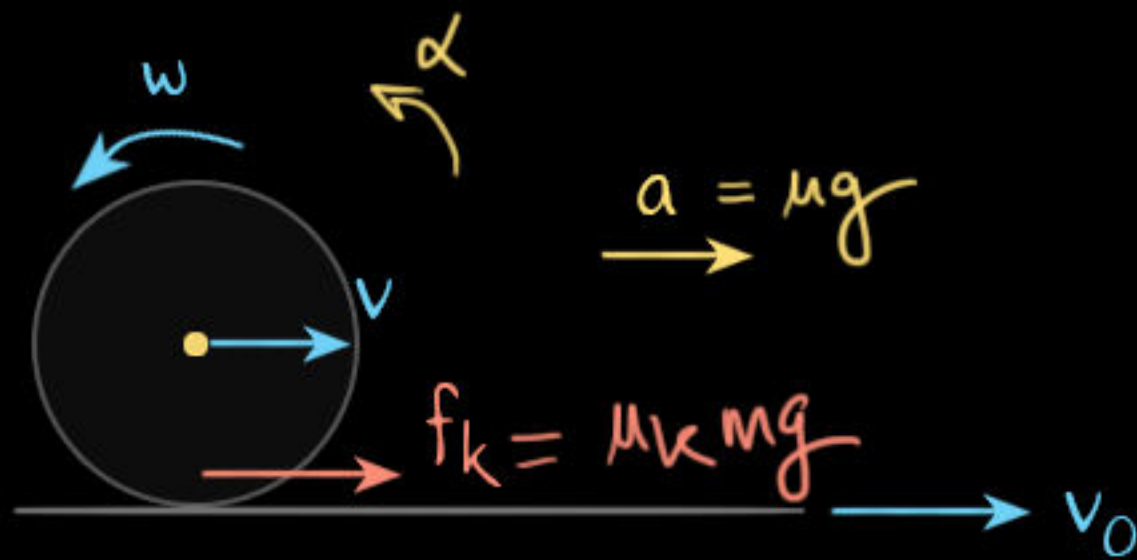
w.r.t belt



Velocity reduces from v_0 to $(v_0 - v)$

i.e you are looking at north

w.r.t ground



$$v = at = \mu_k g t \quad (1)$$

$$\begin{aligned} \omega &= \alpha t = \left(\frac{\tau}{I} \right) t \\ &= \frac{\mu_k mg R}{\frac{2}{5} m R^2} \cdot t \\ &= \frac{5 \mu_k g t}{2 R} \quad (2) \end{aligned}$$

21. After a time t from the instant the sphere was placed on the belt, it starts pure rolling and moves with a velocity v relative to the ground. Which of the following are correct expressions for t and v ?

✓ (b) $t = \frac{2v_0}{7\mu_k g}$

✓ (d) $v = \frac{2v_0}{7}$

When it starts rolling, bottom of sphere will travel with v_0 :

$$\therefore v + \omega R = v_0 \quad (3)$$

From 1, 2, 3:

$$t = \frac{2v_0}{7\mu_k g} //$$

$$v = \frac{2v_0}{7} //$$

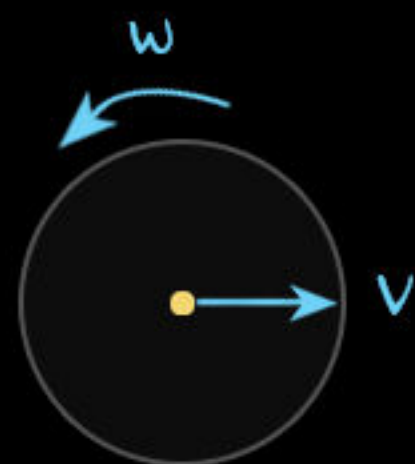
$$\omega = \frac{5v_0}{7R}$$

Method 1



When it starts to roll:

w.r.t ground



$$W_f = \Delta KE$$

$$= \left(\frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} m v^2 \right) - 0$$

$$= \frac{1}{2} \cdot \frac{2}{5} m R^2 \left(\frac{5v}{7R} \right)^2 + \frac{1}{2} m \left(\frac{2v}{7} \right)^2$$

$$= m v_0^2 \left(\frac{5}{7^2} + \frac{2}{7^2} \right)$$

$$= \frac{m v_0^2}{7} //$$

22. Work done by the kinetic friction on the sphere relative to the ground is W_G and relative to the belt is W_B . Which of the following are correct expressions for W_G and W_B ?

✓ (a) $W_G = \frac{1}{7} M v_0^2$

✓ (d) $W_B = -\frac{1}{7} M v_0^2$

w.r.t belt

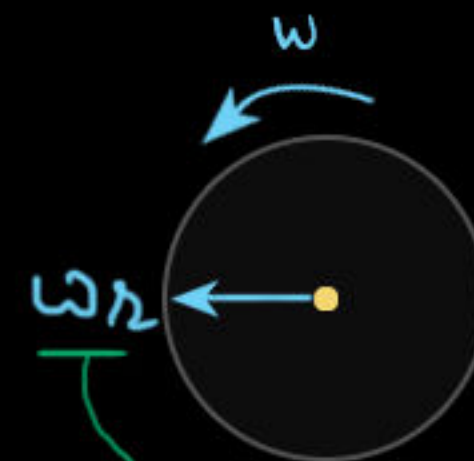
$$W_f = \Delta KE$$

$$= \frac{1}{2} \left(1 + \frac{2}{5} \right) m v^2 - \frac{1}{2} m v_0^2$$

$$= \frac{1}{2} \cdot \frac{7}{5} m \left(\frac{5v_0}{7} \right)^2 - \frac{1}{2} m v_0^2$$

$$= \frac{1}{2} m v_0^2 \left(\frac{4}{5} \cdot \frac{5^2}{7^2} - 1 \right)$$

$$= -\frac{m v_0^2}{7} //$$



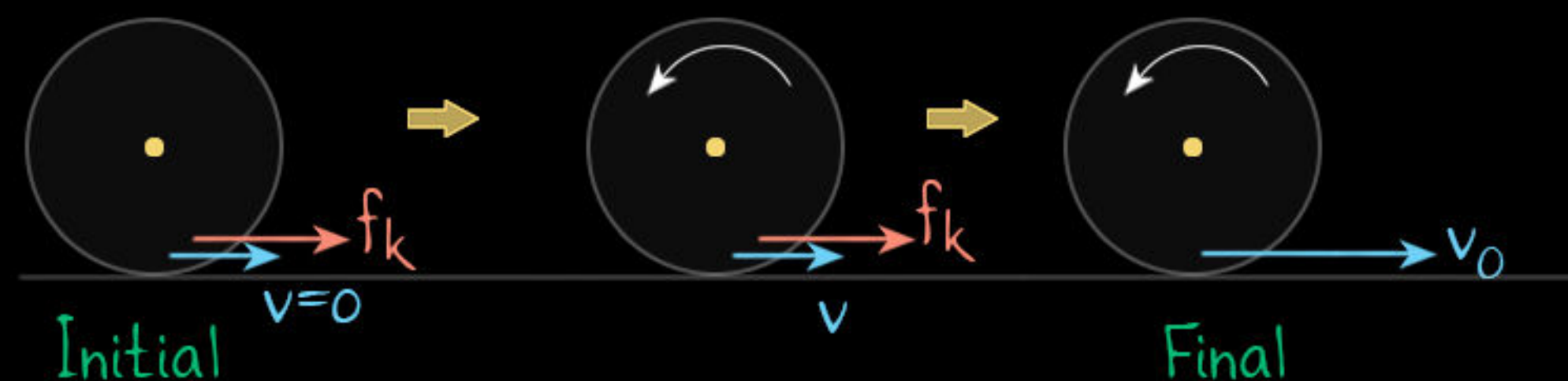
Method 2

22. Work done by the kinetic friction on the sphere relative to the ground is W_G and relative to the belt is W_B . Which of the following are correct expressions for W_G and W_B ?

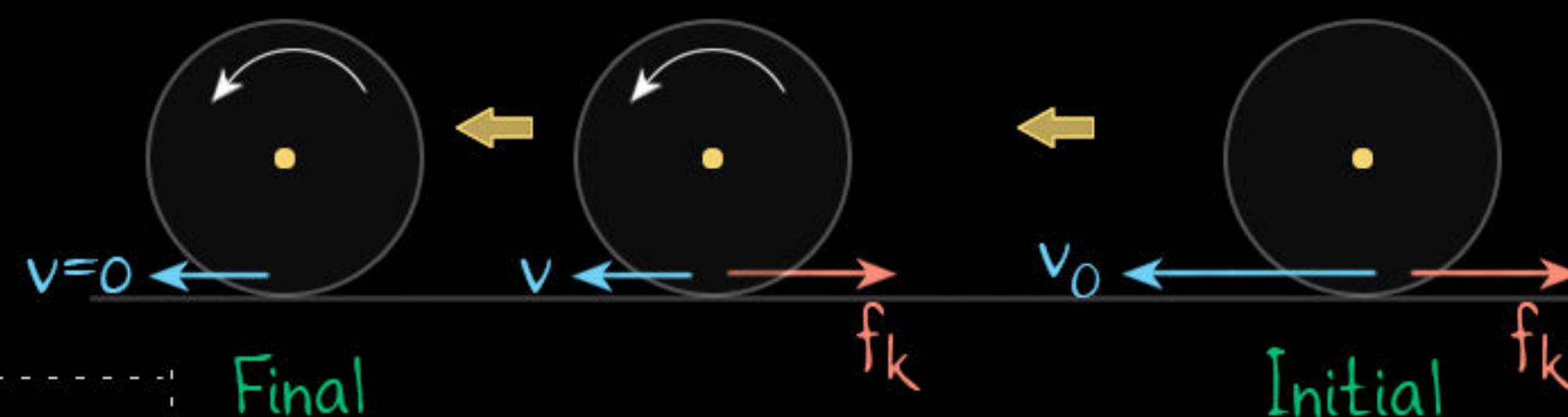
✓ (a) $W_G = \frac{1}{7} M v_0^2$

✓ (d) $W_B = -\frac{1}{7} M v_0^2$

w.r.t ground



w.r.t belt



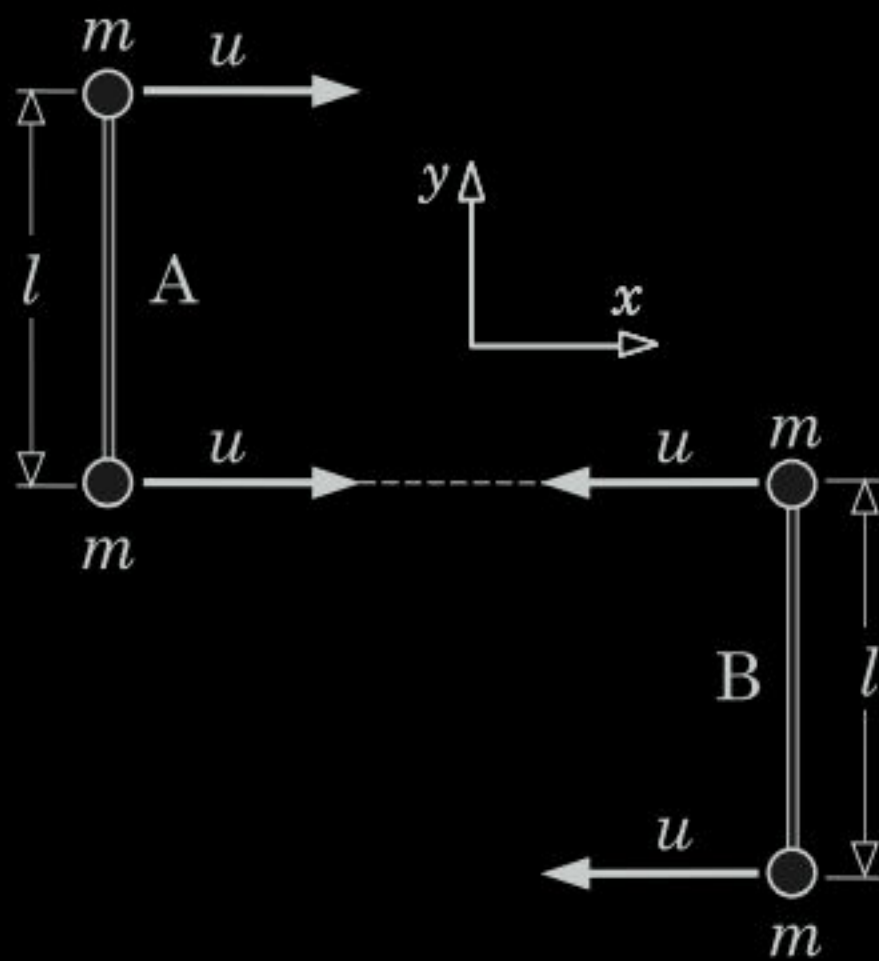
$W_f = f_k \times \text{distance traveled by bottom most point of sphere}$

$$\rightarrow \left(\frac{u+v}{2} \right) t = \frac{v_0 t}{2} = \frac{v_0}{2} \left(\frac{2v_0}{7\mu_k g} \right)$$

$$= \cancel{\mu_k m g} \times \frac{v_0}{2} \cdot \frac{2v_0}{\cancel{7\mu_k g}} = \frac{m v_0^2}{7} //$$

$W_f = -f_k \times \text{distance traveled by...}$

$$= -\frac{m v_0^2}{7} //$$

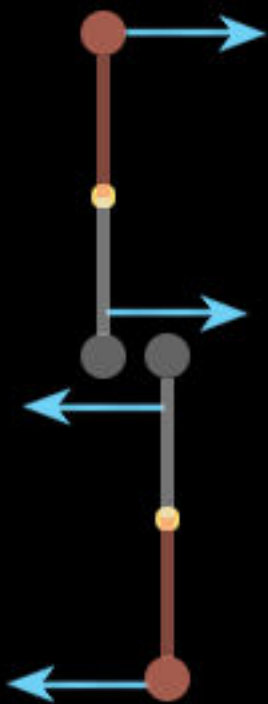


Two identical dumbbells are fabricated by welding small identical particles each of mass m at the ends of light identical rods each of length l . The dumbbells are moving in free space with equal speeds u towards each other without rotation as shown in the figure. Collisions between the particles are perfectly elastic.

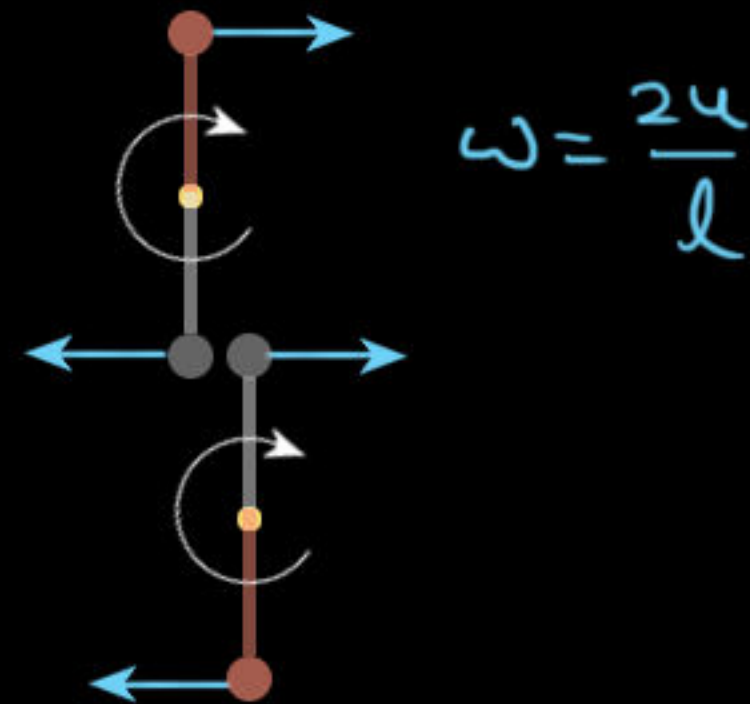
23. Angular velocity of each dumbbell after the first collision is ✓ (c) $\frac{2u}{l}$

24. How many times will the dumbbells collide? ✓ (b) Two

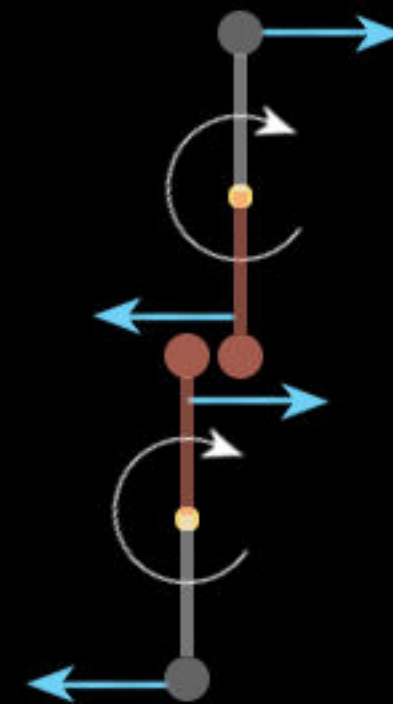
Just before first collision:



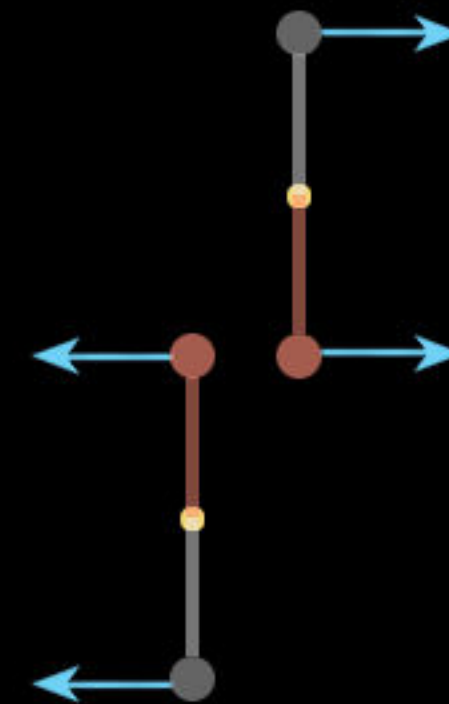
After first collision:



Just before second collision:



After second collision:

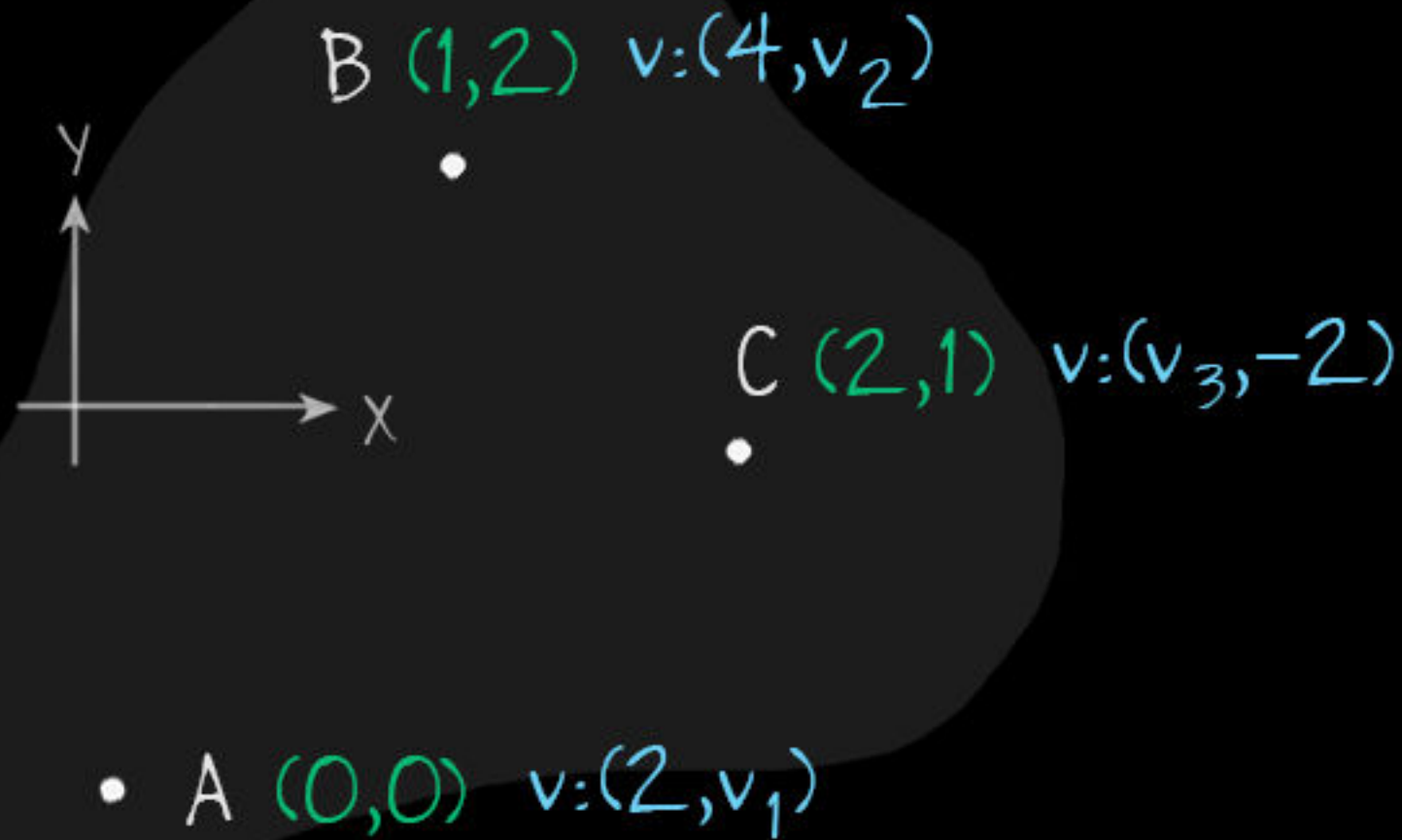


For any two points on rigid body (it can be IOR as well):

$$\vec{v}_1 - \vec{v}_2 = \vec{\omega} \times (\vec{r}_1 - \vec{r}_2)$$

Let IOR be at (x, y)

And, $\vec{\omega}$ be $\omega \hat{k}$



1. A rigid lamina is sliding on a horizontal floor with one of its flat faces on the floor. The floor is represented by $x-y$ plane of a coordinate system. At an instant, some of the velocity components of three particles A, B and C of the lamina, when they occupy positions $(0, 0)$, $(1 \text{ m}, 2 \text{ m})$ and $(2 \text{ m}, 1 \text{ m})$ are $v_{Ax} = 2 \text{ m/s}$, $v_{Bx} = 4 \text{ m/s}$ and $v_{Cy} = -2 \text{ m/s}$. Find coordinates of the instantaneous centre of rotation at this instant.

For A and O: $\vec{v}_A - \vec{v}_O = \vec{\omega} \times (\vec{r}_A - \vec{r}_O)$

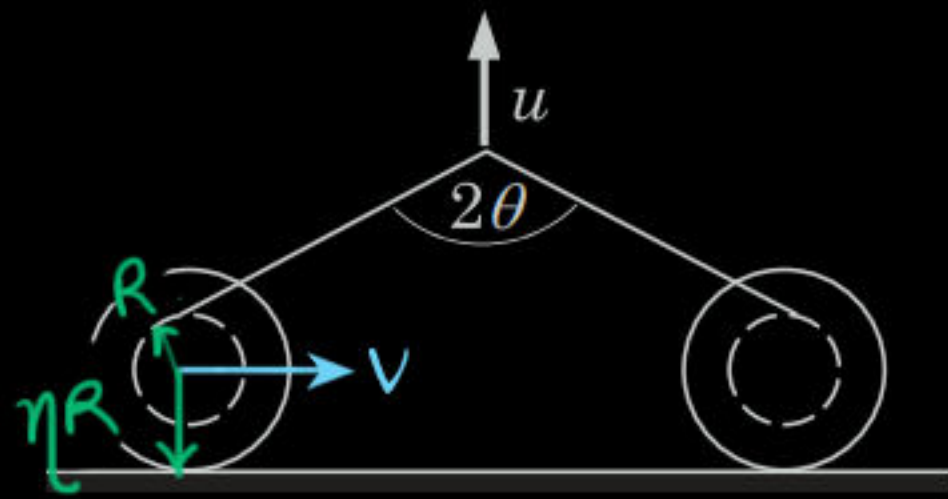
$$\Rightarrow 2\hat{i} + v_1\hat{j} = \omega\hat{k} \times (-x\hat{i} - y\hat{j}) \Rightarrow 2 = \omega y \quad (1)$$

Similarly:

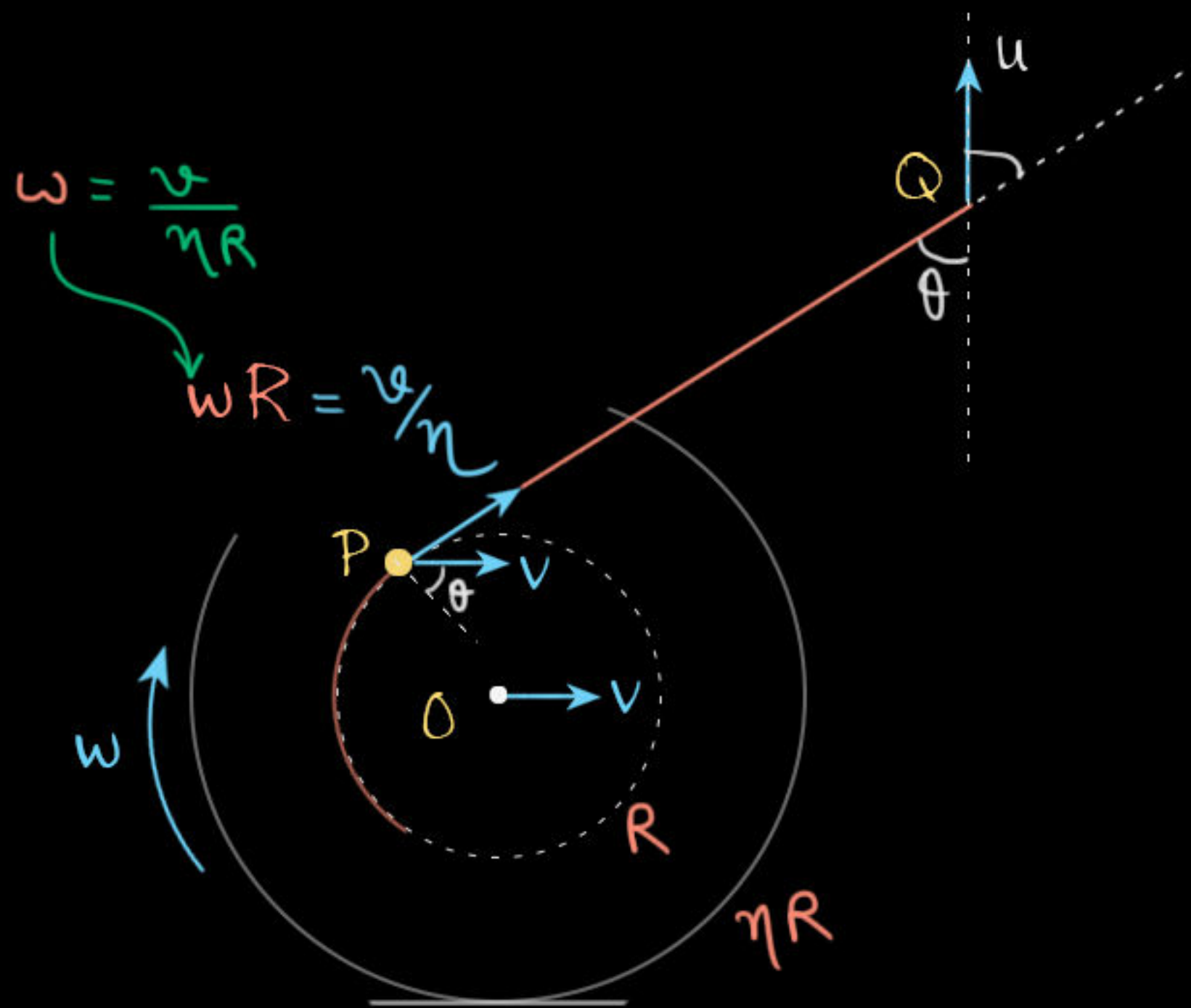
For B and O: $\vec{v}_B = \vec{\omega} \times (\vec{r}_B - \vec{r}_O) \Rightarrow 4 = \omega(y-2) \quad (2)$

For C and O: $\vec{v}_C = \vec{\omega} \times (\vec{r}_C - \vec{r}_O) \Rightarrow 2 = \omega(x-2) \quad (3)$

From 1, 2, 3: $x = 0$
 $y = -2$



2. A thread is wound on two identical bobbins placed on a horizontal floor with their axes parallel. Radius of the outer flanges of the bobbins is η times of that of the inner spools. The midpoint of the thread is pulled vertically upwards with a constant velocity u . If the bobbins roll on the floor without slipping, find velocity of approach of their centres when angle between thread segments becomes 2θ .



As there is no slipping at P point on spool:

$$\vec{V}_P = \vec{v} + \vec{\omega} \times \vec{r}$$

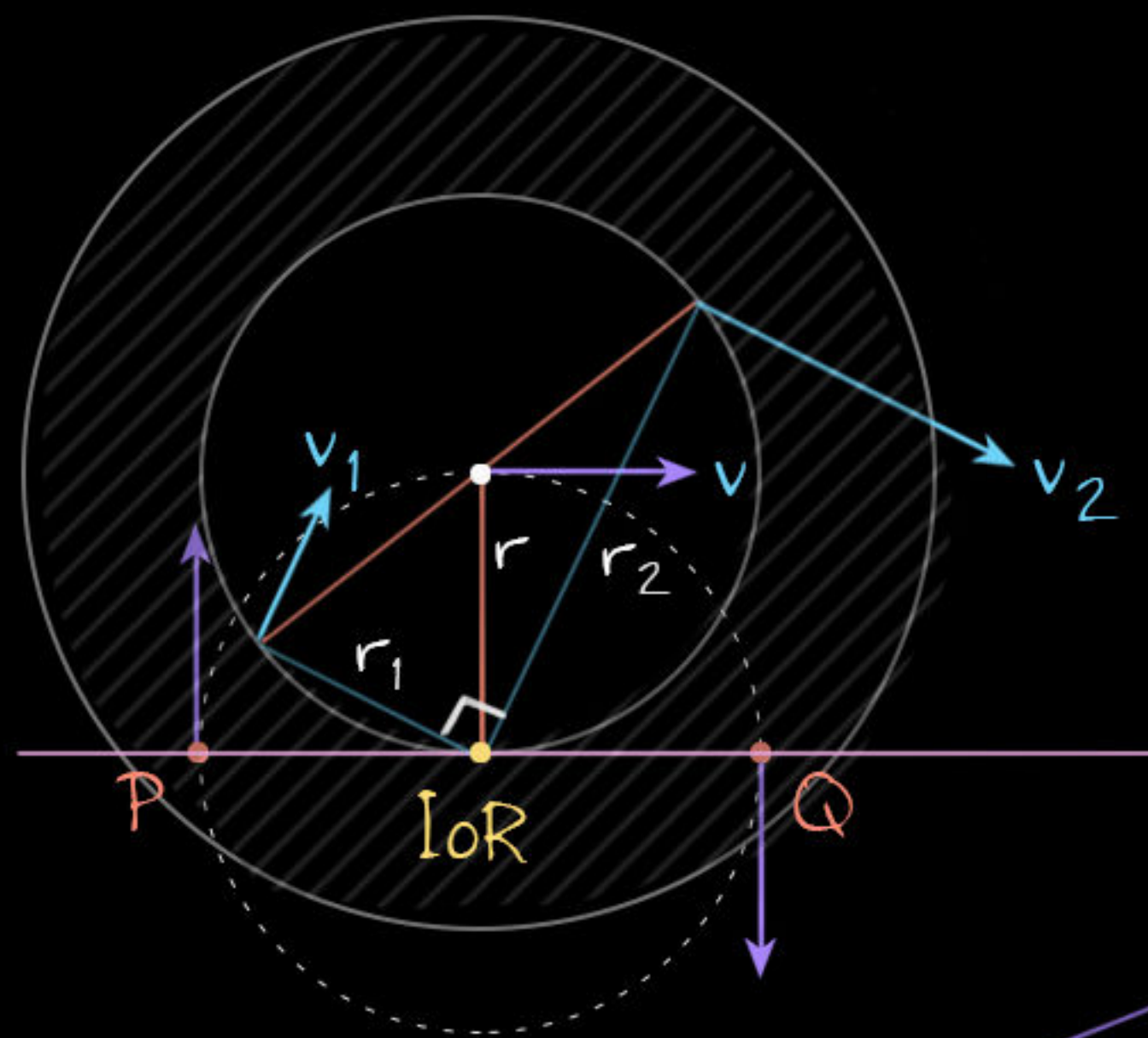
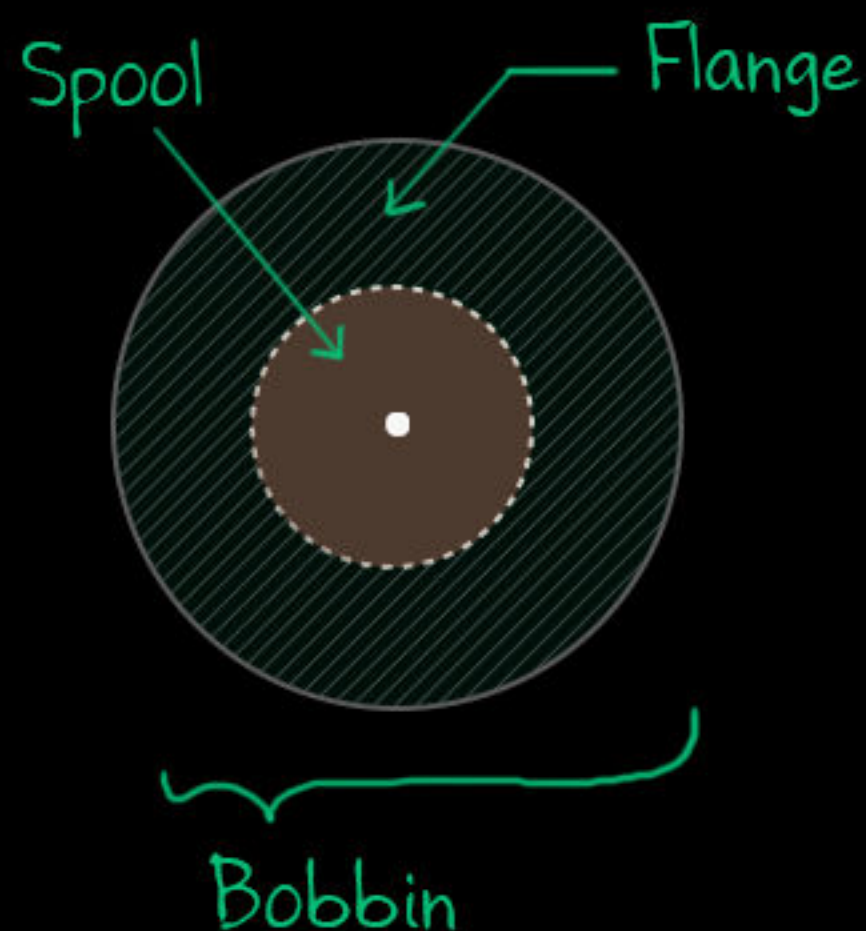
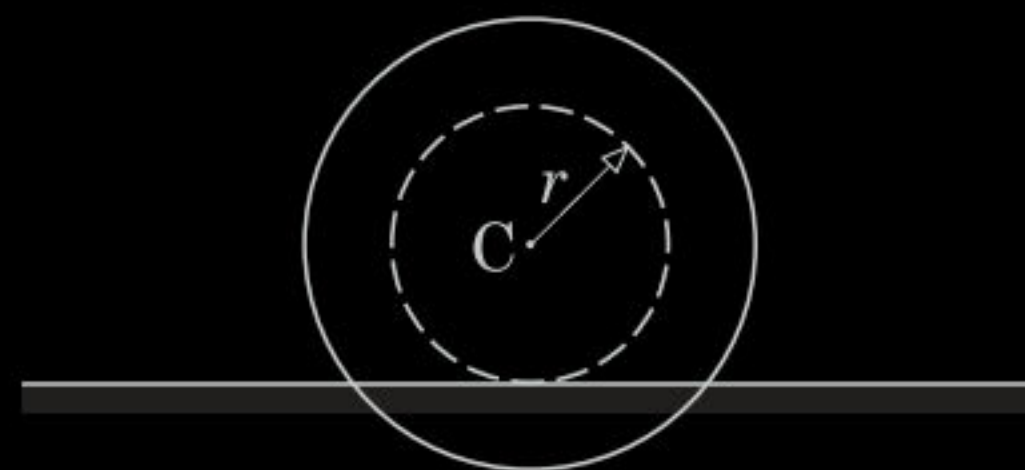
$$V_P \text{ along the rope} = V_Q \text{ along the rope}$$

$$\Rightarrow v \sin \theta + \frac{v}{\eta} = u \cos \theta$$

$$\Rightarrow v = \frac{\eta u \cos \theta}{1 + \eta \sin \theta}$$

Required velocity of approach = $2v$

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So Q lies on the flange

3. Spool of a bobbin rolls without slipping on a horizontal track. The radius of the spool is r and that of the flange is significantly more than r . At some instant speeds of two diametrically opposite points on the spool are v_1 and v_2 .

- (a) What is the speed of the centre of the bobbin?
 (b) Locate points on the flange that are moving vertically either upwards or downwards with a speed equal to that of the centre.

$$A \quad \omega = \frac{v_1}{r_1} = \frac{v}{r} = \frac{v_2}{r_2} \quad 1$$

$$r_1^2 + r_2^2 = 4r^2 \quad 2$$

$$\text{From 1,2: } \frac{v_1^2}{\omega^2} + \frac{v_2^2}{\omega^2} = 4 \frac{v^2}{\omega^2}$$

$$\Rightarrow v = \frac{\sqrt{v_1^2 + v_2^2}}{2} //$$

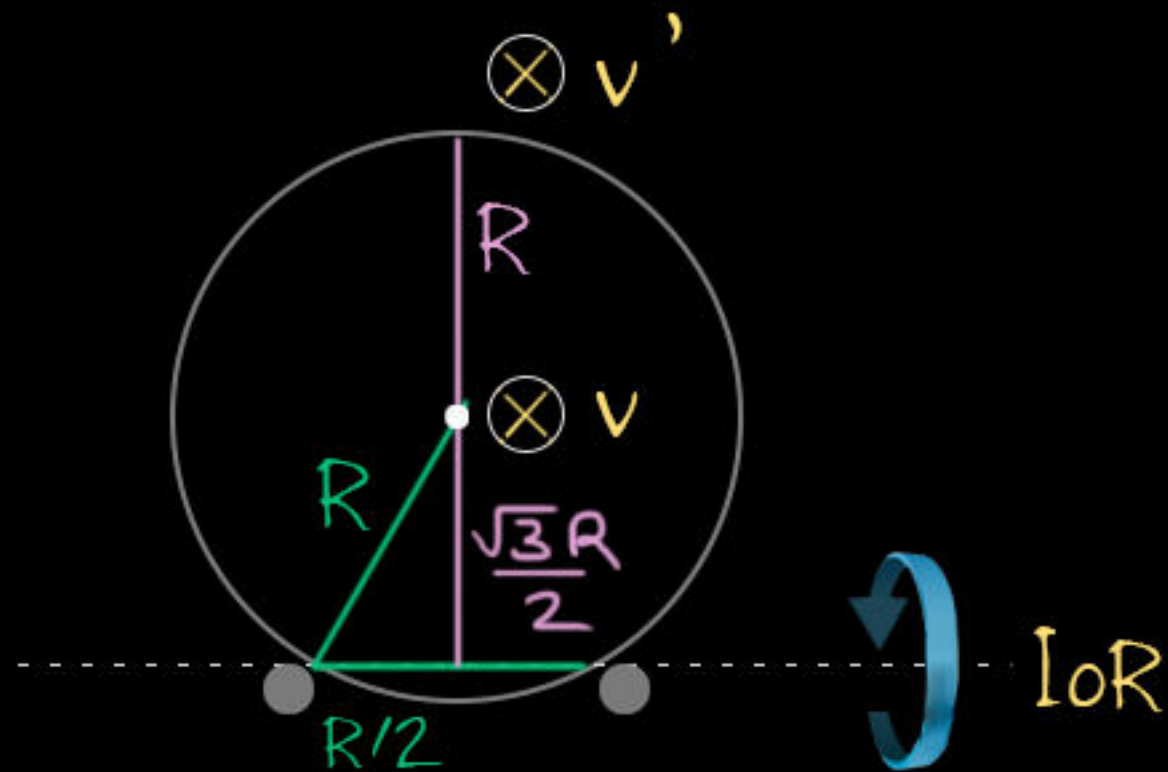
B P and Q



4. A sphere is rolling without slipping on two horizontal and parallel rails that are not in the same level. Distance between the rails is equal to radius of the sphere. If speed of the centre of the sphere is v , find greatest speed of a point on the sphere.

Approach: Lets straighten it first.
Looks better and situation is same.

|||

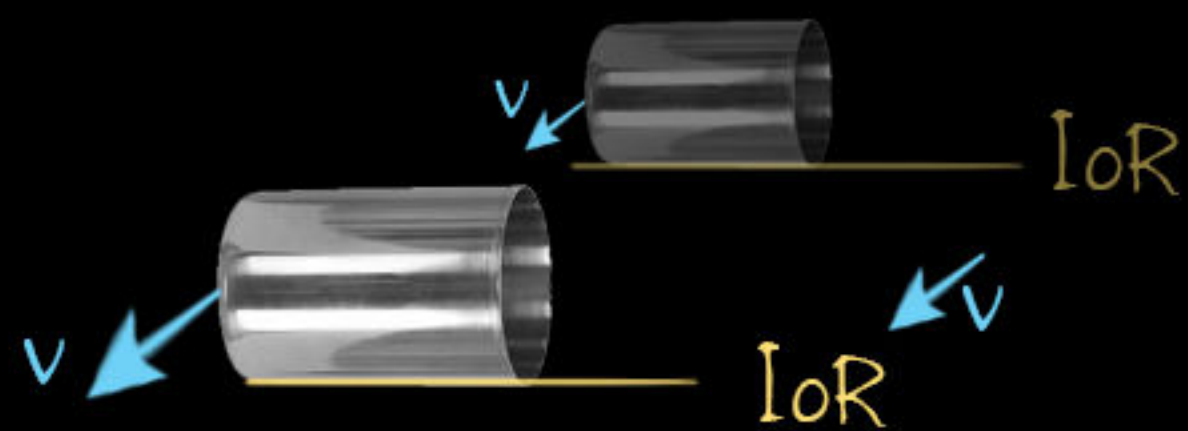


$$\omega = \frac{v}{\sqrt{3}R/2} = \frac{v'}{R + \frac{\sqrt{3}R}{2}}$$

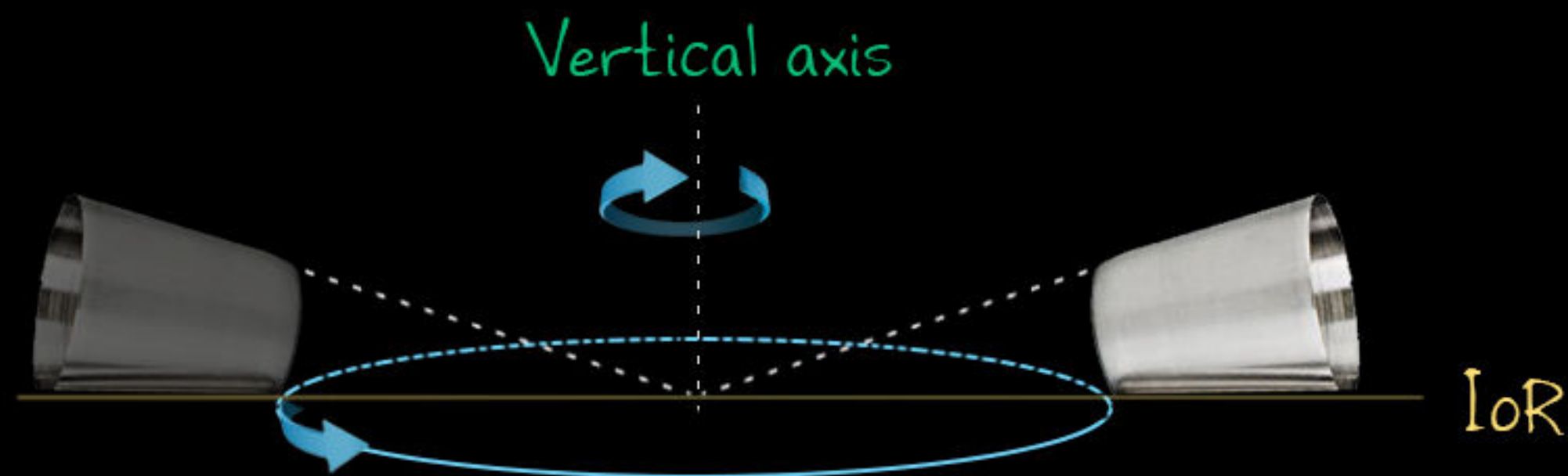
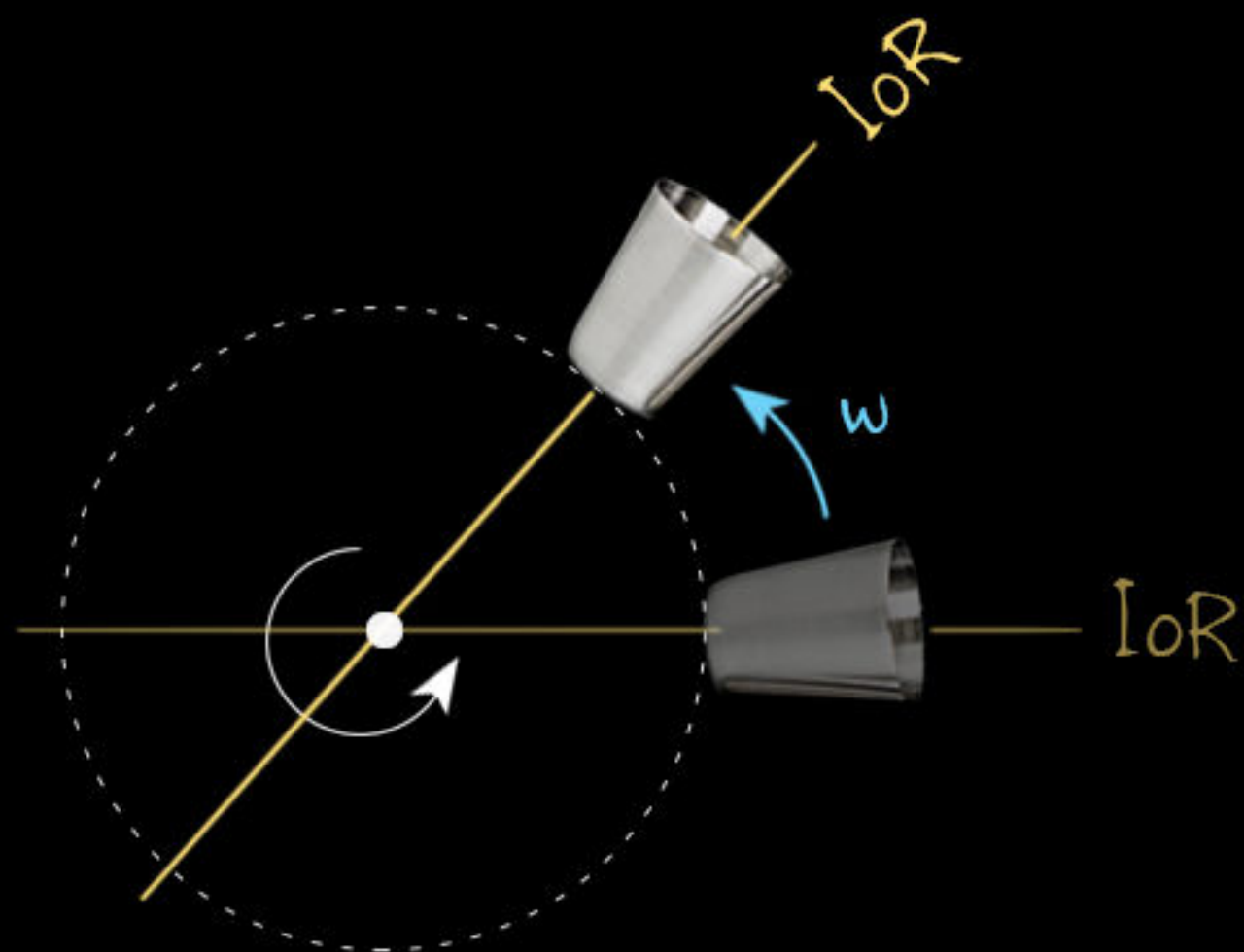
$$\Rightarrow v' = \frac{(2 + \sqrt{3})v}{\sqrt{3}}$$

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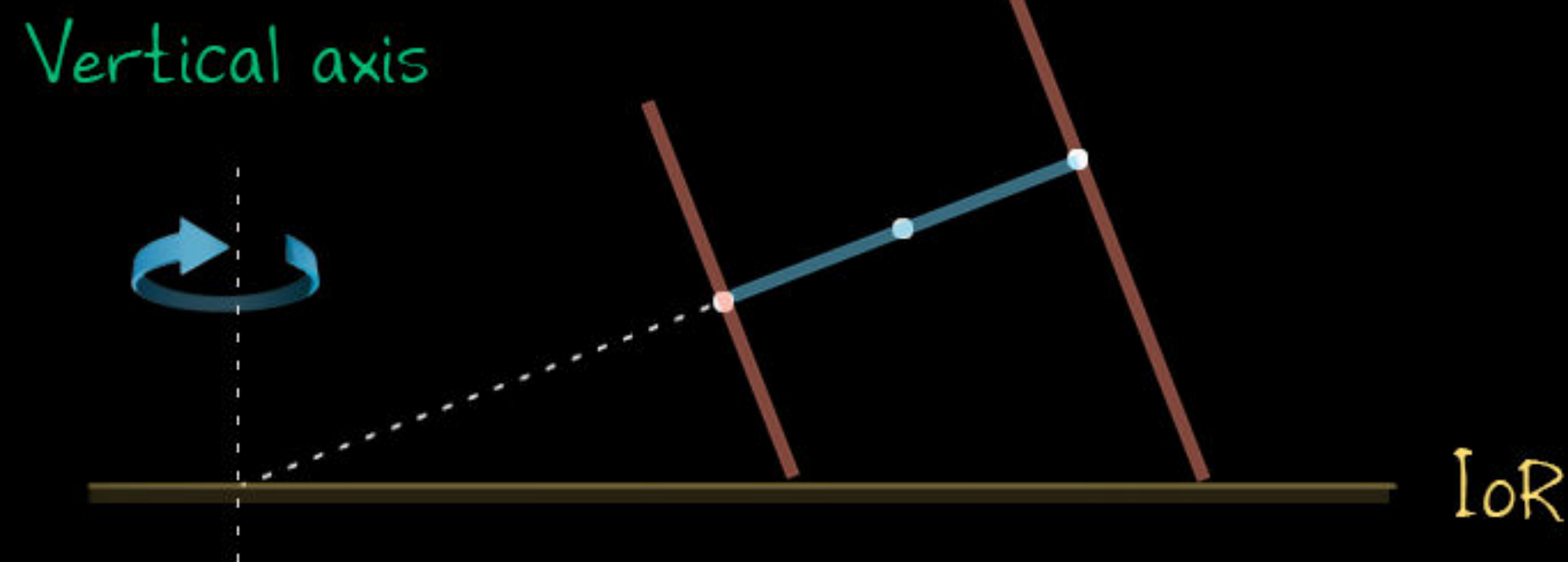
Concept: When a cylinder rolls, IoR shifts parallelly at v m/s.

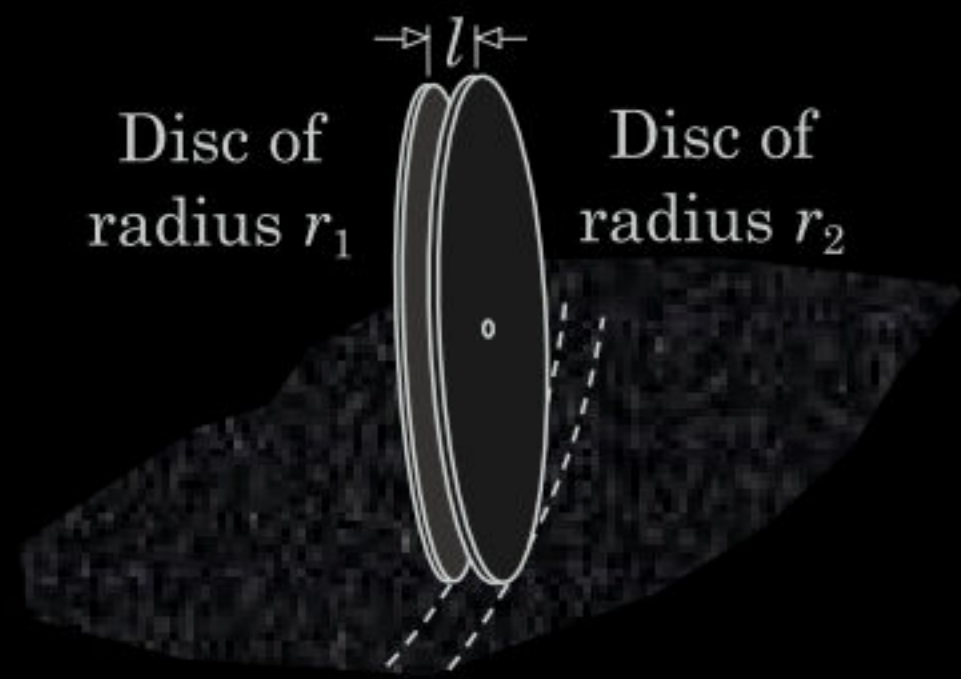


Concept: When a cone rolls, IoR itself rotates about vertical axis at w rad/s.



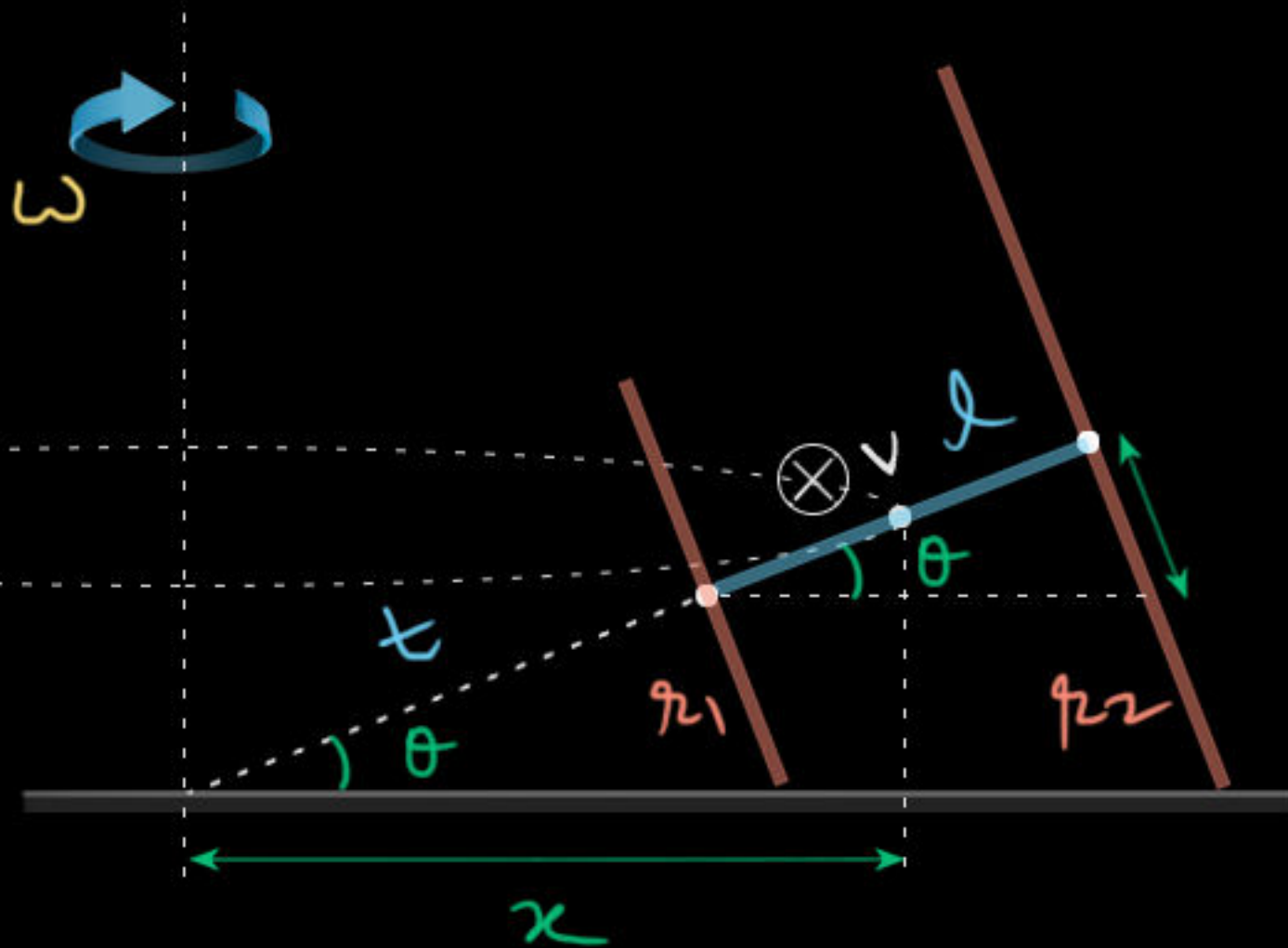
|||





5. A toy is made by affixing coaxially two thin discs of radii $r_1 = 9.95 \text{ cm}$ and the other $r_2 = 10.05 \text{ cm}$ at the ends of a rod of length $l = 1.00 \text{ cm}$. When this toy rolls on a horizontal ground without slipping, it rotates about the rod, advances on a circular path and also rotates about a vertical axis. If centre of the rod moves with a speed $v_C = 10.0 \text{ cm/s}$, find angular velocity of the toy about the vertical axis.

Given: r_1, r_2, l, v



$$\omega = \frac{v}{x} = \frac{v}{\left(t + \frac{l}{2}\right) \cos \theta} \cdot \frac{l}{\sqrt{l^2 + (r_2 - r_1)^2}}$$

$$\frac{r_1}{t} = \frac{r_2}{t + l} \Rightarrow t = \frac{r_1 l}{r_2 - r_1}$$

$$\Rightarrow \omega = \frac{v}{l \left[\frac{r_1}{r_2 - r_1} + \frac{l}{2} \right]} \cdot \frac{l}{\sqrt{l^2 + (r_2 - r_1)^2}}$$

$$\approx \frac{2v(r_2 - r_1)}{l(r_2 + r_1)} //$$

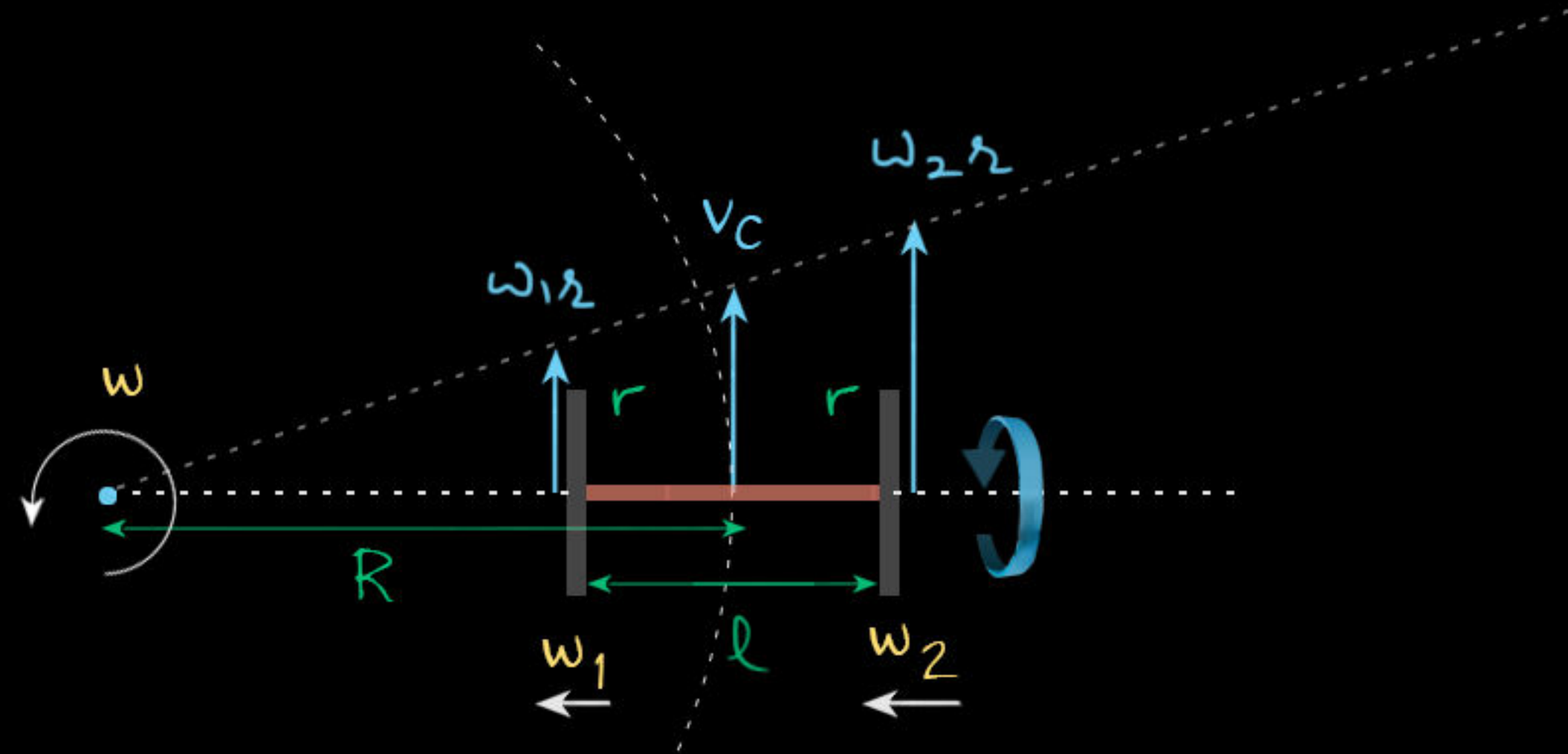
6. A car is moving on a circular path of radius $R = 20$ m and its wheels are rolling without slipping. Wheels of the car are of radius $r = 25$ cm and length of an axle that is distance between the wheels on the same axle is $l = 1.5$ m. If speed of midpoint of an axle is $v_c = 36$ km/h, find amount with which angular velocity of the outer wheel on this axle exceeds that of the inner wheel.

Top view

Given: R, v_c, r, l

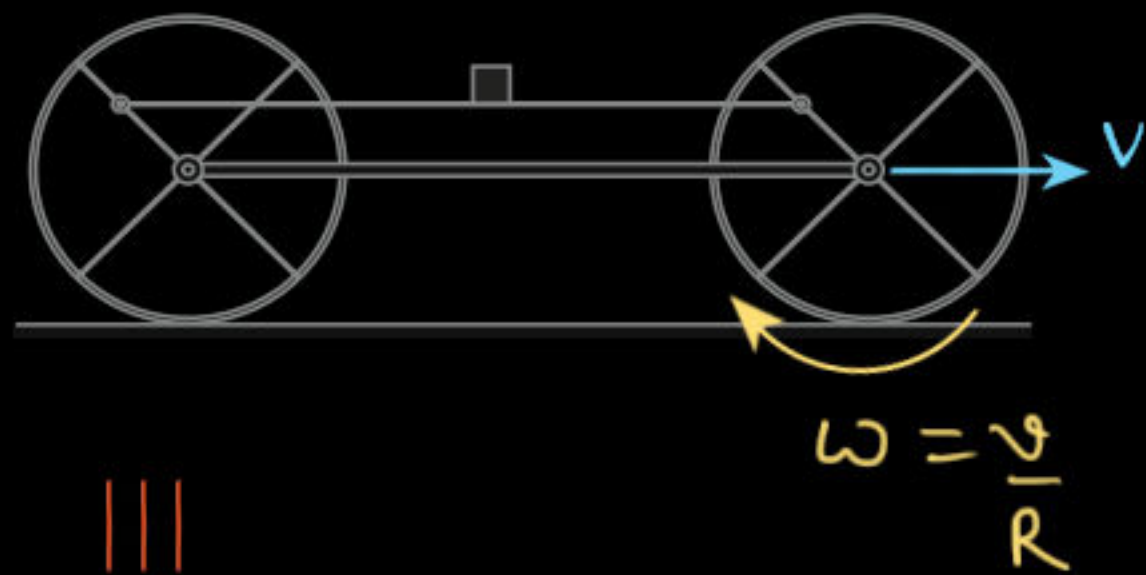
Lets assume: $\omega, \omega_1, \omega_2$

of circular path of wheels



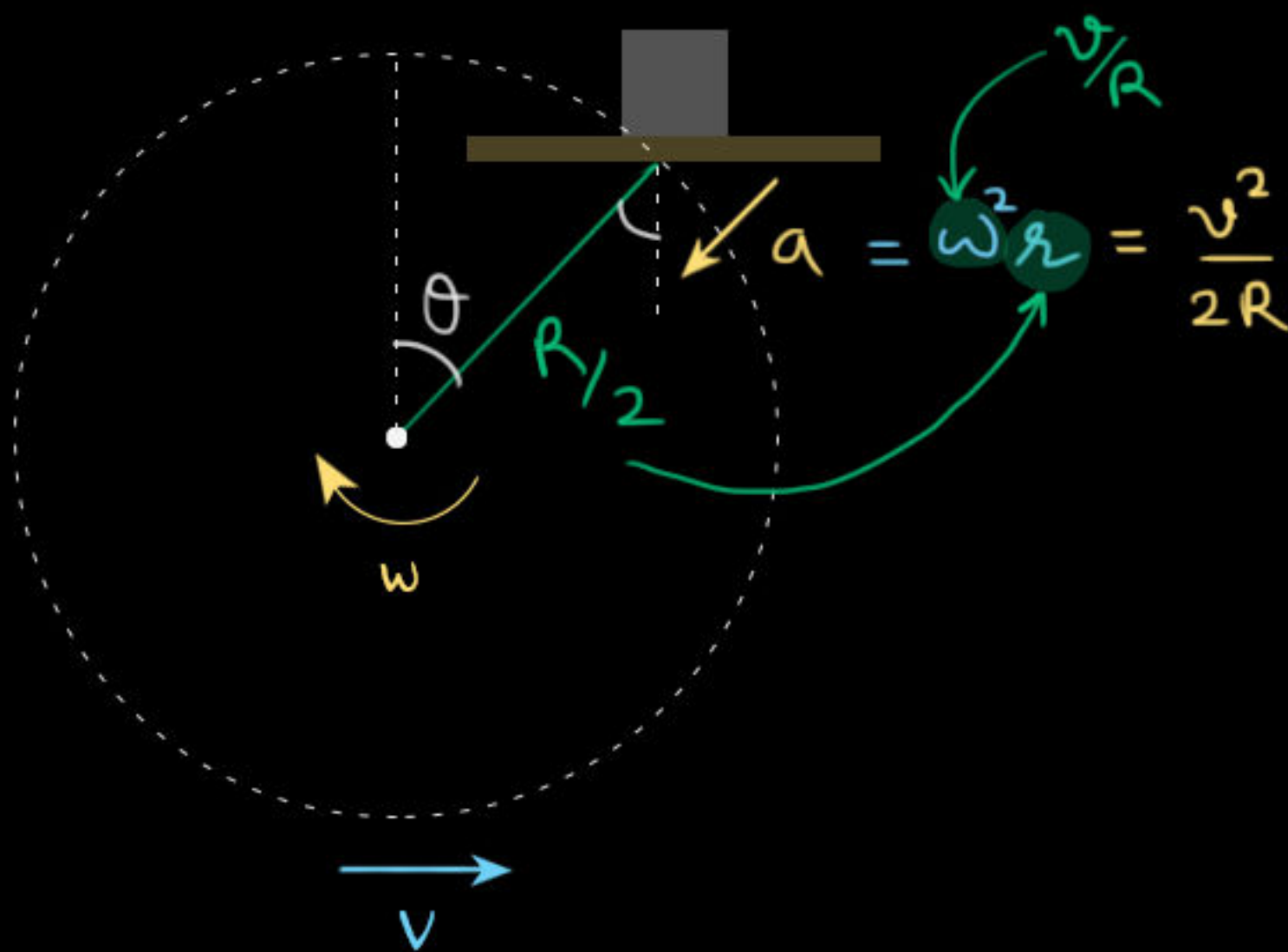
$$\omega = \frac{\omega_2 r - \omega_1 r}{l} = \frac{v_c}{R}$$

$$\Rightarrow \omega_2 - \omega_1 = \frac{v_c l}{R r}$$

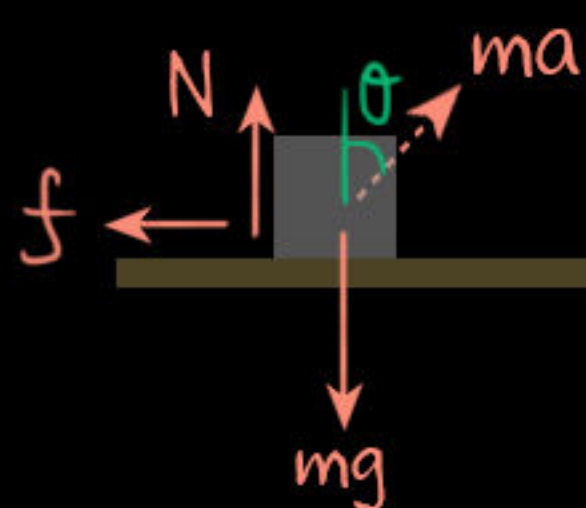


7. Two identical wheels each of radius R are harnessed by connecting their axles with a rigid rod and pivotally connecting midpoints of two parallel spokes with a bar. A box is placed on the bar and the assembly is made to move on a horizontal track with a gradually increasing speed maintaining rolling without slipping. Coefficient of friction between the bar and the box is μ and acceleration due to gravity is g . Find speed of the centre of the wheels at which the box begins sliding with respect to the bar.

w.r.t ground:



w.r.t plate



To not slip:

$$ma \sin \theta = f < \mu (mg - ma \cos \theta)$$

$$\downarrow a = \frac{v^2}{2R}$$

$$v^2 < \frac{2R\mu g}{\sin \theta + \mu \cos \theta}$$

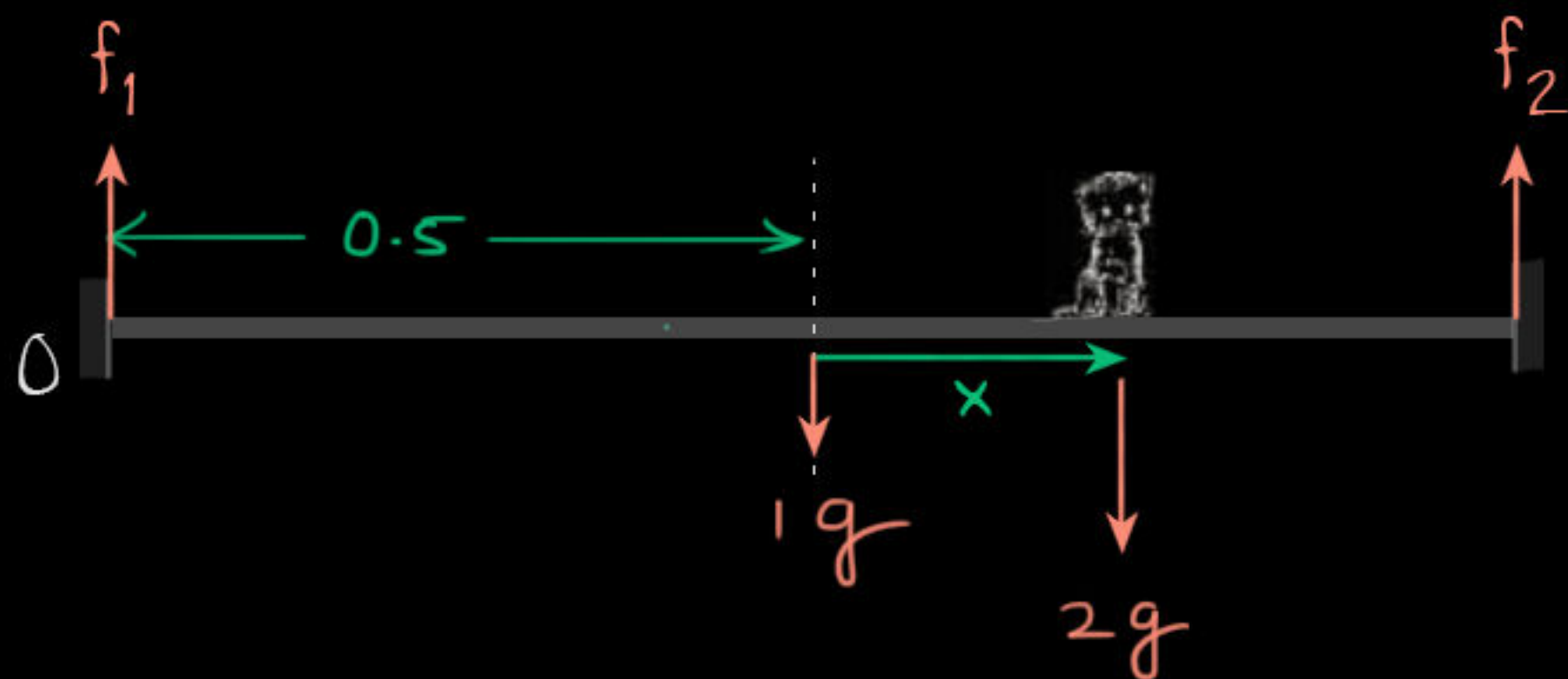
$$a \sin \theta + b \cos \theta \xrightarrow{\text{max}} \sqrt{a^2 + b^2}$$

$$\Rightarrow v^2 < \frac{2R\mu g}{\sqrt{1 + \mu^2}}$$

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8. A uniform rod of length 100 cm and mass 1.0 kg is supported between two walls with the help of frictional forces. If maximum frictional force between an end of the rod and the wall is 20 N, where on the rod can a kitten of mass 2.0 kg sit safely?



$$\sum \tau_o: 1g(0.5) + 2g(0.5 + x) = f_2 \cdot 1 < 20$$

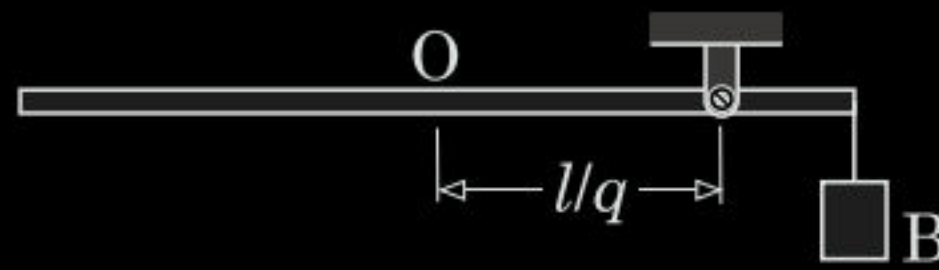
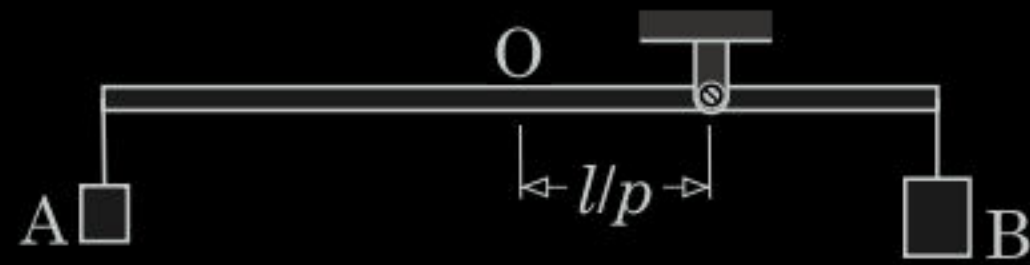
$$\Rightarrow x < 0.25$$

Concept: Both f_1 and f_2 will act upwards.

Concept: When kitten is sitting on right half of rod, then $f_2 > f_1$

So if f_2 is within limits, then f_1 will definitely be.

∴ By symmetry kitten should sit within 25 cm from center of rod.



9. If a load A of unknown mass is suspended at one end of a rod of length l and another load B of mass $m = 30$ kg at the other end, the rod stays horizontal provided that the rod is pivoted l/p away from its centre O towards the load B as shown in the first figure. In the absence of the load A, the rod stays horizontal, if it is pivoted l/q away from the centre towards the load B as shown in the second figure. If $p = 4$, $q = 3$, find mass of the load A.

$\Rightarrow$ Rod has mass

Let rod's mass be m_r

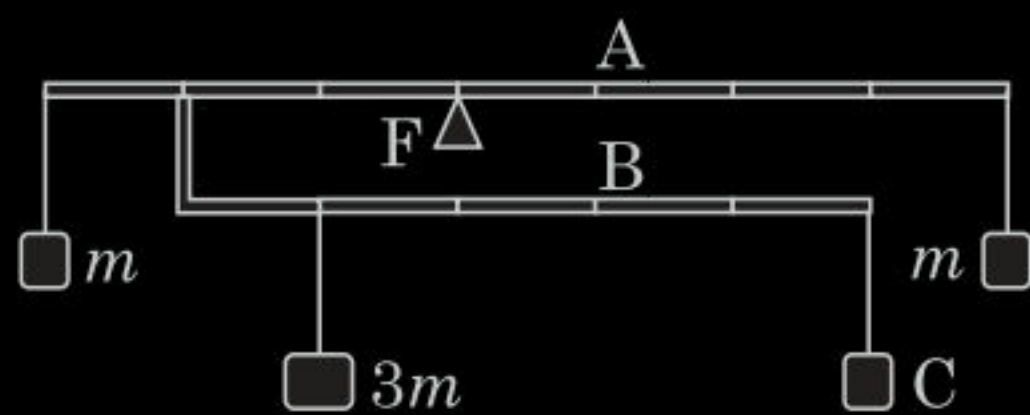
$$m_A \left(\frac{l}{2} + \frac{l}{p} \right) + m_r \frac{l}{p} = m_B \left(\frac{l}{2} - \frac{l}{p} \right) \quad (1)$$

$$m_r \frac{l}{q} = m_B \left(\frac{l}{2} - \frac{l}{q} \right) \quad (2)$$

From 1,2:

$$m_A = m_B \frac{(p-q)}{(p+2)}$$

Youtube / Arihant Senior



10. A balance consists of a straight light rod A, a right-angled light rod B rigidly welded with rod A and a fixed fulcrum F. Four loads are suspended from the balance with the help of light threads as shown in the figure. The rods have equidistant marks on them. If the masses of three loads are known ($m = 6 \text{ kg}$), find the mass of the load C.

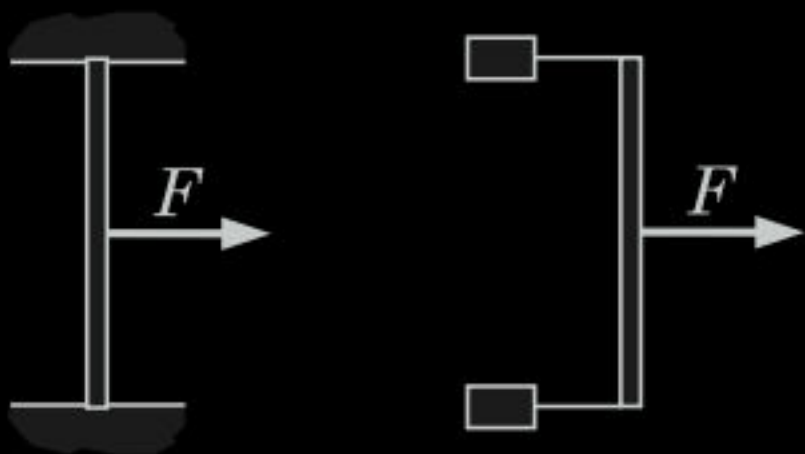
As its in equilibrium. We can balance the torque on this rigid body about F.

$$\sum \tau_F : m(3) + 3m(1) = m(4) + m_C(3)$$

$$\Rightarrow m_C = \frac{2m}{3} //$$

Concept: This rigid body consisting of rods has total five external forces. One on pivot and four due to tension.

Rest are internal forces which we wont take into account.



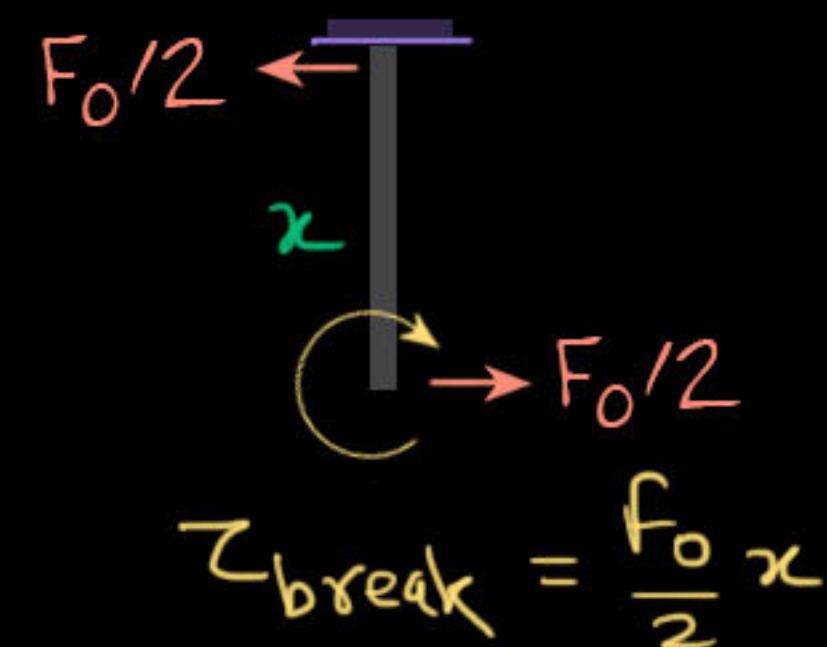
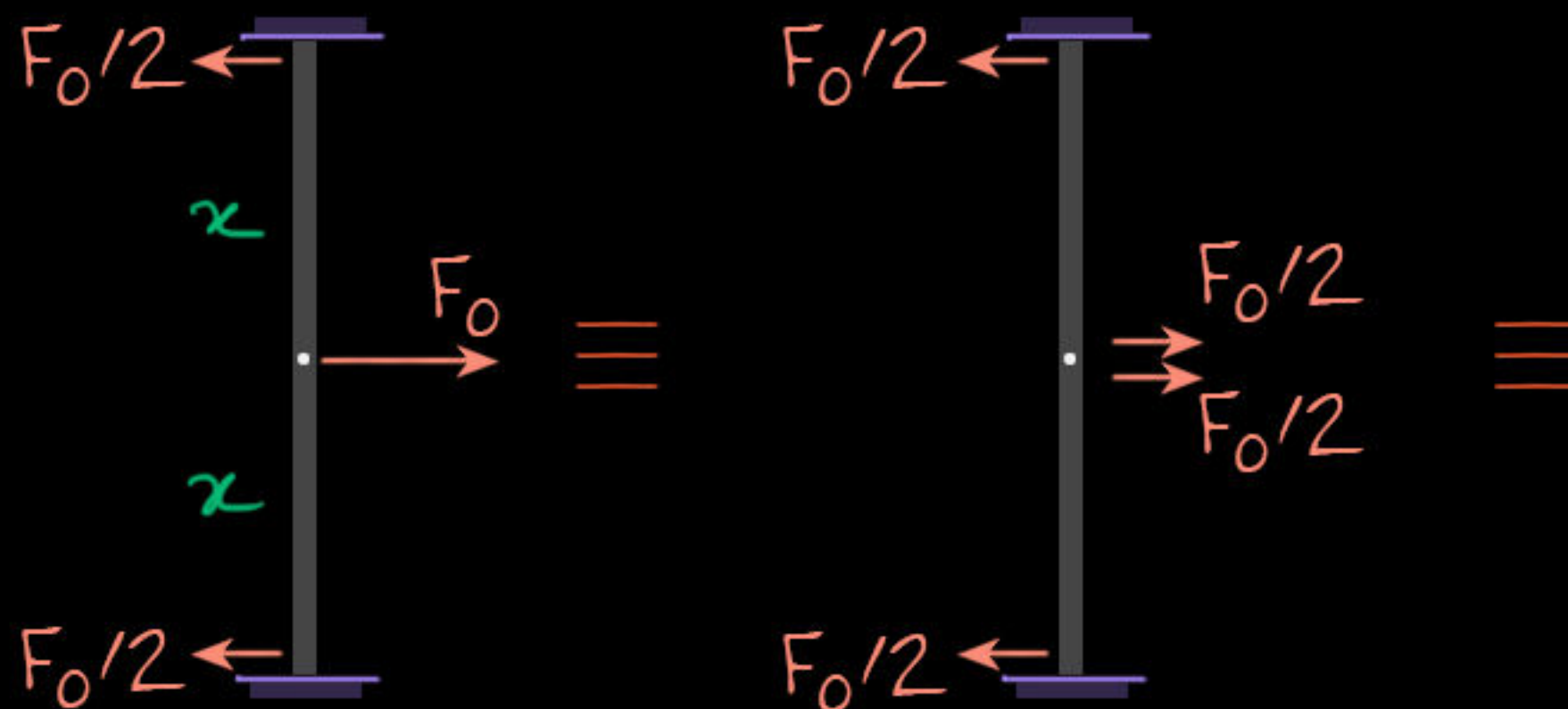
11. A uniform bar of mass $m_0 = 10 \text{ kg}$ is placed on a frictionless horizontal floor and its ends are fixed. If midpoint of the bar is pulled by a gradually increasing force F applied perpendicular to the bar as shown in the first figure, the bar breaks at its midpoint when the force F becomes $F_0 = 150 \text{ N}$. Now another identical bar is placed on the floor and identical blocks of masses $m = 5 \text{ kg}$ are attached at the ends with the help of light inextensible cords as shown in the second figure. What maximum force F can be applied perpendicular to the bar at its midpoint so that the bar does not break?

Approach: Take only one half of the rod. (when its about to break)

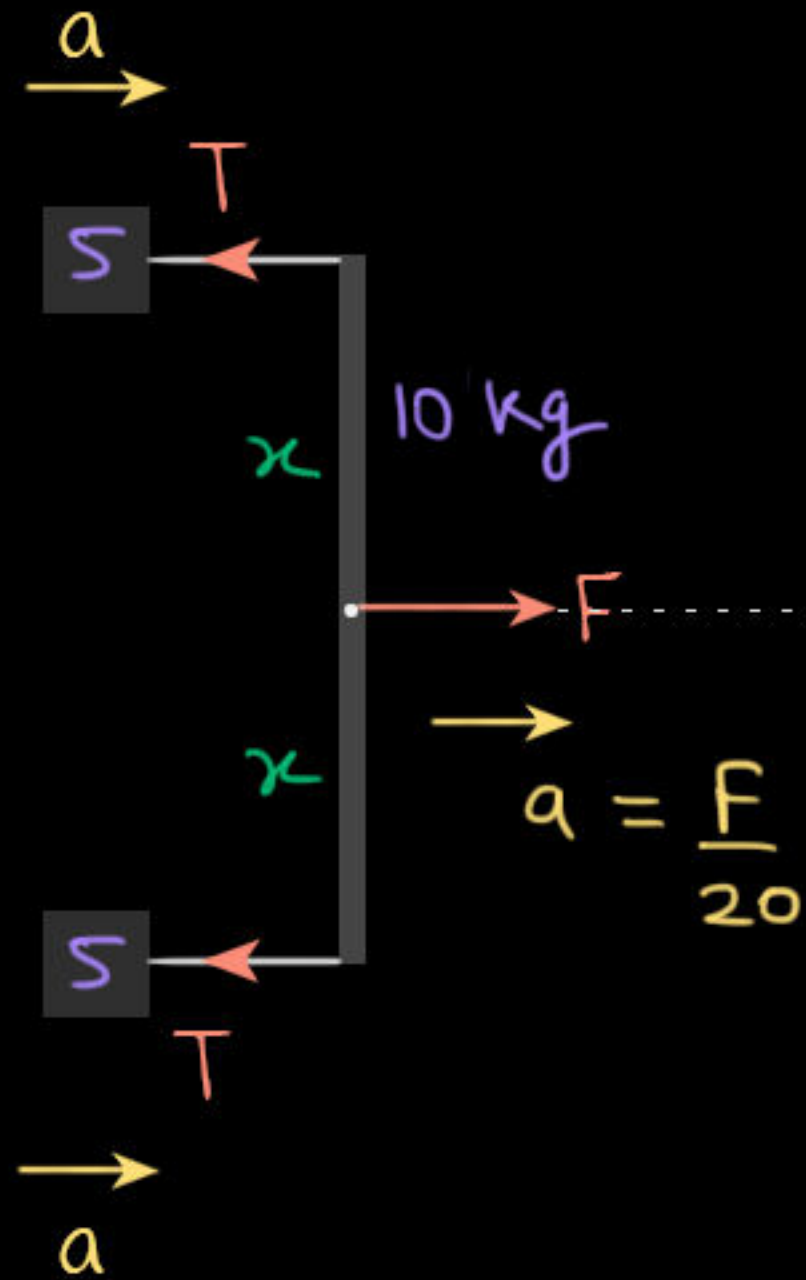
Then show contact (breaking) torque on top half of rod due to lower half.

Then do the same to top half of rod in second case, w.r.t rod (so its at equilibrium)

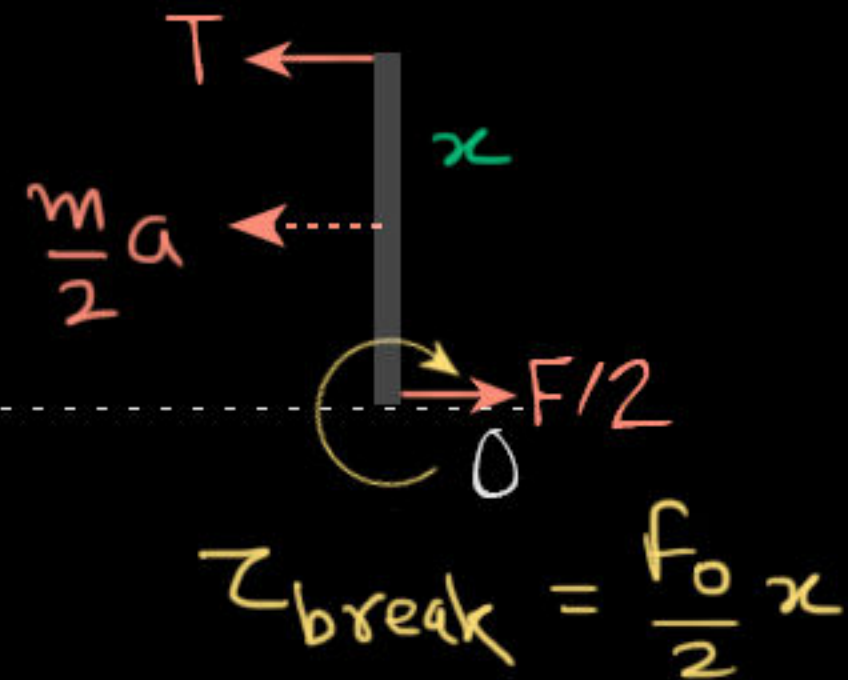
When about to break:



w.r.t ground



Top half, w.r.t rod

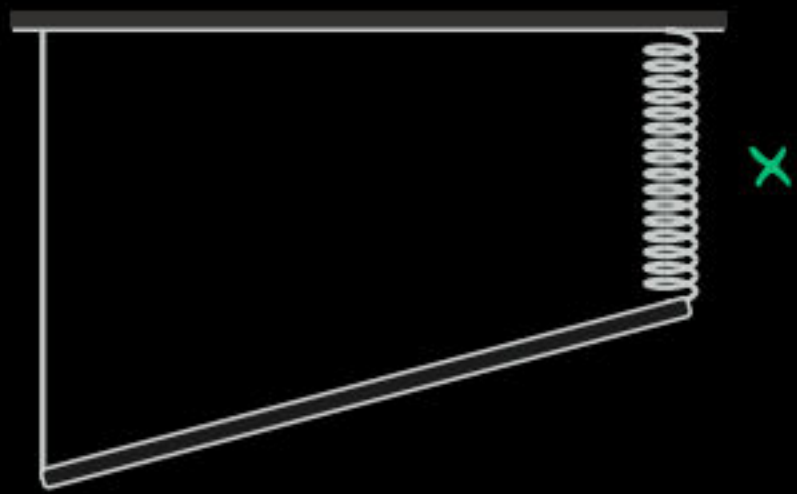


Balancing torque about O :

$$5a = \frac{F}{4}$$
$$Tx + \frac{ma}{2} \cdot \frac{x}{2} = \tau_{\text{break}} = \frac{F_0 x}{2}$$

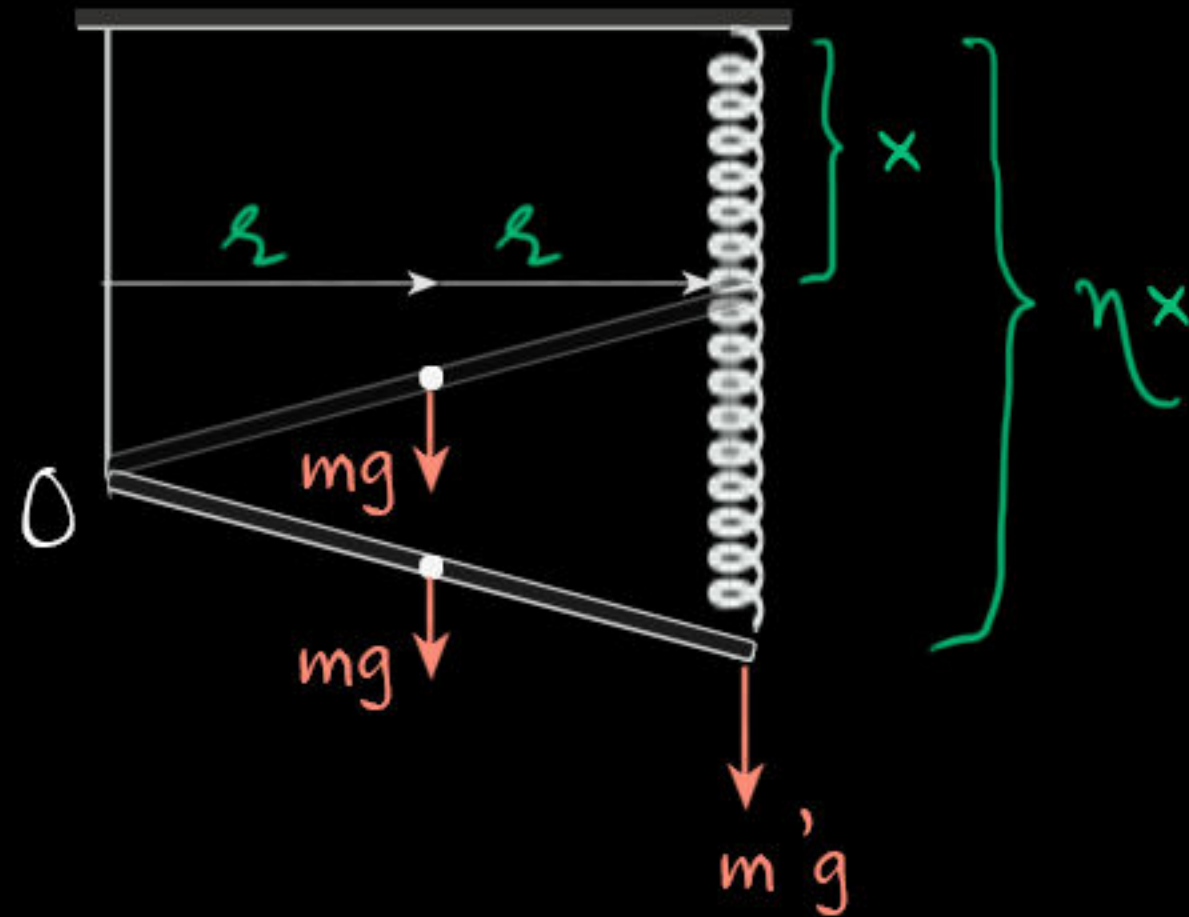
$$\Rightarrow \frac{Fx}{4} + \frac{m}{2} \cdot \frac{F}{20} \cdot \frac{x}{2} = \frac{F_0 x}{2}$$

$$\Rightarrow F = \frac{4F_0}{3} = \frac{4(150)}{3} = 200\text{ N}$$



12. A rod of mass $m = 4.0$ kg is suspended from the ceiling with the help of an inextensible cord and a spring. In equilibrium, the cord and the spring are vertical and the rod stays at an angle 15° above the horizontal as shown in the figure. When an unknown load is suspended from the rod at the end connected with spring, the rod turns clockwise and stays at angle 15° below the horizontal. If in the second case extension in the spring is $\eta = 2$ times of that in the first case, find mass of the unknown load.

Let x be initial extension.

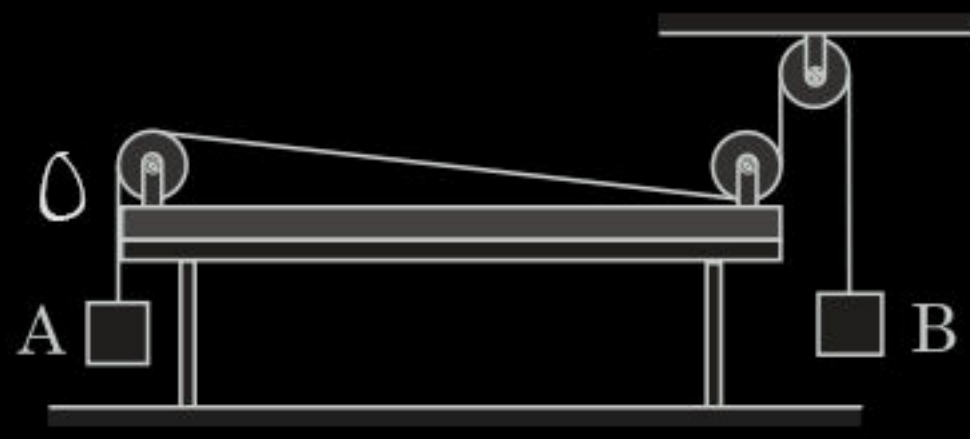


$$\text{Initial } \tau_O: mg \cancel{r} = kx(2\cancel{r}) \quad (1)$$

$$\text{Later } \tau_O: mg \cancel{r} + m'g(2\cancel{r}) = k\eta x(2\cancel{r}) \quad (2)$$

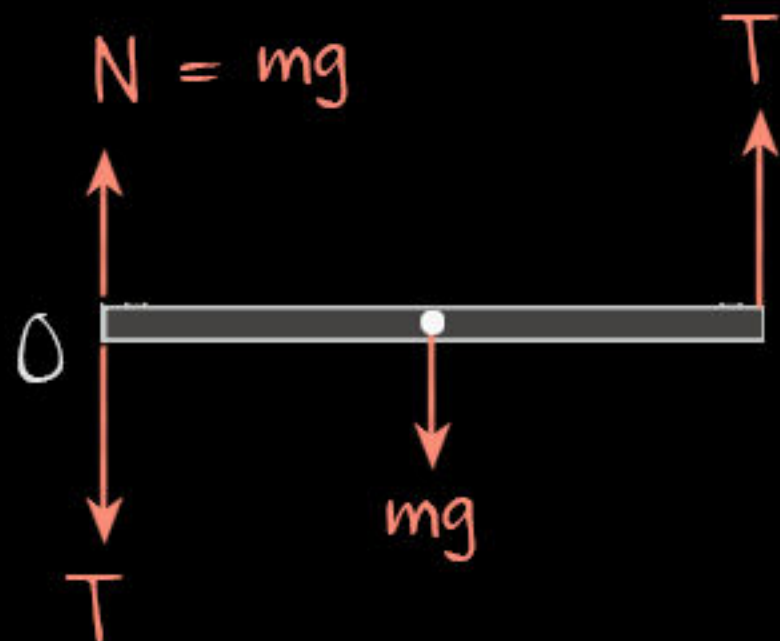
$$\text{From 1 and 2: } m' = (\eta - 1) \frac{m}{2} //$$

Note: Angle is immaterial.
Youtube / Arihant Senior



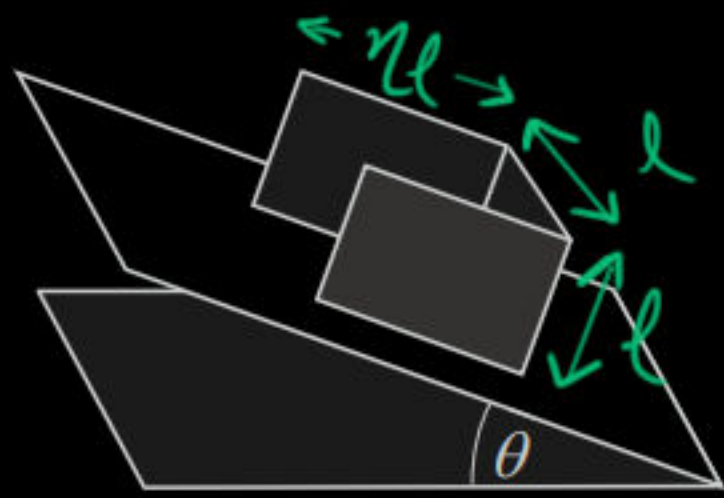
13. A uniform bar is placed on a horizontal **frictionless tabletop**. Two pulleys are rigidly attached at the ends of the bar and two loads A and B of masses m_1 and m_2 are suspended with the help of a light inextensible cord as shown in the figure. The pulleys attached to the bar and on the ceiling are to be considered ideal. **If the bar stays horizontally on the tabletop in equilibrium, find its mass.**

- 1 As rod, and hence pulleys are at rest: $T = \frac{2m_1m_2g}{m_1+m_2}$ (Just like atwood machine)
- 2 Rod will always be in translational equilibrium, as T are couple force.
- 3 When its about to topple, N will pass through O.



$$z_o: Tl = mg \frac{l}{2}$$

$$\Rightarrow m = \frac{2T}{g} = \frac{4m_1m_2}{m_1+m_2} //$$



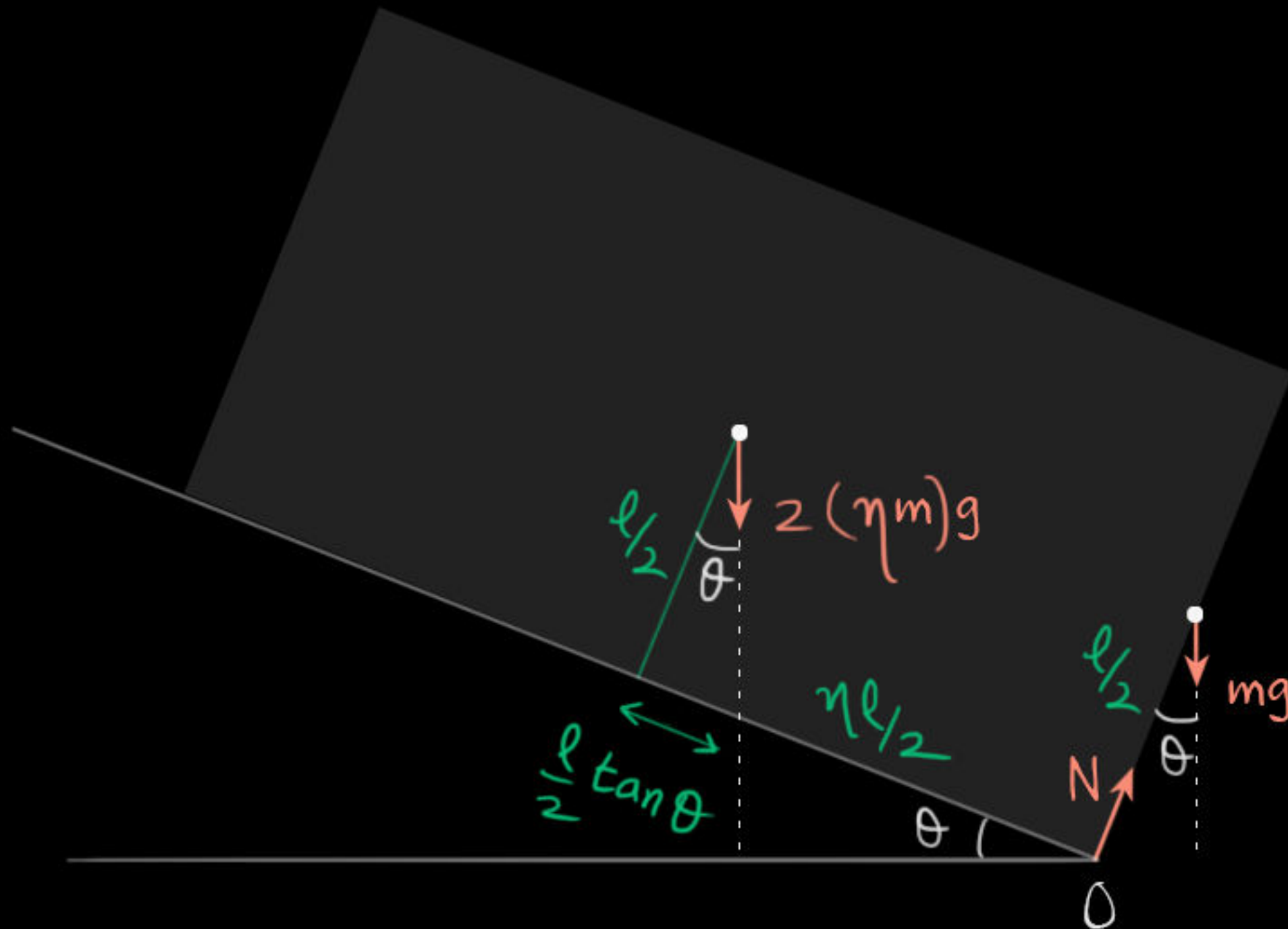
14. A cardboard strip, folded in U shape, is placed on an inclined plane, as shown in the figure. Length of the two parallel sections is $\eta = 1.5$ times of the length of the middle section, that is a square. At what minimum angle of inclination θ of the inclined plane with the horizontal, will the cardboard topple? Friction is sufficient to prevent sliding.

When its about to topple, N will pass through O.

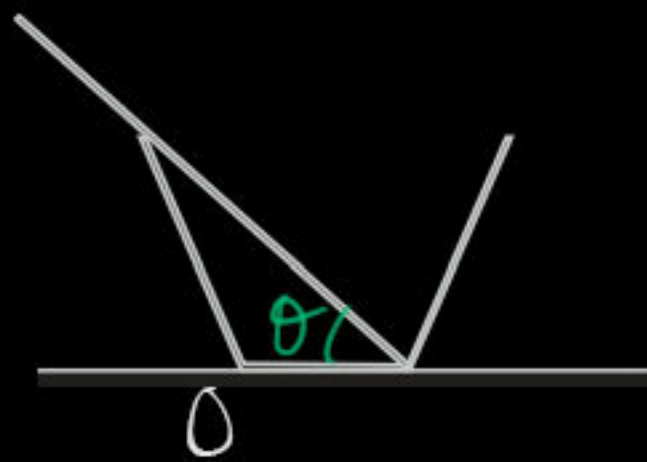
$$\tau_O: 2\eta m/g \left[\frac{\eta l}{2} - \frac{l}{2} \tan \theta \right] \cos \theta = mg \frac{l}{2} \sin \theta$$

$$\Rightarrow 2\eta (\eta \cos \theta - \sin \theta) = \sin \theta$$

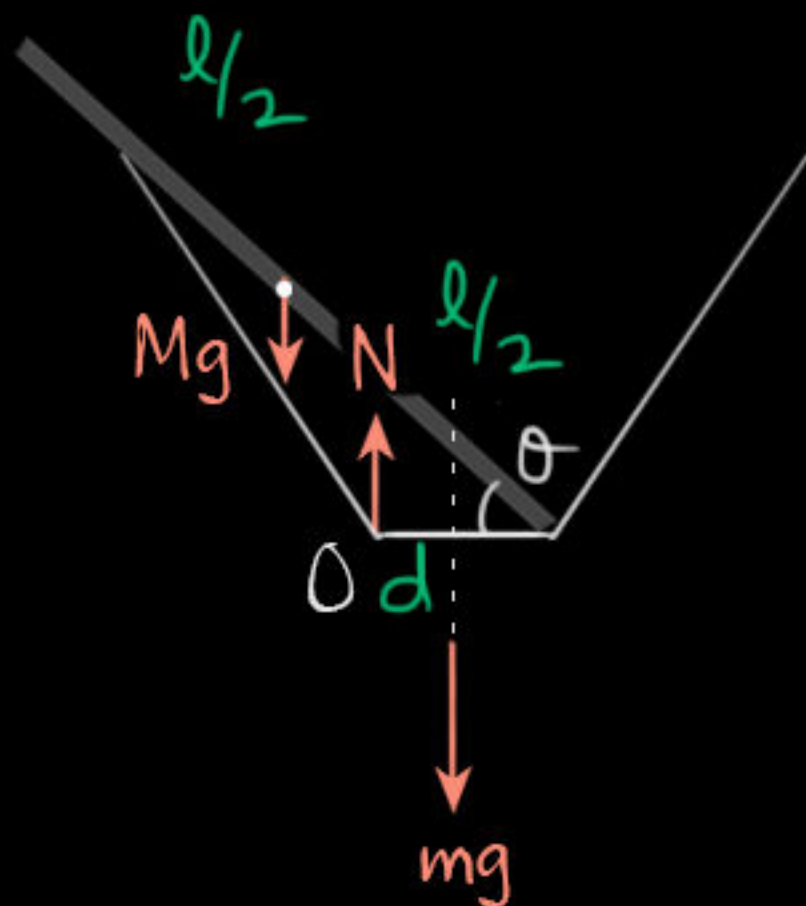
$$\Rightarrow \tan \theta = \frac{2\eta^2}{1+2\eta}$$



Note: Basically, CoM will be above O when about to topple.



When its about to topple,
N will pass through O.



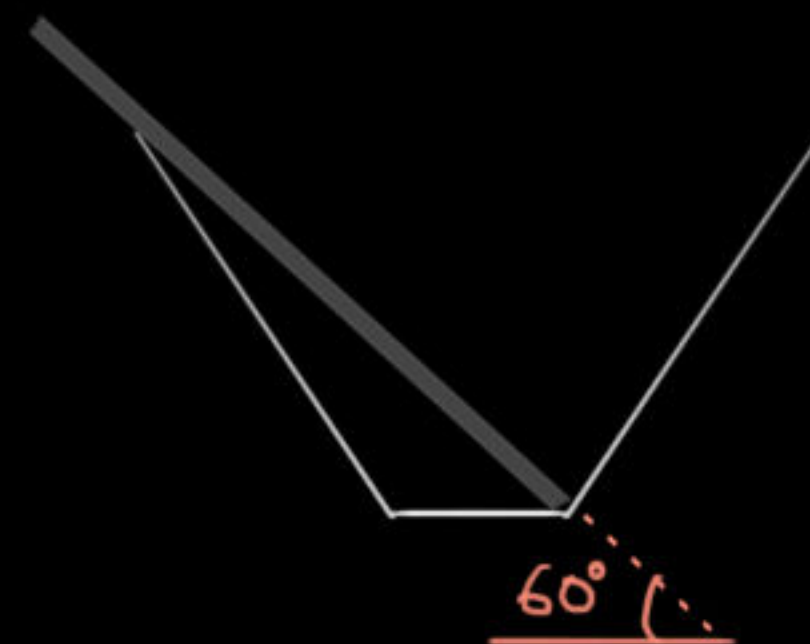
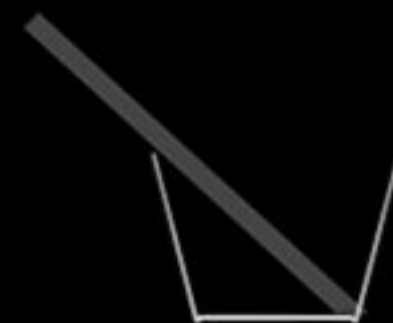
$$\sum \tau_O: Mg \left(\frac{l}{2} \cos \theta - d \right) = mg \frac{d}{2}$$

$$\Rightarrow l = 8d = 40 \text{ cm}$$

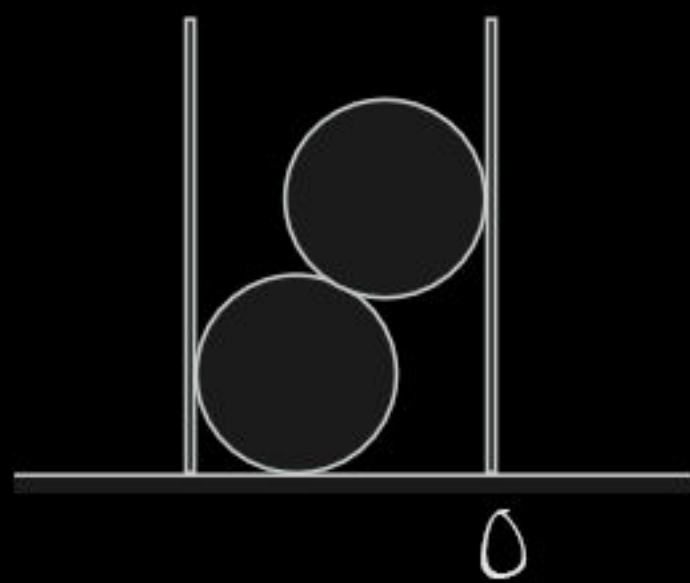
by Ankit Singhvi

15. A cup having shape of a frustum of a cone is placed on a horizontal tabletop. Mass of the cup is $m = 20 \text{ g}$ and diameter of its bottom is $d = 5 \text{ cm}$. A uniform rod of mass $M = 10 \text{ g}$ is placed in the cup as shown. The rod is inclined at an angle $\theta = 60^\circ$ to the horizontal. Find maximum length l of the rod so that the system will not turn over.

Assume: slant side of glass is big enough for rod to not topple by itself!



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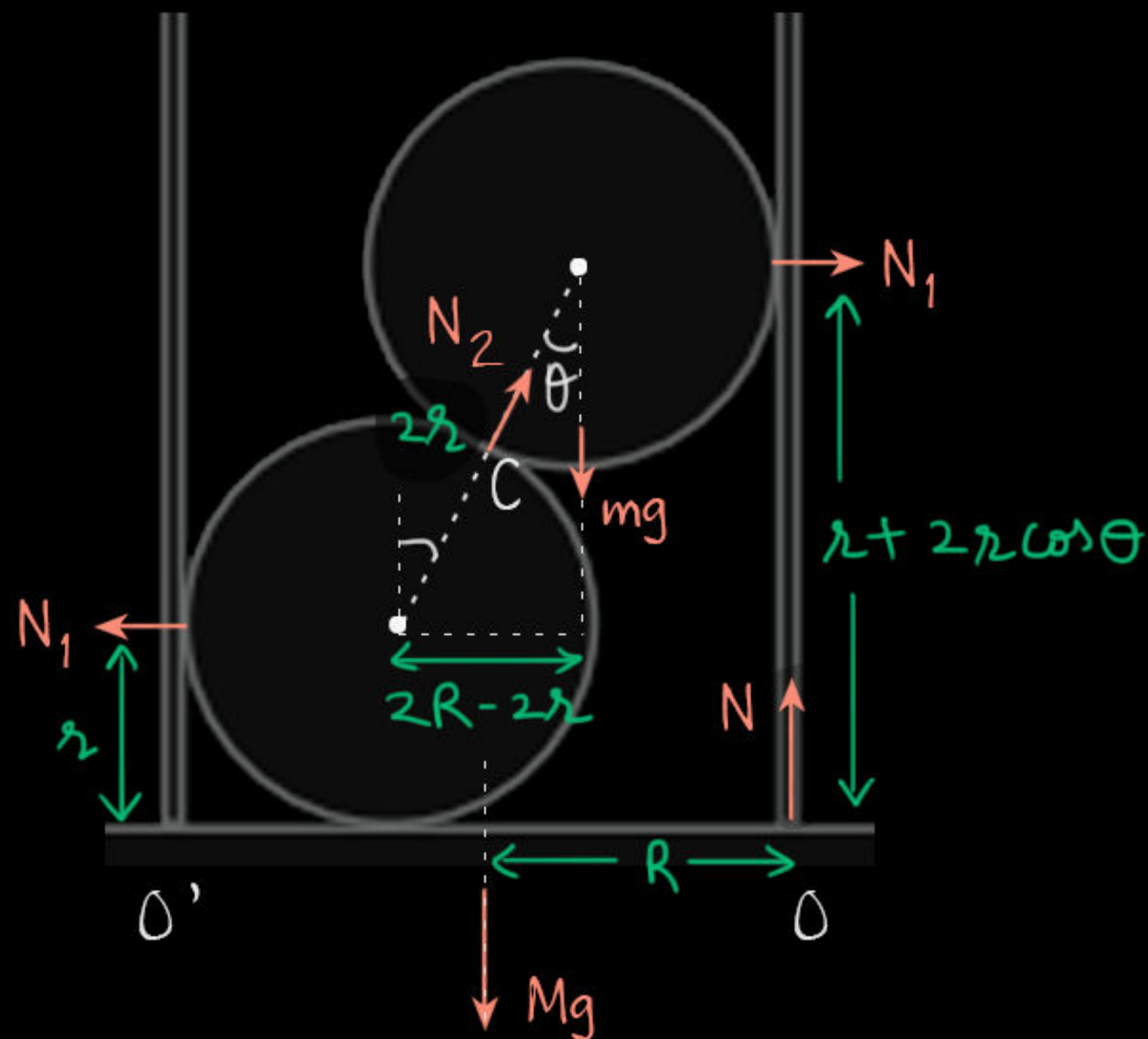


16. A cylindrical pipe of radius R and mass M is placed vertically on a horizontal floor. Two identical spheres each of radius r and mass m are inserted in the cylinder as shown in the figure. Radius of the spheres is more than $0.5R$. At what minimum value of m/M will the arrangement topple? There is no friction between the spheres and between a sphere and inner wall of the cylinder.

It won't have a base.

Concept: It will topple about O.

Hence, N from ground will pass through O.



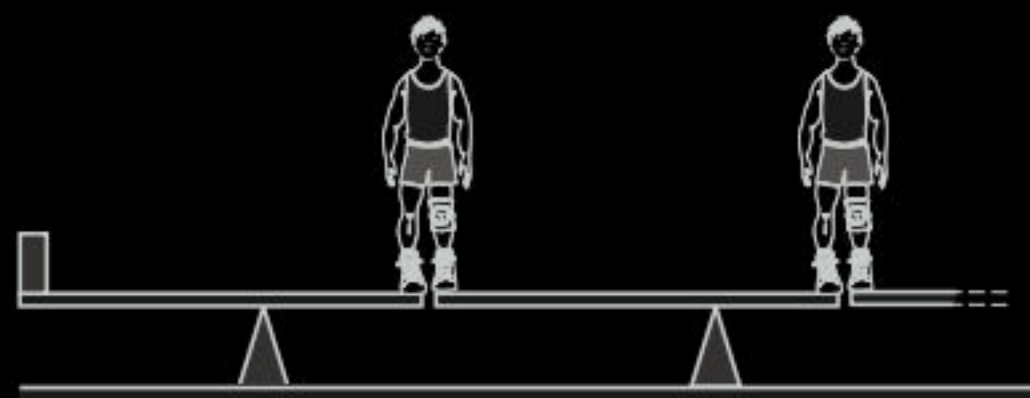
$$N_2 \cos \theta = mg$$

$$N_1 = N_2 \sin \theta \Rightarrow N_1 = mg \tan \theta \quad (1)$$

$$\sin \theta = \frac{2R - 2r}{2r} = \frac{R - r}{r} \quad (2)$$

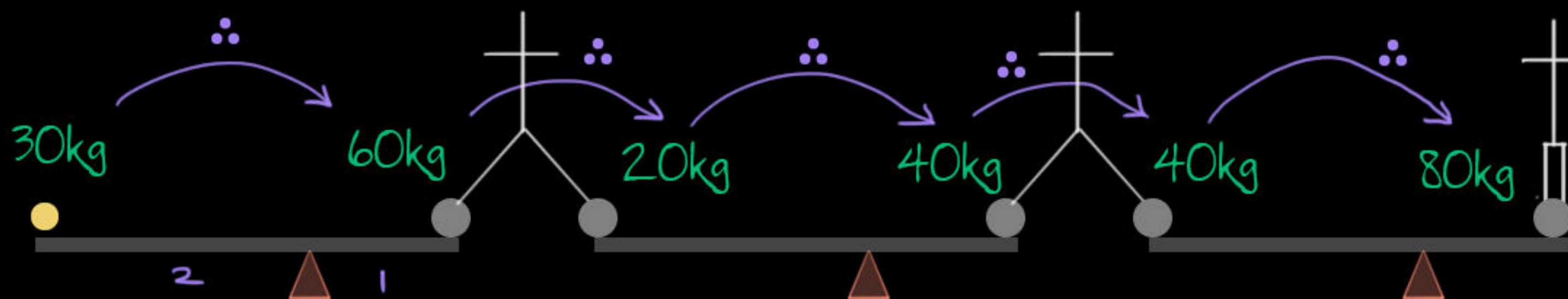
$$\text{To: } N_1 (r + 2r \cos \theta) > N_1 r + MgR \quad (3)$$

From 1, 2, 3: $\frac{m}{M} > \frac{R}{2(R-r)}$
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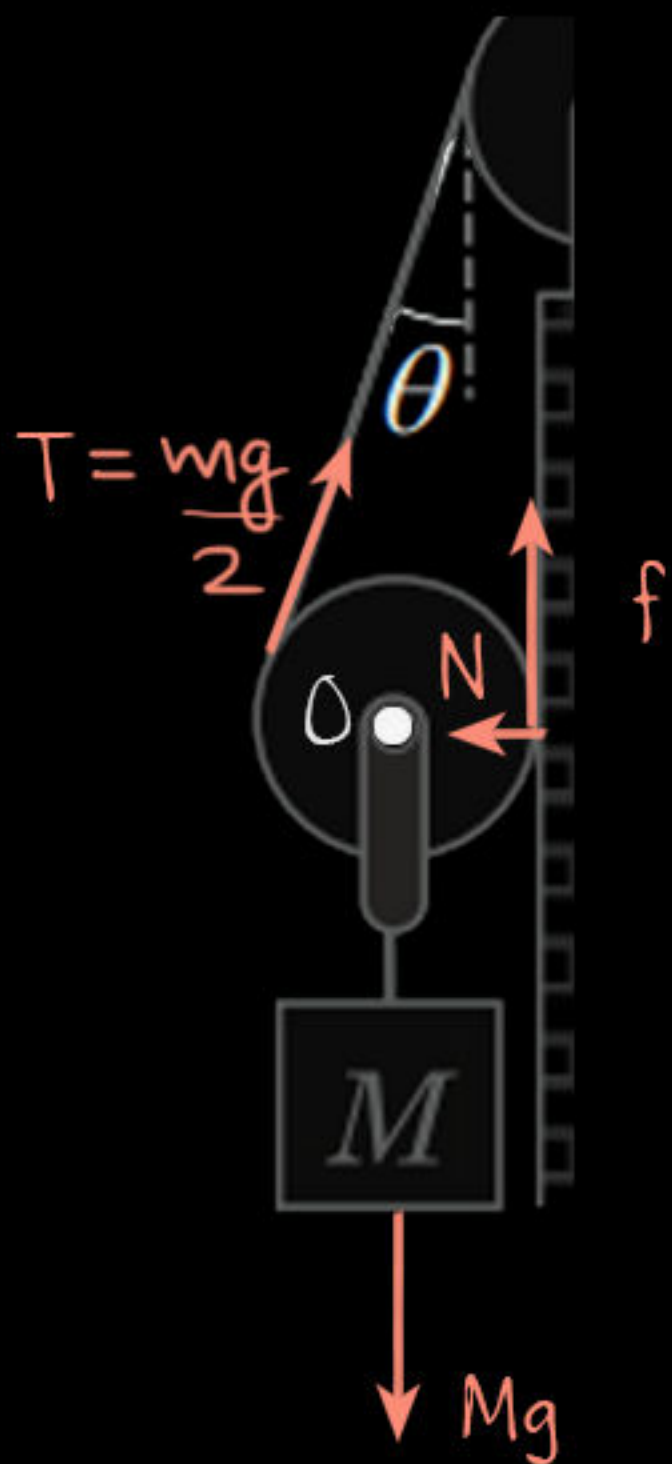
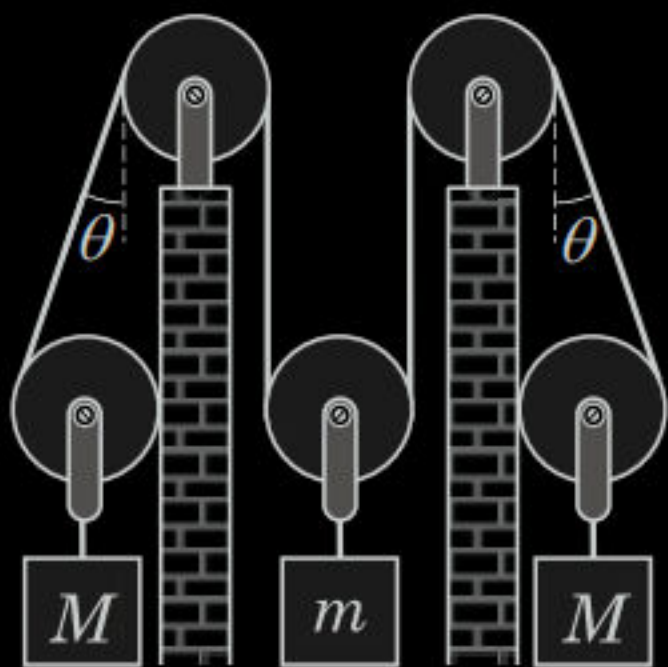


17. In a circus show, several clowns stand on wooden boards arranged in a queue. Each board can rotate about a **fixed fulcrum that divides its length in ratio 2:1**. A portion of the scheme is shown in the figure. A **small load of mass 30 kg** is placed on the end of the leftmost board and the clowns stand keeping their feet at the ends of adjacent boards. If **mass of a clown is 80 kg** and that of a board is negligible, **find the maximum number of clowns that can keep balance in this way.**

distribute weight between feet
(it can be unequal).



$\therefore 3$ can be supported



18. In the arrangement shown three loads are suspended in **equilibrium** from three pulleys. The segment of the rope between each of the uppermost pulleys and the outermost pulleys leans at **angle θ** with the vertical.

- What is the **ratio of masses $m:M$** ?
- Find **range of coefficient of friction** between the walls and the outermost pulleys.
- What is the **nature of the equilibrium**?

$$\Sigma \tau_O : f \cancel{x} = \frac{mg}{2} \cancel{x}$$

$$\Rightarrow f = mg/2$$

$$\textcircled{A} \quad Mg = \frac{mg}{2} \cos \theta + f \quad \leftarrow \frac{mg}{2}$$

$$\Rightarrow \frac{m}{M} = \frac{2}{1 + \cos \theta} //$$

$$\textcircled{B} \quad f = \frac{mg}{2} < \mu N \quad \leftarrow \frac{mg}{2} \sin \theta$$

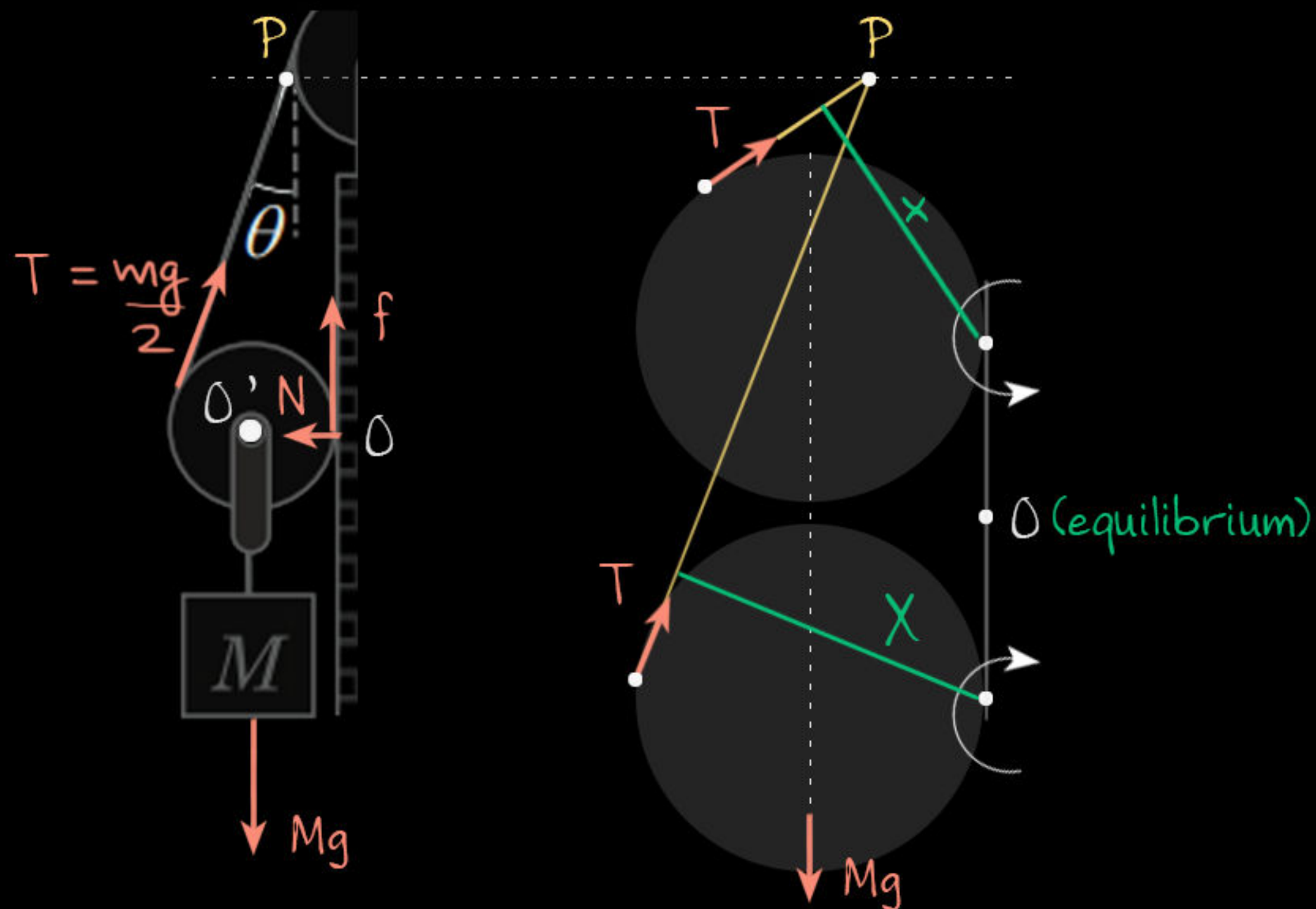
$$\Rightarrow \mu > \frac{1}{\sin \theta} //$$

C

Torque about O due to Mg is constant.

So if torque due to T is restoring in nature, its stable equilibrium.

By geometry: $X > x$



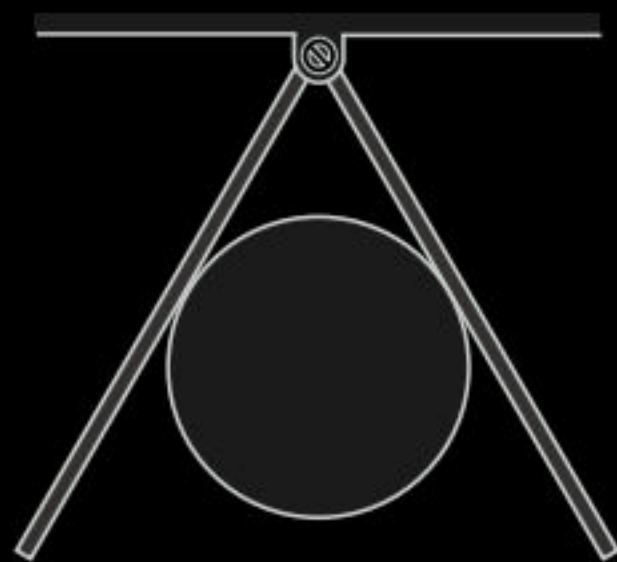
Case A Its moved up from equilibrium.

Torque due to T reduces (as x reduces), so restoring CCW torque will appear.

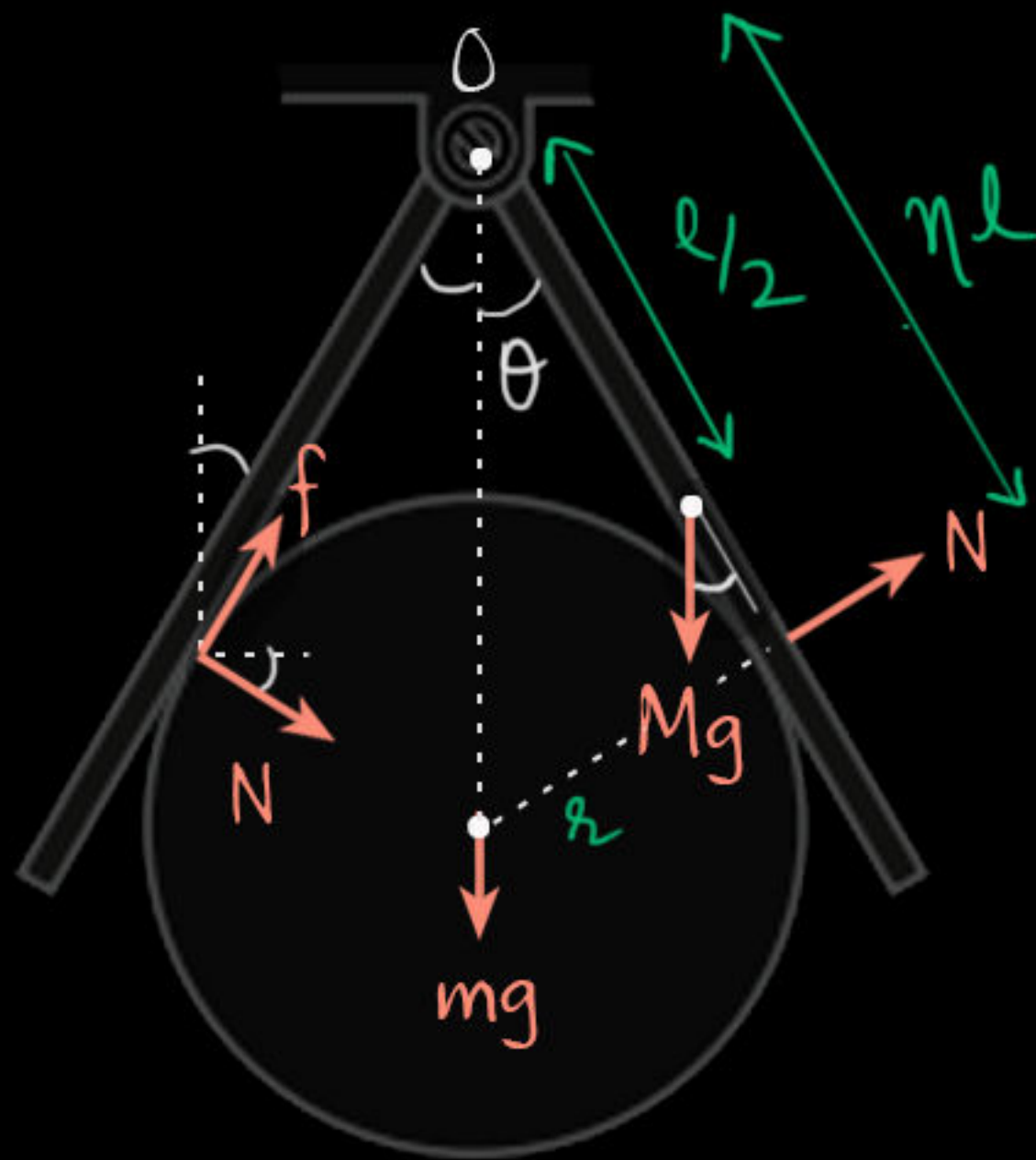
Case B Its moved down from equilibrium

Torque due to T increases (as X increases), so restoring CW torque will appear.

Hence, its in stable equilibrium.



19. Upper edges of two identical uniform planks each of mass $M = 87$ kg and length $l = 1.0$ m are hinged to a horizontal axle affixed to the ceiling. A cylinder of mass $m = 20$ kg and radius $r = 0.30$ m is inserted horizontally between the planks as shown. The cylinder touches the planks at a distance $\eta = 0.75$ times the length of the plank from the hinge. Find range of coefficient of friction between the planks and the cylinder to keep the system in equilibrium.



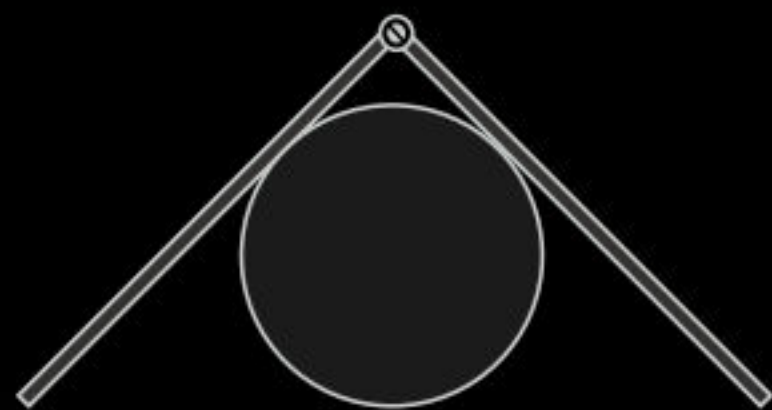
$$\tan \theta = r / \eta l \quad (1)$$

$$\tau_O : Mg \frac{l}{2} \sin \theta = N \eta l \quad (2)$$

$$F_{\text{cylinder}} : 2f \cos \theta = mg + 2N \sin \theta$$

$$\Rightarrow f = \frac{mg + 2N \sin \theta}{2 \cos \theta} < \mu N \quad (3)$$

$$\text{From 1, 2, 3: } \frac{m}{M \eta l} (r^2 + \eta^2 l^2) + \frac{r}{\eta l} < \mu$$



20. A structure consisting of two identical square plates of side $l = 40$ cm hinged to a common axle at one of their edges is placed on a fixed horizontal cylinder of radius $r = 10$ cm as shown in the figure. There is no friction between the cylinder and the plates. At what angle with vertical will the plates lean in equilibrium?

On one rod τ_O : $mg \frac{l}{2} \sin \theta = N r \cos \theta$

On both rods F_y : $2mg = 2N \sin \theta$

Dividing: $l \sin^3 \theta = 2r \cos \theta$

$\downarrow l = 40, r = 10$

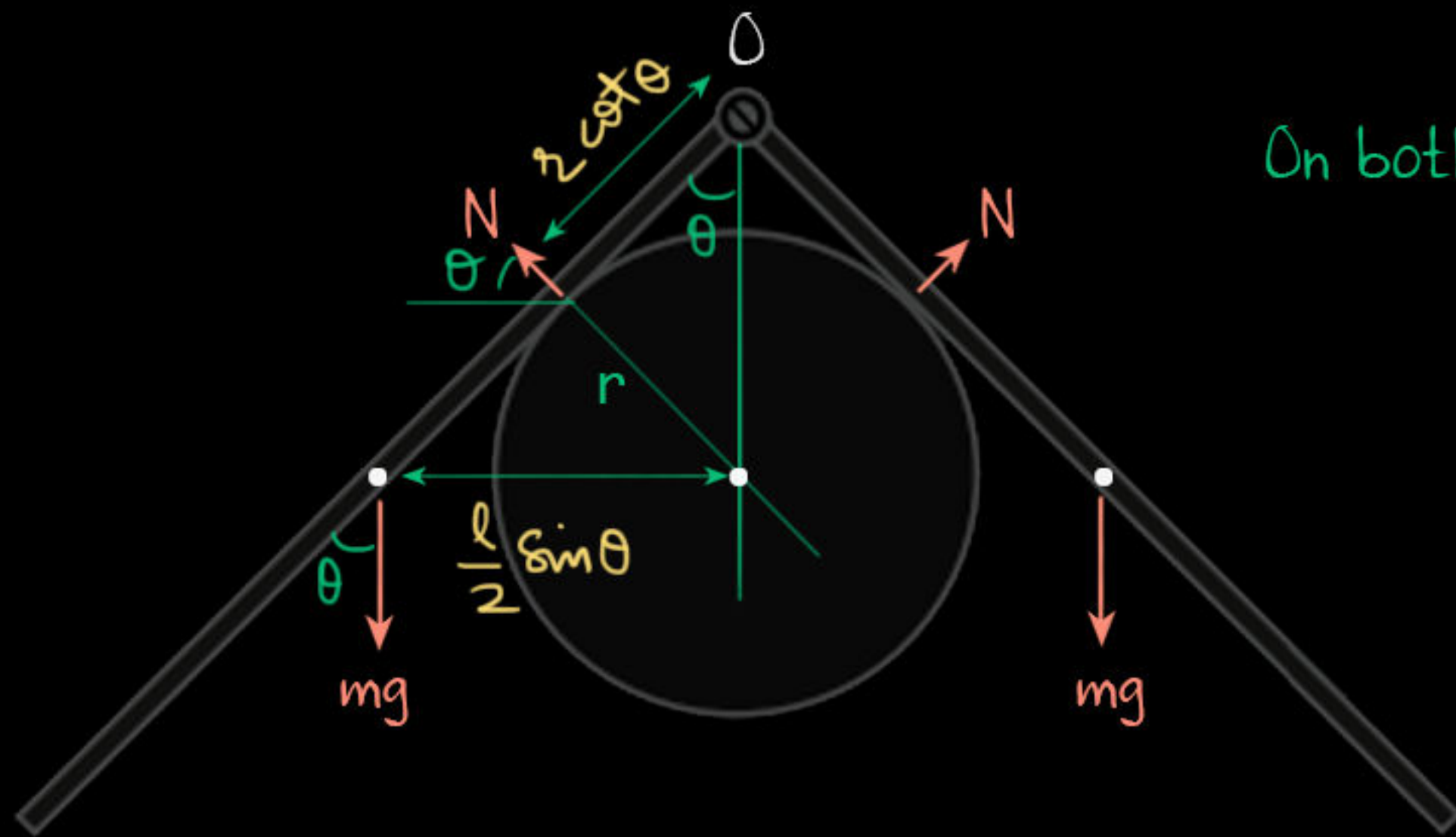
$2 \sin^3 \theta = \cos \theta$

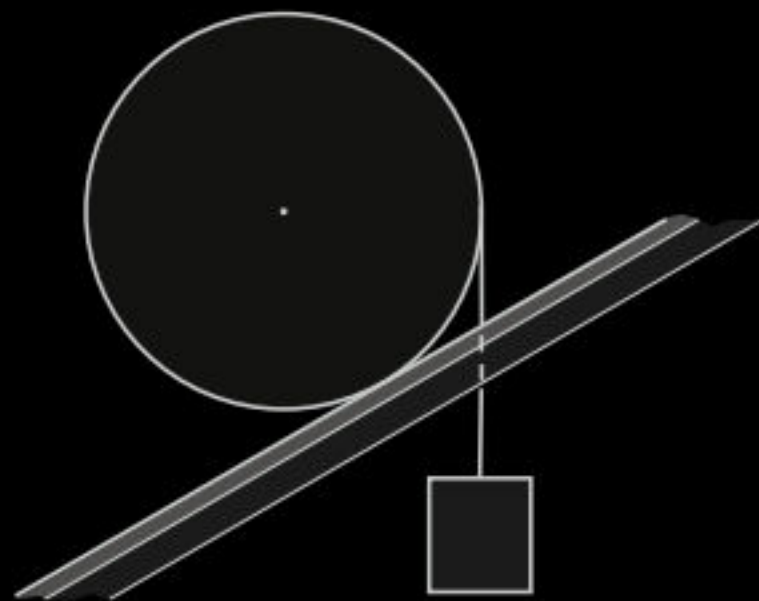
$\Rightarrow \cos^3 \theta + \cos \theta - 2 = 0$

$\Rightarrow (\cos \theta - 1)(\cos^2 \theta + \cos \theta + 2) = 0$

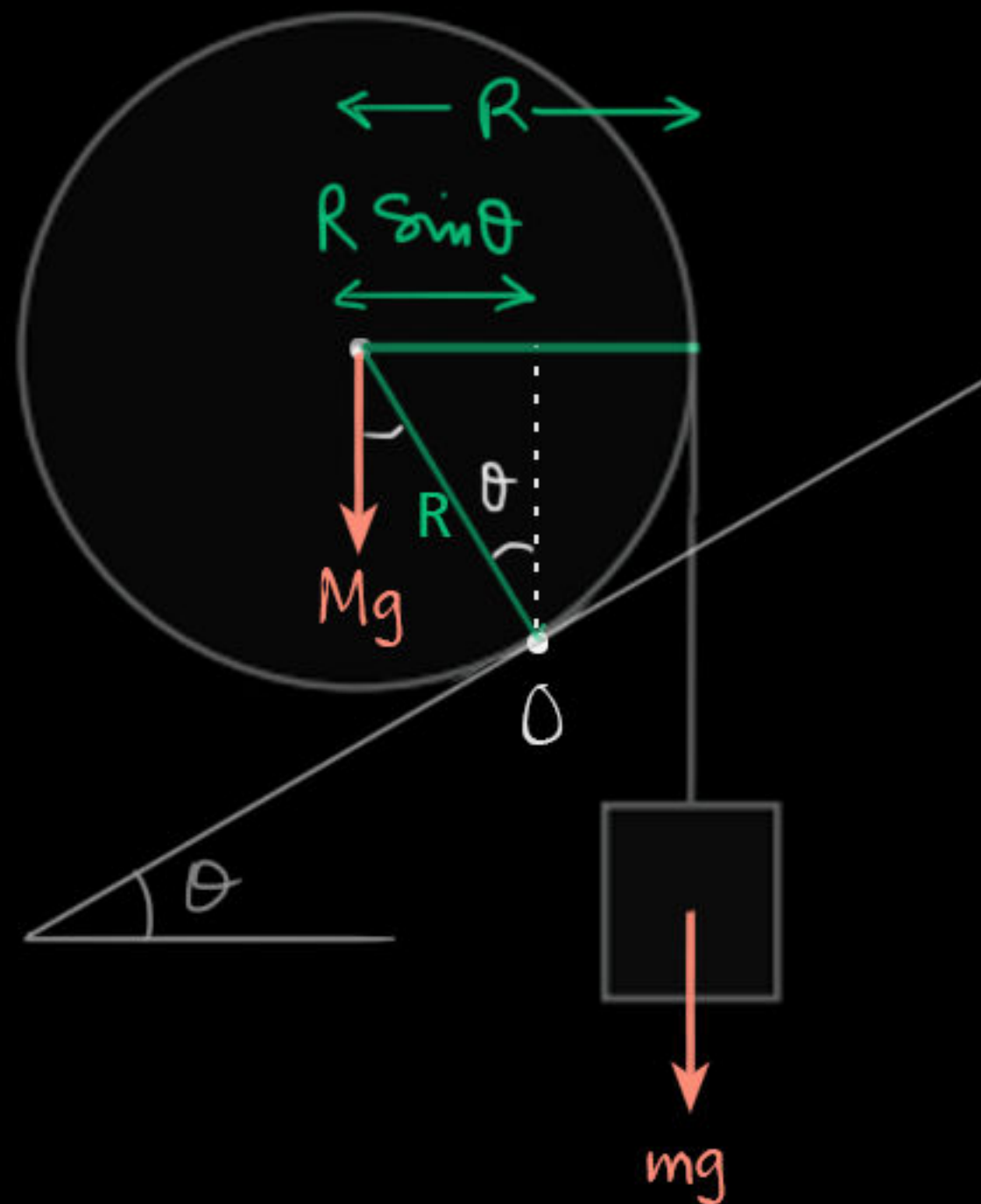
$\Rightarrow \cos \theta = 1$

$\Rightarrow \theta = 45^\circ //$





21. A cylinder of mass $M = 2.0 \text{ kg}$ is placed on two parallel rails inclined at an angle $\theta = 37^\circ$ to the horizontal (side view shown in the figure). A load of unknown mass is suspended from a light inextensible cord wound on the cylinder. The cylinder is held initially at rest and then released. If the cylinder starts rolling up the rails without slipping, what should mass of the load be?



It will roll up, if torque about O is CW.

$$\therefore mg[R - R \sin \theta] > MgR \sin \theta$$

$$\Rightarrow m > \frac{M \sin \theta}{1 - \sin \theta}$$

22. A thin uniform rod of mass m stays in equilibrium on the top of a fixed horizontal cylinder perpendicular to the axis of the cylinder. Length of the rod is equal to circumference of the cylinder. A beetle of mass $0.2m$ starts crawling slowly from the centre of the rod and reaches an end of the rod. As the beetle crawls on the rod, the rod rotates slowly without slipping on the cylinder. Find coefficient of friction between the cylinder and the rod for which the above movement is possible.

Eventually, CoM of Rod and man lies here at the touching point as system is at equilibrium. $\Rightarrow$

$\therefore$ To not fall: $\mu > \tan \theta$ 3

$\Rightarrow \theta = \frac{x}{r}$ 2

As there is no slipping, distance moved on the curved surface by point of contact = distance between CoM of rod and final point of contact.

From 1, 2: $\theta = \pi/6$

$\therefore$ From 3: $\mu > \frac{1}{\sqrt{3}}$

by Ankit Singhvi

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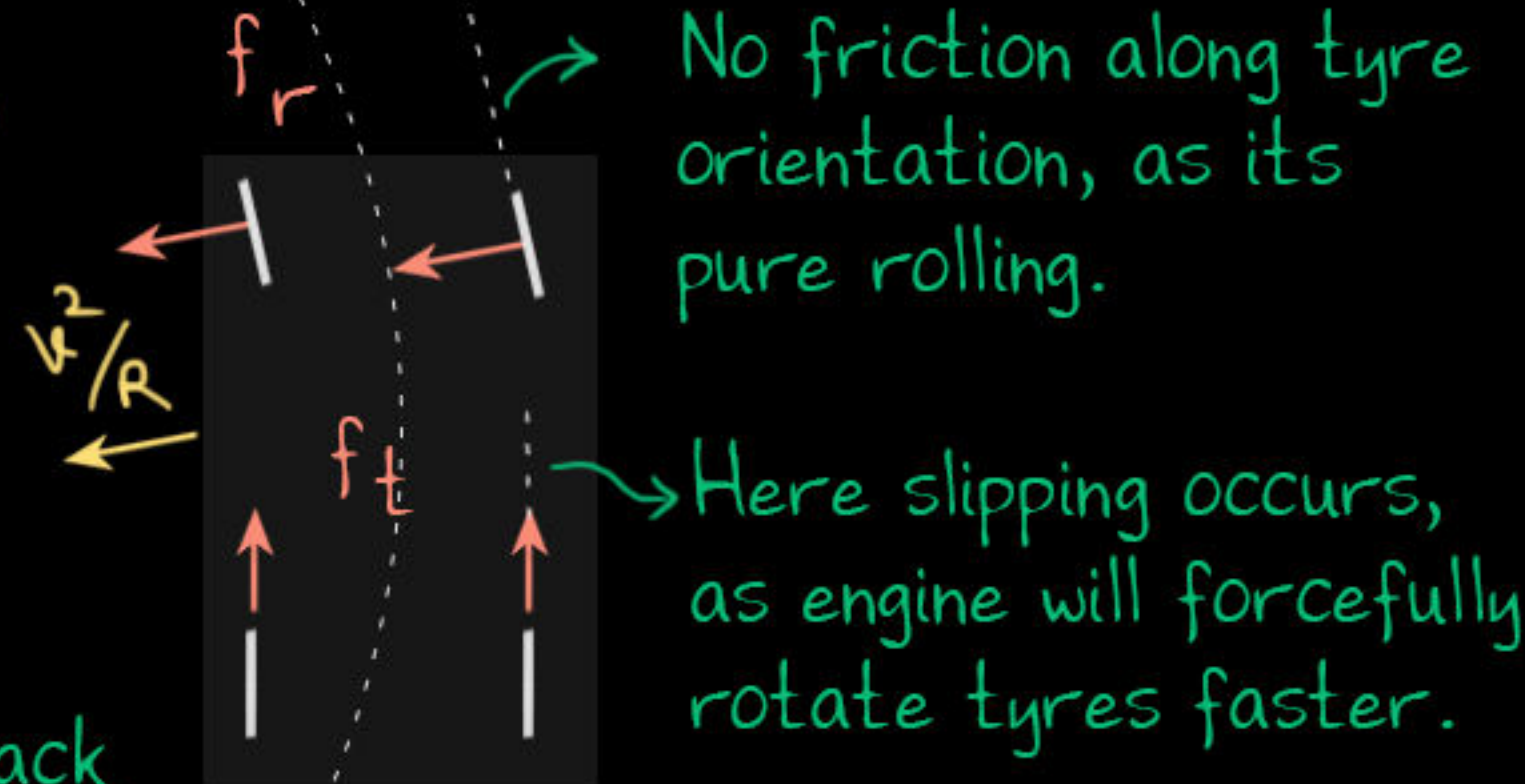
23. A rear wheel drive car moves with a constant speed $v = 20$ m/s on a circular path of radius $r = 200$ m on a horizontal ground. Centre of gravity of the car is close to the ground and equidistant from all the four wheels that have negligible mass as compared to rest of the car.

- (a) What should the minimum coefficient of friction between the tyres and the ground be to make this movement of the car possible?
- (b) If coefficient of friction is made double, in how much minimum time can the driver of the car increase speed from 20 m/s to 20.5 m/s on the circular path?

Rear wheel drive mechanics:

Concept: Centripetal (radial) acceleration is provided by front tyres friction.

Forward (transverse) acceleration is provided by back tyres friction.



A $f_t = 0$
 $f_r = \frac{mv^2}{R} < \mu N$

$\therefore N$ on front two tyres is $mg/2$.

$\Rightarrow \mu > \frac{v^2}{Rg} = 0.4$

B ✓ μ doubles.

✓ v increases marginally.

✓ Path radius is same.

$\therefore$ So we can safely assume centripetal friction will be sufficient and won't reach limiting value.

$f_t \neq 0$ $\frac{\mu' N}{m} = (\cancel{2\mu}) \frac{mg}{2} \frac{1}{m}$

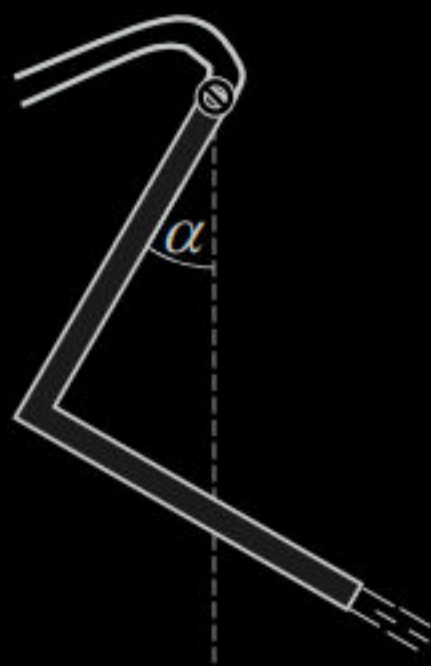
$v = u + a_{\max} t_{\min}$

$\Rightarrow 20.5 = 20 + 4t$

$\Rightarrow t = 0.125$

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24. A metal pipe bent at right angle at its midpoint is pivoted to a fixed support at one of its ends so that it can rotate freely in its plane about a horizontal axis through the pivoted end. A flexible hose is connected at the pivoted end. When water is fed at a particular rate into the pipe, the upper arm of the pipe makes an angle α with the vertical as shown in the figure. If the rate of feeding water is gradually made η times, find an expression for the angle β , which the upper arm of the pipe will make with the vertical.

Concept: $F_{\text{reaction}} \propto v^2 \Rightarrow \tau_{\text{reaction}} \propto v^2$

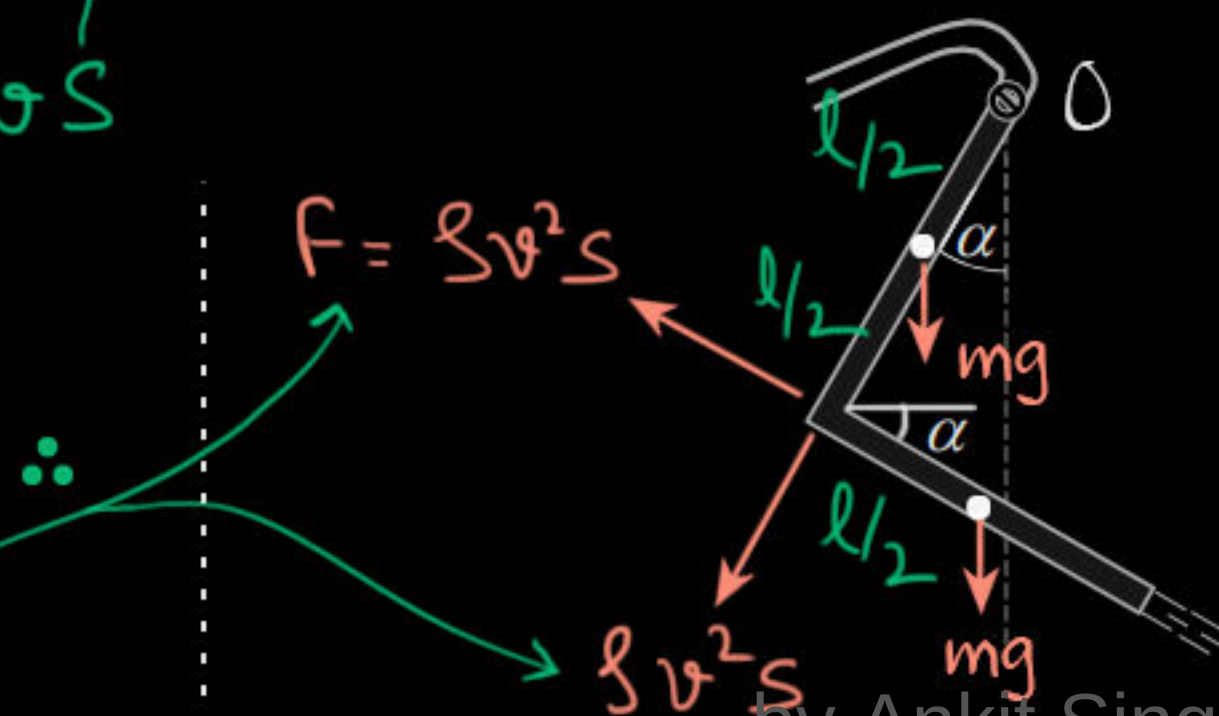
Proof:

$$F_{\text{reaction}} = \frac{dp}{dt} \quad \text{--- } dm(v-0)$$

$$= v \frac{dm}{dt} \quad \text{--- } \rho dV$$

$$= v \rho \frac{dV}{dt} \quad \text{--- } \text{Cross section area}$$

$$= \rho v^2 S$$



Sol: At equilibrium, τ_0 at a general angle θ :

$$\rho v^2 S l = mg \frac{l}{2} \sin \theta + mg \left(l \sin \theta - \frac{l}{2} \cos \theta \right)$$

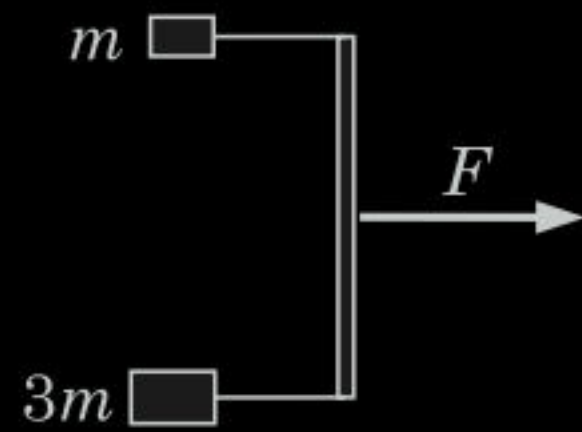
$$\Rightarrow 2 \rho v^2 S = mg (3 \sin \theta - \cos \theta)$$

$$\text{@ } \alpha \quad 2 \rho v^2 S = mg (3 \sin \alpha - \cos \alpha)$$

$$\text{@ } \beta \quad 2 \rho (\eta v)^2 S = mg (3 \sin \beta - \cos \beta)$$

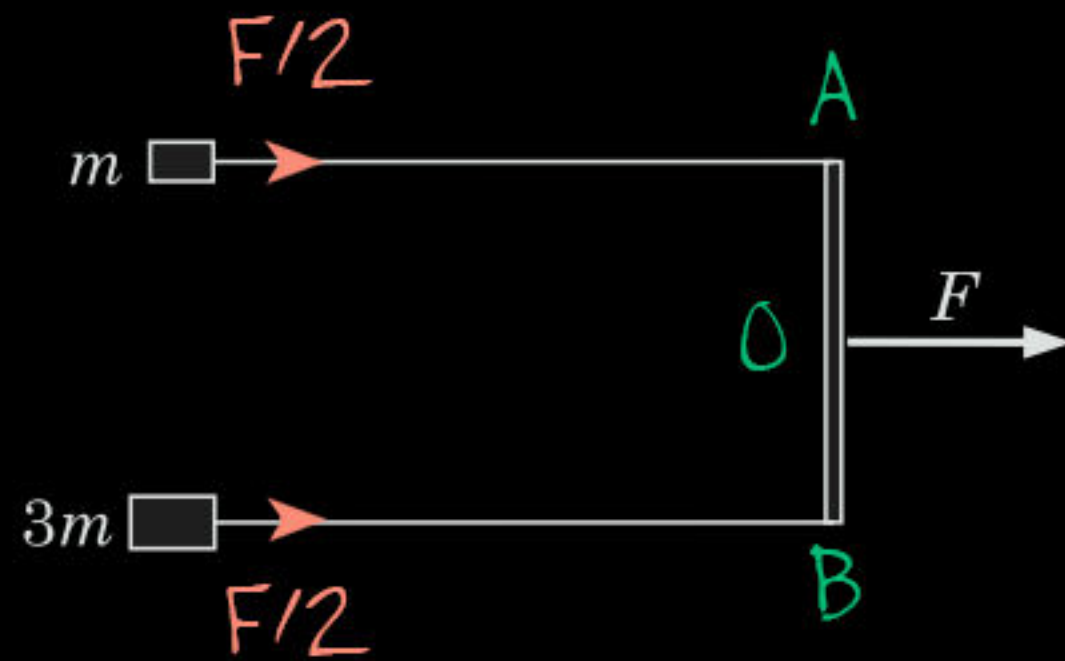
$$\text{dividing} \quad 3 \sin \beta - \cos \beta = \eta^2 (3 \sin \alpha - \cos \alpha)$$

$$\Rightarrow \sin(\beta - \tan^{-1} 1/3) = \eta^2 \sin(\alpha - \tan^{-1} 1/3)$$



25. A light rigid rod is placed on a frictionless horizontal floor and blocks of masses of m and $3m$ are attached to the ends of the rod with the help of light inextensible cords. If a horizontal force F is applied perpendicular to the rod at its midpoint as shown in the figure, what will acceleration of the midpoint of the rod be?

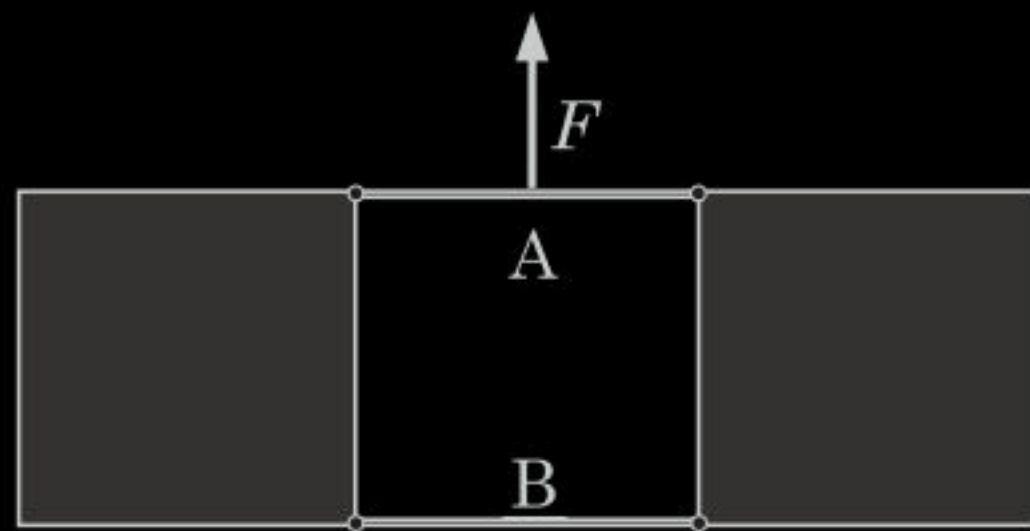
Concept: As net force or torque on rod is zero. So Tension will be $F/2$ each.



$$a_A = a_m = \frac{F}{2m}$$

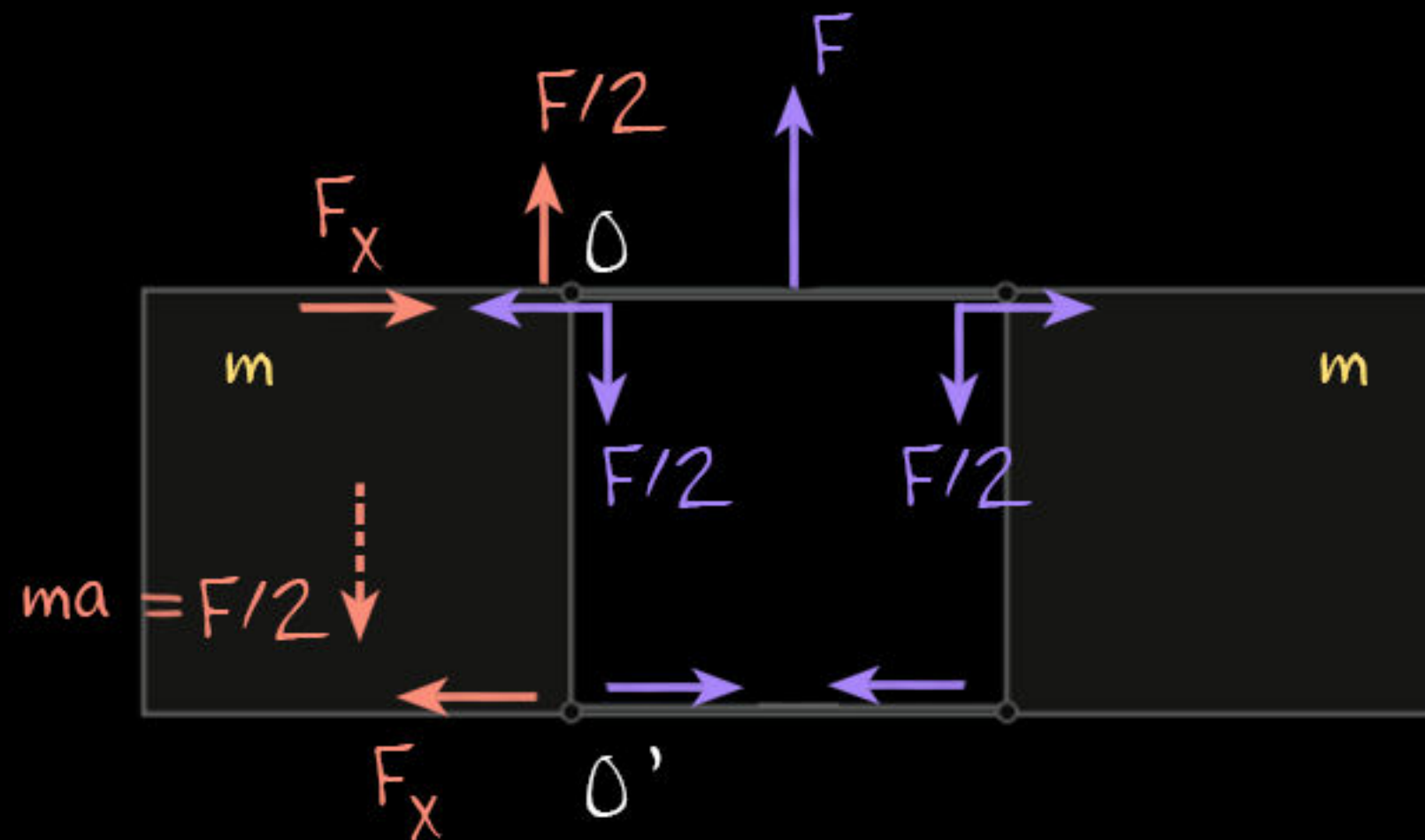
$$a_B = a_{3m} = \frac{F}{6m}$$

$$\therefore a_O = \frac{a_A + a_B}{2} = \frac{\frac{F}{2m} \left(1 + \frac{1}{3}\right)}{2} = \frac{F}{3}$$



26. Two identical square laminae connected by two rods A and B are placed on a frictionless horizontal floor with their flat faces on the floor. The rods are free to rotate at the joints where they are connected with the laminae. The rod A is pulled horizontally at its midpoint perpendicular to the rod by a constant force F as shown in the figure. Find moduli of the reaction forces F_A and F_B between one of the laminae and the rods A and B respectively. Consider the following cases.

A $\uparrow a = \frac{F}{2m}$

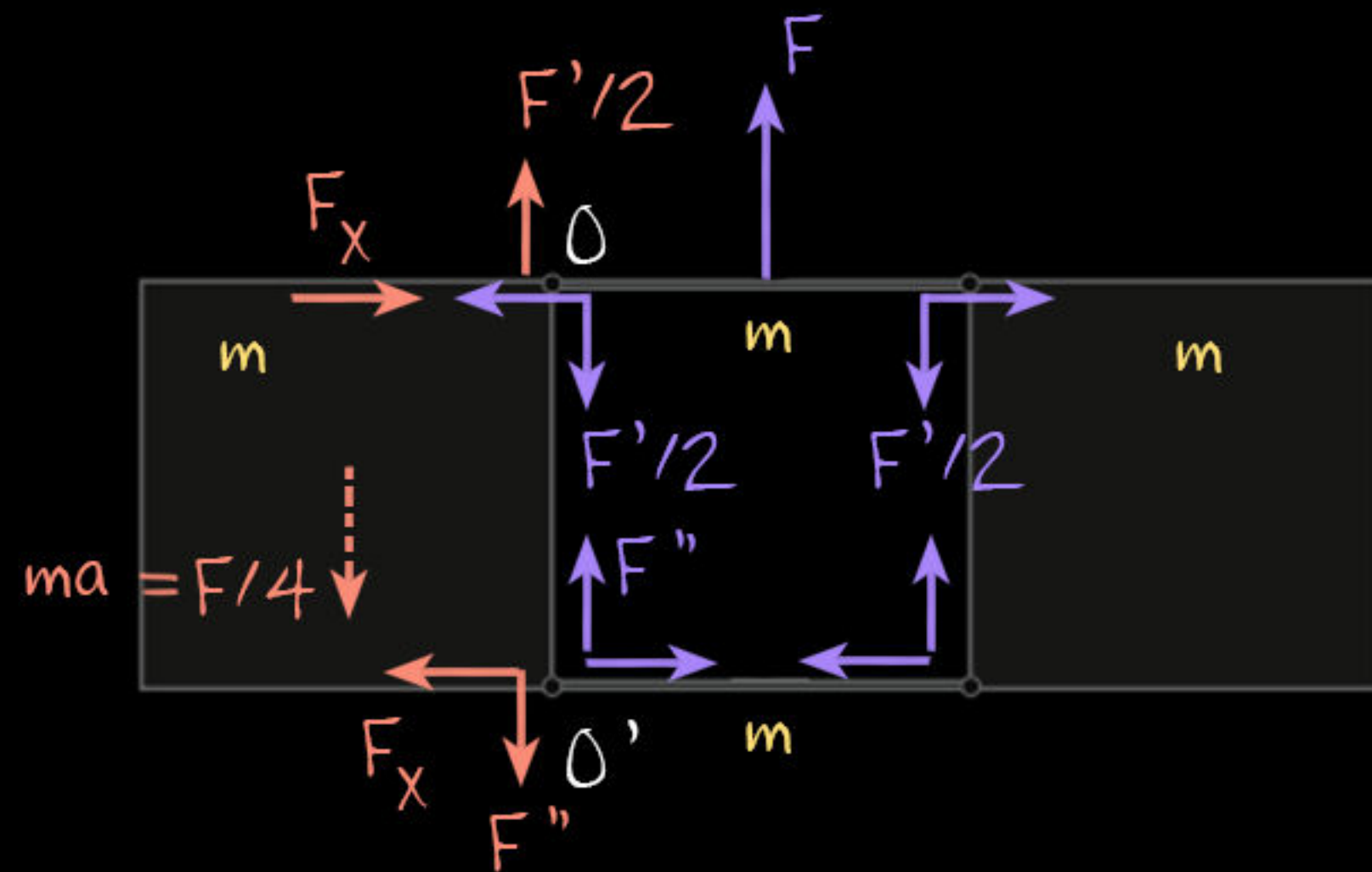


Forces on rod are shown such that, net force and torque on each of them is zero.

On plate, τ_0 : $\frac{F}{2} \cdot \frac{l}{2} = F_x l$
 $\Rightarrow F_x = F/4$

$F_0 = \sqrt{\left(\frac{F}{2}\right)^2 + F_x^2} = \sqrt{\frac{5}{4}} F$
 $F'_0 = F_x = F/4$

B $a = \frac{F}{4m}$



On top rod, F_y : $F - F' = ma$
 $\Rightarrow F' = F - m\left(\frac{F}{4m}\right)$
 $\Rightarrow F' = 3F/4$

On plate, τ_o : $\frac{F}{4} \cdot \frac{l}{2} = F_x l$
 $\Rightarrow F_x = F/8$

On bottom rod, F_y : $2F'' = ma$
 $\Rightarrow F'' = \frac{1}{2}m\left(\frac{F}{4m}\right) = \frac{F}{8}$

$$F_o = \sqrt{\left(\frac{F'}{2}\right)^2 + F_x^2} = \sqrt{\left(\frac{3F}{4 \cdot 2}\right)^2 + \left(\frac{F}{8}\right)^2} = \frac{\sqrt{10} F}{8} //$$

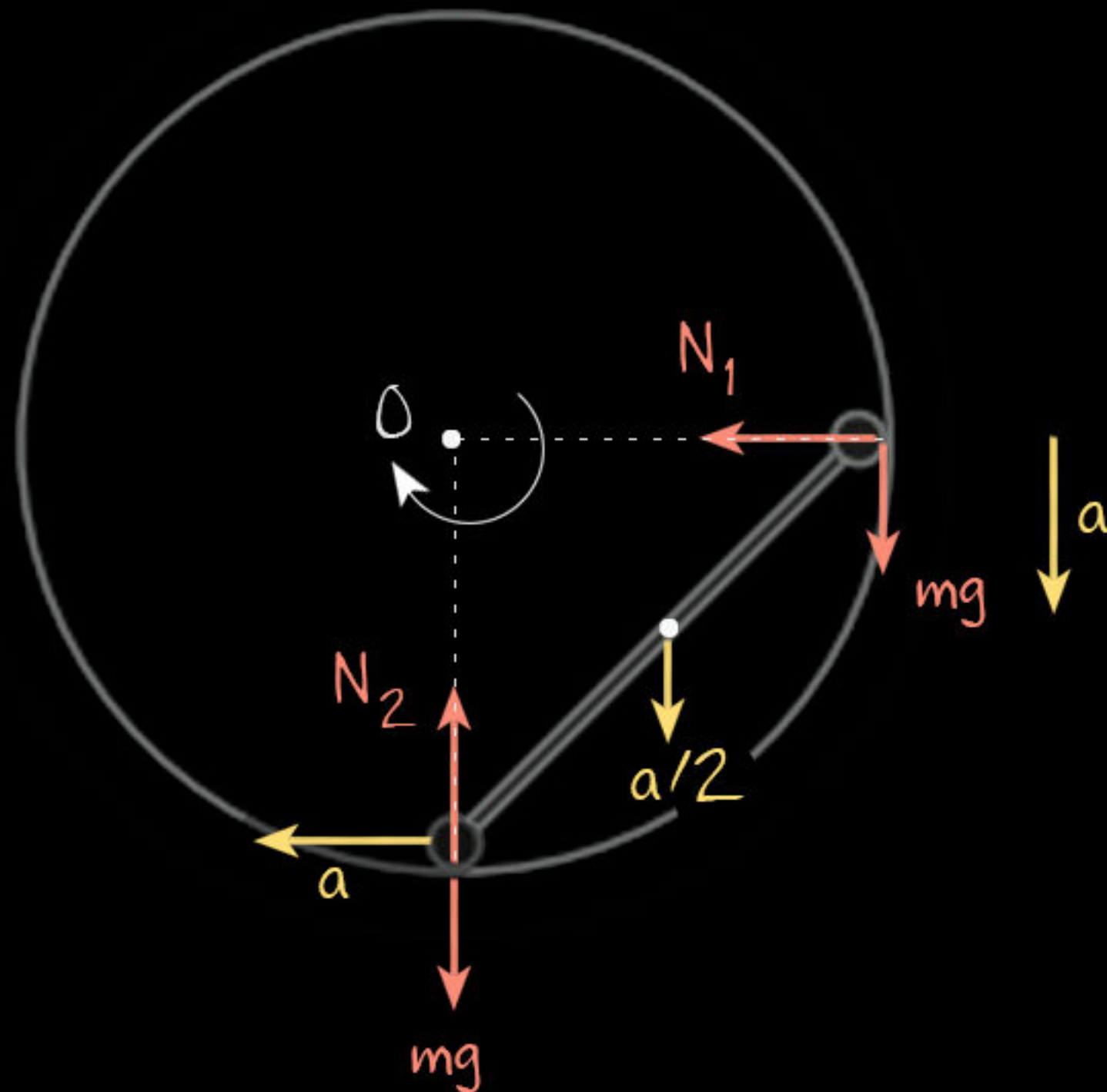
$$F_o' = \sqrt{F''^2 + F_x^2} = \sqrt{\left(\frac{F}{8}\right)^2 + \left(\frac{F}{8}\right)^2} = \frac{\sqrt{2} F}{8} //$$



27. Inside a fixed **frictionless** spherical cavity of **radius r** , a dumbbell is held touching the bottom of the cavity at its lower end. The dumbbell consists of two identical small **balls each of mass m** affixed at the ends of a **light rod of length $r\sqrt{2}$** . **What will the force of interaction between the lower ball and the cavity be immediately after the dumbbell is released?**

Three variables: N_1 N_2 a

But by writing two equations for τ_0 and F_y , we won't need N_1 at all.



$$\tau_0 = I\alpha$$

$$\Rightarrow \cancel{mg/2} = (\cancel{2m} \cancel{r^2}) \frac{a}{\cancel{2}} \Rightarrow a = g/2 \quad (1)$$

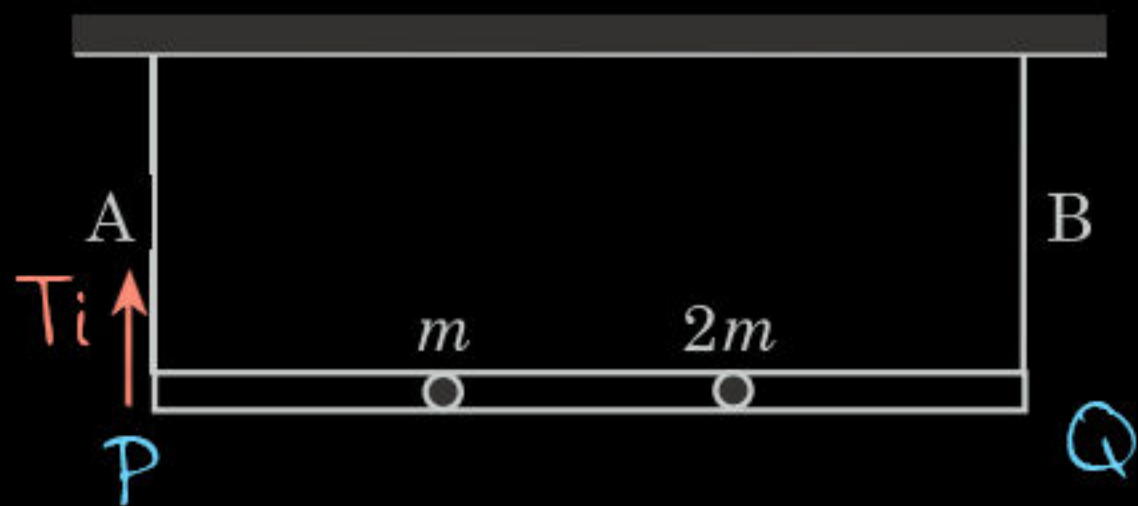
$$F_y = 2m a_{cm}$$

$$\Rightarrow 2mg - N_2 = 2m a/2 \quad (2)$$

From 1,2:

$$N_2 = 3mg/2$$

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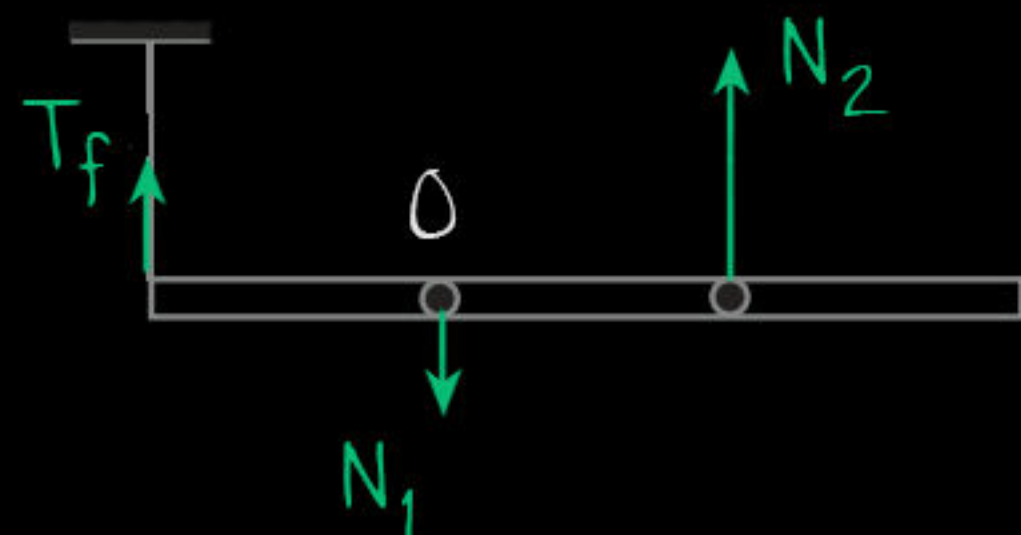


$$\tau_Q: T_i (3l) = mg \cdot 2l + 2mg \cdot l$$

$$\Rightarrow T_i = \frac{4mg}{3}$$

After B is cut:

FBD of light pipe:



$$F: T_f + N_2 = N_1 \quad (1)$$

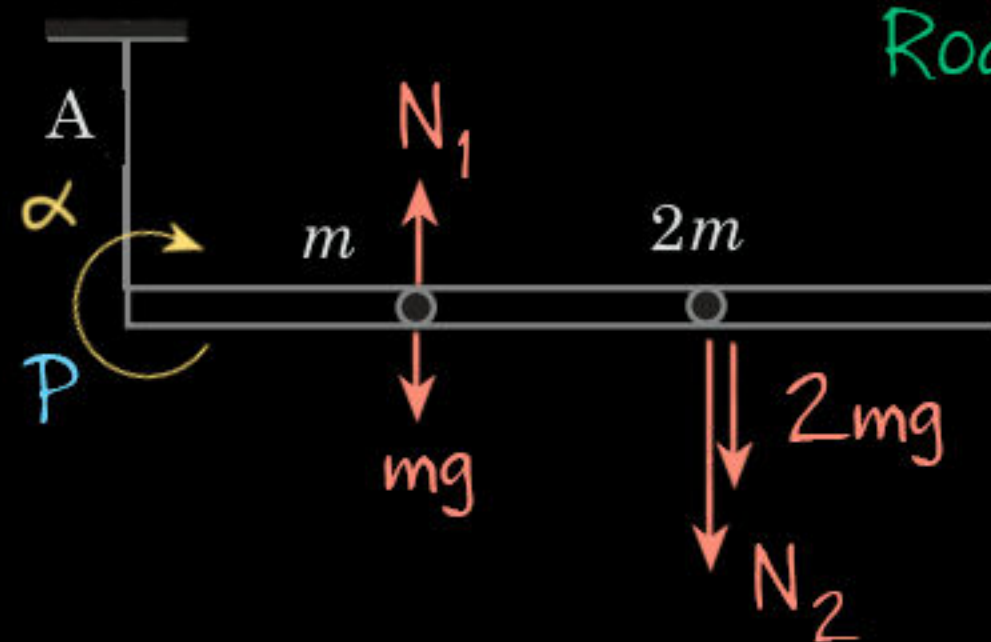
$$\tau_O: T_f = N_2 \quad (2)$$

28. A light rigid pipe is suspended with the help of two light inextensible vertical threads. Inside the pipe, two balls one of mass m and other of mass $2m$ can slide without friction. Initially the pipe is horizontal and the balls stay equidistant from each other and from the ends of the pipe as shown in the figure. How many times will the tension force in the thread A change, immediately after the thread B is cut?

Without thread A, rod would free fall.

Here, thread will prevent point P to move down and will be taut as a result. So P becomes the IoR.

FBD of masses:



$$\text{Rod's } \alpha = \frac{a}{l} = \frac{\left(\frac{mg - N_1}{m}\right)}{l} = \frac{\left(\frac{N_2 + 2mg}{2m}\right)}{2l}$$

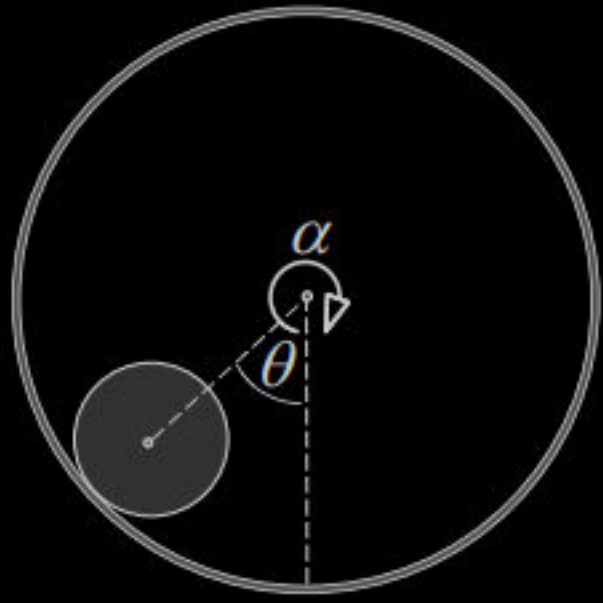
$$\Rightarrow 2mg = 4N_1 + N_2 \quad (3)$$

$$\text{From 1, 2, 3: } T_f = \frac{2mg}{g}$$

$$\therefore \frac{T_f}{T_i} = \frac{2/g}{4/3} = \frac{1}{6}$$

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29. A uniform cylinder is held motionless in a horizontal stationary pipe at an angular position $\theta = 30^\circ$ touching inner surface of the pipe. Radii of the cylinder and the pipe are $r = 0.2 \text{ m}$ and $R = 2.0 \text{ m}$ respectively. The pipe is now made to rotate about its fixed horizontal axis with a constant angular acceleration and the cylinder is released. If the axis of the cylinder remains motionless, find the angular acceleration of the pipe.

Assume cylinder is rolling without slipping:

Concept: Cylinder is at translational equilibrium but NOT rotational equilibrium.

As the cylinder is rolling without slipping: $\alpha R = \alpha_1 r$ (1)

Forces along the surface: $f = mg \sin \theta$ (2)
(To avoid variable N)

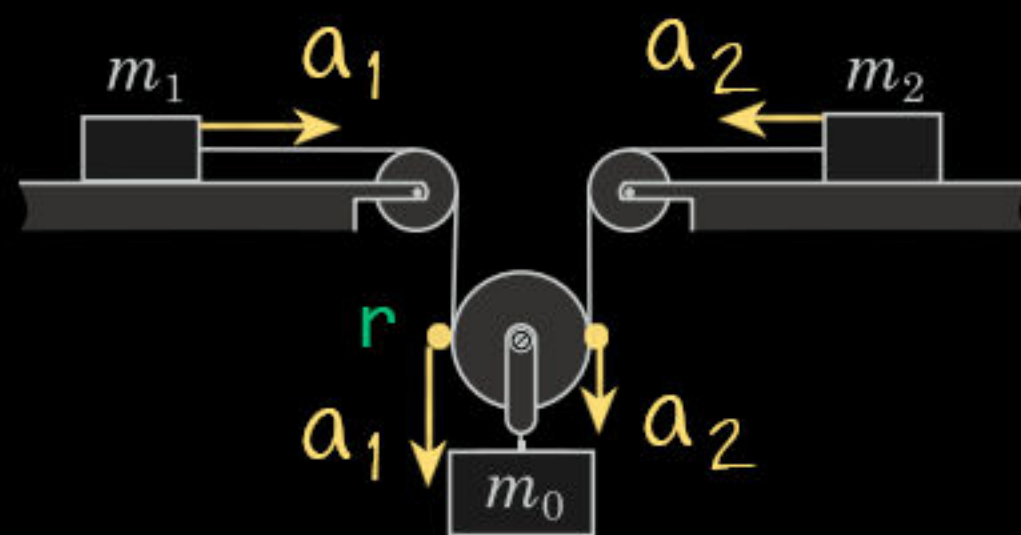
$$\tau_O: f r = \left(\frac{m r^2}{2} \right) \alpha_1 \quad (3)$$

From 1, 2, 3:

$$\alpha = \frac{2g \sin \theta}{R}$$

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30. In the arrangement shown, masses of blocks are $m_0 = 5 \text{ kg}$, $m_1 = 5 \text{ kg}$ and $m_2 = 10 \text{ kg}$ and radius of the movable pulley is $r = 10 \text{ cm}$. The threads and the pulleys are ideal and friction between the blocks and the horizontal surface is negligible. Find change in angular velocity of the movable pulley in a time interval $\Delta t = 0.22 \text{ s}$. Acceleration of free fall is $g = 10 \text{ m/s}^2$.

$$T = m_1 a_1 = m_2 a_2 \quad (1) \text{ and } (2)$$

$$m_0 g - 2T = m_0 \left(\frac{a_1 + a_2}{2} \right) \quad (3)$$

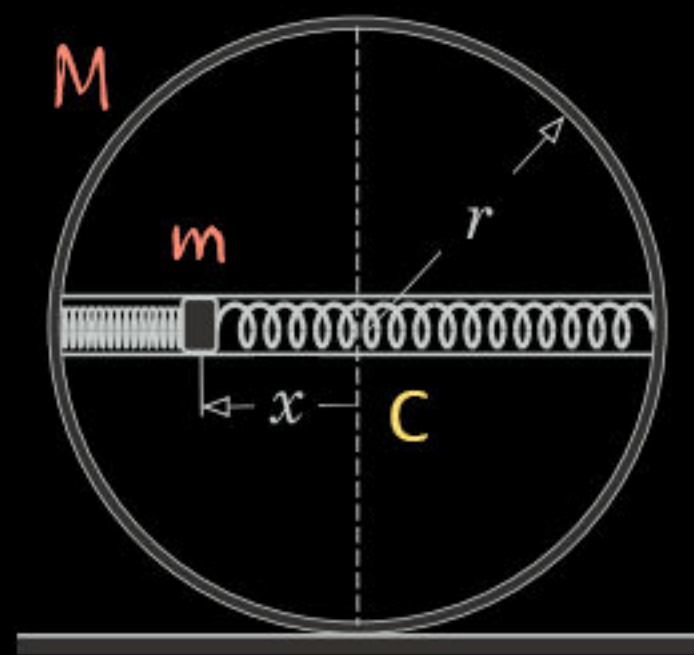
From 1, 2, 3:

$$a_2 = \frac{2m_1 m_0 g}{4m_1 m_2 + m_0(m_1 + m_2)}, \quad a_1 = \frac{m_2}{m_1} a_2$$

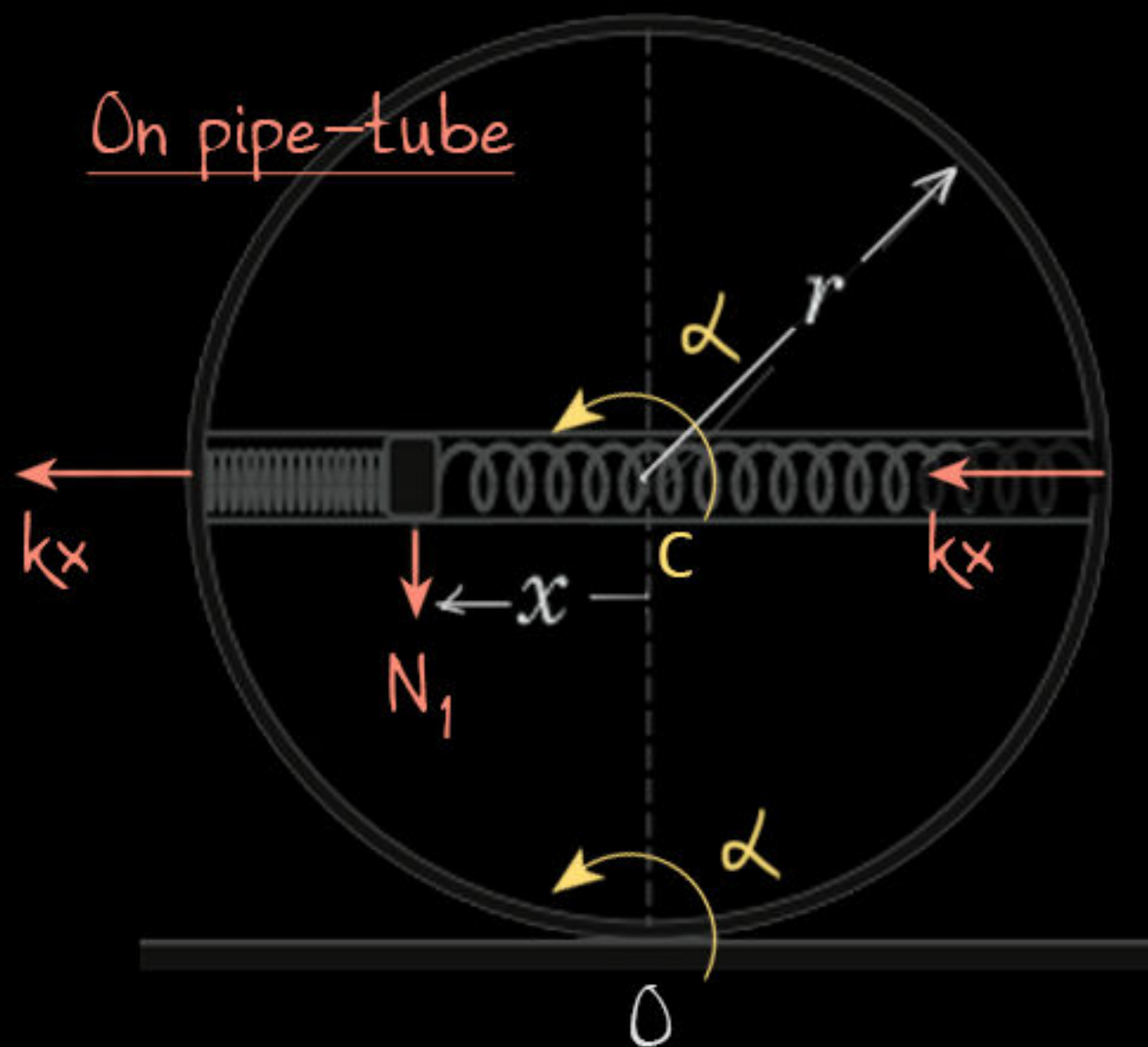
$$\Delta \omega = \alpha \Delta t = \left(\frac{a_1 - a_2}{2r} \right) \Delta t = \frac{m_0(m_2 - m_1) g \Delta t}{[4m_1 m_2 + (m_1 + m_2) m_0] r}$$

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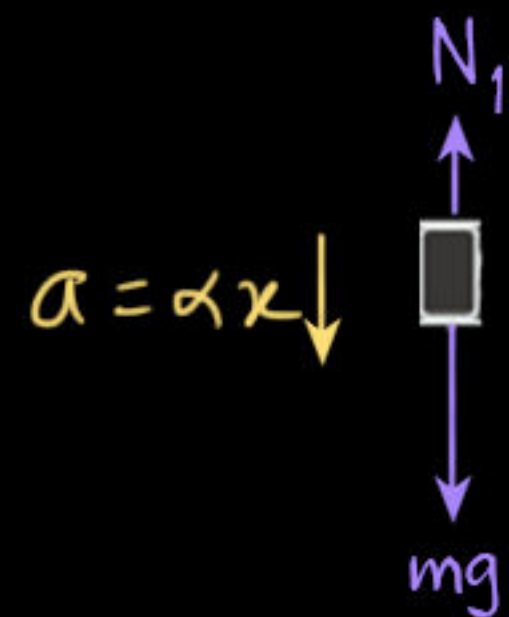
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31. A light rigid tube is welded along a diameter of a uniform cylindrical pipe of mass M and radius r . A bead of mass m connected by two identical light springs each of force constant k can slide inside the tube. Other ends of the springs are connected to the pipe as shown in the figure. The pipe is held motionless on a horizontal floor, the bead is pulled aside a distance x away from the centre of the pipe and then the pipe and the bead both are released simultaneously. If the cylinder rolls without slipping on the ground, find acceleration of its centre immediately after the system is released.



w.r.t C, F_y On block



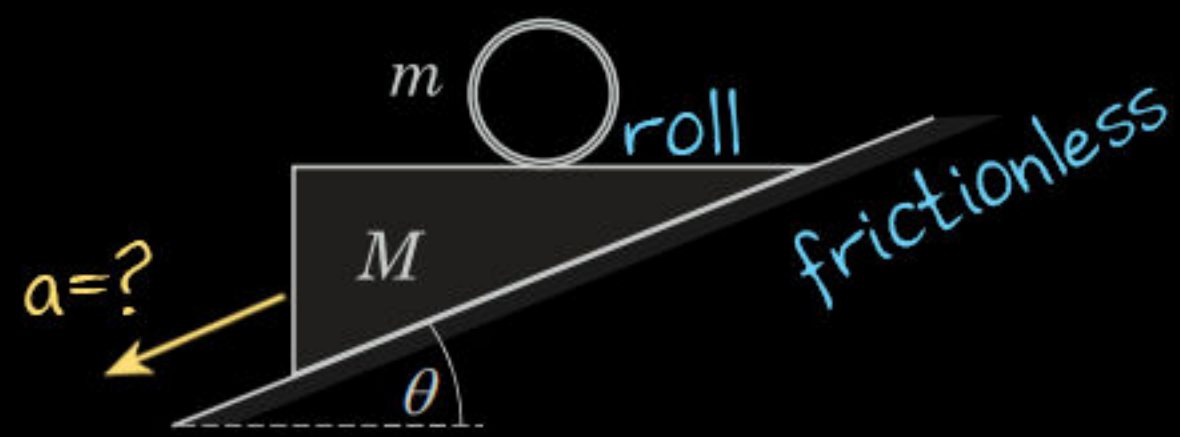
No pseudo force as C only accelerates in X direction.

$$F_y: mg - N_1 = m \alpha r \quad (1)$$

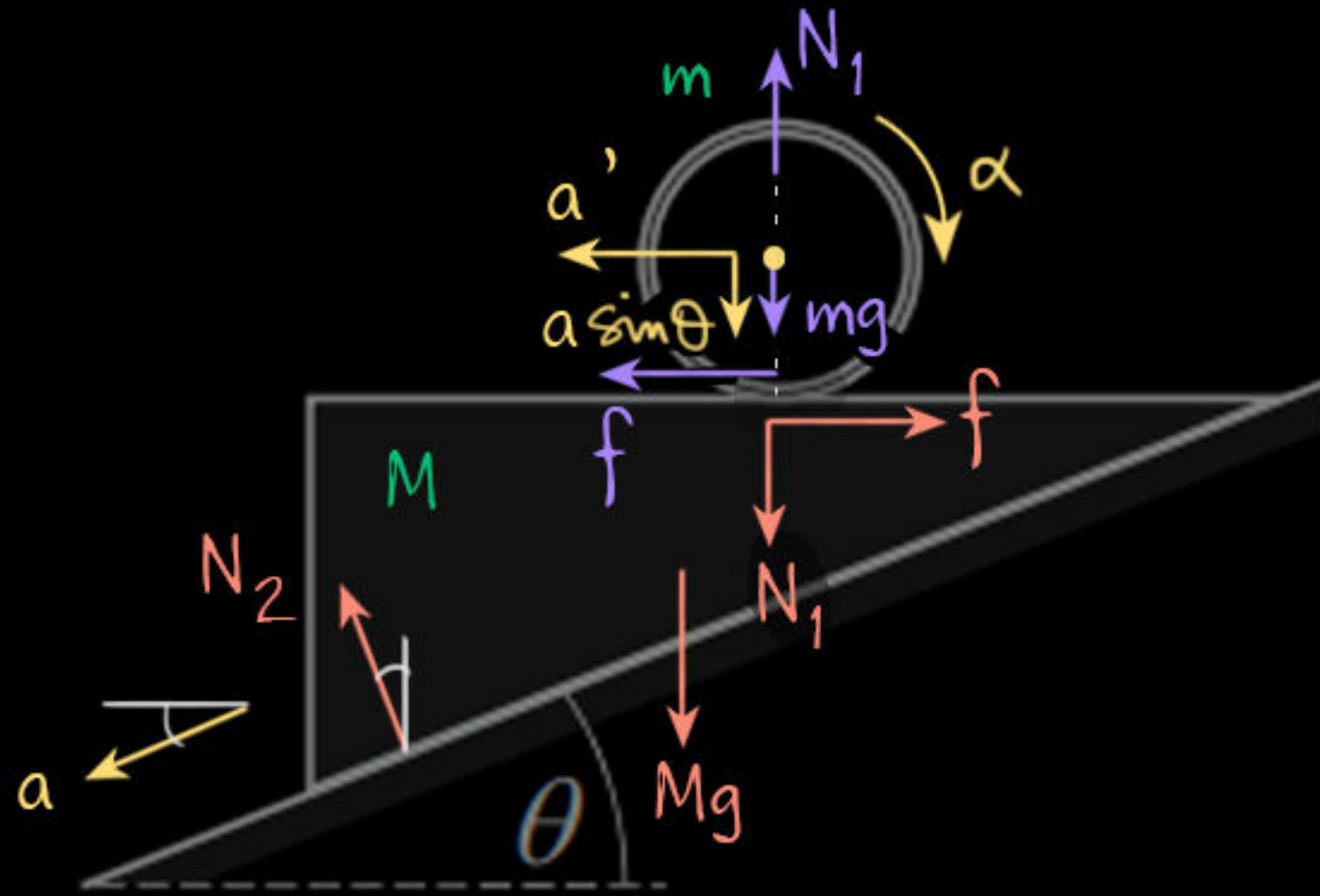
$$\tau_O: 2kx r + N_1 x = (2Mr^2) \alpha \quad (2)$$

$$\text{From 1,2: } \alpha = \frac{(2kr + mg)}{2Mr^2 + m r^2}$$

$$a = \alpha r //$$



32. A uniform hollow cylinder of mass m rests on a wedge of mass M that is held on a frictionless slope of inclination θ as shown in the figure. Friction between the cylinder and the top horizontal surface of the wedge is sufficient to prevent sliding of the cylinder after the wedge is released. Find acceleration of the wedge after it is released.



$$F_{x,M} : N_2 \sin \theta - f = Ma \cos \theta \quad (1)$$

$$F_{y,M} : Mg + N_1 - N_2 \cos \theta = Ma \sin \theta \quad (2)$$

$$F_{x,m} : f = ma' \quad (3)$$

$$F_{y,m} : mg - N_1 = ma \sin \theta \quad (4)$$

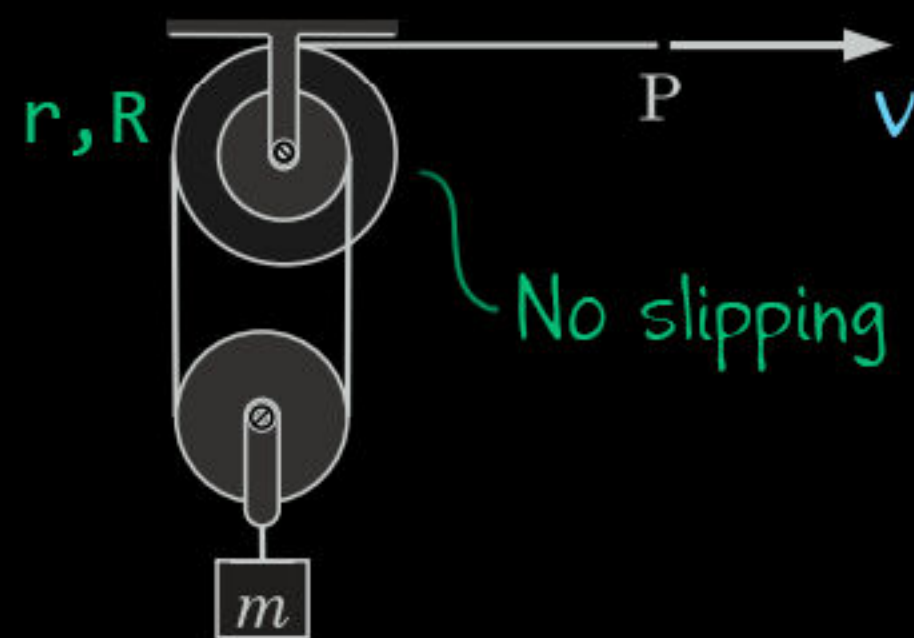
$$\text{Rotation: } fR = mR^2 \alpha \quad (5)$$

$$a' + \alpha R = a \cos \theta \quad (6)$$

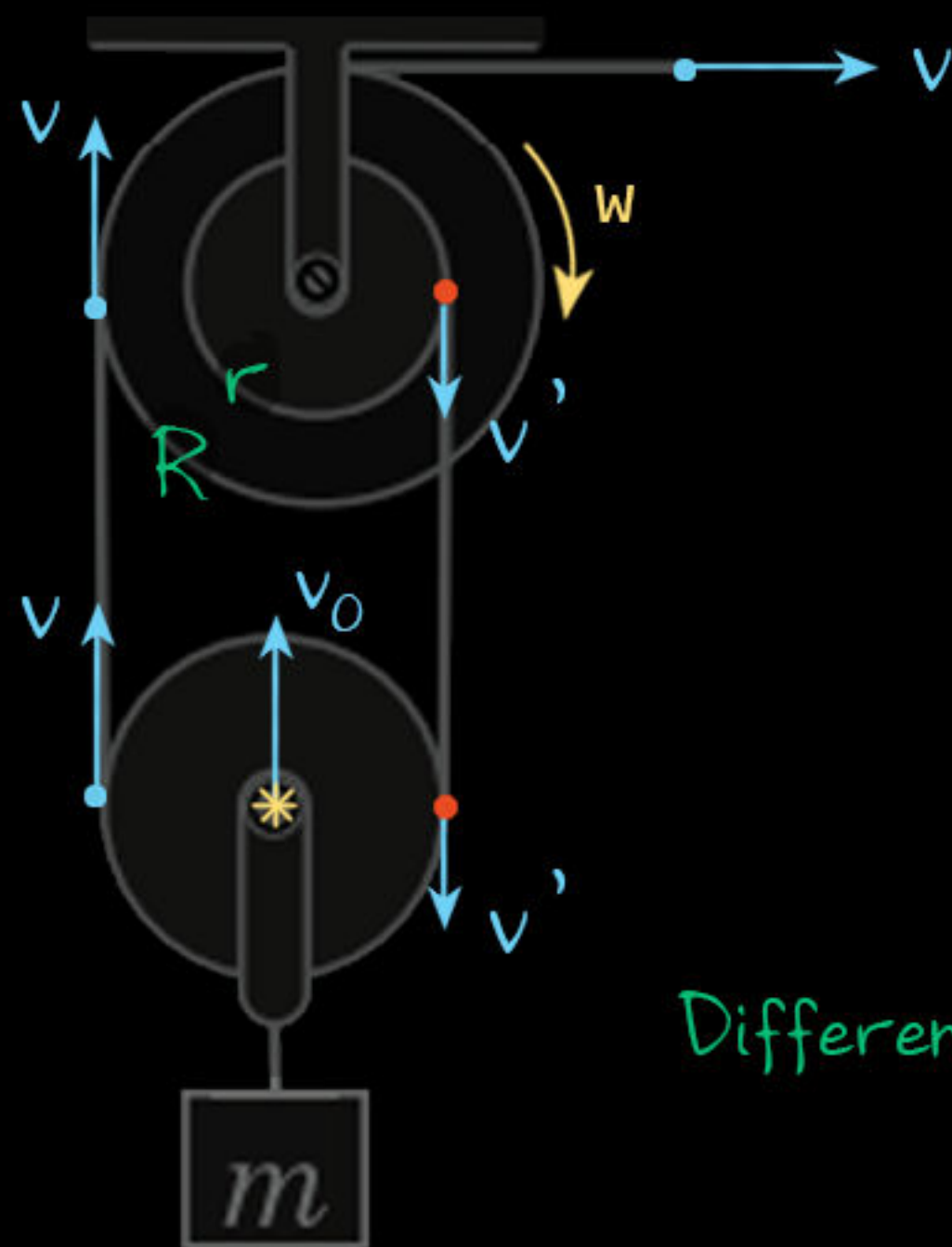
Variables: $a, a', N_1, N_2, f, \alpha$

$$\text{From 1,2,3,4,5,6: } a = \frac{2(m+M)g \sin \theta}{2M + m + m \sin^2 \theta} //$$

Note: Advantage of writing torque about CoM is that we don't have to worry about any pseudo force as it would pass through CoM and won't contribute in torque.



33. In the setup shown, masses of the pulleys and the thread are negligible as compared to mass m of the load hanging from the lower pulley. Radius of the smaller drum of the two-step pulley (drums of a two-step pulley are coaxially fixed preventing them to rotate independently) is r and that of the larger drum is R . There is no friction at the axles of the pulleys whereas friction between the thread and peripheries of the pulleys is sufficient to prevent slipping. Acceleration of free fall is g . If the free end P of the thread is pulled horizontally with a constant velocity v , find power delivered by the pulling agency.



Method 1 Without force

$$\omega = \frac{v}{R} = \frac{v'}{r}$$

$$\text{velocity of } m: v_0 = \frac{v - v'}{2} = \frac{(R - r)v}{2R} \checkmark$$

Differentiate

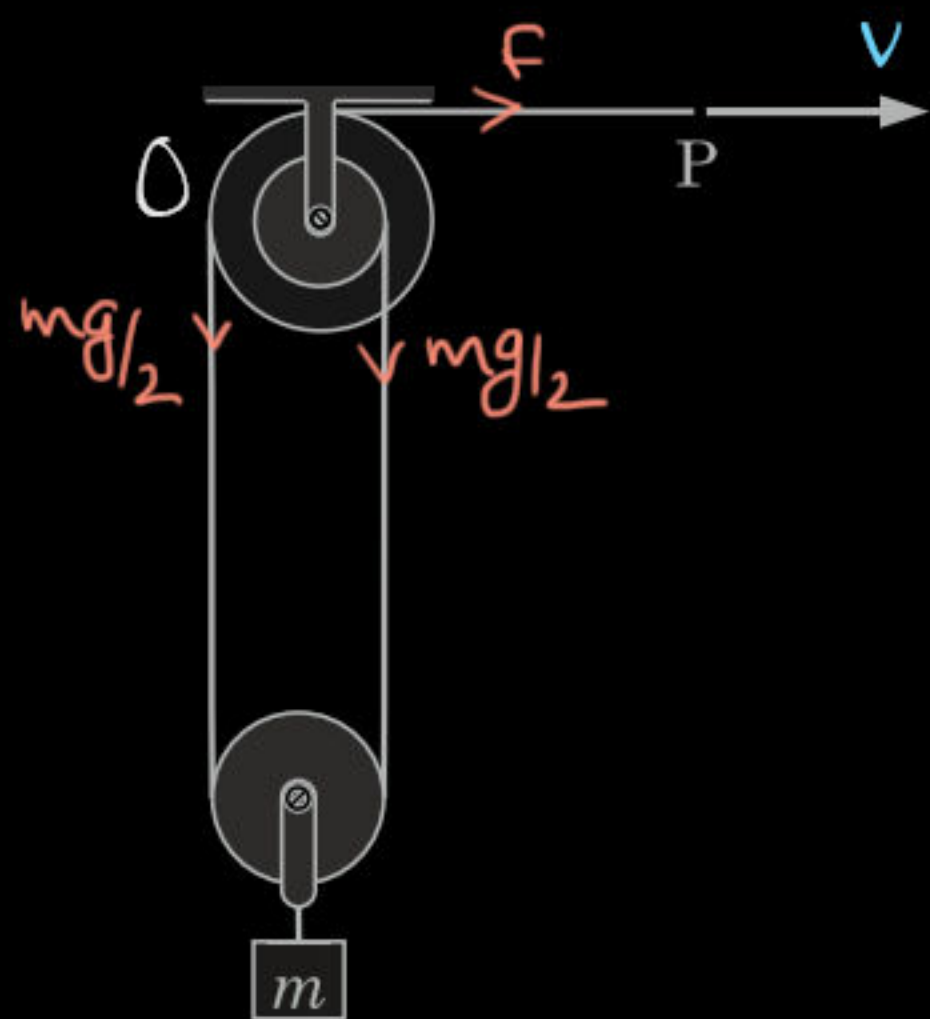
$$W_{\text{agency}} + W_g = \Delta KE \quad (\text{As whole system is moving steadily})$$

$$P_{\text{agency}} + P_g = 0 \Rightarrow P_{\text{agency}} = \frac{mg(R - r)v}{2R}$$

$-mgv_0$

by Ankit Singhvi

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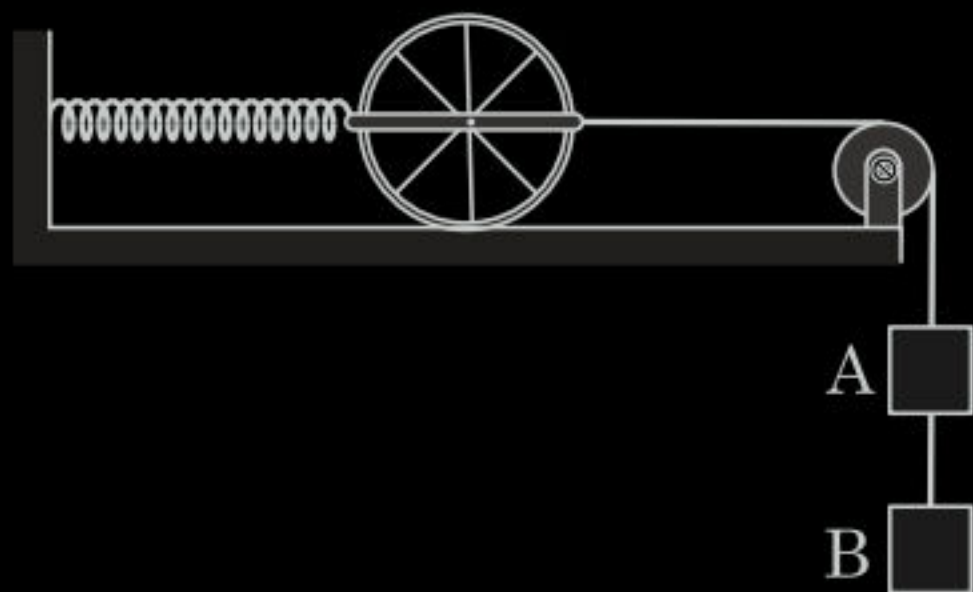


Method 2 With force

$$\sum \tau_O: FR + \frac{mg}{2}r = \frac{mg}{2}R$$

$$\Rightarrow F = \frac{mg}{2} \left(\frac{R-r}{R} \right)$$

$$\therefore P = F \cdot v //$$



34. In the setup shown, the pulley and the threads are ideal, the spring of force constant k is massless, the wheel has all its mass m uniformly distributed in its rim, the harness used to connect wheel axle with the spring and with the thread is massless and the loads A and B are of masses m and $3m$ respectively. Initially the harness is held motionless keeping the spring relaxed and then released. If the wheel rolls on the horizontal tabletop without slipping, find maximum tension developed in the thread connecting the loads.

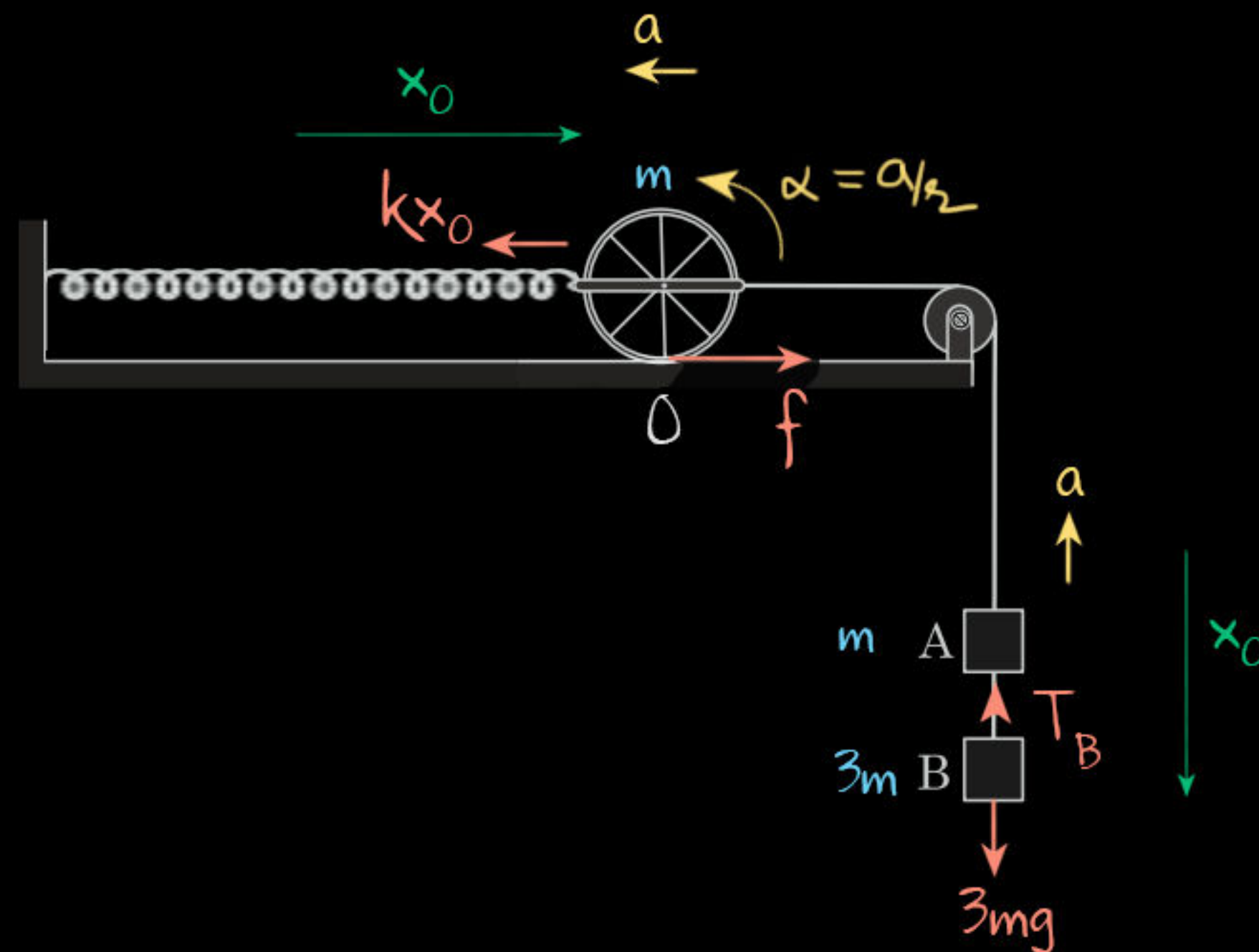
System will perform SHM, with:

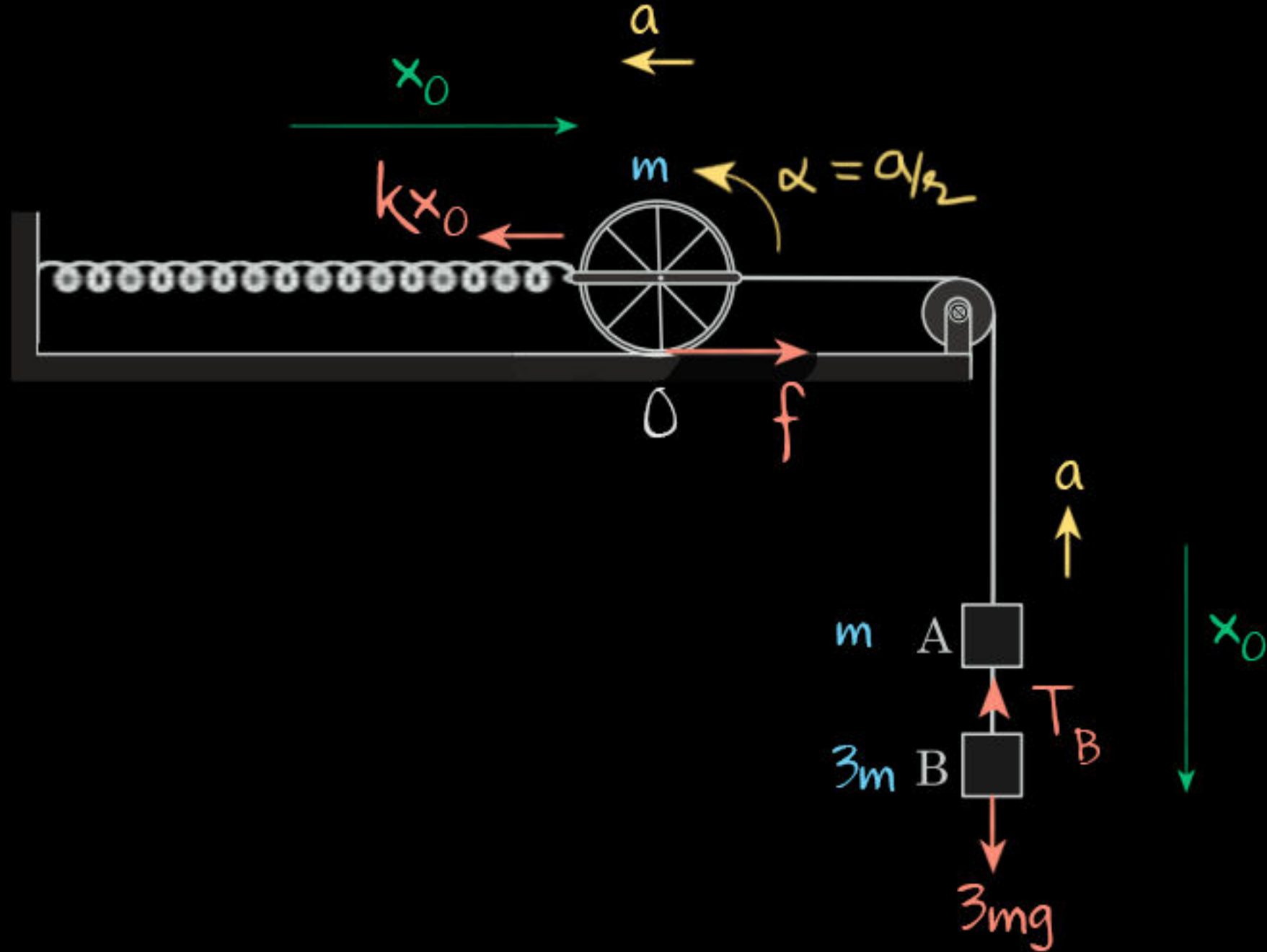
$$\omega = \sqrt{\frac{k}{(m + m_A + m_B) + \frac{I}{r^2}}}$$

Not needed in this problem though!

$$T_B - m_B g = m_B a$$

$\Rightarrow T_B$ is maximum, when a is maximum, which should be when system is pulled by spring with maximum force. i.e. when B is at its lowermost.





From one extreme to another:

$$\Delta PE + \Delta KE = 0 \quad \text{with } (0-0)=0$$

$$\Rightarrow \frac{1}{2} k x_0^2 - (m + 3m) g x_0 = 0$$

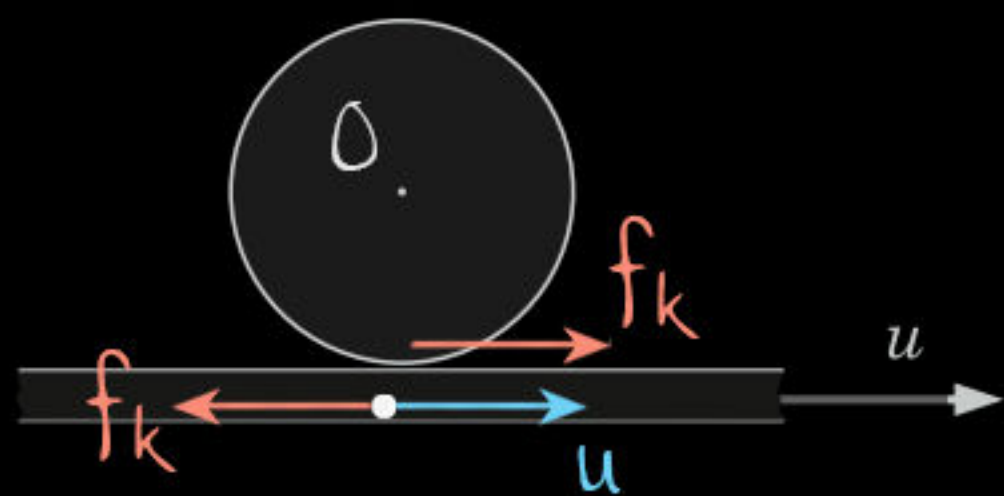
$$\Rightarrow k x_0 = 8mg \quad (1)$$

$$F_{x, \text{system}}: k x_0 - f - 4mg = 5ma \quad (2)$$

$$\tau_{\text{center}}: f r = I \alpha = m r^2 \cdot \frac{a}{2} \quad (3)$$

$$\text{From 1, 2, 3: } a = 2g/3$$

$$\therefore T_B - 3mg = 3ma \Rightarrow T_B = 5mg //$$



35. A uniform wheel of mass m and radius r is free to rotate about a horizontal massless axle that passes through its centre. A horizontal conveyor belt is running underneath the wheel with the help of a motor at a constant velocity u perpendicular to the axle of the wheel. Radius of gyration of the wheel is k .

In both (a) and (b):

Energy is supplied by motor to compensate for friction losses on belt.

Lets say, rolling starts at t_0 .

$$W_{\text{motor}} + \underbrace{(W_{\text{friction on belt}})}_{-f_k(ut_0)} = \Delta KE_{\text{belt}} \quad \rightarrow 0$$

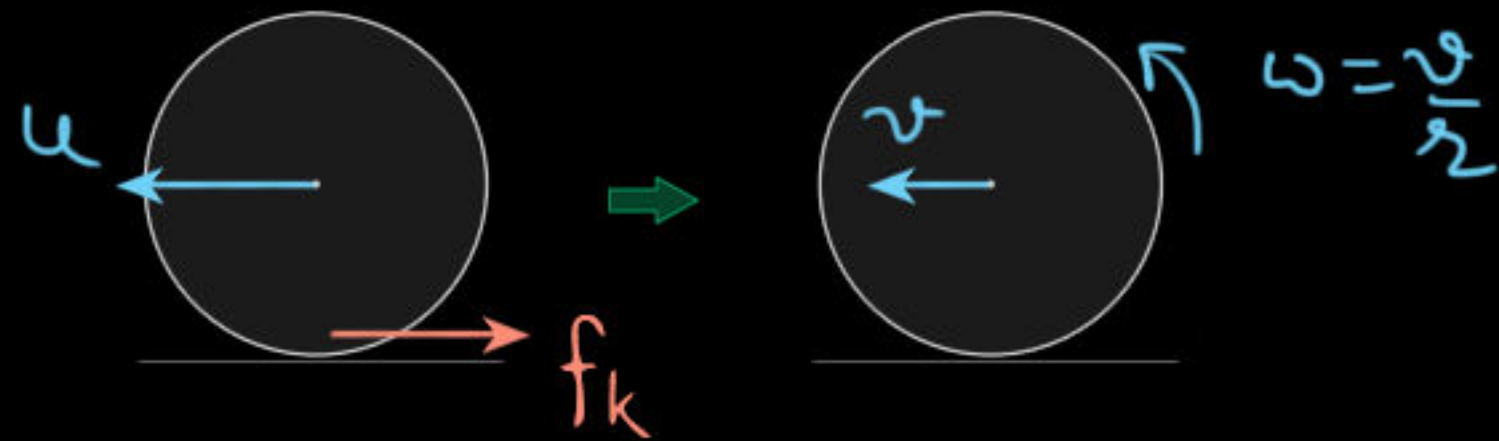
$$\Rightarrow W_{\text{motor}} = f_k(ut_0) \quad (1)$$

$$\text{A } \tau_0: f_k r = I \alpha = I \left(\frac{\omega}{t_0} \right) = I \left(\frac{u}{r} \right) t_0 = (mk^2) \frac{u}{rt_0} \quad (2)$$

$$\text{From 1,2: } W_{\text{motor}} = \frac{mk^2 u^2}{r^2} //$$

- (a) The wheel is lowered on the belt and then pressed down holding the axle stationary. Find additional energy delivered by the motor during the process of slipping.
- (b) The wheel is gently released on the belt. Find additional energy delivered by the motor during the process of slipping.

B w.r.t belt



Translation: $v = u - \frac{f_k}{m} t_0$ ①

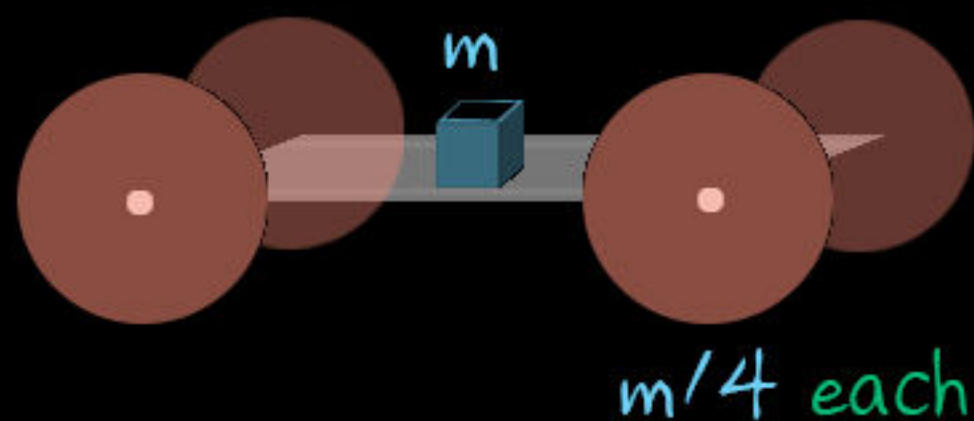
Rotation: $\omega = \alpha t_0$

$\frac{v}{r}$ $\frac{r}{I}$

$\Rightarrow \frac{v}{r} = \left(\frac{f_k r}{mk^2} \right) t_0$ ②

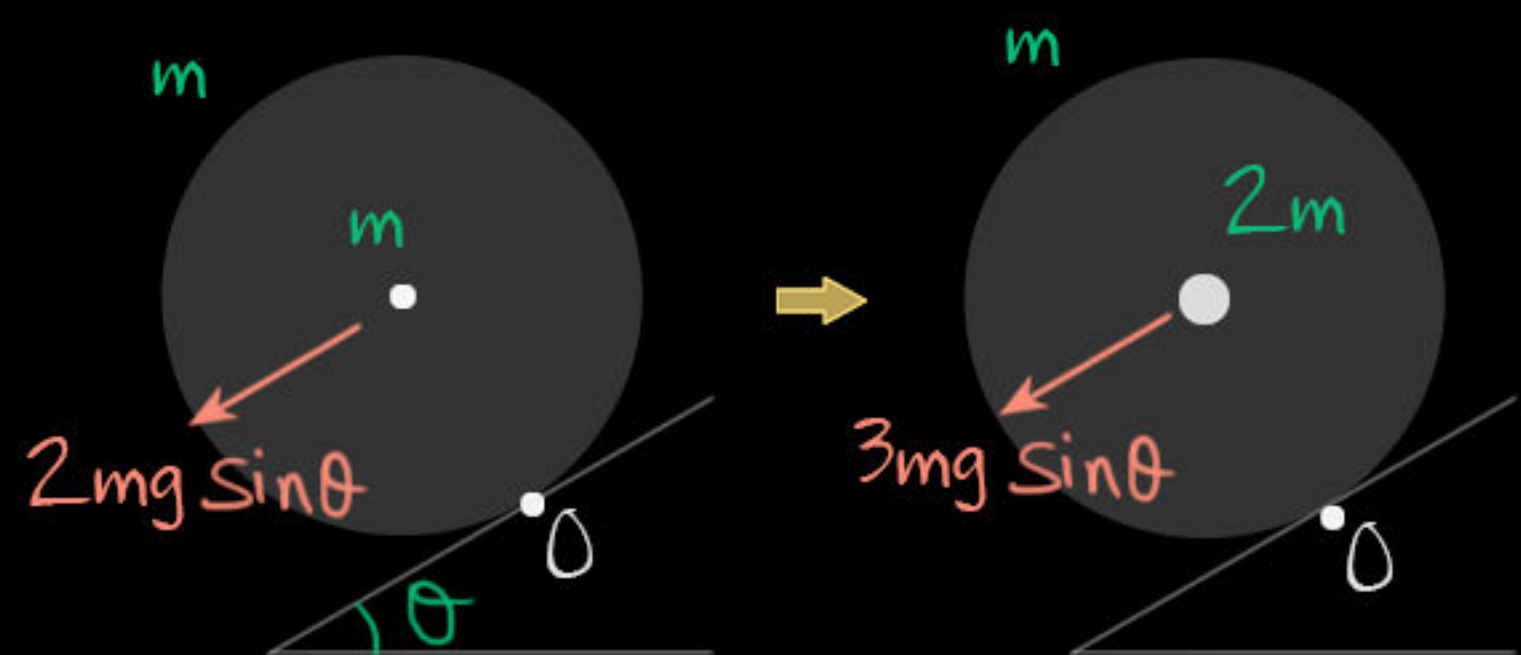
From 1,2: $f_k t_0 = \frac{mk^2 u}{r^2 + k^2}$

$\therefore W_{\text{motor}} = f_k u t_0 = \frac{mk^2 u^2}{r^2 + k^2}$



Approach:

- 1) Merge all wheels into one.
- 2) Merge rest of cart into a point object at center



Let I of combined tyres be $k m r^2$

36. A cart consists of a box of mass m mounted on four identical wheels. Total mass of all the four wheels is also m . When released from rest on a uniform slope of inclination $\theta = 30^\circ$, the cart rolls down a distance $l = 15$ m in time $t_0 = 3.0$ s. Now a load of mass m is put in the box of the cart and the cart is again released on the slope. If the load does not slide in the box, calculate time taken by the cart to roll down the same distance again. Acceleration due to gravity is $g = 10$ m/s².

1st run:

$$l = \frac{1}{2} a t_1^2 = \frac{1}{2} (\alpha_1 r) t_1^2$$

$$\tau_0: 2mg \sin \theta r = [m r^2 + (k m r^2 + m r^2)] \alpha_1$$

$$\Rightarrow 2mg \sin \theta r = m r^2 (k+2) \frac{2l}{r t_{1,2}^2}$$

$$\Rightarrow 2(10)\left(\frac{1}{2}\right) = (k+2) 2 \frac{(15)}{g} \Rightarrow k=1$$

2nd run
 $2m \rightarrow 3m$
 $m \rightarrow 2m$

$$3(10)\left(\frac{1}{2}\right) = (k+3) 2 \frac{(15)}{g} \Rightarrow t_2 = 2\sqrt{2}$$

by Ankit Singhvi

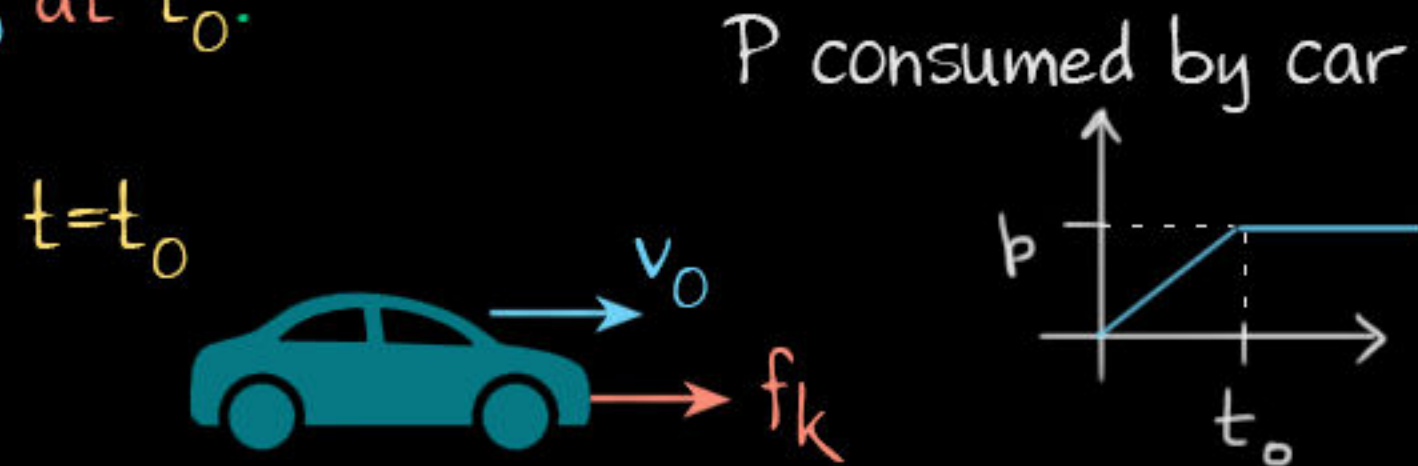
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Concepts:

Initially, tyres will slip and f_k will determine the velocity.

After rolling starts, p will determine the velocity.

Lets say it starts rolling with v_0 at t_0 .



Now, power consumed by car can not exceed p .

So once $f_k v_0 = p$ (1) kinetic friction stops and rolling begins.

37 A rear-wheel drive car of mass m , engine of which delivers a constant power p starts from rest on a straight level road. Coefficient of friction between tyres and the road is μ . If centre of gravity of the car is close to the road and equidistant from all the four wheels and masses of the wheels are negligible as compared to mass of the rest of the car, how does speed of the car vary with time?

Initially ($t < t_0$) $\mu(m/2)g$ ($\because$ rear wheel drive)

$$v = at = \frac{f_k}{m} t = \frac{\mu g t}{2}$$

$$\Rightarrow v_0 = \frac{\mu g t_0}{2} \quad (2)$$

When rolling begins

From 1,2:

$$t_0 = \frac{4p}{\mu^2 g^2 m} \quad (3)$$

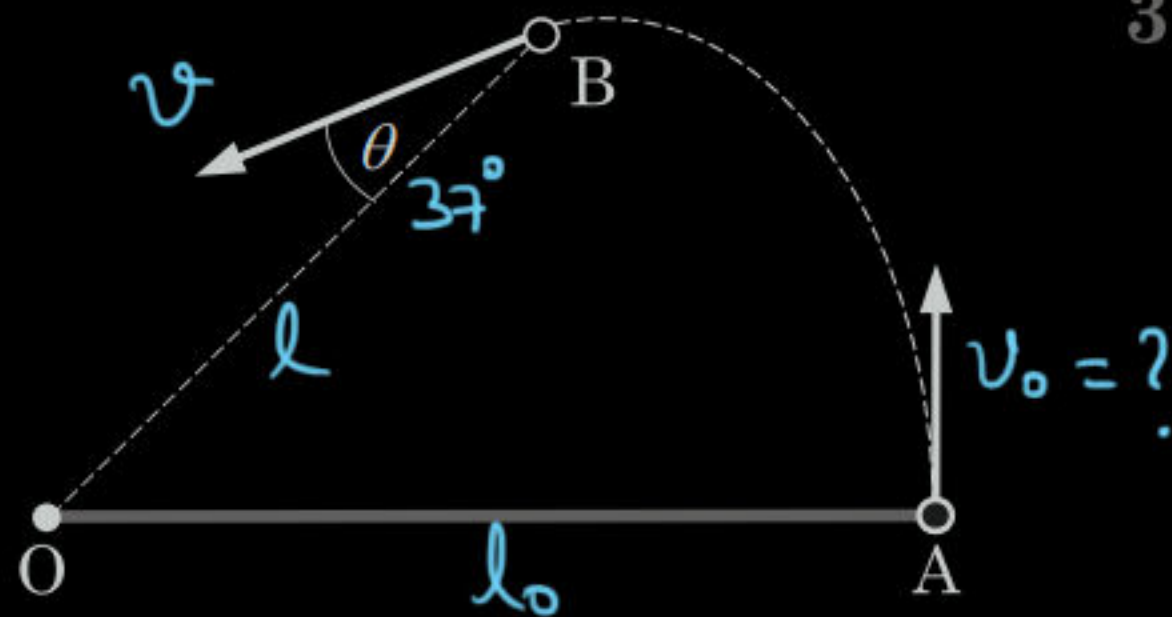
Once rolling begins ($t > t_0$)

$$p(t - t_0) = \frac{1}{2} m (v^2 - v_0^2)$$



$$\Rightarrow v = \sqrt{\frac{2pt}{m} - \frac{4p^2}{\mu^2 m^2 g^2}}$$

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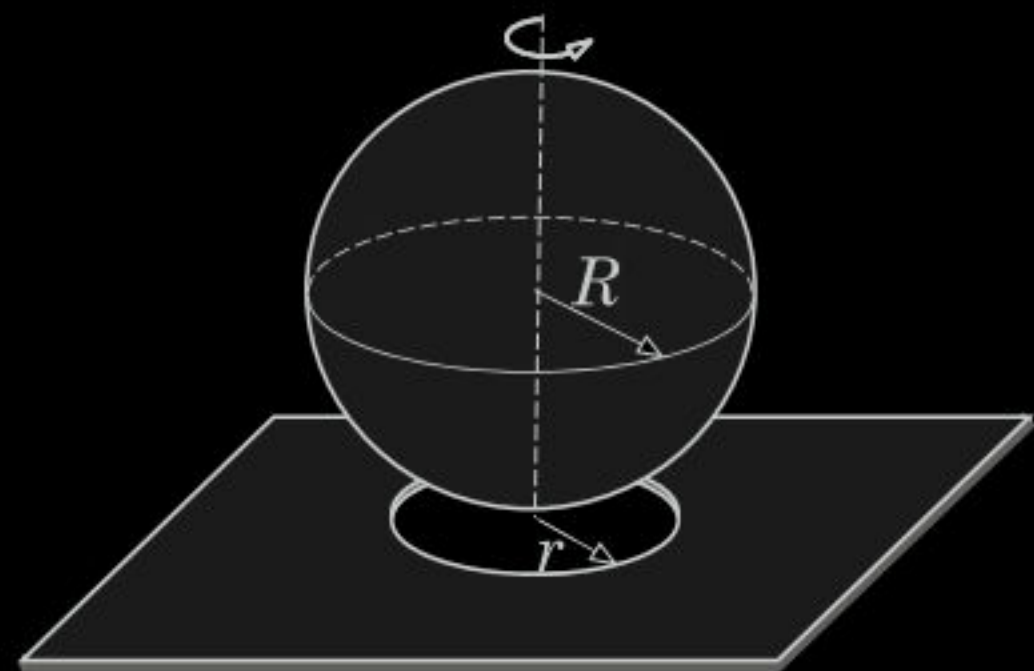


38. A particle of mass m placed on a frictionless horizontal floor is connected with the help of a light elastic cord to a nail O driven in the floor. The particle is pulled stretching the cord to a point A on the floor at a distance $l_0 = 12$ cm from the nail and projected horizontally perpendicular to the cord. If the particle is observed moving with a velocity $v = 24$ m/s at a point B a distance $l = 10$ cm away from the nail at an angle $\theta = 37^\circ$ with the cord as shown, find the velocity of projection of the particle.

Its central force.
So L_0 will remain constant.

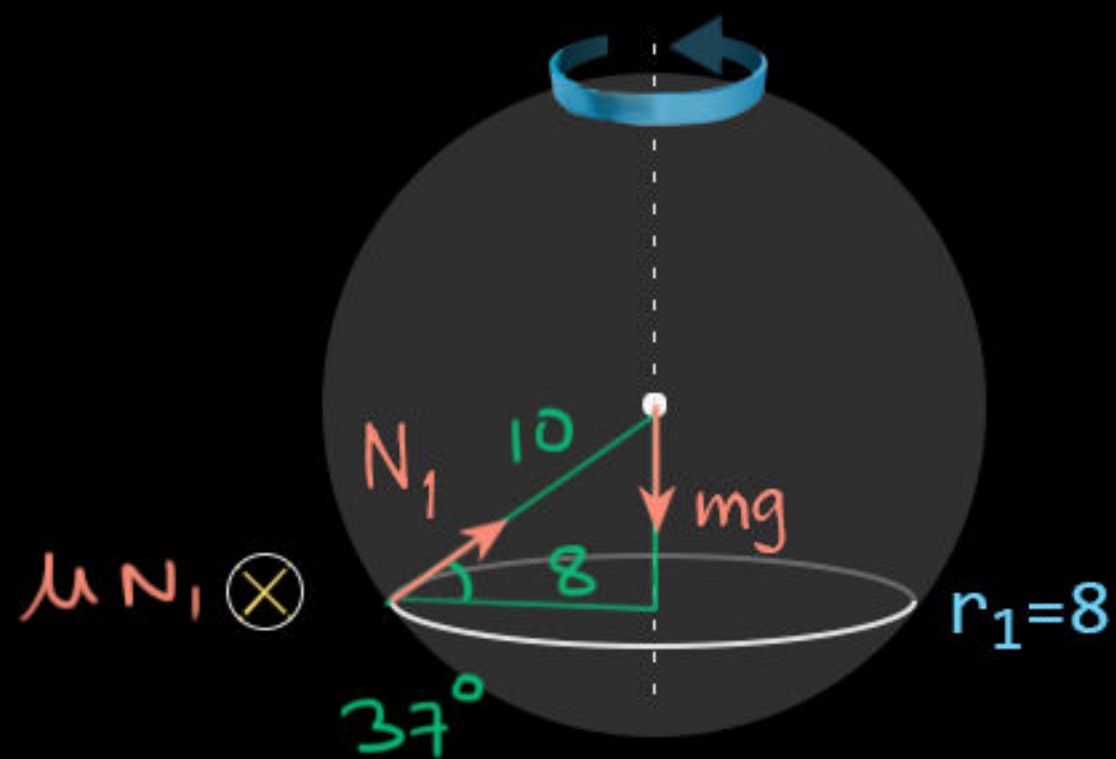
$$m l_0 v_0 = m v \sin 37^\circ l$$

$$\Rightarrow v_0 = (24) \frac{3}{5} \cdot \frac{10}{12} = 12 \text{ m/s} //$$



39. A homogeneous sphere of radius $R = 10$ cm rotating about its vertical diameter is gently released on a hole of radius $r_1 = 8.0$ cm made in a fixed horizontal slab. Centre of the hole is vertically below the centre of the sphere. The sphere stops in a time interval $\Delta t_1 = 9.0$ s. Calculate time interval required to stop the same sphere, if it is placed rotating with the same angular velocity on a hole of radius $r_2 = 6.0$ cm made in another identical horizontal slab.

As ΔL is same in both the cases, angular impulse provided by friction is same too.



$$mg = N_1 \sin 37^\circ$$

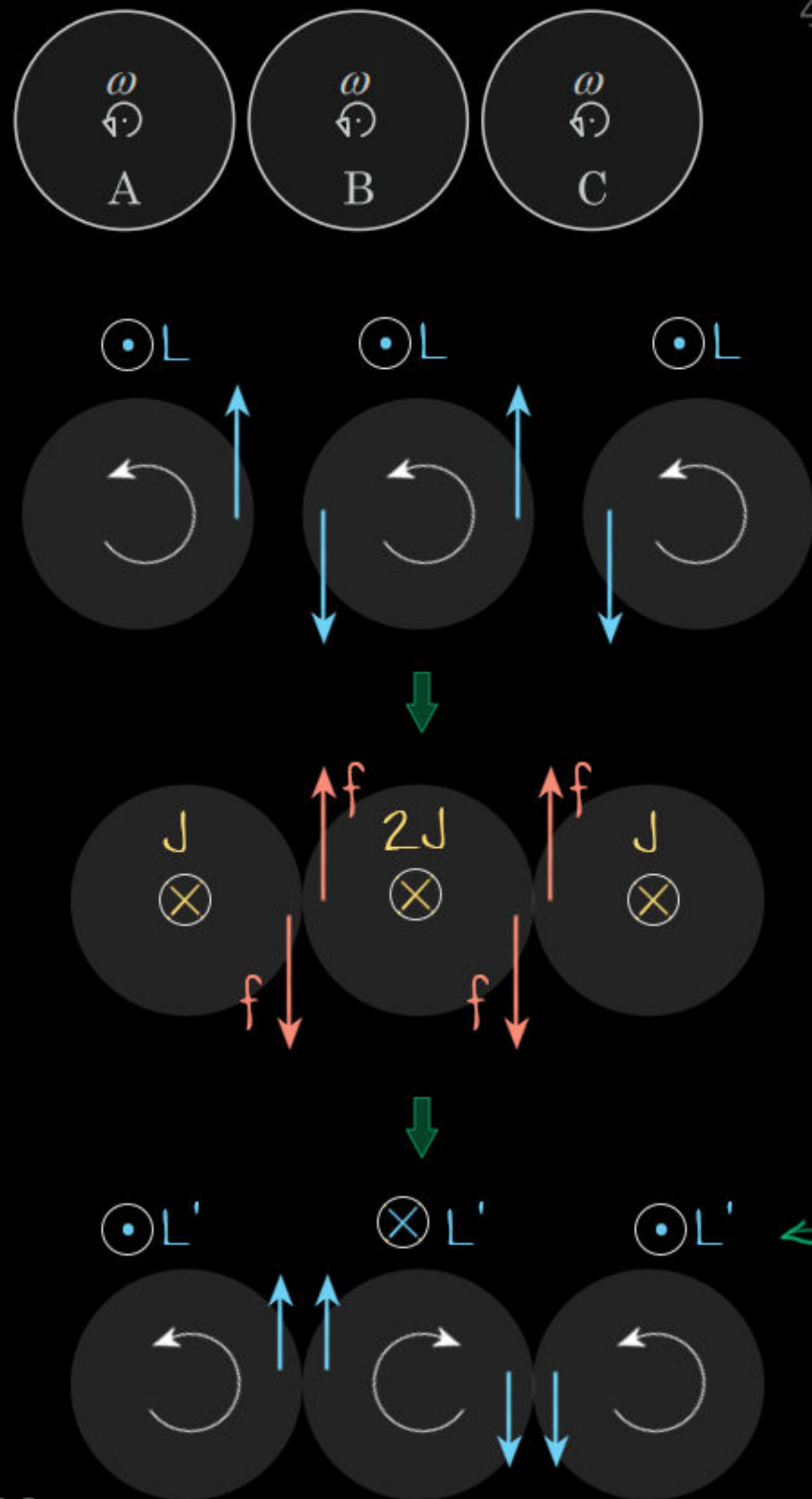
$$\text{Angular impulse, } H = \tau_1 \Delta t_1 = \mu N_1 (8) \Delta t_1 = \mu mg \left(\frac{8 \Delta t_1}{\sin 37^\circ} \right) \quad (1)$$

$$\text{Similarly } H = \mu mg \left(\frac{6 \Delta t_2}{\sin 53^\circ} \right) \quad (2)$$

$$\text{Dividing 1,2: } \Delta t_2 = \frac{8 \times 9}{\sin 37^\circ} \frac{\sin 53^\circ}{6} = 12 \text{ sec} //$$

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40. Three identical uniform cylinders are mounted on frictionless horizontal axles passing through their centres. They are given equal angular velocities ω_0 each and then gradually brought into contact preventing any vertical movement and rotation of their axles.

- (a) Find angular velocities of the cylinders after they stop slipping.
 (b) Can you use principle of conservation of angular momentum to analyse the situation?

A Angular impulse = Change in angular momentum

$$\Rightarrow \vec{J} \Delta t = \vec{L}_f - \vec{L}_i$$

Disc 1 & 3: $J \Delta t = -L' - (-L)$

Disc 2: $2J \Delta t = L' - (-L)$

Dividing $L' = L/3 //$

B No, as whatever axis we take, reaction forces at axles will cause torque that changes angular momentum of system.

Note: We can assume these directions as we are dealing with variables. -ve sign will appear in answer if our assumptions were wrong.

Concept: The insects always need to overcome centrifugal force to walk towards center.

So the effort is maximum

when $m\omega^2 r$ is maximum



$$L_i = L_f$$

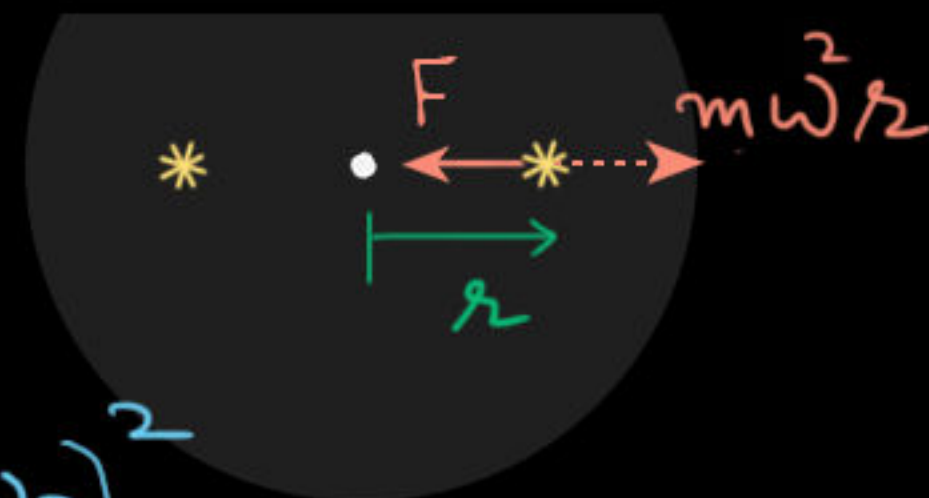
$$\Rightarrow (I + 2mR^2)\omega_0 = (I + 2mr^2)\omega$$

41. Two insects each of mass m are at opposite ends of a diameter of a turntable, moment of inertia of which is I about its central vertical axis. The turntable is rotating freely and both the insects start crawling slowly towards each other on the turntable. At what distance r from the centre of the turntable, do the insects have to apply maximum horizontal force on the turntable?

w.r.t turntable:

$$F = m\omega^2 r$$

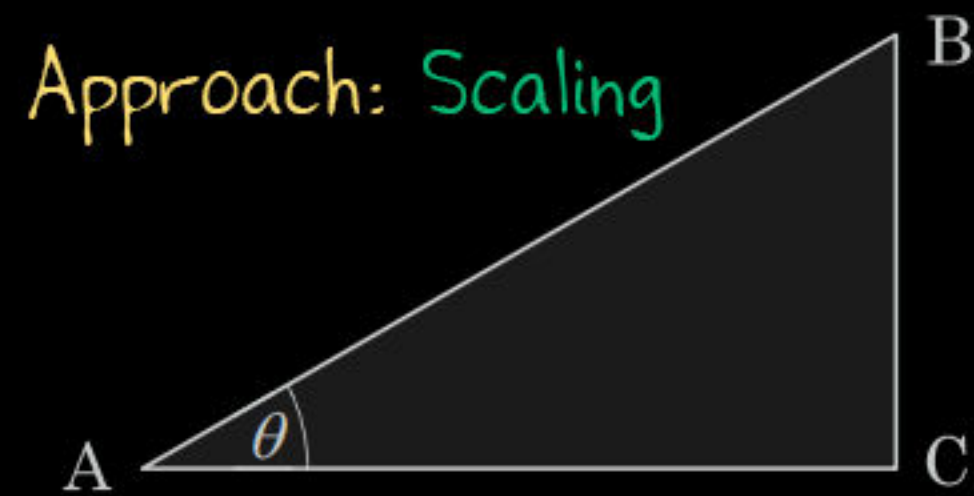
$$= m \left[\frac{(I + 2mR^2)\omega_0}{(I + 2mr^2)} \right]^2 r$$



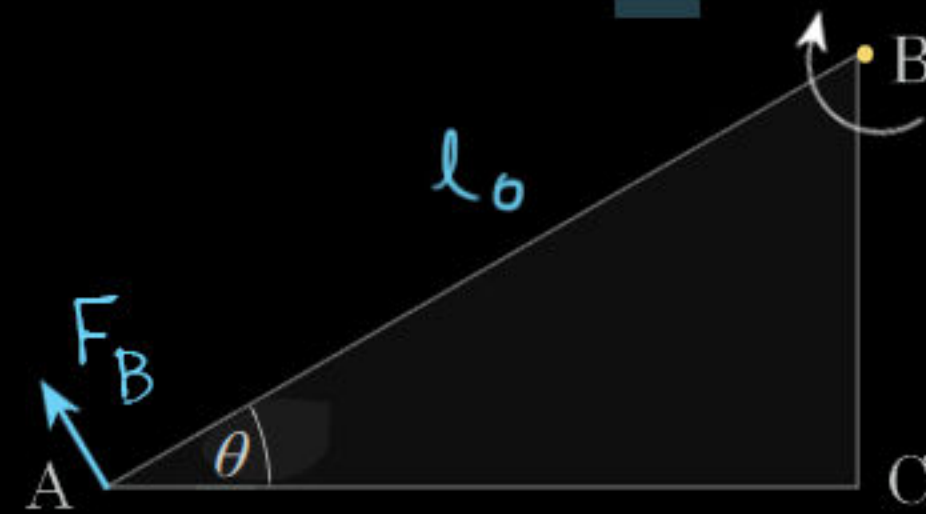
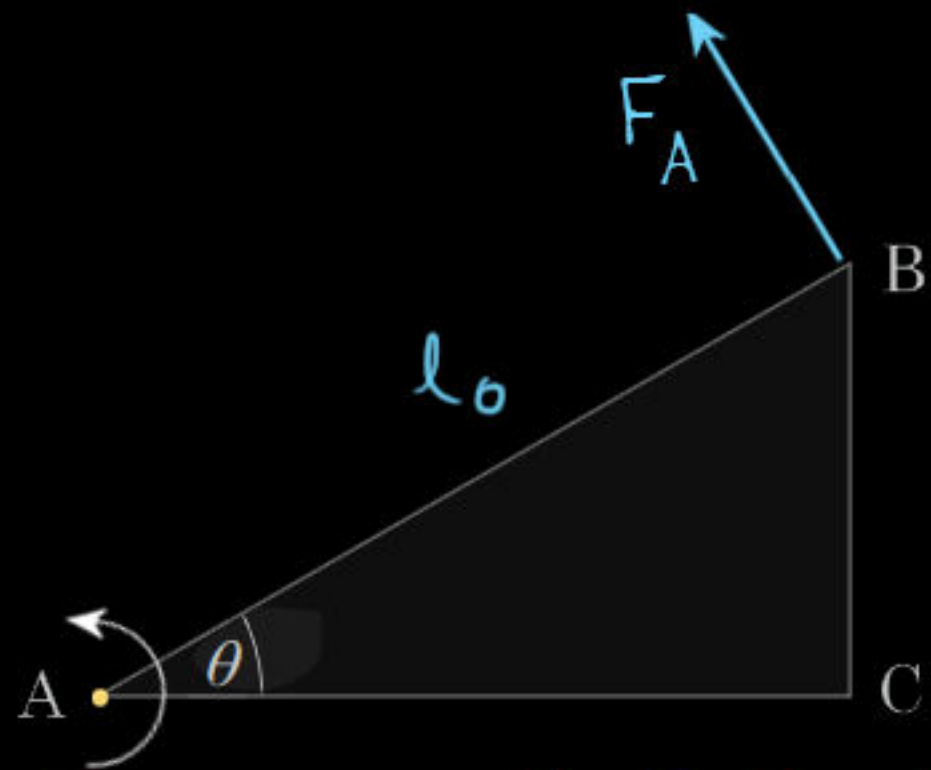
$$@ F_{\max}: \frac{dF}{dr} = 0$$

$$\Rightarrow (I + 2mr^2)^{-2} - r \cdot 2(I + 2mr^2)^{-3} (4mr) = 0$$

$$\Rightarrow I + 2mr^2 - 8mr^2 = 0 \Rightarrow r = \sqrt{\frac{I}{6m}}$$



10. A uniform right-angled triangular lamina is placed on a horizontal floor, which is not frictionless. One of the acute angle of the lamina is θ as shown in the figure by the top view of the situation. If F_A and F_B are the minimum forces required to rotate the lamina about stationary vertical axes through the vertices A and B respectively, find the minimum force required to rotate the lamina about a stationary vertical axis through the vertex C.



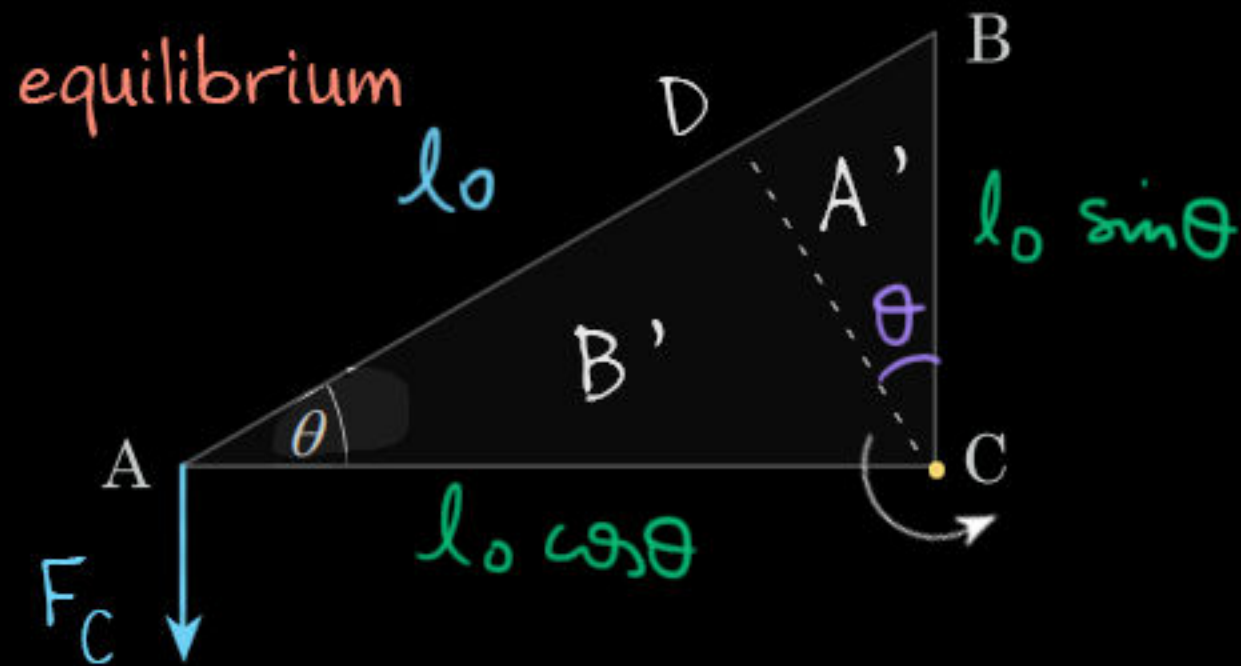
For minimum F_A, F_B, F_C , its rotational equilibrium and these forces must be applied perpendicular to the farthest distance from pivot to maximise torque.

$$\tau_A = F_A l_0 \propto (mg) l_0 = k_1 l_0^3$$

$\propto l_0^2$

Similarly: $\tau_B = F_B l_0 = k_2 l_0^3$

Rotational equilibrium once again



$$\tau_C = \tau_{A'} + \tau_{B'}$$

$$\Rightarrow F_C l_0 \cos \theta = k_1 (l_0 \sin \theta)^3 + k_2 (l_0 \cos \theta)^3$$

$$\Rightarrow F_C \cancel{l_0} \cos \theta = F_A \cancel{l_0} \sin^3 \theta + F_B \cancel{l_0} \cos^3 \theta$$

$$\Rightarrow F_C = F_A \sin^2 \theta \tan \theta + F_B \cot^2 \theta$$

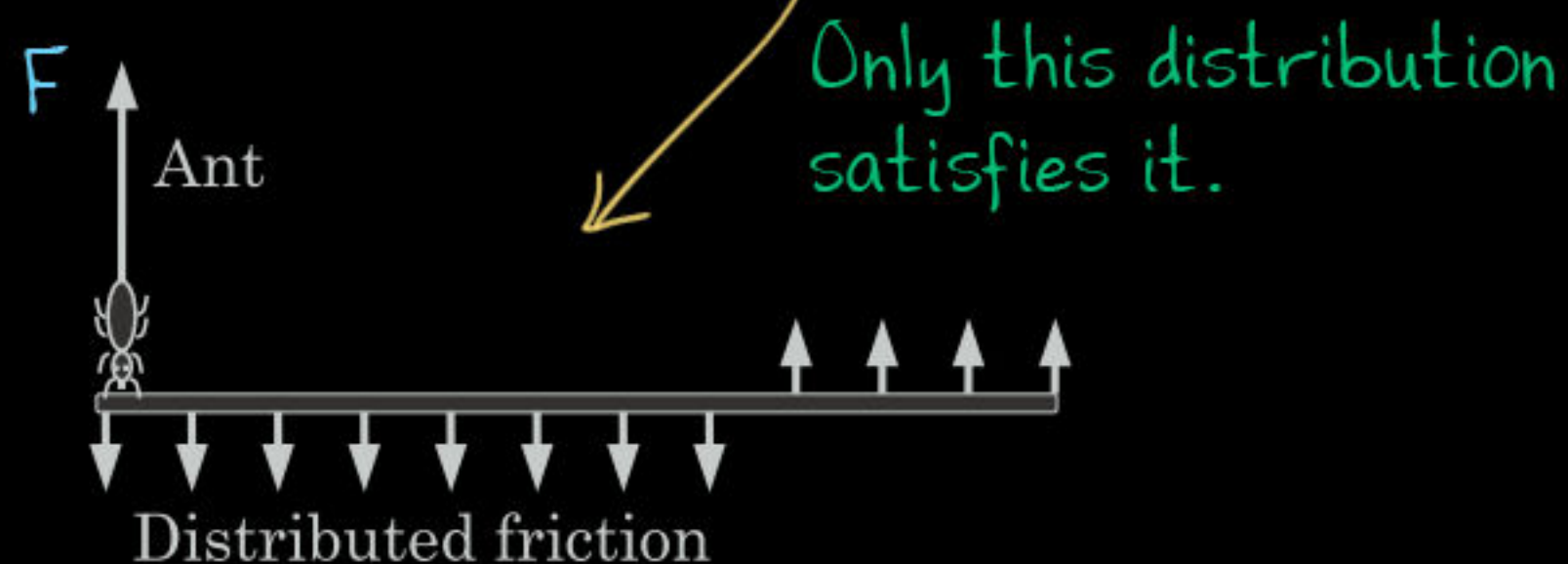


22. Very often, you have seen an insect dragging an object much larger and heavier than it. The figure is a top view of an ant dragging a straw that is much larger and heavier than the ant. The ant pulls the straw at one end rotating it slightly then it does the same action at the other end and so on.

(a) How does this technique work?

(b) An ant is dragging a straw of mass m on a horizontal floor by this technique. Coefficient of friction between the straw and the floor is μ . How much force F does it have to apply?

Hint: Consider the straw at the verge of rotational as well as translational equilibrium under the action of distributed frictional forces and the pull of the ant.

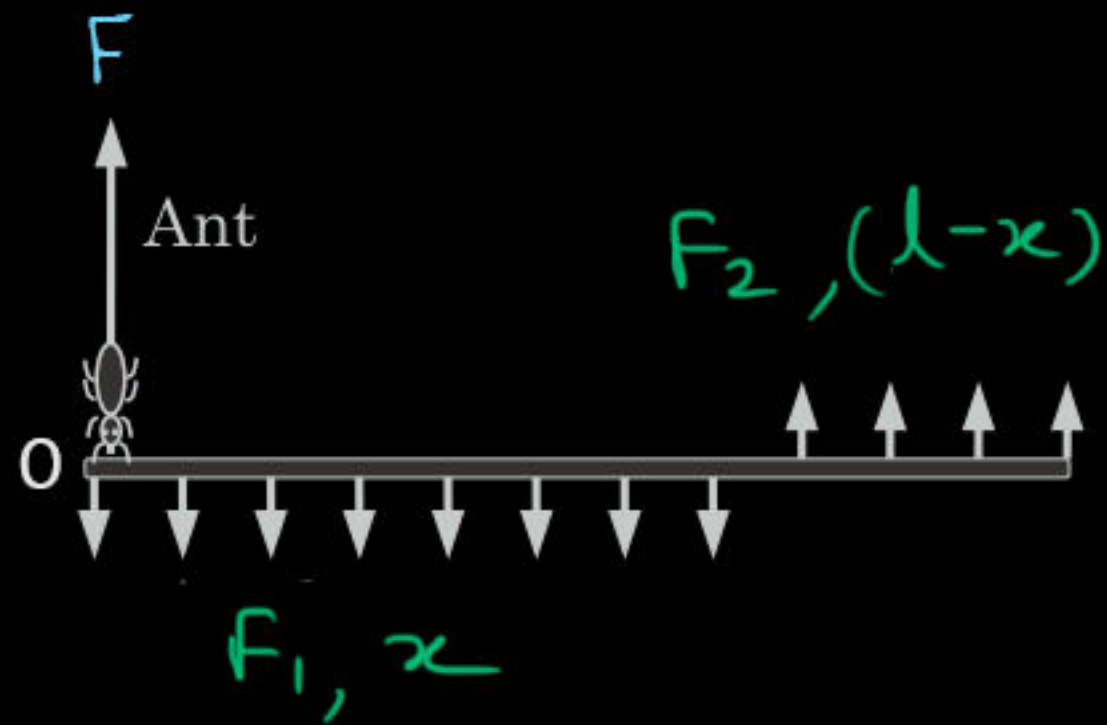


(a) When the ant pulls the straw at one end, the straw rotates about an off centre point nearer to the other end. Distribution of the frictional forces is shown in the following figure.

The net frictional force opposing the pull of the ant is reduced as some friction supports the efforts of the ant.

(b) $F \geq \mu mg(\sqrt{2} - 1)$

Finding F



Translational equilibrium: $F + F_2 = F_1$ ①

Rotational equilibrium (About O): $F_1 \left(\frac{x}{2} \right) = F_2 \left(x + \frac{l-x}{2} \right)$ ②

Net frictional force: $F_1 + F_2 = \mu mg$ ③

Friction per unit length: $\frac{F_1}{x} = \frac{F_2}{l-x}$ ④

From 1,2,3,4:

$$x = \frac{l}{\sqrt{2}}$$

$$\& F = (\sqrt{2} - 1) \mu mg //$$

23. A uniform disc rotating about its vertical axis of symmetry is gently placed on a horizontal floor with one of its flat face on the floor. The floor has two portions, where coefficients of frictions between the disc and the floor are $\mu_1 = 0.78$ and $\mu_2 = 0.67$. Consider the line dividing the two portions as y -axis and the point where centre of the disc is placed as the origin of a coordinate system as shown in the figure. Find acceleration of the centre of the disc immediately after it is placed on the floor. Assume acceleration of free fall to be $g = 10 \text{ m/s}^2$ and $\pi = 22/7$.



On right half: $dF_{y_1} = dF \cos \theta = \mu_1 dm g \cos \theta \rightarrow \frac{dr (r d\theta)}{\pi R^2} M$

$$\Rightarrow F_{y_1} = \frac{\mu_1 M g}{\pi R^2} \int_0^R r dr \int_{-\pi/2}^{+\pi/2} \cos \theta d\theta = \frac{\mu_1 M g}{\pi} \uparrow$$

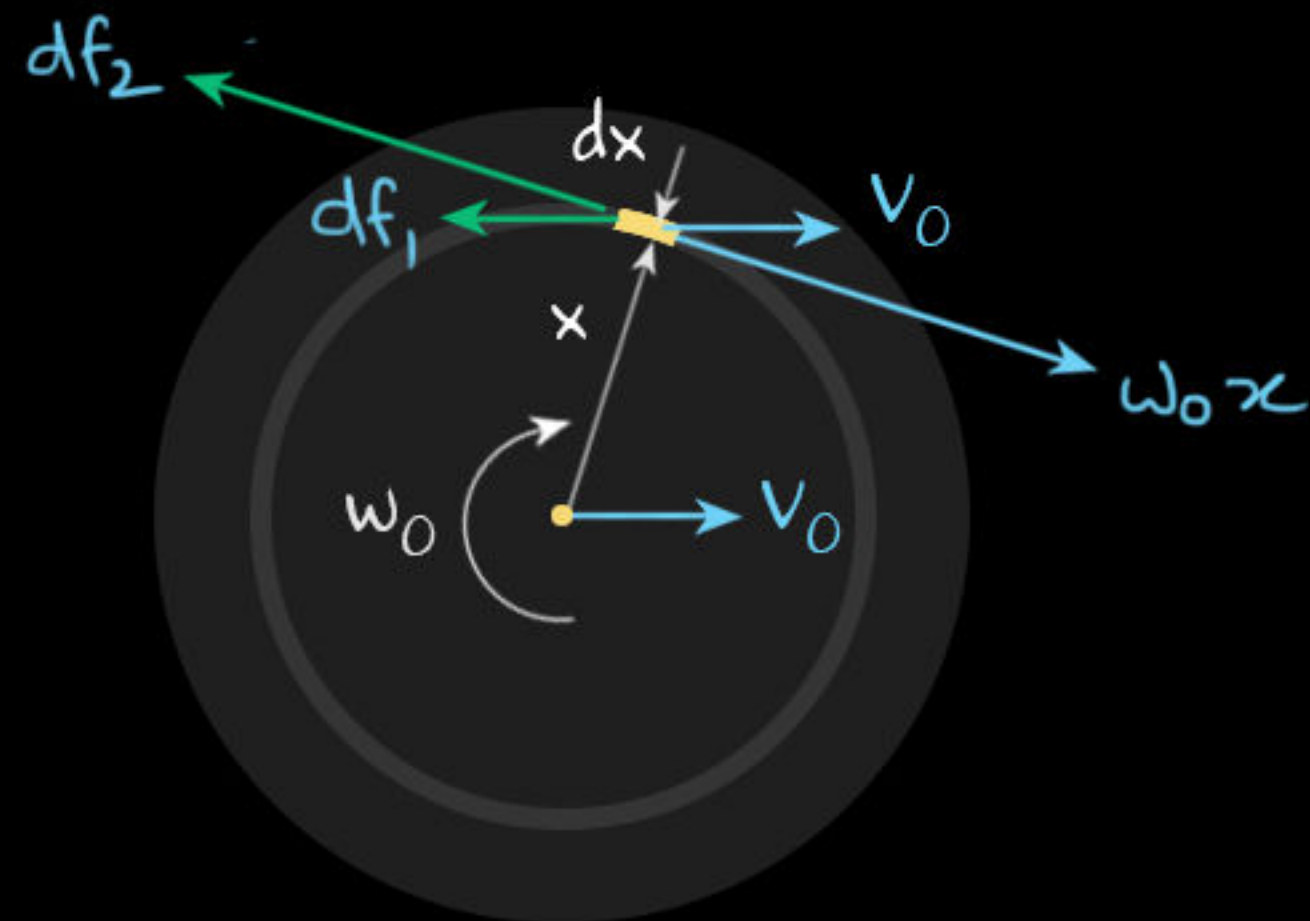
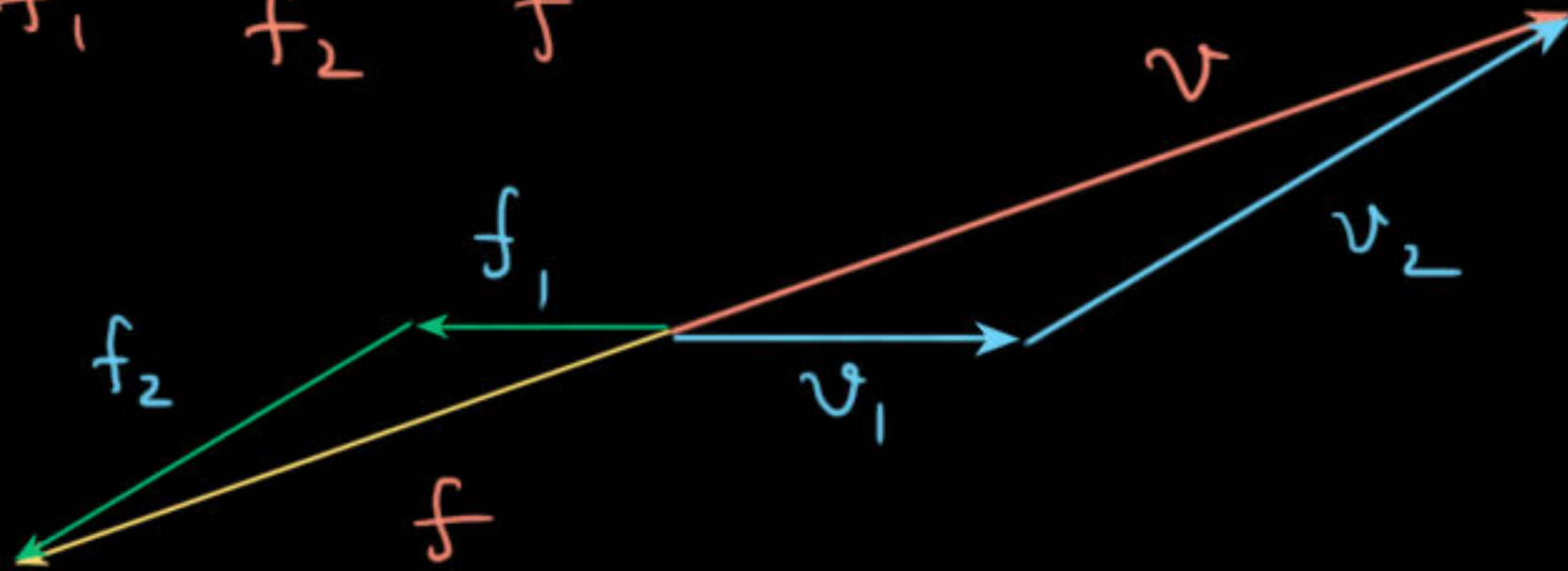
Similarly, on left half $F_{y_2} = \frac{\mu_2 M g}{\pi} \downarrow$

$$\therefore F_{\text{net}} = F_{y_1} - F_{y_2} = (\mu_1 - \mu_2) \frac{M g}{\pi} \uparrow$$

F_{net} on right half will be upwards and left half, downwards.

Approach: Relating velocity components to friction components.

$$\frac{v_1}{f_1} = \frac{v_2}{f_2} = \frac{v}{f}$$

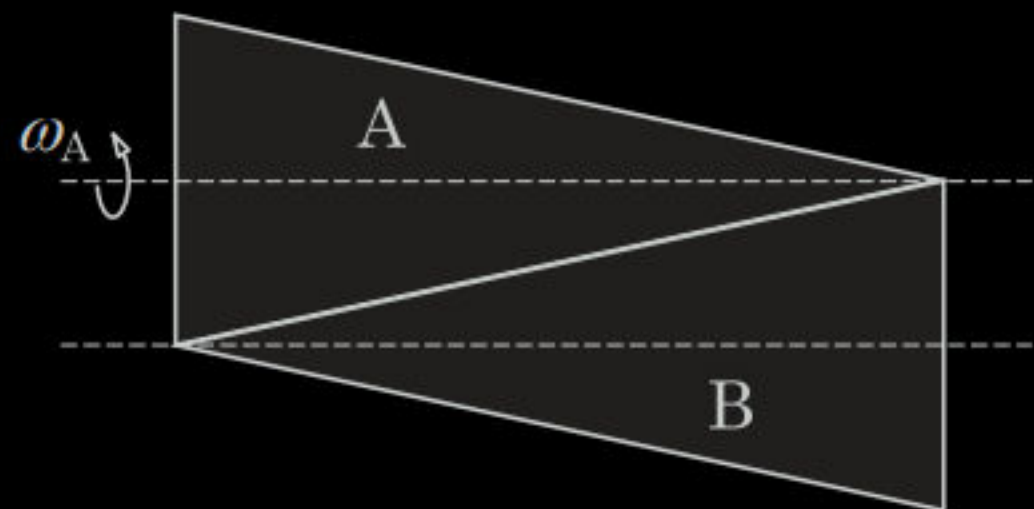


24. A uniform disc of radius R rests with one of its flat faces on a horizontal floor. Coefficient of friction between the disc and the floor is μ . The disc is given an angular velocity ω_0 about its central vertical axis and simultaneously a horizontal velocity v_0 ($v_0 \ll R\omega_0$). Find suitable expression for initial acceleration of the mass centre of the disc.

$$|\vec{v}_0 + \vec{\omega}_0 \times \vec{r}| \approx \omega_0 x$$

$$\frac{v_0}{df_1} = \frac{\omega_0 x}{df_2} = \frac{\sqrt{v_0^2 + \omega_0^2 x^2 + 2v_0(\omega_0 x) \cos \theta}}{\mu dm g}$$

$$\begin{aligned} \therefore F_{\text{net}} &= \int d\vec{f}_1 + \int d\vec{f}_2 = \frac{\mu g v_0}{\omega_0} \int \frac{dm}{x} \\ &= \frac{\mu g v_0}{\omega_0} \int_0^R \frac{2\pi x dx}{\pi R^2} \cdot M \cdot \frac{1}{x} \\ &= \frac{\mu g v_0}{\omega_0} \frac{2M}{R} = \frac{2\mu M g v_0}{\omega_0 R} // \end{aligned}$$



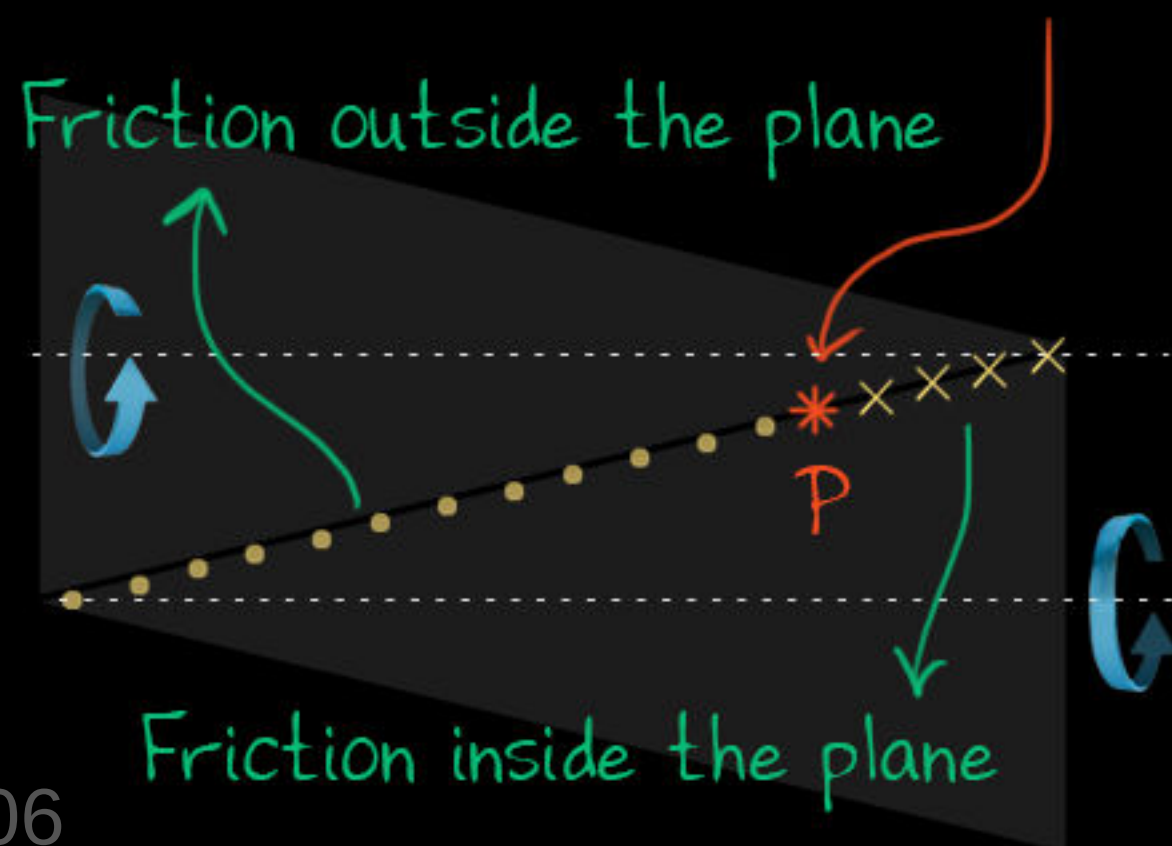
25. Two identical uniform cones A and B can rotate freely about their fixed central axes, pressing each other uniformly on their entire tapered length as shown in the figure. If the cone A is rotated with a constant angular velocity ω_A , how much angular velocity will the cone B ultimately acquire due to friction between the cones?

Concept: At steady state, net torque on B about its axis will be zero.

Shown force distribution satisfies it.

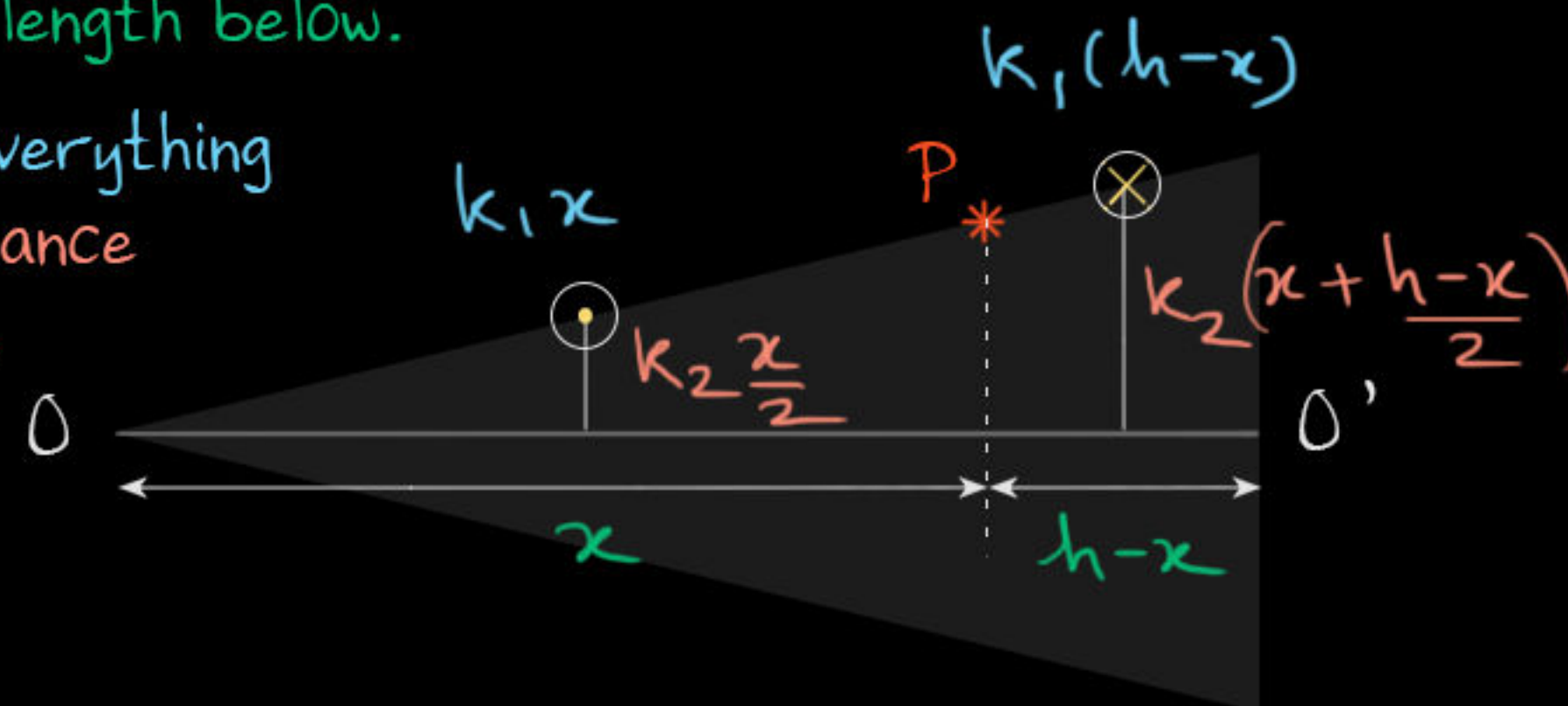
No slipping here

Friction outside the plane



We will replace the uniformly distributed forces with a single force proportional to axis length below.

Approach: Express everything (f, r) in terms of distance from vertex of cone.



$$\text{Balancing the torque about } OO': \quad k_1 x \cdot k_2 \frac{x}{2} = k_1 (h-x) \cdot k_2 \left(\frac{h+x}{2} \right)$$

$$\Rightarrow x = h/\sqrt{2}$$

v_P is same in both cones

$$\therefore v_P = \omega_A \cdot k_2 \left(\frac{h-x}{2} \right) = \omega_B \cdot k_2 x \Rightarrow \omega_B = (\sqrt{2}-1) \omega_A$$

by Ankit Singhvi

Youtube / Arihant Senior

1. A satellite revolves in an equatorial plane in west to east direction around the earth with a period of 8 hrs. At an instant, it is observed vertically overhead a longitude. After how much minimum time will the satellite again be overhead the same longitude?

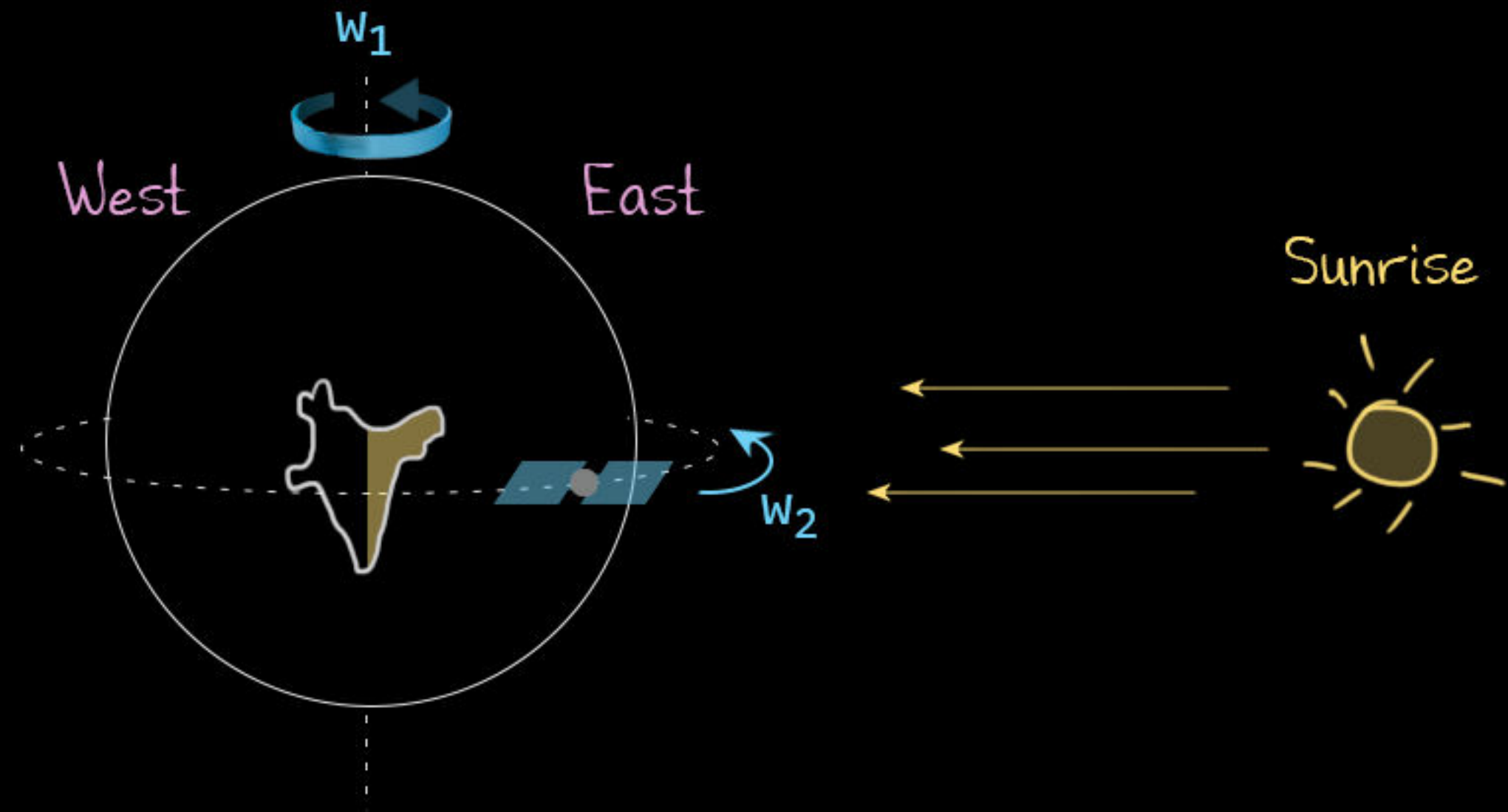
(a) 8 h

✓(b) 12 h

(c) 24 h

(d) 36 h

Earth rotates from west to east too.
That's how we get sunrise from east.



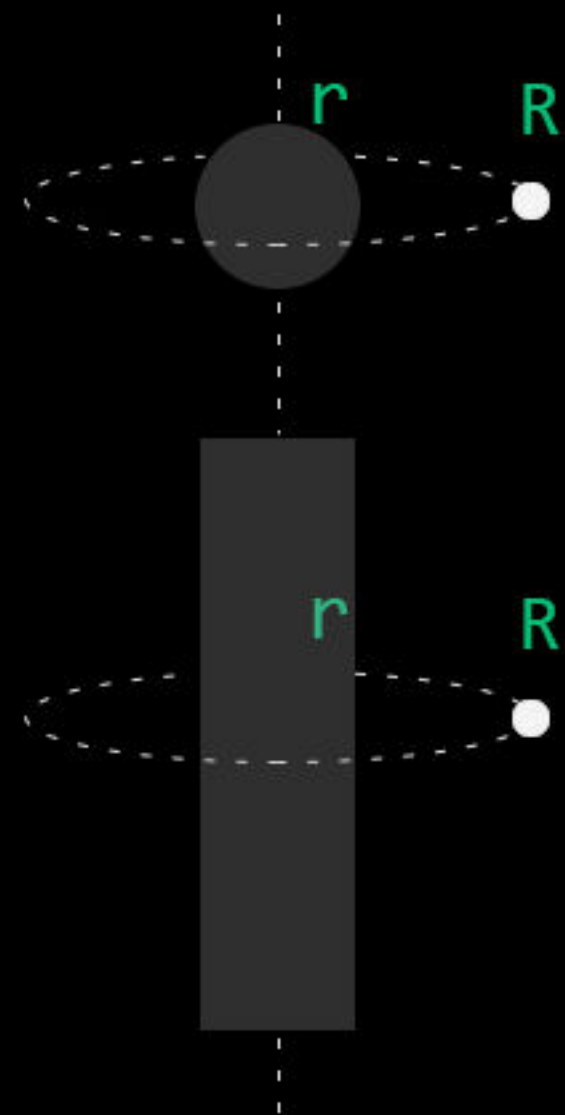
$$W_2 > W_1$$

$$\therefore |W_{\text{relative}}| = W_2 - W_1$$

$$\therefore T_{\text{required}} = \frac{2\pi}{W_{\text{relative}}} = \frac{2\pi}{\frac{2\pi}{T_2} - \frac{2\pi}{T_1}} = \frac{T_1 T_2}{T_1 - T_2} = \frac{(24) 8}{24 - 8} = 12 \text{ hrs},$$

For a body undergoing circular motion: $f = \frac{mv^2}{R}$

If $m \neq R$ are same: $f \propto v^2$



2. Assume that the Earth changes its shape and turns into an infinite cylinder whose radius and density are the same as those of our real Earth and the distance of the Moon from the central axis of this cylindrical earth remains unchanged. What can you say about the speed of the Moon (which remains spherical) in its orbit around the cylindrical Earth?

- (a) It will slightly increase. (b) It will slightly decrease.
(c) It will remain unchanged. ✓(d) It will increase several times.

Here too, moon's radius of orbit and mass are same.

$$\therefore \frac{v_c^2}{v_s^2} = \frac{F_c}{F_s} = \frac{\cancel{m} \cancel{2G} \cancel{1} / R}{\cancel{G} \cancel{m} \cancel{M} / R^2} = \frac{\frac{8\pi R^2 \rho}{2}}{\frac{8 \cdot \frac{4}{3} \pi R^3 \rho}{3} / R} = \frac{3R}{2R} \gg 1 \quad (\text{As } R \gg R)$$

3. Suppose the Saturn rings consist of football size particles orbiting in circular orbits. If the maximum speed of these particles is 1.05 times of the minimum speed, find ratio of radial width to the inner radius of the ring.

(a) 0.01

(b) 0.05

✓(c) 0.10

(d) 0.25

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow v^2 R = \text{constant}$$

$$\Rightarrow v^2 dR + 2v dv R = 0$$

$$\Rightarrow \frac{dR}{R} = -2 \frac{dv}{v} = -2 \left(\frac{+0.05}{1} \right) = -0.1 //$$

As expected. Since if $v \uparrow$ then $R \downarrow$

4. Two planets A and B are orbiting around their sun in circular orbits of radii r and $4r$. In a quarter year of planet B, how many revolutions will the planet A make?

(a) 1

✓(b) 2

(c) 4

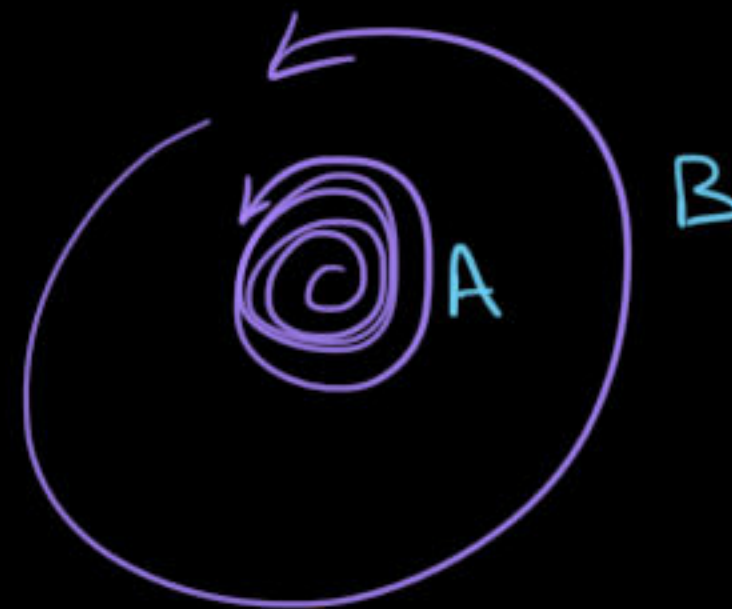
(d) 8

Year on a planet = Time that planet takes to complete a revolution.

$$T^2 \propto R^3$$

$$\Rightarrow \frac{T_A^2}{T_B^2} = \left(\frac{\cancel{r}}{\cancel{4r}} \right)^3$$

$$\Rightarrow T_B = 8T_A \Rightarrow$$



If B makes one round,
in same time A makes 8.

⇒ In quarter year of B,
A makes 2 rounds.

5. An asteroid orbiting around a planet in **circular orbit** suddenly **explodes** into two fragments **in mass ratio 1:4**. If immediately after the explosion, the **smaller fragment** starts orbiting the planet in **reverse direction** in **the same orbit**, **what will happen with the heavier fragment?**

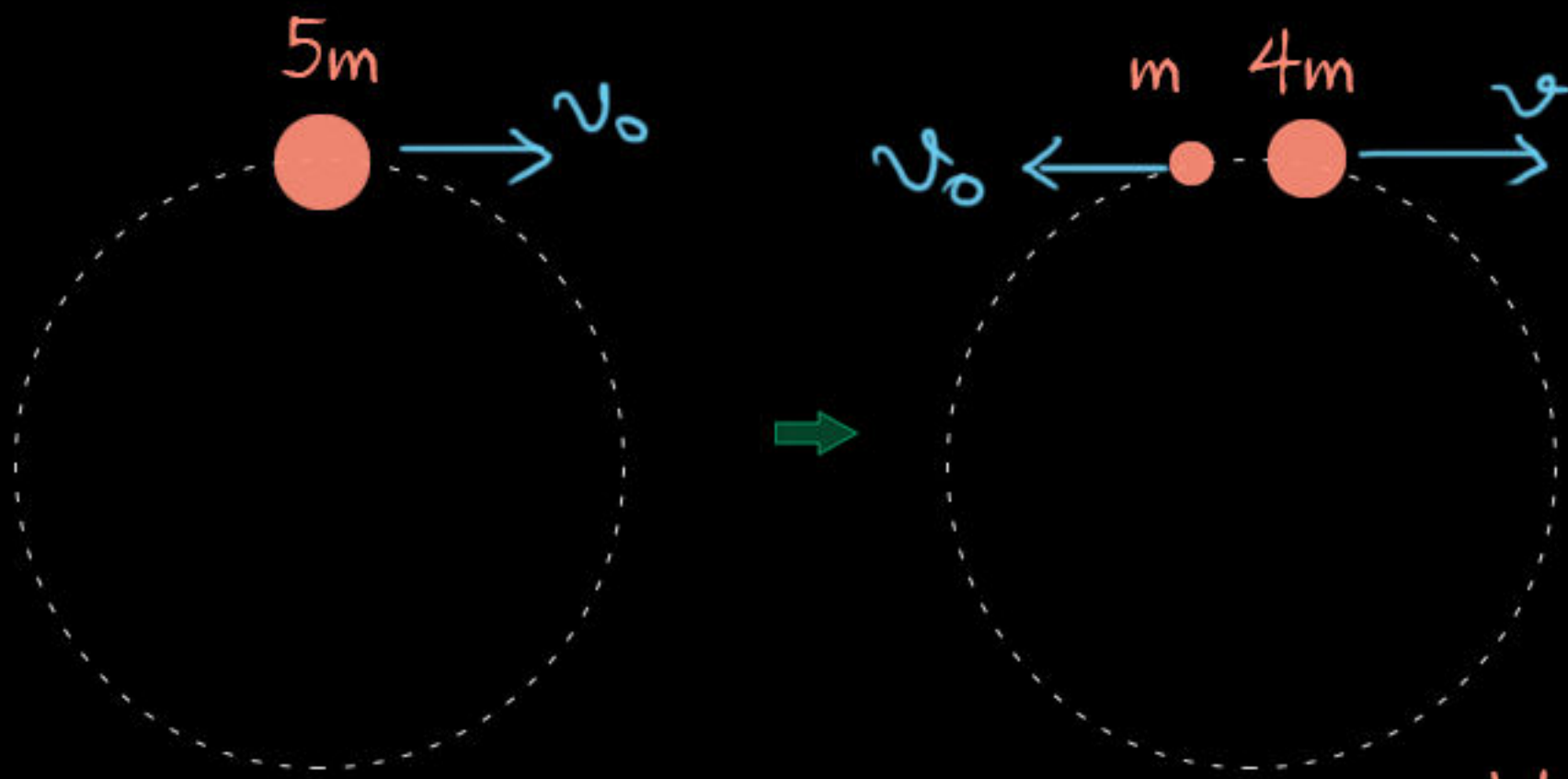
(a) Fall on the planet.

(b) Start orbiting the planet in a larger orbit.

(c) Start orbiting the planet in a smaller orbit.

✓(d) Escape from the gravitational interaction with the planet.

Let orbital velocity be v_0



p conservation: $5mv_0 = 4mv - mv_0$

$$\Rightarrow v = \frac{3}{2}v_0$$

We know, $v_{esc} = \sqrt{2}v_0$

$v > v_{esc} \Rightarrow$ will escape. //

6. The sun orbits the centre of the galaxy (Milky-Way) in almost circular path of radius R in a period T and the earth also orbits the sun in an almost circular path of radius r in a period t . Assume whole mass of the galaxy concentrated at its centre and find an expression for the ratio of the mass of the galaxy to that of the sun.

$$T = \frac{2\pi R}{v}$$

$$= \frac{2\pi R \sqrt{R}}{\sqrt{GM}}$$

$$\Rightarrow T^2 \propto \frac{R^3}{M}$$

$$\Rightarrow \frac{MT^2}{R^3} = \text{constant}$$

Mass of object around which other body is rotating.

More generic relation than well known: $T^2 \propto R^3$

$$\checkmark (a) \left(\frac{R}{r}\right)^3 \left(\frac{t}{T}\right)^2$$

$$(c) \left(\frac{R}{r}\right)^2 \left(\frac{t}{T}\right)^3$$

$$(b) \left(\frac{R}{r}\right)^3 \left(\frac{T}{t}\right)^2$$

$$(d) \left(\frac{R}{r}\right)^2 \left(\frac{T}{t}\right)^3$$

$$\therefore \frac{M_g T^2}{R^3} = \frac{M_s t^2}{r^3}$$

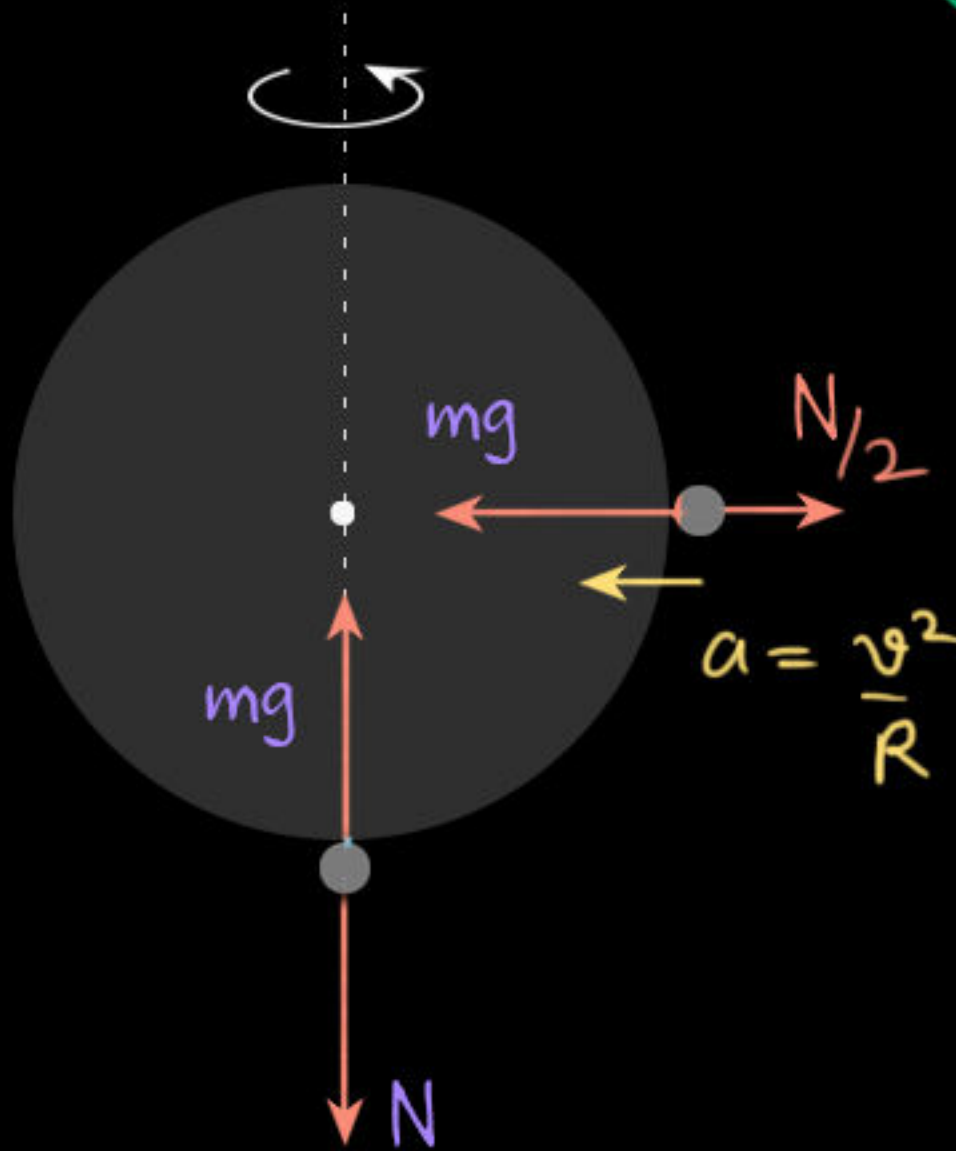
$$\Rightarrow \frac{M_g}{M_s} = \left(\frac{R}{r}\right)^3 \left(\frac{t}{T}\right)^2 //$$

7. Weight of a body at the equator of a planet is half of that at the poles. If peripheral velocity of a point on the equator of this planet is v_0 , what is the escape velocity of a polar particle?

(a) v_0
(c) $3v_0$

✓(b) $2v_0$
(d) $4v_0$

⇒ Weight variation is due to rotation of planet, and NOT shape of planet.



Weight (N) here is as measured by a weighing machine

@ pole $N = mg$

@ equator $mg - \frac{N}{2} = m \frac{v^2}{R}$

⇒ $\frac{mg}{2} = \frac{mv^2}{R}$

⇒ $g = \frac{2v^2}{R}$ ①

We know, at pole: $v_{esc} = \sqrt{2gR}$

↓ From 1

$= 2v_0 //$

at equator, w.r.t planet:

$v_{esc} = 2v_0 - v_0 = v_0 //$

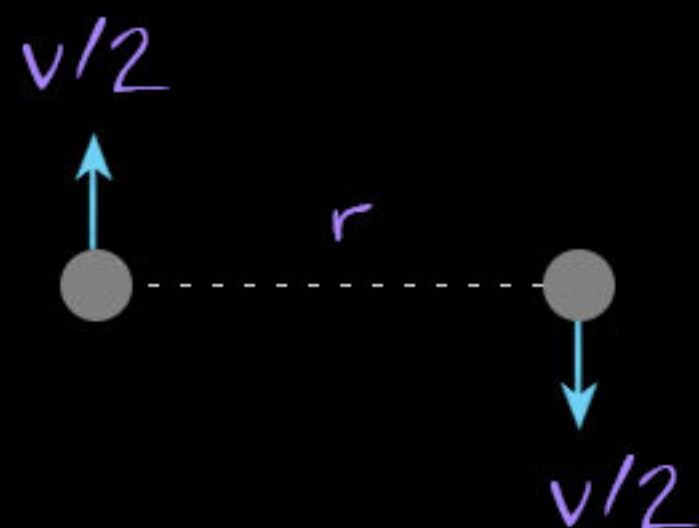
Concept: In bound system,
Total Energy (w.r.t CoM) < 0 .

NOT " $\leq$ "

w.r.t ground



w.r.t CoM



8. Consider two identical particles each of mass m held at a separation r_0 in free space. One of them is given a velocity v_0 perpendicular to r_0 and the other one is simultaneously released. For what range of velocity v_0 will the masses be bound in orbital motion under their mutual gravitational forces.

(a) $v_0 \leq \sqrt{\frac{2Gm}{r_0}}$

(b) $v_0 < \sqrt{\frac{2Gm}{r_0}}$

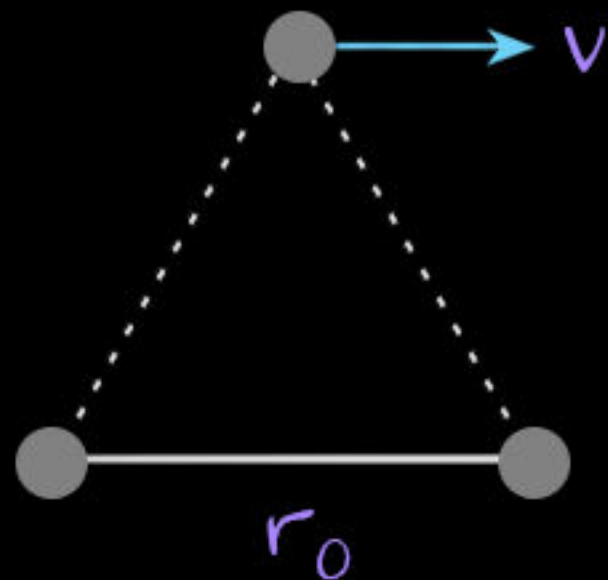
(c) $v_0 \leq 2\sqrt{\frac{Gm}{r_0}}$

✓ (d) $v_0 < 2\sqrt{\frac{Gm}{r_0}}$

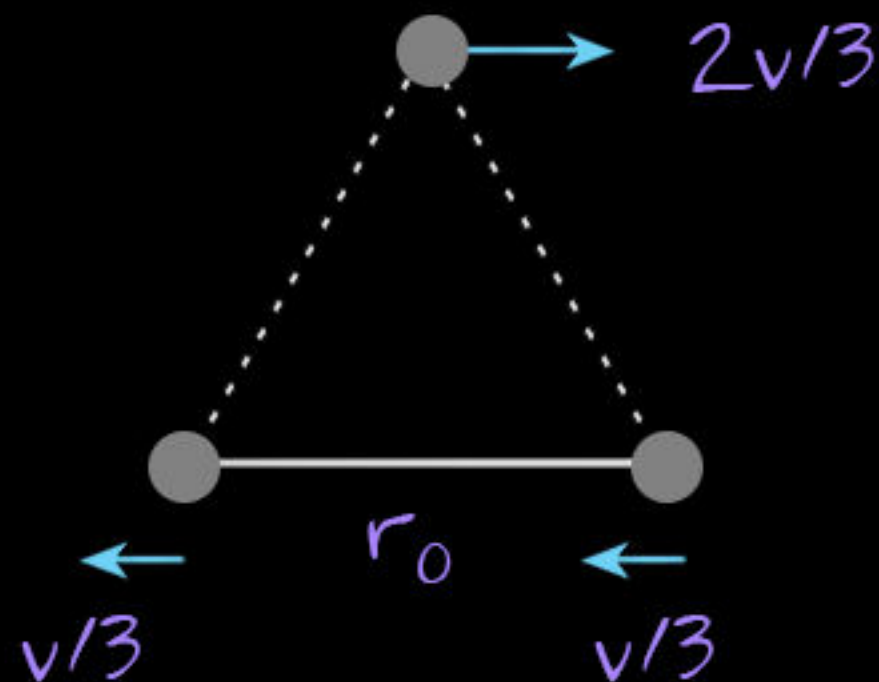
$$\therefore \text{Total energy} = 2 \left[\frac{1}{2} m \left(\frac{v}{2} \right)^2 \right] - \frac{Gm^2}{r} < 0$$

$$\Rightarrow v < 2\sqrt{\frac{Gm}{r}}$$

w.r.t ground



w.r.t CoM



9. In free space, two identical particles A and B are rigidly attached at the ends of a light rigid rod of length r_0 and another identical particle C is held at a position in such a way that the locations of the particles make an equilateral triangle. The particle C is given a velocity v_0 tangential to the circumscribing circle of the triangle and the other two are simultaneously released. For what range of velocity v_0 will the particles be bound under their mutual gravitational forces?

In bound system: Total Energy = KE + PE < 0. ✓ (d) $v_0 < \sqrt{\frac{6Gm}{r_0}}$

w.r.t CoM

(As K.E is frame dependent)

Disregard Self-energy of rigid bodies.
In this case, dumbell.

NOT " $\leq$ "

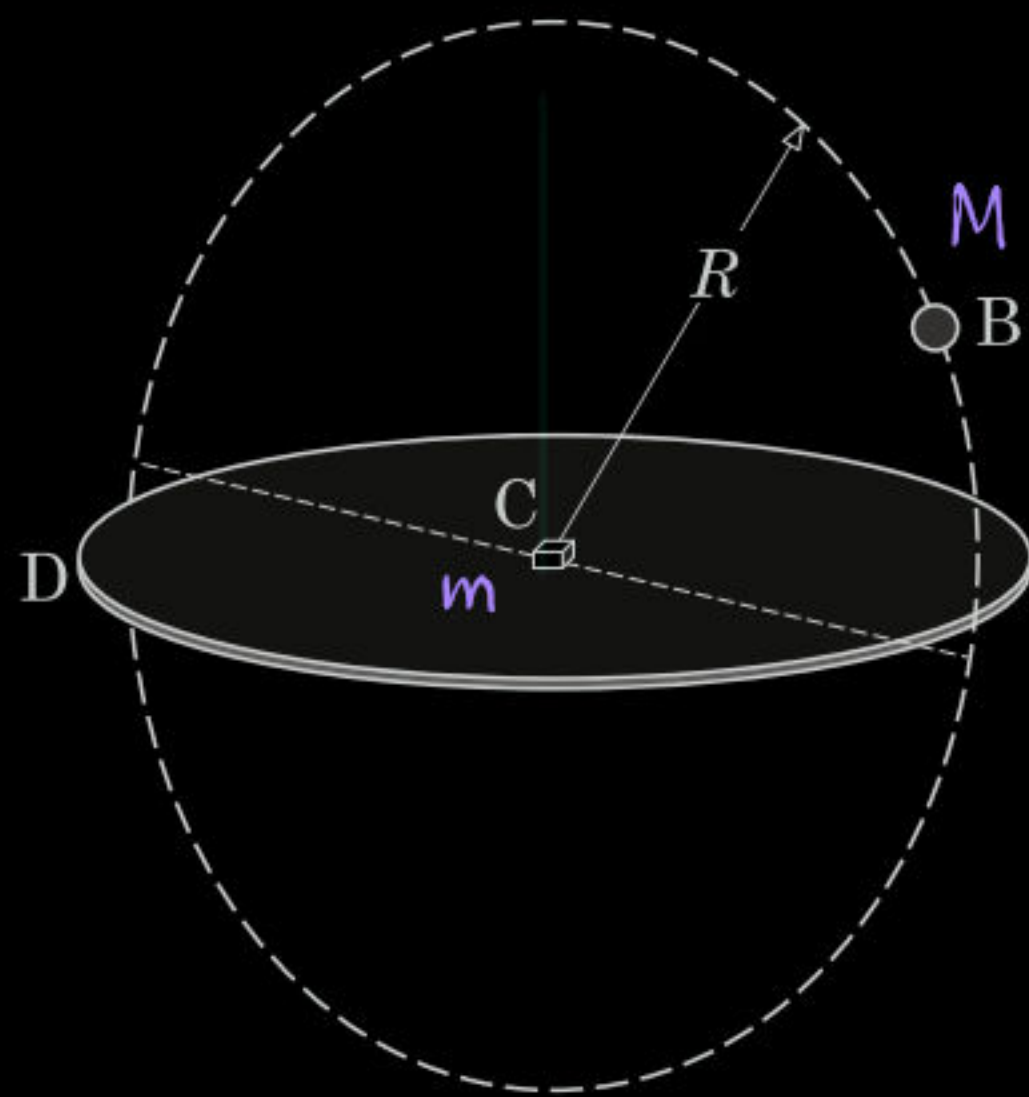
$$\therefore \frac{1}{2} m \left(\frac{2v}{3} \right)^2 + 2 \left[\frac{1}{2} m \left(\frac{v}{3} \right)^2 \right] + 2 \left[-\frac{Gm^2}{r_0} \right] < 0 \Rightarrow v < \sqrt{\frac{6Gm}{r_0}}$$

Note: We can NOT solve this by placing particles of rod at its CoM. As PE between third particle with these two will change.



by Ankit Singhvi

Youtube / Arihant Senior



10. Consider a thought experiment performed in free space. In the experiment, a small cube C of mass m is placed at the centre of a large and highly massive disc D and a ball B of mass M revolves in circular path of radius R around the centre of the disc. The plane of the circle is perpendicular to the plane of the disc as shown in the figure. Intensity of the gravitational field of the disc near its centre is g . What should be range of coefficient of friction between the cube and the disc so that the cube remains motionless?

$$\checkmark (c) \mu \geq \frac{GM}{\sqrt{(gR^2)^2 - (GM)^2}}$$

$$f = F \sin \theta \leq \mu N$$

$$\Rightarrow F \sin \theta \leq \mu (mg - F \cos \theta)$$

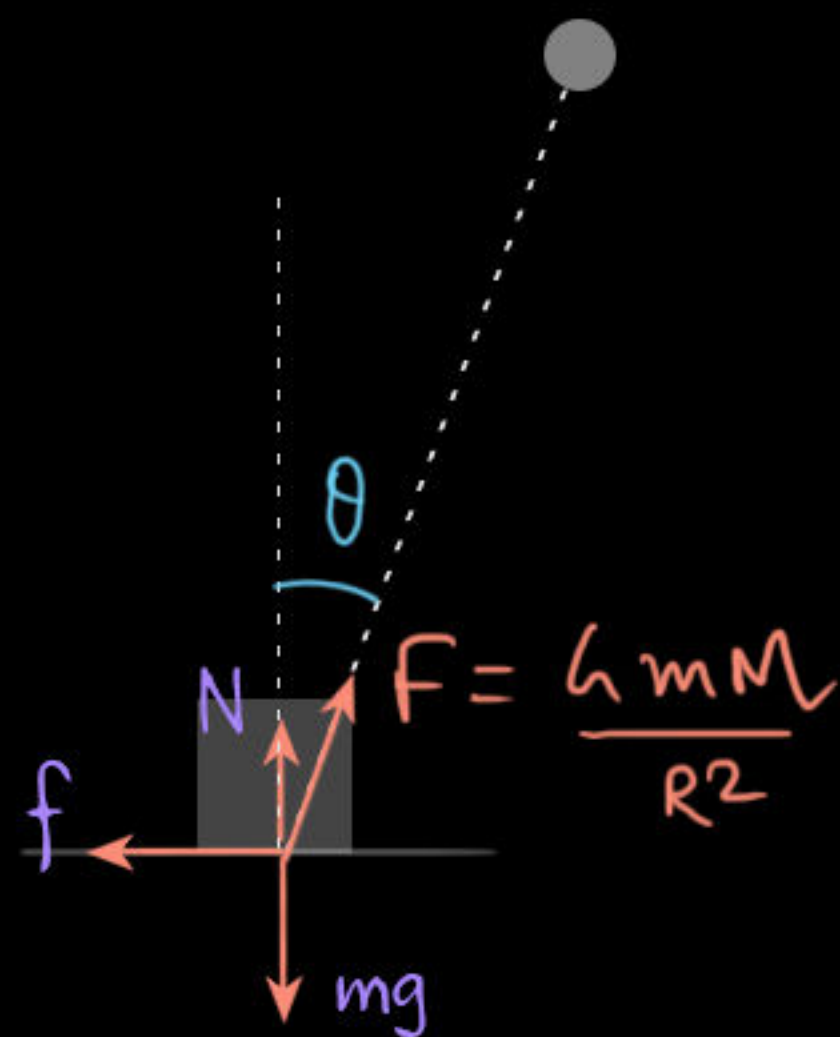
$$\Rightarrow f(\sin \theta + \mu \cos \theta) \leq \mu mg$$

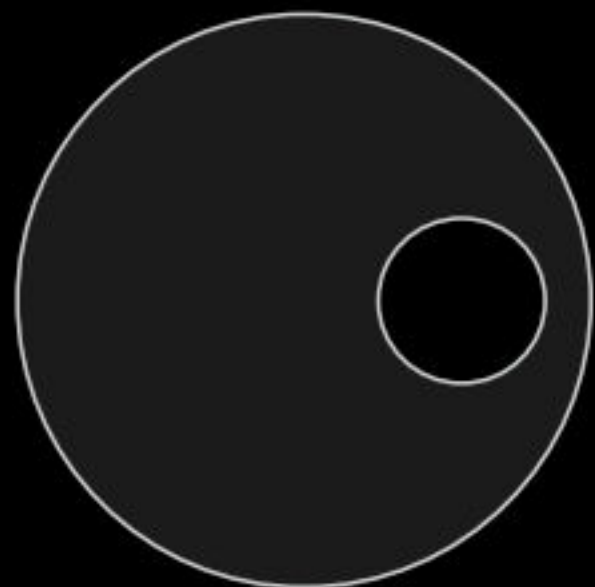
Maximum (doesn't matter what μ is)

$$\Rightarrow F \sqrt{1 + \mu^2} \leq \mu mg$$

$$\Rightarrow F^2 (1 + \mu^2) \leq \mu^2 m^2 g^2 \Rightarrow \mu \geq \frac{F}{\sqrt{m^2 g^2 - F^2}}$$

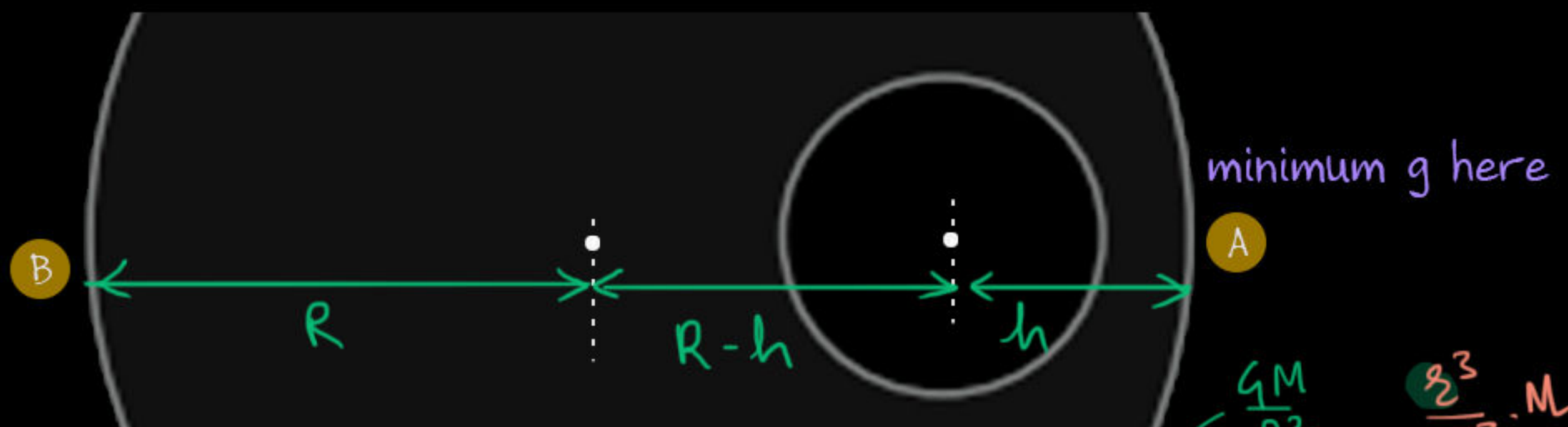
$$f \rightarrow \frac{GmM}{R^2}$$





1. Acceleration due to gravity everywhere on the equator of a spherical asteroid of radius R is g_0 . A spherical cavity with its centre in the equatorial plane inside the asteroid is made and then acceleration due to gravity is measured everywhere on the equator. The minimum accelerations due to gravity is observed smaller than g_0 by η_1 fraction of g_0 at a point and at its diametrically opposite point, acceleration due to gravity is observed smaller than g_0 by η_2 fraction of g_0 . Find radius of the cavity and depth of its centre.

i.e. $= g_0 - \eta_1 g_0 = (1 - \eta_1) g_0$



$$\text{@ A: } g_0(1 - \eta_1) = \frac{GM}{R^2} - \frac{Gm}{h^2} \Rightarrow \eta_1 g_0 = \frac{Gm}{h^2} \quad (1)$$

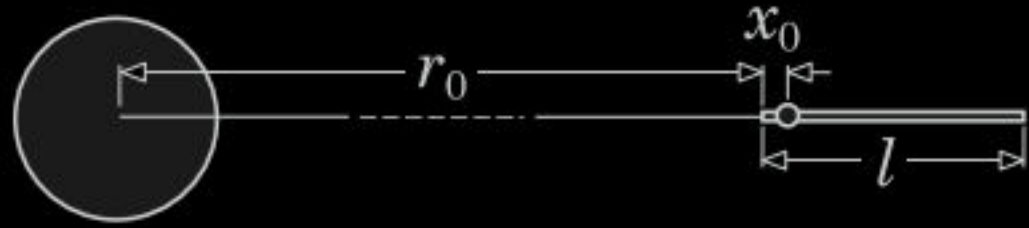
$$\text{Similarly @ B: } \eta_2 g_0 = \frac{Gm}{(2R - h)^2} \quad (2)$$

Dividing 1 and 2:

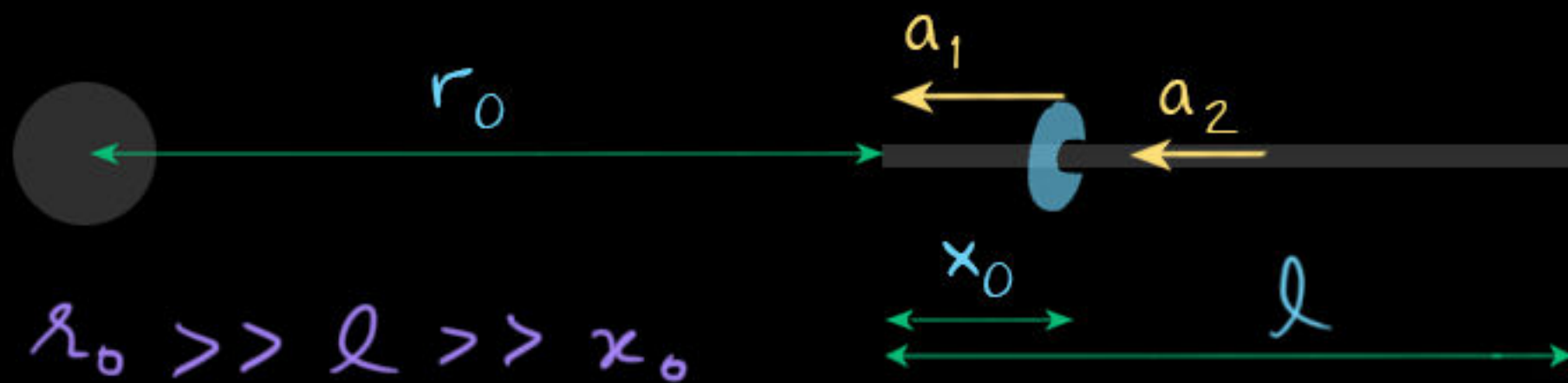
$$h = \frac{2\sqrt{\eta_2} R}{\sqrt{\eta_1} + \sqrt{\eta_2}}$$

$$r^3 = \frac{4\eta_1 \eta_2 R^3}{(\sqrt{\eta_1} + \sqrt{\eta_2})^2}$$

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2. A small bead can slide without friction on a wooden rod of length $l = 10.0$ m. Initially the rod and the bead both are held motionless with the rod aligned radially with the earth. The left end of the rod is at a distance $r_0 = 4 \times 10^8$ m from the earth centre and the bead is at a distance $x_0 = 2.0$ cm away from the left end. Both the bodies are released simultaneously. Considering gravitational interaction only with the earth, find how long after the release, will the bead separate from the rod? Radius of the earth is $R = 6400$ km and acceleration due to gravity on the earth is $g = 10 \text{ m/s}^2$.



Assumption 1: Rods mass can be taken at CoM for force calculation.

Assumption 2: Initial $a_{\text{relative}} = a_1 - a_2$ can be assumed constant.

$$a_1 - a_2 = \frac{GM}{r_0^2} \left[\frac{1}{(r_0 + x_0)^2} - \frac{1}{(r_0 + l/2)^2} \right]$$

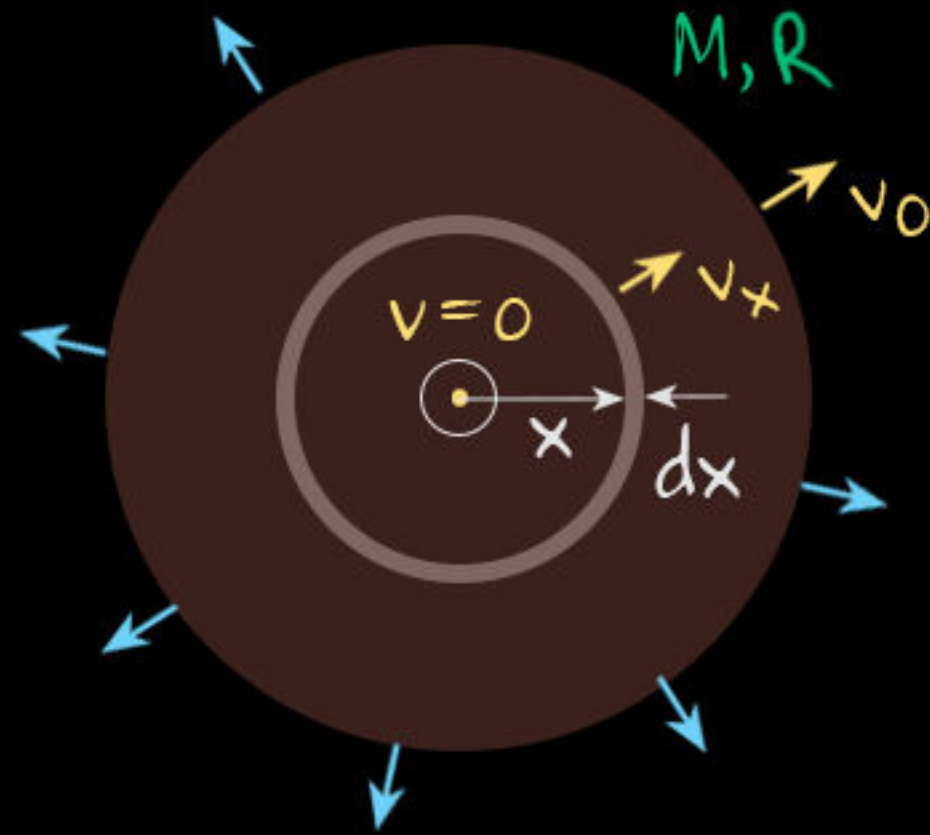
$$\approx gR^2 \frac{r_0^2 \left[\left(1 + \frac{2l}{2r_0}\right) - \left(1 + \frac{2x_0}{r_0}\right) \right]}{r_0^4}$$

$$= \frac{gR^2(l - 2x_0)}{r_0^3} \approx \frac{gR^2 l}{r_0^3}$$

Lets say bead slips in t time.

$$\therefore x_0 = \frac{1}{2} (a_1 - a_2) t^2 \Rightarrow t = \sqrt{\frac{2x_0}{a_1 - a_2}}$$

spherical sponge ball



$$dKE = \frac{1}{2} dm v_x^2$$

$$= \frac{1}{2} \frac{4\pi x^2 dx}{\frac{4}{3}\pi R^3} M \left(\frac{x v_0}{R} \right)^2$$

$$\Rightarrow KE = \frac{3M v_0^2}{2 R^5} \int_0^R x^4 dx$$

$$= \frac{1}{2} \left(\frac{3M}{5} \right) v_0^2$$

3. An asteroid of mass M explodes into a spherical homogeneous cloud in free space. Due to energy received by the explosion, the cloud expands and the expansion is spherically symmetric. At an instant, when radius of the cloud is R_0 , all of its particles on the surface are observed receding radially away from the centre of the cloud with a velocity v_0 . What will the radius of the cloud be, when its expansion ceases.

Concept: Given situation is similar to a homogeneous sponge ball with center at rest and outer surface moving out at v_0 .

Self energy of a sphere: $-\frac{3}{5} \frac{Gm^2}{R}$ (of a shell: $-\frac{1}{2} \frac{Gm^2}{R}$)

Sol:

$$U_i + KE_i = U_f + KE_f$$

$$\Rightarrow -\frac{3}{5} \frac{Gm^2}{R_0} + \frac{1}{2} \left(\frac{3M}{5} \right) v_0^2 = -\frac{3}{5} \frac{Gm^2}{R_f}$$

$$\Rightarrow \frac{1}{R_f} = \frac{1}{R_0} \left(\frac{Gm}{R_0} - \frac{v_0^2}{2} \right) //$$

(of a massive spring: $\frac{1}{2} \left(\frac{m}{3} \right) v_0^2$)

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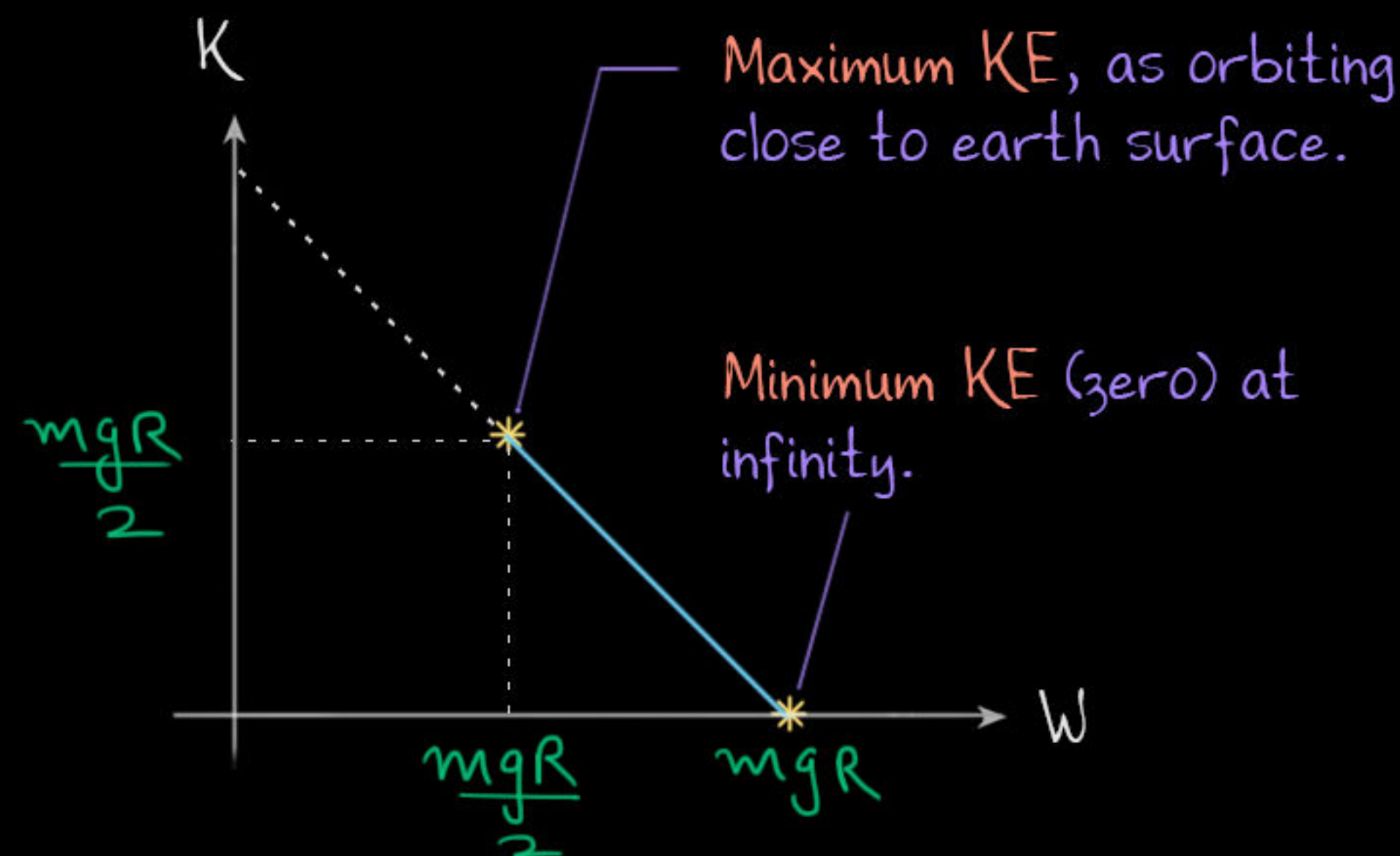
4. Rockets are used to launch satellites. Draw a graph to show how kinetic energy K imparted to a satellite varies with work done W by the rocket used for all possible circular orbits. Ignore effects of rotation of the earth and resistance of the atmosphere. Denote the mass of the satellite by m , radius of the earth by R and acceleration of free fall near the earth surface by g .

$$W_{\text{by rocket}} = W_{\text{on satellite}} = E_f - E_i$$

$$\Rightarrow W = -\frac{GmM}{2R} - \left(-\frac{GmM}{R}\right)$$

$$\Rightarrow W = -K + mgR$$

$$\Rightarrow K = -W + mgR$$



5. A reconnaissance satellite (spy satellite) of mass m orbiting the earth at very low altitude experiences a retarding force. Assuming magnitude of this retarding force F a constant, find the time rate at which radius of the orbit changes when speed of the satellite is v . Acceleration due to gravity near the earth surface is g .

Correct approach ✓ $dW_{ext} = dE_{system}$ $-\frac{GmM}{r^2}$

$$\Rightarrow \frac{dW_{ext}}{dt} = P_{ext} = -fv = \frac{d}{dt} (E_{system})$$

$$\Rightarrow -fv = -\frac{GmM}{r^2} \left(-\frac{1}{r^2} \frac{dr}{dt} \right)$$

$$\Rightarrow fv = \frac{mg}{2} \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{-2fv}{mg} //$$

✗ Wrong approach

$$v^2 = \frac{GM}{r} \quad \Rightarrow \quad 2v dv = -\frac{GM}{r^2} dr$$

This equation is **correct**, if dv and dr represent difference in velocities and radii of two different particles in nearby circular orbits.

$$2v \frac{dv}{dt} = -\frac{GM}{r^2} \frac{dr}{dt}$$

In other words, this equation is
✓ fine for 'difference' in two particles
✗ but not for 'change' in a single particle

$$2v \left(\cancel{\frac{F}{m}} \right) = \cancel{-g} \frac{dr}{dt}$$

This equation is however, **meaningless**.

As time has no significance when talking of two different particles.

$$\frac{dr}{dt} = + \frac{2Fv}{mg} \quad \text{✗}$$

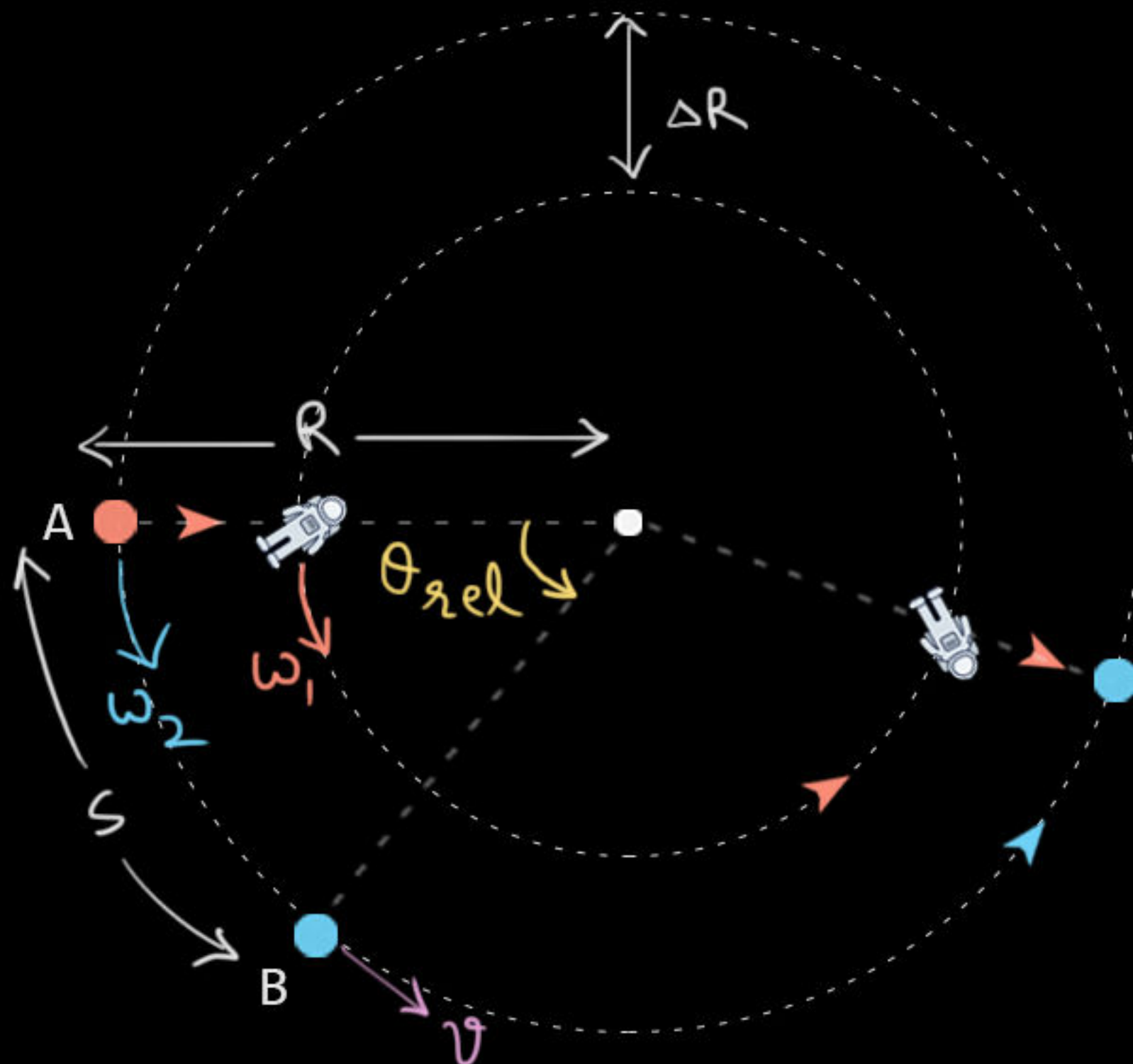
→ Correct answer has a negative sign.

6. Two space stations A and B are orbiting the earth with speed v in the same orbit of radius R . Station B is a distance s ahead of station A. Two astronauts from station A have to join the station B. For this purpose, they leave station A in a spaceship and move radially to a slightly lower orbit and stay in that orbit and when the station B comes closest to them, they immediately move radially to join the station B. If time spend in the lower orbit is Δt , find separation between the orbits.

$$\theta_{rel} = \omega_{rel} \Delta t$$

$$\frac{s}{R} = (\omega_1 - \omega_2) \Delta t$$

$$\omega = \frac{v}{R} = \frac{\sqrt{GM}}{R^{3/2}}$$



$\Rightarrow$

$$\frac{s}{R} = \sqrt{GM} \left(\frac{1}{(R - \Delta R)^{3/2}} - \frac{1}{R^{3/2}} \right) \Delta t$$

$$= \frac{\sqrt{GM}}{R^{3/2}} \left[\left(1 - \frac{\Delta R}{R} \right)^{-3/2} - 1 \right] \Delta t$$

$\Rightarrow$

$$\frac{s}{R} \approx \frac{\sqrt{GM}}{R^{3/2}} \frac{3 \Delta R}{2R} \Delta t$$

$\Rightarrow$

$$\Delta R = \frac{2sR^{3/2}}{3\sqrt{GM} \Delta t} = \frac{2sR}{3v \Delta t} //$$

Hint: Irodov 1.203

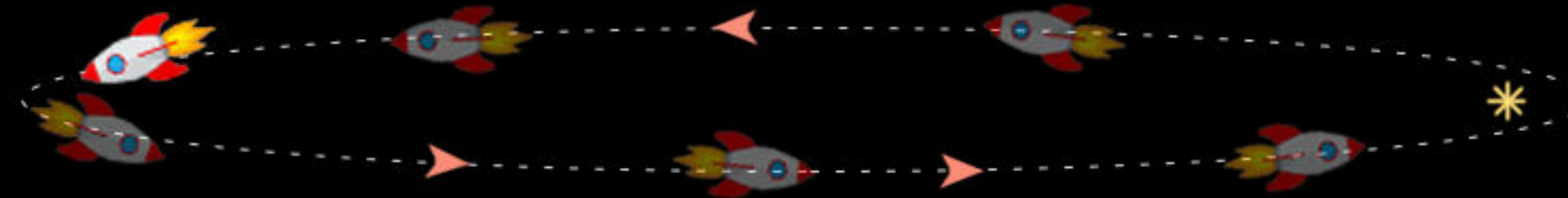
7. Astronauts in a spaceship moving with a constant velocity v_0 observe a massive planet at a great distance also moving with velocity v_0 at an angle of 120° with the direction of motion of the spaceship. The astronauts have to reverse the direction of motion of their spaceship. Determine how much maximum speed in reverse direction, the spaceship can acquire without expenditure of fuel with the help of gravitational field of the planet?

Before and after the end of maneuver, assume the distance of spaceship from planet to be same.

Approach: For maximum reverse velocity, planet must be very heavy. So it will pull the spaceship in an almost flat elliptical orbit, with planet at focus. With near zero velocity (compared to its velocity near the planet), spaceship lies at other end of this flat ellipse.

∴ At other extreme, if we planet is very light, spaceship won't reverse direction at all.

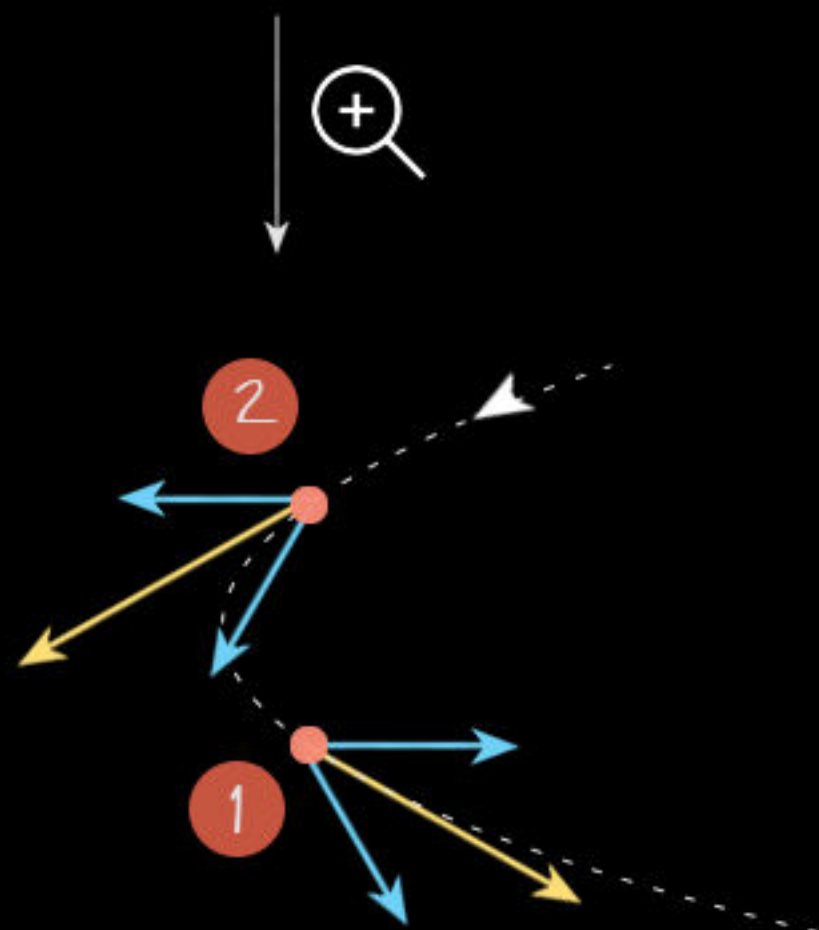
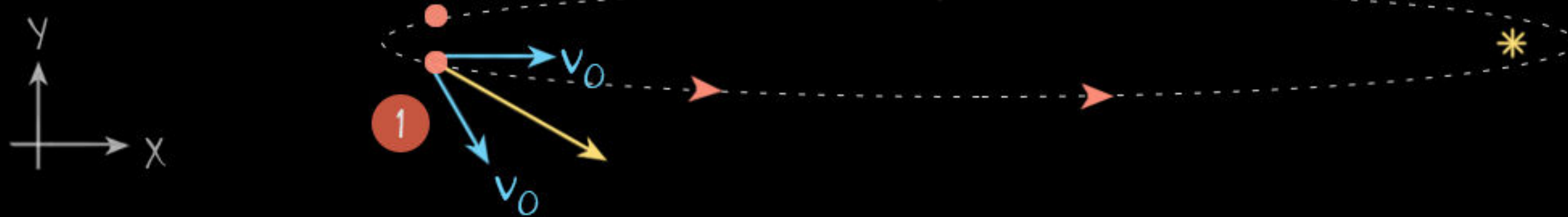
w.r.t planet:



w.r.t given reference frame:

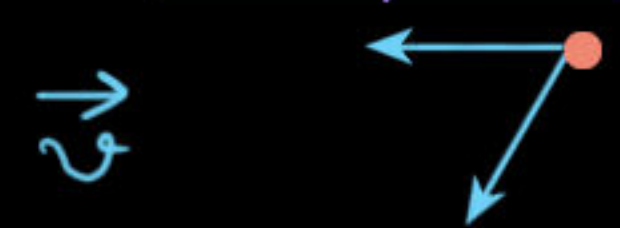


w.r.t planet

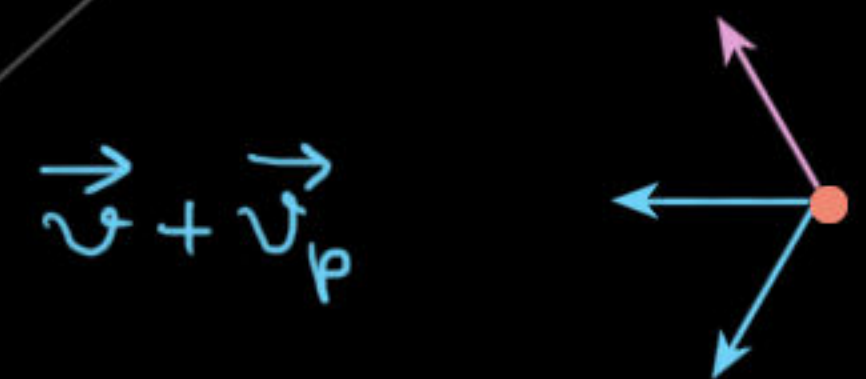


∴ At 2 on returning:

w.r.t planet:



w.r.t given frame:

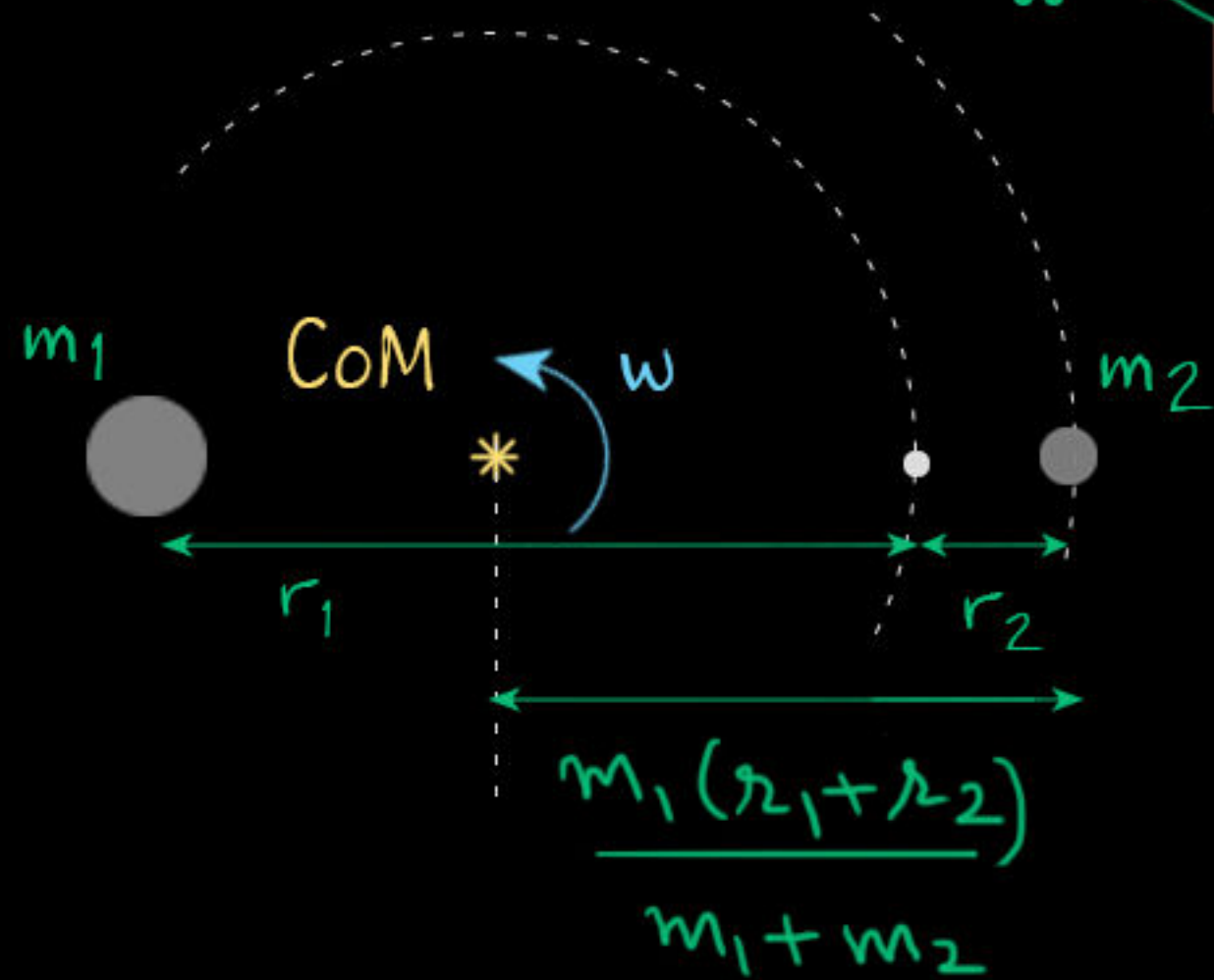


$$\begin{aligned} \therefore v_{\text{reverse}} &= v_0 + \cancel{2v_0 \cos \pi/3} \\ &= 2v_0 \end{aligned}$$

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by Ankit Singhvi

When its between planets:



8. A space ship sent to study a binary star system is set into an orbit in order to remain always collinear with the stars. If distances of the spaceship from the stars are r_1 and r_2 , find ratio of masses of the stars. Is this orbit a stable one?

@ m_2 : $F = m \omega^2 R$

$$\frac{G m_1 m_2}{(r_1 + r_2)^2} = m_2 \omega^2 \frac{m_1 (r_1 + r_2)}{(m_1 + m_2)}$$

$$\Rightarrow \omega^2 = \frac{G (m_1 + m_2)}{(r_1 + r_2)^3}$$

@ satellite: $F = m \omega^2 R$

$$\frac{G m_1}{r_1^2} - \frac{G m_2}{r_2^2} = \frac{G (m_1 + m_2)}{(r_1 + r_2)^3} \left[\frac{m_1 (r_1 + r_2)}{m_1 + m_2} - r_2 \right]$$

$$\left(\frac{m_1}{m_2} \Rightarrow t \right) \Rightarrow \frac{t}{r_1^2} - \frac{1}{r_2^2} = \frac{t r_1 - r_2}{(r_1 + r_2)^3}$$

$$\Rightarrow t = \left[\frac{(r_1 + r_2)^3 - r_2^3}{(r_1 + r_2)^3 - r_1^3} \right] \frac{r_1^2}{r_2^2}$$

Unstable as if shifted right it will fall towards m_2 .

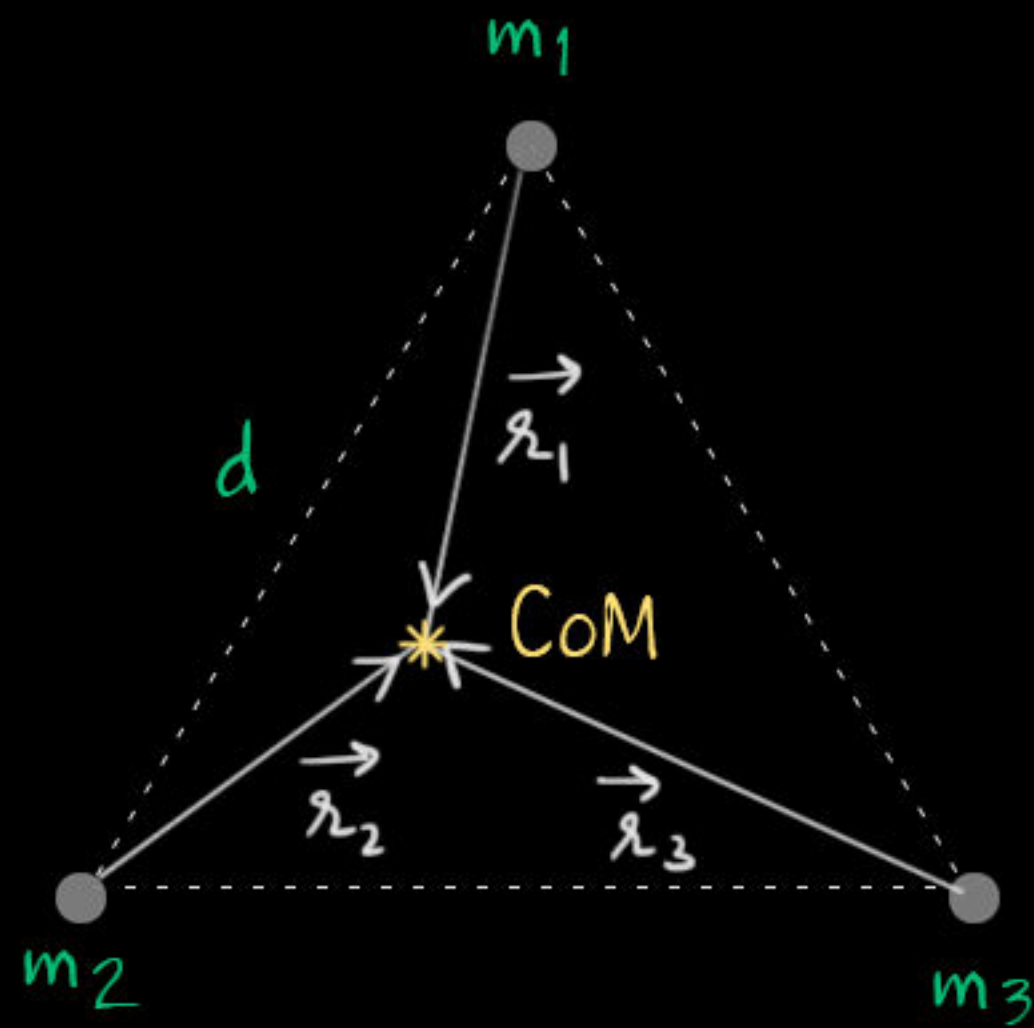
When its not between planets: (and is to the right of m_2)

$r_2 \rightarrow -r_2$ X

sign should not change here and it doesnt happen with $r_2 \rightarrow -r_2$

So must do calculations again to get:

$$t = \left[\frac{r_2^3 - (r_1 - r_2)^3}{(r_1 - r_2)^3 - r_1^3} \right] \frac{r_1^2}{r_2^2}$$



1 CoM:

$$m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 = 0$$

9. Three material points of masses m_1 , m_2 and m_3 are at the vertices of an equilateral triangle of side d . The system is rotating in free space in such a way that under the mutual gravitational interaction of the three particles, the system is neither expanding nor contracting. Find angular velocity ω of this rotation. Universal gravitational constant is G .

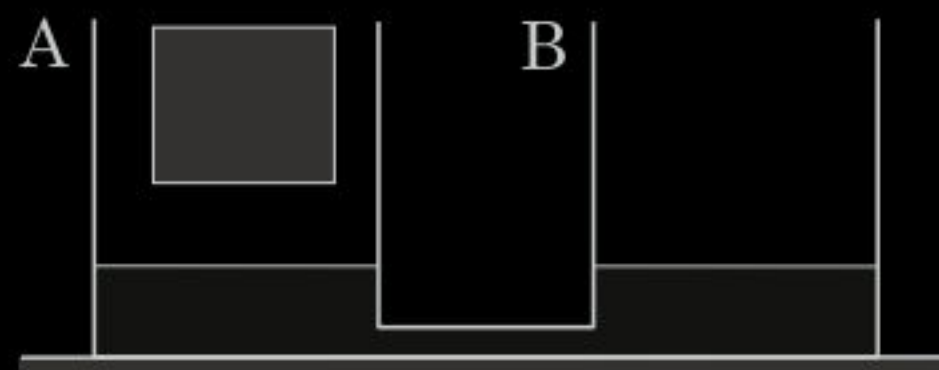
2 @ m_1 $F = m\omega^2 R$

$$\Rightarrow \cancel{G} \frac{\cancel{m_1} m_2}{d^2} \left(\frac{\vec{r}_1 - \vec{r}_2}{d} \right) + \frac{\cancel{G} \cancel{m_1} m_3}{d^2} \left(\frac{\vec{r}_1 - \vec{r}_3}{d} \right) = \cancel{m_1} \omega^2 \vec{r}_1$$

$$\Rightarrow \frac{G}{d^3} \left[(m_2 + m_3) \vec{r}_1 - \underbrace{(m_2 \vec{r}_2 + m_3 \vec{r}_3)}_{\text{From 1}} \right] = \omega^2 \vec{r}_1$$

$$\Rightarrow \frac{G}{d^3} \left[(m_2 + m_3) \vec{r}_1 + \cancel{m_1} \vec{r}_1 \right] = \omega^2 \vec{r}_1$$

$$\Rightarrow \omega^2 = \frac{G(m_1 + m_2 + m_3)}{d^3}$$



1. On a horizontal floor, two identical cylindrical vessels A and B connected by a thin tube near their bottoms contain some water. When an ice block of volume 100 cm^3 is gently put inside the vessel A, it gets half-submerged in the water. How much mass of water will flow through the connecting tube during the process of melting of the ice cube?

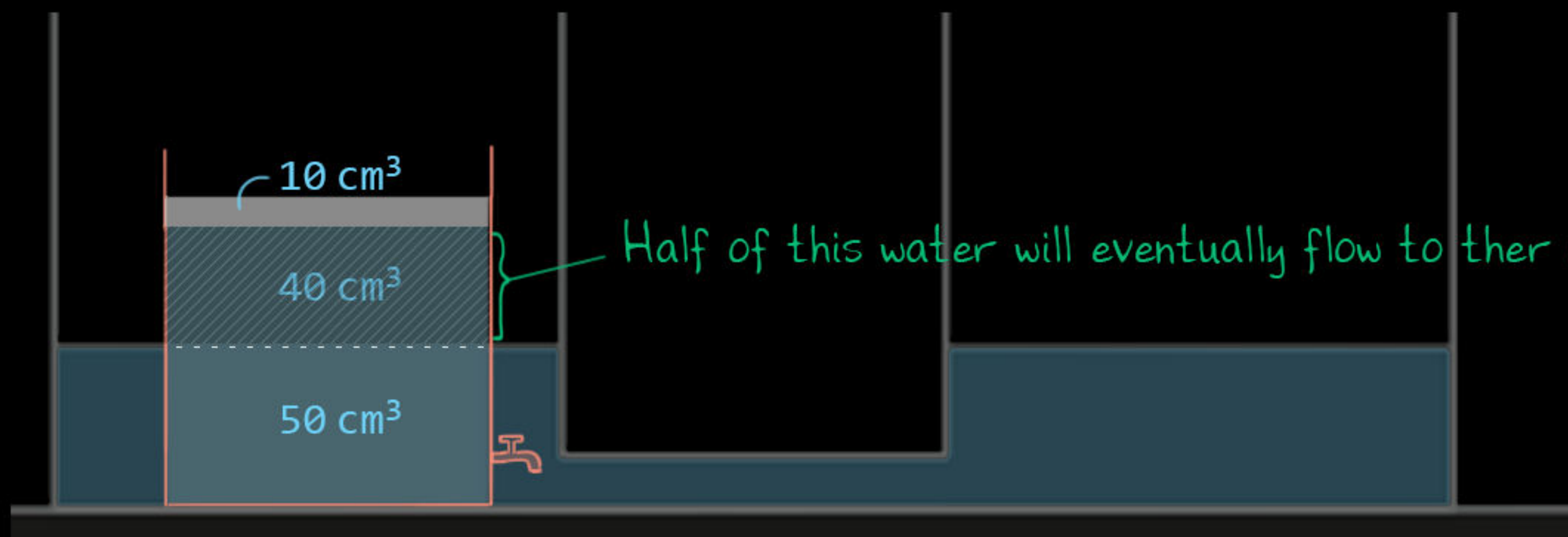
Density of water is 1000 kg/m^3 and density of ice is 900 kg/m^3 .

Only $\frac{9}{10}$ th of ice is water

Lets think of ice being kept in a bucket. Let it melt and then open the valve.

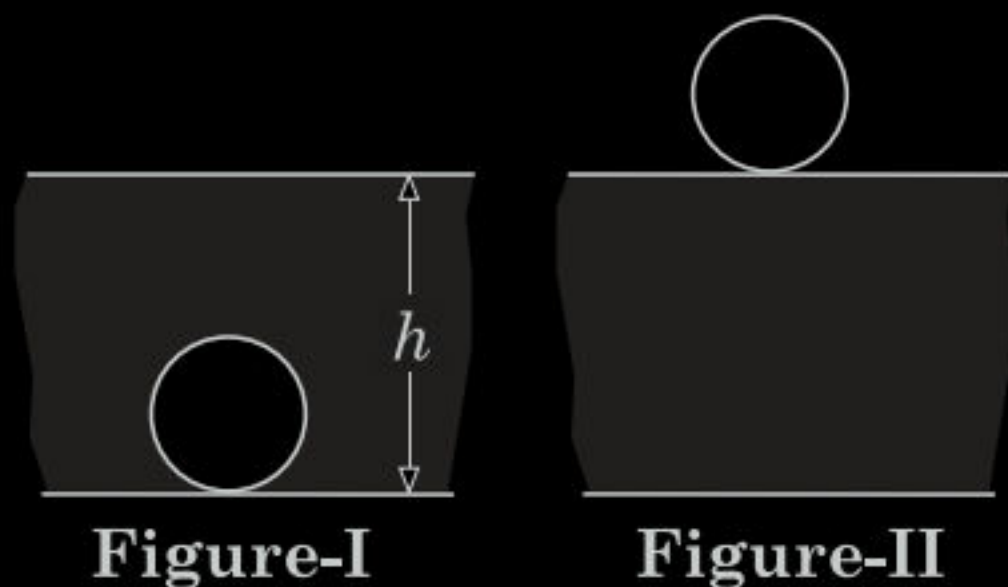
✓ (a) 20 g
(c) 45 g

(b) 25 g
(d) 50 g



Half of this water will eventually flow to ther other side, i.e $\frac{90}{2} \text{ cm}^3$ water

$= 20 \text{ gm water}$



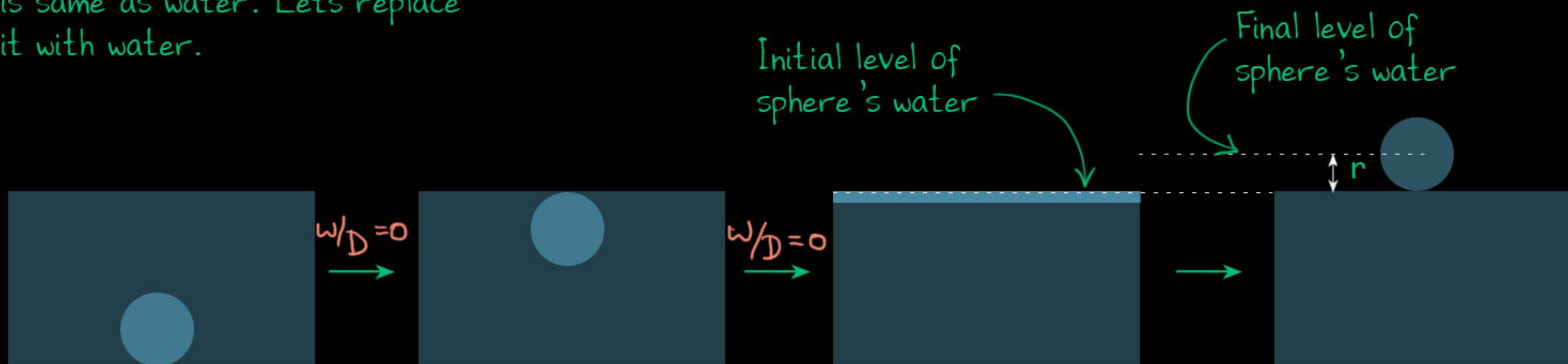
2. A sphere of mass m and radius r rests at the bottom of a large reservoir of water as shown in figure-I. Depth of the reservoir is h . Density of material of the sphere is the same as that of water. Now the sphere is slowly pulled completely out of water as shown in figure-II.

Which of the following is a correct expression for work done by the agent pulling the sphere?

- ✓(a) mgr (b) $0.5mgr$
 (c) $mg(r + h)$ (d) $mg(0.5r + h)$

Similar: WPE, MCQ 4

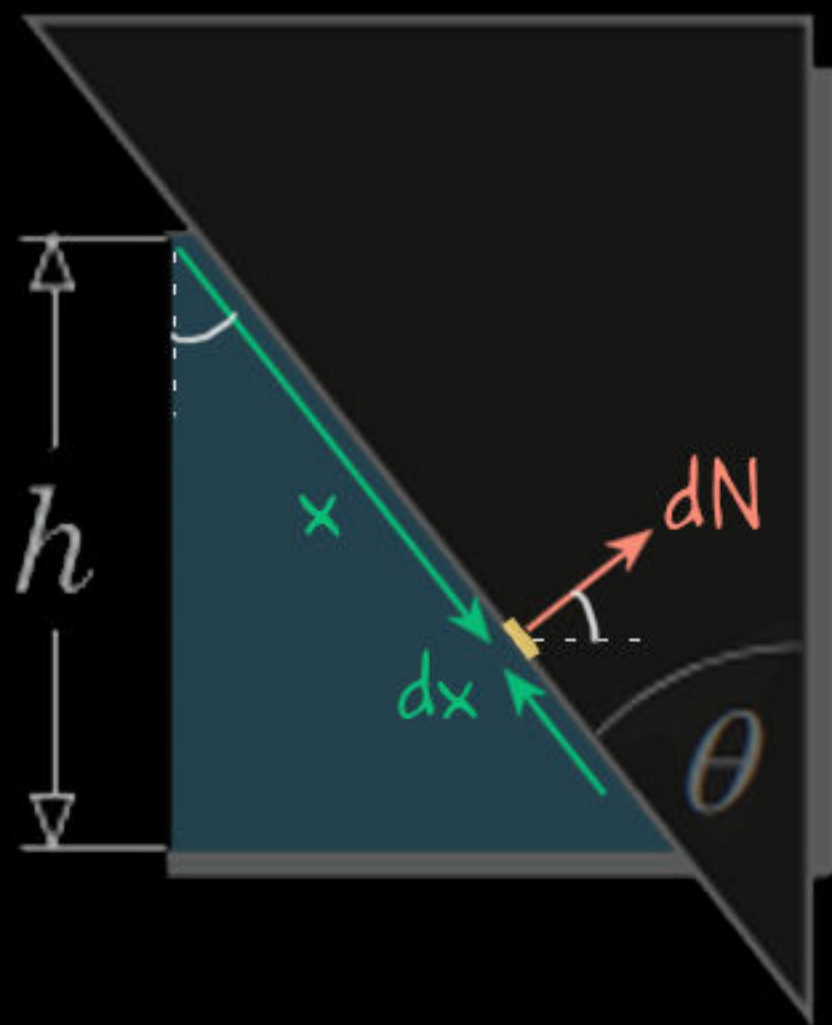
Approach: As sphere's density is same as water. Let's replace it with water.



∴ Work done to pull the 'water' sphere out = mgr



3. A slit is cut along the right bottom edge of a rectangular tank. The slit is closed by a wooden wedge of mass m and apex angle θ as shown in the figure. The vertical plane surface of the wedge is in contact with the right vertical wall of the tank. Coefficient of static friction between these surfaces in contact is μ . To what maximum height, can water be filled in the tank without any leakage through the slit? The width of tank is b and density of water is ρ .



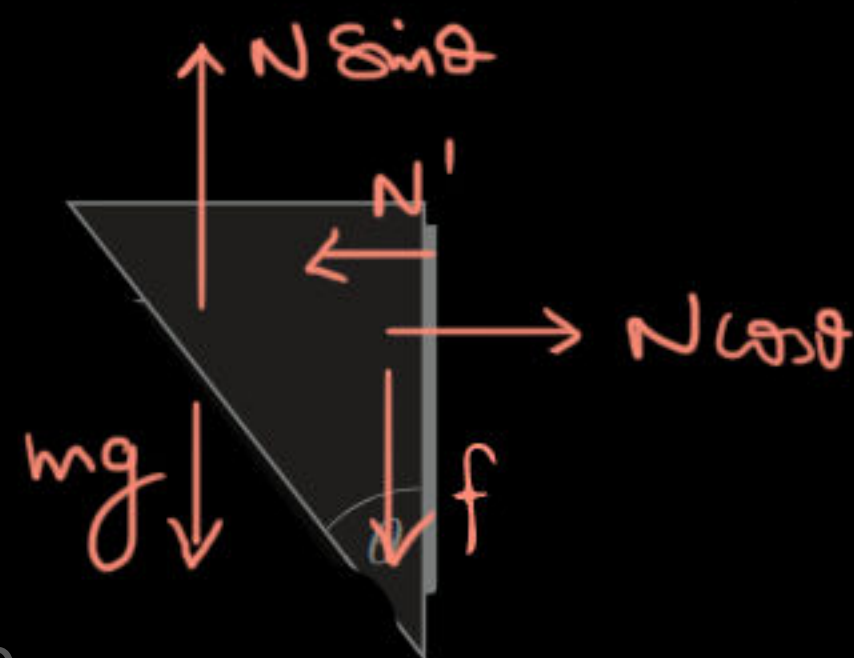
Method 1

$$dN = P dA$$

$$N = \int_0^{h/\cos\theta} \rho g x \cos\theta b dx$$

$$= \rho g \cos\theta b \cdot \frac{h^2}{2 \cos^2\theta}$$

$$= \frac{\rho g b h^2}{2 \cos\theta}$$



$$\checkmark (a) \sqrt{\frac{2m}{\rho b (\tan\theta - \mu)}}$$

Force on wedge:

$$f = N \sin\theta - mg < \mu N'$$

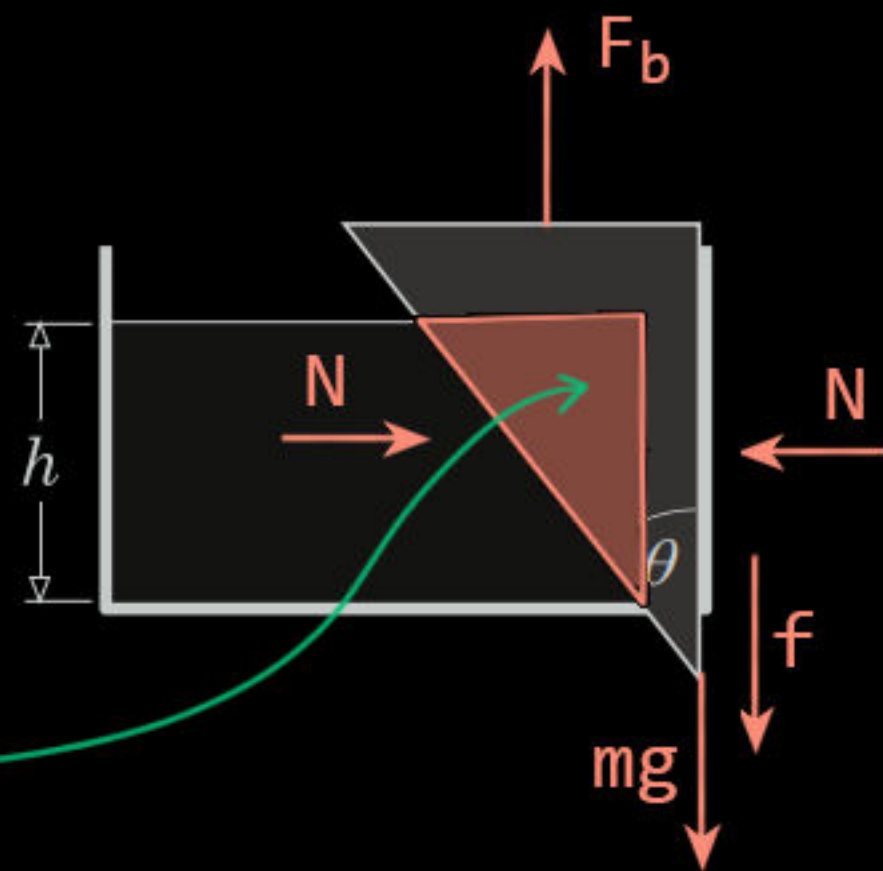
$$\Rightarrow N(\sin\theta - \mu \cos\theta) < mg$$

$$\Rightarrow h^2 < \frac{2mg \cos\theta}{\rho g b (\sin\theta - \mu \cos\theta)}$$

$$\Rightarrow h^2 < \frac{2m}{\rho b (\tan\theta - \mu)}$$

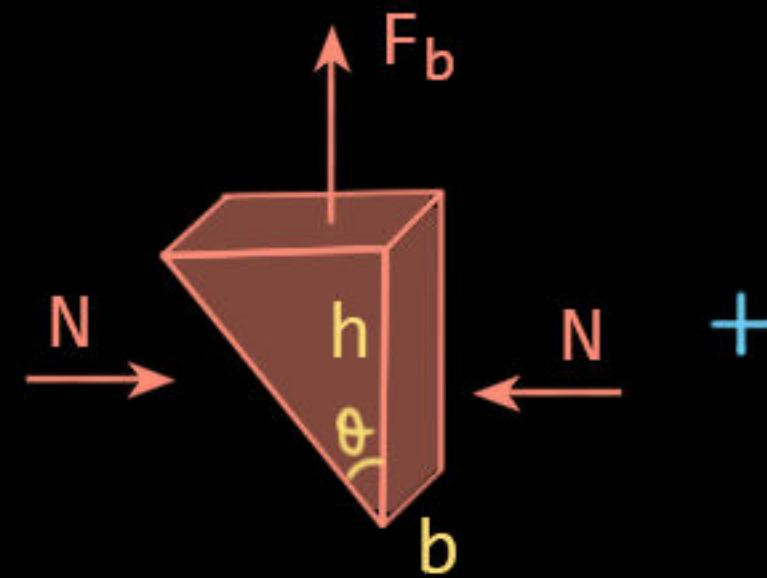
Youtube / Arihant Senior

Method 2



Only Shown part of the whole wedge is responsible for both N and F_b .

$\equiv$



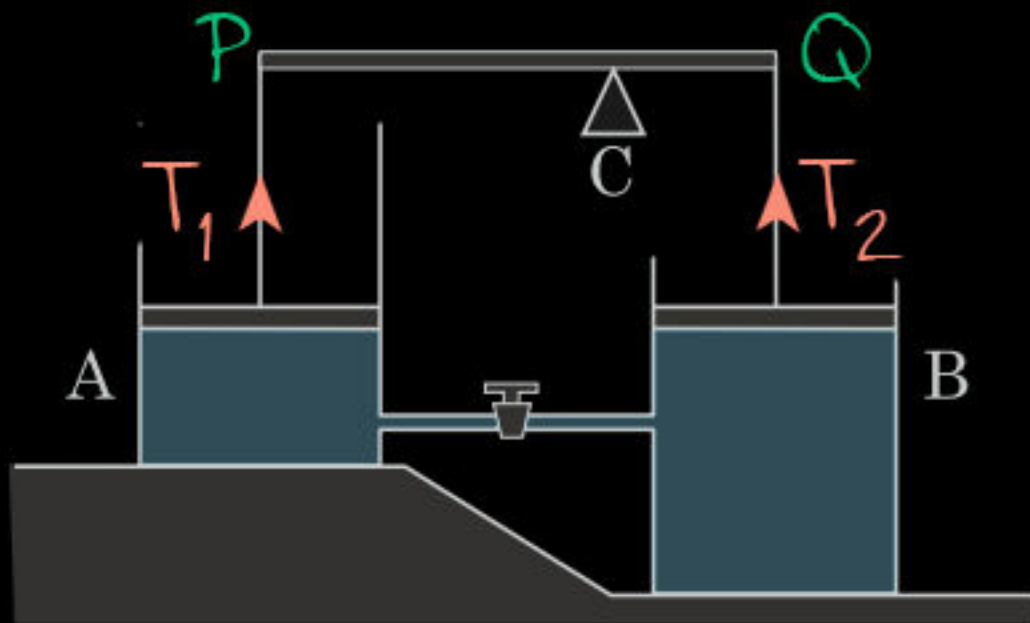
$$N = \frac{\rho g h}{2} \cdot b h \quad (1)$$

$$F_b = \rho \left(\frac{1}{2} h \cdot h \tan \theta \right) b g \quad (2)$$

$$f = F_b - mg < \mu N$$

$$\Rightarrow \frac{\rho g h^2 b \tan \theta}{2} - mg < \frac{\mu \rho g h^2 b}{2}$$

$$\Rightarrow h^2 < \frac{2m}{\rho b (\tan \theta - \mu)}$$



4. In the setup shown, two identical vessels A and B filled with water up to same level are connected by a horizontal tube equipped with a valve that is closed. There are identical leak proof pistons in contact with the water in both the vessels. The centres of the pistons are connected with the help of **two taut elastic cords** to the ends of a uniform lever arm supported on a fulcrum C that is fixed somewhere on the right half of the lever arm. The cords are vertical and there is no friction between the pistons and the vessels. **If the valve is opened, in which direction will the water flow** through the connecting tube?

- ✓ (a) From A to B (b) From B to A
(c) Water will not flow (d) Insufficient information

Rod is in equilibrium



$$T_1 < T_2 \quad (\because PC > QC)$$



$$P_A > P_B \Rightarrow \text{Water will flow from A to B when valve is opened.}$$

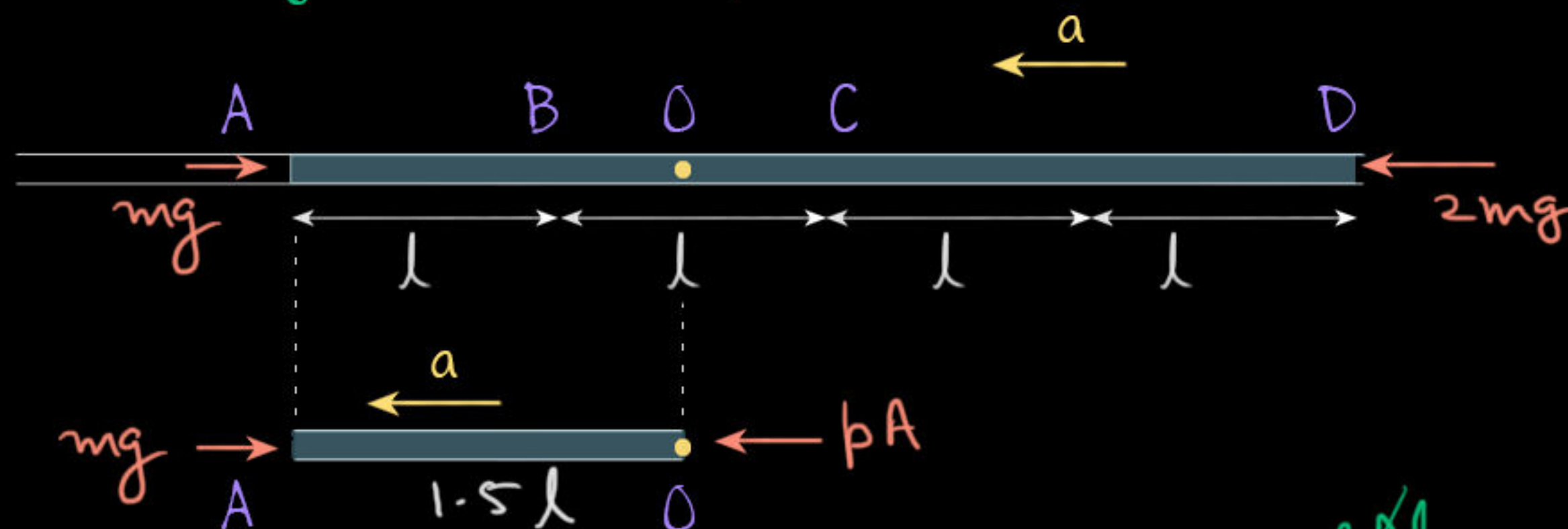


5. Bottom arm of a U-tube of uniform section has length l and the vertical arms have length $3l$ each. The tube is held with its parallel arms vertical and filled with a liquid of density ρ to occupy a total length of $4l$ of the tube and then it is made to accelerate horizontally in the plane of the tube with an acceleration g that is modulus of acceleration of free fall. What will gauge pressure at the midpoint of the bottom arm be immediately after acceleration of the tube is made abruptly zero?

→ Pressure that exceeds 1 atm.

✓ (d) $1.375\rho gl$

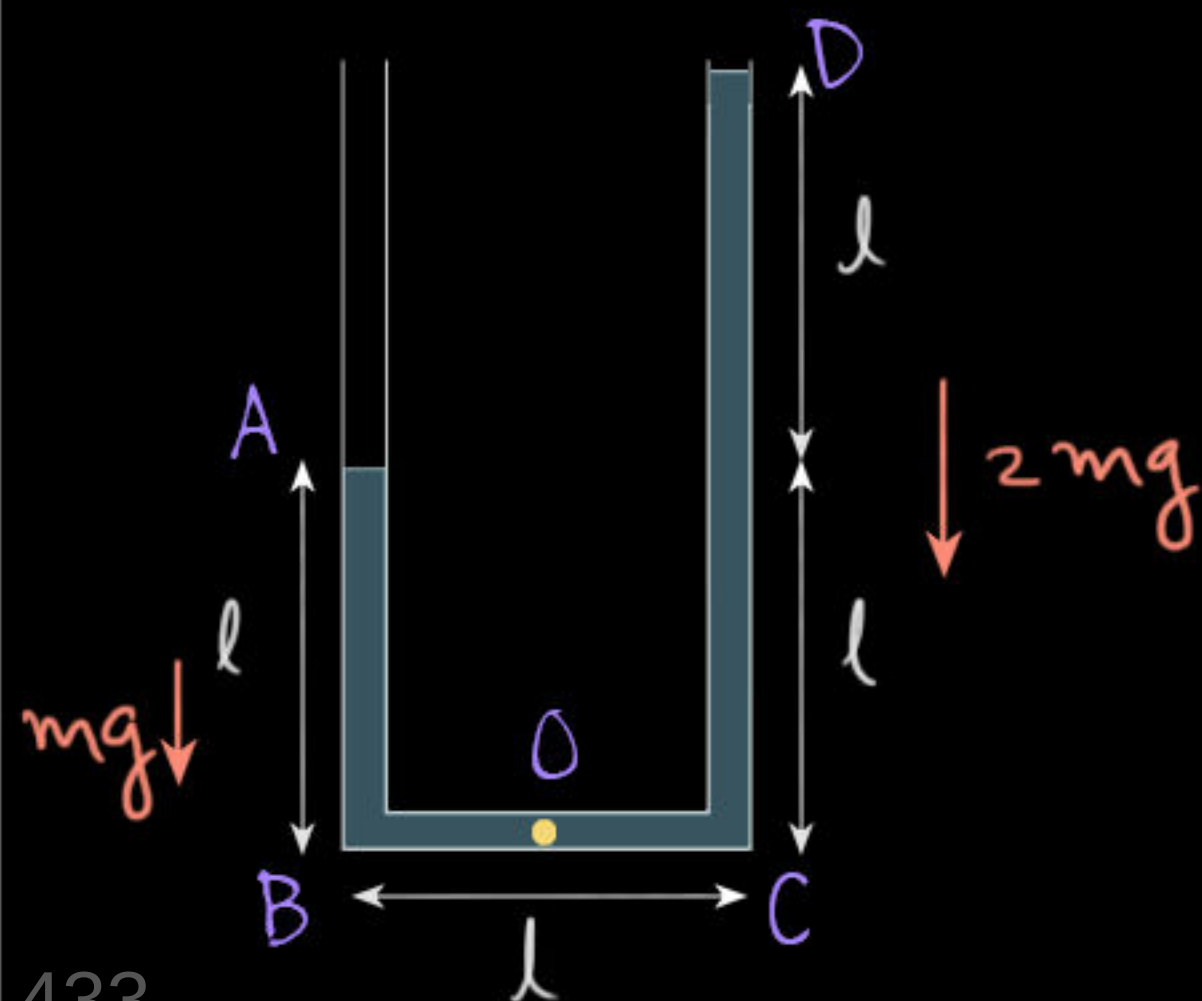
Lets straighten the column. Acceleration be a .

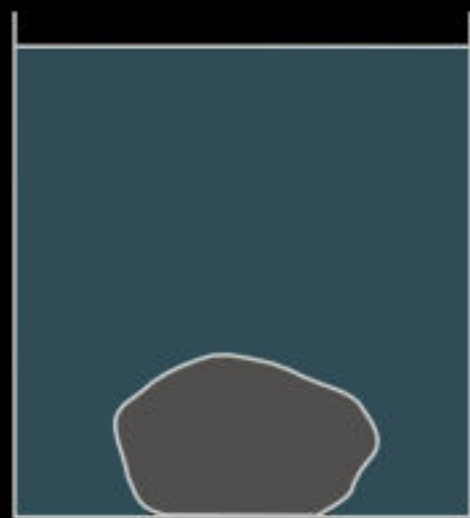


$$a = a$$

$$\Rightarrow \frac{2mg - mg}{4m} = \frac{pA - mg}{1.5m} \Rightarrow p = \frac{5.5}{4} mg = 1.375 \rho gl$$

When stopped:





6. An object of volume 0.5 m^3 and density 500 kg/m^3 is glued to the flat bottom of a vessel containing water. The flat base of the object glued to the bottom has an area 100 cm^2 and density of water is 1.0 g/cm^3 . If the maximum force that the glue can withstand is 2000 N , to what ~~maximum~~ ^{minimum} height above the base can water be filled in the vessel so that the object is not torn apart from the base?

For given answer 5m, there is a correction and 2 assumptions:

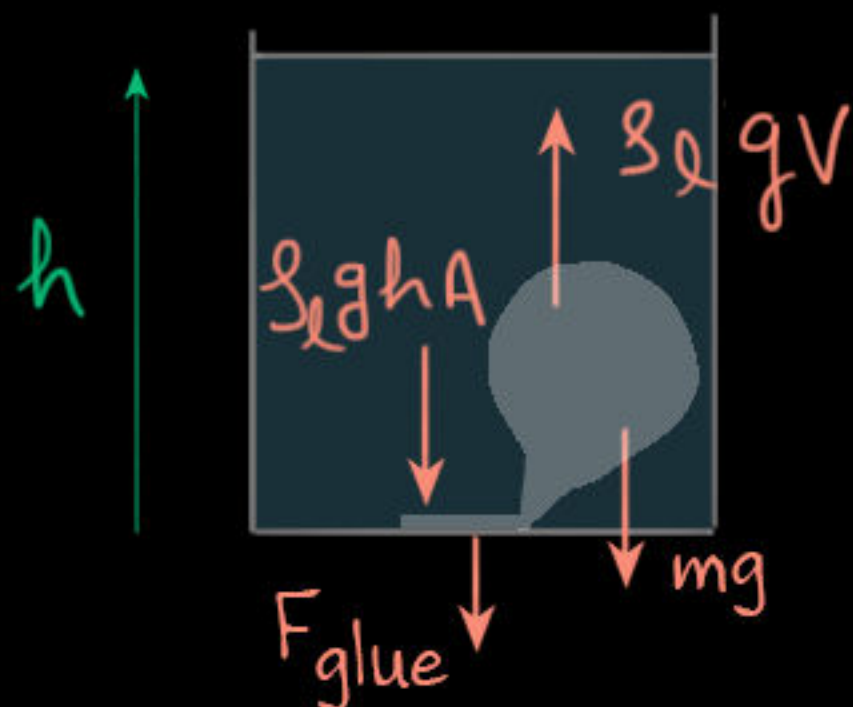
- ✓ (a) 5 m
(c) Up to any height

- (b) 10 m
(d) Insufficient information

Above glue there is very little part of object

- Shape matters. A cylindrical shape for example will never get torn apart.

Sol: Glue remains at P_{atm}



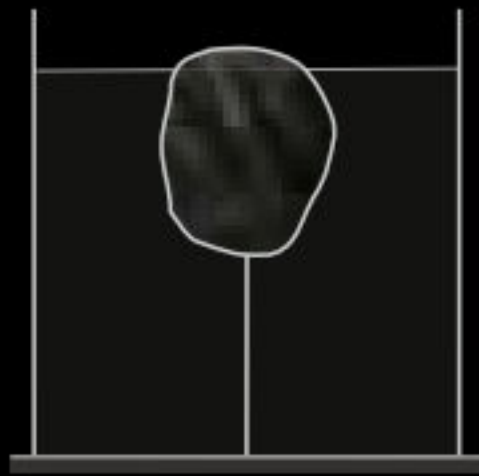
$$F_{\text{glue}} = \rho_w g V - \rho_w g h A - mg < 2000$$

$$\Rightarrow \rho_w g (V - Ah) - mg < 2000$$

$$\Rightarrow (1000) 10 (0.5 - 10^{-2} h) - (0.5)(500) 10 < 2000$$

$$\Rightarrow 5 - \frac{h}{10} - 2.5 < 2$$

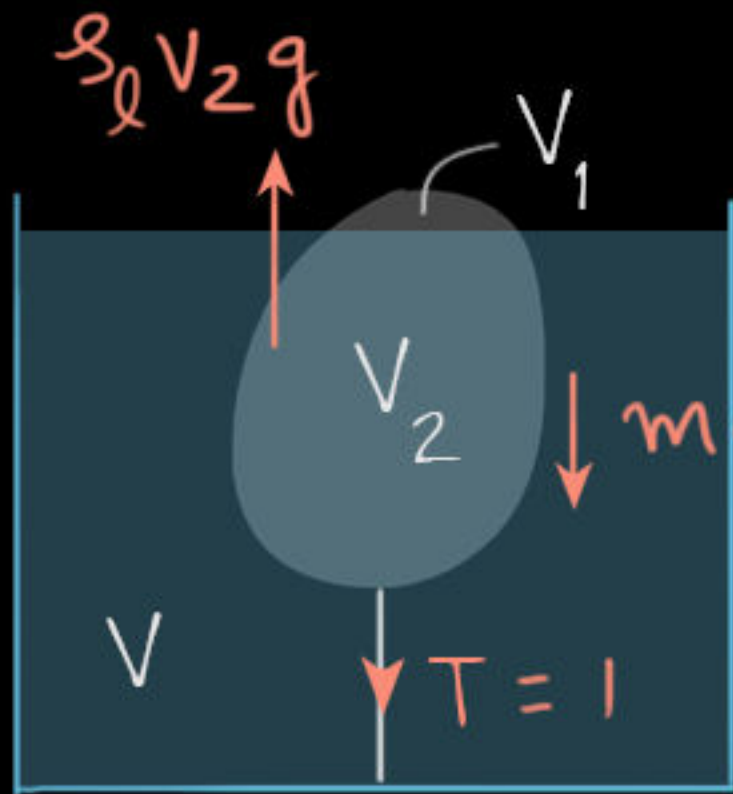
$$\Rightarrow h > 5 \text{ m}$$



7. A piece of ice tied with the help of a thread to bottom of a cylindrical container having some amount of water. Length of the thread is so adjusted that the ice is not fully submerged. Tension force in the thread is 1.0 N. How much and in what way will the water level in the container shift, when all the ice melts? Area of the bottom of the vessel is 400 cm², density of water is 1.0 g/cm³ and density of ice is 0.9 g/cm³.

- (a) 2.5×10^{-3} m upwards
(c) 2.8×10^{-3} m upwards

- ✓ (b) 2.5×10^{-3} m downwards
(d) 2.8×10^{-3} m downwards

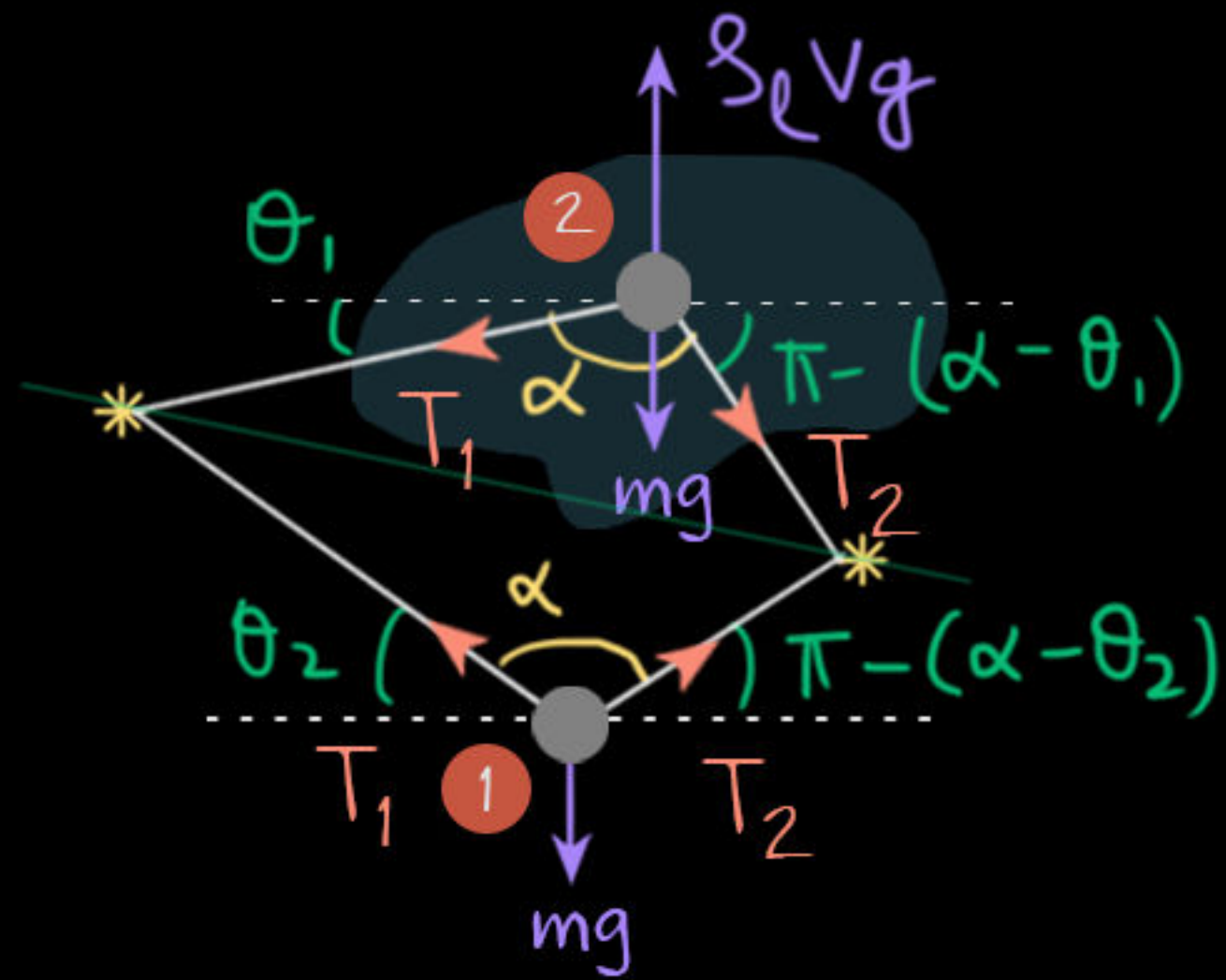


$$m g = \left(\frac{\rho_{ice}}{\rho_{water}} \right) (V_1 + V_2) g$$

$$1 \text{ } F_y: \rho_{ice} V_2 g = \frac{\rho_{ice}}{\rho_{water}} (V_1 + V_2) g + 1$$

Level rise:

$$2 \text{ } \Delta h = \frac{\Delta V}{A} = \frac{(V + (V_1 + V_2) \frac{\rho_{ice}}{\rho_{water}}) - (V + V_2)}{A} = \frac{\left(\frac{-1}{\rho_{ice} g} \right)}{A} = \frac{-1}{1000 \cdot 10 \cdot 400} = -2.5 \times 10^{-3} \text{ m}$$



8. A ball is suspended with the help of two threads inside an empty cylindrical vessel without touching its walls, from two points on the walls of the vessel. If the vessel is filled with water up to such a height that the ball remains submerged and both the threads become taut, tension forces in the threads remain the same as they were in the empty vessel. Density of water is 1000 kg/m^3 . Which of the following is a correct statement?

- (a) Density of the ball is 500 kg/m^3 , if threads are of equal length.
- (b) Density of the ball is 500 kg/m^3 , if the threads are of equal length and affixed on walls of the vessel at equal heights.
- (c) Density of the ball is 500 kg/m^3 , if the threads are of equal length and affixed on walls of the vessel at equal heights on diametrically opposite points.
- (d) Density of the ball is 500 kg/m^3 , irrespective of lengths of the threads and location of point of suspension.

$$\begin{aligned} 1 \quad T_1 \cos \theta_1 &= T_2 \cos [\pi - (\alpha - \theta_1)] \\ 2 \quad T_1 \cos \theta_2 &= T_2 \cos [\pi - (\alpha - \theta_2)] \end{aligned}$$

Need not be diametrically opposite, but at same height though.

Dividing and simplifying: $\tan \theta_1 = \tan \theta_2 \Rightarrow \theta_1 = \theta_2 \Rightarrow$ Points of suspension are at same height

$$T_{\text{net}, y} = mg = \rho_l V g - mg \Rightarrow \rho_l = \frac{2mg}{Vg} \Rightarrow \rho_{\text{ball}} = \frac{\rho_l}{2} = 500 \text{ kg/m}^3$$

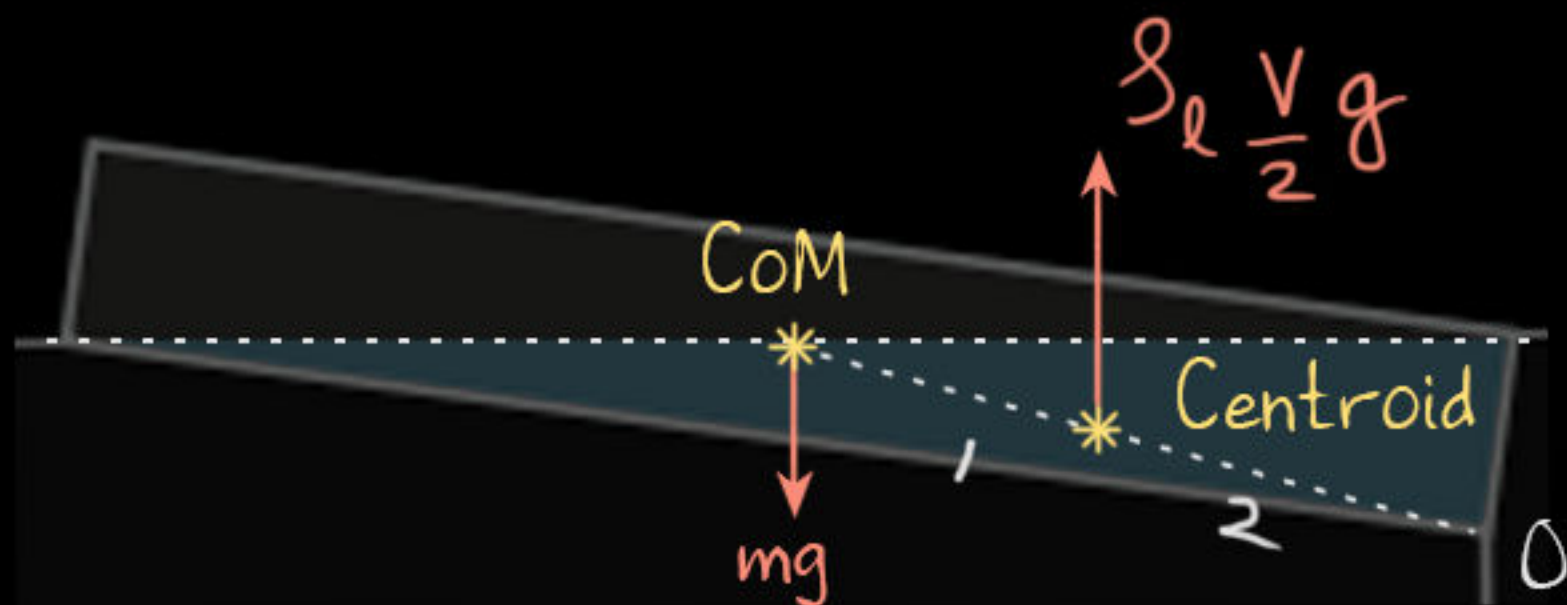


9. A wooden plank is floating on water in a pool. It is tethered to the bottom of the pool by a string attached at mid-point of an edge of its bottom face, which causes it to float with a diagonal of one of its vertical cross-section coinciding with the level surface of the water, as shown in the figure. What is the specific gravity of the wood?

- (a) $1/5$ (b) $1/4$
 ✓ (c) $1/3$ (d) $1/2$

F_g will apply at CoM of whole plank.

Whereas F_b will apply at Centroid of immersed triangular wedge.



Balancing torque about O, so we need not worry about reaction forces at hinge.

$$\tau_O: \rho_p V = \rho_l \frac{V}{2} g$$

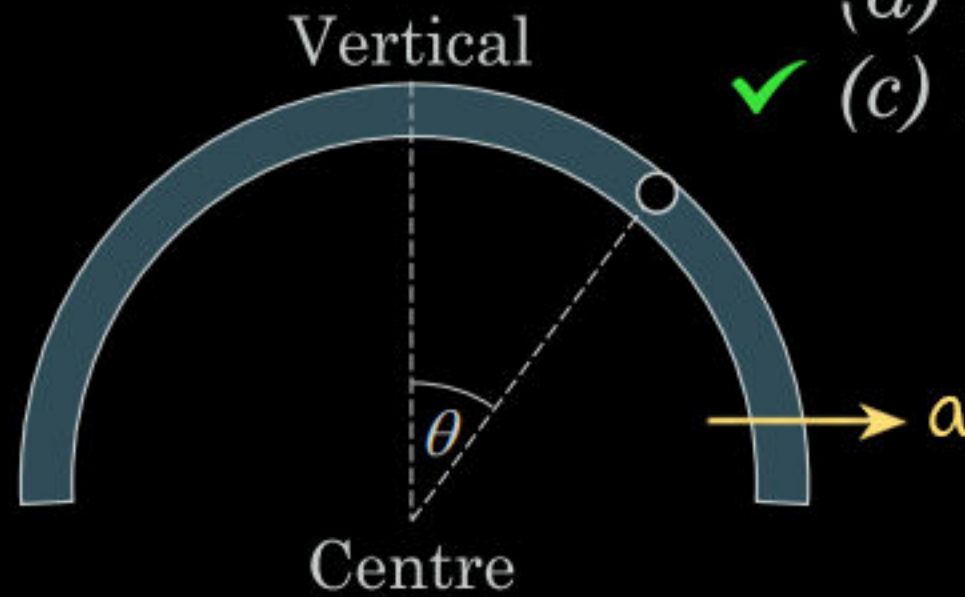
$$\Rightarrow 3 \rho_p V = \rho_l V$$

$$\Rightarrow \rho_p = \frac{\rho_l}{3}$$

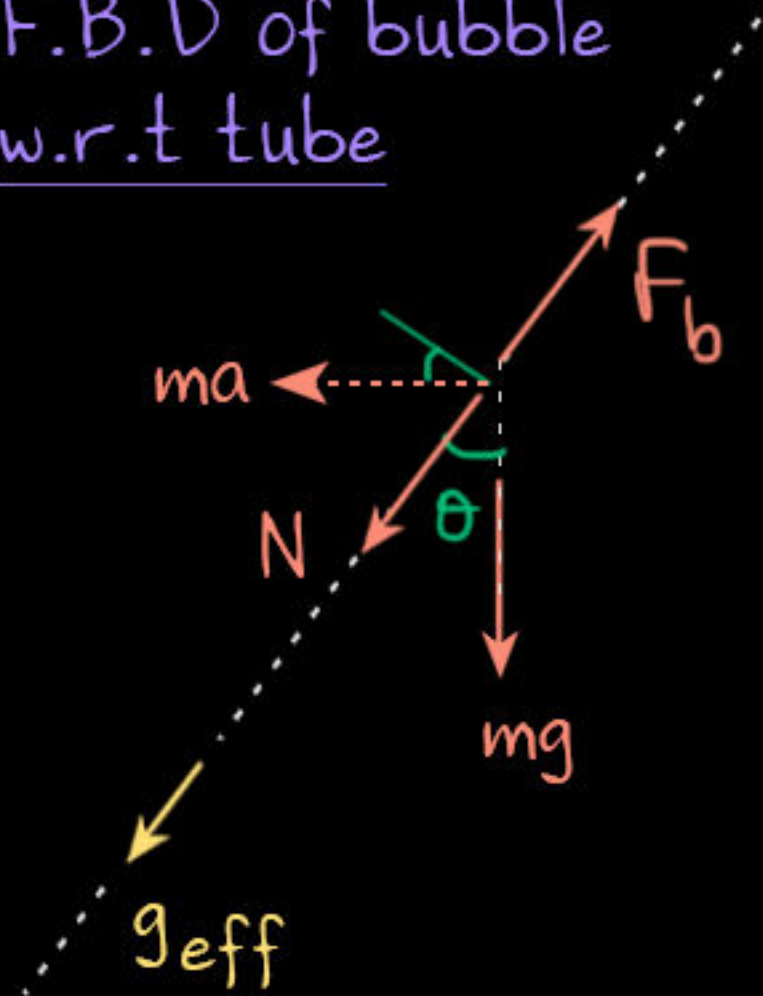
10. A semicircular glass tube filled with water containing an air bubble is sealed at its ends. If the tube is held with its plane vertical and made to move in its plane with a constant acceleration, the bubble stays aside of the highest point as shown in the figure. What can you conclude about acceleration vector of the tube?

g_{eff} is in opposite direction of F_b

- (a) It points towards the left. ✓ (b) It points towards the right.
✓ (c) Its magnitude is $g \tan \theta$. (d) Its magnitude is $g \cot \theta$.



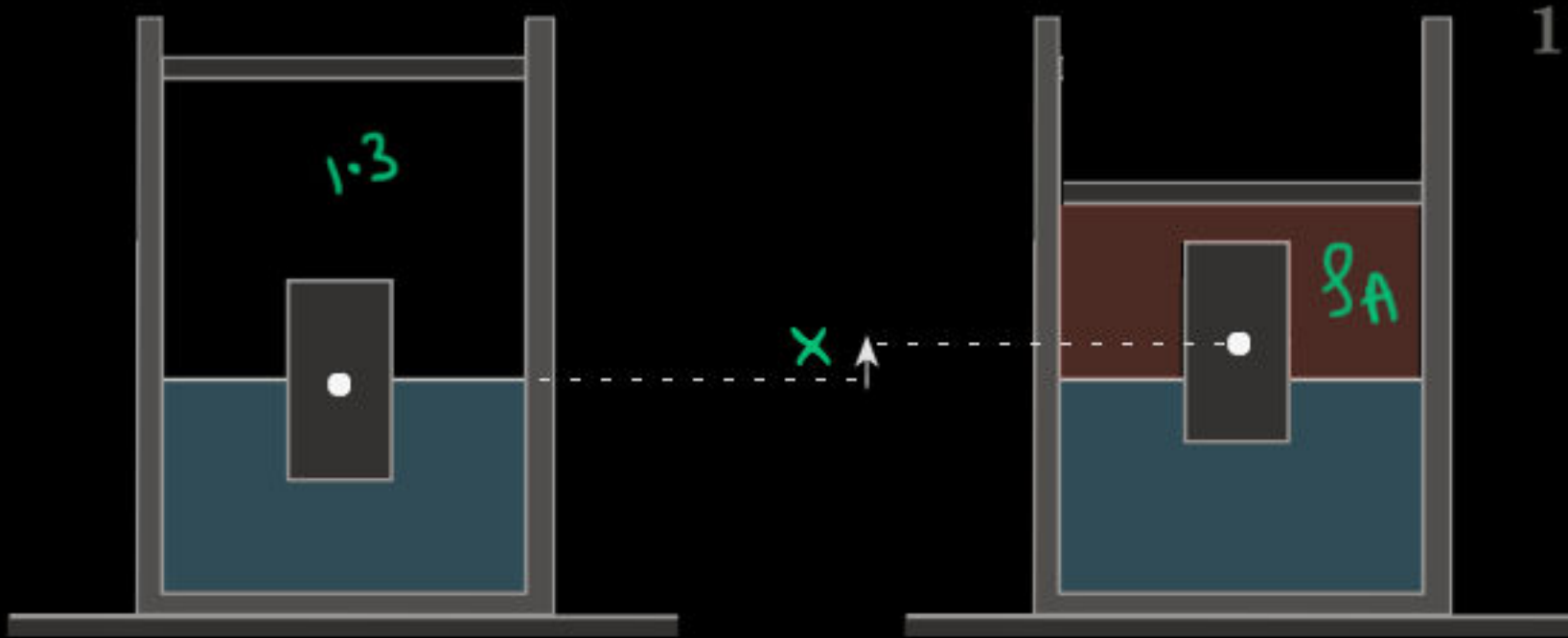
F.B.D of bubble w.r.t tube



Pseudo force must be towards left,
 $\therefore$ Tubes acceleration is towards right.

$$F_{\text{tangential}}: \cancel{mg} \sin \theta = \cancel{ma} \cos \theta$$

$$\Rightarrow a = g \tan \theta //$$



11. A cylinder of length 10 cm made of material of density 0.65 g/cm^3 floats half submerged in an ideal liquid kept in closed vessel as shown in the figure. Air in the vessel has density 1.30 kg/m^3 . If pressure of air in the vessel is made 100 times of its initial value, what will you observe?

- ✓ (a) The cylinder moves upwards.
- (b) The cylinder moves downwards.
- ✓ (c) Displacement of the cylinder is 0.56 cm.
- (d) Displacement of the cylinder is 0.60 cm.

Lets say the center moves up by x
Initially:

$$\rho_l \frac{h}{2} g = (\rho_s h) g$$

$$\Rightarrow \rho_l = 2 \rho_s = 1300$$

Lets assume isothermal compression

$$P = \frac{\rho R T}{M} \Rightarrow P \propto \rho$$

As the air is now pretty heavy, it will act as a liquid with finite variation of pressure with height.

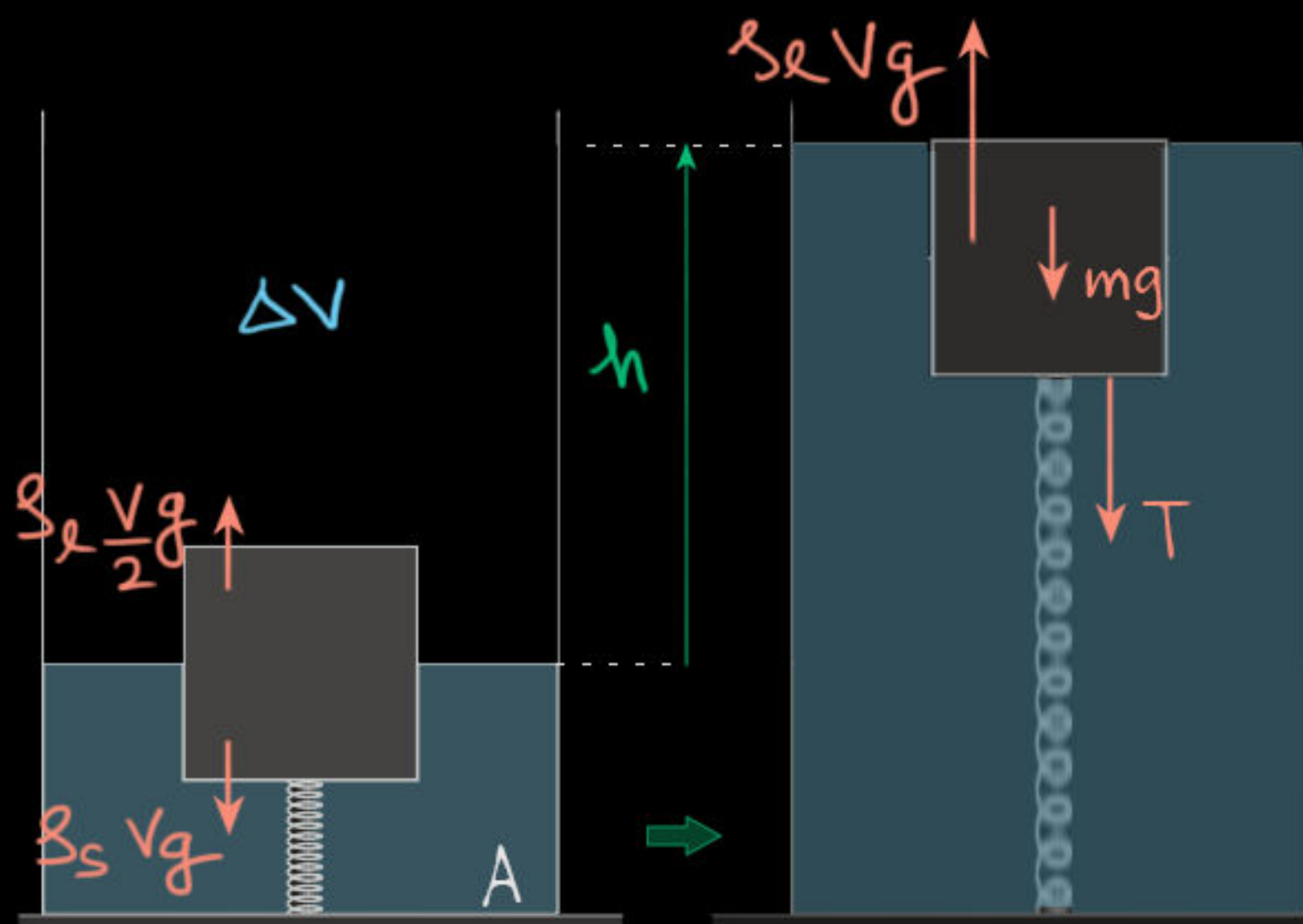
Finally:

$$100(1.3) \left(\frac{h}{2} + x \right) A g + \rho_l \left(\frac{h}{2} - x \right) A g = (\rho_s h A) g$$

$$\Rightarrow 130 \left(\frac{h}{2} + x \right) + 1300 \left(\frac{h}{2} - x \right) = 650 \cdot h$$

$$\Rightarrow \frac{h}{2} + x + 5h - 10x = 5h$$

$$\Rightarrow x = \frac{h}{18} = \frac{10}{18} = 0.55 \text{ cm}$$



12. A wooden cube of mass 2048 g is attached to the bottom of a cylindrical vessel with the help of a spring. The vessel is filled with some amount of water so that the cube floats half submerged and the spring is relaxed. Area of the bottom of the cylinder is 600 cm², density of water is 1000 kg/m³ and acceleration of free fall is 10 m/s². Now additional water is slowly poured in the vessel until the cube is fully submerged. In this process, the water level rises by 20.48 cm. Which of the following statements are correct?

- ✓(a) Length of a side of the cube is 16 cm.
- ✓(b) Amount of additional water is 10.24 L.
- ✓(c) Spring force when the cube is fully submerged is 20.48 N
- ✓(d) Displacement of the cube when it is fully submerged is 12.48 cm.

$$F_y: \rho_l V_{\frac{1}{2}} g = \rho_s V g \Rightarrow \rho_s = \frac{\rho_l}{2} = 500$$

$$\text{Mass of block} = 2.048 = \rho_s l^3 \Rightarrow l = 0.16 //$$

$$\text{Final } F_y: mg + T = \rho_l V g$$

$$\Rightarrow (2.048) 10 + T = (1000) l^3 g$$

$$\Rightarrow T = 20.48 \text{ N} //$$

$$\text{Water added: } \Delta V = Ah - \frac{V}{2} \rightarrow l^3$$

$$= 600 \times 20.48 - 2048 \text{ cm}^3 = 10.24 \text{ L} //$$

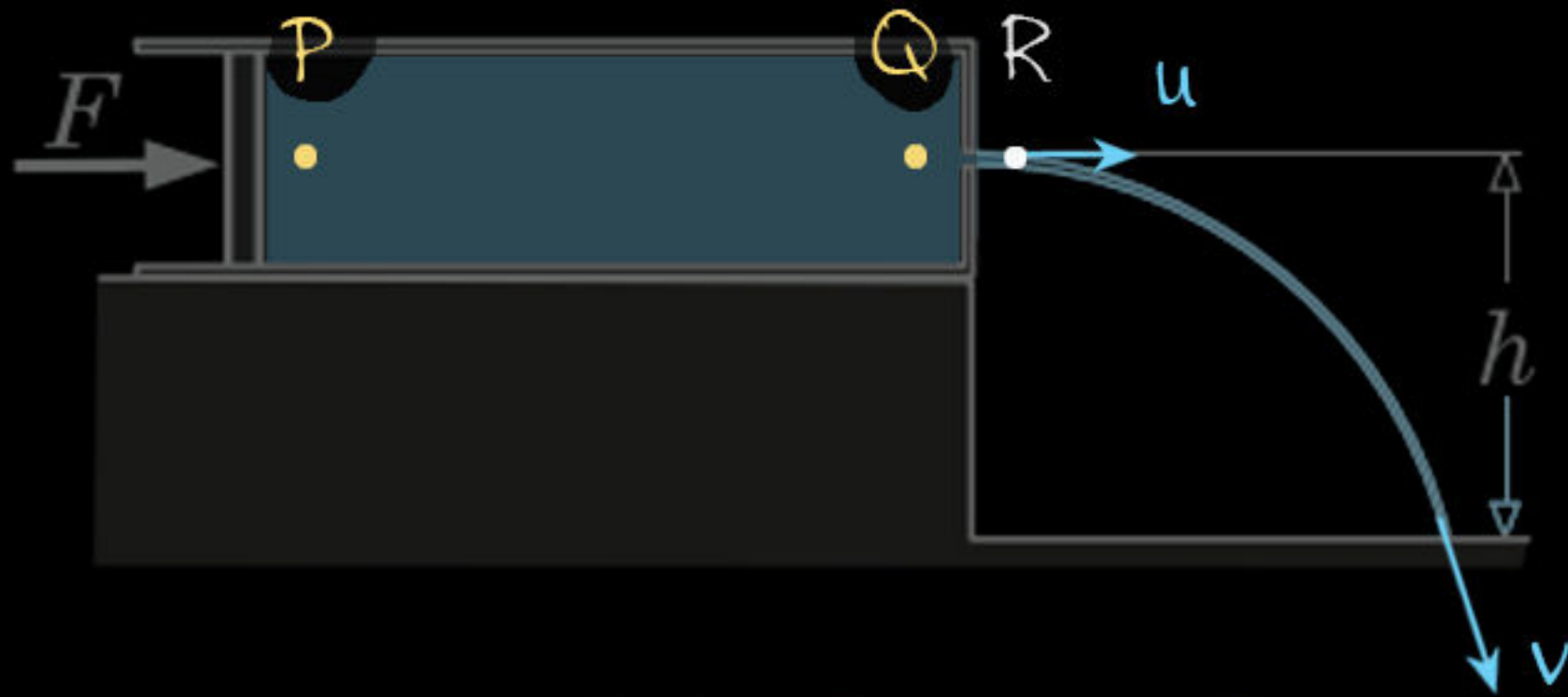
$$\text{Displacement of cube} = h - l/2 = 12.48 //$$



An ideal liquid of density ρ is filled in a horizontally fixed syringe fitted with piston. There is no friction between the piston and the inner surface of the syringe. Cross-section area of the syringe is A . An orifice is made at one end of the syringe. When the piston is pushed into the syringe, the liquid comes out of the orifice and then following a parabolic path falls on the ground.

13. With what velocity does the liquid come out of the orifice? (b) $\sqrt{\frac{2F}{\rho A}}$

14. With what velocity does the liquid strike the ground? (d) $\sqrt{\frac{2(F + \rho ghA)}{\rho A}}$



$$P_Q = P_P = \frac{P_0 A + F}{A} = P_0 + \frac{F}{A}$$

∴ Piston doesn't accelerate, so forces on it must be equal.

Bernoulli's between Q and R:

$$P_Q = P_R + \frac{1}{2} \rho u^2$$

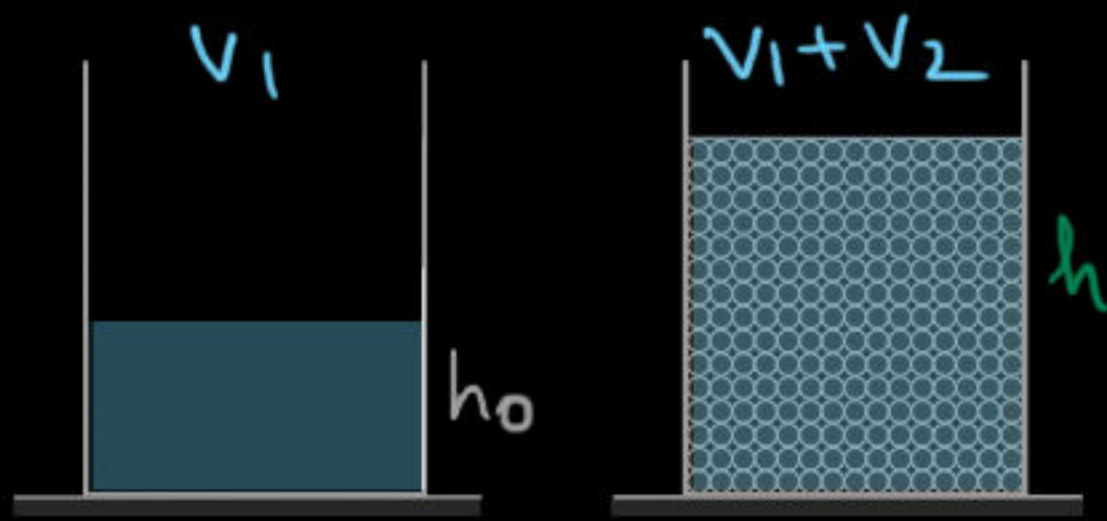
$$\Rightarrow P_0 + \frac{F}{A} = P_0 + \frac{1}{2} \rho u^2$$

$$\Rightarrow u = \sqrt{\frac{2F}{\rho A}}$$

$$v^2 = u_x^2 + u_y^2$$

$$v^2 = u^2 + (\sqrt{2gh})^2$$

$$\Rightarrow v = \sqrt{\frac{2F}{\rho A} + 2gh}$$



- 1 Water
- 2 Iron

1. A cylindrical vessel is filled with water up to a height $h_0 = 1.0$ m and then a large number of small iron balls are gently dropped in it until the topmost layer of the balls becomes completely submerged in water as shown in the figure. Density of iron is $\rho_i = 7140$ kg/m³ and density of water is $\rho_0 = 1000$ kg/m³. If average density of the contents is $\rho = 4070$ kg/m³, find the height of the water level in the vessel with the iron balls.

Geometry: Bottom area = $\frac{V_1}{h_0} = \frac{V_1 + V_2}{h}$

$$\Rightarrow h = h_0 \left(1 + \frac{V_2}{V_1} \right) \quad 1$$

Conserving mass: $m_1 + m_2 = m_{\text{mix}}$

$$\rho_1 V_1 + \rho_2 V_2 = \rho_{\text{mix}} (V_1 + V_2)$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{\rho_1 - \rho_{\text{mix}}}{\rho_{\text{mix}} - \rho_2} = \frac{1000 - 4070}{4070 - 7140} = 1 \quad 2$$

From 1,2:

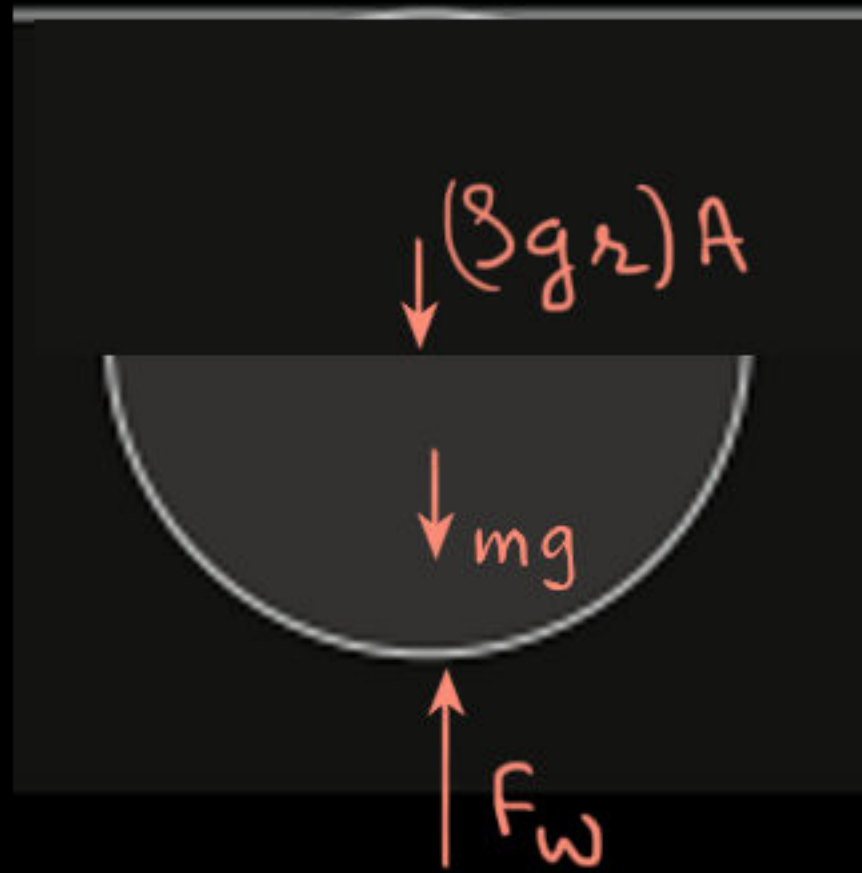
$$h = 2h_0 //$$



2. A plastic sphere of radius r floats almost fully submerged in a liquid as shown in the figure. Density of the liquid is ρ , acceleration of free fall is g . Find the force exerted by the liquid on the lower half of the sphere excluding contribution of the atmospheric pressure.

M 1

Lets remove top half. Bottom half will be unaffected
As density of sphere and liquid is same.



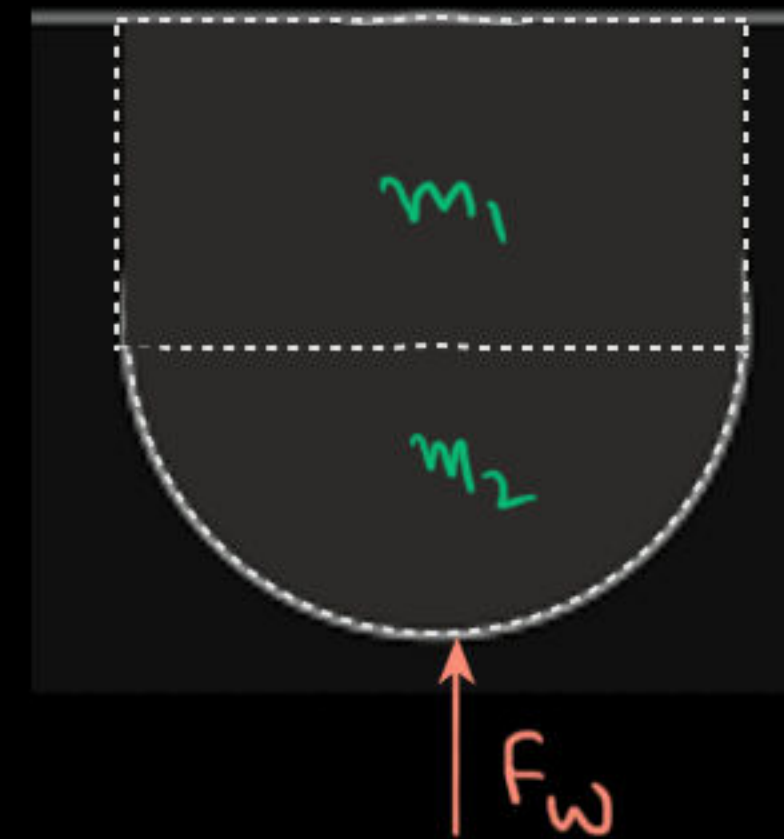
$$F_w = (3gr)A + \left[\frac{1}{2} \rho \frac{4}{3} \pi r^3 \right] g$$

$$\Rightarrow F_w = 3g \pi r^3 \left(1 + \frac{2}{3} \right)$$

$$= \frac{5\pi}{3} 3gr^3 //$$

M 2

Force by water on bottom half supports all the weight above it.



$$F_w = (m_1 + m_2)g$$

$$= \left(3 \pi r^2 \cdot r + \frac{1}{2} \rho \frac{4}{3} \pi r^3 \right) g$$

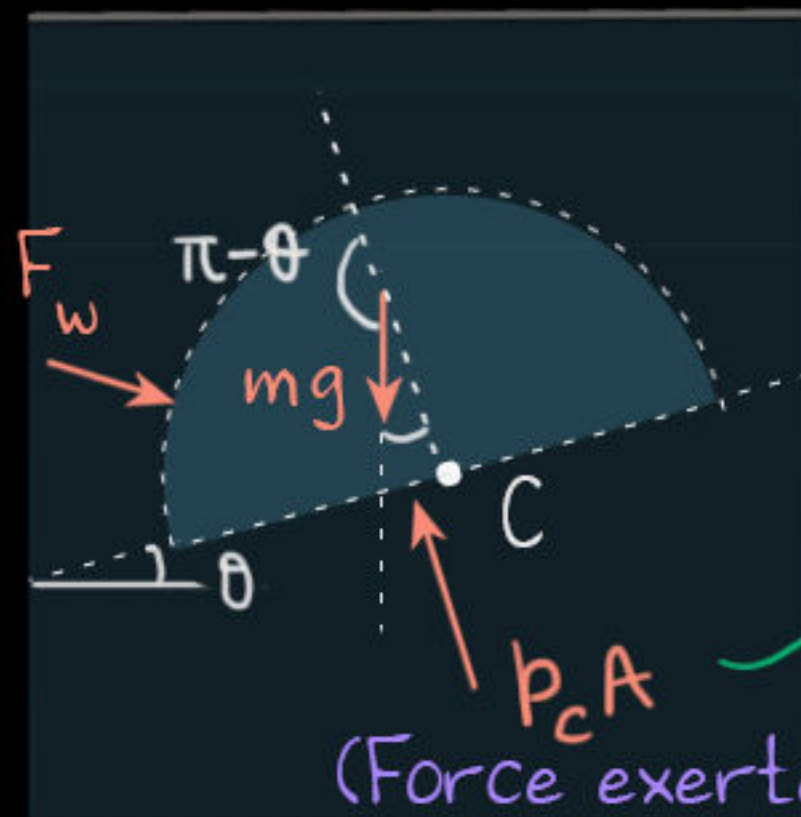
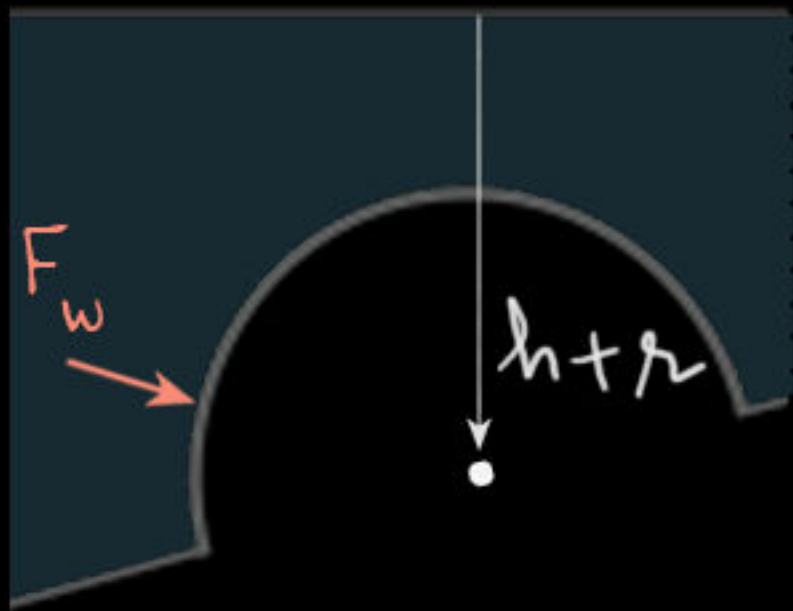
$$= \frac{5\pi}{3} 3gr^3 //$$

Youtube / Arihant Senior



3. Bottom of a vessel is inclined at an angle θ and has a hemispherical dent of radius r as shown in the figure. If a liquid of density ρ is filled in the vessel to a height h above the highest point of the dent, find a suitable expression for magnitude of force of liquid pressure on the dent excluding contribution of the atmospheric pressure.

F_w on hemispherical dent = F_w on a hemispherical water surface



Proof

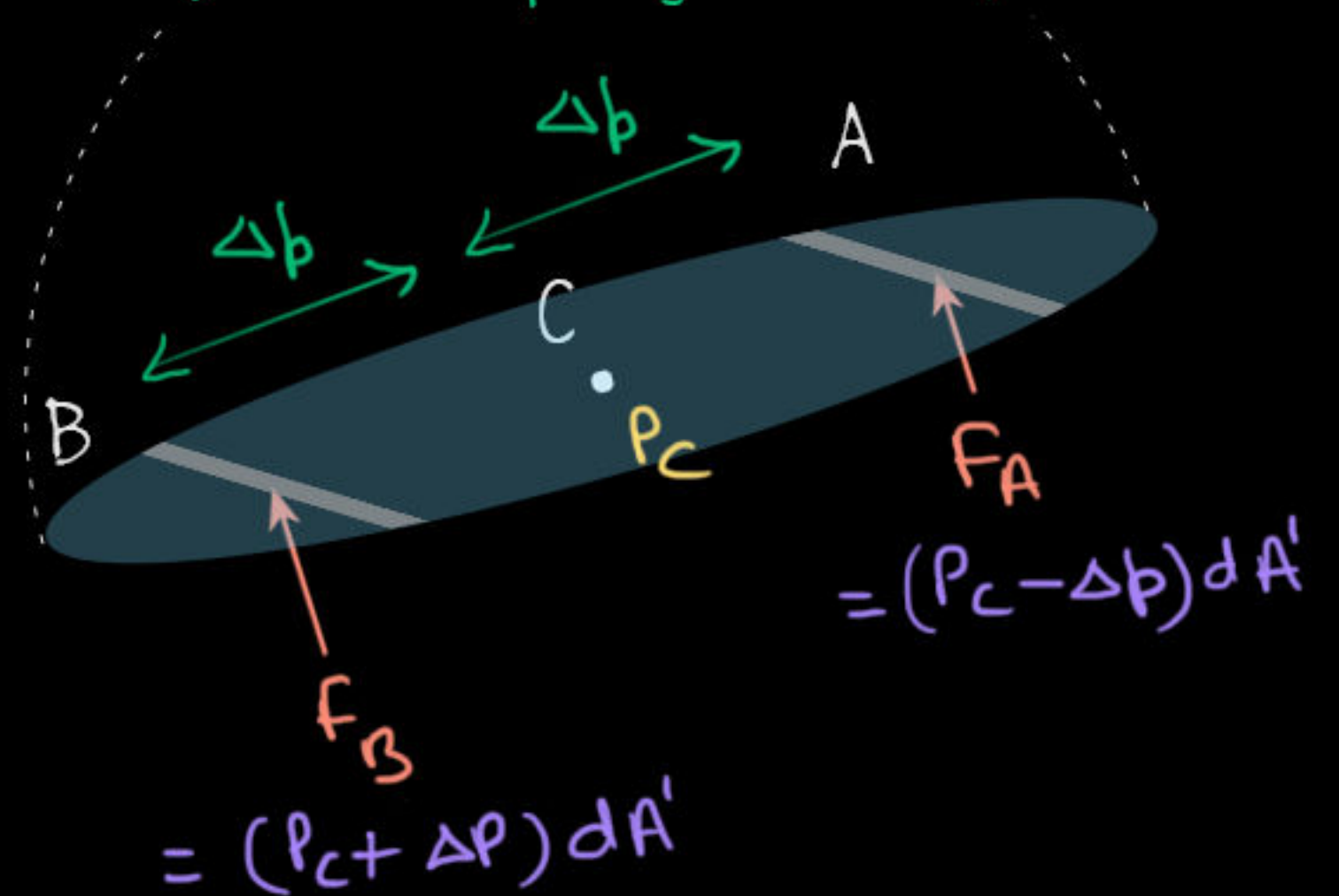
(Force exerted by water on bottom surface)

All three forces are balanced

$$\therefore F_w = \sqrt{(mg)^2 + (p_c A)^2 + 2mg p_c A \cos(\pi - \theta)}$$

$$m \rightarrow \frac{1}{2} \rho \cdot \frac{4}{3} \pi r^3, \quad p_c \rightarrow \rho g(h+r), \quad A \rightarrow \pi r^2$$

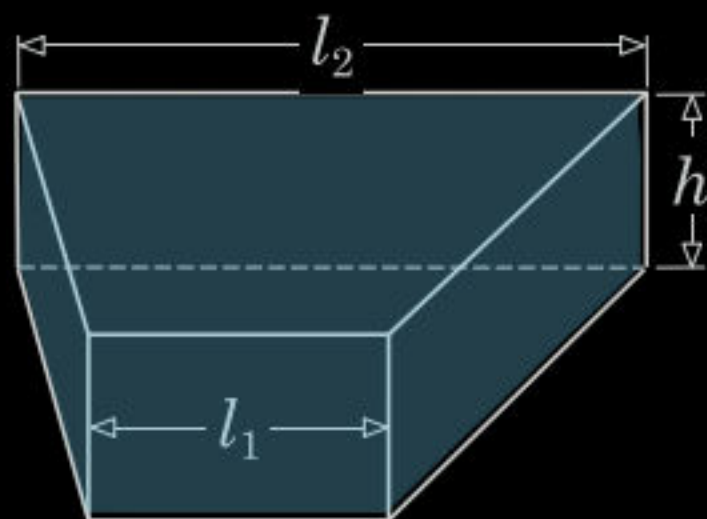
On the tilted bottom circular surface, take two horizontal strips symmetric from center.



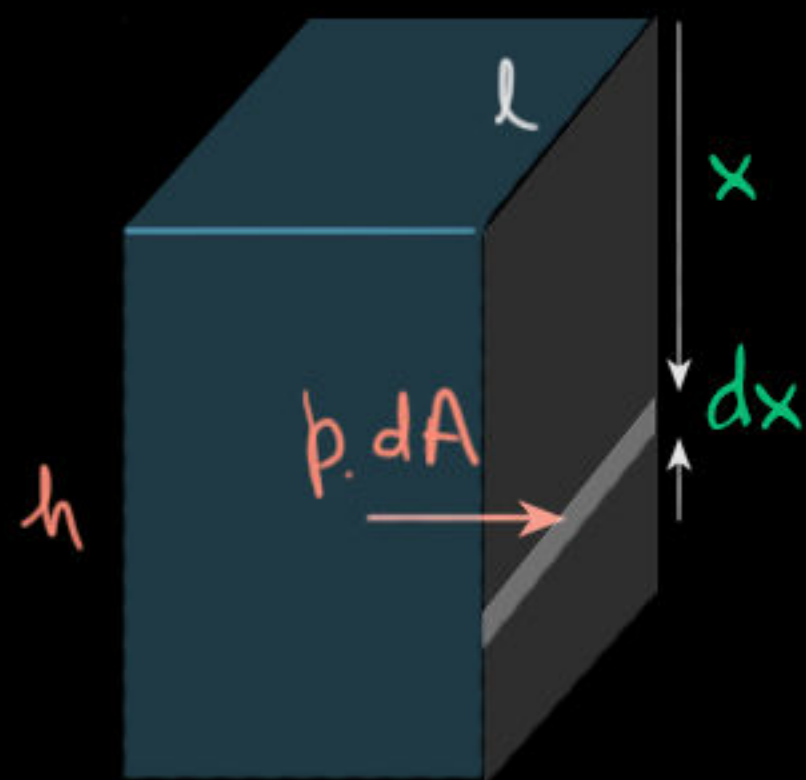
$$dF = F_A + F_B = p_c (2dA')$$

$$\Rightarrow F = p_c \int 2dA' = p_c A$$

Youtube/Arihant Senior



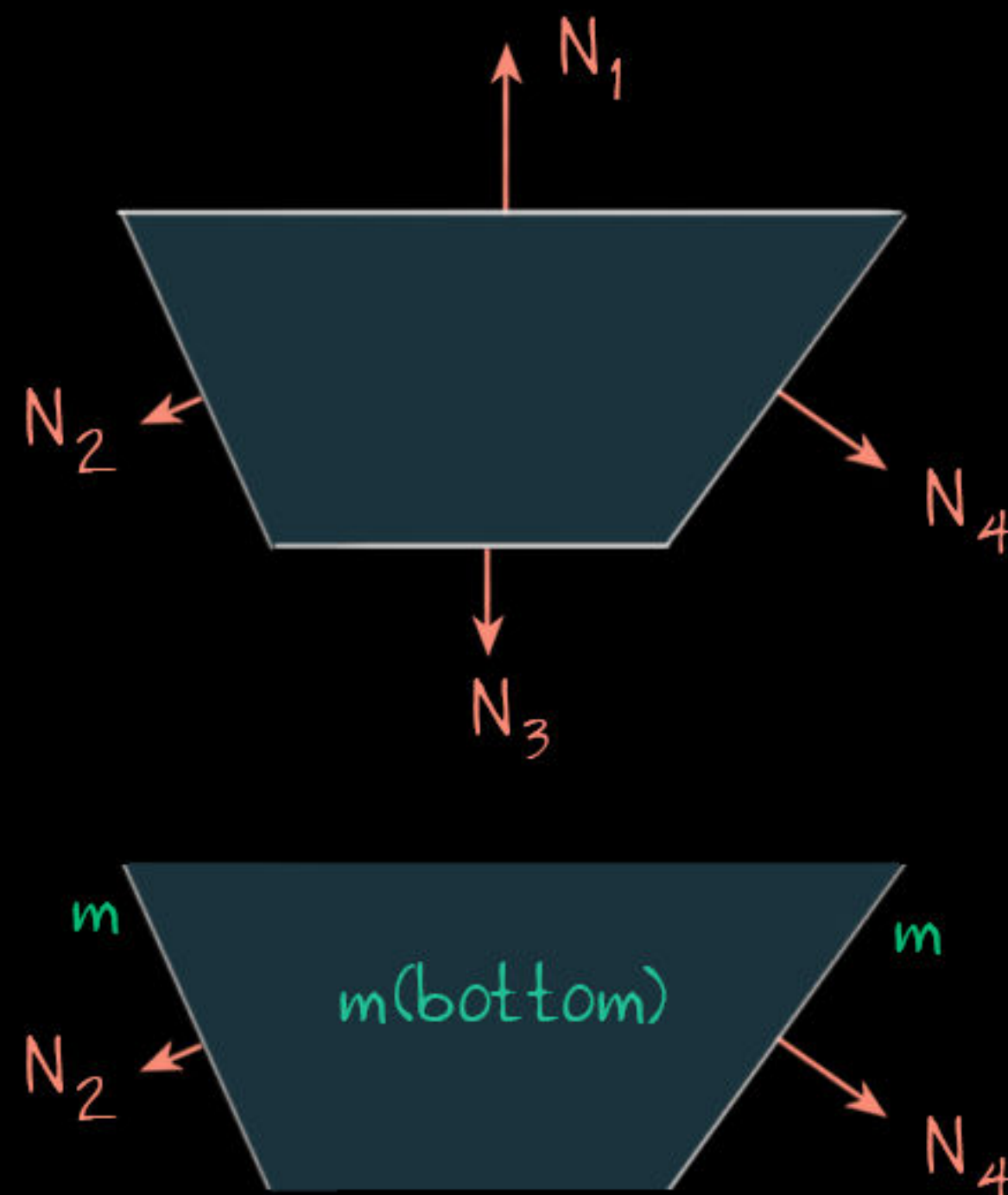
Force on a general wall



$$\begin{aligned}
 F_w &= \int p dA \\
 &= \int \rho g x l dx \\
 &= \frac{\rho g l h^2}{2} \quad (1) \\
 &\text{(Standard result)}
 \end{aligned}$$

4. Walls and base of an open trapezoidal tank each are of mass m . Some of the dimensions of the tank are shown in the figure. The tank is placed on a frictionless horizontal floor and completely filled with water. At some point of time, the front and the back walls are removed. Find acceleration of the tank immediately after removal of the walls. Density of water is ρ and acceleration due to gravity is g .

Sol: Top view:

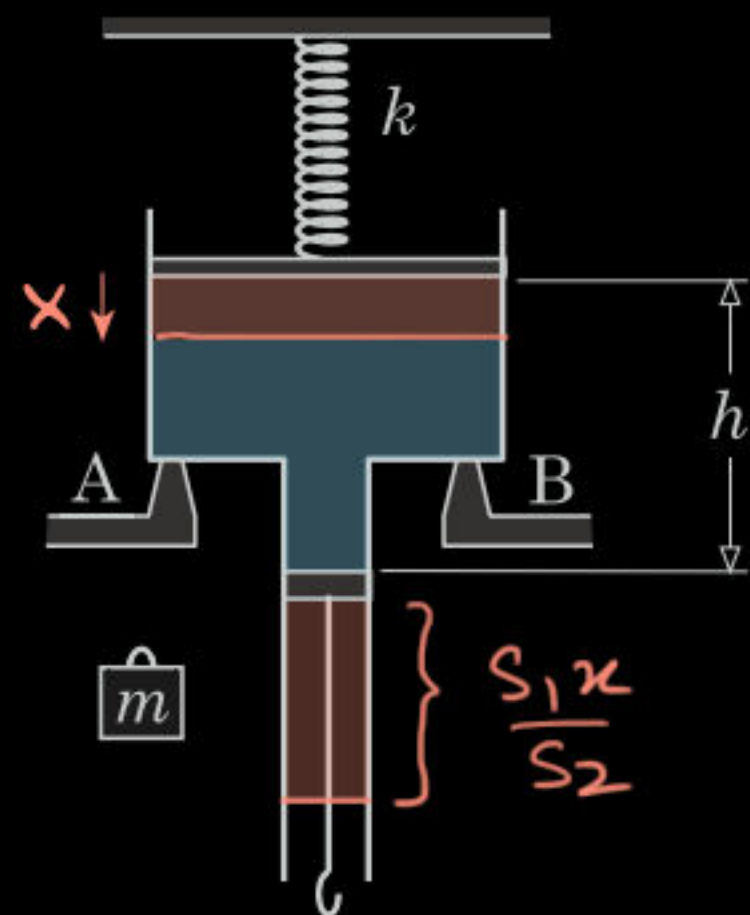


Lets write the forces exerted by liquid on each wall.

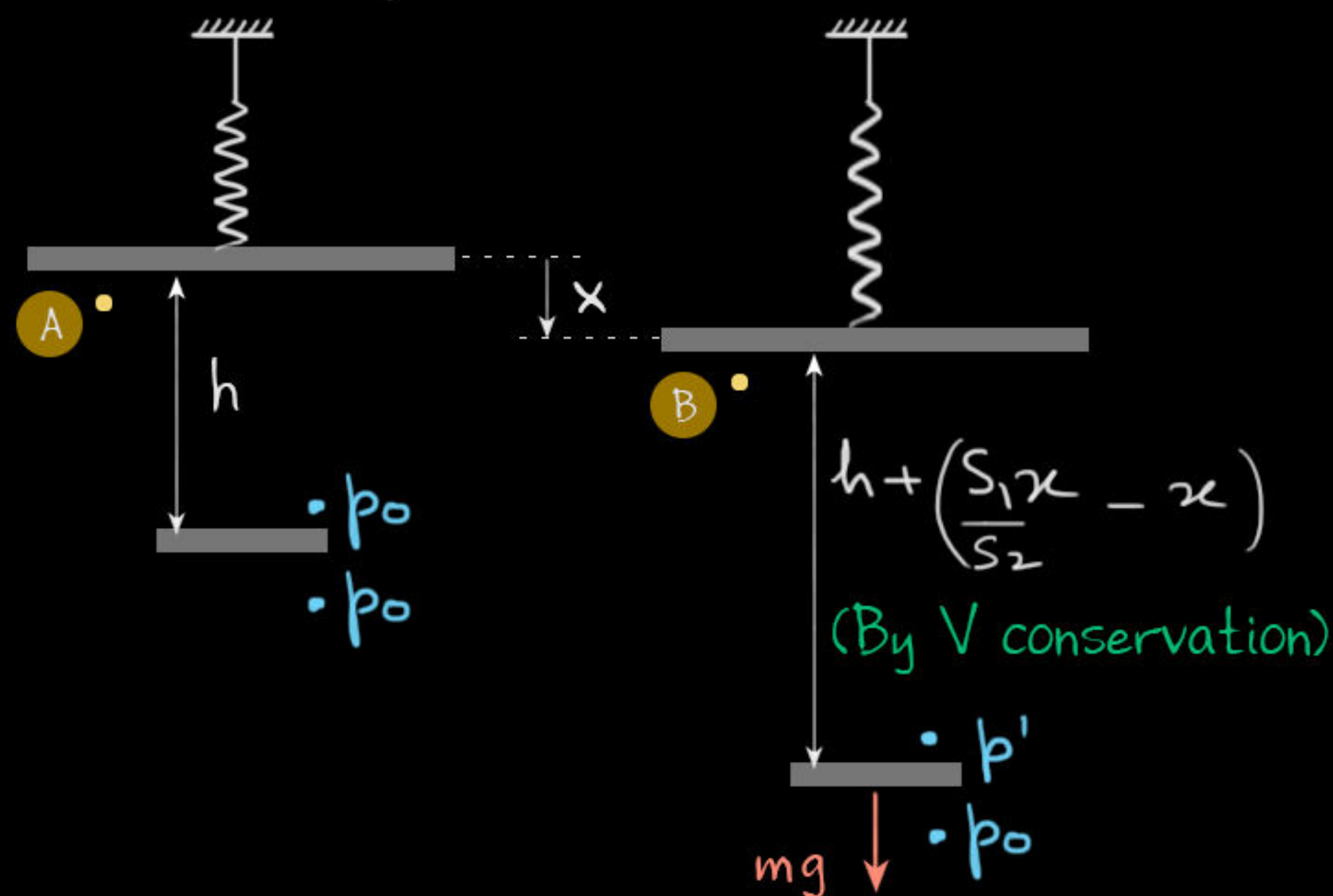
$$\vec{N}_1 + \vec{N}_2 + \vec{N}_3 + \vec{N}_4 = 0 \quad (2)$$

When parallel walls are removed:

$$\begin{aligned}
 a &= \frac{\vec{N}_2 + \vec{N}_4}{3m} = \frac{-\vec{N}_1 - \vec{N}_3}{3m} \quad (\text{From 2}) \\
 &= \frac{N_1 - N_3}{3m} \\
 &= \frac{\rho g h^2 (l_1 - l_2)}{6m} \quad (\text{From 1})
 \end{aligned}$$



5. In a vessel that consists of two vertical portions of unequal sections, some water is trapped between two airtight light pistons that can slide without friction. The vessel is placed on two fixed supports A and B. The upper piston is suspended from the ceiling with the help of a spring of force constant k and a hook is attached to the lower piston. Area of the upper and the lower sections are S_1 and S_2 respectively, distance between the pistons is h , density of water is ρ and acceleration due to gravity is g . Find change in extension of the spring after a block of mass m is suspended from the hook.



Top piston comes down because of lowering of pressure in liquid $\therefore \Delta p S_1 = kx$

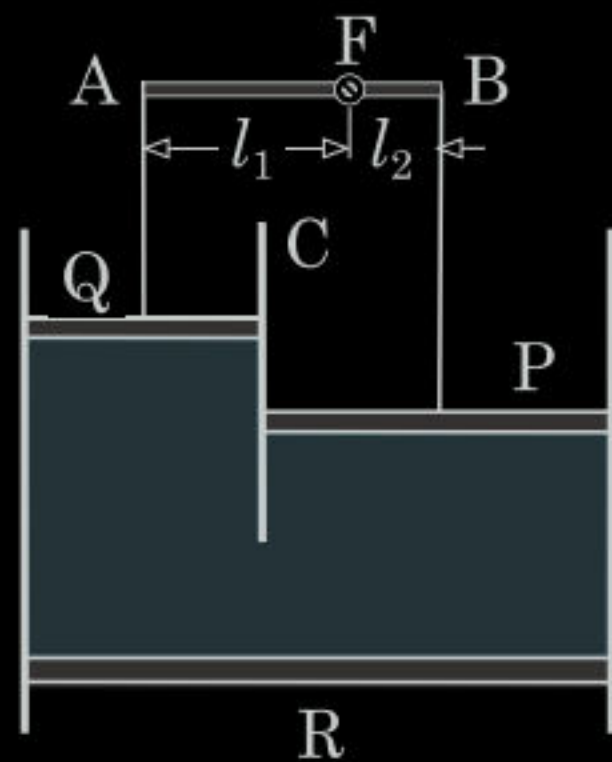
$$p_A - p_B$$

$$p_0 - \rho g h$$

$$p' - \rho g \left[h + \left(\frac{S_1 - S_2}{S_2} \right) x \right]$$

$$p' S_2 + mg = p_0 S_2$$

Solving: $x = \frac{mg S_1}{k S_2 - \rho g (S_1 - S_2) S_1}$



6. A volume V of water is trapped in a fixed vertical pipe of uniform section having a fixed partition C with the help of three light pistons P (of area S), Q and R that can slide without friction. The piston Q and R are connected to a light lever arm AB with the help of two vertical light cords. The lever arm has a fixed fulcrum that divides it in two portions of lengths l_1 and l_2 as shown in the figure. If the system is in equilibrium, find the height of the water column between the pistons P and R .

$$\tau_f: T_1 l_1 = T_2 l_2 \quad (1)$$

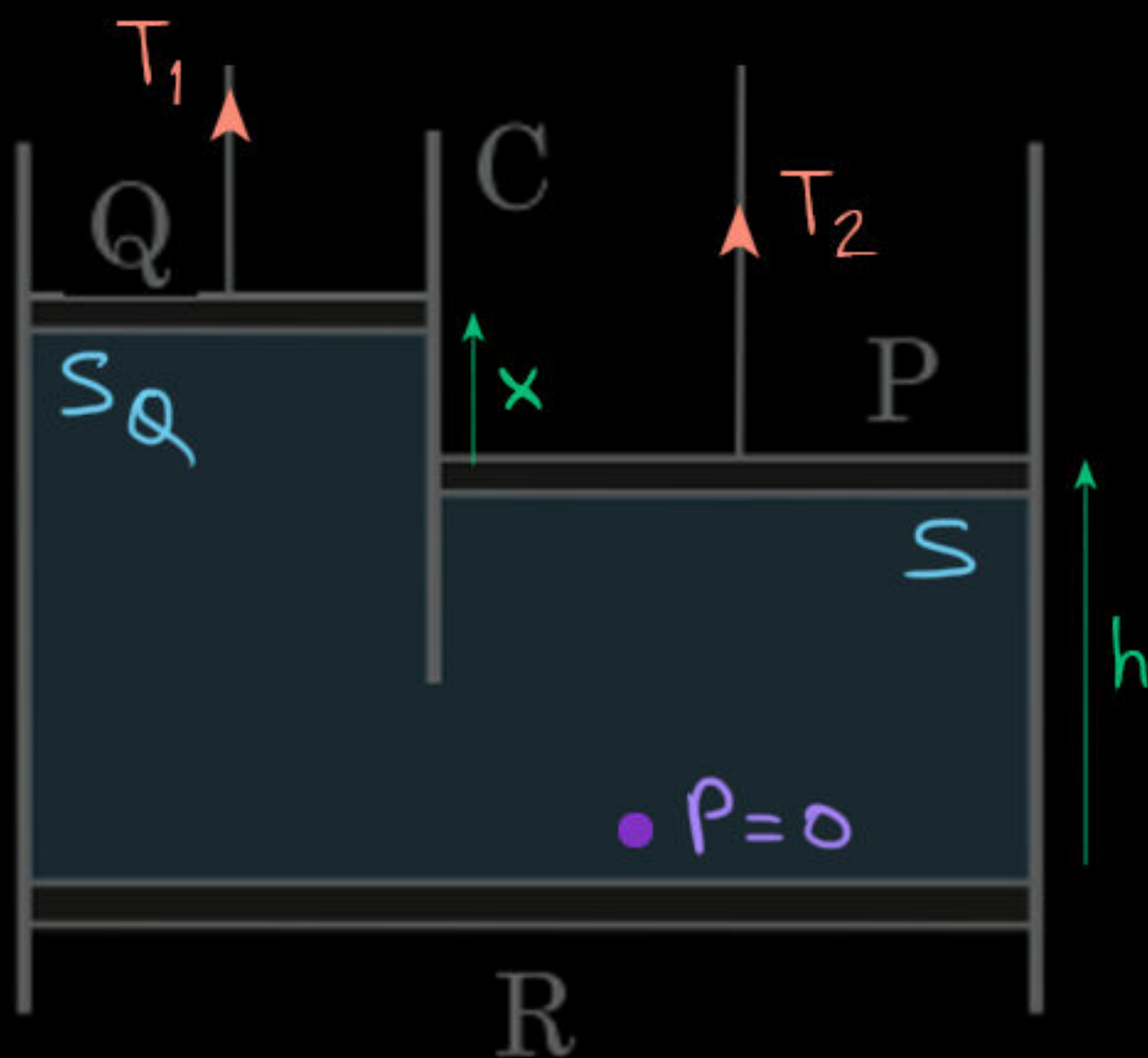
Dealing with gauge pressure only:

$$T_2 = (\rho g h) S \quad (2)$$

$$T_1 = \rho g (h+x) S_Q \quad (2)$$

$$V_1 + V_2 = V$$

$$T_1 + T_2 = \rho g V \quad (3)$$

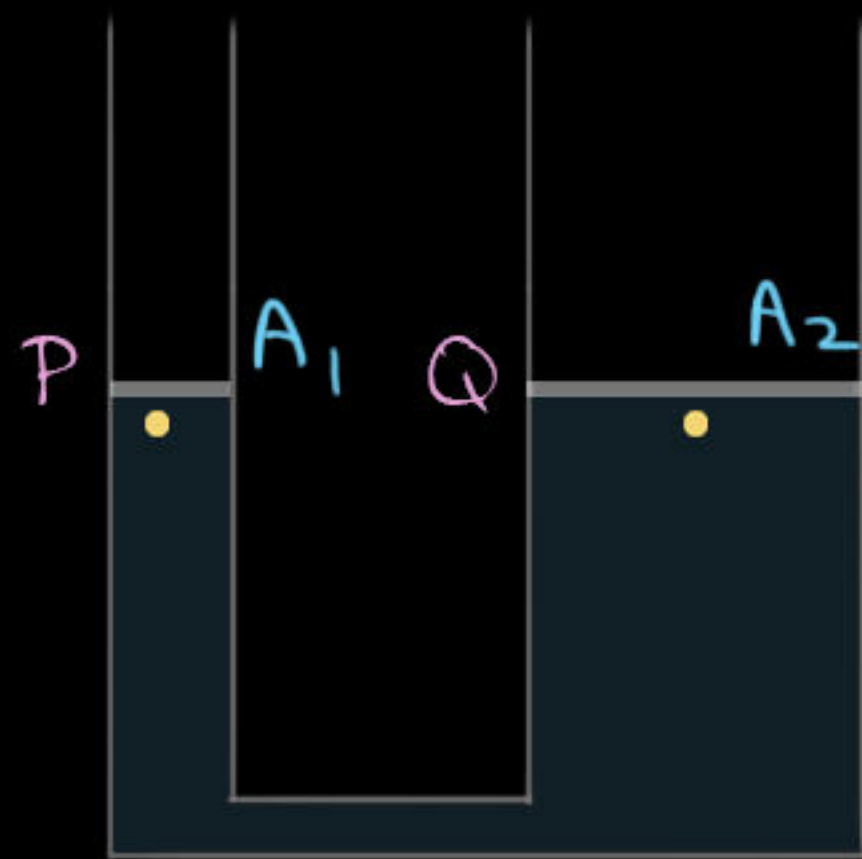


Equations 1, 2, 3,
variables T_1, T_2, h :

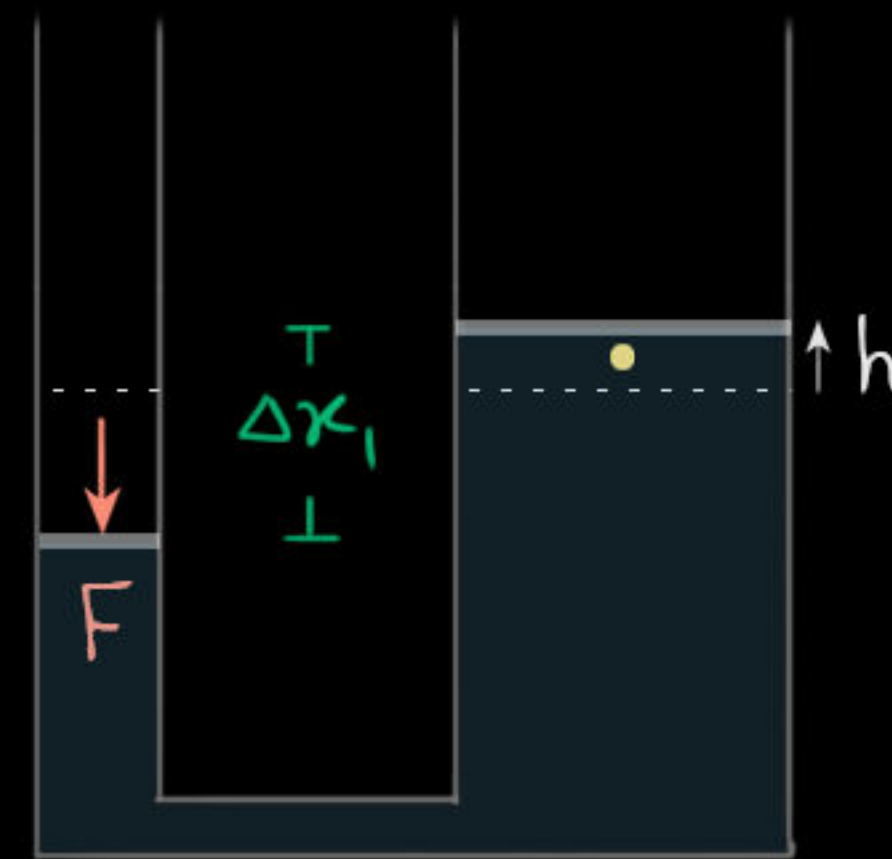
$$h = \frac{V l_1}{S(l_1 + l_2)}$$

7. Two vertical cylindrical vessels containing water are connected by a thin horizontal tube at their bottom. Water in the vessels is enclosed by pistons P and Q of masses m and αm that stay in equilibrium in the same level. A gradually increasing downward force is applied on the piston P. When magnitude of this force becomes F , the piston Q moves a height h above its initial level. Now the force on the piston P is gradually removed and applied on the piston Q. How high will the piston P move above its initial equilibrium level when magnitude of the force on the piston Q becomes βF ? Neglect all dissipative effects.

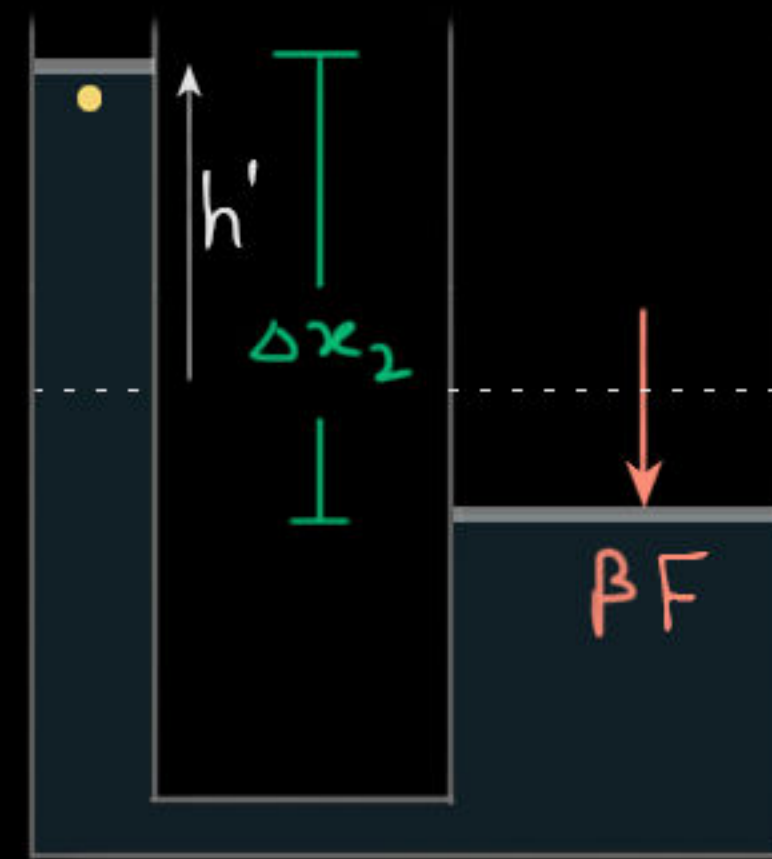
• $\rightarrow$ are at same pressure



External force will be balanced by pressure change developed below piston.



$$\therefore F = \rho g \Delta x_1 A_1 \quad (1)$$



$$\beta F = \rho g \Delta x_2 A_2 \quad (2)$$

Volume conservation:

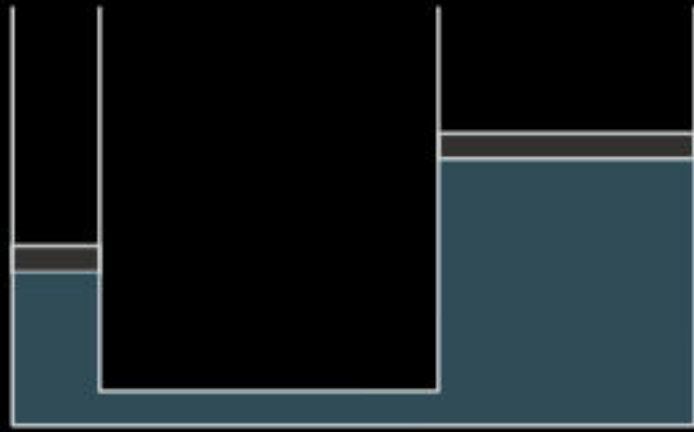
$$\Delta x_1 = h + \frac{A_2}{A_1} h \quad (3)$$

$$\Delta x_2 = h' + \frac{A_1}{A_2} h' \quad (4)$$

From four equations:

$$\beta = \frac{\Delta x_2 A_2}{\Delta x_1 A_1} = \frac{h' (A_1 + A_2)}{h (A_1 + A_2)}$$

$$\Rightarrow h' = \beta h$$

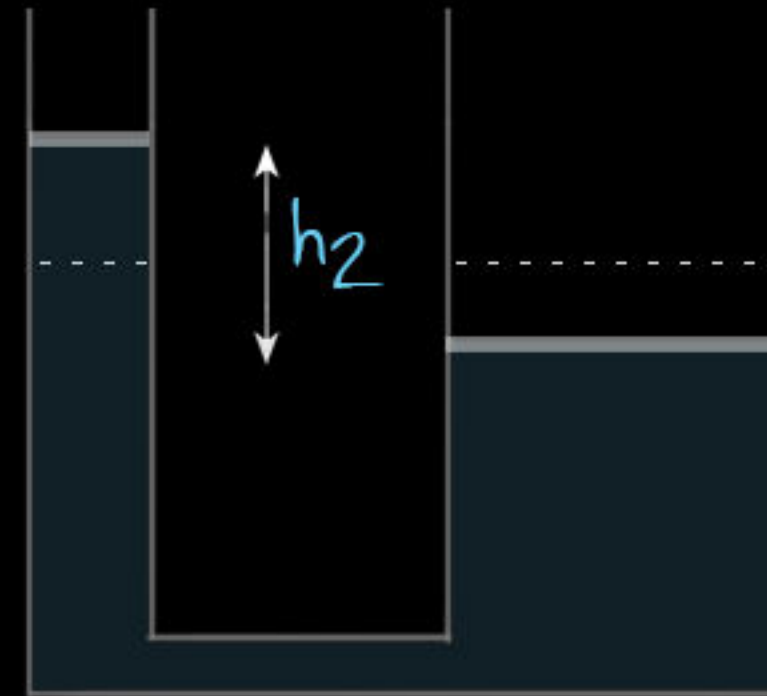
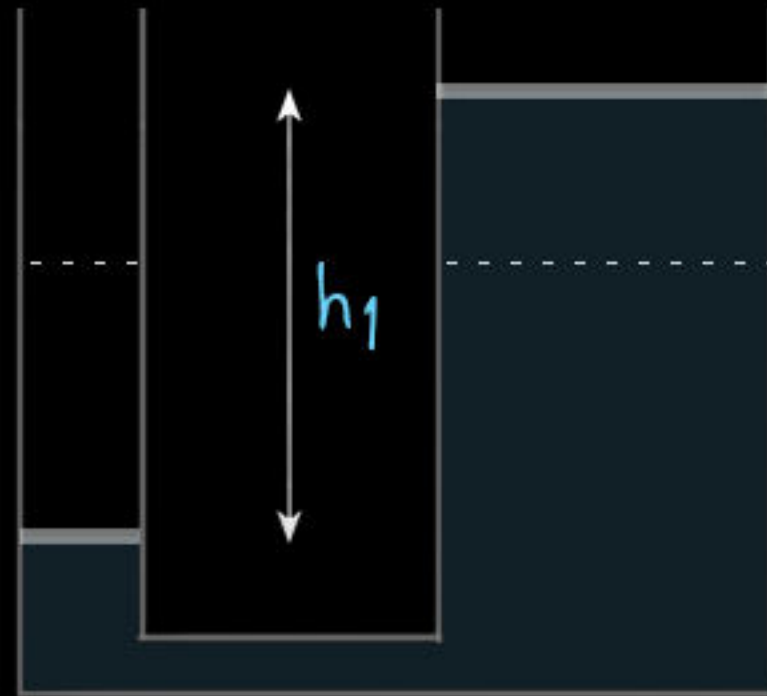


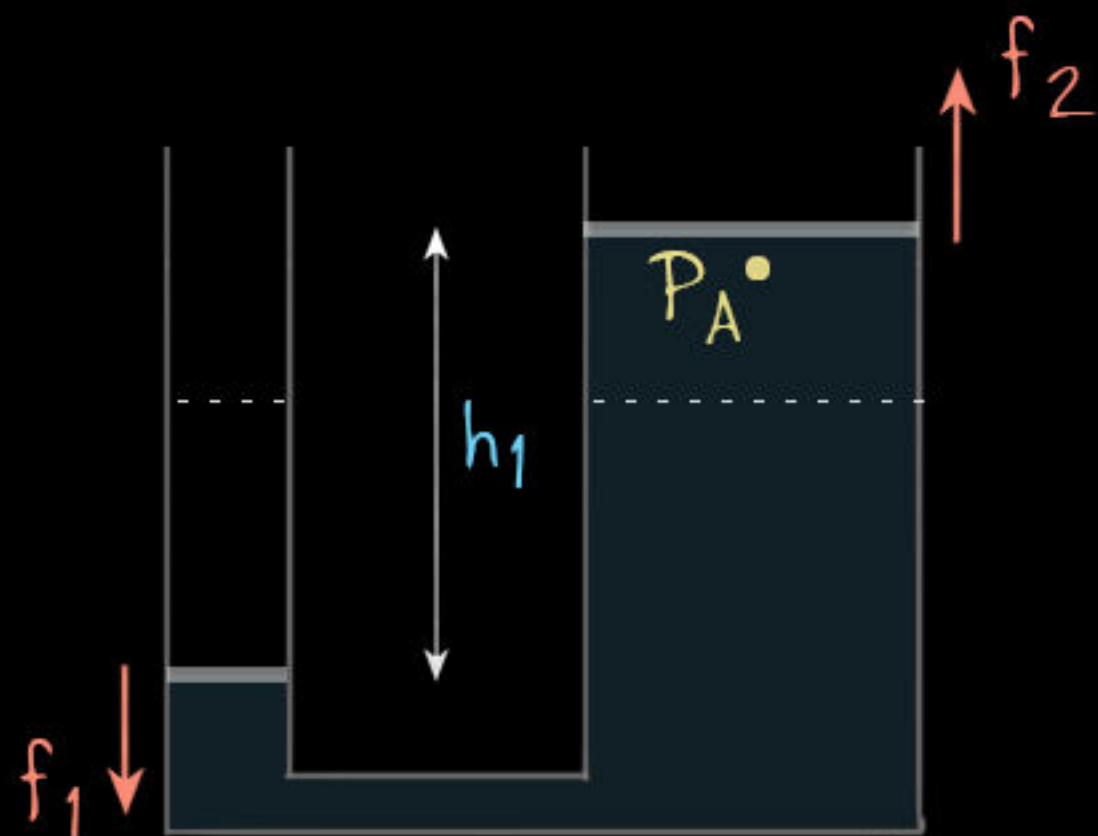
8. In a real hydraulic press, frictional forces between the piston and the inner surface of tubes cannot be neglected. In a particular real hydraulic press shown in the figure, piston in the wider arm can stay in equilibrium above the piston in the narrower arm at any height that ranges from $h_{\min} = 1.0 \text{ cm}$ to $h_{\max} = 2.0 \text{ cm}$. Density of material of the pistons is $\eta = 3$ times of that of the fluid used. If thickness of the piston in the narrower arm is $t = 5.0 \text{ cm}$, find the thickness of the piston in the wider arm.

Modified question: When smaller piston is at its lowest equilibrium, gap is h_1 .

When its at its highest equilibrium, gap is h_2 .

If thickness of first is t_1 , find t_2 .





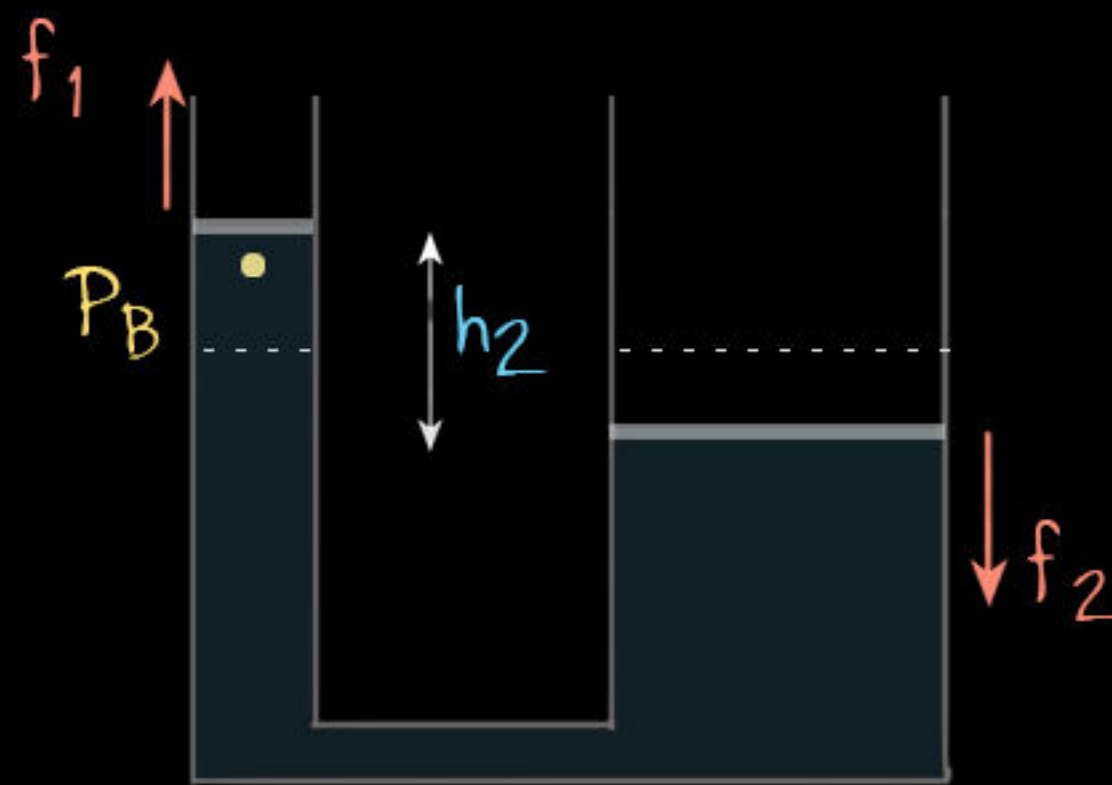
$$p_0 S_1 + f_1 + m_1 g = (P_A + \rho g h_1) S_1$$

$$\Rightarrow p_0 + \frac{f_1}{S_1} + \rho t_1 g = P_A + \rho g h_1$$

$$p_0 S_2 + m_2 g = P_B S_2 + f_2$$

$$\Rightarrow p_0 + \rho t_2 g = P_B + \frac{f_2}{S_2}$$

$$\frac{f_1}{S_1} + \frac{f_2}{S_2} + \rho(t_1 - t_2)g = \rho g h_1 \quad \text{Subtract} \quad (1)$$



$$p_0 S_2 + m_2 g = f_1 + P_B S_2$$

$$\Rightarrow p_0 + \rho t_2 g = \frac{f_1}{S_2} + P_B$$

$$p_0 S_2 + f_2 + m_2 g = (P_B + \rho g h_2) S_2$$

$$\Rightarrow p_0 + \frac{f_2}{S_2} + \rho t_2 g = P_B + \rho g h_2$$

$$\frac{f_2}{S_2} + \frac{f_1}{S_1} + \rho(t_2 - t_1)g = \rho g h_2 \quad \text{Subtract} \quad (2)$$

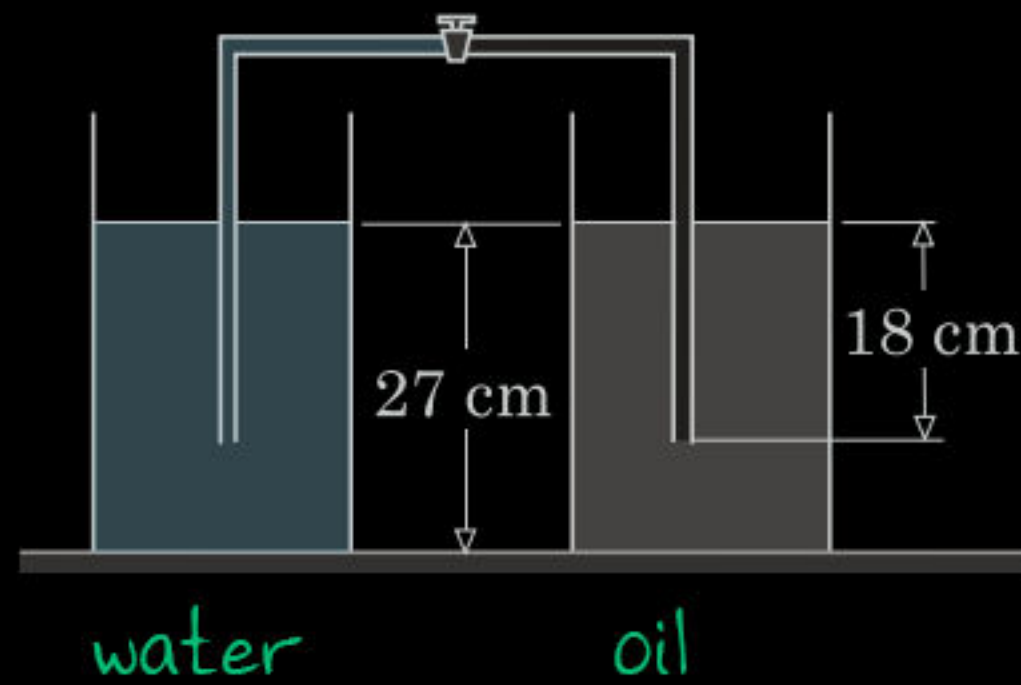
Subtracting 1 and 2:

$$\rho \frac{f_1}{S_1} - \rho \frac{f_2}{S_2} = \rho g (h_1 - h_2)$$

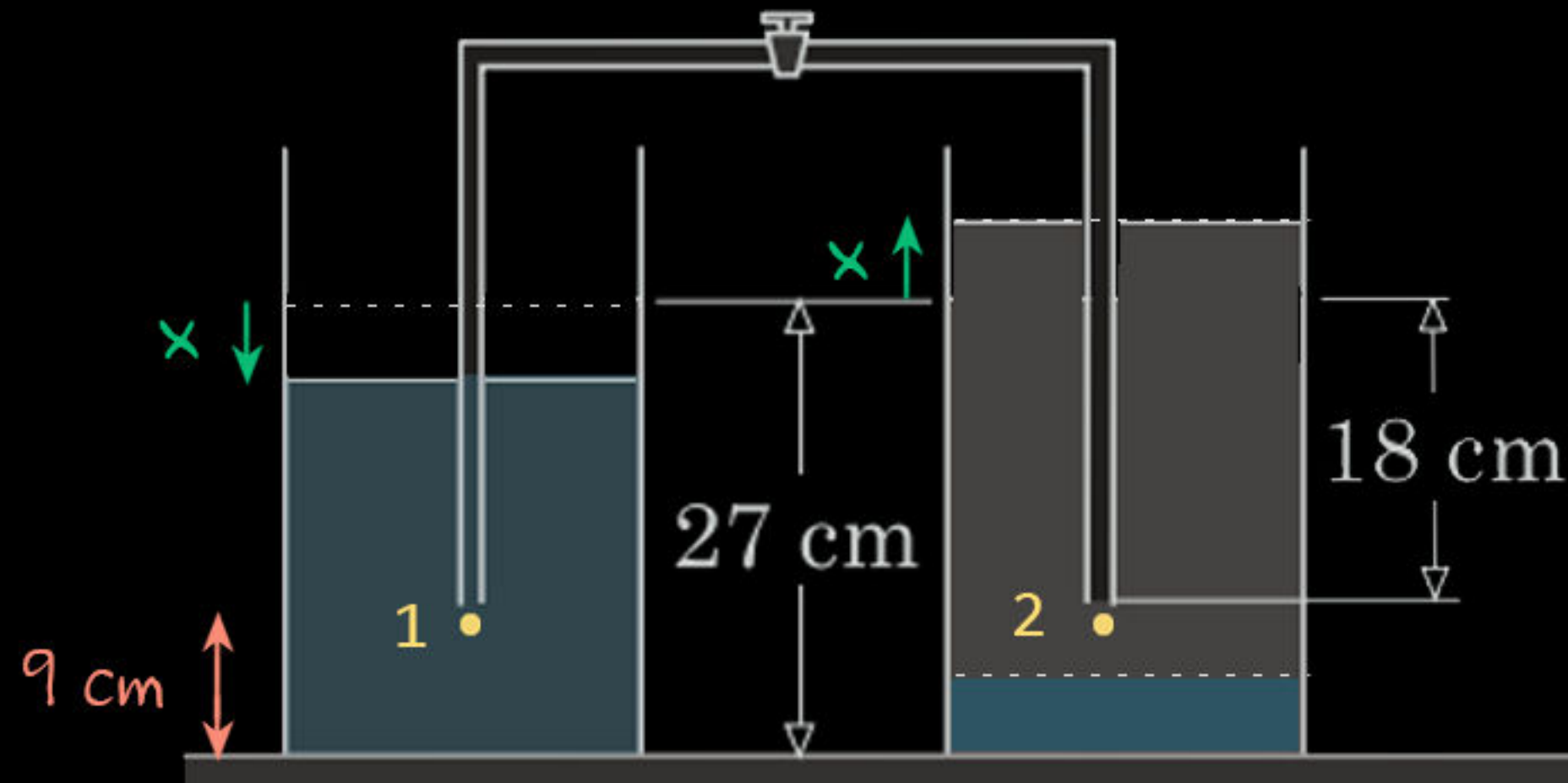
$$\Rightarrow t_2 = t_1 - \frac{(h_1 - h_2)}{2}$$

$\therefore$ if $h_1 > h_2$, t_2 is thinner

& if $h_1 < h_2$, t_2 is thicker



9. Two identical vessels each of height 36 cm are filled with water and oil each to a height 27 cm. A thin U tube equipped with a valve at its centre is filled with water and the valve is closed. The tube is held vertically and lowered in the vessels until its open ends reach a depth 18 cm in each liquid as shown in the figure. Density of water is 1.0 g/cm^3 and that of the oil is 0.8 g/cm^3 . At what heights will the water and the oil levels settle in the vessels after the valve is opened?



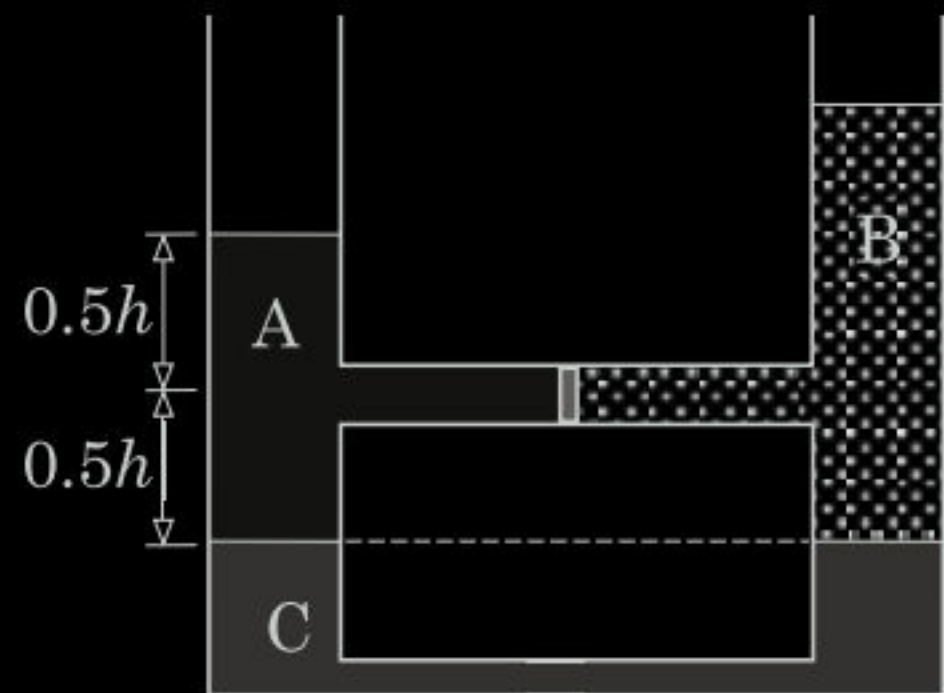
Water will flow until pressure at both ends of tube is equal.

$$\therefore P_1 = P_2$$

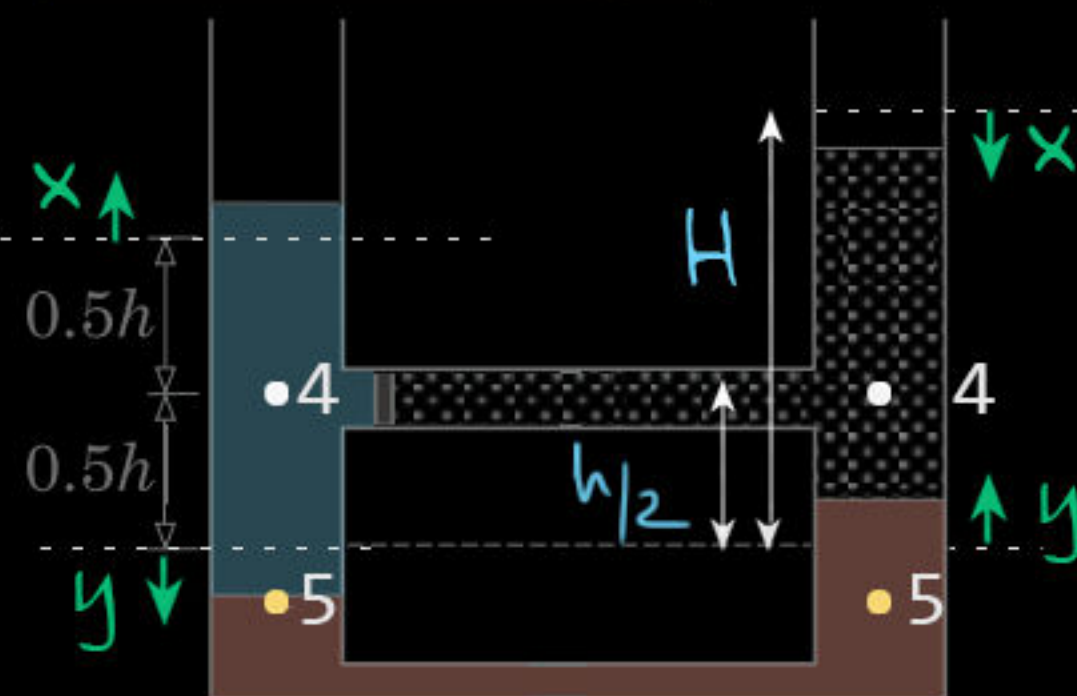
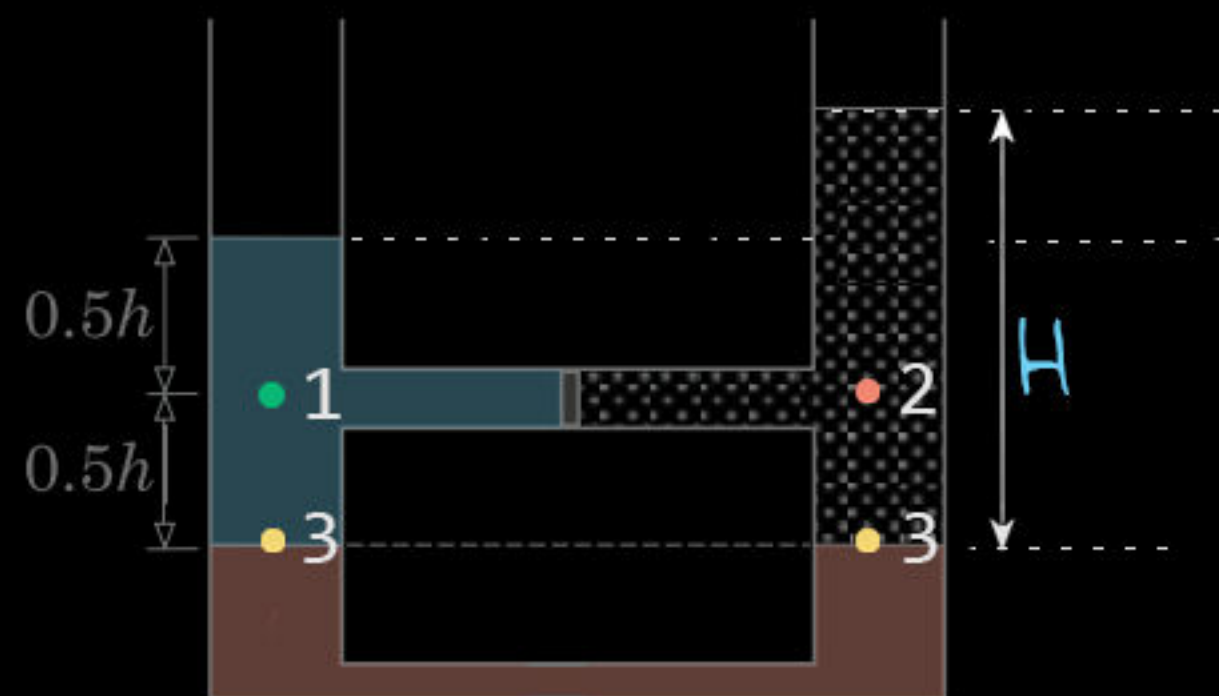
$$\Rightarrow \cancel{\rho_w} g (27 - 9 - x) = \cancel{\rho_o} g (18 + x)$$

$$\Rightarrow 18 - x = 18(0.8) + 0.8x$$

$$\Rightarrow x = 2 \text{ cm} //$$



10. Vertical arms of a special U-tube are connected by a thin transparent rubber tube having a piston inside. The tube contains three immiscible liquids A, B and C of densities ρ_A , ρ_B and ρ_C ($\rho_C > \rho_A > \rho_B$). Initially when the piston is held in the middle, lengths of columns of liquid C in both the arms are equal, length of column of liquid A in the left arm is h , the connecting tube meets liquid A in the middle and in the right arm is liquid B. If after release, the piston stops before reaching an end of the connecting tube, find by what amounts will the liquid levels in both the arms change?



$$P_3 = P_3 \Rightarrow \rho_A g h = \rho_B g H \Rightarrow H = \frac{\rho_A h}{\rho_B} \quad (1)$$



$P_1 < P_2$ (As A is denser than B)

∴ Piston goes left when released.

$$4: \rho_A g \left(x + \frac{h}{2} \right) = \rho_B g \left(H - \frac{h}{2} - x \right)$$

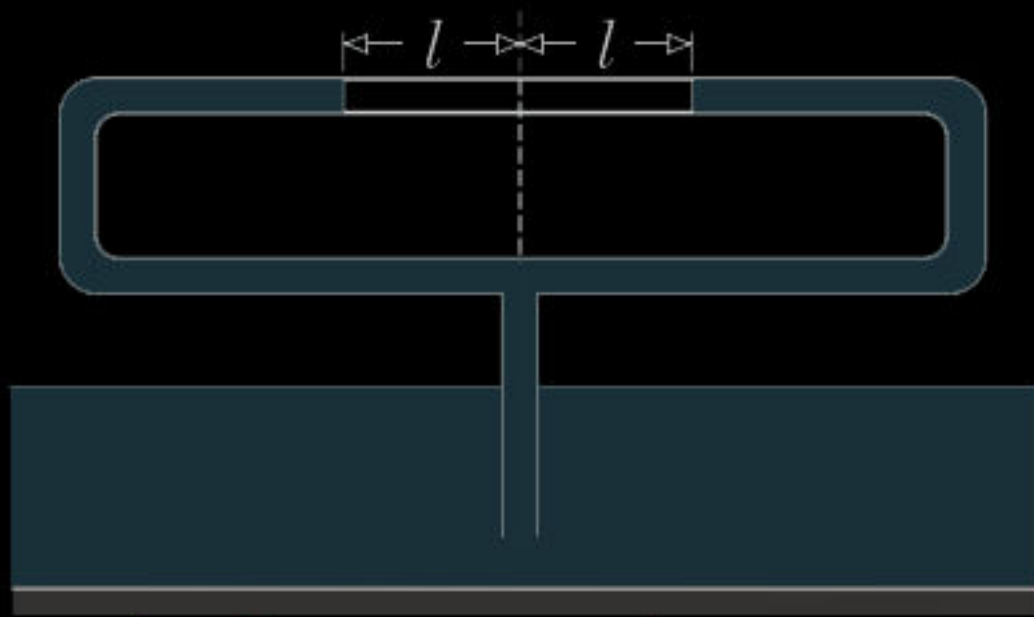
$$\Rightarrow x = \frac{\rho_A - \rho_B}{\rho_A + \rho_B} \cdot \frac{h}{2} //$$

$$5: P_4 + \rho_A g \left(\frac{h}{2} + y \right) = P_4 + \rho_B g \left(\frac{h}{2} - y \right) + \rho_C g (2y)$$

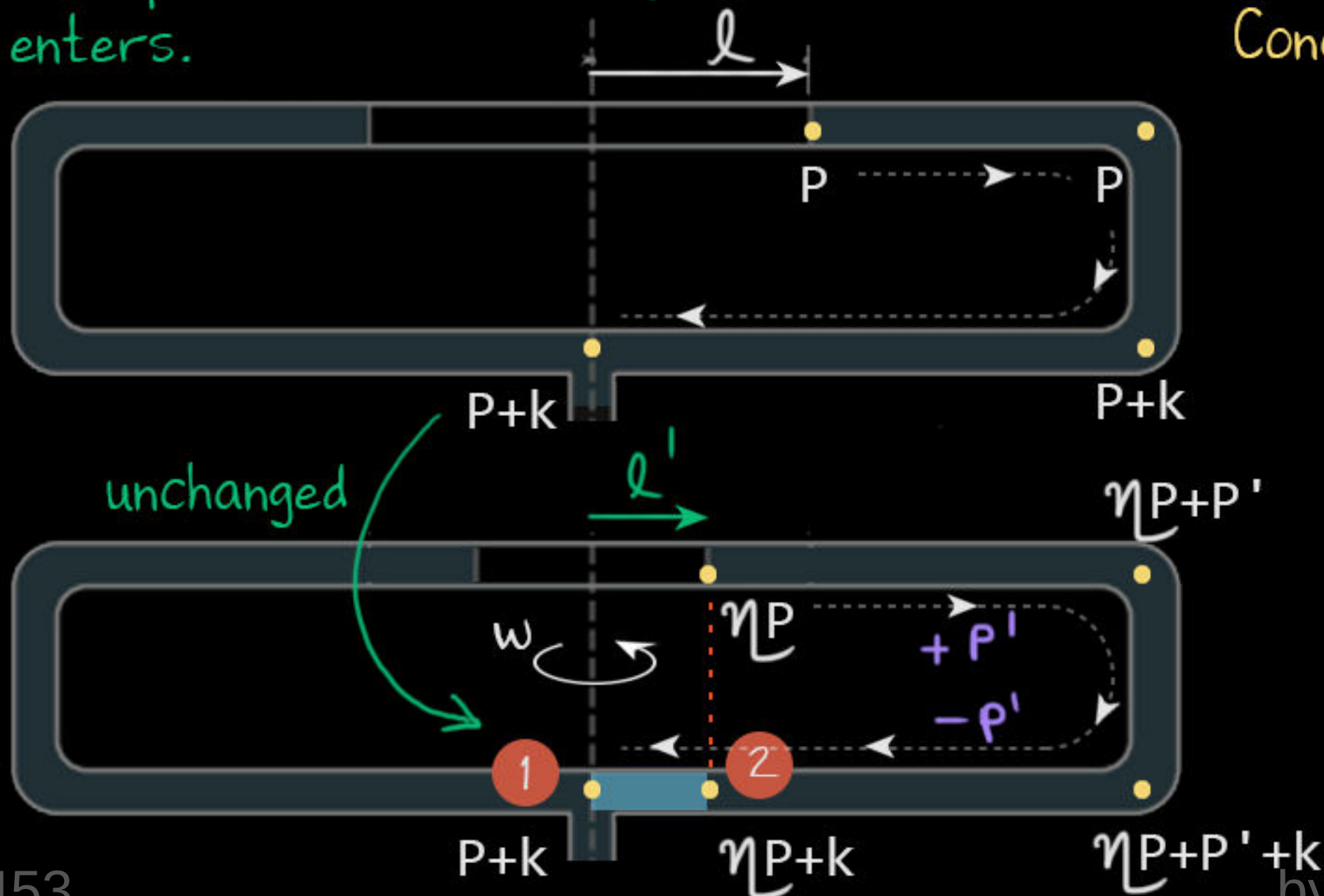
$$\Rightarrow y = \frac{(\rho_A - \rho_B) h}{2(2\rho_C - \rho_A - \rho_B)} //$$

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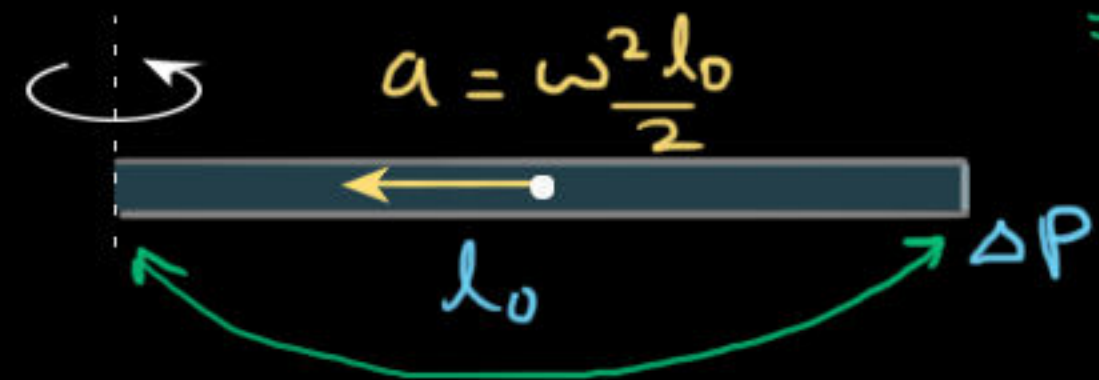


Concept: Pressure at opening is unchanged on rotation, but increases everywhere else, including at top. So air will shrink, and water enters.



11. A specially designed glass tube has a rectangular loop and only one opening through a tube extending from centre of a longer arm of rectangular loop. It is filled with mercury and held with its plane vertical and its opening is dipped in mercury filled in a large vessel. There is some air in the middle of upper horizontal arm of the tube. When the tube is stationary, ends of the air packet are at distance l from the axis of symmetry of the tube. Pressure in the air packet is p and density of mercury is ρ . At what angular velocity must the tube be rotated about its vertical axis of symmetry so that pressure in the air packet becomes η times of its value in stationary tube?

Concept: For a rotating fluid



$$F = ma$$

$$\Rightarrow \Delta P A = (\rho A l_0) \omega^2 \frac{l_0}{2}$$

$$\Rightarrow \Delta P = \frac{\rho \omega^2 l_0^2}{2}$$

•• glass, isothermal
Sol: $P_1 V_1 = P_2 V_2$
 $\Rightarrow \rho l A = \eta \rho l' A \Rightarrow l' = \frac{l}{\eta}$

Between 1 & 2:

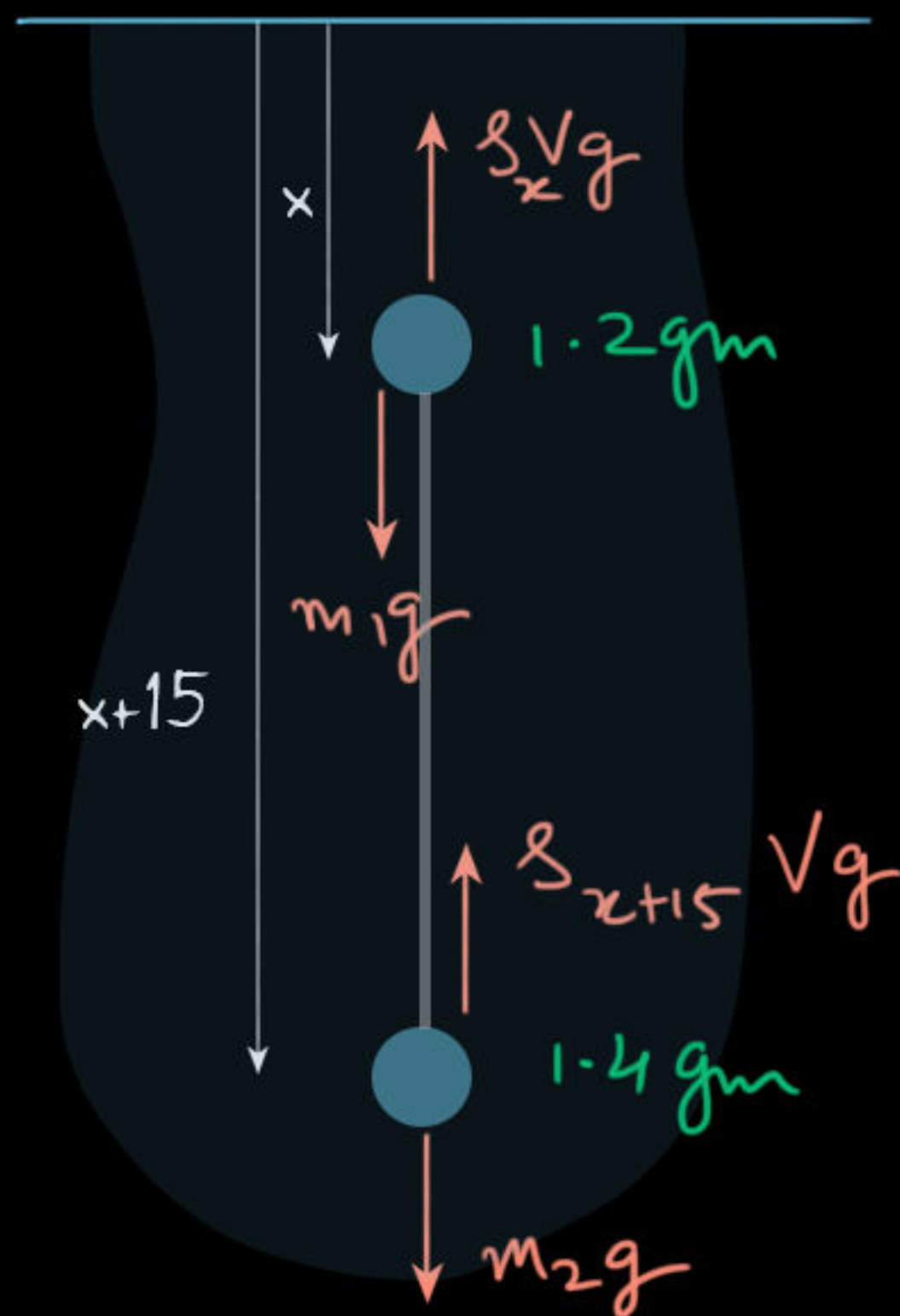
$$\Delta p = \frac{\rho \omega^2 l'^2}{2}$$

$$\Rightarrow (\eta - 1)P = \frac{\rho \omega^2 l^2}{2 \eta^2}$$

$$\Rightarrow \omega = \frac{\eta}{l} \sqrt{\frac{2(\eta - 1)P}{\rho}}$$

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12. In a vessel is a liquid, density of which increases with depth y from its surface according to the law $\rho = \rho_0 + ky$, where $\rho_0 = 1.0 \text{ g/cm}^3$ and $k = 0.01 \text{ g/cm}^4$. Two balls connected with an inextensible thread of length 15 cm are dropped in the liquid. Volume of each ball is 1.0 cm^3 and their masses are 1.2 g and 1.4 g. At what depth will these balls settle in equilibrium?
- depths

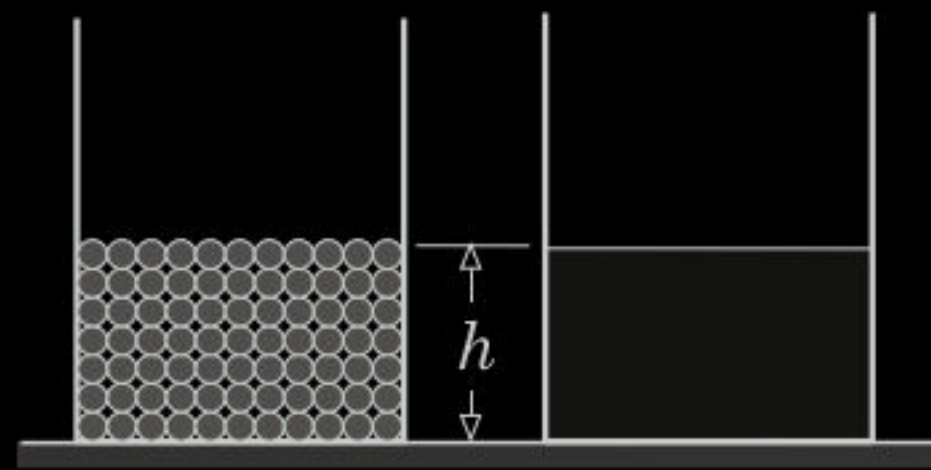
$$F_y = 0$$

$$\rho_x V g + \rho_{x+15} V g = (m_1 + m_2) g$$

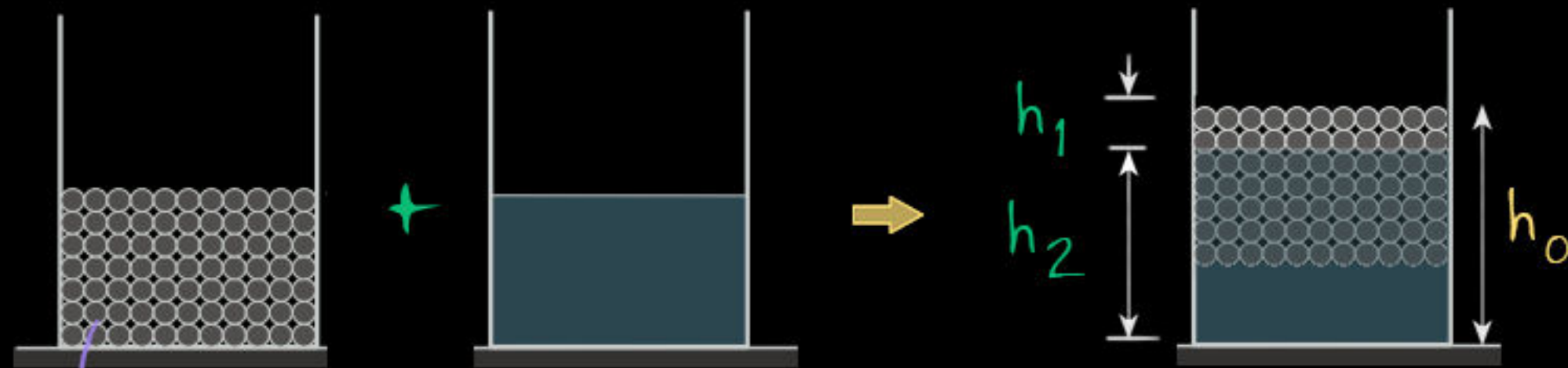
$$\Rightarrow (\rho_0 + kx) + (\rho_0 + k(15 + x)) = 2.6$$

$$\Rightarrow x = \frac{2.6 - 2 - 15(0.01)}{2(0.01)}$$

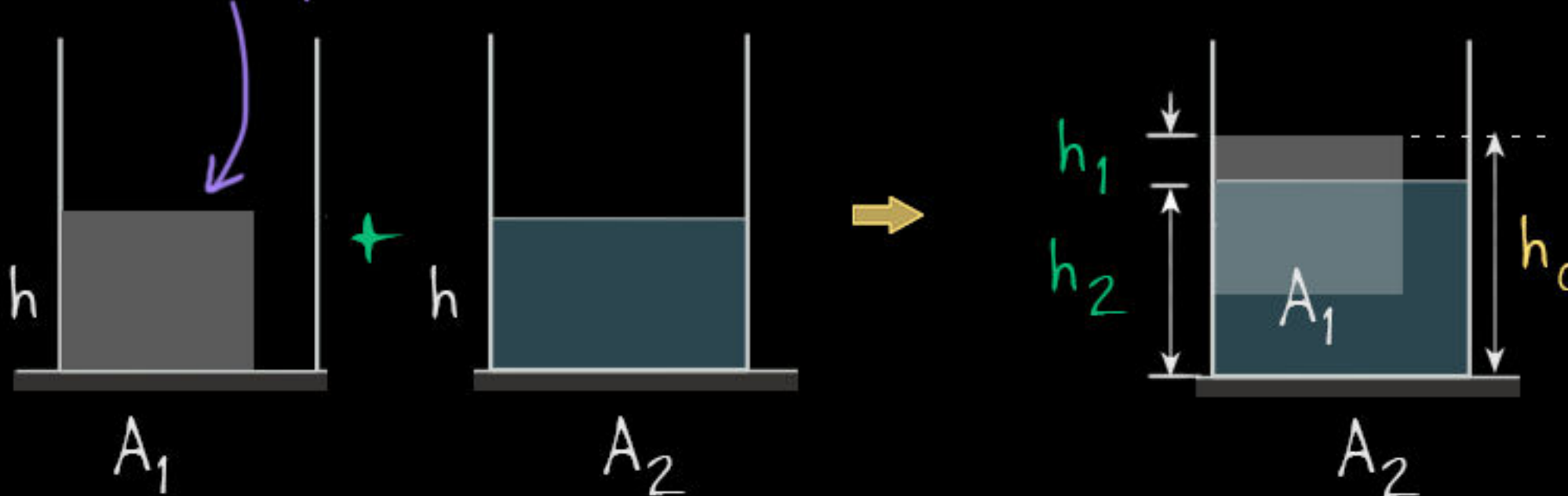
$$= 22.5 \text{ cm}$$



13. Consider two identical cylindrical vessels, one filled up to a height $h = 1.0$ m by a large number of plastic beads and the other filled up to the same height by water. Total mass of the beads is $m = 500$ kg and that of the water is $M = 1000$ kg. If all the water is gradually transferred in the first vessel, find height attained by the top layer of the beads. Density of the plastic is $\rho = 800$ kg/m³ and that of water is $\rho_0 = 1000$ kg/m³.



||| can be compressed horizontally to solid plastic



Floating part: $h_1 = \left(1 - \frac{\rho}{\rho_0}\right) h$ ①

Volume conservation for plastic + water:

$$A_1 h + A_2 h = A_2 h_2 + A_1 h_1$$

$$\Rightarrow A_1 (h - h_1) = A_2 (h_2 - h)$$

$$\Rightarrow \frac{m_p}{\rho h} \left(\frac{\rho h}{\rho_0} \right) = \frac{m_w}{\rho_0 h} (h_2 - h)$$

$$\Rightarrow h_2 = \left(1 + \frac{m_p}{m_w}\right) h$$
 ②

From 1,2:

$$h_0 = h_1 + h_2 = \left[2 + \frac{m_p}{m_w} - \frac{\rho}{\rho_0} \right] h$$



14. At every metre on a very thin rod, small air-filled balloons are attached except at the top end. Length of the rod is integral multiple of a metre and mass of every metre of the rod including balloons is 2.7 kg. The rod is lowered vertically into water. Volume of a balloon in air is 0.003 m^3 and it reduces by 100 cm^3 for every 10 kPa increase in pressure. How much maximum length of such a rod can float in water with its upper end in level with the water surface? Density of water is 1000 kg/m^3 .

Concept: 10kPa pressure increases every 1 meter depth.
and balloons are also at gaps of 1 meter.

⇒ Balloons volume reduce by 100 cm^3 every 1m.

On system: $m g = \rho V g$ $\text{cm}^3 \rightarrow \text{m}^3$

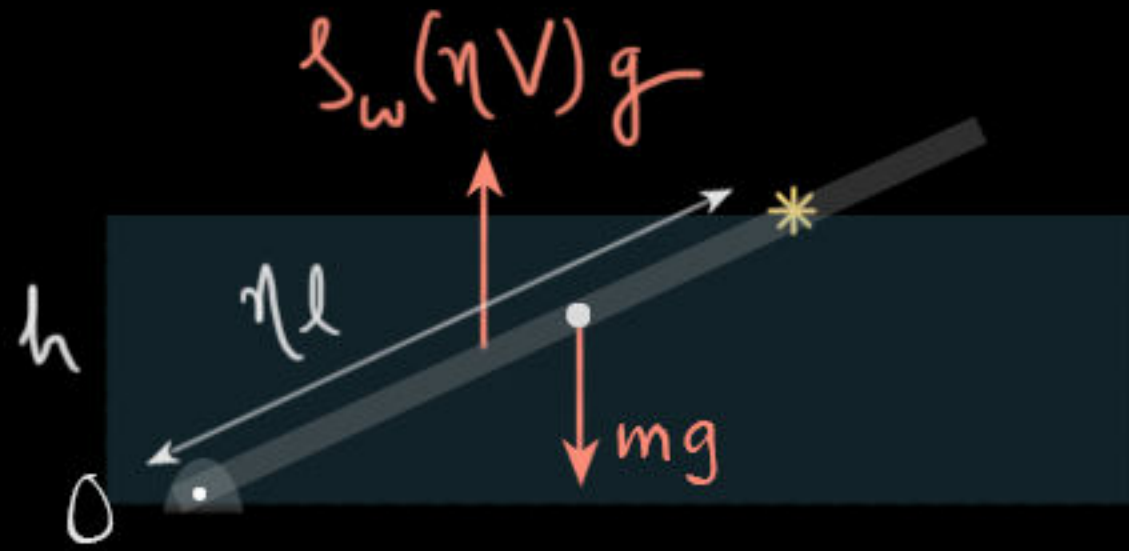
$$\Rightarrow (2.7) n = 1000 \left[n 3000 - 100 (1+2+3 \dots n) \right] \times 10^{-6}$$

$$\Rightarrow 2.7n = 3n - \frac{(n)(n+1)}{20}$$

$$\Rightarrow n = 5 //$$

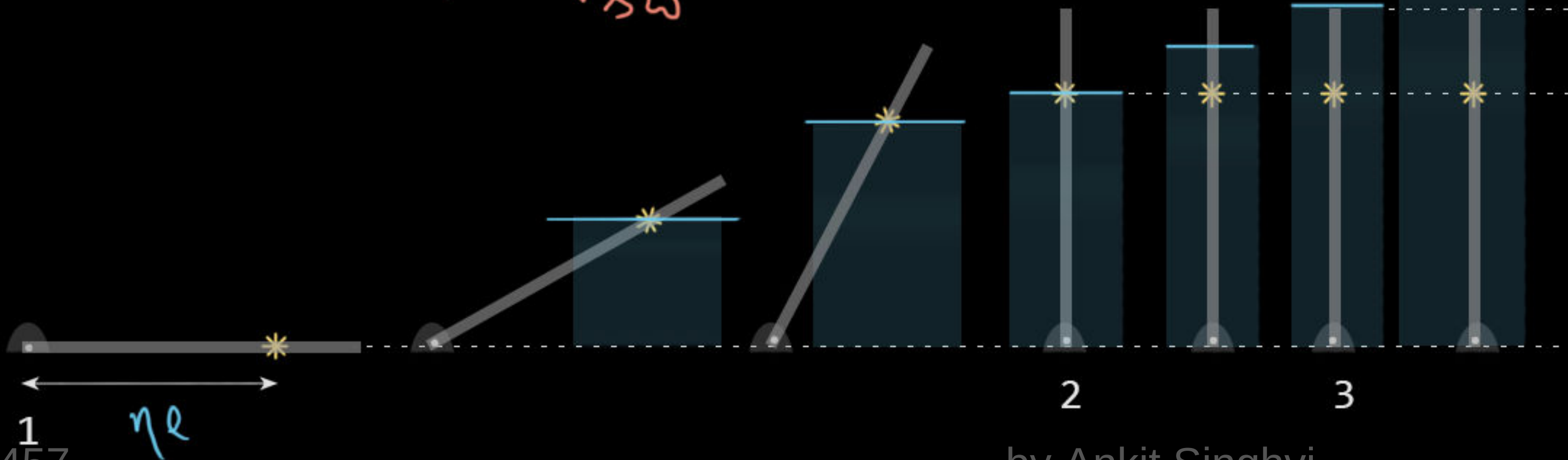
Let submerged fraction be η

15. A uniform rod of length l is lying on horizontal bottom of an empty container and one end of the rod is hinged there. The container is now slowly filled with water. Draw a graph to show how length l_i of rod immersed in water varies with height h of the water level in the container. Density of material of the rod is ρ and that of water is $\rho_w > \rho$.

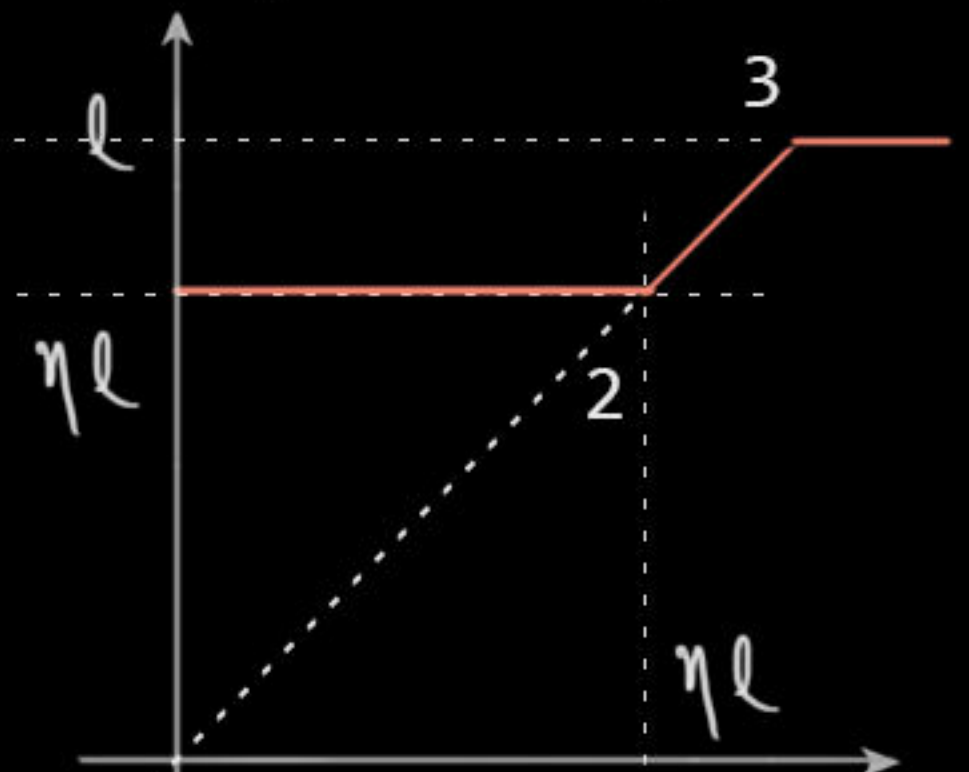


$$\tau_0: \rho \frac{V}{2} g \frac{l}{2} = S_w(\eta V) g \cdot \frac{\eta l}{2}$$

$$\Rightarrow \eta = \sqrt{\frac{\rho}{\rho_w}} = \text{constant}$$

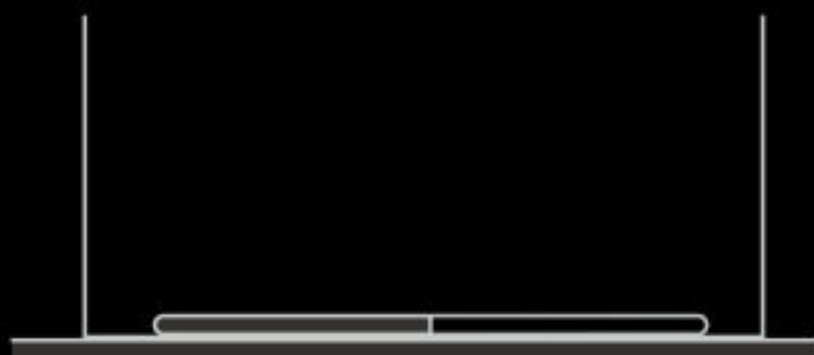


Submerged rod length



Height of water
Youtube / Arihant Senior

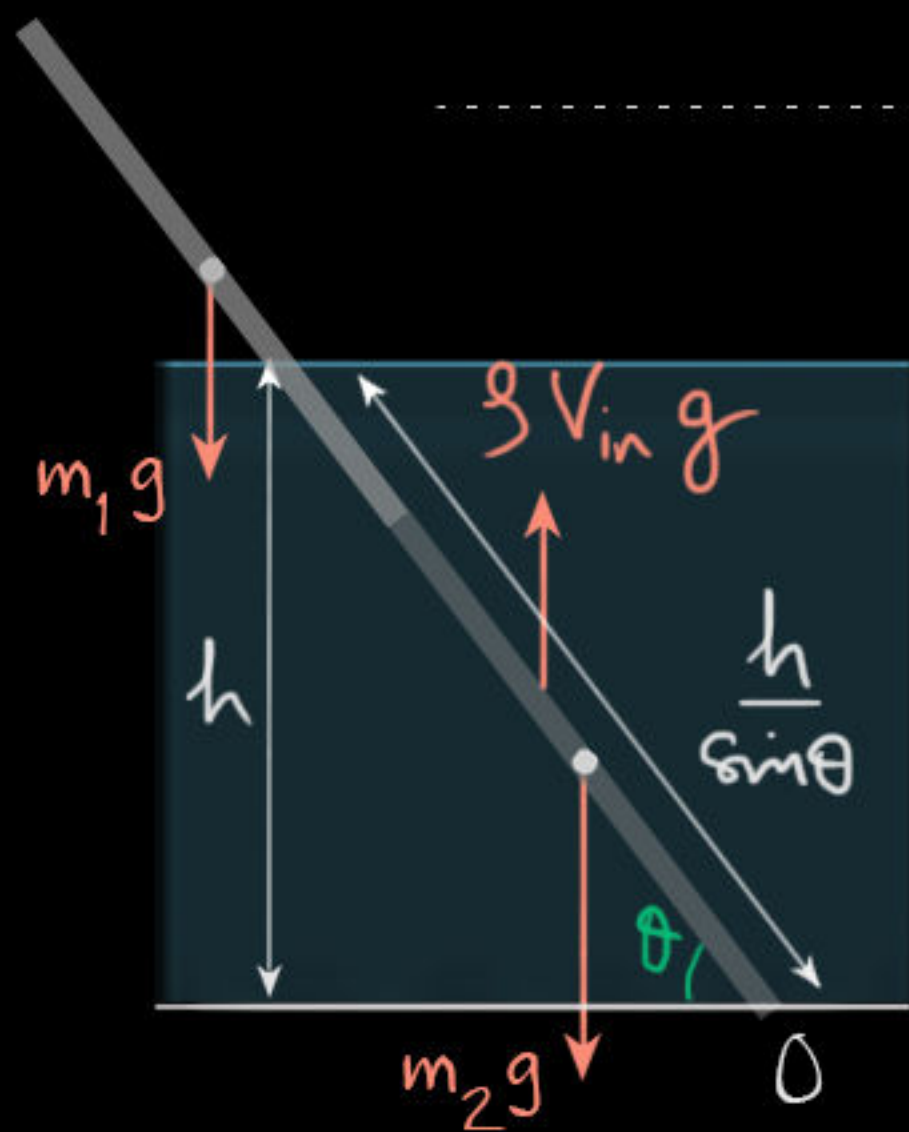
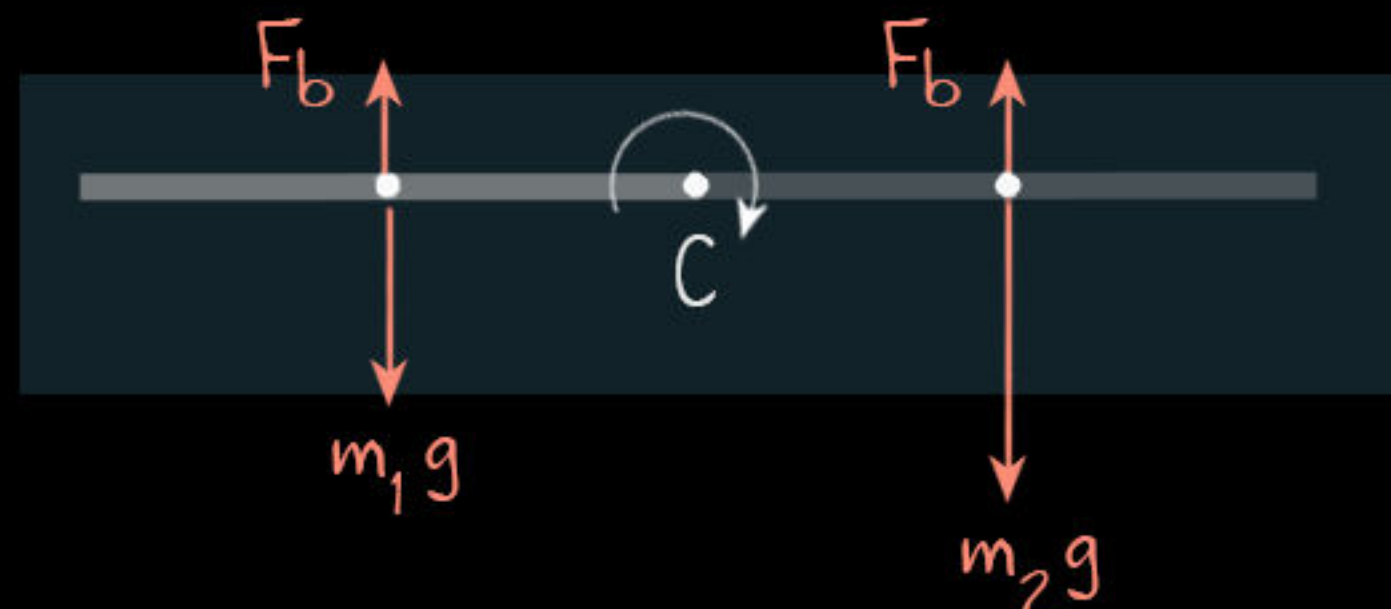
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16. One-half of a rod of length $l = 100$ cm has density $\rho_1 = 0.50$ g/cm³ and the other half has density $\rho_2 = 2.50$ g/cm³. The rod is placed on horizontal bottom of a large tub and then water is gradually poured in the tub. Find the depth of water in the tub, when the rod makes angle $\theta = 53^\circ$ with the horizontal. Density of water is $\rho = 1.00$ g/cm³.

Denser part will stay down.

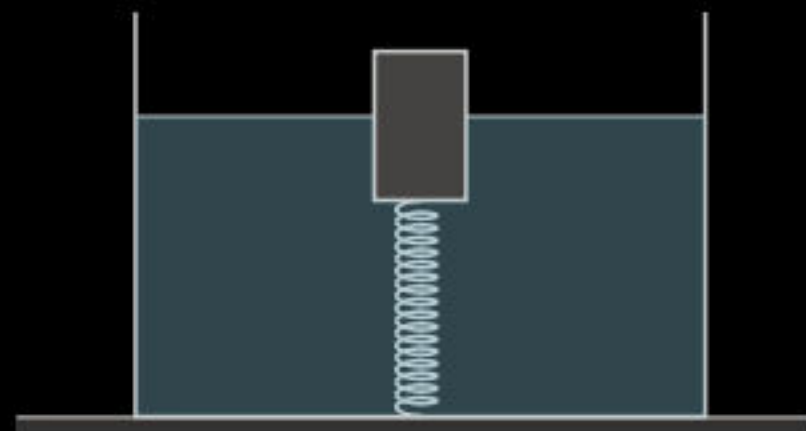
As torque about geometrical center of rod will keep heavier half down. $\rightarrow$



$$\tau_O: \overbrace{\left(\rho_1 \frac{V}{2} \right)}^{m_1} g \frac{3l}{4} + \overbrace{\left(\rho_2 \frac{V}{2} \right)}^{m_2} g \frac{l}{4} = \rho \left[\frac{V_{in}}{l} \cdot l \right] g \frac{h}{\sin \theta}$$

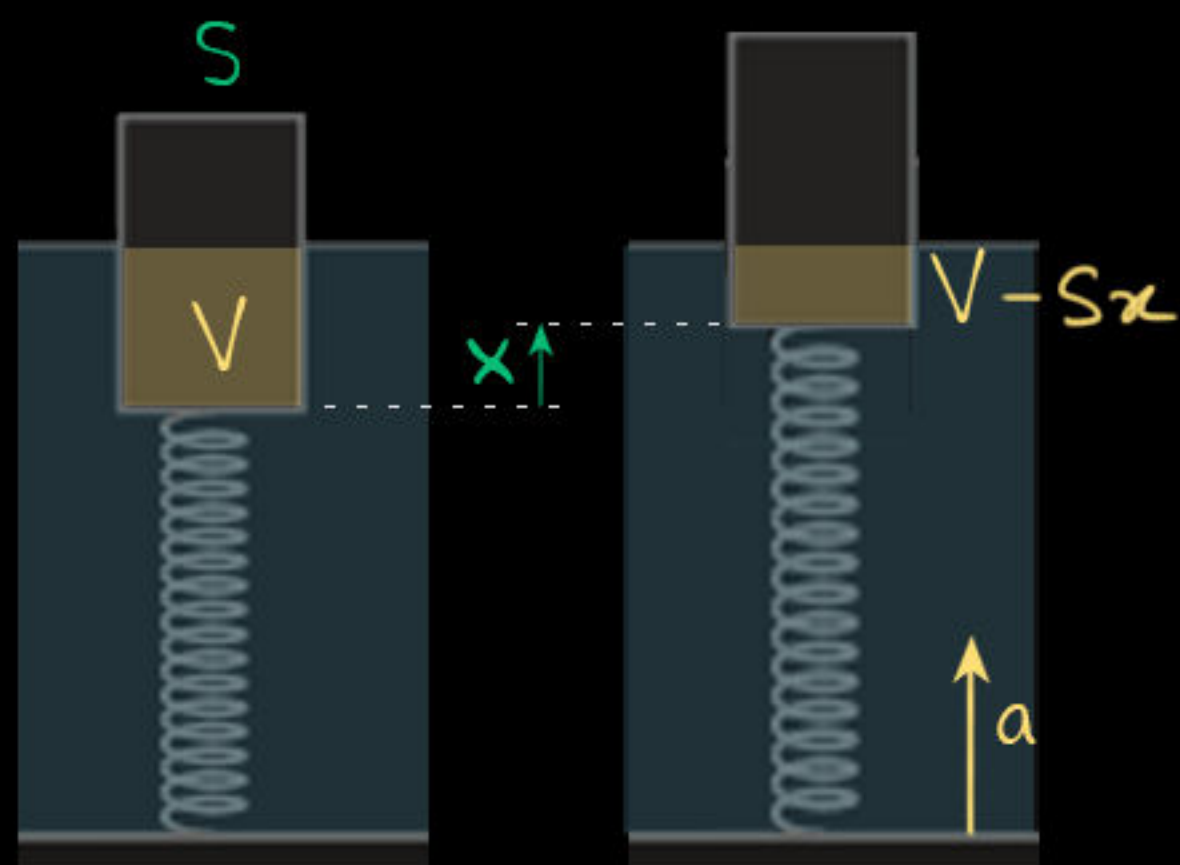
$$\Rightarrow \frac{l}{4} (3\rho_1 + \rho_2) = \rho \frac{h^2}{2 \sin^2 \theta}$$

$$\Rightarrow h = \frac{l \sin \theta}{2} \sqrt{\frac{3\rho_1 + \rho_2}{\rho}} //$$



17. A wooden cylinder of cross-section area S is connected with the bottom of a vessel by a spring of stiffness $k = 10 \text{ N/m}$. When some water is filled in the vessel, the cylinder floats partially submerged and extension in the spring becomes x_0 . Density of water is ρ_w and acceleration of free fall is g . If the vessel is given an upwards acceleration a , by what amount will the length of the submerged portion of the cylinder change?

Lets take initial submerged volume as V .
And lets assume it goes up by x later.



Initially:

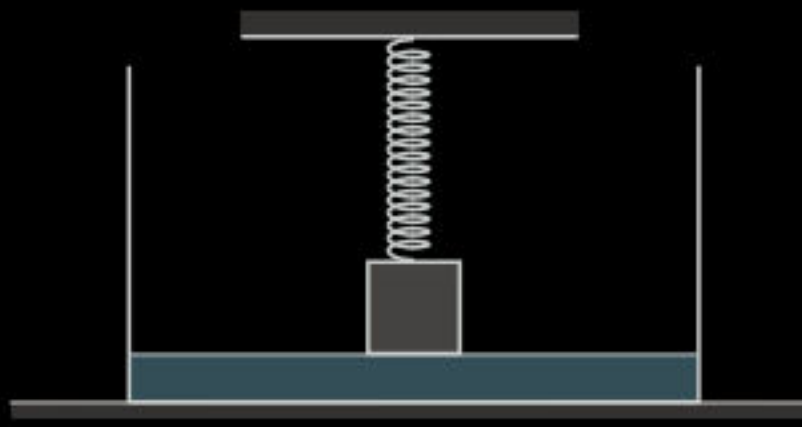
$$mg + kx_0 = \rho Vg \quad (1)$$

Later (w.r.t vessel):

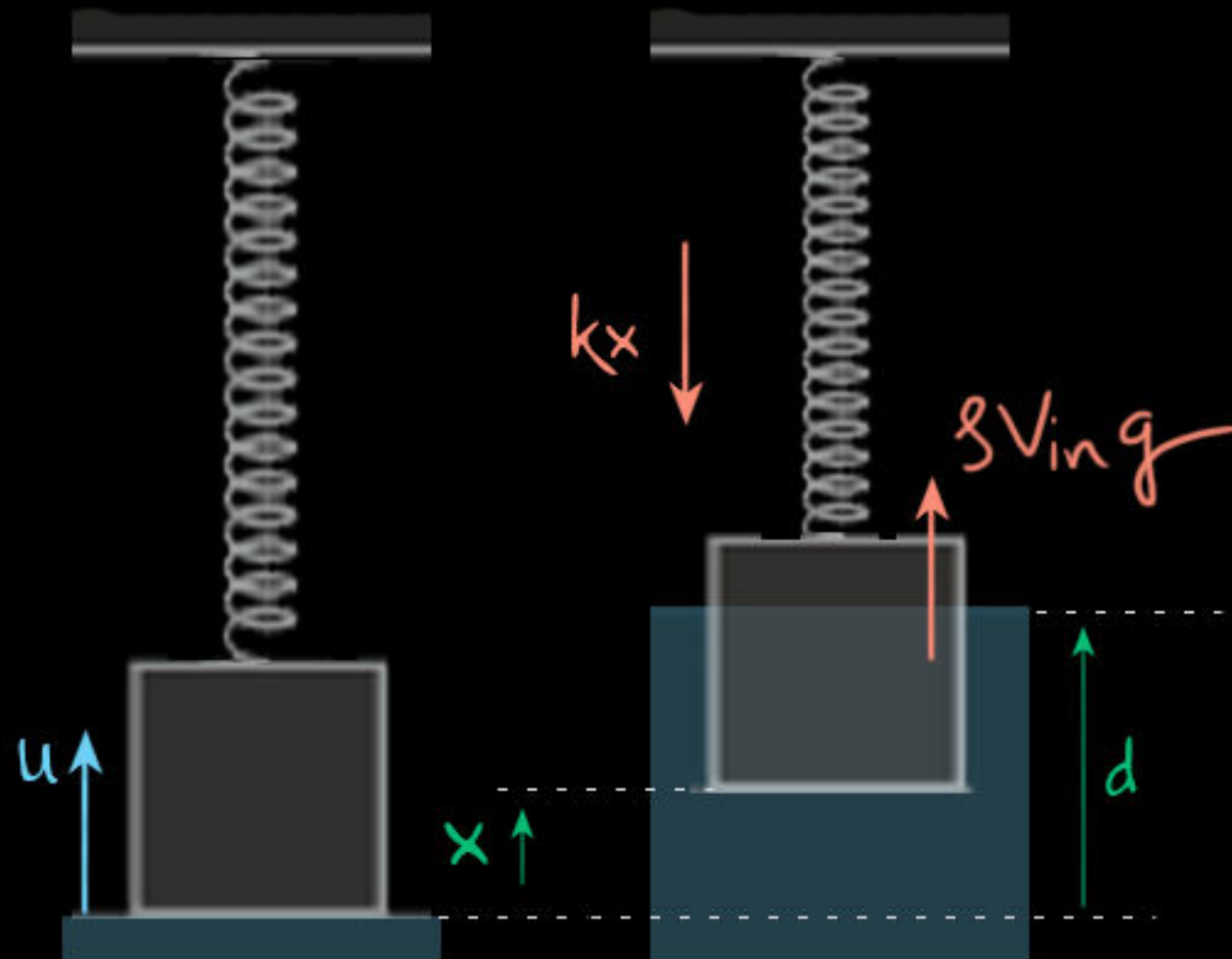
$$m(g+a) + k(x_0+x) = \rho(V-Sx)(g+a) \quad (2)$$

From 1,2:

$$x = \frac{kx_0 a}{g(k + \rho S(g+a))} \quad \text{// (+ve, so our assumption was correct, that it goes up)}$$



Being light, net force on cube will always be zero.



Cube will stop moving, once its completely submerged.

18. A light iron cube of side l suspended from a spring of stiffness k touches water level in a vessel as shown in the figure. If more water is added in the vessel at such a rate that water level in the vessel rises with a small and steady speed u , how will speed of the iron cube vary with time t ? The density of water is ρ_w and acceleration of free fall is g .

Balancing two new forces that have appeared:

$$kx = 8V_{in}g$$

$$\Rightarrow kx = 8(d-x)l^2g$$

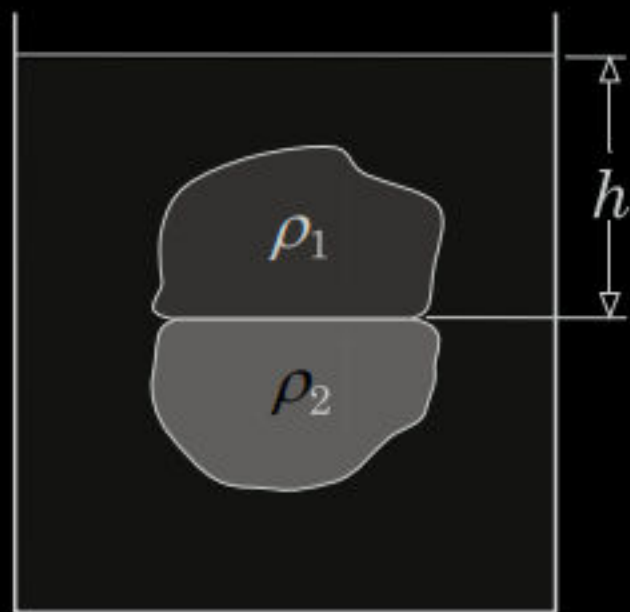
$$\Rightarrow x = \frac{8l^2g}{k+8l^2g}d$$

differentiate $v_{block} = \frac{8l^2g}{k+8l^2g}u$

Until t , when it submerges completely after t , block will be at rest.

Finding t :

At t : $d - x = l \Rightarrow t = \frac{l}{ku}(k+8l^2g)$

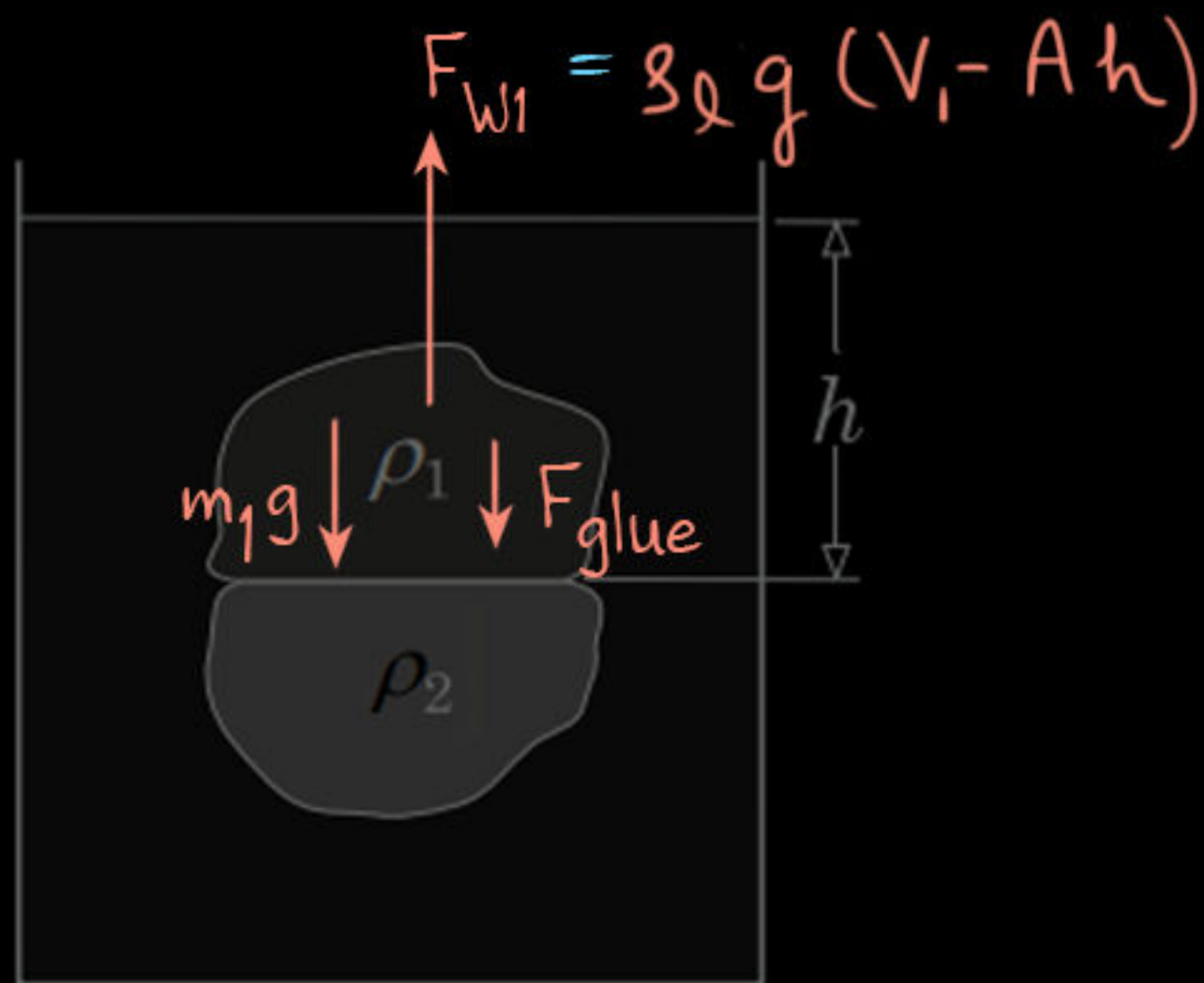


19. Two objects of equal volume $V = 1.0 \text{ m}^3$ and densities $\rho_1 = 400 \text{ kg/m}^3$ and $\rho_2 = 600 \text{ kg/m}^3$ have identical flat portions of area $S = 100 \text{ cm}^2$ on their surfaces. These flat portions are glued to each other. The composite body thus formed, floats fully submerged in a liquid with the common flat portion horizontal as shown in the figure. If the glue can withstand a maximum force $F = 500 \text{ N}$, at what minimum depth h in the liquid can the common flat portion be in equilibrium keeping the objects intact? Acceleration of free fall is g

Similar: MCQ 6

Sol: Glue remains at P_{atm}

Let force exerted by water on top part be F_w



$$(m_1 + m_2)g = \rho_l (V_1 + V_2)g$$

$$\Rightarrow (\rho_1 + \rho_2)V = \rho_l (2V)$$

$$\Rightarrow \rho_l = \frac{\rho_1 + \rho_2}{2}$$

$$F_y: F_{\text{glue}} = F_{w1} - m_1g < F$$

$$\Rightarrow \rho_l g (V - Ah) - \rho_1 V g < F$$

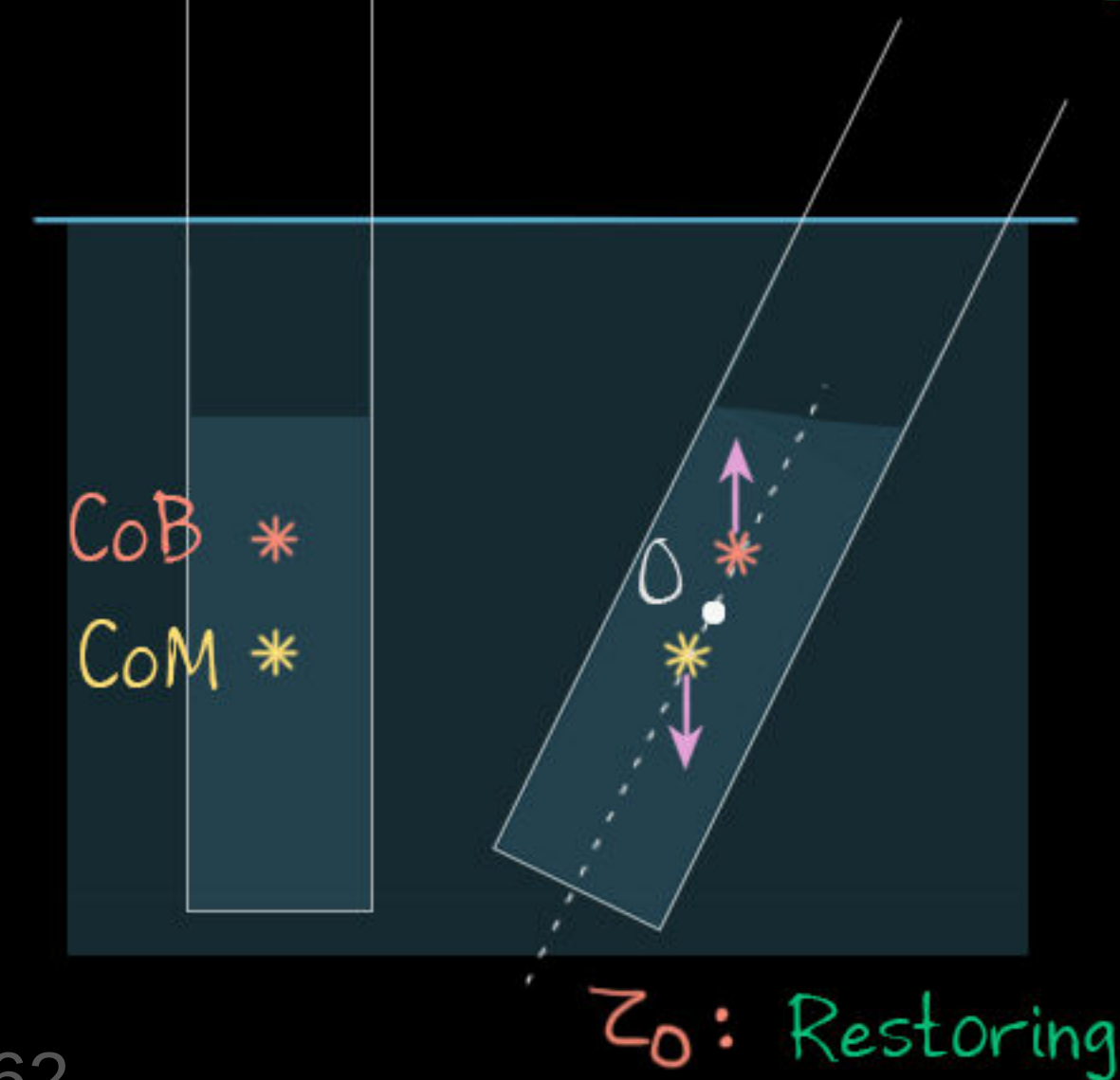
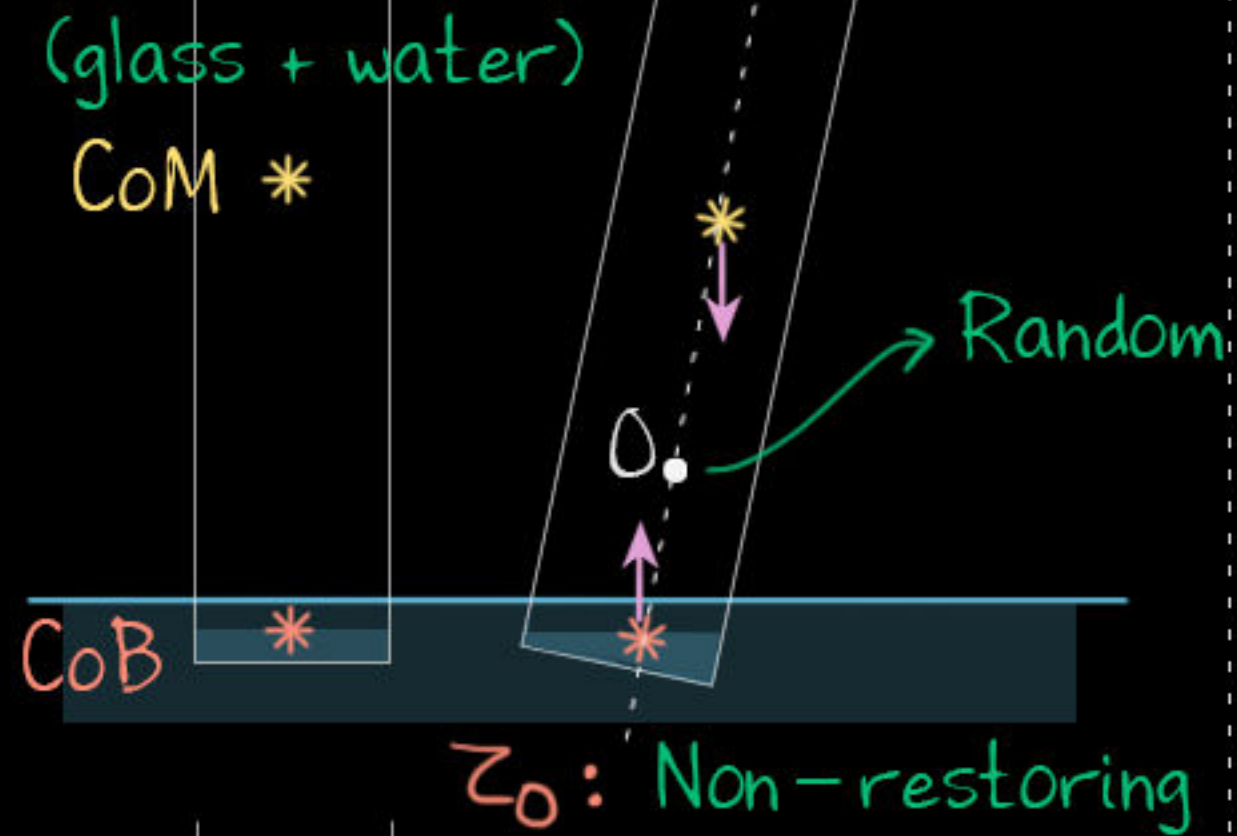
$$\frac{\rho_1 + \rho_2}{2} \Rightarrow h > \frac{(\rho_2 - \rho_1)Vg - 2F}{(\rho_1 + \rho_2)Ag}$$

by Ankit Singhvi

Youtube // Arihant Senior

20. A tube made of very thin glass sheet has mass m , length l and cross section area S . Its lower end is closed. Up to what height h should water be filled in the tube so that it can float vertically in water? Density of water is ρ_0 .

in equilibrium

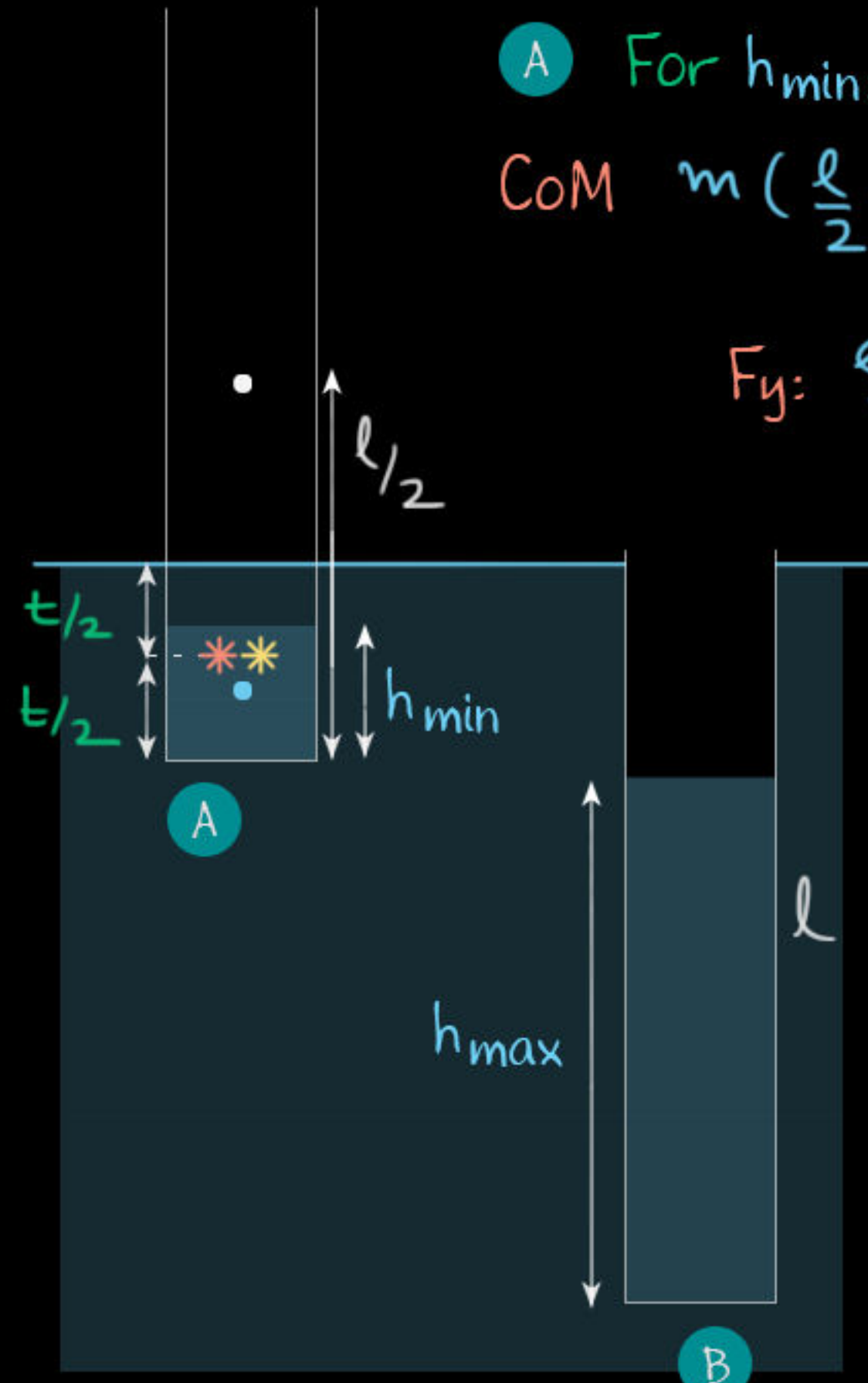


A For $h_{\min}$, CoM and CoB (at $l/2$ depth) will coincide.

$$\text{CoM } m \left(\frac{l}{2} - \frac{t}{2} \right) = \rho_0 S h \left(\frac{t}{2} - \frac{h}{2} \right) \quad (1)$$

$$F_y: \rho_0 S t = m + \rho_0 S h \quad (2)$$

$$\text{From 1,2: } h_{\min} = \frac{\rho_0 S l - m}{2 \rho_0 S} //$$



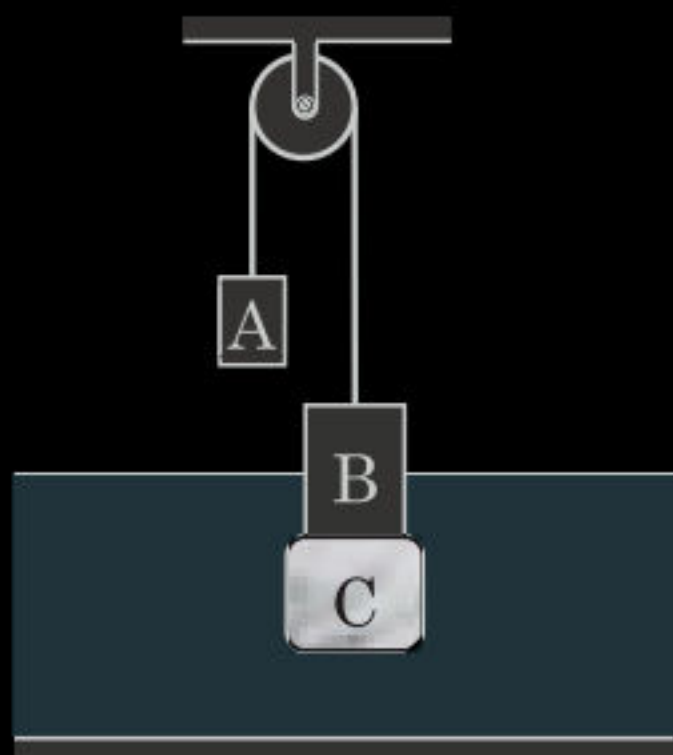
B For $h_{\max}$, it's just about to sink.

$$\therefore m + \rho_0 S h = \rho_0 S l$$

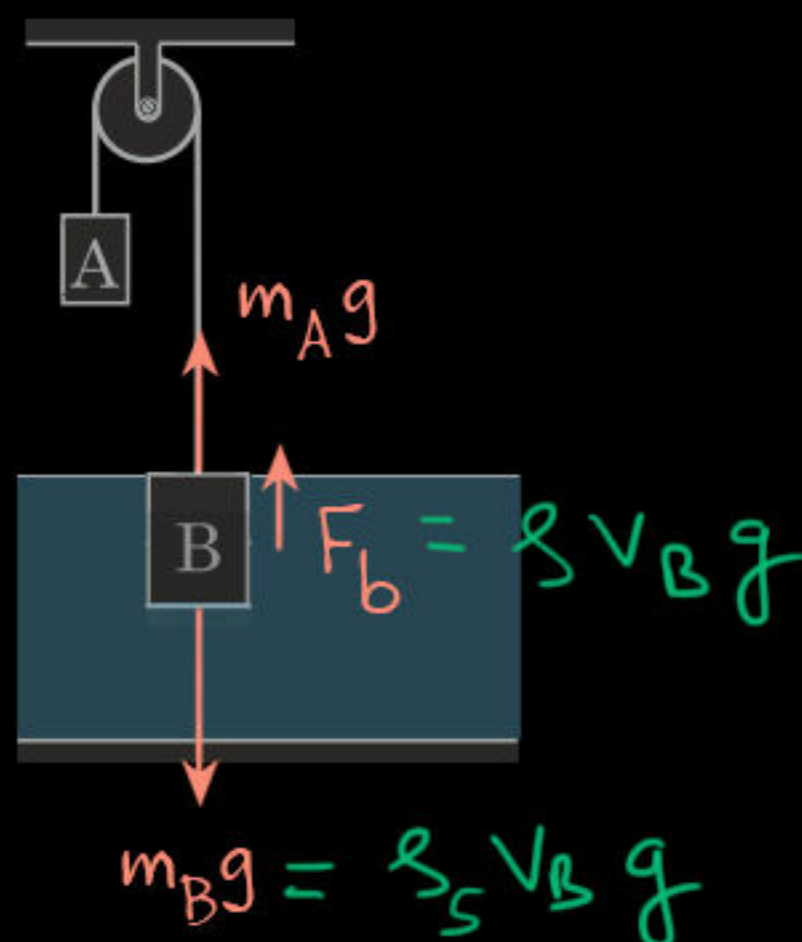
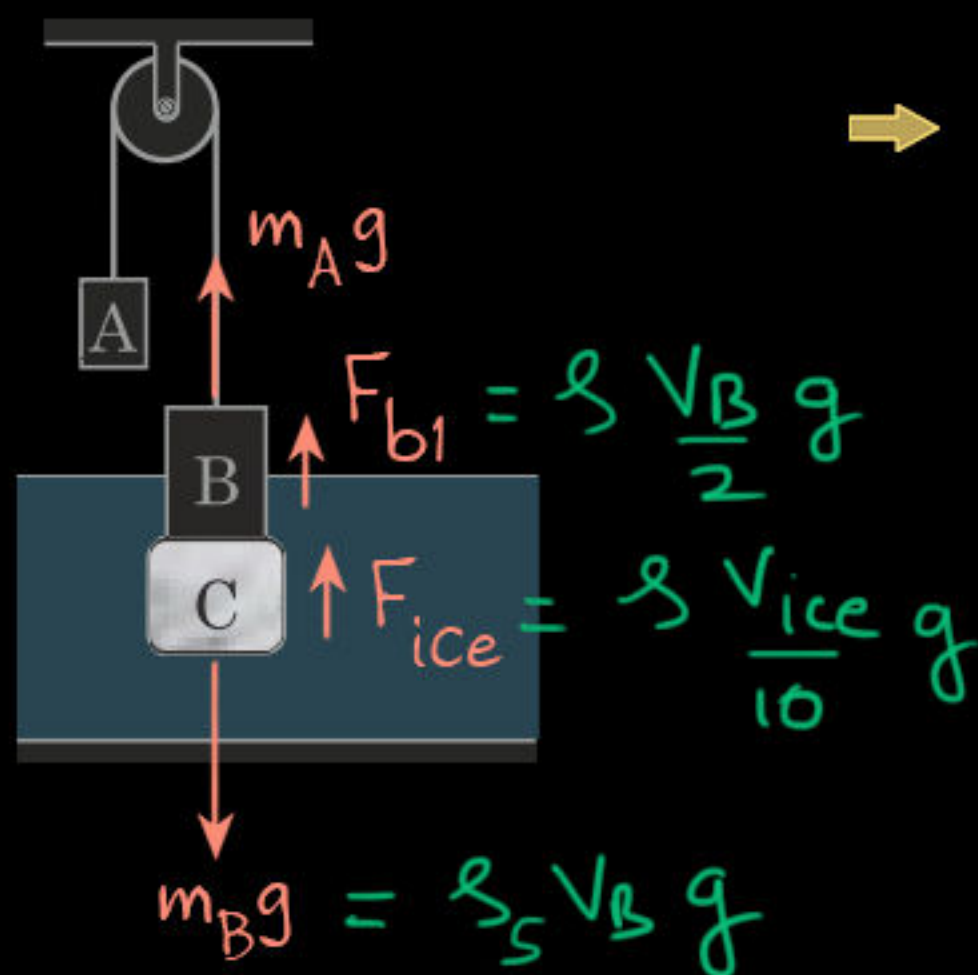
$$\Rightarrow h_{\max} = \frac{\rho_0 S l - m}{\rho_0 S} //$$

by Ankit Singhvi

Youtube / Arihant Senior



21. Two steel loads A of mass $m_A = 0.68 \text{ kg}$ and B of unknown mass are suspended from an ideal fixed pulley with the help of a light inextensible cord. An unknown amount of ice C is affixed on the bottom of the load B and the setup is in equilibrium with the load A in air and half volume of the load B submerged in water of a large tank as shown in the figure. When all the ice melts, the setup still remains in equilibrium but the load B is now completely submerged in water. Find mass of the load B as well as initial mass of the ice C. Density of water is $\rho_w = 1000 \text{ kg/m}^3$, density of ice is $\rho_i = 900 \text{ kg/m}^3$ and density of steel is $\rho_s = 7800 \text{ kg/m}^3$.

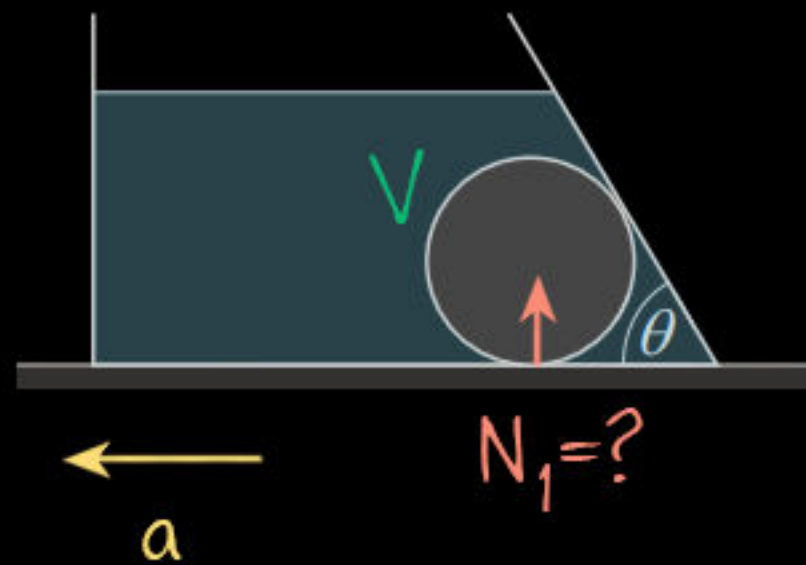


$$\text{Initially: } m_A + \frac{\rho V_B}{2} + \frac{\rho V_{ice}}{10} = \rho_s V_B \quad (1)$$

$$\text{Finally: } m_A + \rho V_B = \rho_s V_B \quad (2)$$

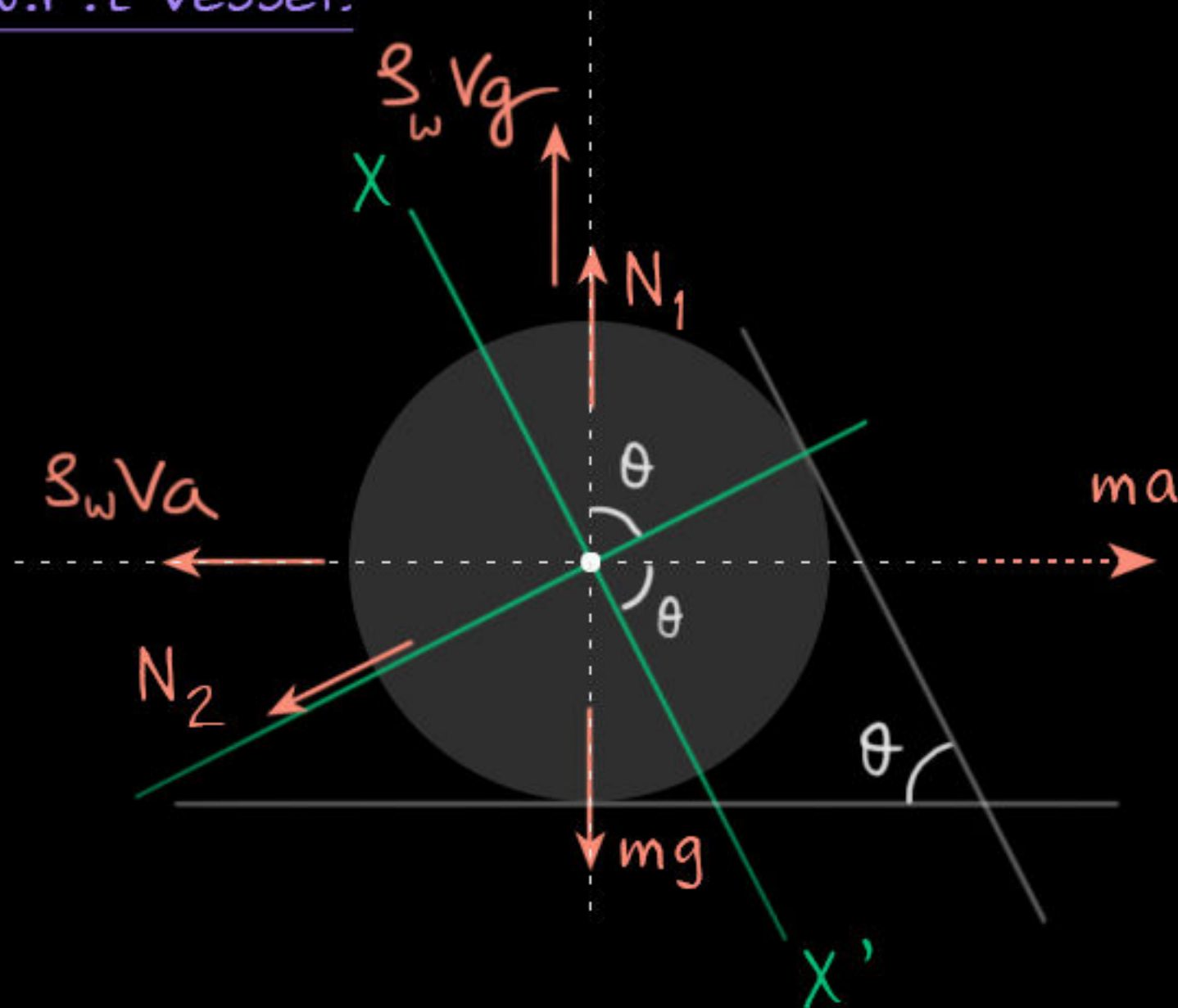
$$\text{From 2: } m_B = \rho_s V_B = \frac{\rho_s m_A}{\rho_s - \rho} //$$

$$\text{From 1,2: } m_{ice} = \rho_i V_i = 4.5 \text{ kg} //$$



22. A glass sphere of volume V is placed in a vessel filled with water. A side wall of the vessel is inclined at an angle θ with horizontal bottom of the vessel. The container is moving with uniform leftward acceleration a . Find force of normal reaction between the bottom of the vessel and the glass sphere. Density of water is ρ_w , density of the glass is ρ and acceleration of free fall is g .

w.r.t vessel:

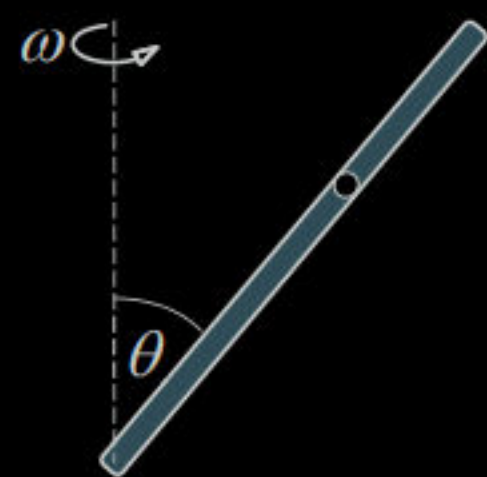


$$F_{XX'}: (\rho_w V g + N_1 - mg) \sin \theta = (ma - \rho_w V a) \cos \theta$$

$$\Rightarrow N_1 = a \cot \theta (m - \rho_w V) + g(m - \rho_w V)$$

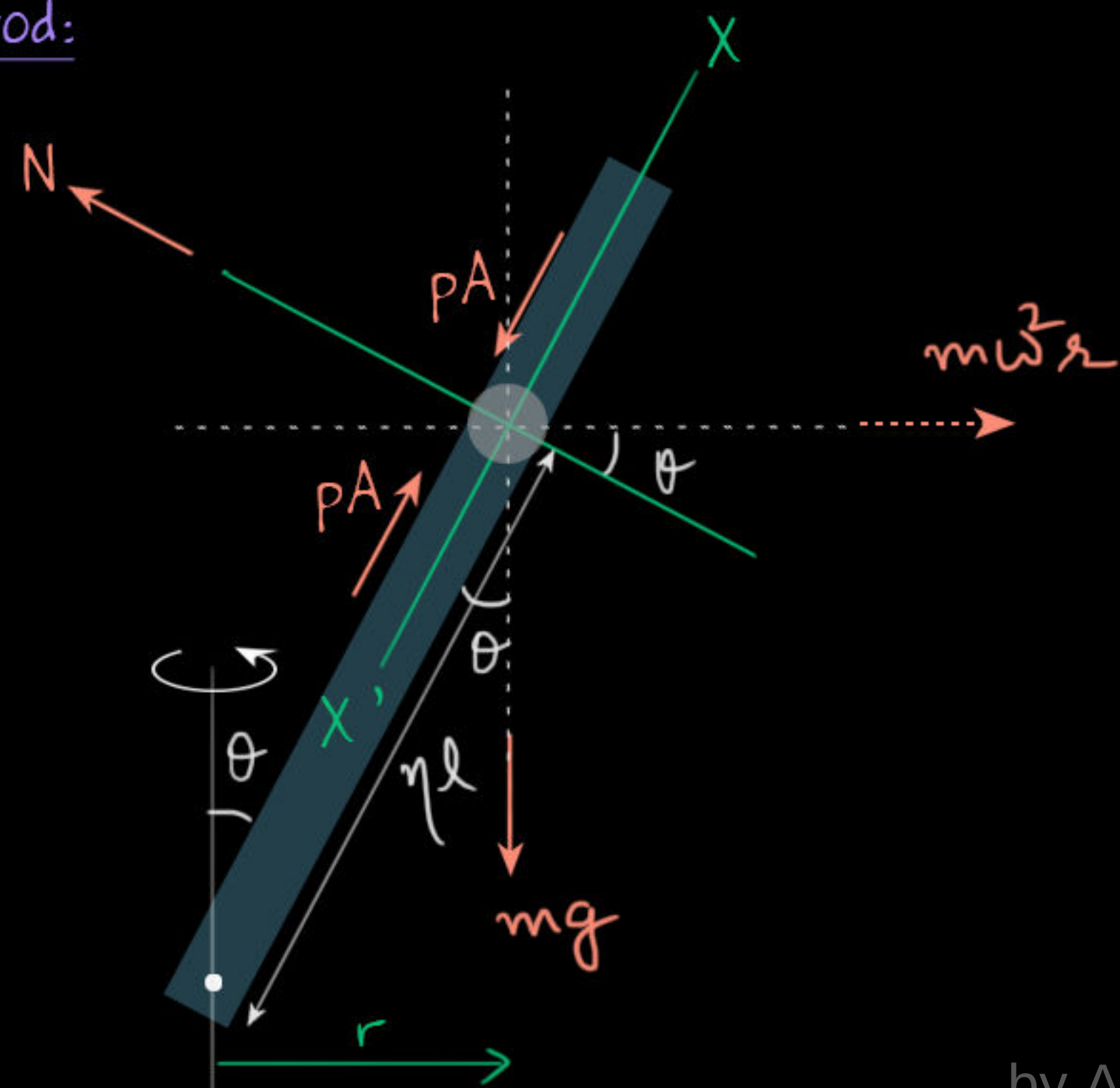
$$= (a \cot \theta + g) (\underbrace{m - \rho_w V}_{\rho V})$$

$$= V (a \cot \theta + g) (\rho - \rho_w) //$$



23. A closed tube of length l completely filled with water has a small air bubble trapped in it. When the tube is held at an angle θ with the vertical and rotated at a constant angular velocity ω about the vertical axis through its lower end, the bubble settles at some intermediate position in the tube. What fraction η of length of the tube is the distance of the bubble from the lower end? Acceleration due to gravity is g .

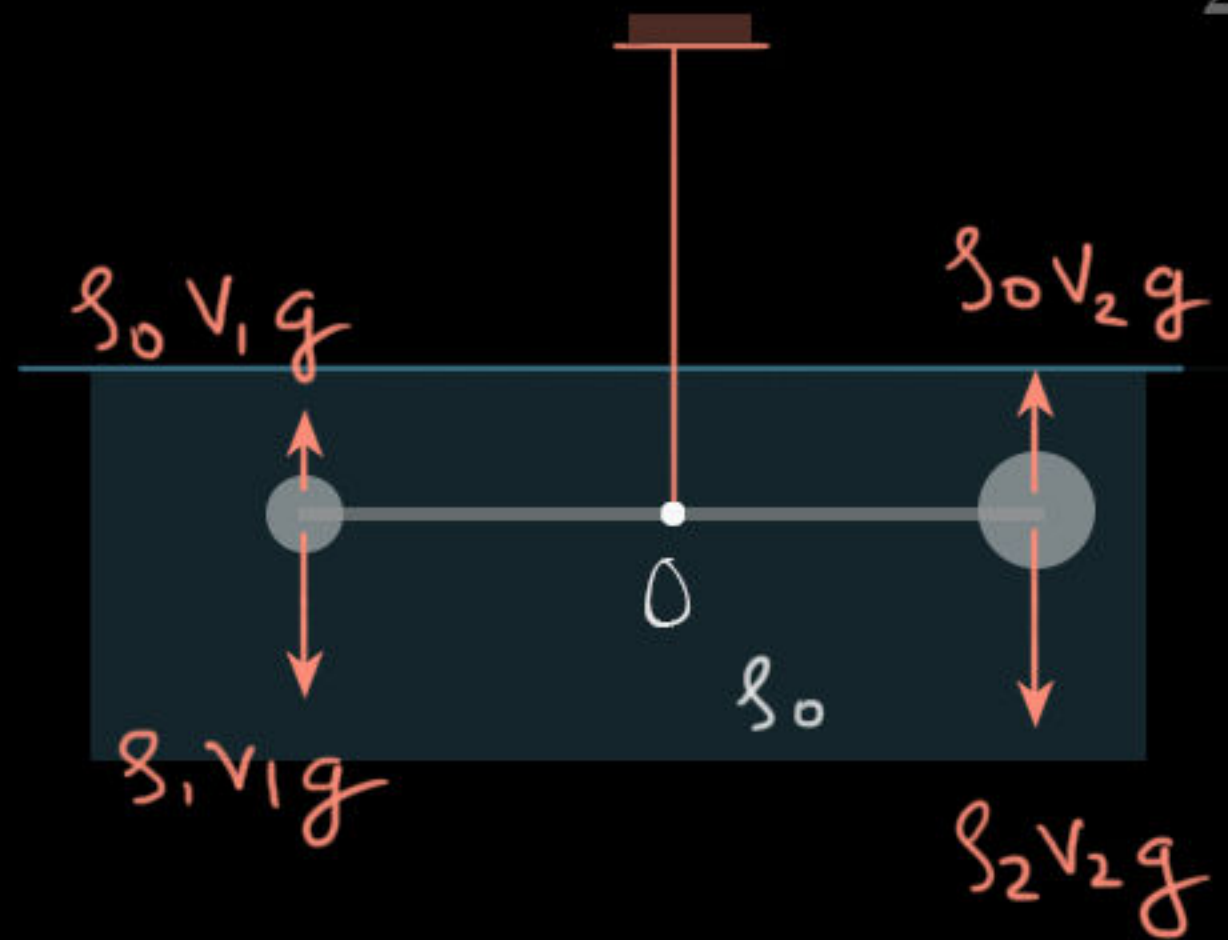
w.r.t rod:



$$F_{XX'}: \cancel{m}g \cos\theta = \cancel{m}\omega^2 r \sin\theta \quad \rightarrow \eta l \sin\theta$$

$$\Rightarrow g \cos\theta = \omega^2 \eta l \sin^2\theta$$

$$\Rightarrow \eta = \frac{g \cos\theta}{\omega^2 l \sin^2\theta} //$$



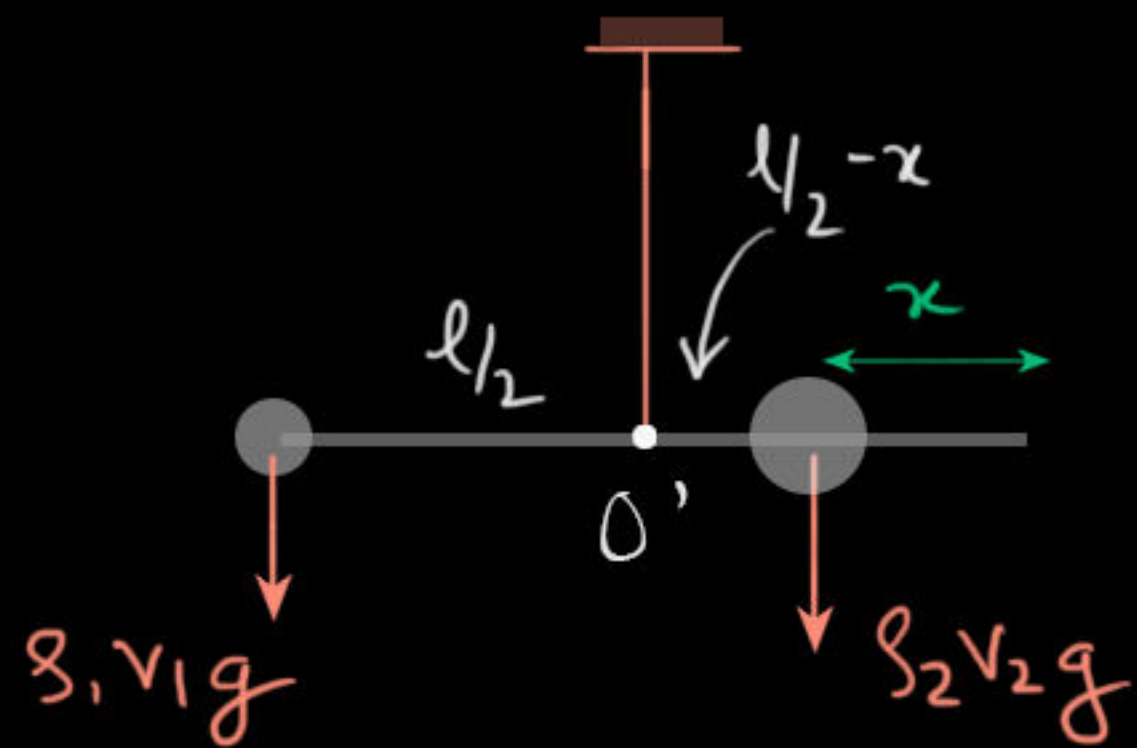
24. Two balls A and B made of materials of densities $\rho_A = 7140 \text{ kg/m}^3$, and $\rho_B = 1740 \text{ kg/m}^3$ are affixed at the ends of a rod of length $l = 40 \text{ cm}$. A thread is tied at the middle of the rod. When holding the thread the balls are lowered in water, both the balls get fully submerged and the rod remains horizontal. When the thread is pulled upwards and both the balls come out of water, the rod does not stay horizontal. By what distance should the ball B be shifted on the rod to make the rod horizontal when both the balls are in air? Density of water is ρ_0 .

In water, $\Sigma \tau_O$: $(\rho_1 - \rho_0) V_1 g = (\rho_2 - \rho_0) V_2 g$

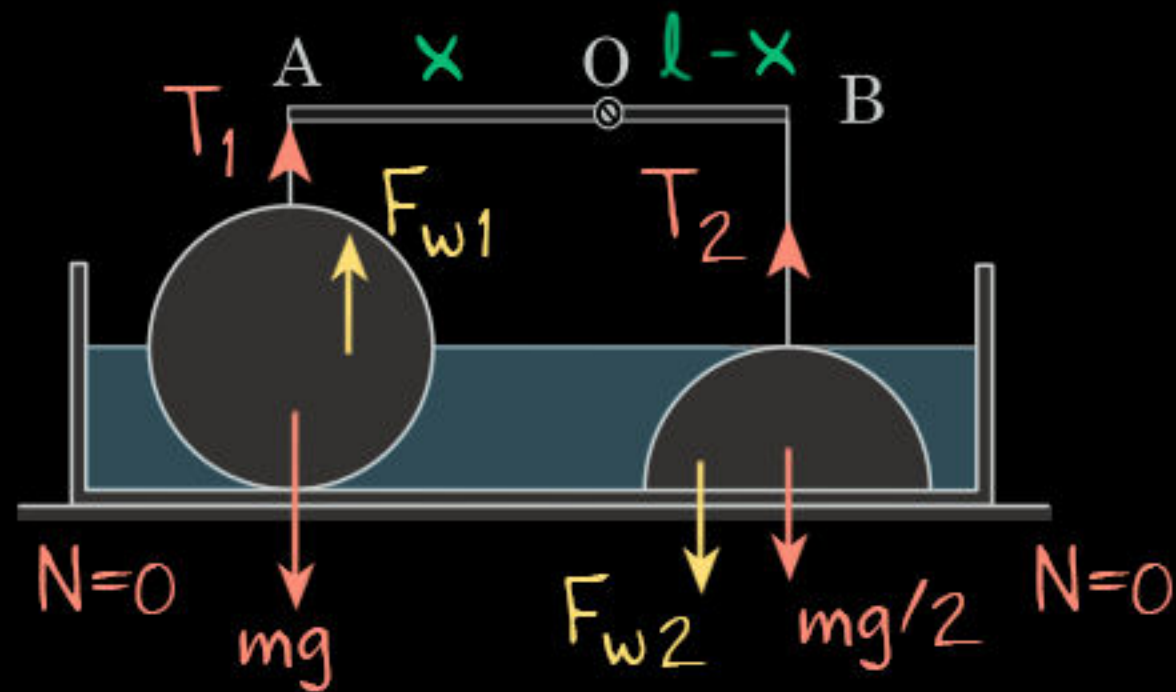
In air, $\Sigma \tau_{O'}$: $\rho_1 V_1 g \frac{l}{2} = \rho_2 V_2 g \left(\frac{l}{2} - x \right)$

Dividing: $\frac{\rho_1 - \rho_0}{\rho_1 l} = \frac{\rho_2 - \rho_0}{\rho_2 (l - 2x)}$

$$\Rightarrow x = \frac{l}{2} \frac{\rho_0 (\rho_1 - \rho_2)}{\rho_2 (\rho_1 - \rho_0)}$$



Let F_w be force exerted by water on curved surface.
About to lift:



25. A uniform sphere of mass m and another uniform hemisphere of the same material and radius r are placed in a vessel. In the vessel water is filled up to height $r = 30.0$ cm as shown. There is no water or air under the hemisphere. The sphere and the hemisphere both are connected with vertical threads to the ends of a light rod AB of length $l = 116$ cm having a hinge O. Find the distance of the hinge from the end A so that when an upward minimum pull is applied at the hinge, the sphere and the hemisphere both will simultaneously leave the bottom of the vessel. Density of the material of the sphere is $\rho = 5.0$ g/cm³ and that of water is $\rho_0 = 1.0$ g/cm³.

When about to lift both, rod will be at translational as well as rotational equilibrium.

Calculating F_{w1}

$$F_{w1} = \rho \frac{V}{2} g = \frac{2}{3} \rho \pi r^3 g$$

Calculating F_{w2} : Let's say there is water below

$$F_b = \rho \left(\frac{1}{2} \frac{4}{3} \pi r^3 \right) g = \rho g r (\pi r^2) - F_{w2}$$

$$\Rightarrow F_{w2} = \left(1 - \frac{2}{3} \right) \rho \pi r^3 g$$

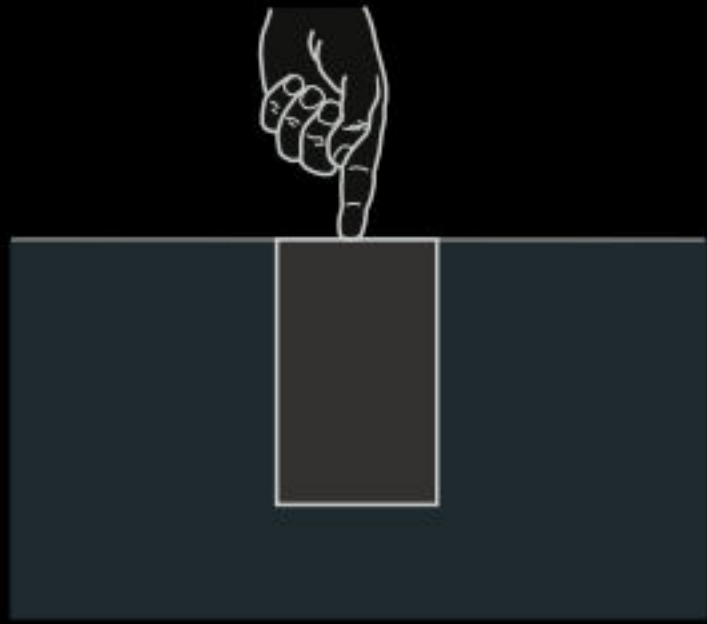
$$= \frac{1}{3} \rho \pi r^3 g$$

$$\tau_O: T_1 x = T_2 (l - x) \quad \frac{4}{3} \rho_s \pi r^3$$

$$\Rightarrow (mg - F_{w1}) x = \left(\frac{mg}{2} + F_{w2} \right) (l - x)$$

$$\Rightarrow \left(\frac{4}{3} \rho_s - \frac{2}{3} \rho \right) x = \left(\frac{2}{3} \rho_s + \frac{1}{3} \rho \right) (l - x)$$

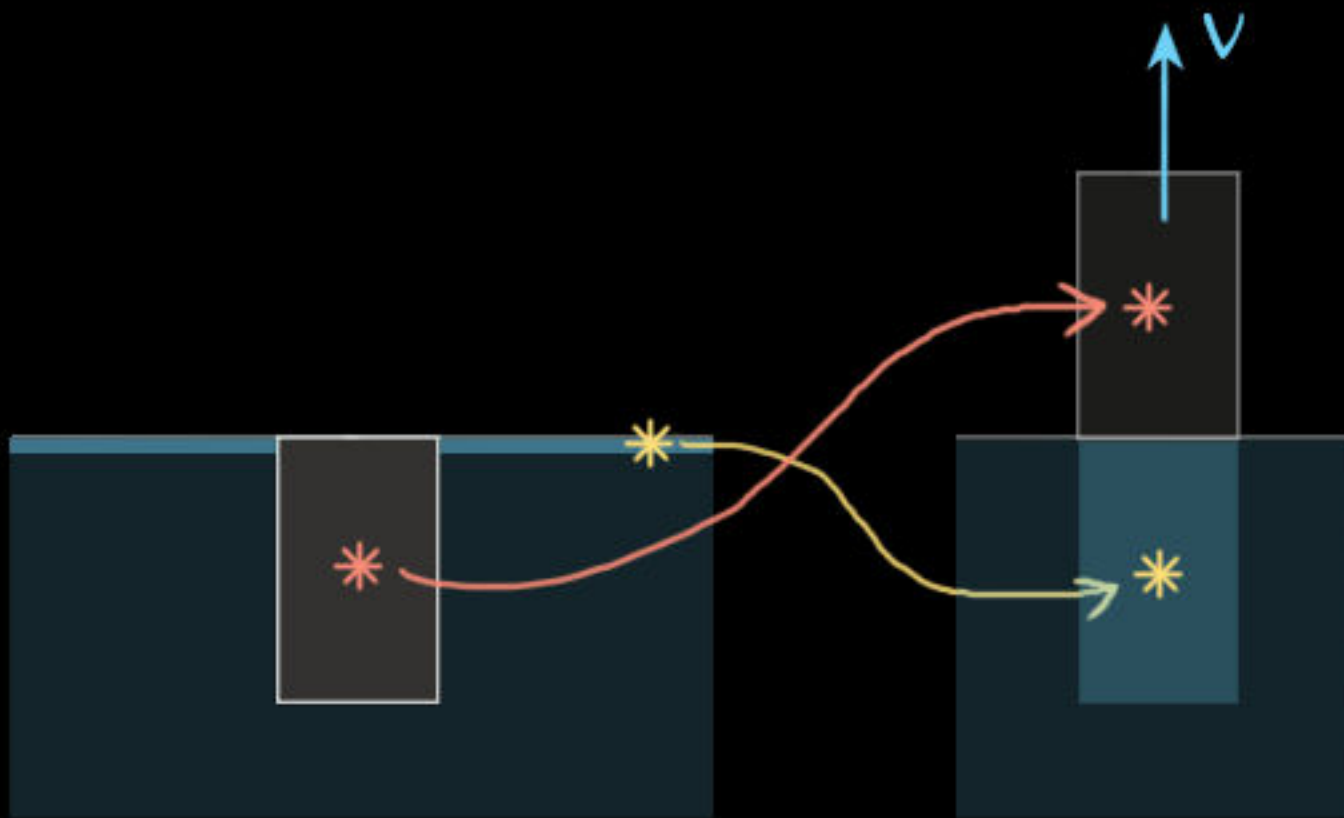
$$\Rightarrow x = \left(\frac{2\rho_s + \rho}{6\rho_s - \rho} \right) l //$$



26. A uniform cylinder made of a material of density $\rho = 250 \text{ kg/m}^3$ is held at rest in a pool so that its upper circular surface is in level with the water surface. Length of the cylinder is $l = 20 \text{ cm}$ and radius is $r = 3.0 \text{ cm}$. When released, the cylinder jumps out of water moving vertically. Find velocity with which the cylinder leaves the water surface. Density of water is $\rho_0 = 1000 \text{ kg/m}^3$ and acceleration due to gravity is $g = 10 \text{ m/s}^2$. Neglect all dissipative effects.

Method 1 Energy

Concept: Water on the surface will fill up the space left empty by the cylinder.

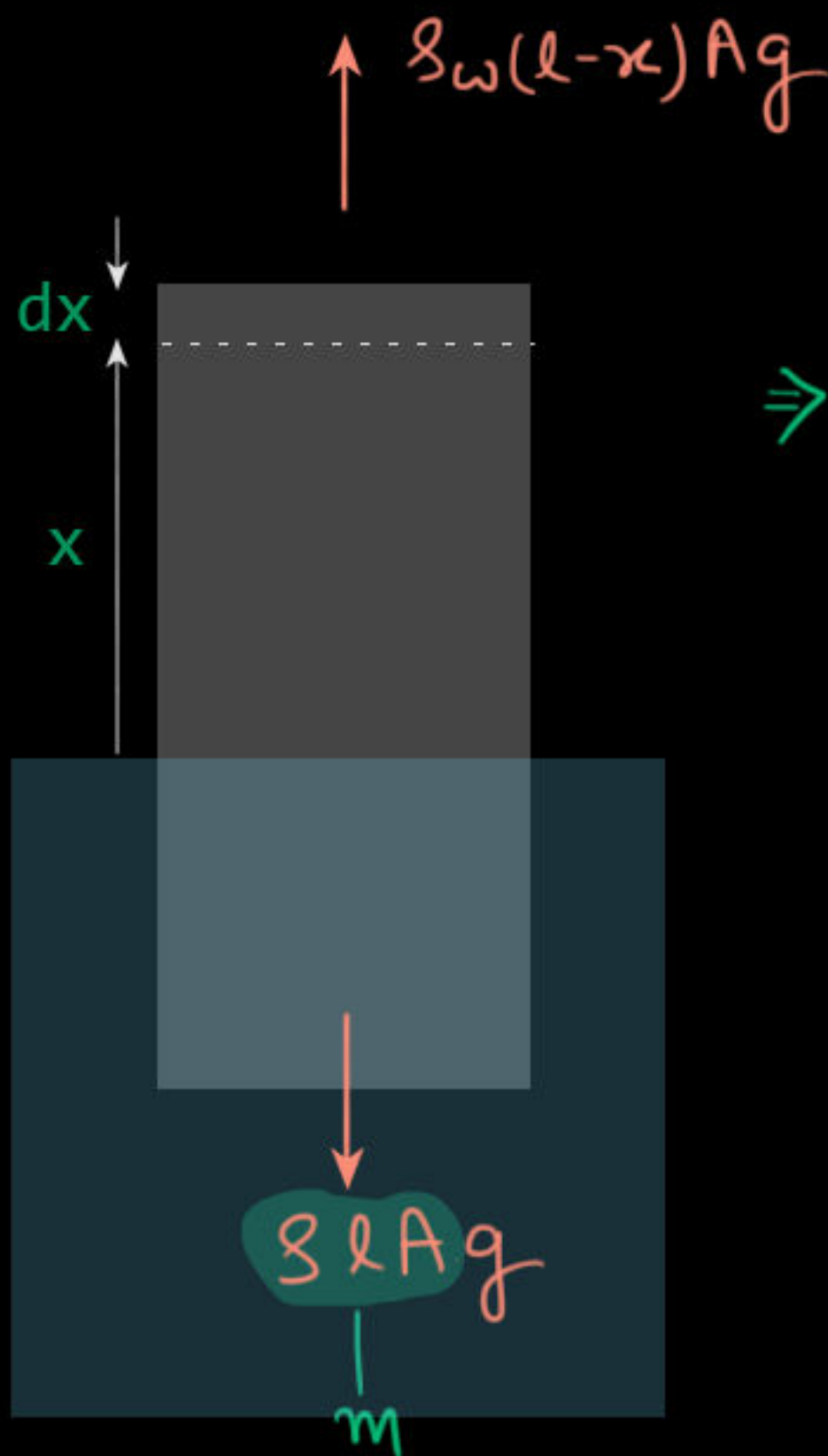


$$W_g = \Delta KE + \text{Heat}$$

$$\Rightarrow \cancel{8000} g \frac{l}{2} - \cancel{8000} g l = \frac{1}{2} \cancel{8000} v^2$$

$$\Rightarrow v = \sqrt{\frac{(8000 - 2500) g l}{8000}}$$

Method 2 Force



$$W_{ext} = \Delta KE$$

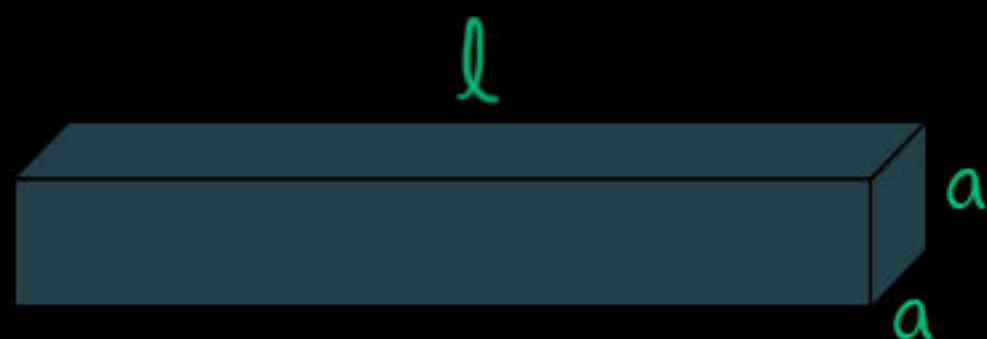
$$\Rightarrow W = \int F dx = \int_0^l (\rho_w(l-x)Ag - \rho l Ag) dx = \frac{1}{2}mv^2$$

$$\Rightarrow (\rho_w - \rho)lAg \int_0^l dx - \rho_w Ag \int_0^l x dx = \frac{1}{2}mv^2$$

$$\Rightarrow (\rho_w - \rho)l^2Ag - \frac{1}{2}\rho_w l^2Ag = \frac{1}{2}\rho l A v^2$$

$$\Rightarrow (2\rho_w - 2\rho - \rho_w)l^2Ag \cdot \frac{1}{\cancel{\rho l A}} = v^2$$

$$\Rightarrow v = \sqrt{\frac{(\rho_w - 2\rho)gl}{\rho}}$$

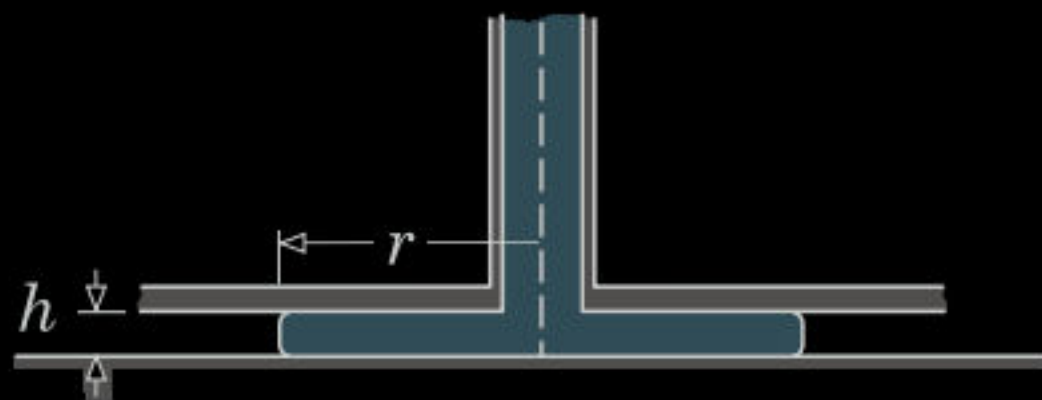


27. A homogeneous beam of length l and square section of side length a is held horizontally so that one of its long faces touches water in a large reservoir. Density of material of the beam ρ is equal to density of water. The beam is released. Find the amount of heat developed until all the perturbations cease. Acceleration of free fall is g .

Since density of beam is same as water, let's assume a slab of water is dropped instead.
Situation is similar. As water level rise at top is same.



$$\begin{aligned}
 Wg &= \cancel{\Delta KE} + \text{Heat} \\
 \Rightarrow \text{Heat} &= mg \frac{a}{2} \\
 &= (\rho a^2 l) g \frac{a}{2} \\
 &= \frac{\rho a^3 l g}{2} //
 \end{aligned}$$

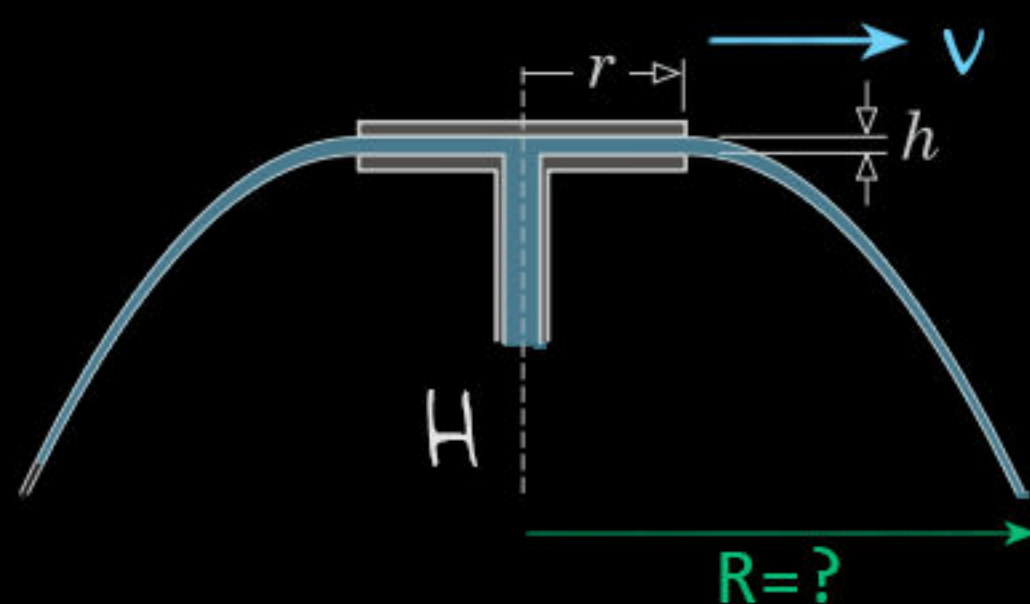


28. A cylindrical tube is fitted coaxially at one of its ends into a hole made at the centre of a large disc. The structure is held firmly at a height h above a horizontal floor, keeping plane of the disc horizontal. When an incompressible non-viscous fluid of density ρ is fed at a constant rate μ (kg/s) into the tube completely filling the tube, the fluid spreads evenly in all directions in the gap between the disc and the floor. Find radius r of the fluid spreading in the gap as a function of time t .

Mass passed through pipe in time t = Mass collected in disc in time t

$$\Rightarrow \mu t = \rho \pi r^2 h$$

$$\Rightarrow r = \sqrt{\frac{\mu t}{\pi \rho h}}$$



29. One end of a long cylindrical tube is fitted coaxially into a hole made at the centre of a thin disc of radius r . The structure is held firmly under another horizontally fixed disc of the same radius. Gap between the discs is h . Water fed into the tube comes out evenly everywhere from the opening between the discs and spreads in a dome-like shape. If water is fed into the tube at a rate μ (kg/s), find the radius of the dome at a depth H below the opening between the discs. Neglect capillary effects and the distance h between the discs as compared to the depth H . Acceleration of free fall is g .

Mass flow rate through pipe = Mass flow rate through disc

$$\begin{aligned} \Rightarrow \mu &= \rho A v \\ &= \rho (2\pi r h) v \end{aligned}$$

Cross sectional area

$$\Rightarrow v = \frac{\mu}{2\pi r \rho h}$$

$$\begin{aligned} \therefore R &= r + v T \\ &= r + \frac{\mu}{2\pi r \rho h} \sqrt{\frac{2H}{g}} \end{aligned}$$



At constant speed, net force on a piston is zero.

So pressure on either side of it will be equal.

30. A horizontal cylindrical pipe consists of two coaxial sections, the section to the left has radius r_1 and that to the right has radius r_2 . Inside the pipe, there is a piston in each section trapping an incompressible fluid of density ρ filled in the complete space between them. Constant pressures p_1 and p_2 ($p_1 > p_2$) are maintained outside the pistons in the sections to the left and to the right respectively. When both the pistons acquire constant speeds towards the right, find their speeds.

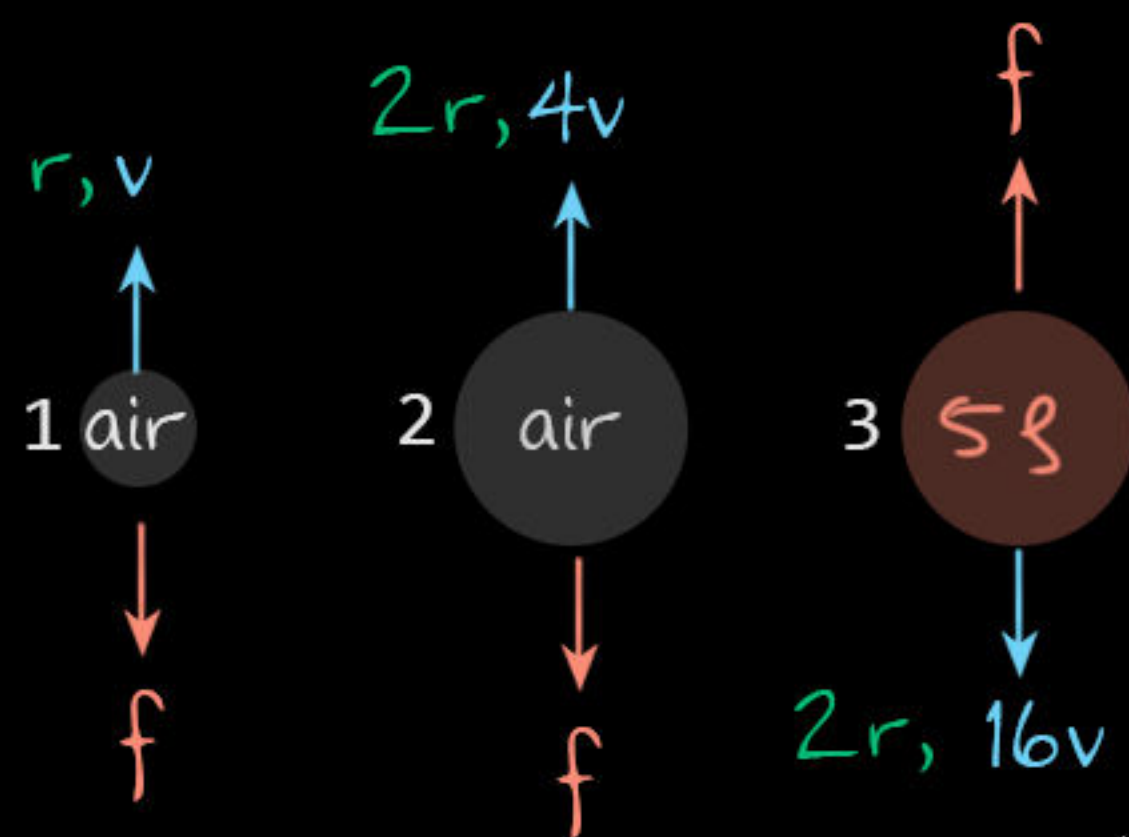
$$\text{Bernoulli's: } p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2 \quad (1)$$

$$\text{Continuity: } v_1 (\pi r_1^2) = v_2 (\pi r_2^2) \quad (2)$$

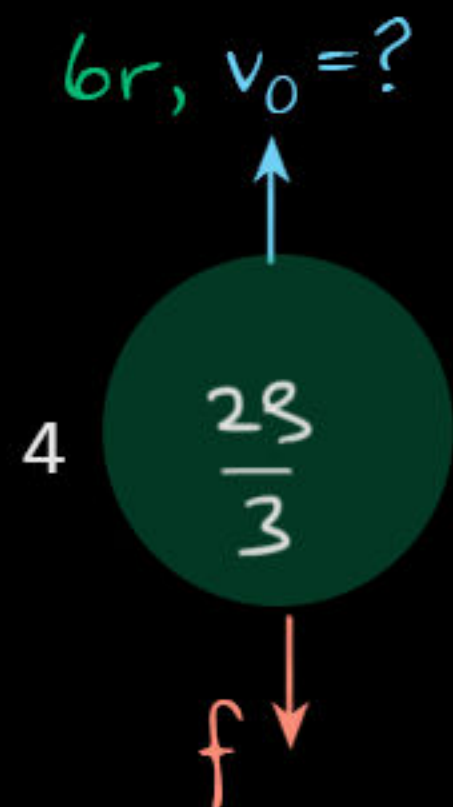
$$\text{From 1,2: } v_1 = \sqrt{\frac{2(p_1 - p_2) r_2^4}{\rho (r_1^4 - r_2^4)}} //$$

Given: $f = k r^\alpha v^\beta$

Let water density be ρ



Find: $6r, v_0 = ?$



31. A spherical air bubble of radius 0.5 mm made under water moves up with a velocity 0.5 cm/s. Another spherical air bubble of double radius moves up in water four times faster than the smaller bubble. A metal ball of radius 1.0 mm and density 5.0 g/cm^3 when released in water sinks down with a velocity 8.0 cm/s. Knowing that the force of water resistance on a spherical body is proportional to the product $r^\alpha v^\beta$, where α and β are real numbers, r is radius of the body and v is its speed, find the constant velocity with which another plastic ball of density $(2/3) \text{ g/cm}^3$ and radius 3.0 mm will move in water. Density of water is 1.0 g/cm^3 and density of the air in the bubbles is negligible as compared to the density of water. Acceleration of free fall is $g = 10 \text{ m/s}^2$.

Sol:

At terminal velocity, forces are balanced.

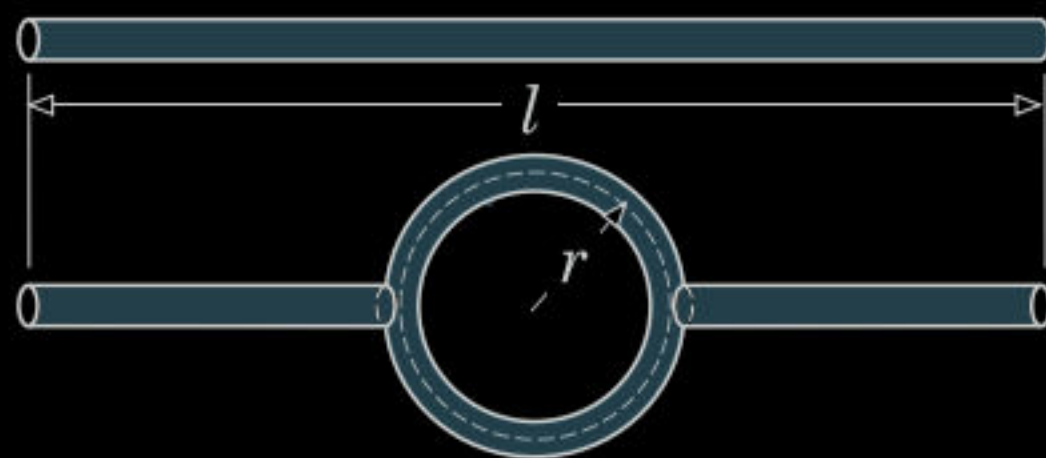
$$\therefore 1: \rho k_1 r^3 g = k r^\alpha v^\beta \quad \begin{matrix} \xrightarrow{4/3\pi} \\ 8 = 2^\alpha 4^\beta \end{matrix} \quad \begin{matrix} \alpha = 1 \\ \beta = 1 \end{matrix}$$

$$2: \rho k_1 (2r)^3 g = k (2r)^\alpha (4v)^\beta \quad \begin{matrix} \xrightarrow{4/3\pi} \\ 4 = 4^\beta \end{matrix}$$

$$3: (5\rho - \rho) k_1 (2r)^3 g = k (2r)^\alpha (16v)^\beta$$

$$4: \left(\rho - \frac{2\rho}{3}\right) k_1 (6r)^3 g = k (6r)^\alpha (v_0)^\beta$$

$$\therefore \text{From 4/1: } \frac{1}{3} 6^3 = 6^\alpha \left(\frac{v_0}{v}\right)^\beta \xrightarrow[\beta=1]{\alpha=1} v_0 = 12v = 6 \text{ cm/s} \quad \nearrow 0.5 \text{ cm/s}$$



Concept: Flow of water through pipes is similar to flow of current through wires.

$$V = IR \quad \rightarrow \frac{8\ell}{A}$$

||| Volume flow rate

$$\Delta p = qR \quad \rightarrow \frac{8\ell}{A}$$

Resistances get added in series and parallel similar to circuits

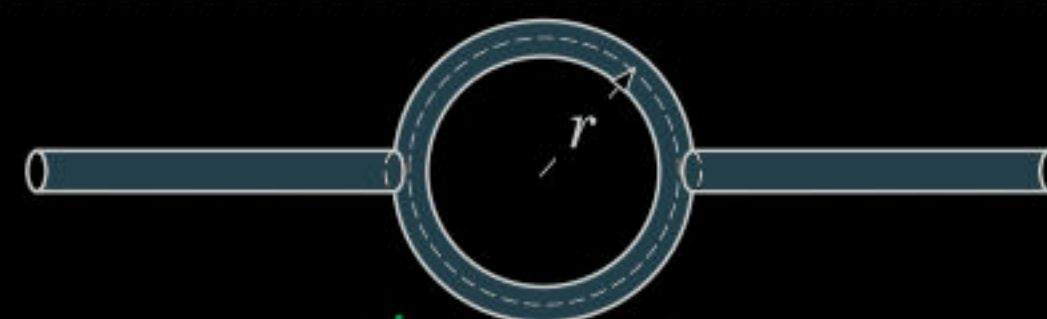
32. A pump circulates water at a time rate q_0 through a straight pipe of length l . Now a ring-shape section of mean radius r is connected in the middle of the pipe. Inner section of the ring is equal to that of the pipe as shown in the figure. If the same pump is connected to this new pipe, how much time rate q of water flow will the pump maintain? Assume that the pressure difference at the ends of the tube remains unchanged.



$$\Delta p = q_0 R = q_0 \frac{8\ell}{A} \quad (1)$$

Dividing 1,2:

$$q = \frac{2\ell q_0}{2(\ell - 2r) + \pi r}$$



combined: $\ell - 2r$



$$\Delta p = q(R_1 + R_2) = q \left[\frac{8(\ell - 2r)}{A} + \frac{8(\pi r)}{2A} \right] \quad (2)$$

1. In colder regions like the poles, two identical open containers one filled with hot water and the other filled with cold water, if left exposed to atmosphere, it is observed that **usually the hot water freezes earlier** than the cold water.

More radiation in hot water → ✓ I. The hot water has more heat to lose.
More evaporation in hot water → ✓ II. The hot water initially loses heat at a faster rate.
III. More evaporation causes more heat loss in hot water.

Which of **the above reasons can be** responsible for this effect?

(a) Reason *II*

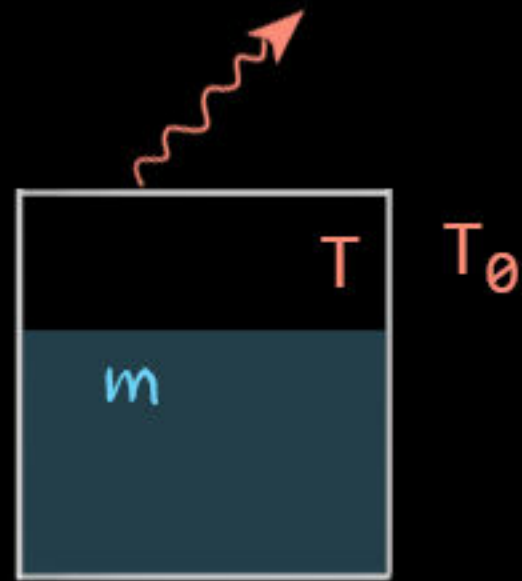
(b) Reason *III*

(c) Reasons *II* and *III*

(d) All the above reasons

Statements II and III are correct in themselves, but the premise (mpemba effect) itself is controversial.

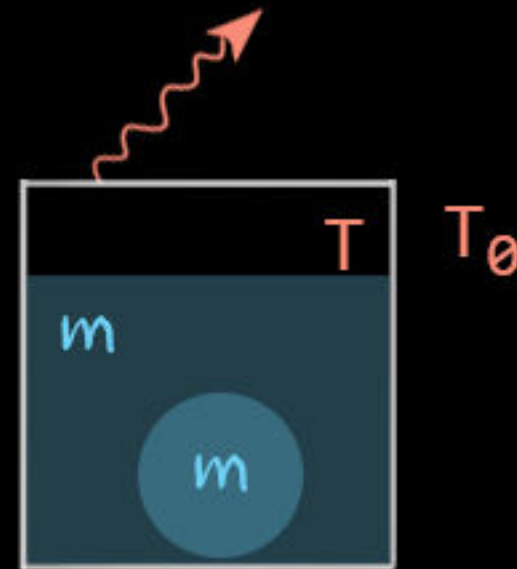
2. Two identical light metal containers filled with equal amounts of water are placed in a room maintained at a constant temperature 25°C . A small metal ball suspended by a thin nonconducting string is submerged into one of the containers. Mass of the ball is equal to that of water. Both the containers are heated to temperature 40°C and then allowed to cool. The container with the ball takes k times longer to cool down to the room temperature than the container without ball. Specific heat of water is s_w . Specific heat of the material of the ball s_b is closest to



Internal energy reduction rate = Radiation loss rate.

$$\Rightarrow -m s_w \frac{dT}{dt} = C(T - T_0)$$

$$\Rightarrow \int_{40}^{25} \frac{dT}{(T - T_0)} = -\frac{C}{m s_w} \int_0^t dt \quad (1)$$



$$(-m s_w - m s_b) \frac{dT}{dt} = C(T - T_0)$$

$$\Rightarrow \int_{40}^{25} \frac{dT}{(T - T_0)} = -\frac{C}{m(s_w + s_b)} \int_0^{kt} dt \quad (2)$$

✓(c) $s_b = (k - 1)s_w$

Dividing 1,2:

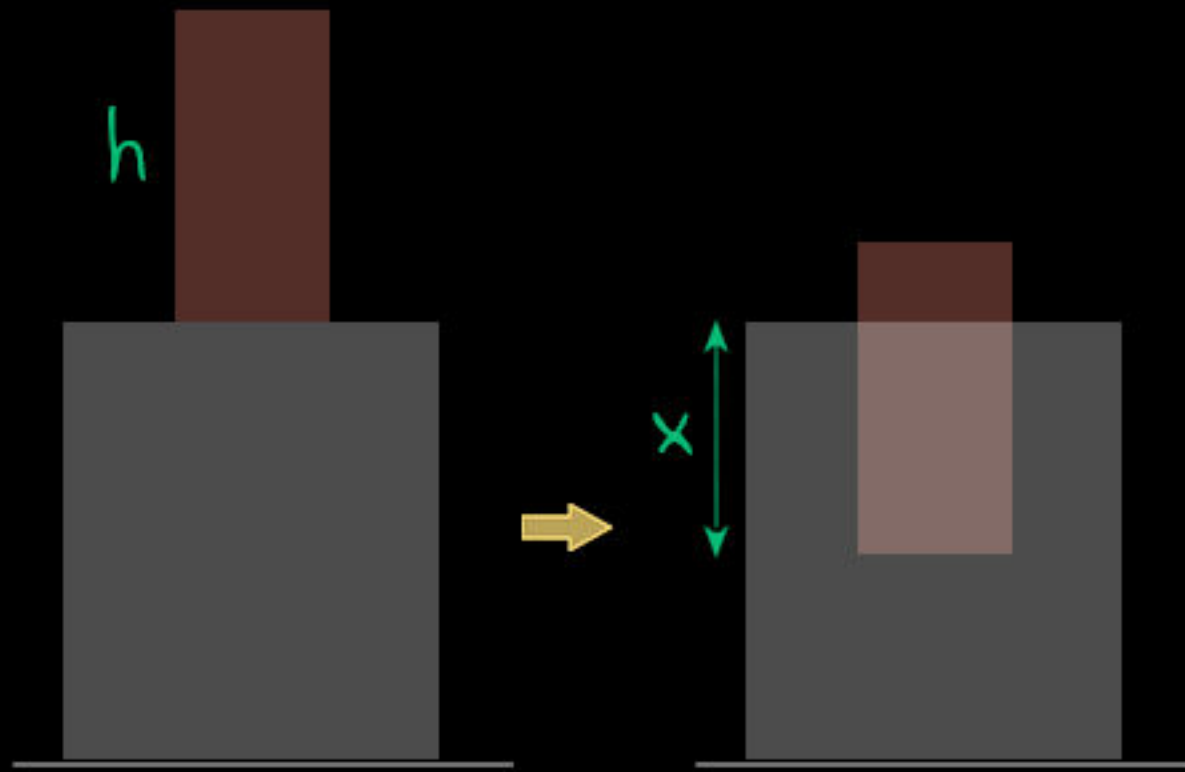
$$\frac{1}{s_w} = \frac{k}{s_w + s_b}$$

$$\Rightarrow s_b = (k - 1)s_w //$$

3. A metal cylinder is placed upright on horizontal top of an ice slab of temperature 0°C . Initial temperature of the cylinder is 50°C . The vertical curved surface and top of the cylinder are adiabatic while its base is perfectly conducting. Ice below the base of the cylinder melts and the cylinder sinks into the hole thus formed. What fraction of height of the cylinder will ultimately sink?

Specific heat of material of cylinder is $400 \text{ J/(kg}\cdot^\circ\text{C)}$ and density is 9.0 g/cm^3 . Specific latent heat of melting of ice is $4.0 \times 10^5 \text{ J/kg}$ and density is 0.9 g/cm^3 . Neglect dip in melting point of ice due to increase in pressure.

✓(b) 50%



Heat lost by cylinder = Heat gained by ice

$$m_{cy} S_{cy} \Delta T = m_{ice} L$$

$$(\cancel{\rho_{cy}} A h) S_{cy} (50) = (\cancel{\rho_{ice}} A x) L$$

$$\Rightarrow \frac{x}{h} = \frac{\rho_{cy} S_{cy} 50}{\rho_{ice} \cdot L} = \frac{9 (400) 50}{0.9 \cdot 4 \cdot 10^5} = 0.5 //$$

Let the constant
heaters power = P' per min

Let the constant
heat dissipation rate = P per min

4. A heater of constant power is used to boil water in a pot. During heating, temperature of water increases from 60°C to 65°C in 1.0 min. When the heater is switched off, temperature of water falls from 65°C to 60°C in 9.0 min. If rate of heat dissipation to the surrounding remains almost constant, what proportion of heat received by water during heating would be dissipated to the surroundings?

(a) 9%

(c) 12.5%

✓ (b) 10%

(d) 15 %

$$P'(1) - P(1) = ms(65 - 60) \quad (1)$$

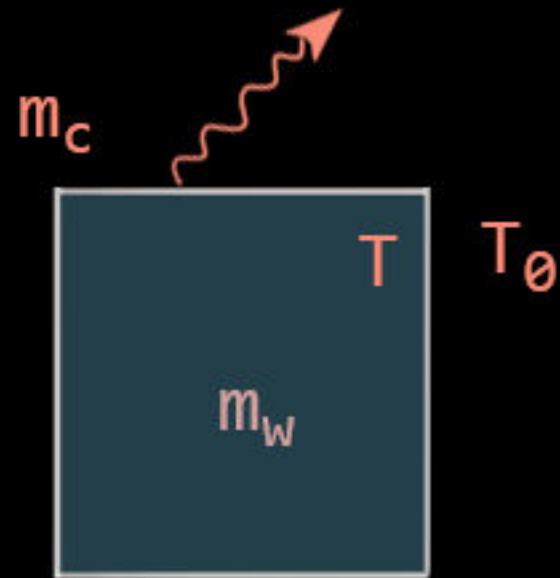
$$P(9) = ms(65 - 60) \quad (2)$$

Dividing 1 and 2: $P' - P = 5P$

$$\Rightarrow \frac{P}{P'} = \frac{1}{10} = 10\%$$

Similar: MCQ 2

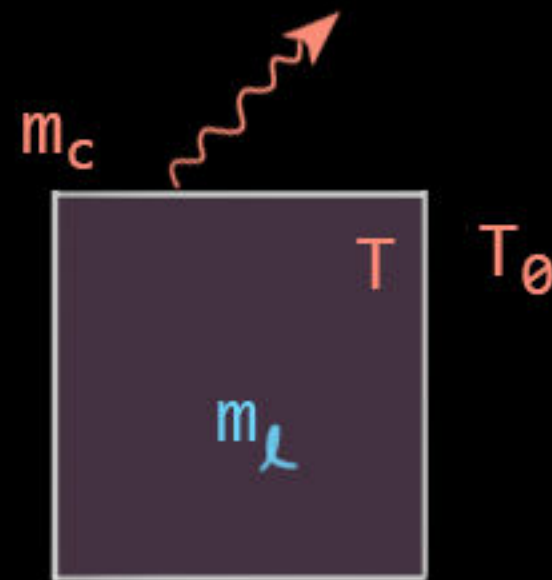
5. Two identical calorimeters one containing water and the other an unknown liquid are left in a room and allowed to cool. Masses of the water, the liquid and the calorimeter are equal. Specific heat of water is $4.20 \text{ J/(g}\cdot^\circ\text{C)}$ and that of the material of the calorimeters is $0.36 \text{ J/(g}\cdot^\circ\text{C)}$. If the calorimeter containing water takes 60 s and that containing the unknown liquid takes 40 s in cooling from 50°C to 45°C , what is the specific heat in $\text{J/(g}\cdot^\circ\text{C)}$ of the unknown liquid? ✓ (b) 2.68



Internal energy reduction rate = Radiation loss rate.

$$\Rightarrow (-m_c s_c - m_w s_w) \frac{dT}{dt} = c(T - T_0)$$

$$\Rightarrow \int_{50}^{45} \frac{dT}{T - T_0} = \frac{-c}{m(s_c + s_w)} \int_0^{60} dt \quad (1)$$



$$(-m_c s_c - m_l s_l) \frac{dT}{dt} = c(T - T_0)$$

$$\Rightarrow \int_{50}^{45} \frac{dT}{T - T_0} = \frac{-c}{m(s_c + s_l)} \int_0^{40} dt \quad (2)$$

Dividing 1,2: $\frac{60}{s_c + s_w} = \frac{40}{s_c + s_l}$

$$\Rightarrow s_l = \frac{2s_w - s_c}{3} = 2.68$$

Youtube / Arihant Senior

by Ankit Singhvi

6. **Speed distribution** of molecules of an ideal mono-atomic gas trapped in a vessel is given in the following table.

Speeds of the molecules	% number of molecules
100 m/s	10
400 m/s	20
600 m/s	40
800 m/s	20
1000 m/s	10

Let there be 100 molecules

∴ Translational KE:

$$\sum \frac{1}{2}mv^2 = \frac{3}{2}nRT$$

Temperature of this gas is **324 K**. If all the molecules of speed 800 m/s escape from the vessel, by what amount will the temperature of the gas change?

✓(b) Decrease by 47 °C

$$\Rightarrow \frac{1}{2}m \left[10 \cdot (100)^2 + 20(400)^2 + 40(600)^2 + 20(800)^2 + 10(1000)^2 \right] = \frac{3}{2} \left(\frac{100}{N} \right) RT_1 \quad (1)$$

$$\text{and } \frac{1}{2}m \left[10 \cdot (100)^2 + 20(400)^2 + 40(600)^2 + 10(1000)^2 \right] = \frac{3}{2} \left(\frac{80}{N} \right) RT_2 \quad (2)$$

Dividing 1 and 2: $\frac{T_2}{T_1} = \frac{2770}{8(405)} \Rightarrow T_2 - T_1 = T_1 \left(\frac{T_2}{T_1} - 1 \right) = 324 \left(\frac{2770}{8 \cdot 405} - 1 \right) = -47 //$

by Ankit Singhvi

Youtube / Arihant Senior

7. A balloon filled with 1.0 g of hydrogen is in thermal equilibrium with air in a room. Root mean square speed of air molecules at the room temperature is 500 m/s, the average molecular mass of air molecules is 29 g/mol. If hydrogen in the balloon is considered an ideal gas; its total internal energy is closest to

(a) 1813 J

(c) 4223 J

✓ (b) 3021 J

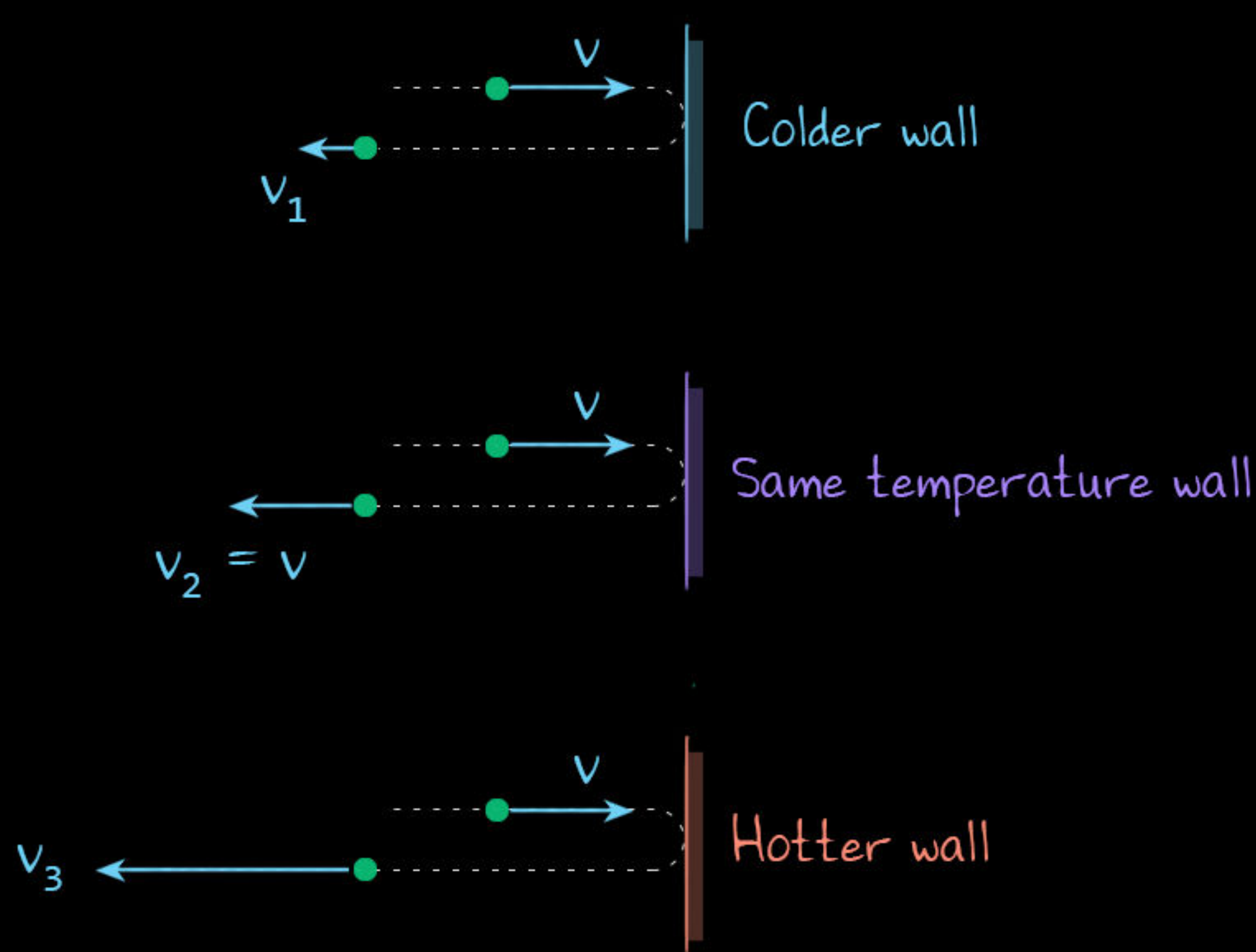
(d) 6042 J

For air: $v_{rms} = \sqrt{\frac{3RT}{M}}$

For hydrogen: $U = n C_v T = \frac{1}{2} \cdot \frac{5}{2} RT = \frac{5}{4} (v_{rms})^2 \frac{M}{3} = \frac{5}{4} (500)^2 \left(\frac{29}{1000} \right) \cdot \frac{1}{3} = 3021 \text{ J} //$

8. In a container, air is filled and maintained at temperature T_0 . Inner surface of walls of the container is maintained at temperature T . At different values of temperatures T that are less than, equal to and greater than T_0 , pressure exerted by the air on the container walls are P_1 , P_2 and P_3 respectively. Which of the following statements is correct?

✓(a) $P_1 < P_2 < P_3$



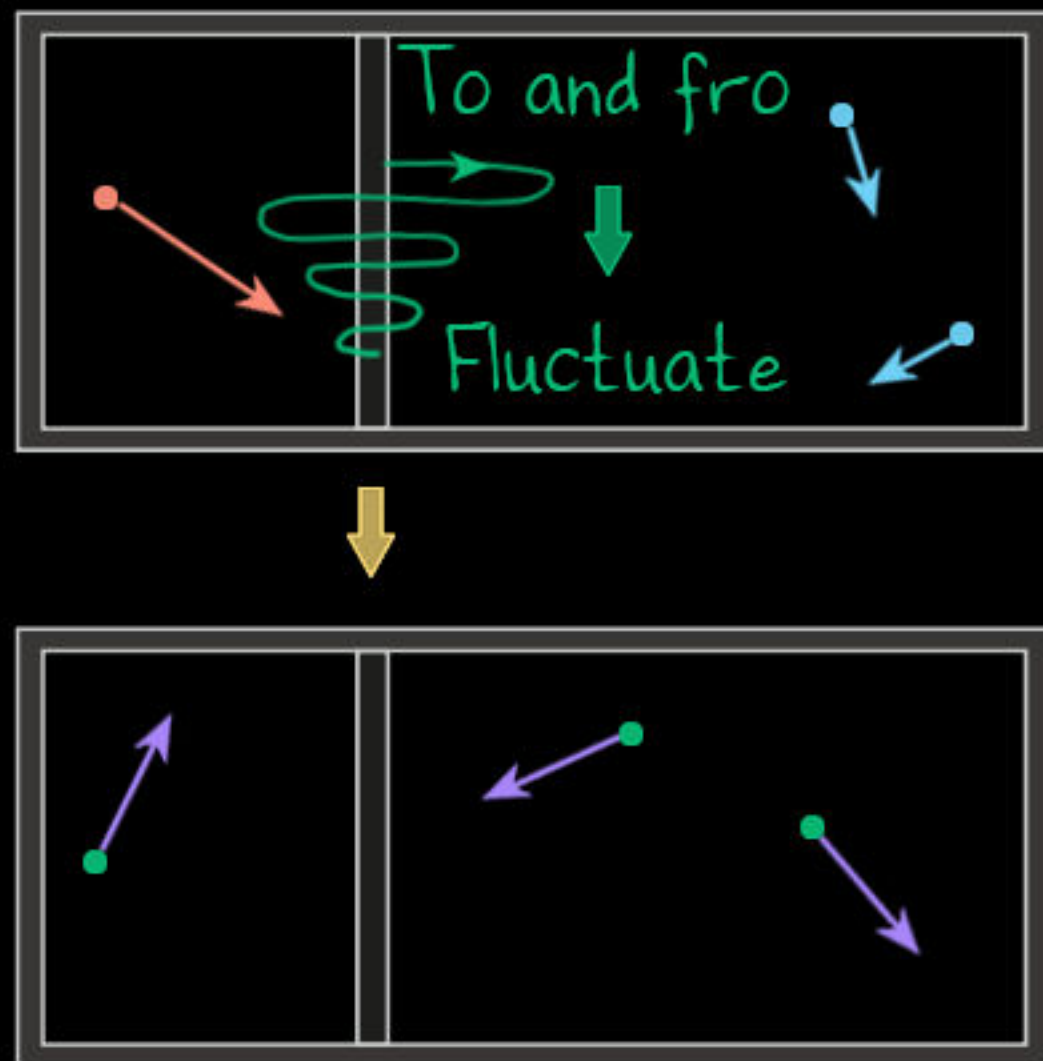
Momentum imparted increases



P exerted on wall increases



$$P_3 > P_2 > P_1 //$$



9. A piston can slide without friction inside a horizontal cylindrical vessel, which contains an ideal mono-atomic gas. The piston and the inner surface of the cylinder are coated with a thin layer of a perfect heat insulating material. Initially, the piston in equilibrium divides the cylinder into two parts A and B, which are not necessarily equal. The temperatures of the gases in both the parts are equal. Now, the piston is held in its initial position and the gas in part A is supplied some amount of heat, then the piston is released. What will the piston do in its subsequent motion?

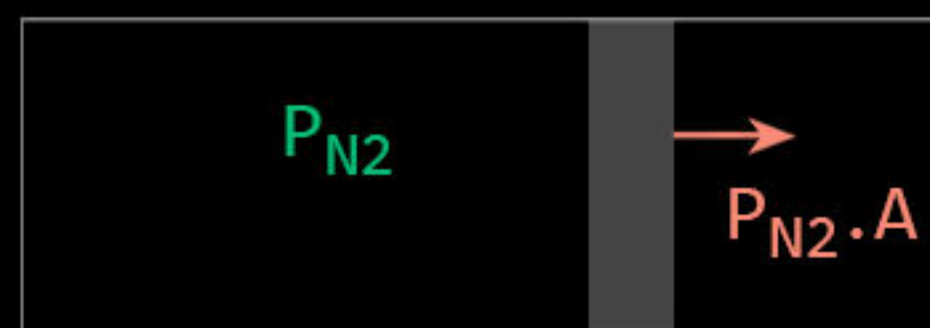
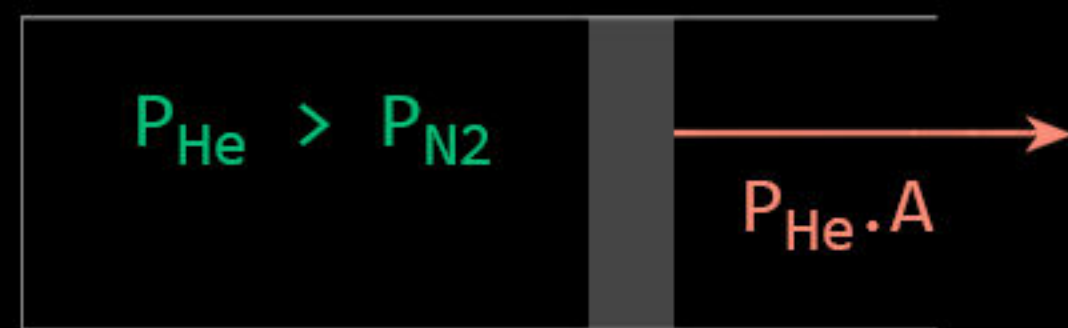
- ✓ (c) It will stop exactly where it had started after several oscillations of decreasing amplitude. Frictionless piston is like a large molecule

https://en.wikipedia.org/wiki/Second_law_of_thermodynamics:

"The second law may be formulated by the observation that the entropy of isolated systems left to spontaneous evolution cannot decrease, as they always arrive at a state of thermodynamic equilibrium where the entropy is highest at the given internal energy."

- Our isolated system here will be at equilibrium when $dS = 0$ (equal pressure is NOT the only condition.) So piston will keep moving to and fro (and later fluctuate) through irreversible states, until entropy of the system is maximum. It does so by momentum transfer and thus equalising internal energy of every molecule bit by bit both sides of piston. Eventual result will be same as if piston was conducting. Temperature of gas everywhere will be equal and piston will settle at its original position.

Situation is similar to...
which piston among these
will be pushed faster.



Clearly Helium will inflate
the balloon faster.

10. Identical balloons are connected through identical valves with two identical cylinders. One cylinder contains gaseous helium and the other gaseous nitrogen. Both the gases may be assumed ideal, and both the cylinders weigh equally. Which balloon will be inflated faster when the valves are opened, and why?

- (a) The nitrogen balloon will be inflated faster because nitrogen is a heavier element, and so the molecules of nitrogen have greater momentum and thus force the balloon to expand at a greater rate.
- (b) The helium balloon will be inflated faster, because helium is a lighter element and so its atoms move faster and can get into the balloon at a greater rate.
- ✓(c) The helium balloon will be inflated faster because the helium must be at higher pressure and hence the gas will be forced into the balloon at a greater rate.
- (d) It will depend on whether the gases have to flow up or down to enter the balloon. Helium being lighter than air as compared to nitrogen will rise faster than nitrogen. Therefore, if the balloons are at the top of the cylinders, the helium balloon will be inflated faster; and if they are at the bottoms, the nitrogen balloon will be inflated faster.

Bag will inflate until inside pressure equalizes to outside pressure of 1 atm.

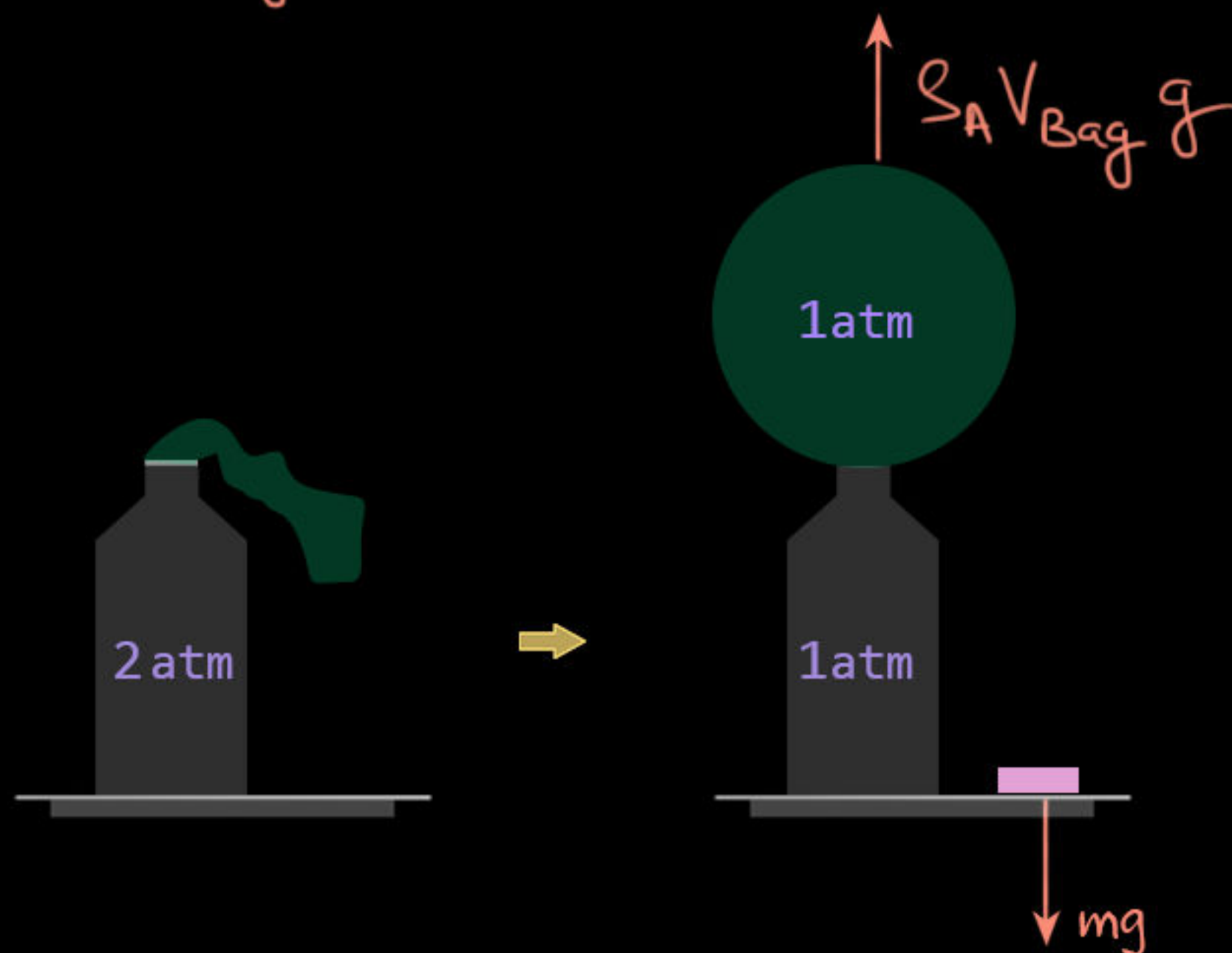
$$P_1 V_1 = P_2 V_2 \leftarrow \text{isothermal}$$

$$\Rightarrow 2 \cdot 2 = 1 (2 + V_{\text{Bag}})$$

$$\Rightarrow V_{\text{Bag}} = 2 \text{ lt}$$

11. In a two-litre metallic bottle, air is pumped to a pressure of 2.0 atm. A thin plastic bag of large capacity (greater than 10 L) with no air inside is connected to the opening of the bottle that is closed. Bottle together with the bag is placed on one pan of balance and weights are put on the other pan to establish balance. When the opening of the bottle is opened, air flows from the bottle into the bag and the balance may be disturbed. How much additional weight is required to reestablish balance? Density of air is 1.3 kg/m^3 and acceleration of free fall is 10 m/s^2 . ✓ (d) 2.6 g

So bag is not 'stretched', and eventually inside Pressure is 1 atm.



Buoyancy is the only change in FBD of bag bottle system.
So additional weight mg should balance it.

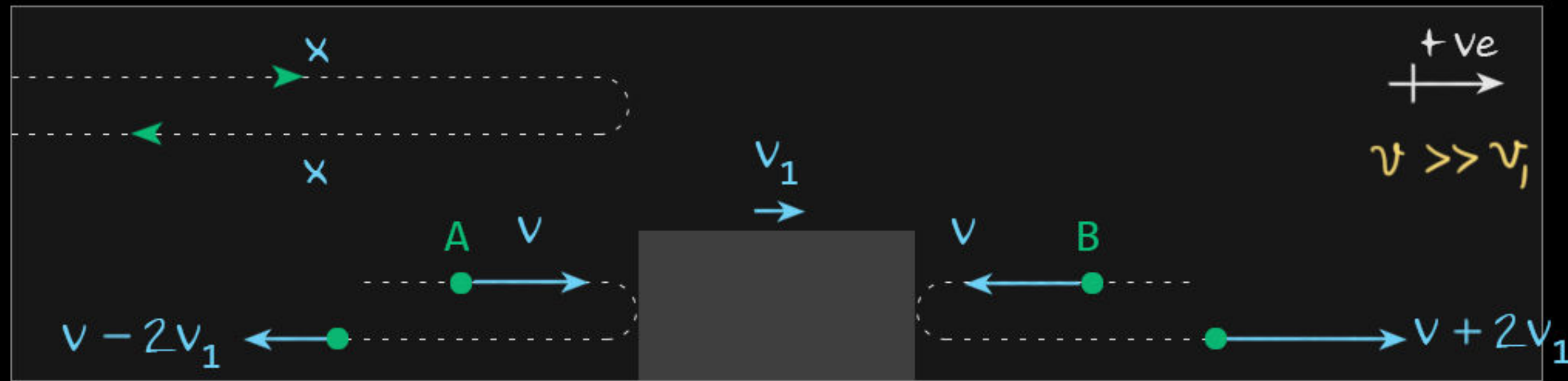
$$\therefore mg = \rho_A V_{\text{Bag}} g$$

$$\Rightarrow m = (1.3) \cdot \frac{2}{1000} = 2.6 \text{ gm}$$

We know, $v = \sqrt{\frac{3RT}{M}}$

$\Rightarrow v \propto \sqrt{T}$

Lets say cylinder is in the middle and, only two molecules hit it from either side every Δt time interval.



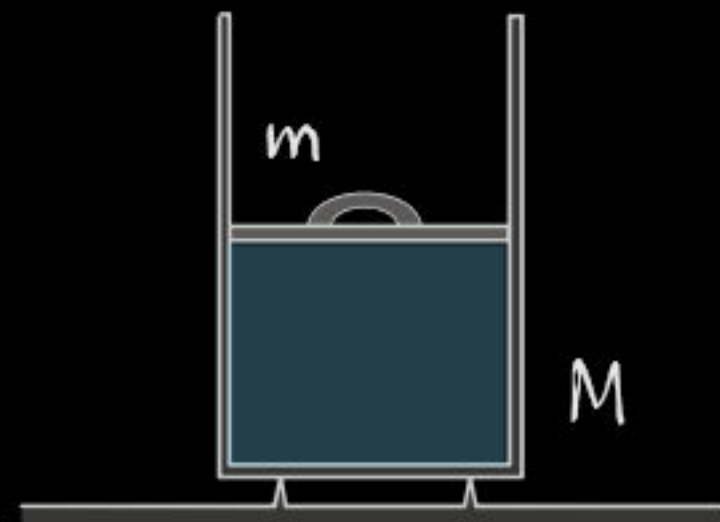
Momentum change of molecules in direction of v_1

$$\text{Drag} = \frac{\text{Momentum change of molecules in direction of } v_1}{\Delta t} = \frac{m[(v + 2v_1) - (-v)] + m[-(v - 2v_1) - v]}{2x/v} = \frac{2mv_1 v}{x} \propto v_1 \sqrt{T}$$

$\therefore \text{Drag}_1 = \text{Drag}_2 \Rightarrow v_1 \sqrt{T} = v_2 \sqrt{2T} \Rightarrow v_2 = \frac{v_1}{\sqrt{2}} = 0.707 v_1$

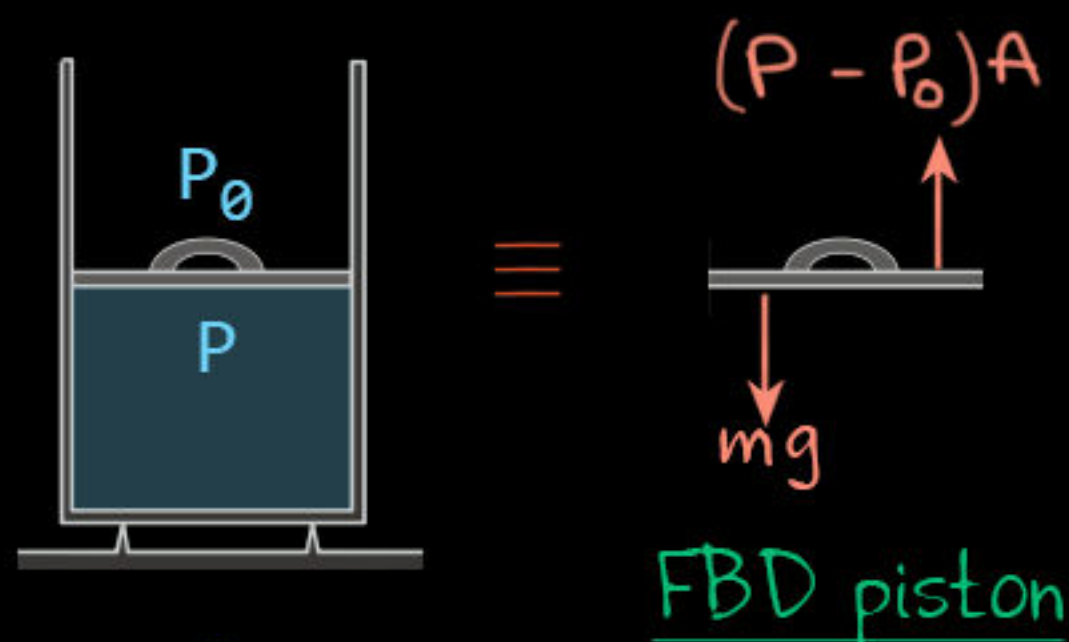
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13. A highly conducting cylindrical vessel placed on elevated small supports contains an ideal gas under a piston of mass m . Area of the base of the cylinder is A . The piston is in equilibrium in the mid of the cylinder. Friction force between the cylinder and the piston can be ignored. Total mass of the cylinder and the gas is M . The atmospheric pressure is p_0 . Now the piston is slowly pulled upwards, find the maximum value of M such that the cylinder can be lifted off the supports.

$$\checkmark (b) \frac{p_0 A - mg}{2g}$$



$$(P - p_0)A = mg \quad (1)$$

To be able to lift:

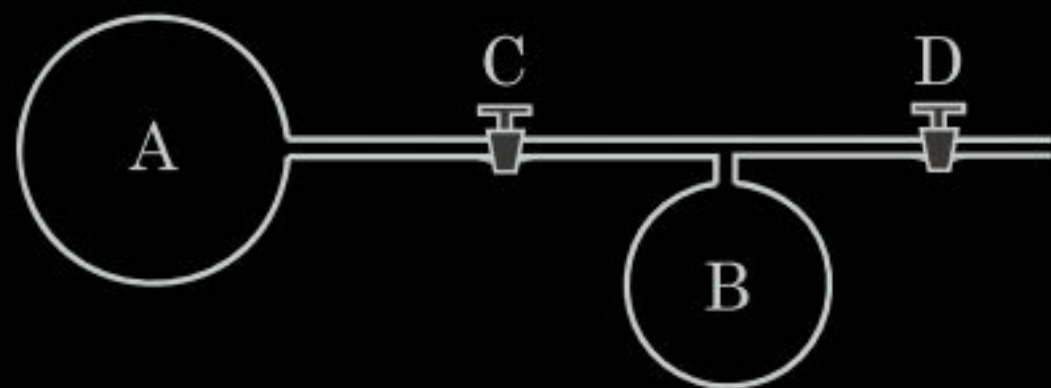
$$N = Mg - \left(p_0 - \frac{P}{2}\right)A \leq 0 \quad (2)$$

From 1,2:

$$M \leq \frac{p_0 A - mg}{2g}$$

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14. Two flasks A and B made of **conducting** materials are connected by a thin tube. Two valves C and D are installed on the tube as shown in the figure. **Initially, both the flasks are evacuated** and the valves are closed. Now the valve D is opened, an **ideal gas is filled in the flask B at pressure p** and then the valve D is closed. When the valve **C is opened**, the **pressure in the flask B drops by amount Δp** . If volume of the flask B is **V_B** , what is the volume of the flask A?

✓ (a) $\frac{\Delta p V_B}{p - \Delta p}$

(b) $\frac{p V_B}{p - \Delta p}$

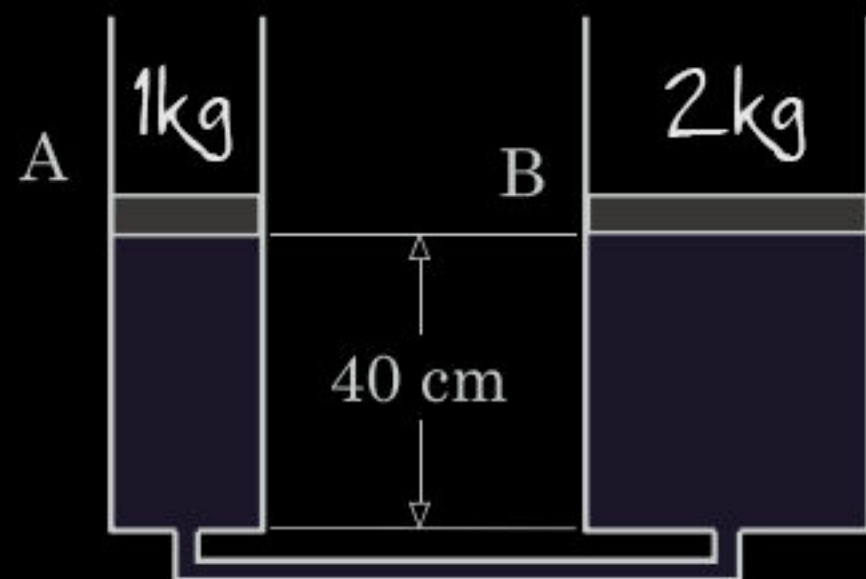
(c) $\frac{(p - \Delta p) V_B}{p}$

(d) $\frac{(p - \Delta p) V_B}{\Delta p}$

Isothermal: $p_1 V_1 = p_2 V_2$

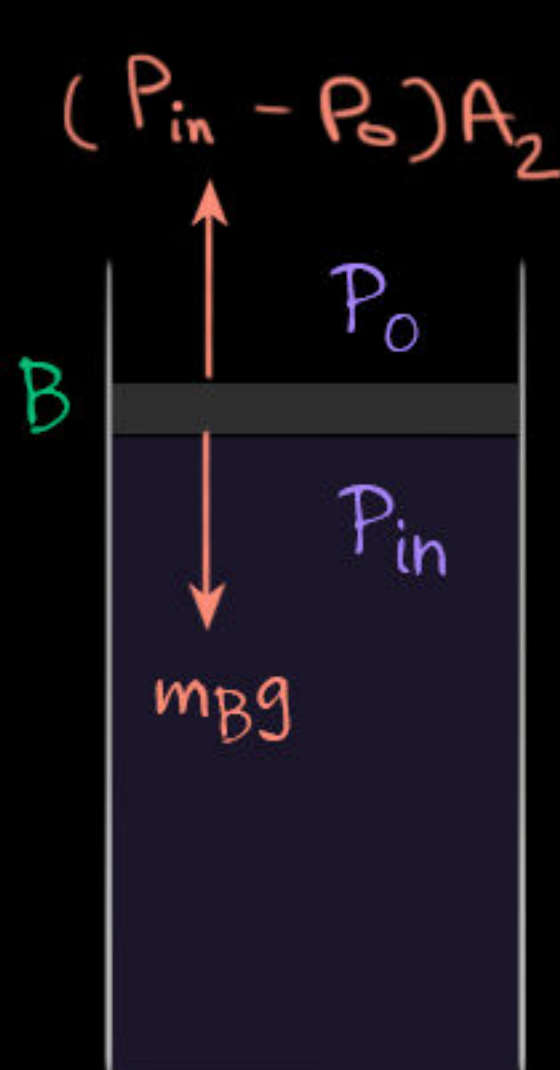
$$\Rightarrow p V_B = (p - \Delta p) (V_A + V_B)$$

$$\Rightarrow V_A = V_B \left[\frac{p}{p - \Delta p} - 1 \right] = \frac{\Delta p}{p - \Delta p} V_B //$$



15. Two vertical **conducting** cylinders A and B of different cross sections are connected by a thin tube as shown. An ideal gas is trapped in the cylinders by two airtight **pistons of masses 1.0 kg and 2.0 kg** respectively. Initially the pistons are at the same height **40 cm** above the base of the cylinders. **If an additional mass of 10 g were gently placed on the piston in the cylinder A,** which of the following statements correctly **describe the new steady state** ultimately reached.

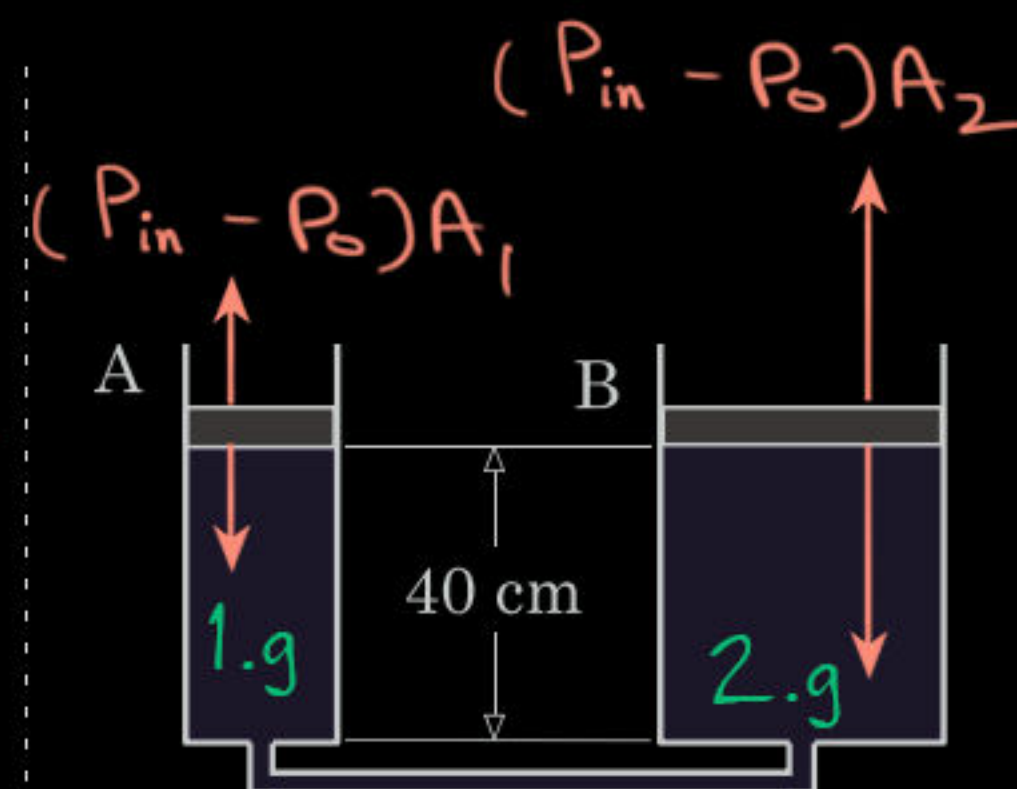
- (a) Pressure of the gas has been increased.
- ✓(b) Pressure of the gas remains unchanged.
- (c) Height difference between the pistons becomes 30 cm.
- ✓(d) Height difference between the pistons becomes 60 cm.



Let the initial pressure be P_{in} .
Piston B is at rest.

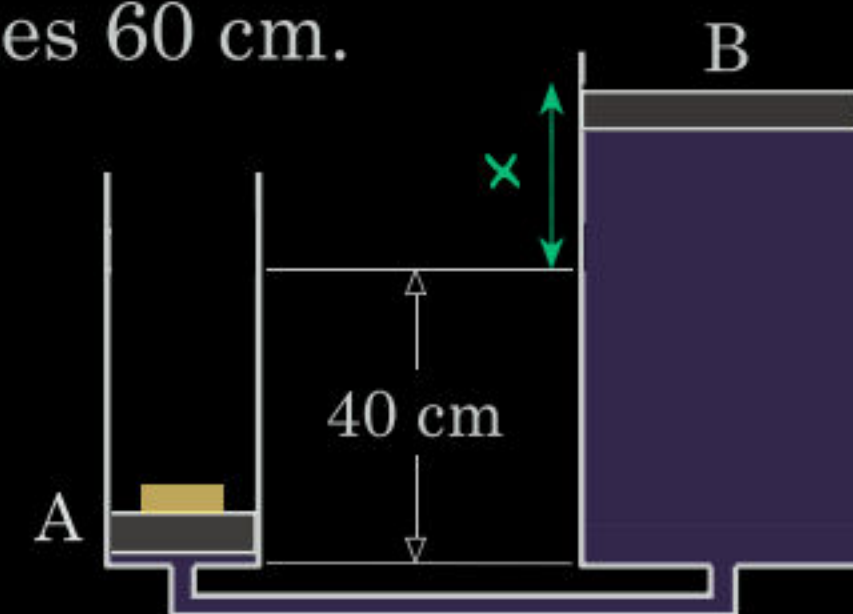
After disturbance, piston B will be at rest again at steady state,
So pressure below it (and everywhere) will be P_{in} again.

And P_{in} is not sufficient to hold piston A, so it will fall to the bottom.



Initial equilibrium

$$\Rightarrow A_2 = 2A_1$$



Volume conservation:

$$\Rightarrow A_1 \cdot 40 = (2A_1)x$$

$$\Rightarrow x = 20$$

$$\Rightarrow H = 40 + 20 = 60$$

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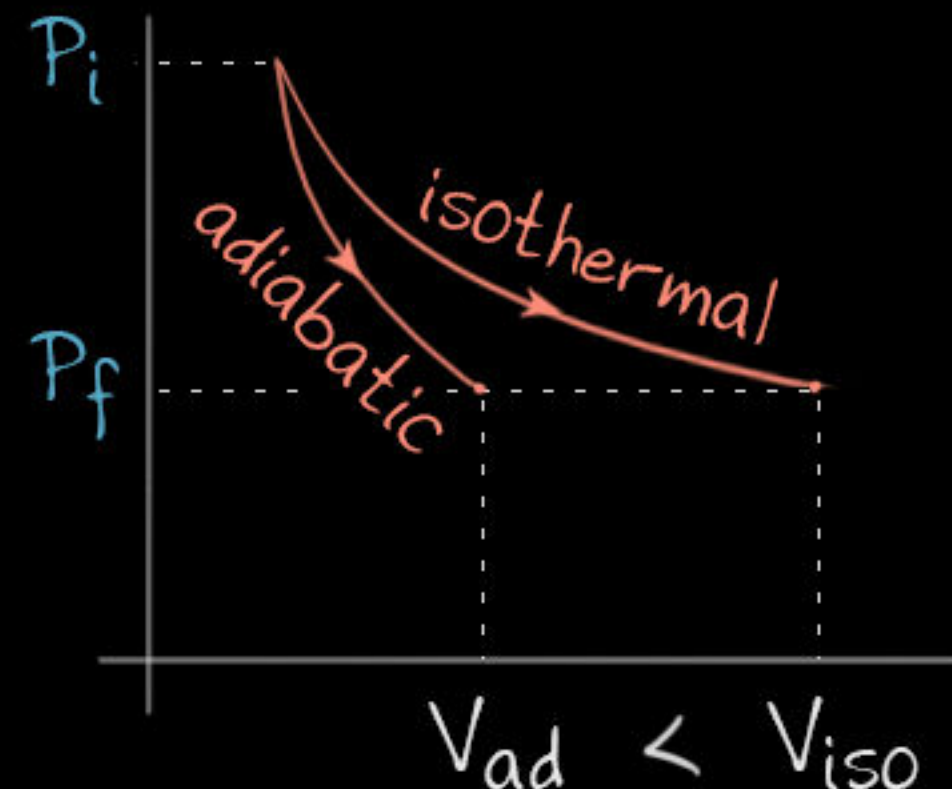
16. Two rubber balloons filled with the same ideal gas when held at the bottom of a lake in thermal equilibrium with the surrounding water occupy equal volumes. Rubber of the first balloon is a good conductor of heat while that of the second balloon is a good insulator of heat. Both the balloons are set free simultaneously. If temperature of water in the lake is uniform, which balloon will occupy more volume when it comes near surface of the lake?

- ✓ (a) The first balloon.
- (b) The second balloon.
- (c) Both the balloons will occupy equal volumes.
- (d) Decision depends on the adiabatic exponent of the gas.

Both balloons are at same depth. So their initial P_i is same.

As balloons go up, they will both expand. And when they are at surface, both will have equal P_f again.

Conducting balloon: isothermal expansion.
Insulated balloon: adiabatic expansion.



We know, for

$$pV^n = \text{constant}$$

$$C = C_v + \frac{R}{1-n}$$

17. One mole of a **mono-atomic** gas is supplied heat in such a way that its **molar specific heat during the heating process is $2R$** , here R is the universal gas constant. If due to heating, **volume of the gas is doubled**, by what factor does its **temperature change**?

(a) $1/4$

(b) $1/2$

(c) 2

✓ (d) 4

$$\therefore \cancel{2R} = \frac{\cancel{3R}}{2} + \frac{\cancel{R}}{1-n}$$

$$\Rightarrow n = -1$$

$$\therefore \frac{p}{V} = \text{constant} \quad \begin{matrix} \nearrow nRT/V \end{matrix}$$

$$\Rightarrow \frac{T}{V^2} = \text{constant}$$

$$\frac{T_1}{\cancel{V_1^2}} = \frac{T_2}{\cancel{V_2^2}} \quad 4$$

$$\Rightarrow T_2 = 4T_1$$

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Formula: Frequency of collisions per unit area

$$f = \frac{N}{V} \frac{\langle v \rangle}{4}$$

$$\frac{N}{V}$$

Molecules per unit volume.

$$\sqrt{\frac{8RT}{\pi M}}$$

Proof: out of scope.

We also know, for

$$PV^n = \text{constant}$$

$$C = C_V + \frac{R}{1-n}$$

18. In a special quasi-static process, an ideal mono-atomic gas is supplied heat in such a way that its volume increases and frequency of collisions of its atoms on unit area of the walls of the container remains constant. What is the molar heat capacity of the gas?

(a) 0

(c) $3R$

✓ (b) $2R$

(d) $4.5R$

Collision frequency is constant

$$\Rightarrow \frac{N}{V} \frac{\langle v \rangle}{4} = \text{constant}$$

$$\Rightarrow \frac{\sqrt{T}}{V} = \text{constant}$$

$$T \propto PV$$

$$PV^{-1} = \text{constant}$$

This is the process gas is going through.

$$\therefore C = C_V + \frac{R}{1-(-1)}$$

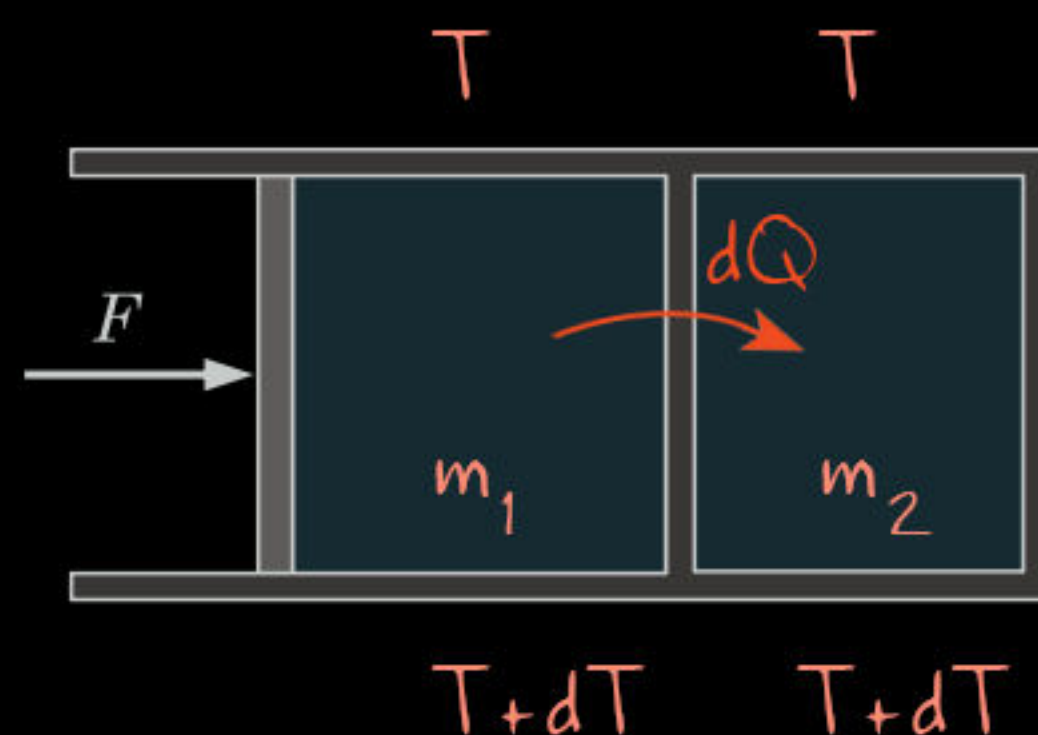
$$= 2R$$

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Approach: Total inside volume is insulated.
So, heat transfer can only occur via partition.



19. An ideal **mono-atomic** gas is trapped in a cylinder closed at its right end. The cylinder is divided into two parts by a **fixed heat-conducting partition** and a piston that is to the left of the fixed partition. **The piston and walls of the cylinder cannot conduct heat.** Masses of the gas in the left and right parts are m_1 and m_2 . A force F applied on the piston is **slowly increased**, starting from some initial value. **What is the molar specific heat of the gas in the left part during this process?**

$$\checkmark (d) - \frac{3m_2 R}{2m_1}$$

Lets say dQ is transfered from first to second cylinder.

For left part: $\frac{m_1}{M} C_1 dT = -dQ$ ①

For right part: $+dQ = \frac{m_2}{M} C_v dT + P dV$ ②

From 1,2: $-\frac{m_1}{M} C_1 dT = \frac{m_2}{M} \cdot \frac{3R}{2} dT$

Gas temperature increases as it loses Heat!

$$\Rightarrow C_1 = - \frac{3m_2 R}{m_1}$$

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We know, for

$$pV^n = \text{constant}$$

$$C = C_v + \frac{R}{1-n}$$

∴ In isobaric: $pV^0 = \text{const}$

$$\Rightarrow C = C_v + R$$

20. If internal energy U of an ideal gas depends on pressure p and volume V of the gas according to equation $U = 3pV$, which of the following conclusions can you make regarding the gas?

- ✓(a) The gas is not a mono-atomic gas.
- (b) The gas can be a di-atomic gas.
- ✓(c) The gas can be a tri-atomic gas.
- ✓(d) Molar specific heat of the gas in an isobaric process is $4R$.

$$U = nC_v T = n \left(\frac{f}{2} R \right) T = \cancel{pV} \frac{f}{2} = \overset{\text{(given)}}{3pV}$$

$$\Rightarrow f = 6$$

Maximum DoF of di-atomic gas is 5.
So gas cant be mono-atomic or di-atomic.

$$C = C_v + R$$

$$= \frac{f}{2} R + R$$

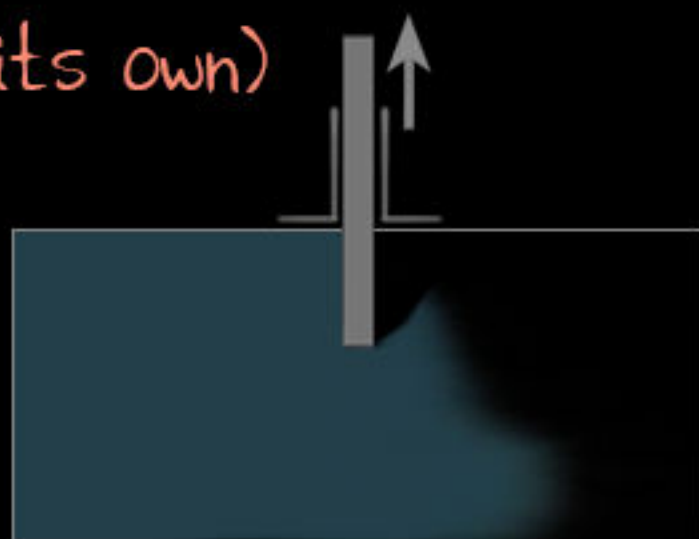
$$= \frac{6}{2} R + R$$

$$= 4R$$

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Two kinds of adiabatic expansions of ideal gas into evacuated chambers.

1 Free expansion (gas goes on its own)



Can be quasistatic or quick, doesn't matter

$$\begin{array}{l} \Delta Q = 0 \\ \Delta W = 0 \end{array} \Rightarrow \Delta U = 0 \Rightarrow T \text{ constant}$$

2 Forced expansion (gas is pushed in)



Quick

$$\begin{array}{l} \Delta Q = 0 \\ \Delta W \neq 0 \end{array} \Rightarrow \Delta U \neq 0 \Rightarrow T \text{ rises}$$

(Work is done on gas by piston)

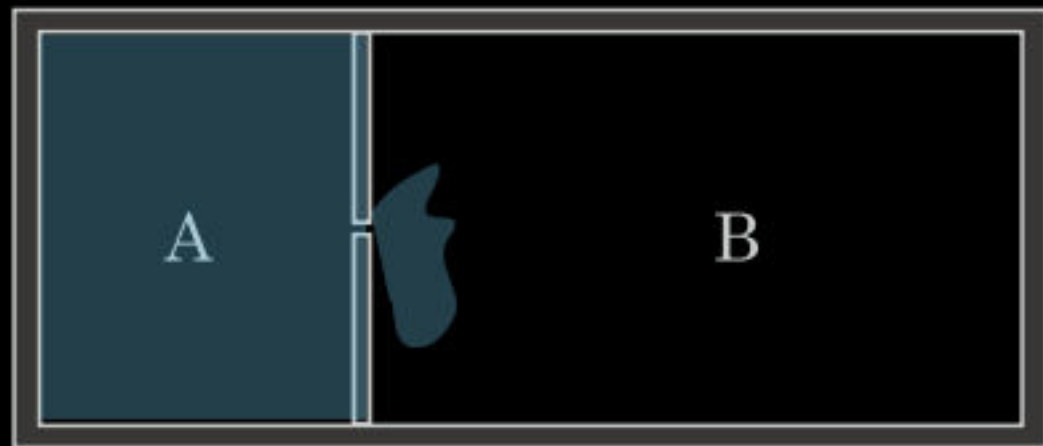
Gas gets ordered directional KE, that gets converted into random motion (heat) at equilibrium

Governing eqn: $\cancel{\Delta Q} + Fx = nC_V\Delta T$

Note: Both 1 and 2 are irreversible

Note: At room temperature, real gases mostly cool down in both (due to attractive intermolecular forces)

Exceptions are Hydrogen and Helium that have weak attraction. These two heat up on expansion.



21. A vessel is divided into two parts A and B by a fixed partition. The walls of the vessel and the partition are made of a perfect heat insulating material. There is a hole in the partition, which can be opened and closed. Initially the hole is kept closed, an ideal gas is filled in the part A and the part B is evacuated. Now the hole is opened for a short time and then closed, as a result some of the gas enters the part B. Now the hole is again opened, and gas is allowed to flow through the hole until net flow of the gas through the hole ceases. How does the temperature of the gas change during the first and the second opening of the hole?

- (a) During both the first and the second openings, the temperature decreases.
- (b) During the first opening, the temperature remains constant but during the second opening, the temperature decreases.
- (c) During the first opening, the temperature remains constant but during the second opening, the temperature increases.
- ✓(d) During both the first and the second openings, the temperature remains constant.

Free expansion (gas goes on its own)

$$\begin{array}{l} \Delta Q = 0 \\ \Delta W = 0 \end{array} \Rightarrow \Delta U = 0 \Rightarrow T \text{ constant}$$

22. A heat-insulated, rigid flask is evacuated and its opening is sealed with a cork. The flask is held at rest in a large chamber filled with an ideal mono-atomic gas at temperature 27°C . The cork is suddenly removed and air quickly fills the flask. If volume of the flask is negligible as compared to the volume of the chamber, what is the temperature of the air inside the flask immediately after a thermodynamic equilibrium is attained in the flask?

✓(c) 227°C

Forced expansion (gas is pushed in)

$$\Delta Q = 0$$

$$\Delta W \neq 0 \Rightarrow \Delta U \neq 0 \Rightarrow T \text{ rises}$$

(Work is done on gas by surrounding P_0)

Volume of gas that's pushed in the flask.

$$\cancel{\Delta Q} + \underbrace{P_0 V}_{\text{Work done on gas} = nRT} = nC_V \Delta T$$

$$\therefore \cancel{nR} \overset{100}{(300)} = \cancel{n} \frac{\cancel{3R}}{2} \Delta T$$

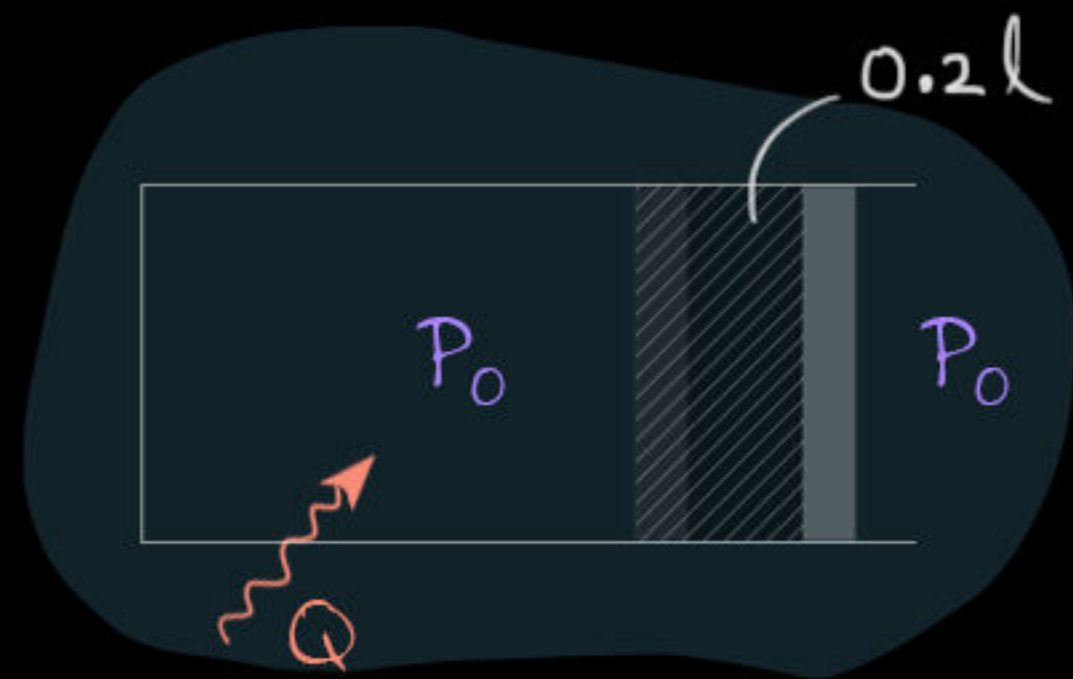
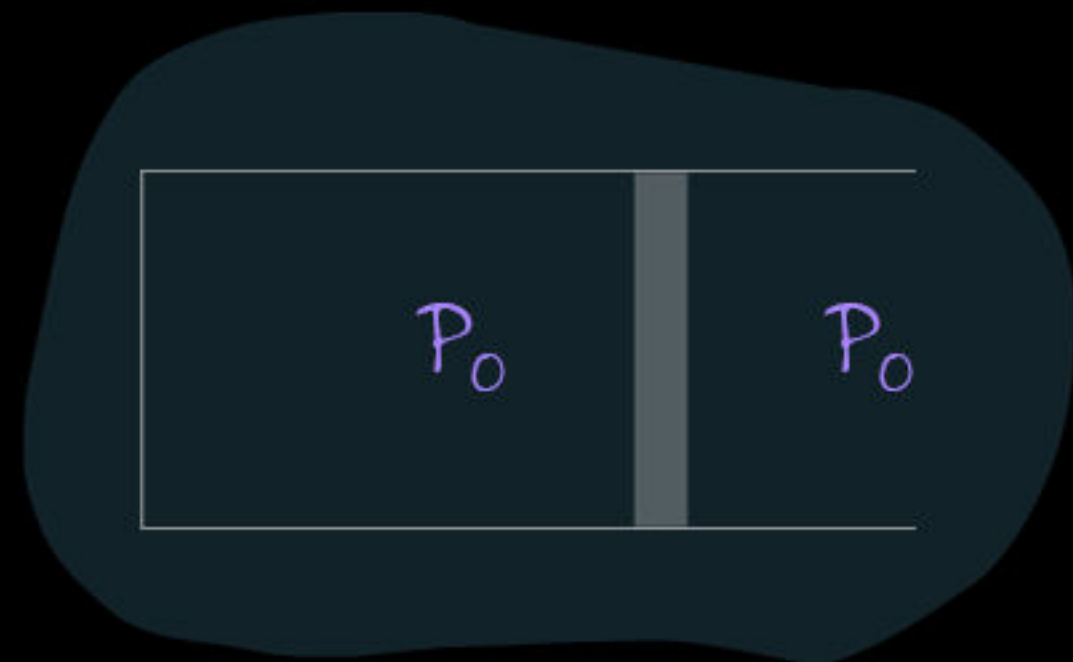
$$\Rightarrow \Delta T = 200 \text{ K}$$

$$\Rightarrow T_f = 200 + 27$$

$$= 227^\circ\text{C}$$

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by Ankit Singhvi



23. An ideal **mono-atomic** gas is confined in a cylinder with the help of a **piston** that can slide inside the cylinder without friction. Walls of the cylinder have **finite conductivity**. Initially the piston is in equilibrium with its external surface exposed to the atmosphere at pressure 10^5 Pa . A transient **pulse of heat** is injected into the cylinder **through its walls** causing the gas to **expand** until its volume changes by 0.2 L . **How much heat was supplied by the pulse?**

✓(c) 50 J

∴ Heat transfer is slow



Piston moves slowly



Net force on piston is negligible



Isobaric process

$$\begin{aligned}
 \therefore Q &= n c_p \Delta T \\
 &= n \left(R + \frac{3R}{2} \right) \Delta T \\
 &= \frac{5}{2} P \Delta V \\
 &= \frac{5}{2} \cdot 10^5 \cdot \frac{0.2}{10^{-3}} = 50 \text{ J} //
 \end{aligned}$$



24. Gaseous **helium** filled in a horizontal fixed cylindrical vessel is separated from its surroundings by a **piston of finite mass**, which can move without friction. Inner surface of the vessel as well as of the piston are coated with a perfectly heat **insulating** material. The **external pressure is increased rapidly to triple** of its initial value **without changing the ambient temperature**. How many times of its initial value will the volume of helium become, when the piston finally stops?

Piston is heavy, so we shall ignore fluctuations of piston, that lead to thermal equilibrium after long time. (like in MCQ 9, where piston was like a large, but light molecule).

∴ Piston after a few to and fro motions, will come to rest where pressure is balanced, even though thermal equilibrium is not reached.

After infinite time however, piston will settle at ambient temperature (ie, thermodynamic equilibrium) and therefore at location $1/3V$

∴ Cant use $PV^\gamma = \text{constant}$ as its for quasistatic process. ✓(b) $\frac{3}{5}$

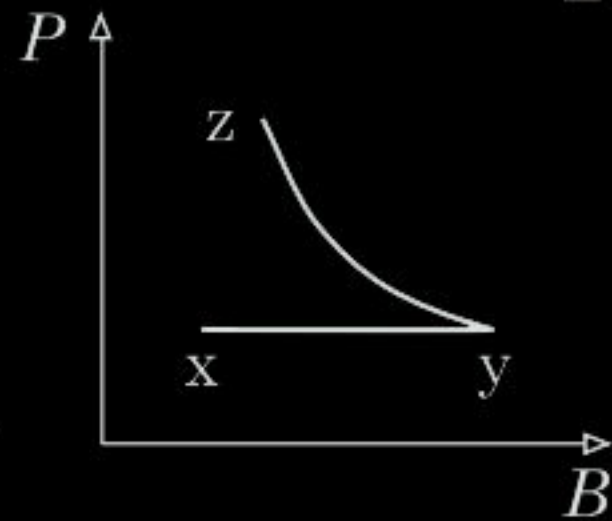
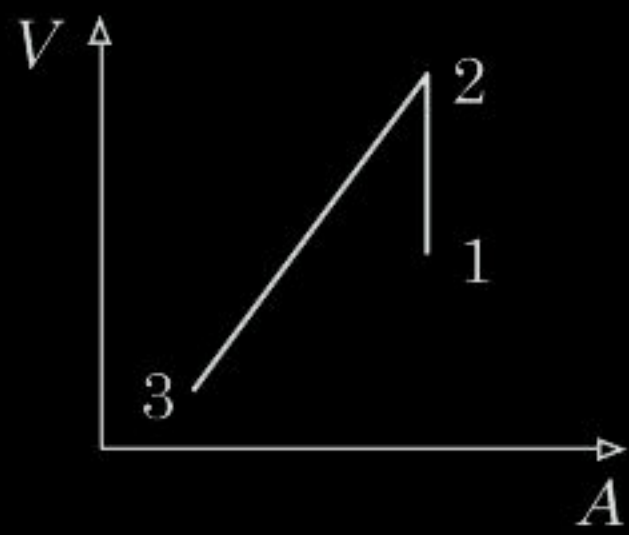
$$\cancel{\Delta Q} + W_{\text{on gas}} = \Delta U$$

$$\Rightarrow 3P_0(V_i - V_f) = n\left(\frac{3R}{2}\right)\Delta T$$

$$\downarrow nR\Delta T = \Delta(PV) = 3P_0V_f - P_0V_i$$

$$\Rightarrow \cancel{3P_0}(V_i - V_f) = \frac{3}{2}\cancel{P_0}(3V_f - V_i)$$

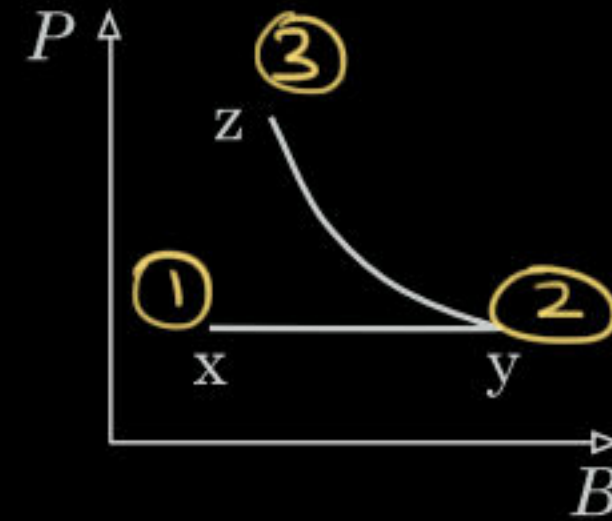
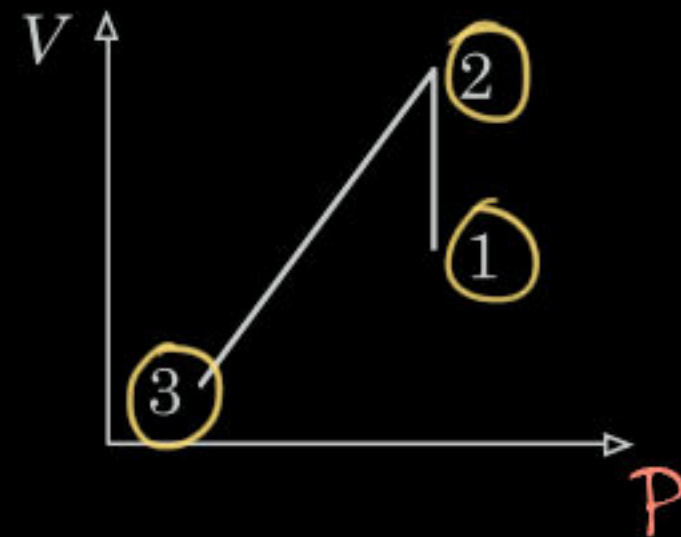
$$\Rightarrow \frac{V_f}{V_i} = \frac{3}{5} //$$



25. A student in an experimental study of a process $1 \rightarrow 2 \rightarrow 3$ on an ideal gas prepares two graphs to show relations between absolute temperature T , pressure P and volume V . Later he found that he has forgotten to specify a coordinate axis in each graph and marking states in the second graph. Unspecified coordinates are shown by letters A and B and unspecified states by letters x , y and z in the second graph. Which of the following combinations best suits these graphs?

Case A

Let $A \equiv P$



- (a) $A \equiv P, B \equiv T, x \equiv 1, y \equiv 2, z \equiv 3$
- (b) $A \equiv P, B \equiv T, x \equiv 3, y \equiv 2, z \equiv 1$
- ✓ (c) $A \equiv T, B \equiv V, x \equiv 3, y \equiv 2, z \equiv 1$
- (d) $A \equiv T, B \equiv T, x \equiv 1, y \equiv 2, z \equiv 3$

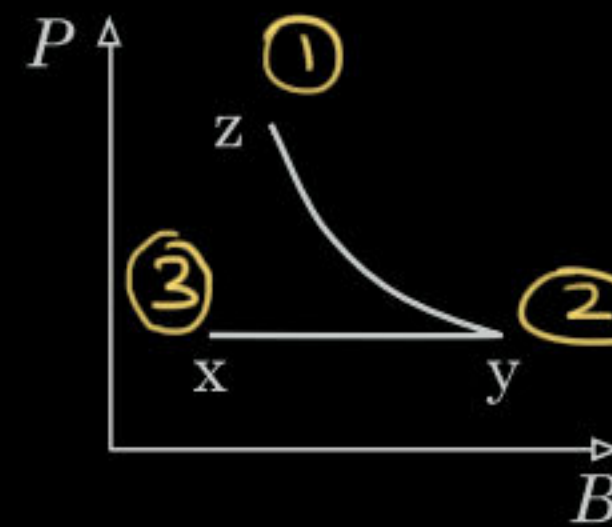
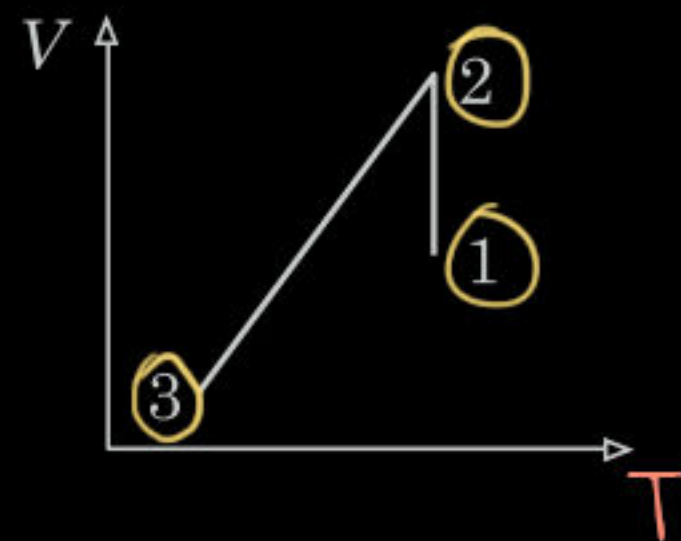
✗ ∵ If, $P_1 = P_2$, P_3 is NOT minimum.

Case B

∴ $A \equiv T$

$V_3 < V_1 < V_2$

$T_3 < T_1 = T_2$



T ✗ ∵ $T_1 \neq T_2$
V ✓



Two heat conducting pistons can slide without friction in a horizontal pipe made of a heat insulating material. Distance between the pistons is 80.0 cm. The space between the pistons is completely filled with water. Densities of water and ice are 1000 kg/m^3 and 900 kg/m^3 respectively, specific heats of water and ice are $4200 \text{ J/(kg}\cdot^\circ\text{C)}$ and $2100 \text{ J/(kg}\cdot^\circ\text{C)}$ respectively, specific latent heat of fusion of ice is 330 kJ/kg and thermal conductivity of ice is four times of that of water. Heat capacity of the pistons and that of the pipe are negligible.

26. The left and the right pistons are maintained at constant temperatures -40°C and 16°C respectively. After a sufficiently long time when a steady state is reached, what will the distance between the pistons become?

✓(c) 88.0 cm

Concept: Volume of ice is $10/9$ times that of water.

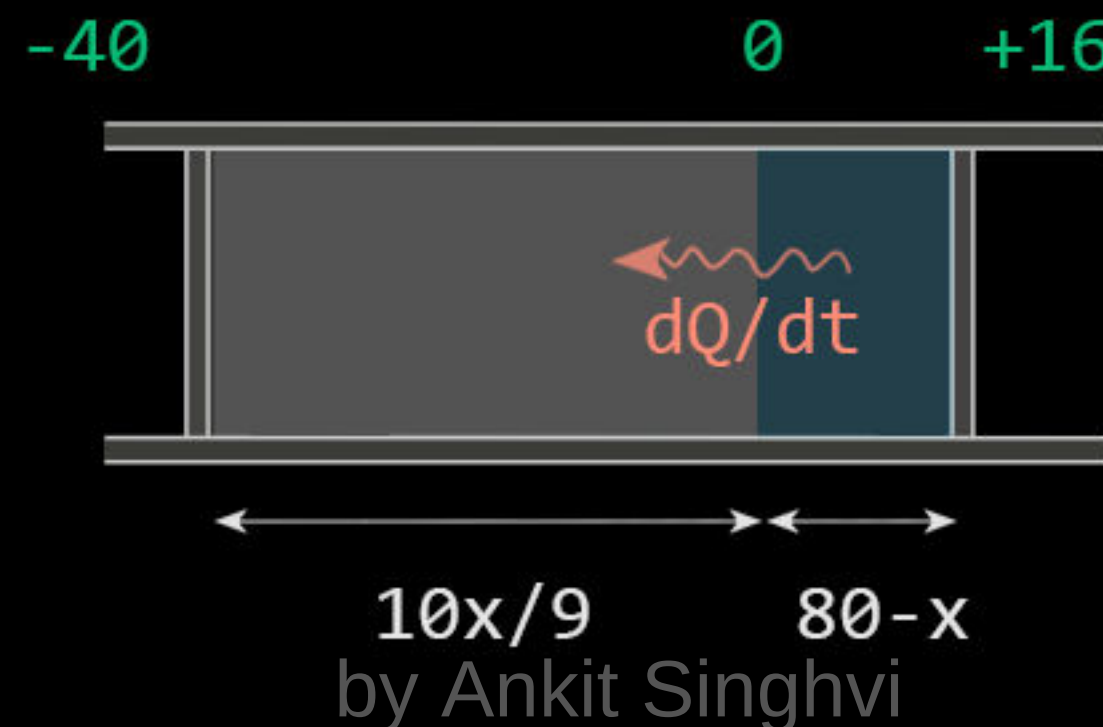
Proof: $m_i = m_w$

$$\rho_i V_i = \rho_w V_w$$

$$\left(\frac{9}{10} \rho_w\right) V_i = \rho_w V_w$$

$$\Rightarrow V_i = \frac{10}{9} V_w$$

Sol: Let x length of water be converted to ice.



by Ankit Singhvi

$$\frac{dQ}{dt} = K_1 A \frac{\Delta T_1}{\Delta l_1} = K_2 A \frac{\Delta T_2}{\Delta l_2}$$

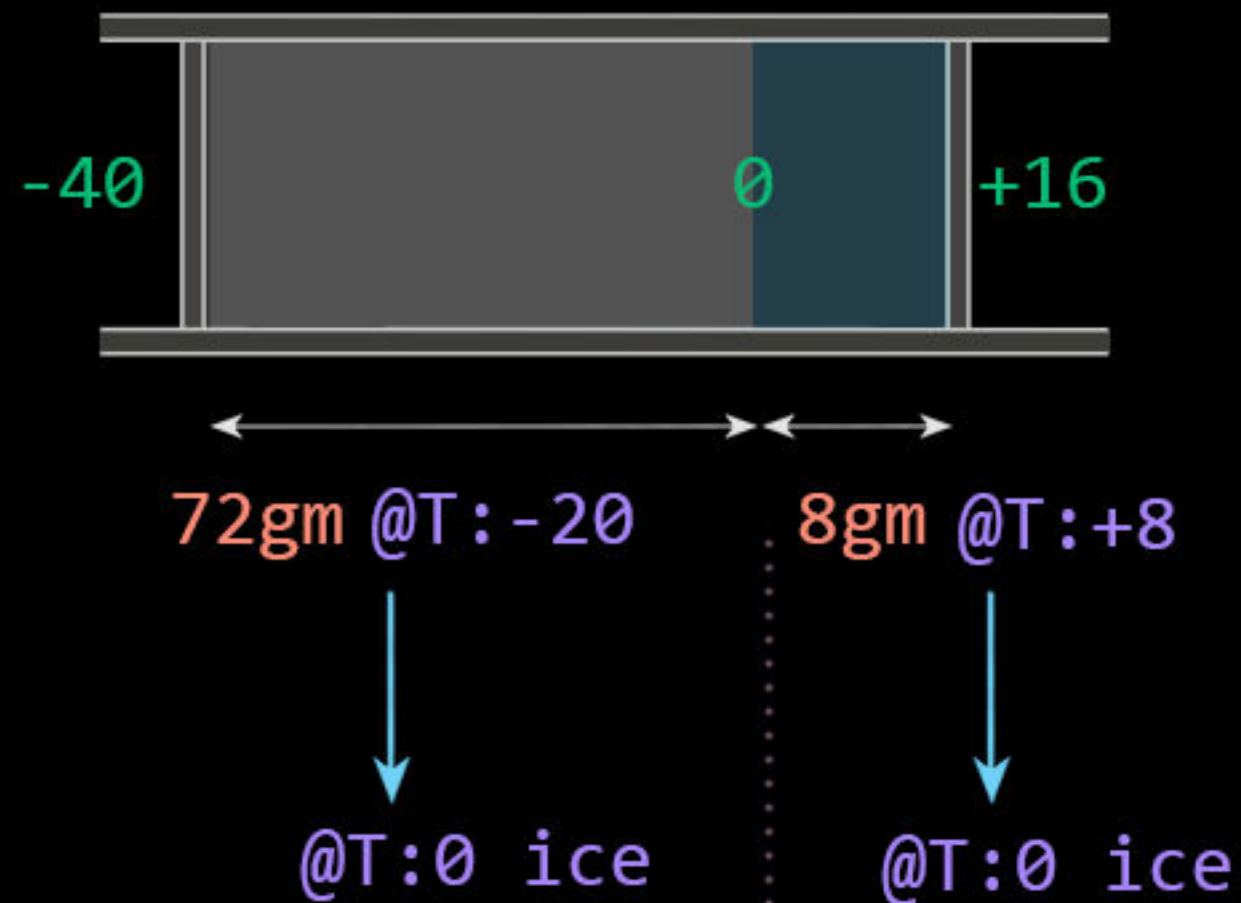
$$\Rightarrow \cancel{K_{ice}} A \frac{40}{10x/9} = \cancel{K_w} A \frac{16}{80-x}$$

$$\Rightarrow x = 72$$

$$\therefore l = \frac{10x}{9} + 80 - x = 88$$

Youtube / Arihant Senior

Sol: Initially:



Heat absorbed

$$= mc\Delta T$$

$$= 72(2100)(20)$$

$$= 3024 \times 10^3$$

Heat lost

$$= mc\Delta T + mL$$

$$= 8(4200)(8) + 8(330 \cdot 10^3)$$

$$= 2909 \times 10^3$$

⇒ Everything becomes ice finally

$$\therefore l = 80 \cdot \frac{10}{9} = 88.9 //$$

27. Now the devices maintaining temperatures of the pistons are removed and the pistons are covered with heat insulating materials. After a sufficiently long time when a steady state is reached again, what will the distance between the pistons become? ✓(b) 88.9 cm

We calculated,

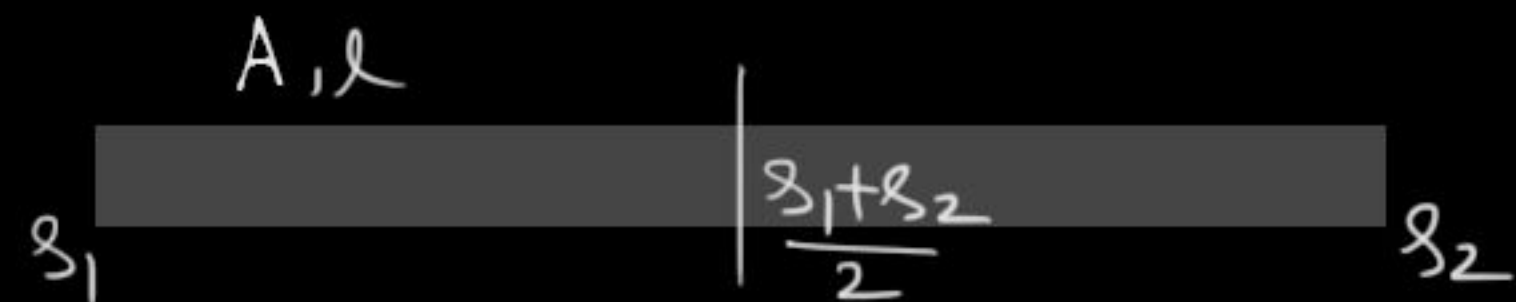
72cm of water gets converted to ice and 8cm remains water.

So let's say:

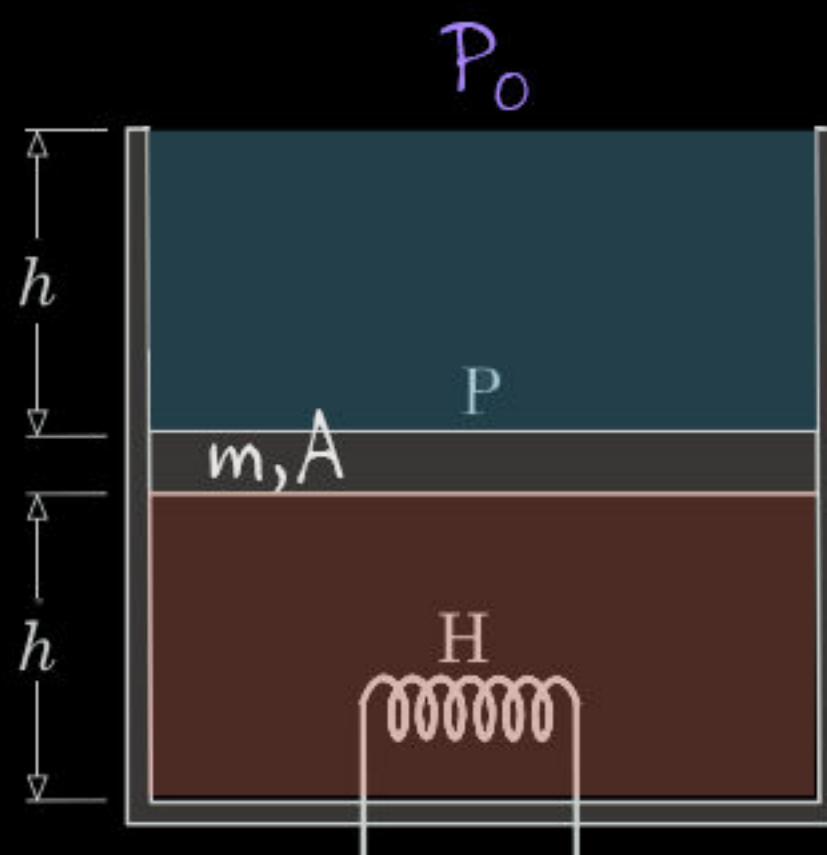
72gm of water gets converted to ice and 8gm remains water.

They are at average temperature of -20 and +8. (We can use T_{average} as T_{midpoint} here as heat gain/loss which we will calculate is proportional to T of a body, and T itself varies linearly over length)

Example: In the shown rod, where density varies linearly we can directly write mass as: $M = \frac{\rho_1 + \rho_2}{2} \cdot A l$



$$M = \int_0^l \rho_2 (A dx) = \int_0^l \left(\rho_1 + \frac{\rho_2 - \rho_1}{l} x \right) A dx = \frac{\rho_1 + \rho_2}{2} \cdot A l$$

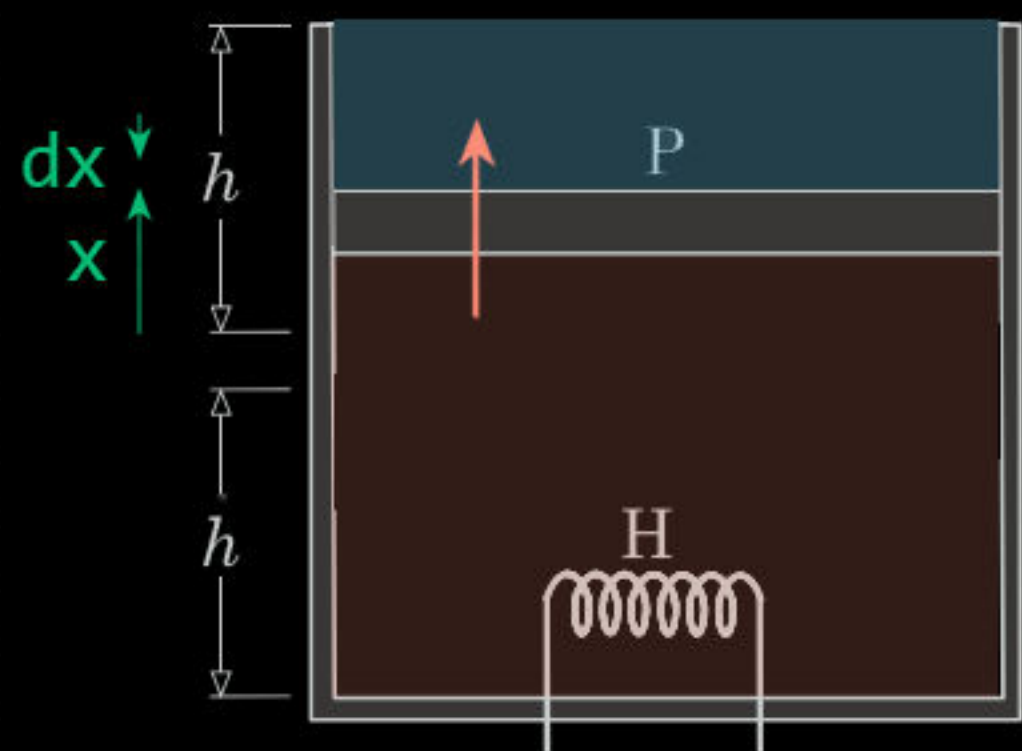


A hollow cylinder made of a thermally insulating material is equipped with a horizontal piston P of mass m and area A . The piston is also made of a thermally insulating material. There is no friction between inner surface of the cylinder and the piston. Above the piston, a liquid of density ρ is filled up to brim of the cylinder. The piston is supported at the position shown due to pressure of an ideal gas filled in the lower portion of the cylinder. Number of moles of the gas is n , atmospheric pressure is p_0 and initial temperature of the gas is T_0 . The heater H is switched on.

28. Work done by the gas on the piston is ✓ (c) $p_0 h A + 0.5 \rho g h^2 A + mgh$

Concept: Heater supplies heat at a finite rate. So piston will rise slowly. i.e net force on piston is always zero.

So net work on piston is zero too.



$\Delta KE = 0$

On piston

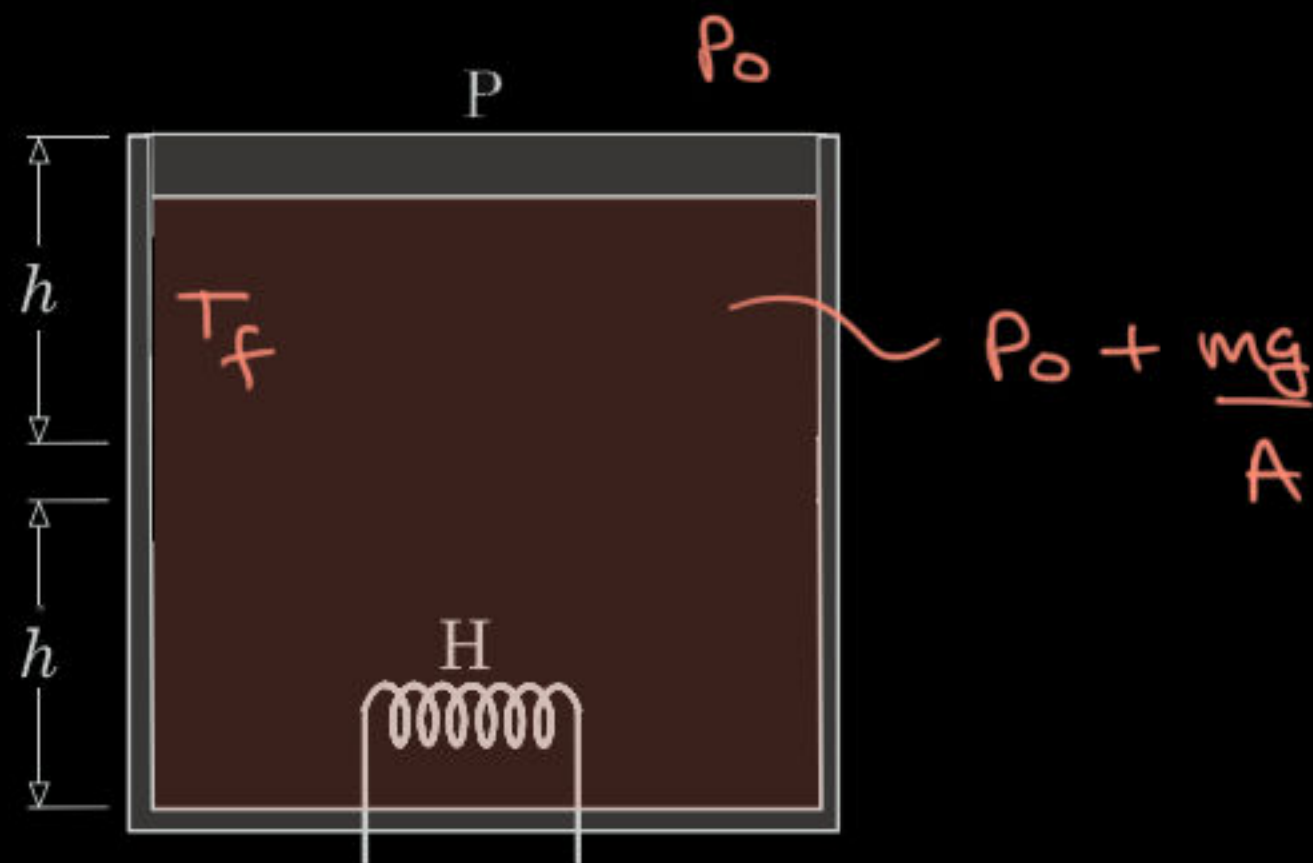
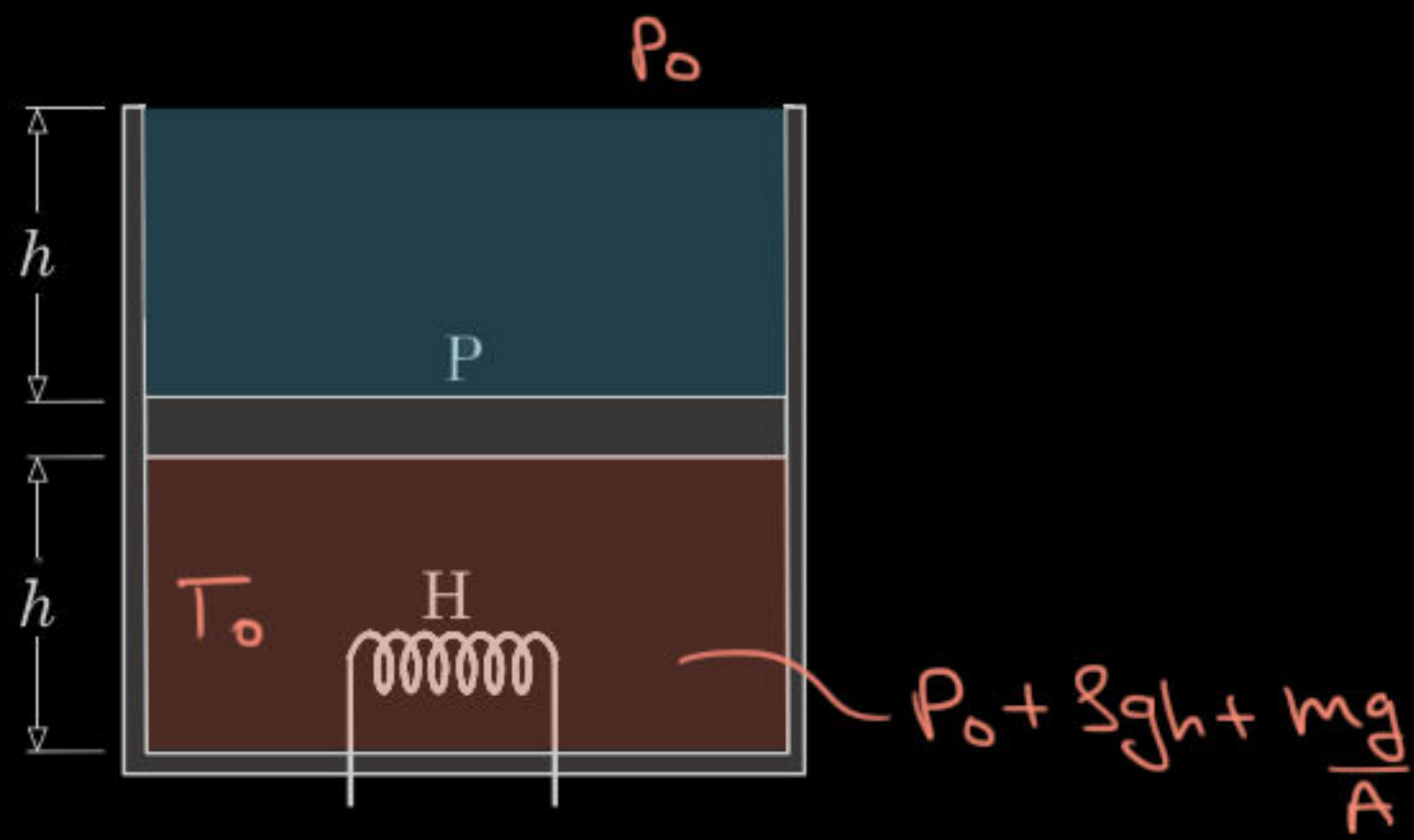
$W_{\text{on piston}} = W_{\text{gas}} + W_{\text{top pressure}} + W_g$

To find $\leftarrow$

$$-\int_0^h (P_0 + \rho g(h-x)) A dx \quad -\int_0^h mg dx$$

$$\Rightarrow W_{\text{gas}} = mgh + (P_0 + \rho gh)Ah - \rho g A \frac{h^2}{2}$$

$$= mgh + P_0 Ah + \rho gh^2 A$$



29. Final temperature of the gas is

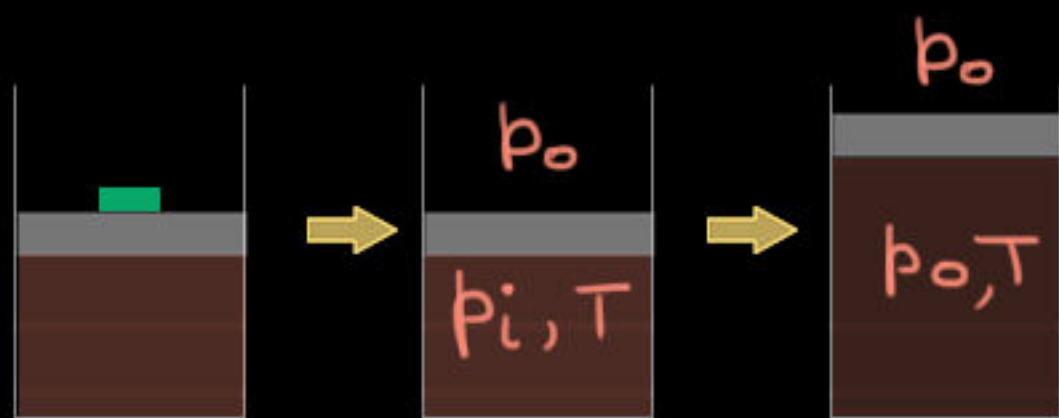
✓(d) $2T_0 \left(\frac{p_0 A + mg}{p_0 A + \rho gh A + mg} \right)$

Sol:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\Rightarrow \frac{(P_0 + \rho gh + \frac{mg}{A}) A h}{T_0} = \frac{(P_0 + \frac{mg}{A}) A (2h)}{T_f}$$

$$\Rightarrow T_f = \frac{2T_0 (P_0 A + mg)}{(P_0 A + \rho gh A + mg)}$$



1 and 4

Even though process is not isothermal, final temperature is again T .

$$W_{\text{on piston}} = W_{p_0} + W_{\text{gas}} + W_{\text{fr}}$$

$$W_{\text{gas}} = -W_{p_0}$$

$$= p_0(V_f - V_i)$$

$$= p_0 V_f \left(1 - \frac{V_i}{V_f}\right)$$

$$NkT$$

$$= NkT \left(1 - \frac{p_0}{p_i}\right)$$

$$\because T_i = T_f$$

In a long cylindrical vessel made of perfectly conducting walls, an ideal mono-atomic gas is confined with the help of a light piston, which can slide inside the cylinder without friction. Number of atoms of the gas in the vessel are N . Initially the piston is kept in equilibrium against the pressure p_i of the gas with the help of a weight placed on the piston. In this state temperature of the gas is T . Atmospheric pressure is p_0 .

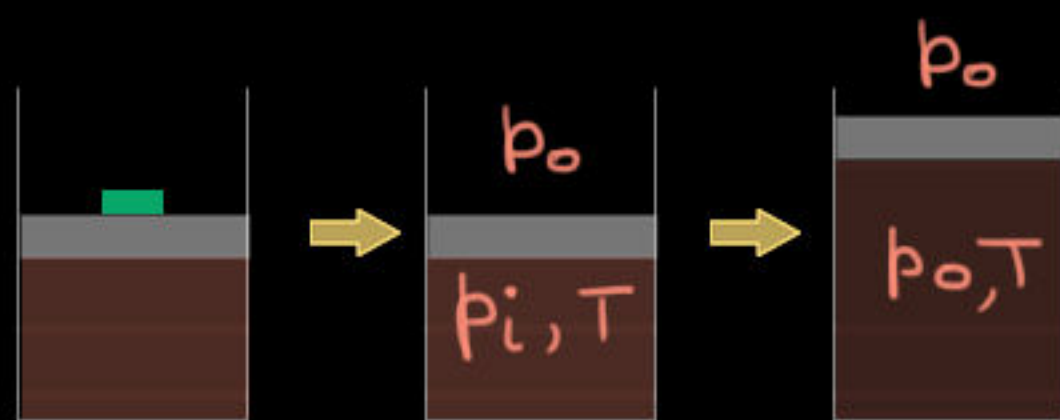
30. When the weight on the piston is removed, the gas expands rapidly and finally the piston settles to a position where the force on the piston due to the gas pressure balances the force due to the atmospheric pressure. Which of the following is a correct description of this process?

(a) The gas does no work.

✓ (b) Work done by the gas is $NkT \left(1 - \frac{p_0}{p_i}\right)$.

(c) Work done by the gas is $NkT \ln \left(\frac{p_i}{p_0}\right)$.

✓ (d) The process is neither isobaric nor isothermal.



Friction force depends on velocity. So initial as well as final friction is zero.

So final pressure is P_o again just like in previous case.

$$W_{\text{on piston}} = W_{P_o} + W_{\text{gas}} + W_{\text{fr}}$$

$$\Rightarrow W_{\text{fr}} = -W_{P_o} - W_{\text{gas}} = NkT \left(1 - \frac{P_o}{P_i}\right) - NkT \ln \frac{P_i}{P_o} //$$

$$NkT \left(1 - \frac{P_o}{P_i}\right)$$

Derived earlier

$$NkT \ln \frac{V_f}{V_i}$$

∴ Isothermal, 2

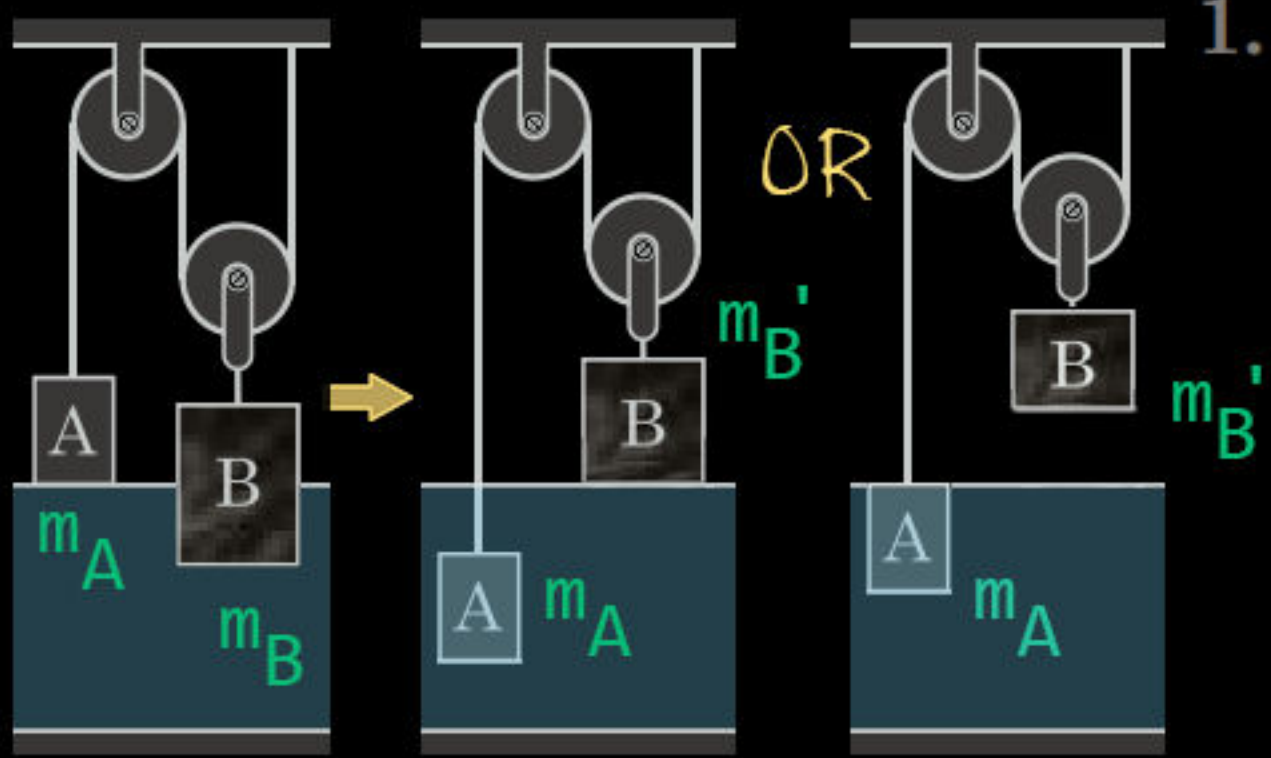
31. In actual practice, considerable frictional forces act between the vessel and the piston. The inner surface of the vessel is lubricated with oil, which makes the frictional forces¹ depend on the velocity² of the piston. Now after removal of the weight, the gas expands quasi-statically² and finally the piston settles to a position where the force on the piston due to the gas pressure balances the force due to the atmospheric pressure. How much work is done by the frictional forces on the piston during the above movement of the piston?

$$(a) -NkT \ln \left(\frac{p_i}{p_o} \right)$$

$$\checkmark (b) NkT \left\{ \left(1 - \frac{p_o}{p_i} \right) - \ln \left(\frac{p_i}{p_o} \right) \right\}$$

$$(c) -NkT \left(1 - \frac{p_o}{p_i} \right)$$

(d) insufficient information to decide.



1. In the setup shown, load A of mass 160 g made from aluminium of density 2.7 g/cm^3 and load B of mass 400 g made from ice of density 0.9 g/cm^3 are suspended from the ceiling with the help of a light inextensible insulating thread and two ideal pulleys as shown in the figure. Specific latent heat of melting of ice is 335 J/g . Initially the setup is in equilibrium with the load A touching the water level and the load B partially submerged in water. How much heat must be given to the load B so that the load A sinks to the bottom of the container?

Initially, A will move bit by bit under water and B will move bit by bit out of water, as ice starts to melt.

A cant sink as long as part of A is outside water OR part of B is inside water, as system is in equilibrium of restoring nature.

Why restoring? Because when A dips further OR B emerges out, upward force on A and downward force on B increase respectively. Either of two counter effects wont allow A to sink.

Therefore for A to sink, A should be completely submerged AND B should be completely out of water. This will get rid of all restoring nature.

Moment both events take place, A will sink if ice melts further. (it doesnt matter which event occurs first, hence dimensions of loads are not given)

∴ Equilibrium is always there.
And at moment of both events:

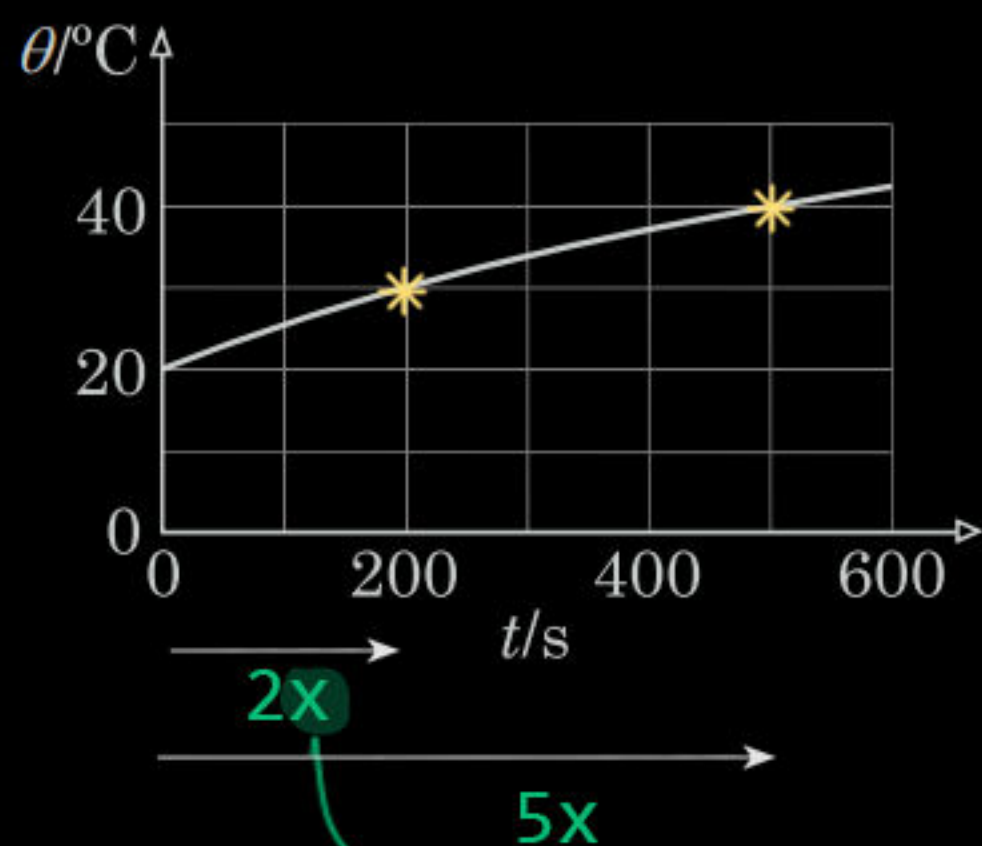
$$\frac{m_B' g}{2} = m_A g - \rho_w V_A g$$

$\swarrow m_A / \rho_A$

$$\Rightarrow m_B' = 2m_A - 2m_A \cdot \frac{\rho_w}{\rho_A} = 201 \text{ gm}$$

$$\begin{aligned} \therefore Q &= (m_B - m_B') L \\ &= (400 - 201) 335 \\ &= 67 \text{ kJ} \end{aligned}$$

Youtube / Arihant Senior



2. Hot water from a reservoir maintained at constant temperature is being added at a very slow and constant rate in a calorimeter initially containing 1.0 kg of water at temperature 20 °C. The water in the calorimeter is being continuously stirred to maintain its temperature uniform. The graph shows how the temperature θ of water in calorimeter changes with time t . Assuming heat loss to the surroundings and work done in the stirring process to be negligibly small, find the temperature of the hot water reservoir.

Let reservoir water be at T , and
Mass of it added every 100 sec be x .

$$\begin{array}{c} 1 \text{ kg, } 20^\circ\text{C} \\ + \\ 2x, T \end{array}$$



$$1 + 2x, 30^\circ\text{C}$$

1

$$\begin{array}{c} 1 \text{ kg, } 20^\circ\text{C} \\ + \\ 5x, T \end{array}$$



$$1 + 5x, 40^\circ\text{C}$$

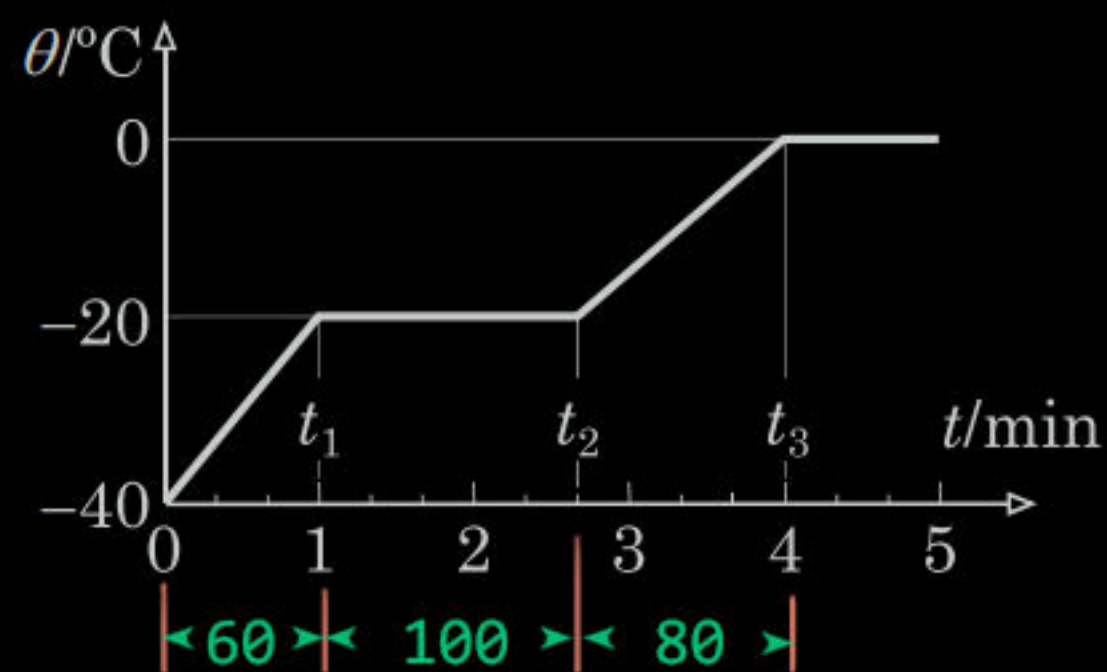
2

$$E_i = E_f$$

$$1: (1) \cancel{S} (20) + 2x \cancel{S} T = (1 + 2x) \cancel{S} (30)$$

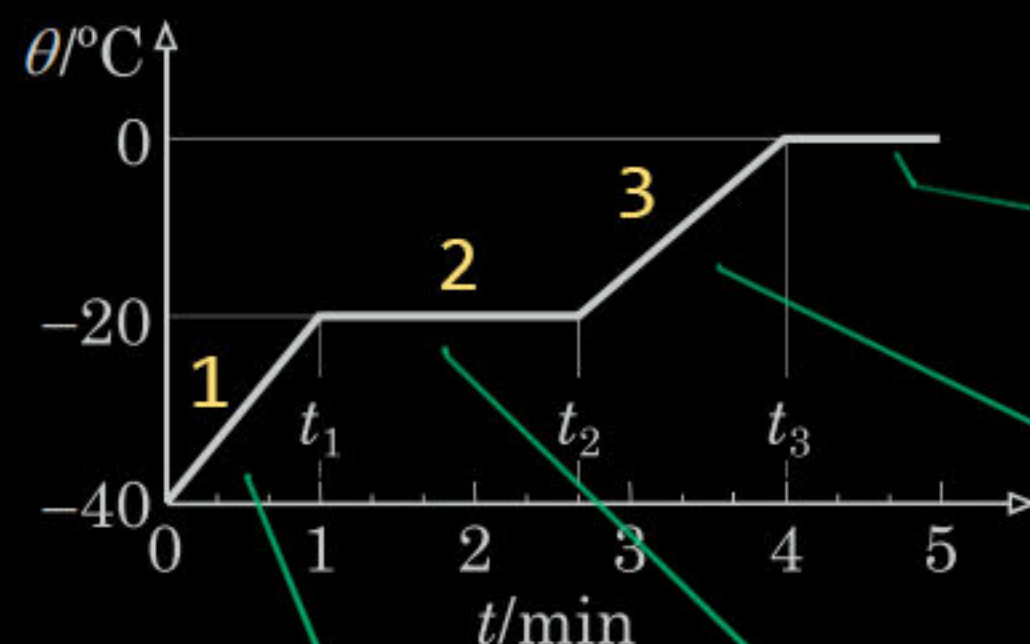
$$2: (1) \cancel{S} (20) + 5x \cancel{S} T = (1 + 5x) \cancel{S} 40$$

$$\text{From 1, 2: } T = 80 //$$



3. Equal masses of an unknown substance in granular form and crushed ice are mixed and kept in an insulated container at -40°C . Specific heat of ice and the substance in solid state are $s_i = 2.1 \times 10^3 \text{ J/(kg}\cdot^\circ\text{C)}$ and $s_s = 900 \text{ J/(kg}\cdot^\circ\text{C)}$ respectively. The substance in liquid state is immiscible with water. Now heat is supplied at a constant rate to the contents in the calorimeter and temperature is recorded at regular intervals of time. The data obtained are shown in the graph. Find specific latent heat L of melting and specific heat s_l in liquid state of the substance.

Let heater power be r .
masses be m each.



Both solid

Substance melting, ice solid

Ice starts to melt
Substance liquid, ice solid

$$1: \cancel{r} (60)^{\cancel{\text{sec}}} = m s_i (20) + m s_s (20)$$

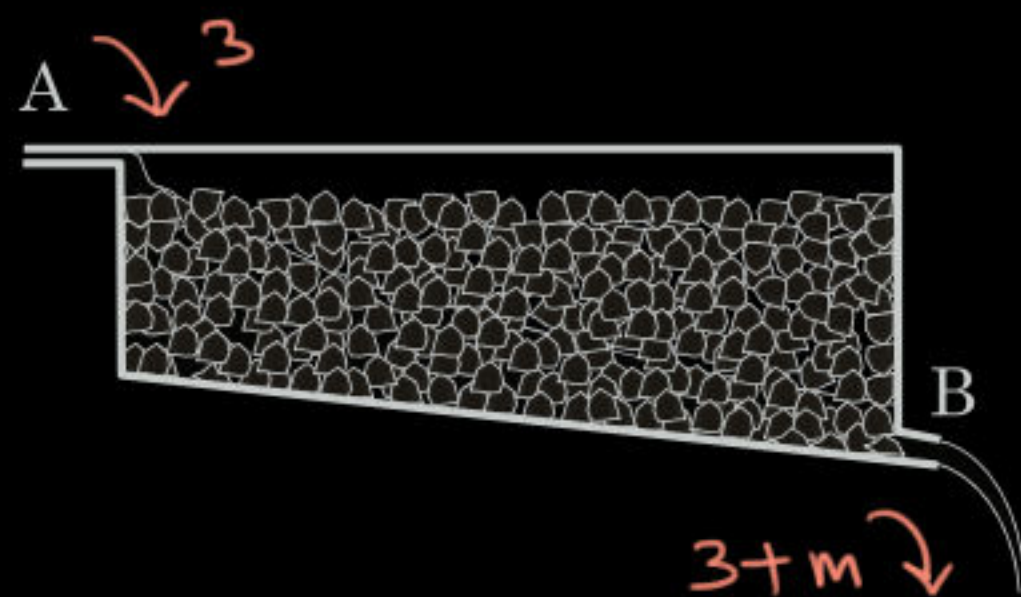
$$\Rightarrow \frac{\cancel{r}}{m} = \frac{(s_i + s_s) 20}{60} = 1000 \text{ SI}$$

$$2: \cancel{r} (100) = m L$$

$$\Rightarrow L = \frac{\cancel{r}}{m} \cdot 100 = 10^5 \text{ J/kg}^\circ\text{C}$$

$$3: \cancel{r} (80) = m s_i (20) + m s_l (20)$$

$$\Rightarrow s_l = 4 \frac{\cancel{r}}{m} - s_i = 4000 - 2100 = 1900 \text{ J/kg}^\circ\text{C}$$



Lets say, m mass of ice melts every second and gets drained along with input water.

4. A special container having tilted bottom of uniform slope has one inlet A at top and one outlet B at the lowest place near the bottom. It contains large amount of crushed ice at $\theta = 0.0^\circ\text{C}$. From the inlet, water of temperature $\theta_1 = 28^\circ\text{C}$ is continuously fed to the container at a rate $r_i = 3.0 \text{ g/s}$. Spaces between pieces of the ice permit water to run down and come out from the outlet. The outlet is made wider than the inlet to prevent accumulation of water in the container and inner surface of the container is adiabatic. Specific heat of water is $s = 4.2 \text{ J/(g}\cdot^\circ\text{C)}$ and that of melting of ice is $L = 336 \text{ J/g}$. Find rate of water coming out from the outlet, if its temperature is $\theta_0 = 1.0^\circ\text{C}$.

In 1 second:

water

$3\text{g}, 28^\circ\text{C}$

↓ Heat released

water

$3+m, 1^\circ\text{C}$

↑ Heat absorbed

ice

$m, 0^\circ\text{C}$

Heat absorbed

$m, 0^\circ\text{C}$

water

Heat released = Heat absorbed

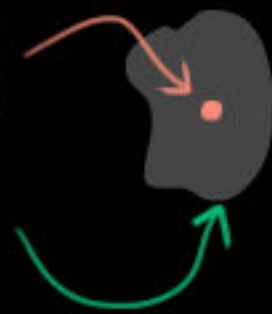
$$\Rightarrow 3C(28-1) = mL + mC(1)$$

$$\Rightarrow m = \frac{3C \cdot 27}{L + C} = \frac{3(4.2) \cdot 27}{336 + 4.2} = 1$$

$$\therefore \text{Outflow rate} = 3+1 = 4\text{gm/s}$$

Fundamentals of super cooled water.

To freeze at 0°C , water molecules need a "seed crystal/nucleus/minerals" to form ice crystal around it.



Pure water, free of nucleation sites won't freeze at 0°C , but at much lesser temperature like -50°C with a different mechanism called homogeneous nucleation.

This water at less than 0°C , is called super cooled water.

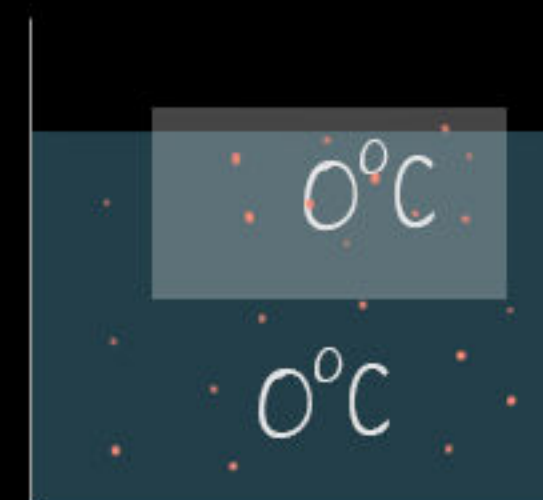
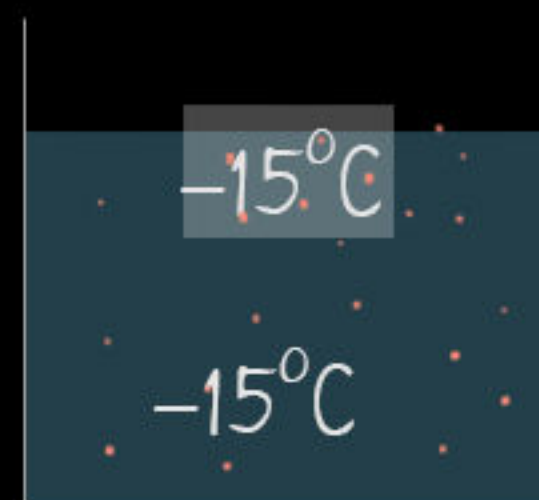
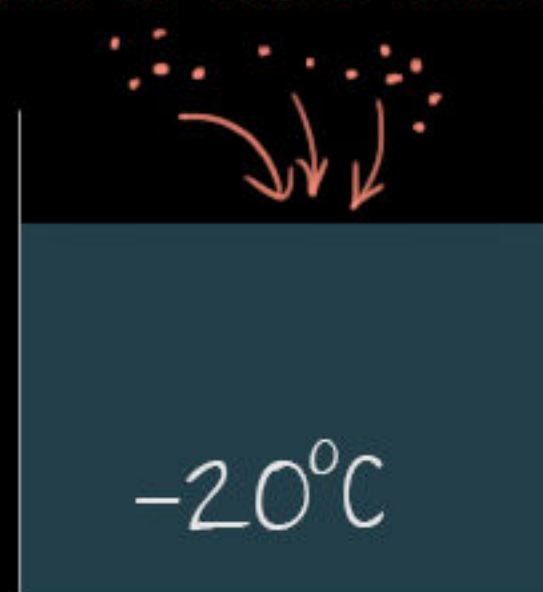
Specific heat of super cooled water is just like of normal water. i.e, $4.2 \text{ J/kg}^{\circ}\text{C}$

Ex. What will happen, if some mineral dust is added to supercooled liquid at -20°C .

Ans. Part of water will turn to ice.

As some water turns to ice, Latent heat is removed and released to surrounding. So, temperature of rest of water will increase to 0°C until normal ice water equilibrium at 0°C is reached and further ice formation stops.

mineral dust added

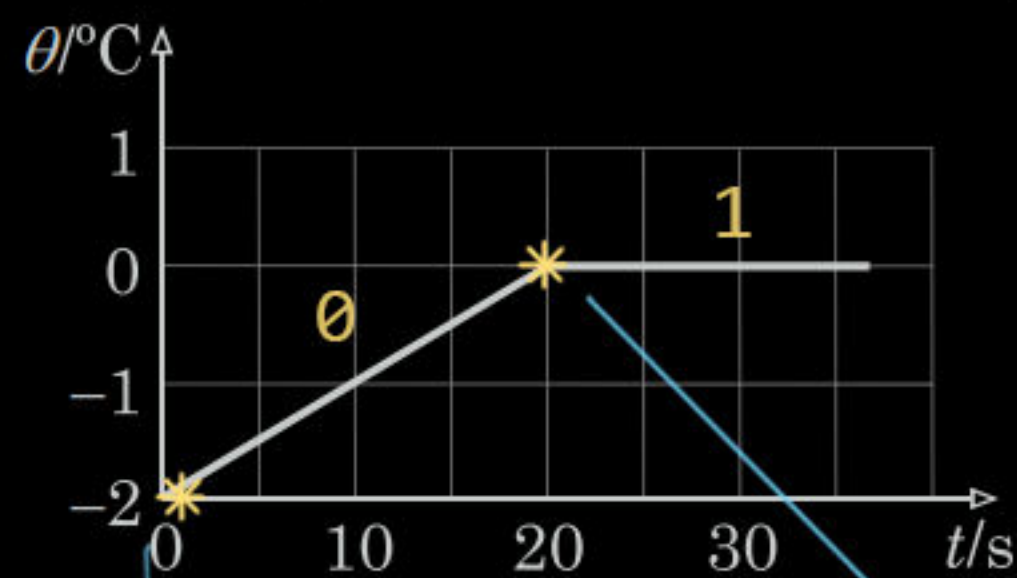
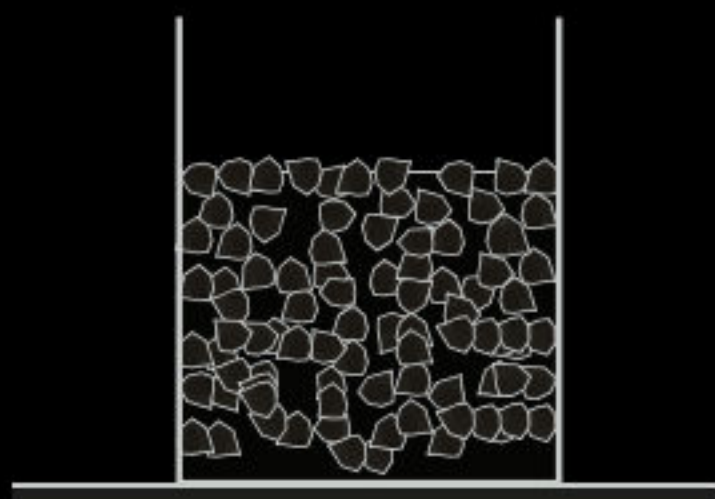


ice continues to form

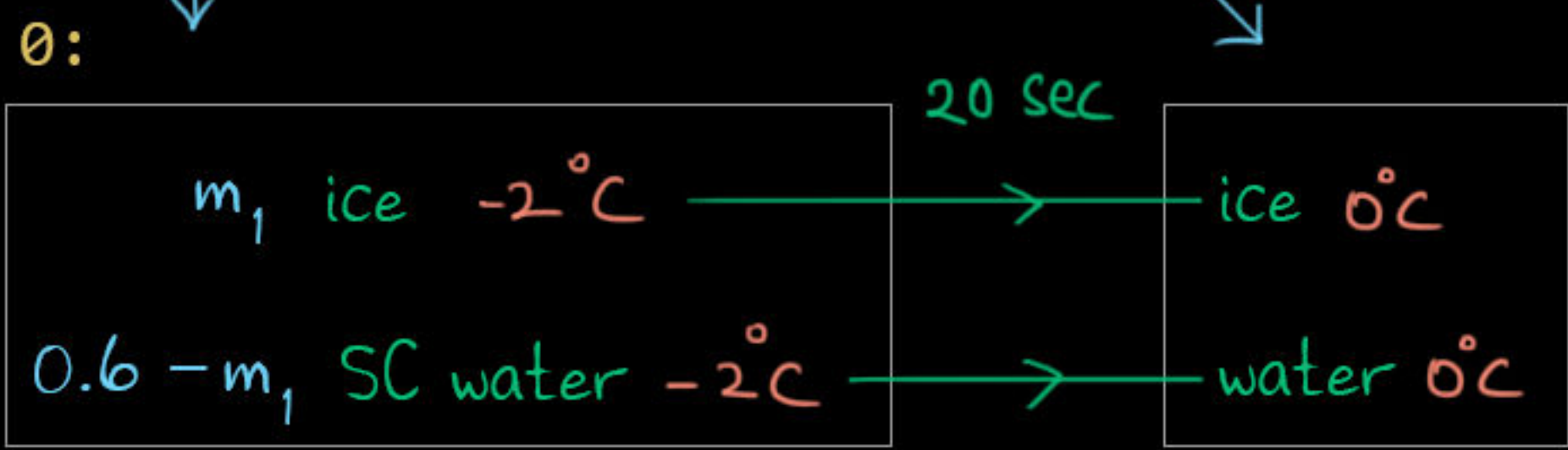
equilibrium

by Ankit Singhvi

Youtube / Arihant Senior



Heater power be p .
Mass of ice be m_1
Total mass: .6



5. Under certain conditions, a mixture of crushed ice and super cooled water at temperature $\theta_1 = -21.0^\circ\text{C}$ is kept in a calorimeter. Total mass of the mixture is $m = 0.60 \text{ kg}$. Heat capacities of water and ice in the calorimeter are equal. Specific heat of ice is $s_i = 2.1 \text{ kJ}/(\text{kg}\cdot^\circ\text{C})$, specific heat of water is $s_w = 4.2 \text{ kJ}/(\text{kg}\cdot^\circ\text{C})$, specific latent heat of fusion of ice is $L = 0.33 \text{ MJ/kg}$ and melting point of ice is $\theta_0 = 0^\circ\text{C}$. Now heat is supplied at a constant rate by a heater to the mixture. The graph shows temperature of the mixture for a portion of a time interval after the heater is switched on.

Assuming heat loss to the surroundings and heat capacity of the calorimeter to be negligible, find power p delivered by the heater to the mixture, time t_1 elapsed in the process of melting of ice and time t_2 elapsed after complete ice melts until temperature reaches $\theta_2 = 20^\circ\text{C}$.

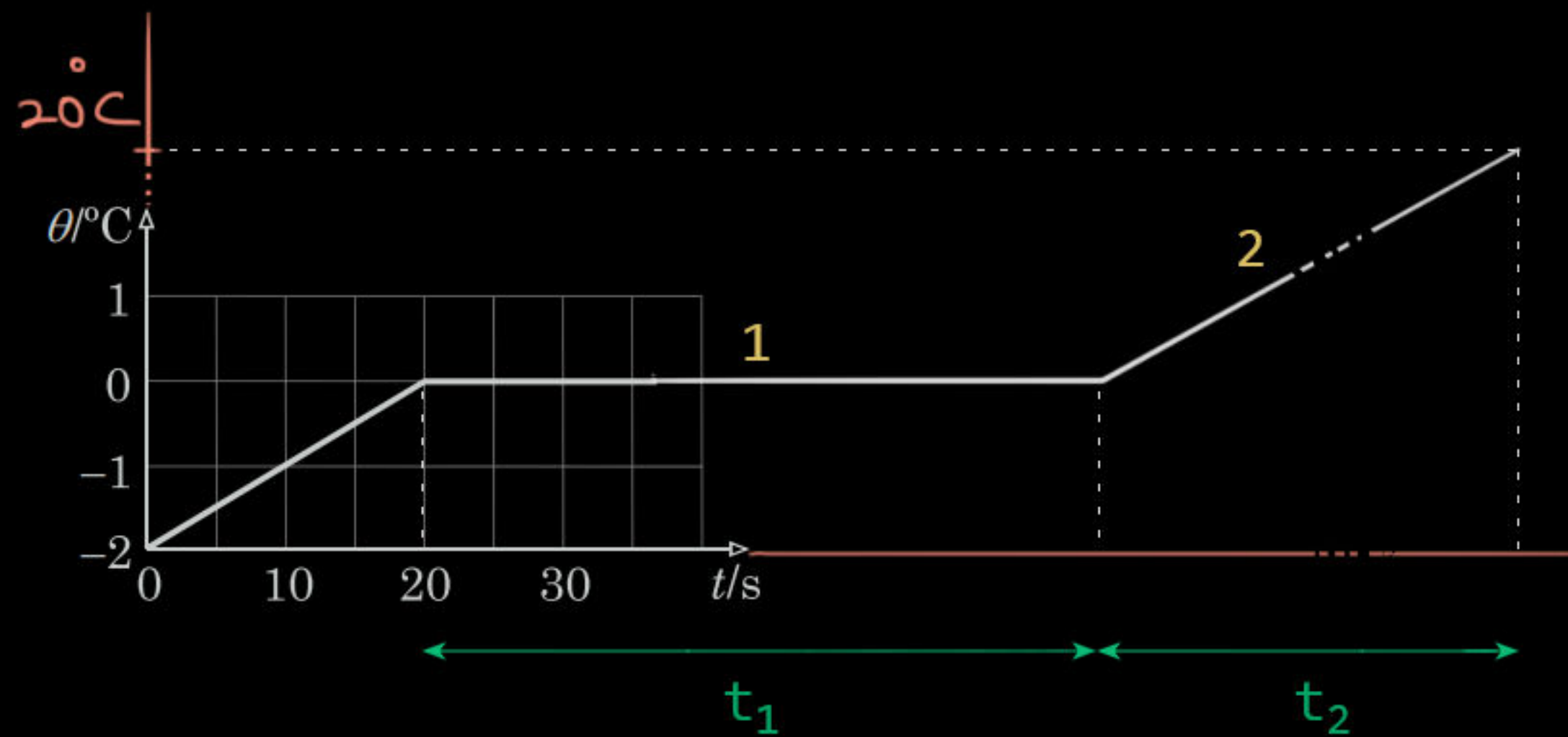
0: Heat absorbed by mixture = Heat released by heater

$$\Rightarrow m_1 s_i (2) + (0.6 - m_1) s_w (2) = 20 p \quad (1)$$

Given: Heat capacity of water = Heat capacity of ice

$$\Rightarrow m_1 s_i = (0.6 - m_1) s_w \Rightarrow m_1 = 0.4 \quad (2)$$

From 1,2: $p = 168 \text{ J/s} //$



1:

$$\begin{array}{l} m_1 \text{ ice } 0^\circ\text{C} \\ 0.6 - m_1 \text{ water } 0^\circ\text{C} \end{array}$$

$\xrightarrow{t_1}$

water 0°C

$$p t_1 = m_1 L$$

$$\Rightarrow t_1 = \frac{m_1 L}{p} = \frac{(0.4) 330 \cdot 10^3}{168} = 785 \text{ s} //$$

2:

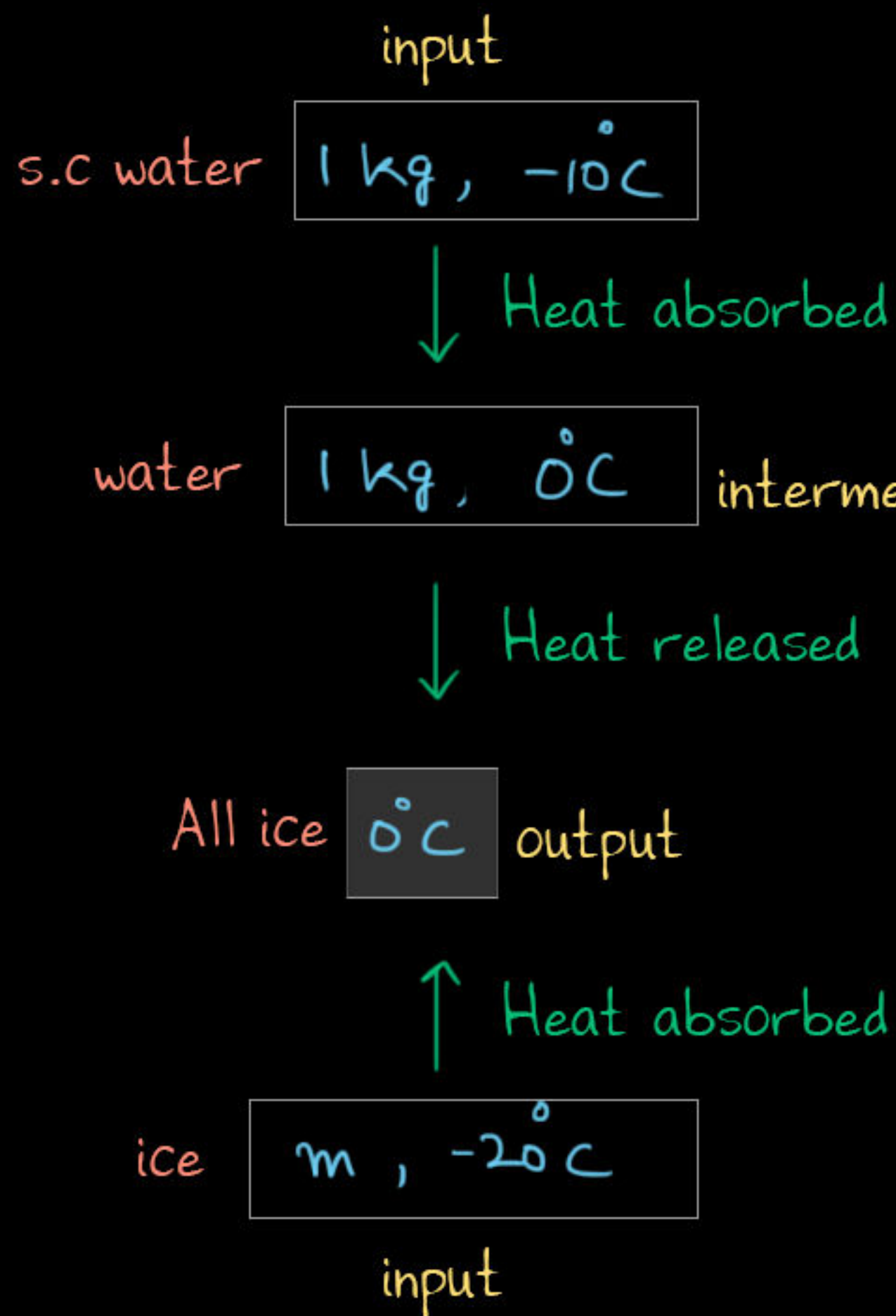
0.6 water 0°C

$\xrightarrow{t_2}$

0.6 water 20°C

$$p t_2 = 0.6 S (20)$$

$$\Rightarrow t_2 = \frac{(0.6) (4200) 20}{168} = 300 \text{ s} //$$



6. You have $m_w = 1.0$ kg of super cooled water at temperature $\theta_w = -10.0$ °C kept in a container and crushed ice at temperature $\theta_i = -20$ °C kept in another container. How much of this ice would you add to the water so that whole water freezes? Specific heat of water is $s_w = 4.2$ kJ/(kg·°C), specific heat of ice is $s_i = 2.1$ kJ/(kg·°C), specific latent heat of melting of ice is $L = 336$ kJ/kg and melting point of ice is $\theta_0 = 0$ °C. Heat capacity of the vessel and heat loss to the surroundings are vanishingly small.

Heat released = Heat absorbed

$$(1) L = (1) s_w (10) + m s_i (20)$$

$$\Rightarrow m = \frac{L - 10 s_w}{20 s_i}$$

$$= \frac{336 - 10(4.2)}{20(2.1)}$$

$$= 7 \text{ kg} //$$

Clearly thermometer itself can store heat.
Let its heat capacity be C .

7. You have two identical small calorimeters and a very accurate thermometer. In one calorimeter is 100 g water at room temperature and in the other is 100 g boiling water. If you put the thermometer in the first calorimeter, it shows 20.00°C . When you remove the thermometer from the first calorimeter and put it in the other, it shows 99.20°C . If you remove the thermometer from the second calorimeter and immediately put it again in the first one, how much would it read? Neglect loss of heat to the environment.

Thermometer 20°C
Water 100 gm, 100°C



99.2°C

Heat released = Heat absorbed

$$100 S_w (100 - 99.2) = C (99.2 - 20) \quad (1)$$

Thermometer 99.2°C
Water 100 gm, 20°C



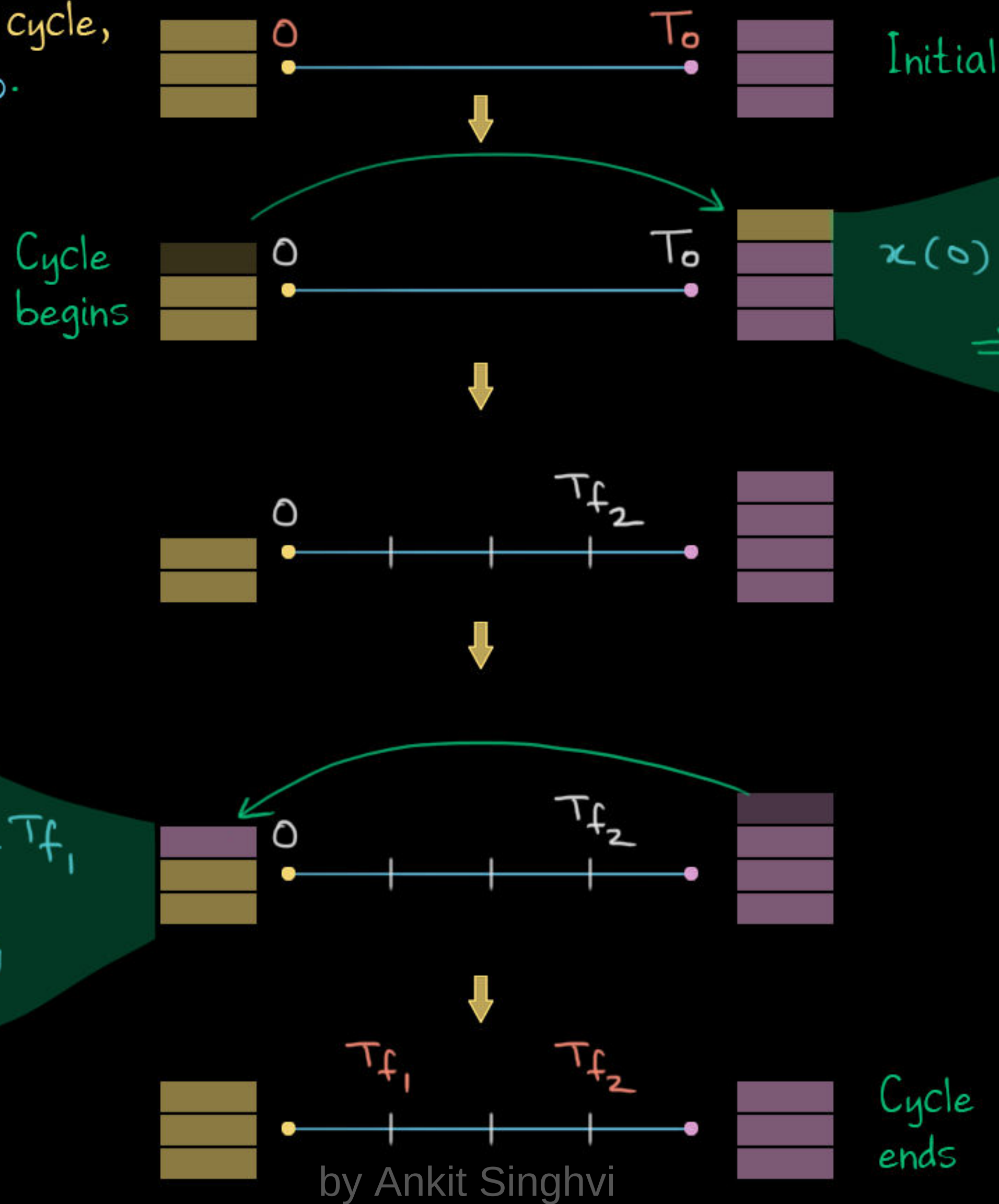
T

$$C (99.2 - T) = 100 S_w (T - 20) \quad (2)$$

Dividing 1,2: $T = 20.8$

Youtube / Arihant Senior

Lets see through one complete cycle,
if temperature difference is T_0 .
Mass of each block be x .



$$E_i = E_f$$

$$x(0) + 3x(T_0) = 4x T_{f2}$$

$$\Rightarrow T_{f2} = 3T_0/4$$

$$x T_{f2} + 2x(0) = 3x T_{f1}$$

$$\Rightarrow T_{f1} = T_0/4$$

$$T_{f2} - T_{f1} = \frac{T_0}{2} //$$

∴ At end of 1st cycle,
 ΔT halves.

⇒ Next cycle it again
halves and so on.

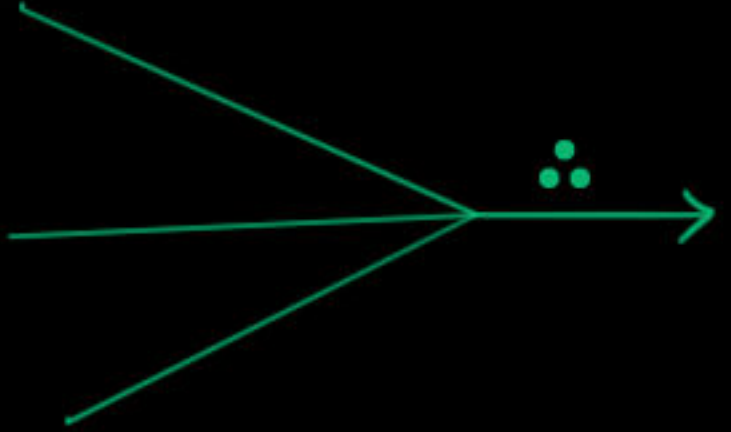
Youtube / Arihant Senior

8. Two calorimeters A and B contain equal amounts of water at temperatures 48°C and 80°C respectively. Mass of water in each of the calorimeters is 300 g . From calorimeter A, 100 g water is transferred into B and stirred until thermal equilibrium is established, then 100 g water is transferred from B to A and stirred until thermal equilibrium is established. How many times this cycle has to be repeated until, the difference between the temperatures of the calorimeters becomes less than 1.0°C . Neglect water equivalent of the calorimeters, work done in transferring the liquid, work done in the stirring and heat loss to the surroundings. (or equal to)

1 Initial $\Delta T = (80 - 48) = 32^\circ\text{C}$

2 At end of every cycle, ΔT halves.

3 Let n be number of cycles that bring the temperature difference less than equal to 1°C .

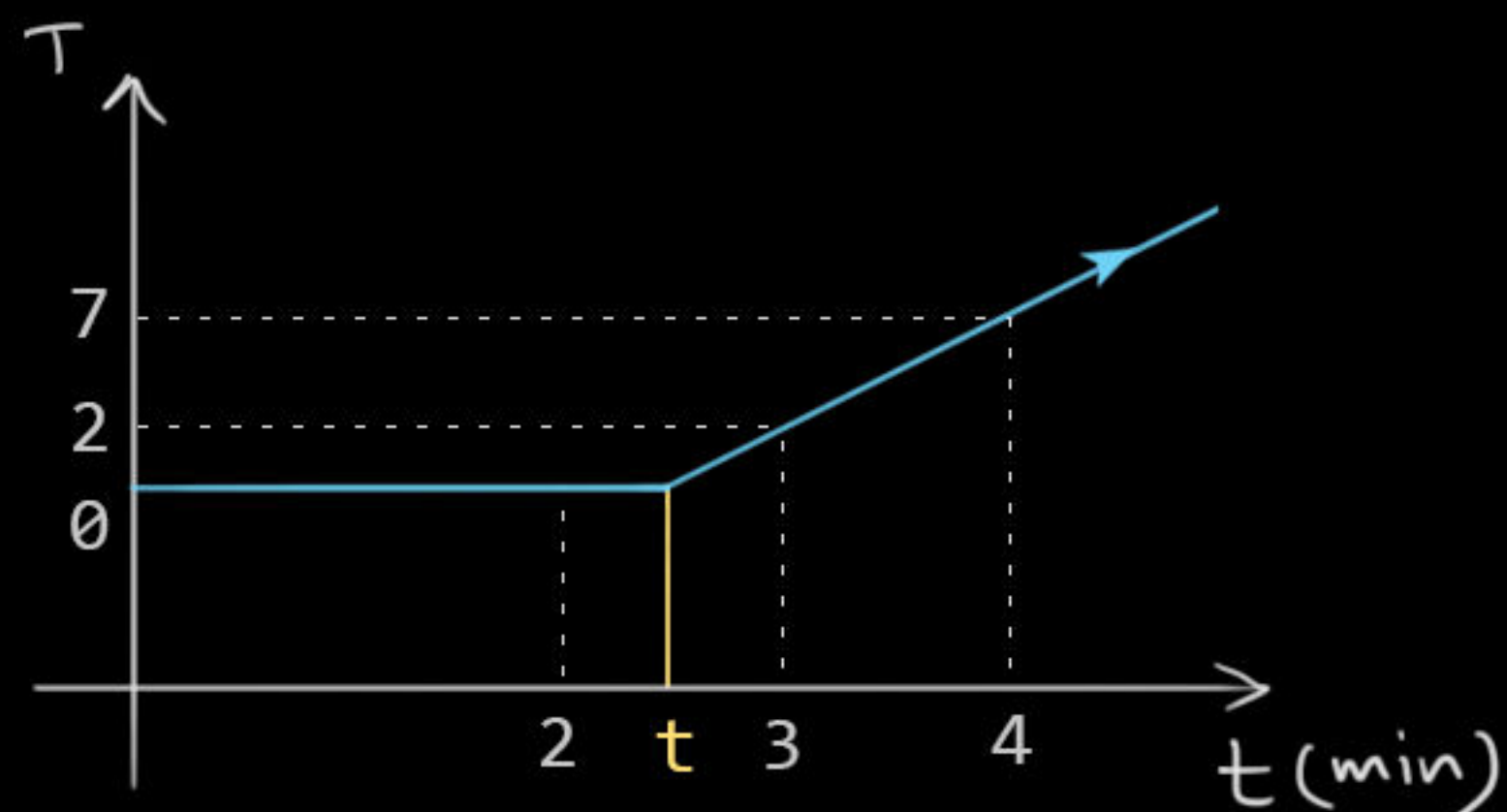

$$\begin{aligned} \therefore \frac{32}{2^n} &\leq 1 \\ \Rightarrow 2^n &\geq 32 \\ \Rightarrow n &\geq 5 \end{aligned}$$

Power of heater be P .

$$\text{and, } M = m_i + m_w$$

9. A mixture of water and ice is kept in a well-insulated calorimeter of negligible heat capacity. A heater, which can supply heat at a constant rate 50 W to the mixture, is switched on. At the ends of second, third and fourth minutes after the heater is switched on, temperature of the mixture becomes 0°C , 2°C and 7°C respectively. How many grams of water and ice were initially present in the calorimeter?

Specific latent heat of melting of ice is 390 J/g and specific heat of water is $4.0 \text{ J/(g}\cdot^\circ\text{C)}$.



If ice melts completely at time t .

$$\Rightarrow \frac{7-2}{4-3} = \frac{7-0}{4-t} \Rightarrow t = 13/5 \text{ min}$$

To melt the ice: $Pt = m_i L$

$$\Rightarrow 50 \left(\frac{13}{5} \cdot 60 \right) = m_i (390)$$

$$\Rightarrow m_i = 20 \text{ gm} //$$

To raise temperature of all water:

$$P(4-3) \cdot 60 = M S_w (7-2)$$

$$\Rightarrow M = 150 \text{ gm}$$

$$\therefore m_w = M - m_i = 130 \text{ gm} //$$

Concept: After lid is opened, at steady state, water will come down to 100°C .

Heat released will be used up in steam/vapour formation also at 100°C .

10. A pressure cooker on a stove contains $m_i = 2.6 \text{ kg}$ of water at temperature $\theta_i = 120^\circ\text{C}$ with its lid and pressure vent both closed. Now the cooker is removed from the stove and the lid is opened. How much mass of water will remain in the cooker after the water stops boiling?

Specific latent heat of vaporization is $L = 2.1 \text{ MJ/kg}$, specific heat of water is $s = 4.2 \text{ kJ/(kg}\cdot^\circ\text{C)}$. Boiling point of water at atmospheric temperature is $\theta_b = 100^\circ\text{C}$. Neglect mass of water evaporated, heat capacity of the cooker and heat loss to the surroundings.

Water $m_i, 120^\circ\text{C}$ input

Heat released

Water $m_i, 100^\circ\text{C}$ intermediate(for calculation)

Heat absorbed

Water $m, 100^\circ\text{C}$

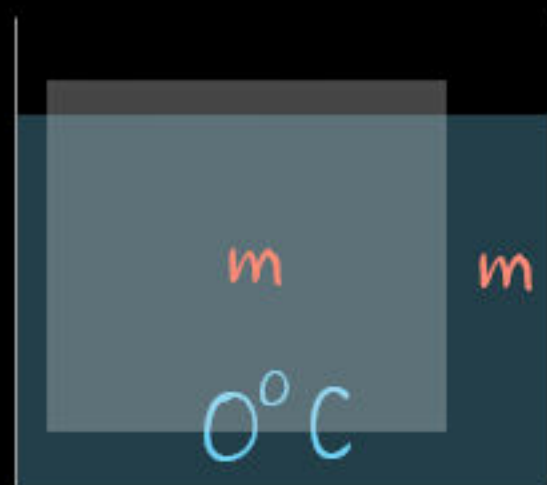
Steam $m_i - m, 100^\circ\text{C}$

output

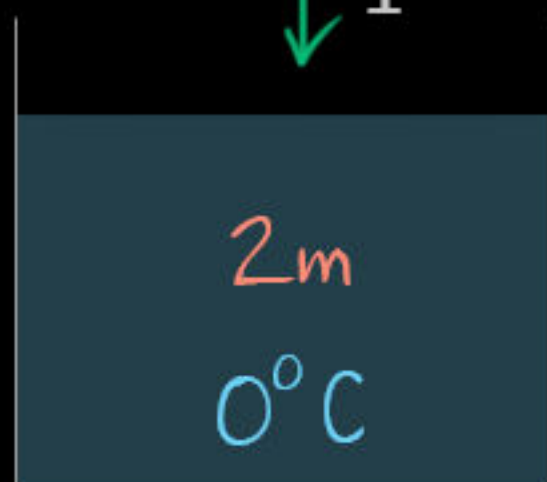
Heat released = Heat absorbed

$$\Rightarrow m_i s_w (120 - 100) = (m_i - m) L$$

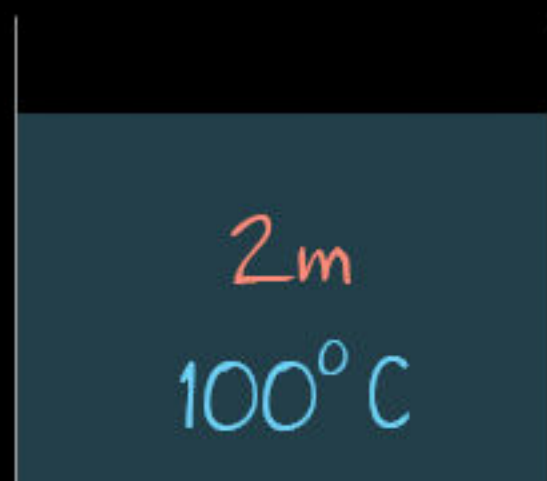
$$\Rightarrow m = \frac{m_i (L - s_w \cdot 20)}{L}$$
$$= 2.5 \text{ kg} //$$



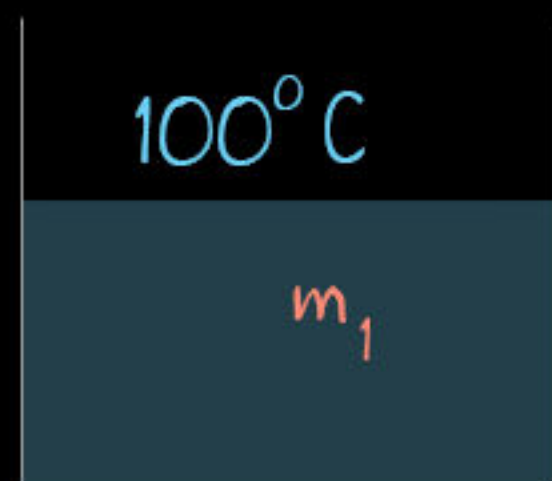
↓ 1



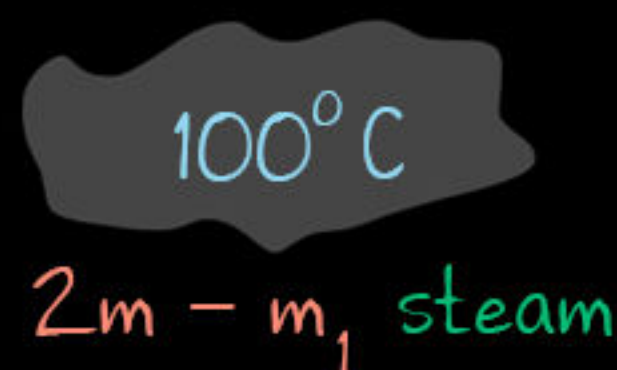
↓ 2



3



+



11. A container of base of area $S = 200 \text{ cm}^2$ and negligible heat capacity contains a mixture of water and ice each of mass $m = 1.0 \text{ kg}$ and placed on a kerosene stove.

Calorific value of kerosene is $q = 7.0 \times 10^4 \text{ kJ/kg}$. Specific heat of water is $s_w = 4.2 \times 10^3 \text{ J/(kg} \cdot ^\circ\text{C)}$, specific latent heat of melting of ice is $L_i = 3.5 \times 10^5 \text{ J/kg}$, specific latent heat of vaporization of water is $L_v = 2.5 \times 10^6 \text{ J/kg}$, density of water is $\rho = 1000 \text{ kg/m}^3$. Boiling point of water is $\theta_b = 100^\circ\text{C}$ and melting point of ice is $\theta_m = 0^\circ\text{C}$.

If $\eta = 50\%$ of the heat obtained from burning the kerosene is transferred to the mixture, find height of water level in the container after burning $m_k = 34 \text{ g}$ of kerosene.

$$Q_{in} = \overset{1}{m}L + \overset{2}{2m} s_w (100 - 0) + \overset{3}{(2m - m_1)} L$$

$$\Rightarrow \frac{1}{2} \left(\frac{34}{1000} \right) 7 \cdot 10^4 \cdot 10^3 = (1) 3.5 \times 10^5 + 2(1) 4.2 \times 10^3 100 + (2 - m_1) 2.5 \times 10^6$$

$$\Rightarrow m_1 = 2 \text{ kg} \parallel (\because \text{Steam doesn't form})$$

$$\text{Height} = \frac{V}{A} = \frac{m}{\rho A} = \frac{2}{1000 \cdot 200 \times 10^{-4}} = 0.1 \text{ m} \parallel$$

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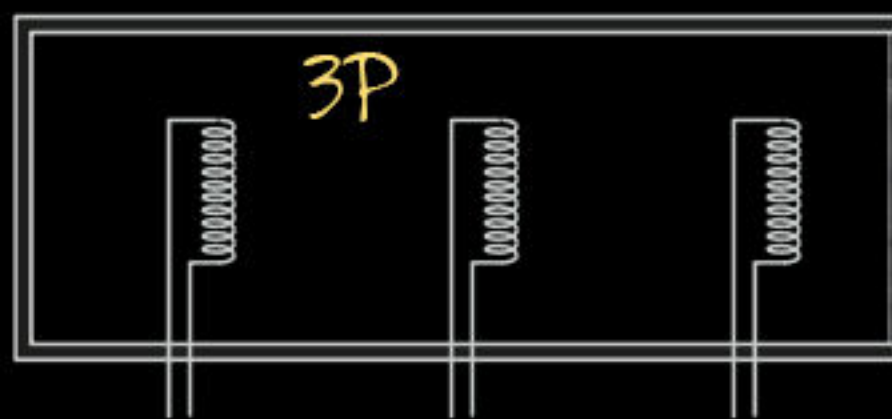


Figure-I

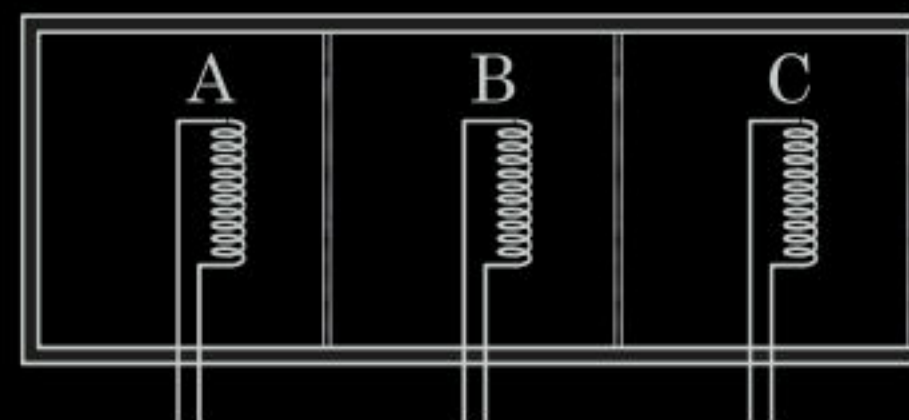
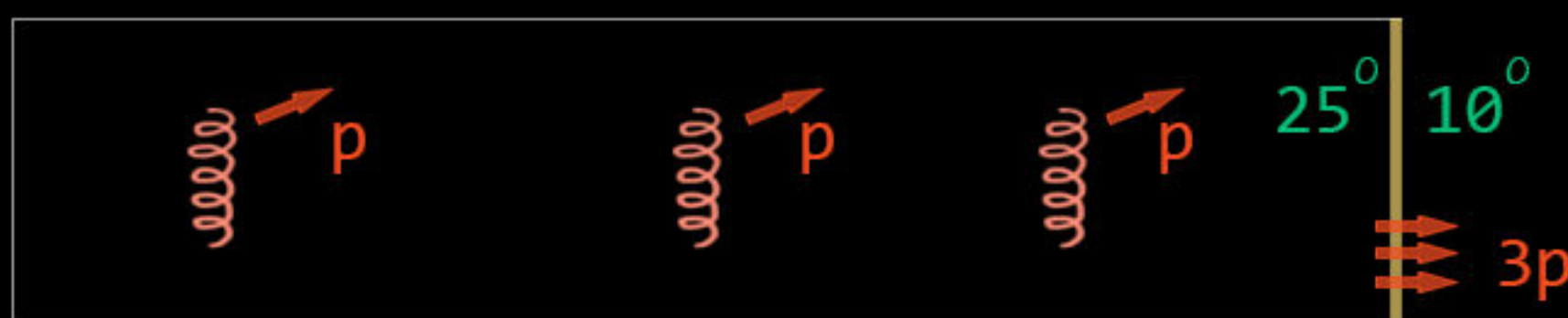


Figure-II

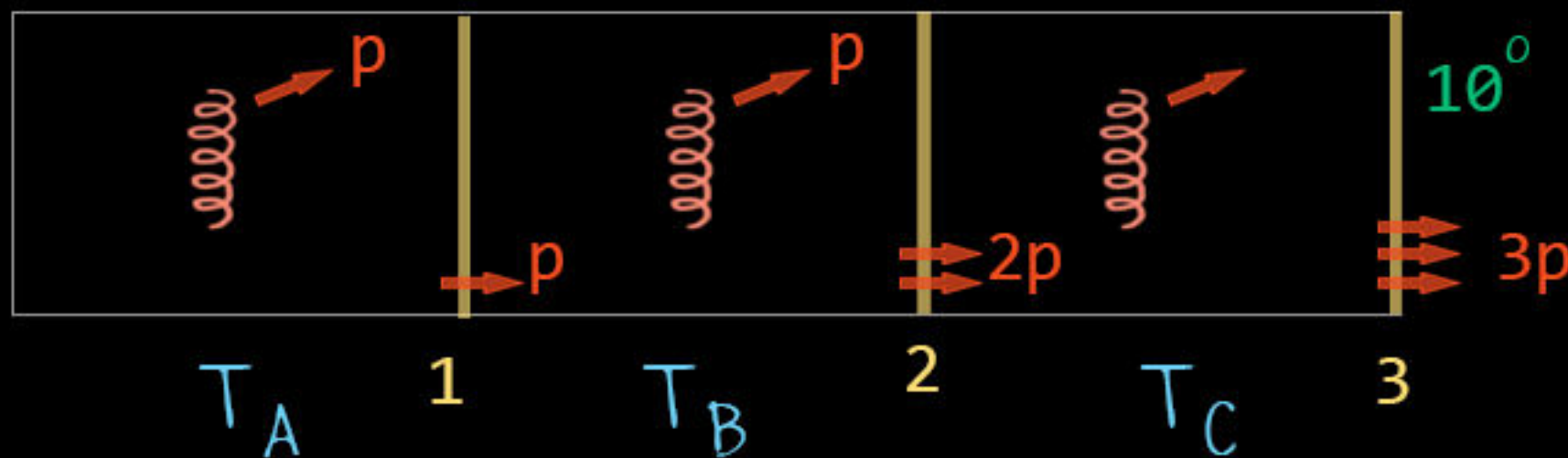
12. Three identical heaters are installed in a chamber that is made of heat insulating walls and is open at one side. The open side is covered with a heat conducting membrane as shown in the figure-I. Outside temperature is 10°C . If all the heaters are switched on, steady-state temperature inside the chamber becomes 25°C . If two more membranes that are similar to the first one were placed between the heaters as shown in the figure-II, what would be the steady-state temperatures of the parts A, B and C of the chamber?

Let power of one heater be p .

For membrane, let: $\frac{dQ}{dt} = c \Delta T$



$$3p = c(25 - 10) \Rightarrow \frac{p}{c} = 5$$



$$3: 3p = c(T_C - 10) \Rightarrow T_C = 25^\circ$$

$$2: 2p = c(T_B - T_C) \Rightarrow T_B = 35^\circ$$

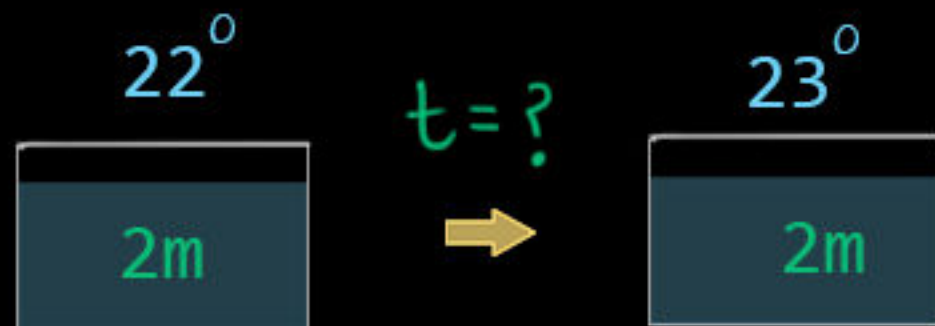
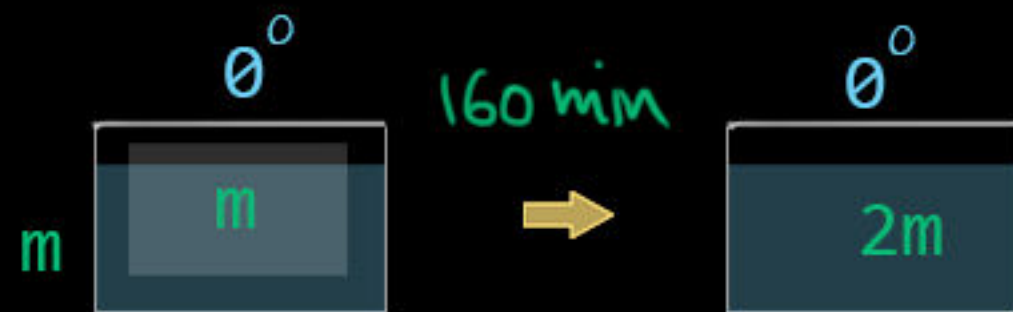
$$1: p = c(T_A - T_B) \Rightarrow T_A = 40^\circ$$

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13. A closed bowl containing a mixture of equal amounts of water and ice at temperature $\theta_0 = 0^\circ\text{C}$, when left in a room, the whole ice melts in time $\Delta t_0 = 160\text{ min}$. Room temperature is $\theta_r = 25^\circ\text{C}$, specific heat of water is $s_w = 4.2\text{ kJ/(kg}\cdot\text{K)}$ and specific latent heat of melting of ice is $L = 320\text{ kJ/kg}$. If the bowl were further left in the room undisturbed, how long would it take to increase temperature of water in the bowl from $\theta_1 = 22^\circ\text{C}$ to $\theta_2 = 23^\circ\text{C}$?

Room: 25°



Newtons law of cooling on closed bowl: $\frac{d\theta}{dt} = k \Delta T$

$$mL = k(25-0)\Delta t_1 \Rightarrow m(320) = k(25)160 \Rightarrow \frac{m}{k} = \frac{25}{2}$$

$$2ms(T_2 - T_1) = k \left[25 - \left(\frac{22+23}{2} \right) \right] \Delta t_2$$

$$\Rightarrow \Delta t_2 = \left(\frac{m}{k} \right) \frac{2s(T_2 - T_1)}{2.5} = \frac{10}{2.5} \times \frac{(4.2)(1)}{2.5} = 42\text{ min} //$$

As T_1 and T_2 are close, $T(t)$ can be approximated to straight line. And,

$$\text{Area} \int T(t) dt \approx T(t)_{av} \Delta t$$

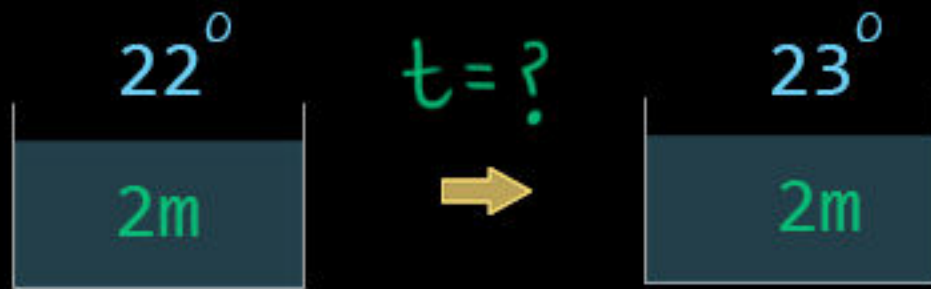
Cant use this approximation within 1° of 25° . M 1

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Method 2 Accurate answer:

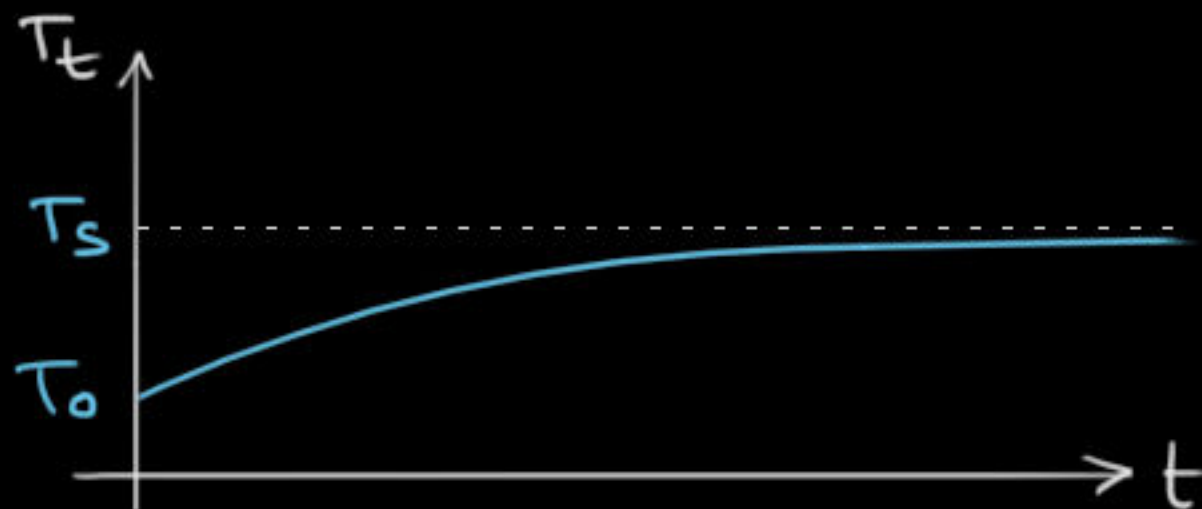
$$\frac{m}{k} = \frac{25}{2}$$



General variation of T with time.

$$\frac{T_t - T_s}{T_0 - T_s} = e^{-kt/ms}$$

$$\Rightarrow T_t = T_s + (T_0 - T_s)e^{-kt/ms}$$



$$-2ms \frac{dT}{dt} = k(T - 25)$$
$$-2ms \int_{T_1}^{T_2} \frac{dT}{T - 25} = \int_0^{\Delta t_2} dt$$

$$\Rightarrow -2ms \ln(T - 25) \Big|_{T_1}^{T_2} = \Delta t_2$$

$$\Rightarrow \Delta t_2 = \frac{2ms}{k} \ln \left(\frac{T_1 - 25}{T_2 - 25} \right)$$

$$\downarrow T_1 = 22, T_2 = 23$$

$$= \cancel{2} \left(\frac{25}{\cancel{2}} \right) (4.2) \ln \left(\frac{3}{2} \right)$$

$$= 42.6 \text{ min} //$$

14. Boiling water filled up to the brim in a tub cools to a desired temperature in a time interval $\Delta t_0 = 20 \text{ min}$. How long would it take to cool boiling water filled up to the brim in another tub of similar shape having linear dimensions $\eta = 8 \text{ times}$ of those of the former to the same desired temperature? Consider walls of the tub perfect insulator of heat.

Newton's law of cooling on all similarly shaped tubs:

$$-\frac{dQ}{dt} = kA(T - T_s)$$

$$\Rightarrow -ms \frac{dT}{dt} = kA(T - T_s)$$

$$\Rightarrow -\frac{s}{k} \int_{T_1}^{T_2} \frac{dT}{T - T_s} = \frac{A}{m} \int_0^{\Delta t} dt$$

$$\Rightarrow \underbrace{\text{constant}} = \frac{A \Delta t}{m}$$

$$\therefore \frac{A_1 \Delta t_1}{m_1} = \frac{A_2 \Delta t_2}{m_2}$$

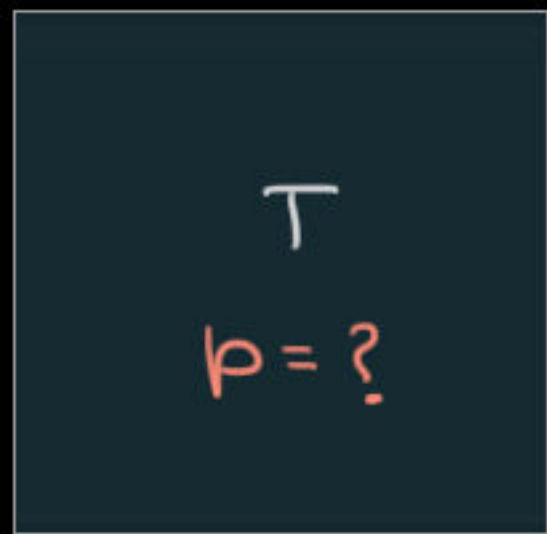
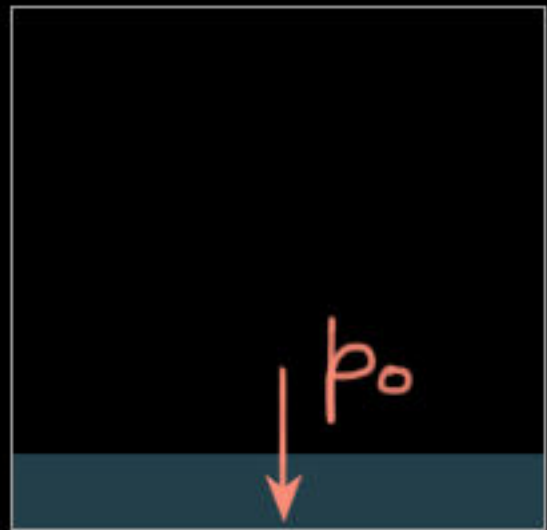
$$\Rightarrow \Delta t_2 = \left(\frac{m_2}{m_1}\right) \left(\frac{A_1}{A_2}\right) \Delta t_1$$

$$= \eta^3 \cdot \frac{1}{\eta^2} \cdot \Delta t_1$$

$$= \eta \Delta t_1 //$$

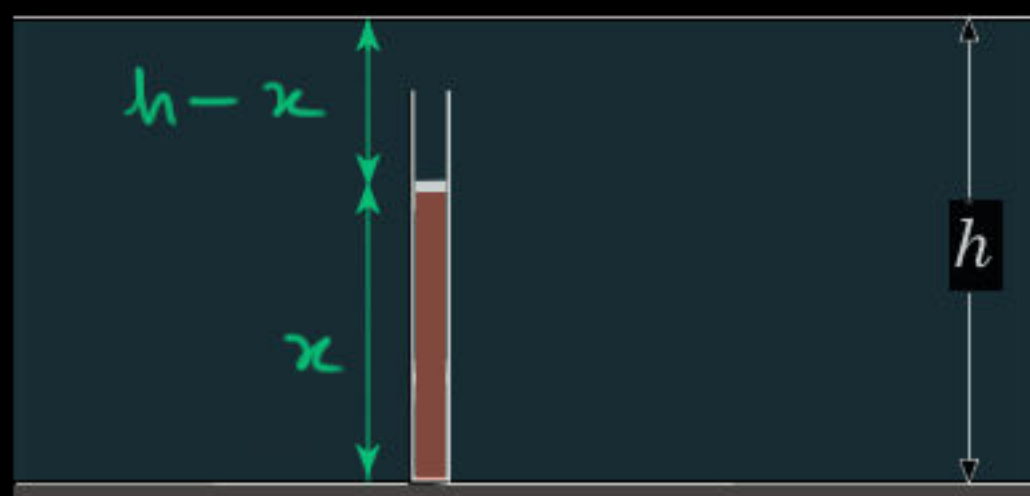
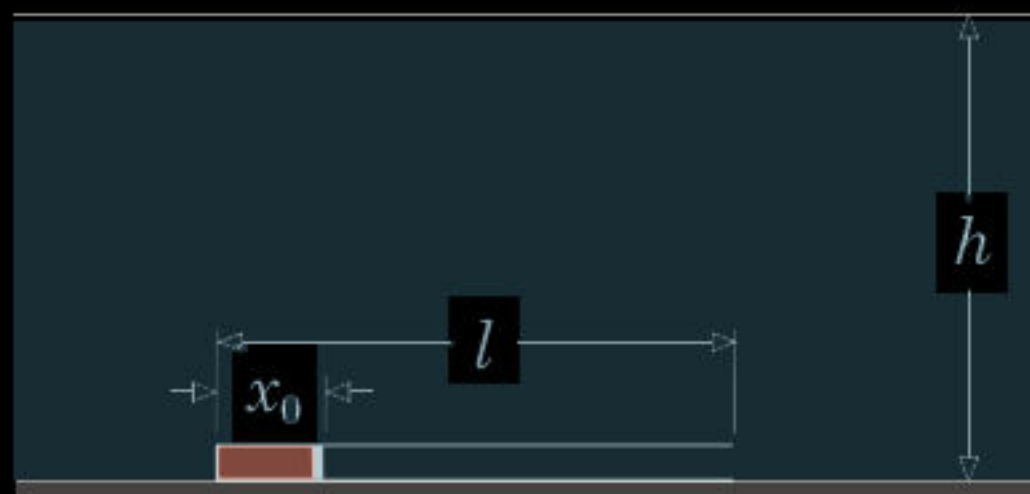
15. Argon in an airtight metal cube of volume V exerts a pressure p_0 only at the bottom at a very low temperature. Find the force exerted on the top of the cube at a temperature T when all the argon is in gaseous state. Acceleration due to gravity is g , molar mass of argon is M and universal gas constant is R .

Its liquid very low temperature.



$$\therefore p_0 = \frac{mg}{A}$$

$$p = \frac{\rho RT}{M} = \left(\frac{m}{V}\right) \frac{RT}{M} = \frac{p_0 A}{gV} \frac{RT}{M} \xrightarrow[A \rightarrow l^2]{V \rightarrow l^3} = \frac{p_0 RT}{Mg l} = \frac{p_0 RT}{Mg V^{1/3}}$$



16. Thin horizontal straight metal pipe of length $l = 80$ m is lying at bottom of a lake that is $h = 100$ m deep. One end of the pipe is closed and an air packet of length $x_0 = 9.0$ m is trapped in it between the closed end and a light piston. The pipe is now slowly rotated about its closed end. At what height above the closed end will the piston be when the tube becomes vertical? In your calculations, neglect the atmospheric pressure.

$$T_1 = T_2 \quad \because \text{metal pipe}$$

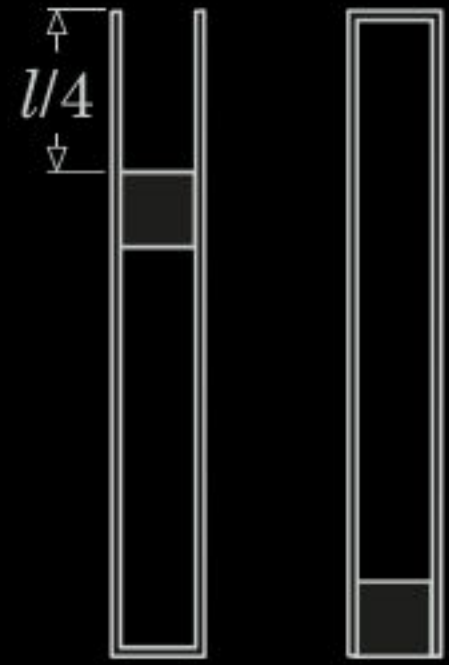
$$\Rightarrow P_1 V_1 = P_2 V_2$$

$$\cancel{\rho} \cancel{g} h \cancel{A} x_0 = \cancel{\rho} \cancel{g} (h-x) \cancel{A} x$$

$$\Rightarrow x^2 - hx + hx_0 = 0$$

$$\therefore x = 10, 90 \quad \nearrow \text{not possible as pipe is not that long}$$

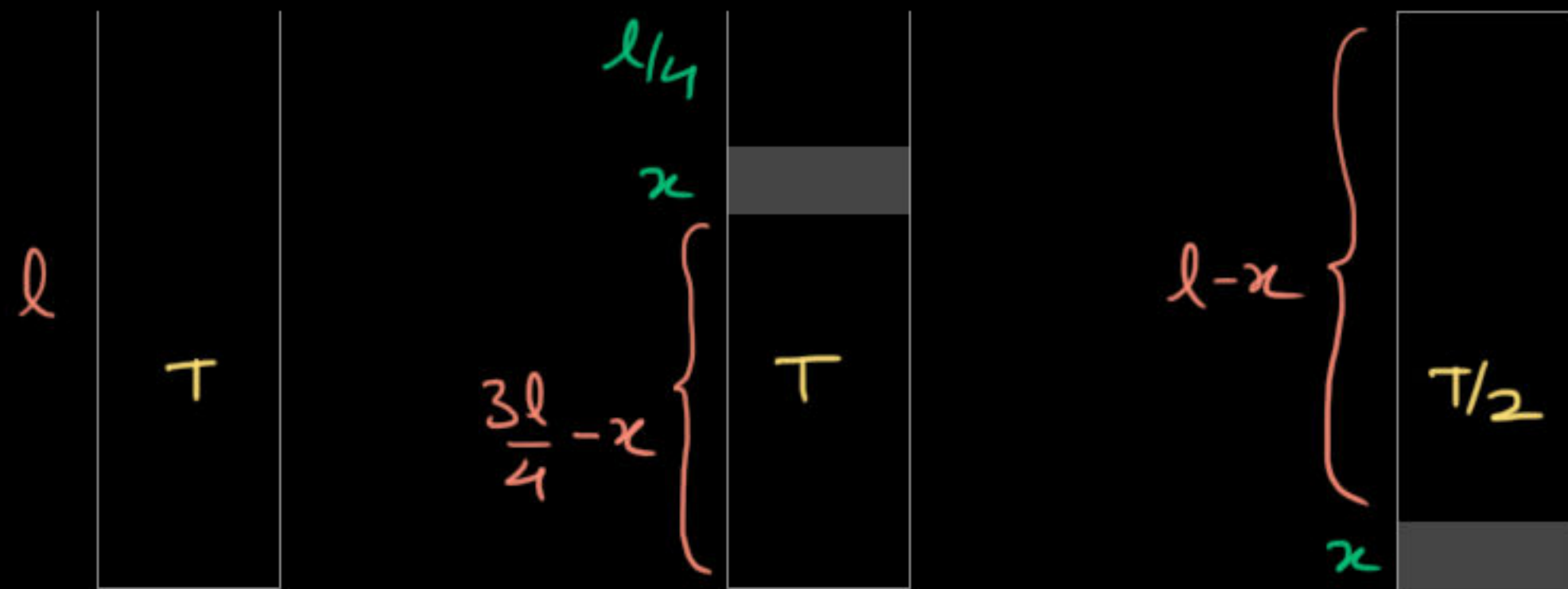
Two 'corrections' to get the desired answer



17. A tube of length l closed at lower end is held vertically. If an airtight piston is placed on its mouth, the piston slides down without friction and stops with its upper face at a depth $l/4$ from the mouth of the tube. If the temperature of air ~~inside~~ the tube is halved and the tube is inverted, the piston will settle with its lower face in level with the mouth of the tube as shown in the figure. Find thickness of the piston.

Conducting Outside

Solving 2 equations, 2 variables



$$\frac{P_1 V_1}{T} = \frac{P_2 V_2}{T} = \frac{P_3 V_3}{T/2}$$

$$\Rightarrow P_0 l A = \left(P_0 + \frac{mg}{A}\right) \left(\frac{3l}{4} - x\right) A = 2 \left(P_0 - \frac{mg}{A}\right) (l - x) A$$

$$l = \left(1 + \frac{mg}{P_0 A}\right) \left(\frac{3l}{4} - x\right) = 2 \left(1 - \frac{mg}{P_0 A}\right) (l - x)$$

$$\Rightarrow l = (1 + y) \left(\frac{3l}{4} - x\right) = 2(1 - y)(l - x)$$

1 2

From 1,2:

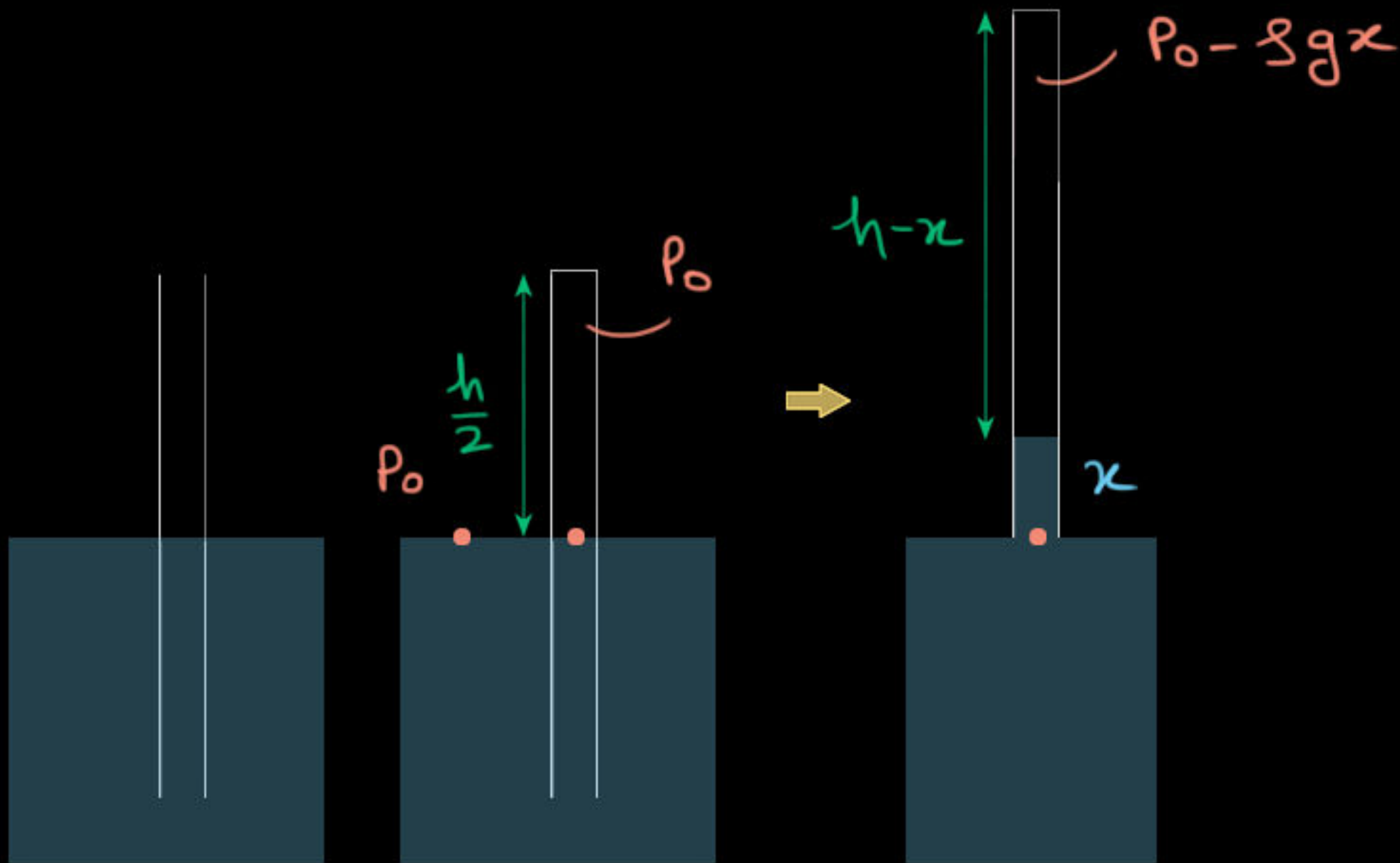
Reject as x can't be greater than $3l/4$

$$x = \frac{2 - \sqrt{3}}{4} l$$

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18. A glass tube of length h open at both the ends is held vertically so that half of its length is submerged in mercury kept in a large container. If the top end of the tube is closed and the tube is slowly pulled out, how much length of mercury column will be left in the tube? The atmospheric pressure is equal to the pressure of the column of mercury of height h_0 . Assume constant temperature and neglect capillary effects.

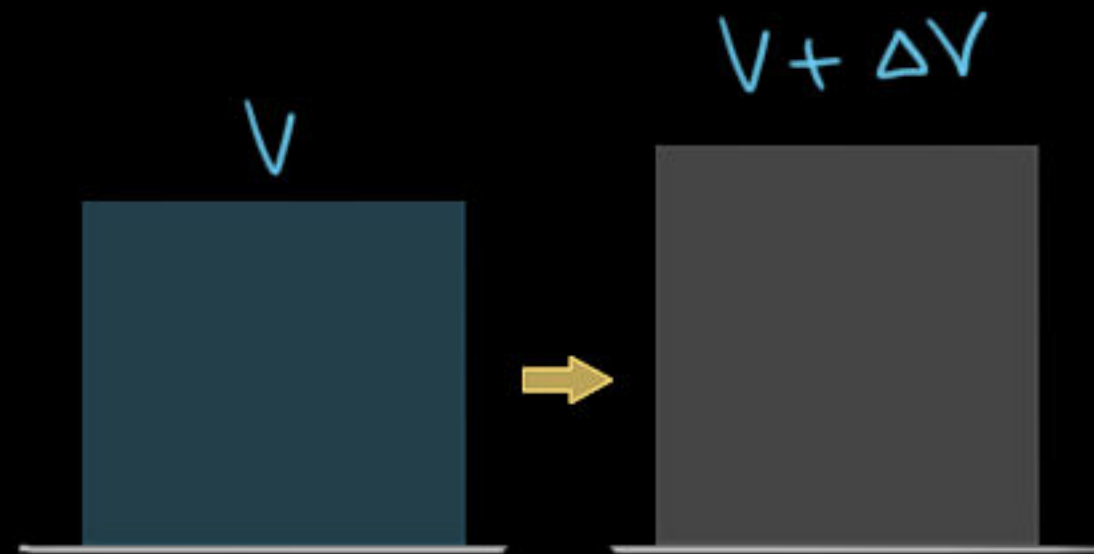


$$P_1 V_1 = P_2 V_2$$

$$\Rightarrow P_0 \frac{h}{2} = (P_0 - \rho g x) (h - x)$$

$$\Rightarrow x = \frac{(h + h_0) - \sqrt{h^2 + h_0^2}}{2}$$

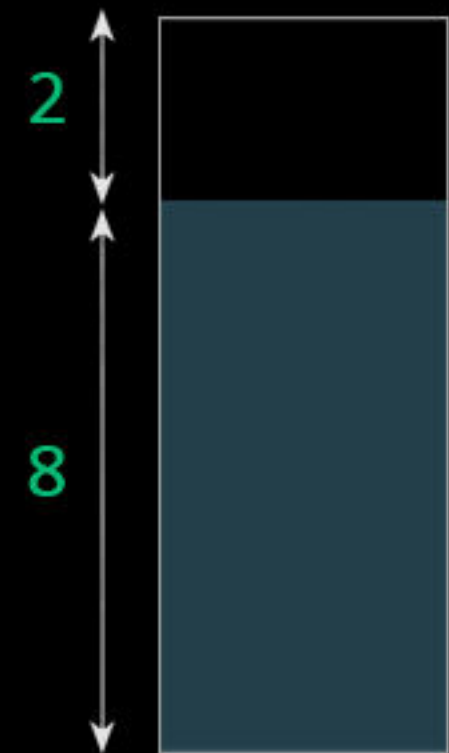
When water freezes:



$$\begin{aligned}
 \Delta V &= \frac{m}{\rho_i} - \frac{m}{\rho_w} \\
 &= \rho_w V \left(\frac{1}{\rho_i} - \frac{1}{\rho_w} \right) \\
 &= V \left(\frac{\rho_w}{\rho_i} - 1 \right) \\
 &= V \left(\frac{10}{9} - 1 \right) = \frac{V}{9}
 \end{aligned}$$

19. Water of volume $V_w = 0.8 \text{ L}$ and dry air at atmospheric pressure $p_0 = 1.03 \times 10^5 \text{ Pa}$ are present at temperature $T_1 = 309 \text{ K}$ in a closed vessel of volume $V = 1.0 \text{ L}$. A thin layer of oil separates the water from the air and prevents evaporation of water. Density of water is $\rho_w = 1.0 \text{ g/cm}^3$ and density of ice is $\rho_i = 0.9 \text{ g/cm}^3$. If temperature in the container is reduced and maintained at a value $T_2 = 240 \text{ K}$, find air pressure after all the water freezes.

Sol:



$$2 - 8 \left(\frac{1}{9} \right) = \frac{10}{9}$$

$$\begin{aligned}
 \frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\
 \Rightarrow \frac{1.03 \times 10^5 \times 2}{309} &= \frac{p_2 \times \frac{10}{9}}{240}
 \end{aligned}$$

$$\Rightarrow p_2 = 1.44 \times 10^5 \text{ Pa} //$$

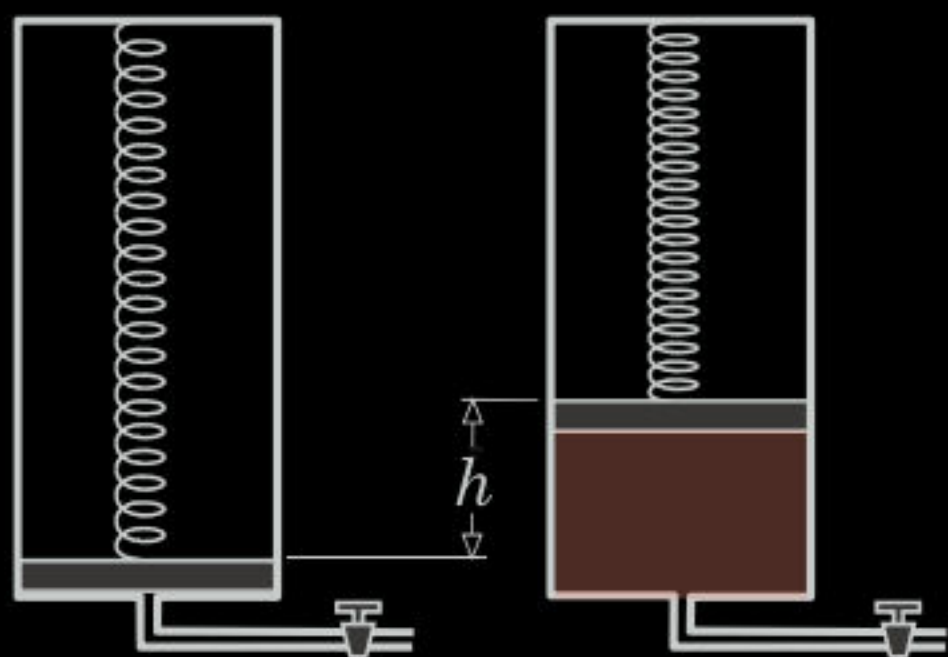
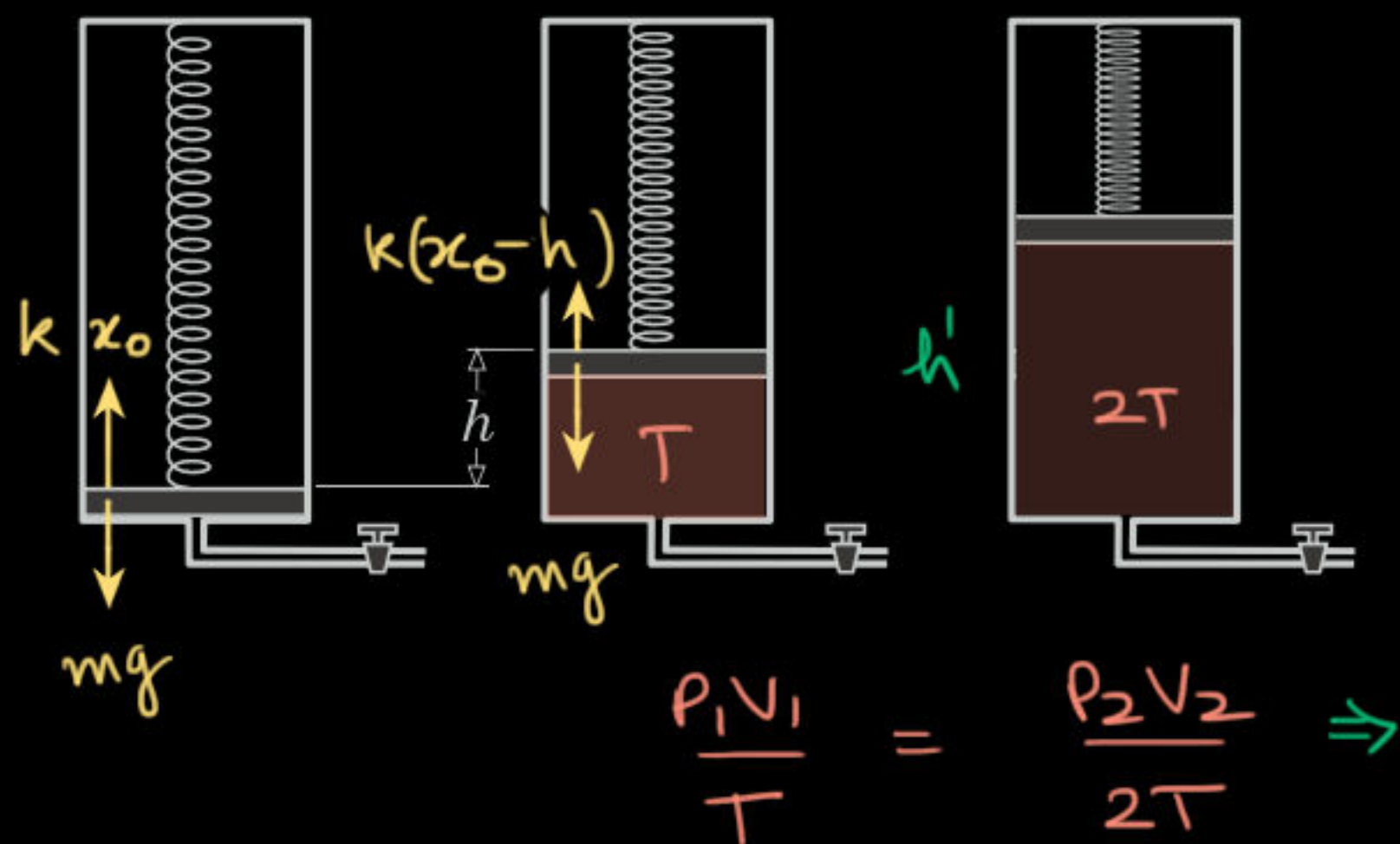


Figure-I

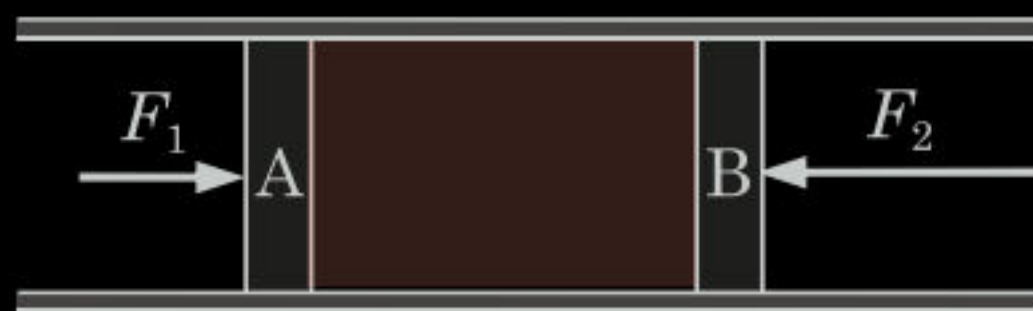
Figure-II

20. A thin tube equipped with a valve is connected at the bottom of a vertical cylinder. A heavy piston is suspended from the top of the cylinder with the help of a spring and the space above the piston is evacuated. If all the air below the piston is pumped out, the piston touches the bottom without exerting any force on the bottom as shown in the figure-I. Now an ideal gas at temperature T is introduced under the piston through the tube until the piston slowly rises to a height h and then the valve is closed as shown in figure-II. What will be the height of the piston above the bottom of the container, if temperature of the gas is increased to $2T$? Assume that the piston moves without friction and that the spring obeys Hooke's law.

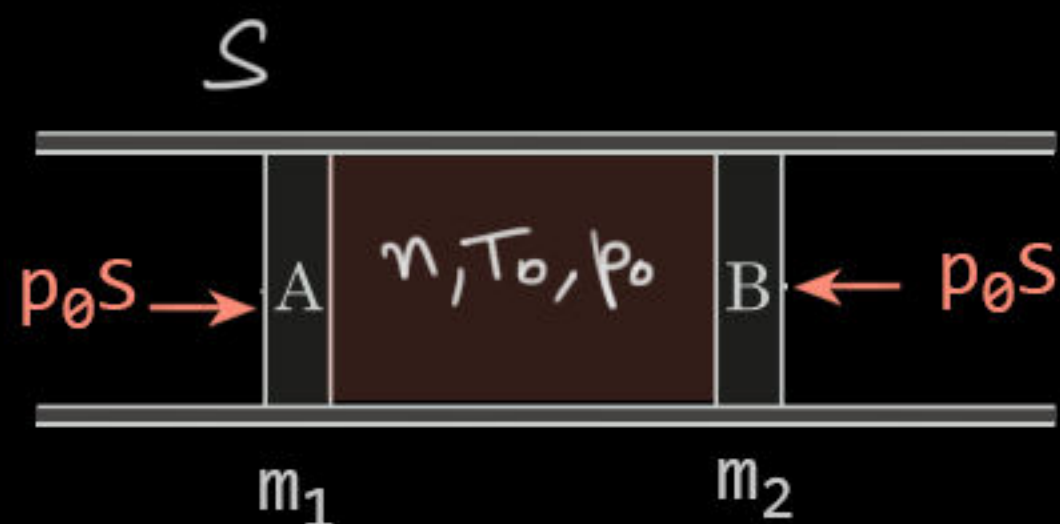


Concept: When piston rises to h , Net force by piston spring assembly will be kh downwards.

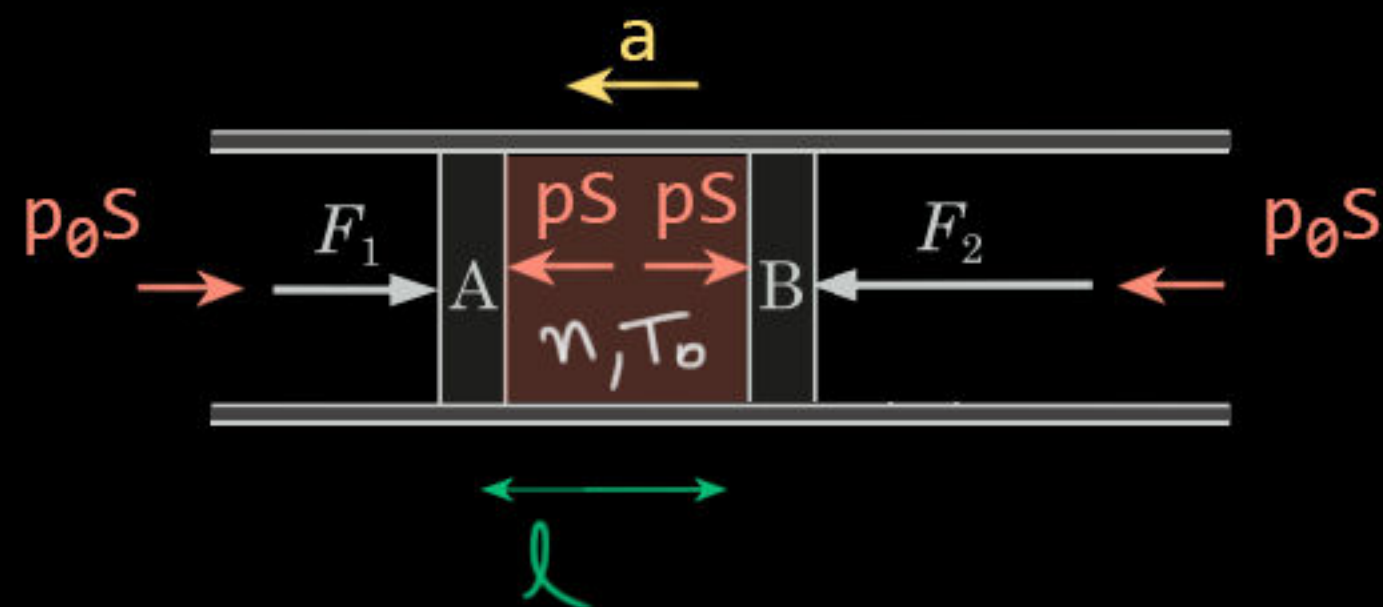
$$\frac{\cancel{k}h}{\cancel{A}} \cdot \cancel{A}h = \frac{\cancel{k}h'}{\cancel{A}} \cdot \cancel{A}h' \Rightarrow h' = \sqrt{2}h //$$



21. Two pistons A and B of masses m_1 and m_2 trapping n moles of an ideal gas between them can slide without friction in a long fixed horizontal pipe of cross-section area S . The pipe and the pistons are perfect conductors of heat; mass of the gas is negligible, outside pressure is p_0 and temperature is T_0 . If constant forces F_1 and F_2 are applied on the pistons along the axis of the cylinder as shown in the figure, what will the distance between the pistons ultimately become?



At steady state let pressure be p .



$$a = \frac{\text{first piston's } pS - (F_1 + p_0S)}{m_1} = \frac{\text{second piston's } (F_2 + p_0S) - pS}{m_2} \quad (1)$$

$$p(Sl) = nRT_0 \quad (2)$$

$$\text{From 1, 2: } l = \frac{nRT_0(m_1 + m_2)}{m_1F_2 + (m_1 + m_2)p_0S + m_2F_1}$$

Frequency of collisions
per unit area

$$f = \frac{1}{4} N/V \langle v \rangle$$

$$N/V$$

Molecules
per unit volume.

$$\sqrt{\frac{8RT}{\pi M}}$$

Proof: out of scope.

Same concept in MCQ 18.

22. An ideal gas is trapped in a stationary container with the help of a piston and there is a small hole in the container through which the gas leaks. If by heating the gas and simultaneously pushing the piston into container, temperature and pressure of the gas is increased to 4 times and 8 times respectively of their initial values, how many times does the rate of leakage of the gas (i.e. number of molecules leaking per second) change?

$$\text{Frequency of collisions per unit area} = \frac{\text{Leakage}}{\text{Area}} \propto \frac{\sqrt{T}}{V} \propto \frac{\sqrt{T}}{T/P} \propto \frac{P}{\sqrt{T}}$$

$$\therefore \frac{L_2}{L_1} = \frac{P_2}{P_1} \sqrt{\frac{T_1}{T_2}} = \frac{8}{1} \sqrt{\frac{1}{4}} = 4 //$$

Lets start with 1 kg of gas.
So heat needed to raise the temperature by 1°C is required specific heat.

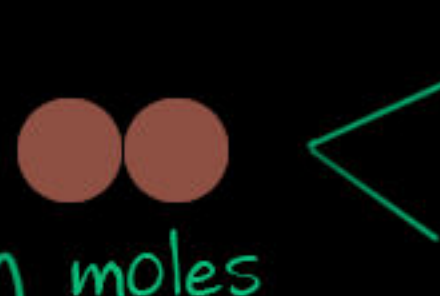
Lets say it has n moles.

23. A diatomic gas is filled in a closed container of constant volume. If temperature of the gas is increased by a significant amount, some of the molecules dissociate into atoms. Do not consider vibrational degrees of freedom while answering the following questions.

- (a) In what way will the specific heat of the mixture change?
(b) If the specific heat changes by 8%, what fraction of initial amount of the diatomic gas has been dissociated?

Before dissociation  n moles

$$\text{Specific heat} = Q_1 = n C_v \Delta T = n \left(\frac{5R}{2} \right) \Delta T = \frac{5nR}{2} \Delta T$$

After η fraction dissociates:  n moles
  $(1-\eta)n$ moles
  $\eta(2n)$ moles

$$\begin{aligned} \text{Specific heat} = Q_2 &= (1-\eta)n \left(\frac{5R}{2} \right) \Delta T + \eta(2n) \left(\frac{3R}{2} \right) \Delta T \\ &= (1-\eta) \frac{5nR}{2} + \eta \frac{6nR}{2} \end{aligned}$$

$$\therefore \frac{Q_2}{Q_1} = \frac{(1-\eta)5 + 6\eta}{5}$$

$$= 1 - \eta + 1.2\eta$$

$$= 1 + 0.2\eta$$

A $Q_2 > Q_1$

B $1.08 = 1 + 0.2\eta$

$$\Rightarrow \eta = 0.4 //$$

24. A diatomic gas is filled in an adiabatic container at temperature T_1 . At this temperature molecules of the gas begin to disassociate. If each molecule absorbs energy ϵ in disassociation and disassociation process terminates at temperature T_2 , find ratio of final pressure to initial pressure of the gas.

Let η fraction of N dissociate



$$U_i = N \left(\frac{5k}{2} \right) T_1$$

$$U_f = 2\eta N \left(\frac{3k}{2} \right) T_2 + (1-\eta)N \left(\frac{5k}{2} \right) T_2$$

Also,

$$\frac{P_1 V_1}{N_1 T_1} = \frac{P_2 V_2}{N_2 T_2}$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{N_2 T_2}{N_1 T_1}$$

$$= \frac{2\eta N + (1-\eta)N}{N} \frac{T_2}{T_1}$$

$$= (\eta + 1) \frac{T_2}{T_1} //$$

Energy absorbed from U_i .
So U_i reduces to U_f .

$$U_i - E = U_f$$

$$\Rightarrow \eta = \frac{5(T_1 - T_2)k}{2\epsilon + kT_2}$$



25. In a long fixed horizontal pipe two identical pistons A and B each of mass $m = 415 \text{ g}$ can slide without friction. The pipe and the pistons are made of perfect heat insulating materials. The space between the pistons is filled with $n = 0.1 \text{ mole}$ of gaseous helium. The piston A is given an initial velocity $u = 12 \text{ m/s}$ towards the piston B. Find the maximum temperature change of the gas in ensuing motion of the pistons.

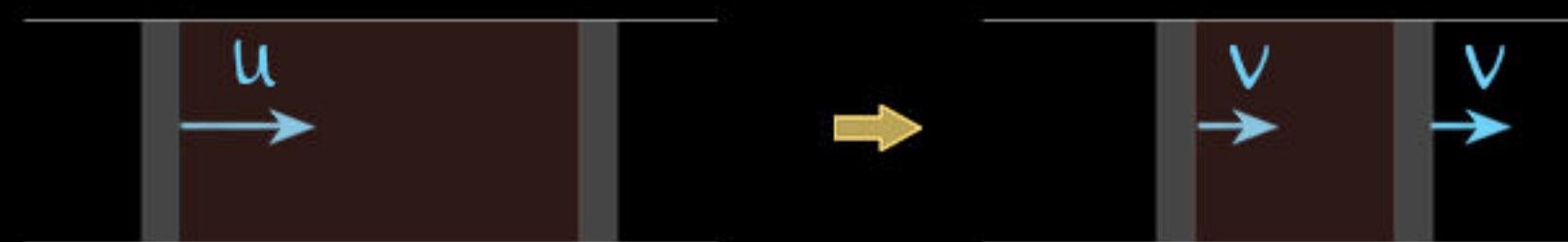
$$nC_V \Delta T = -\int P dV \quad (\because \Delta Q = 0)$$

→ Temperature keeps increasing as long as dV is -ve.

→ i.e, until volume keeps decreasing.

→ or pistons are closest

→ or pistons are moving with same velocity.



$$E: \frac{1}{2} m u^2 = \Delta U + 2 \left(\frac{1}{2} m v^2 \right) \quad (1)$$

$$P: \cancel{m} u = 2 \cancel{m} v \quad (2)$$

$$\text{From 1,2: } \Delta U = \frac{m u^2}{4} = n C_V \Delta T \quad \rightarrow 3R/2$$

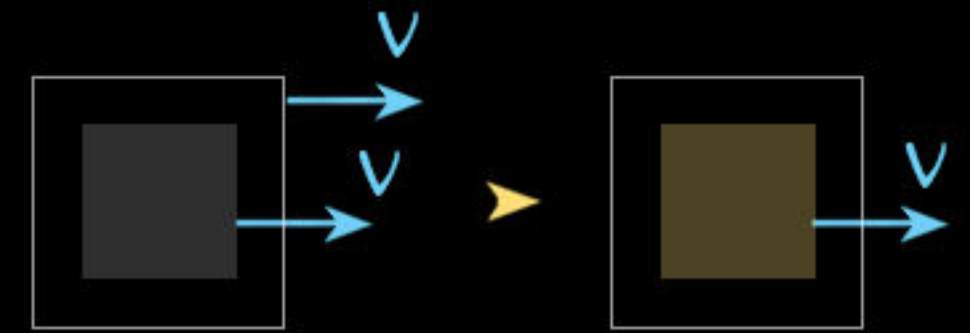
$$\Rightarrow \Delta T = \frac{m u^2}{6 n R} //$$

26. A mono-atomic gas is filled in a heat insulated horizontal cylindrical container at pressure p and temperature T . Density of the gas is ρ . The container is abruptly made to move along its axis with a constant velocity v . Find temperature of the gas when a new steady-state will establish in the container.

Concepts:

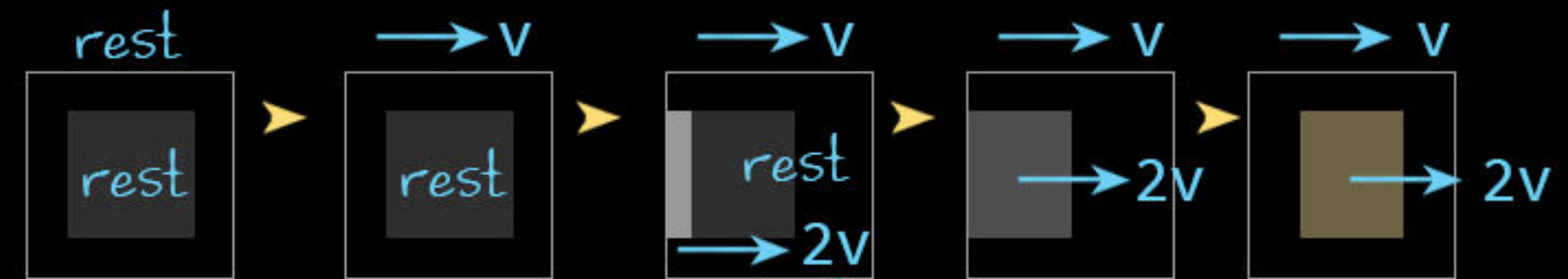
- 1 By first law ($\Delta U = Q + W_{\text{on gas}}$), temperature should be unchanged. But first law doesn't apply when bulk linear or angular momenta of the system change.
- 2 By second law, w.r.t a given container, ordered Kinetic Energy eventually turns to random motion (heat).
- 3 In gases, a compression shock wave travels at supersonic speed as gas molecules 'push' each other. (like iron bar that's pushed at one end, other end almost immediately has the same velocity)
A rarefaction shockwave on the other hand doesn't exist as gas molecules don't 'attract' each other. (unlike iron bar that's pulled at one end, other end almost immediately has the same velocity).

Case A If a container stops



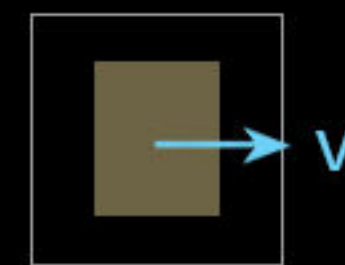
$$\therefore \frac{1}{2}mv^2 = nC_V \Delta T$$

Case B If a container at rest starts with v w.r.t ground:



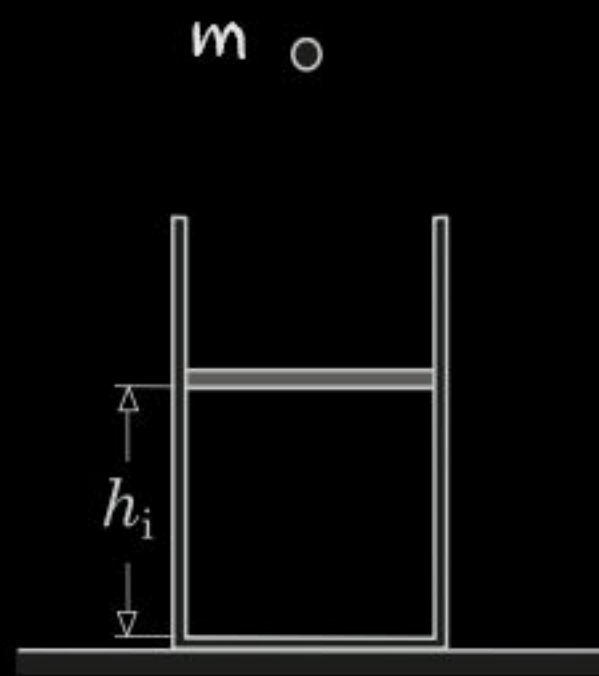
$$v_{\text{separation}} = v_{\text{approach}} = v$$

w.r.t container:



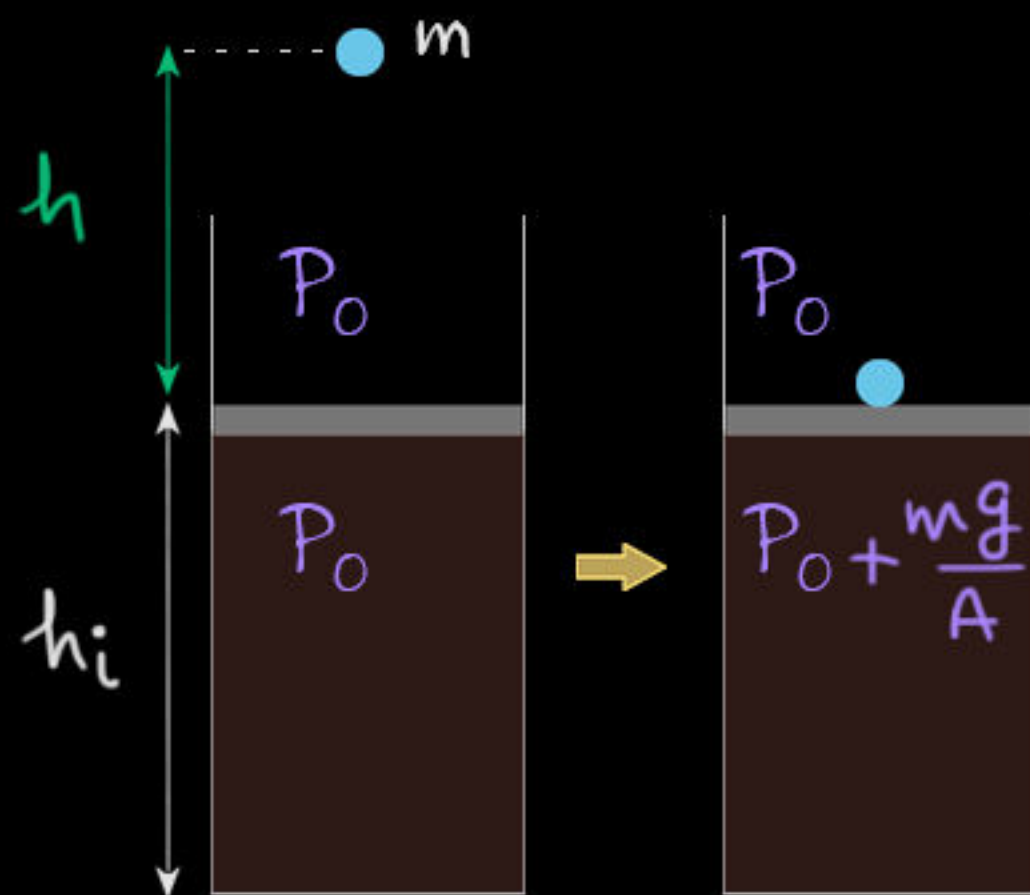
$$\begin{aligned} \frac{1}{2}mv^2 &= nC_V \Delta T \quad \frac{3R}{2} \\ \Rightarrow \frac{1}{2} \left(\frac{3nR}{P} \right) v^2 &= \frac{3R}{2} (T_f - T) \\ \Rightarrow T_f &= T \left(\frac{3v^2}{3P} + 1 \right) \end{aligned}$$

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27. A cylindrical container contains hydrogen in gaseous state under a light piston. The container and the piston both are very good insulators of heat. Initially the piston stays at a height h_i above the bottom of the container in equilibrium. A ball dropped from a height above the piston collides elastically with the piston. When all motion ceases and equilibrium is reestablished, the piston again stays at the height h_i . Find work done by the gas on the piston and the height above the piston from where the ball was dropped. Mass of ball is m .

Sol:



E: On piston, gas system:

$$Q + W_{\text{ext}} = \Delta U + \Delta KE \quad (1)$$

$$\Rightarrow mgh + P_0 \Delta V = n C_V \Delta T$$

$$\Rightarrow mgh = \frac{5}{2} \Delta(PV)$$

$$\Rightarrow \cancel{mg} h = \frac{5}{2} V \cdot \frac{\cancel{mg}}{A}$$

$$\Rightarrow h = \frac{5}{2} \left(\frac{V}{A} \right) = \frac{5h_i}{2} //$$

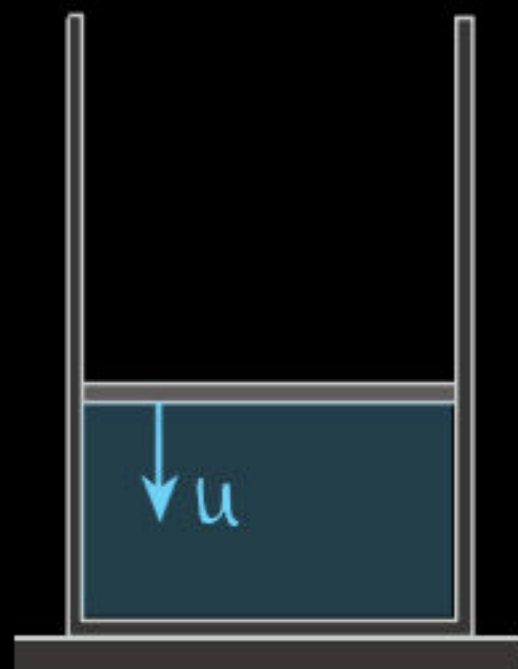
E: On gas alone

$$Q = \Delta U + W_{\text{by gas}} \quad (2)$$

Comparing 1 and 2:

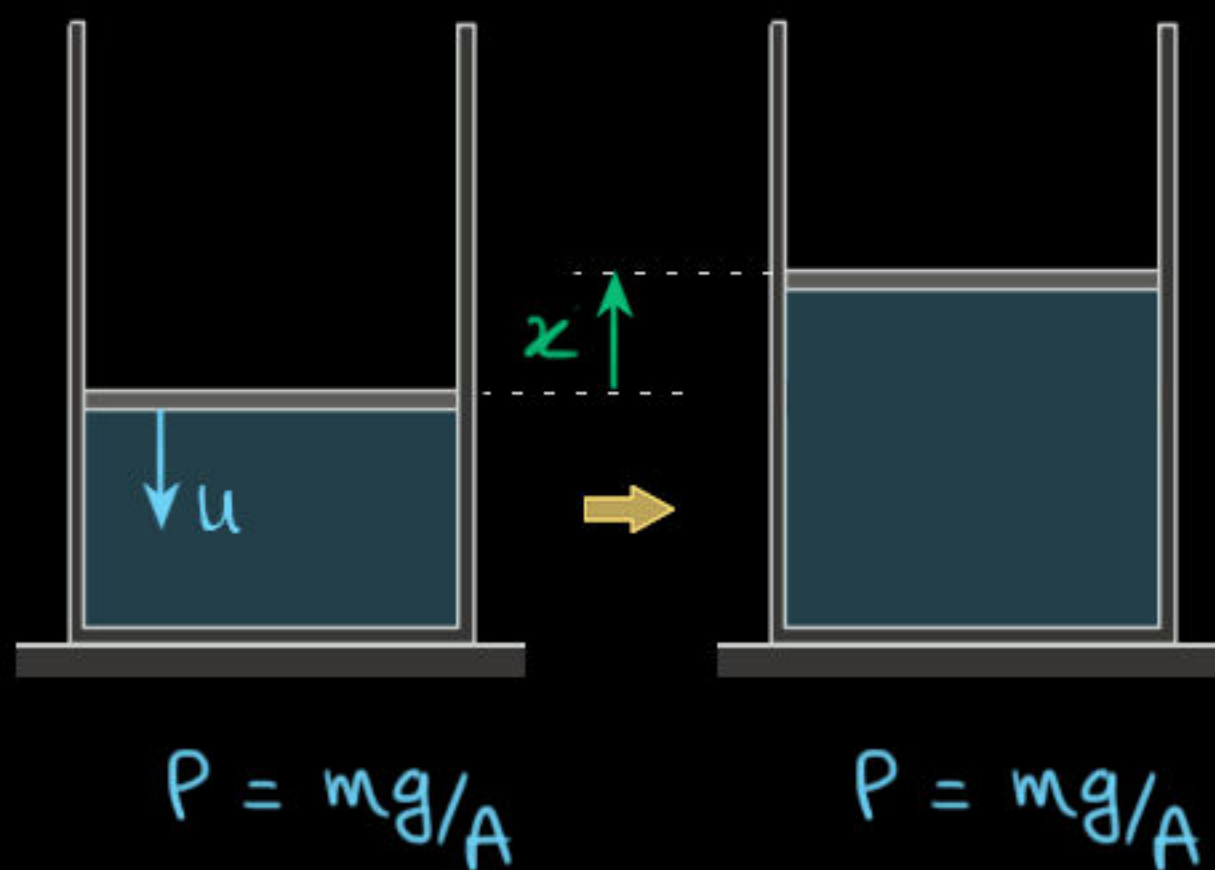
$$W_{\text{by gas}} = -W_{\text{ext}}$$

$$= -mgh //$$



28. Some amount of **nitrogen** gas is filled inside a vertical cylindrical container under a **massive piston** that can slide in the cylinder without friction. The inner surfaces of the piston and the container are coated with a perfect heat **insulating** material. The whole assembly is inside a highly **evacuated chamber**. If the piston is struck to impart it a velocity **u downwards**, after a large number of oscillations, the piston comes to a complete stop. **Find displacement of the piston.**

Sol:



E: On piston, gas system:

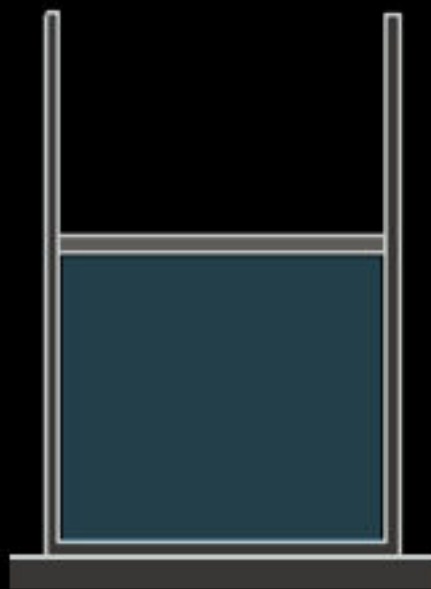
$$Q + W_{\text{ext}} = \Delta U_{\text{gas}} + \Delta KE_{\text{piston}}$$

$$\Rightarrow -mgx = nC_V \Delta T + \left(0 - \frac{1}{2}mv^2\right)$$

$$= \frac{5}{2} \Delta(PV) - \frac{1}{2}mv^2$$

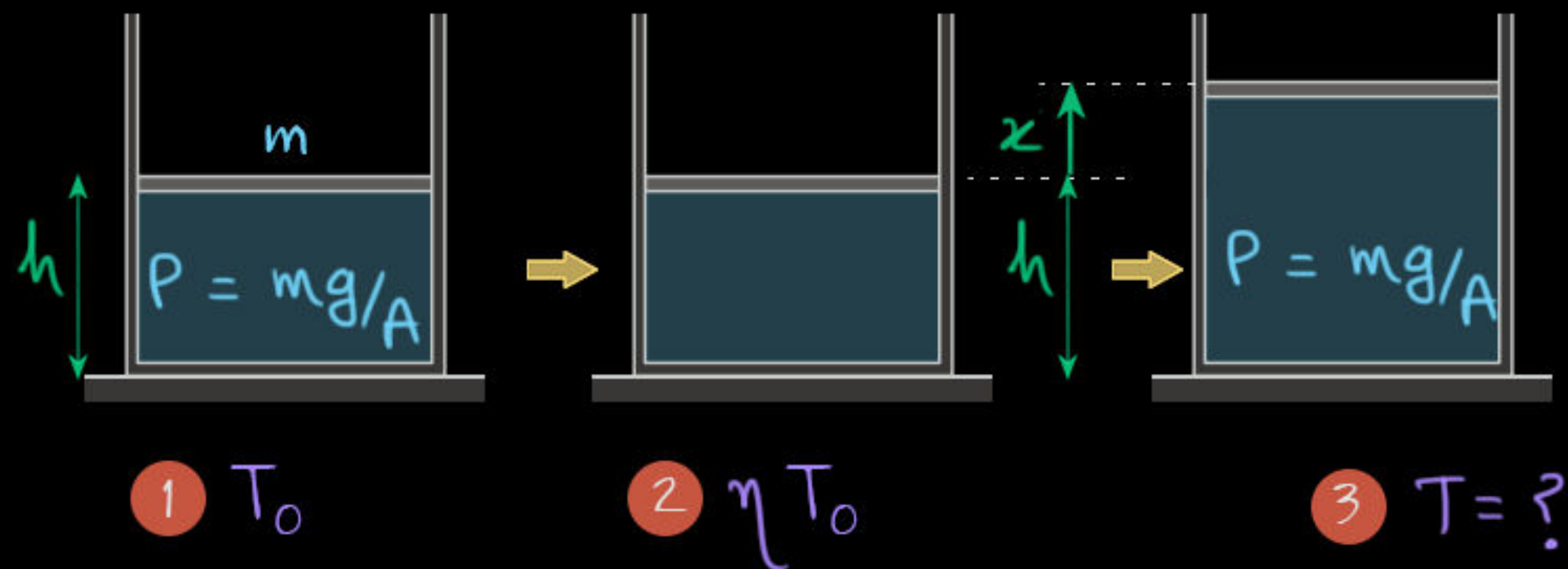
$$= \frac{5}{2} \frac{mg}{A} \Delta V - \frac{1}{2}mv^2$$

$$- \cancel{mg}x = \frac{5}{2} \frac{\cancel{mg}}{\cancel{A}} \cancel{A}x - \frac{1}{2}\cancel{m}v^2 \Rightarrow x = \frac{v^2}{7g} //$$



29. An **ideal mono-atomic** gas is confined in a cylinder with the help of a piston that can slide inside the cylinder without friction. The cylinder is placed on a horizontal platform in a **vacuum** chamber. The walls of the cylinder and the piston are made of heat **insulating** materials. Initially the temperature of the gas is T_0 and the piston stays **in equilibrium** as shown in the figure. The **temperature of the gas is quickly increased η times** and then the gas is allowed to expand. If the piston rises and again acquires an equilibrium state, **find temperature of the gas** in this new equilibrium state.

Sol: Let initial height be h , and displacement be x .



$$1\&3: \frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{Ah}{T_0} = \frac{A(h+x)}{T} \Rightarrow \frac{x}{h} = \frac{T}{T_0} - 1$$

2&3: E: On piston, gas system:

$$\cancel{Q} + W_{ext} = \Delta U_{gas} + \cancel{\Delta KE}_{piston}$$

$$\Rightarrow -mgx = nC_V \Delta T$$

$$\Rightarrow -mgx = \frac{3}{2} nR (T - \eta T_0)$$

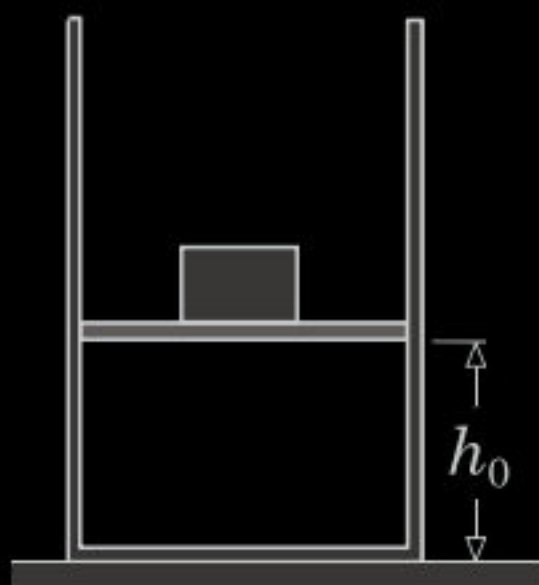
$$\Rightarrow -mgx = \frac{3}{2} nRT_0 \left(\frac{T}{T_0} - \eta \right)$$

$$\Rightarrow \cancel{mg}x = \frac{3}{2} \frac{\cancel{mg}}{\cancel{A}} \cdot \cancel{A} \cdot h (\eta - T/T_0)$$

$$\Rightarrow T/T_0 - 1 = \frac{3}{2} (\eta - T/T_0) \Rightarrow T = \left(\frac{3\eta + 2}{5} \right) T_0$$

by Ankit Singhvi

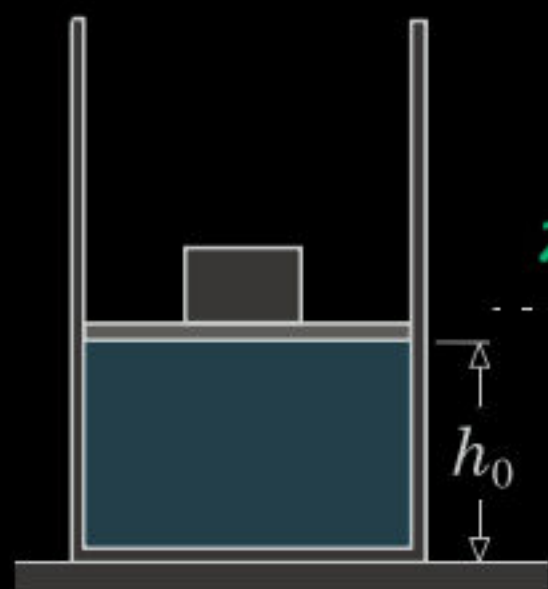
Youtube / Arihant Senior



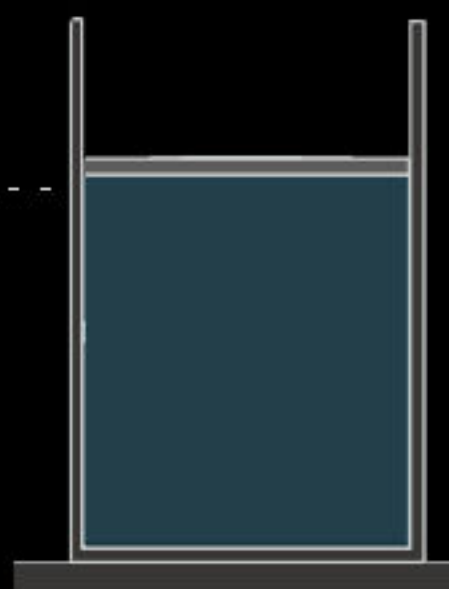
30. Some amount of an ideal gas is confined in a vertical cylinder with the help of a piston of mass m loaded with some extra weight of mass Δm . The walls of the cylinder and the piston are made of heat insulating materials. The piston can slide inside the cylinder without friction. Initially height of the piston is h_0 . If the extra weight on the piston is removed suddenly, at what height will the piston ultimately settle? Ratio of specific heats of the gas (C_P/C_V) is γ . Assume vacuum outside.

We know, $C_V = \frac{R}{\gamma - 1}$

Sol:



$$p = \frac{(m + \Delta m)g}{A}$$



$$p = \frac{mg}{A}$$

E: On piston, gas system:

$$Q + W_{\text{ext}} = \Delta U_{\text{gas}} + \Delta KE_{\text{piston}}$$

$$-mgx = nC_V\Delta T$$

$$-mgx = \frac{1}{\gamma - 1} nR\Delta T$$

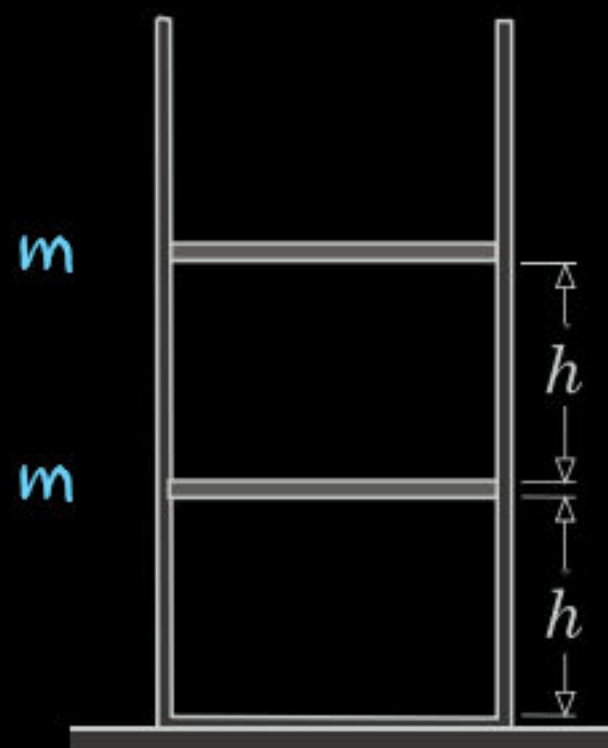
$$mgx = \frac{1}{\gamma - 1} (P_1V_1 - P_2V_2) = \frac{1}{\gamma - 1} \left[\frac{(m + \Delta m)g}{A} Ah_0 - \frac{mg}{A} A(h_0 + x) \right]$$

$$\Rightarrow (\gamma - 1)mx = m h_0 + \Delta m h_0 - m h_0 - mx$$

$$\Rightarrow x = \frac{\Delta m h_0}{m\gamma} \Rightarrow \text{Final height} = h_0 + x = h_0 \left(1 + \frac{\Delta m}{m\gamma} \right)$$

by Ankit Singhvi

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31. An ideal mono-atomic gas is confined in two portions at the same temperature in an adiabatic cylinder with the help of two pistons of equal masses as shown in the figure. The whole assembly is inside a highly evacuated chamber. The pistons can slide inside the cylinder without friction and initial separation between them as well as between the lower piston and bottom of the cylinder is h . The upper piston is made of a heat insulating material and the lower piston is made of such a material that is an insulator of heat and becomes a permanent conductor of heat once its temperature becomes twice of the initial temperature. Heat capacity of both the pistons and of the cylinder is negligibly small as compared to that of the gas.

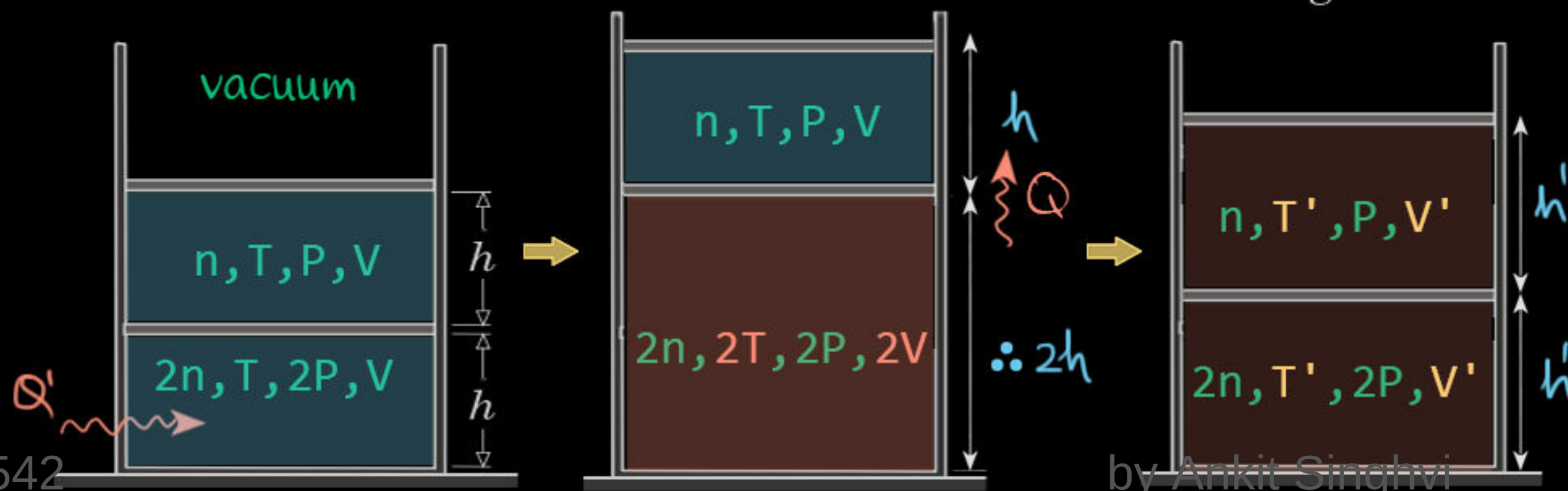
Sol: All processes are slow.
 $\Rightarrow$ Piston has no net force.

$\therefore$ At every moment:

$$P_{\text{top}} = mg/A = p$$

$$P_{\text{bottom}} = 2mg/A = 2p$$

The gas in the lower portion is slowly heated until its temperature becomes double of its initial temperature. What will be the separation between the pistons after a long time when thermal equilibrium is re-established between gases in both the portions?



$$P(Ah) = nRT \quad (1)$$

$$P(Ah') = nRT' \quad (2)$$

$$Q = 2nC_p(2T - T') = nC_p(T' - T) \quad (3)$$

From 1, 2, 3: $h' = 5h/3$

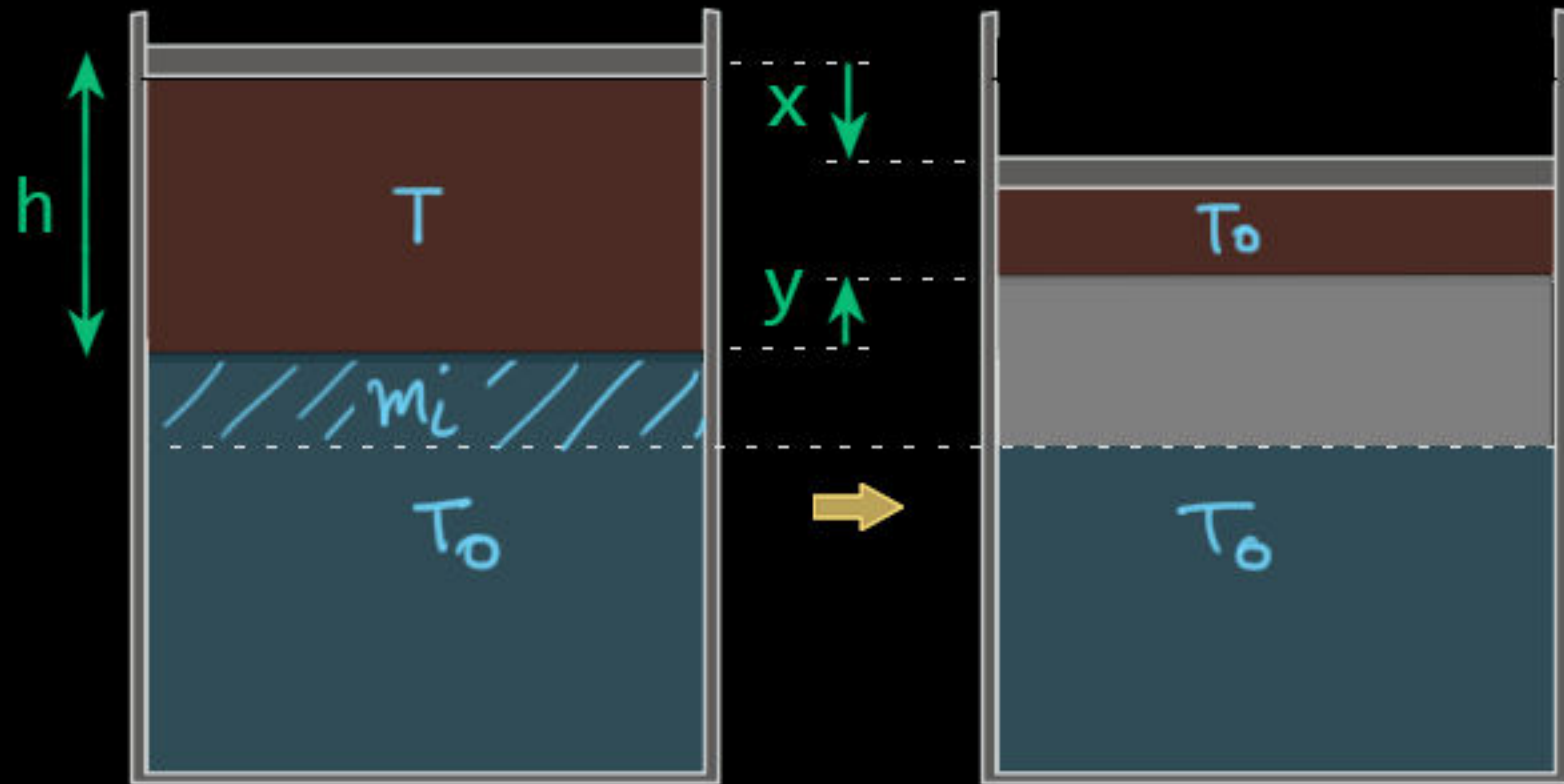
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32. In a vertical cylindrical vessel, water is kept at its freezing point T_0 separated by a thin oil film from n moles of a mono-atomic gas at temperatures $T < T_0$ under a light piston of area A . The walls of the cylinder and the piston are made of insulating materials and there is no friction between the piston and the cylinder. Initially the piston is in equilibrium. Neglecting heat capacities of the vessel and of the piston, find displacement of the piston after a long time, when temperature of all the contents inside the cylinder settles to T_0 . Specific latent heat of melting of ice is L , density of water is ρ_w , density of ice is ρ_i and the atmospheric pressure is p_0 .

Sol: Piston is massless
 $\Rightarrow$ Isobaric (P_0) process.



Let m_i mass of water turns to ice.
 and $\Delta V = A y$ be volume change due to freezing.

Then, $\Delta V = V_i - V_w$

$$\Rightarrow A y = \frac{m_i}{\rho_i} - \frac{m_i}{\rho_w} = m_i \left(\frac{\rho_w - \rho_i}{\rho_w \rho_i} \right) \quad (1)$$

Heat absorbed by gas:

$$Q = n C_p \Delta T = m_i L$$

$$\Rightarrow \sum \frac{5}{2} n R (T_0 - T) = m_i L \quad (4)$$

From 1, 2, 3, 4 (x , y , h , m_i):

$$x = \frac{-n R (T_0 - T)}{A p_0} \left\{ 1 + \frac{5}{2} \frac{p_0 (\rho_w - \rho_i)}{L \rho_w \rho_i} \right\}$$

$$p_0 = \frac{n R T}{A h} = \frac{n R T_0}{A (h - x - y)}$$

by Ankit Singhvi

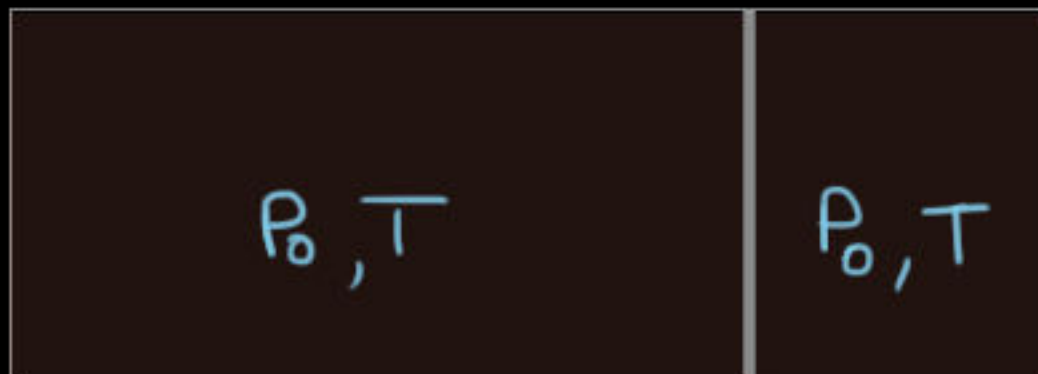
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Its isobaric (P_0) process in both chambers..
(Proof on next slide)

Initially:



Let final temperature be T .



33. A heat insulated cylindrical container is divided into two parts by a piston made from a material of very low thermal conductivity and negligible heat capacity. The cylinder is fixed horizontally and an ideal mono-atomic gas is filled in both the parts to occupy equal volumes V each at equal pressure p_0 but at different temperatures T_1 and $T_2 > T_1$. After a long time, when equilibrium is re-established, what will be the temperature in both the parts? How much heat will pass through the piston from one gas to another?

A

$$U_i = U_f$$

$$\Rightarrow n_1 \cancel{C_V} T_1 + n_2 \cancel{C_V} T_2 = n_1 \cancel{C_V} T + n_2 \cancel{C_V} T$$

$$\Rightarrow T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2} \xrightarrow{\left(\frac{n_1}{n_2} = \frac{T_2}{T_1}\right)} = \frac{2 T_1 T_2}{T_1 + T_2} //$$

B

$$Q = n_1 C_p (T - T_1)$$

$$= \frac{5}{2} n_1 R T_1 \left(\frac{T}{T_1} - 1 \right)$$

$$= \frac{5}{2} P_0 V \left(\frac{2 T_2}{T_1 + T_2} - 1 \right) = \frac{5}{2} P_0 V \left(\frac{T_2 - T_1}{T_2 + T_1} \right) //$$

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Proof of isobaric (P_0) process in both chambers.

The process is slow.

$\Rightarrow F_{\text{net}}$ on piston is always zero.

$\Rightarrow$ Pressure on both sides of piston is always equal.

Let it be P at a general time.

Initially:



Some time later:



E conservation: $dQ_1 = n_1 c_v dT_1 + P dV_1$

$$dQ_2 = n_2 c_v dT_2 + P dV_2$$

$$\text{add } \cancel{dQ_1} + \cancel{dQ_2} = (n_1 dT_1 + n_2 dT_2) c_v + P(\cancel{dV_1} + \cancel{dV_2})$$

$$\Rightarrow 0 = n_1 dT_1 + n_2 dT_2 \quad (1)$$

V conservation:

$$2V \text{ (constant)} = V_1 + V_2 = \frac{n_1 R T_1}{P} + \frac{n_2 R T_2}{P}$$

differentiate $\downarrow$

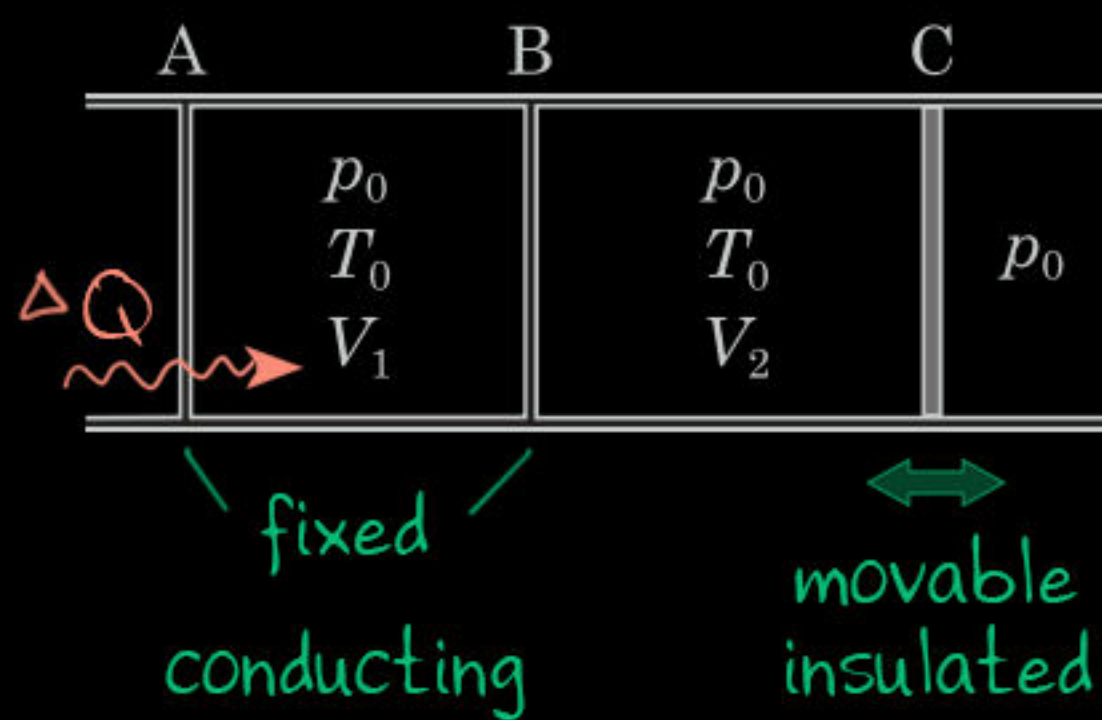
$$0 = P(n_1 dT_1 + n_2 dT_2) + (n_1 T_1 + n_2 T_2) dP \quad (2)$$

From 1,2:

$$dP = 0 \Rightarrow \text{Isobaric } (P_0) \text{ process.}$$

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Let heat transferred through partition B be ΔQ_2



All processes are slow.
 $\Rightarrow$ Piston C has no net force.
 $\therefore$ Second chamber undergoes isobaric (P_0) process.

34. A horizontal cylindrical tube is divided into four parts with the help of two fixed partitions A and B and a movable piston C. The tube and the piston C are made from heat insulating materials whereas the fixed partitions A and B are made from a conducting material. Volume between the fixed partitions A and B is V_1 and initial volume between the fixed partition B and piston C is V_2 . These volumes are filled with an ideal mono-atomic gas at pressure p_0 and temperature T_0 . Outside the piston C, pressure is also p_0 . Quantity ΔQ of heat is slowly transferred through the partition A to the gas trapped between the partitions A and B. Find temperatures of the gases on both sides of the partition B and heat transferred through it.

$$\Delta Q - \Delta Q_2 = n_1 c_v \Delta T = \frac{3}{2} n_1 R (T - T_0) = \frac{3}{2} \frac{p_0 V_1}{T_0} (T - T_0) \quad (1)$$

$$\Delta Q_2 = n_2 c_p \Delta T = \frac{5}{2} n_2 R (T - T_0) = \frac{5}{2} \frac{p_0 V_2}{T_0} (T - T_0) \quad (2)$$

From 1,2: $\Delta Q_2 = \frac{5 V_2 \Delta Q}{5 V_2 + 3 V_1} \quad T = T_0 \left[1 + \frac{2 \Delta Q}{p_0 (5 V_2 + 3 V_1)} \right]$

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35. Two parts of a heat-insulated container are separated by a piston that can move without friction. The part to the left of the piston is filled with n moles of a mono-atomic gas and in the part to the right of the piston is vacuum. The piston is connected to the right end of the container by a spring of relaxed length that is equal to the full length of the container. Find the heat capacity of the system, neglecting the heat capacities of the container, the piston and the spring. Universal gas constant is R .

Sol:



dQ

$T \rightarrow T + dT$

$$nC dT = nC_V dT + P dV$$

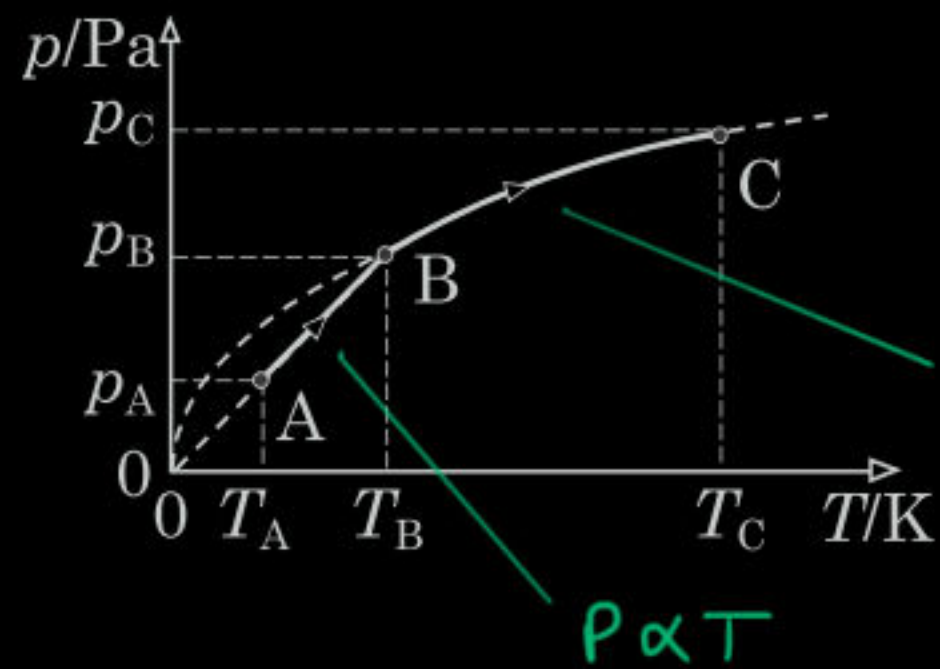
$$\Rightarrow C = C_V + R \frac{P dV}{d(PV)}$$

$$\downarrow P \rightarrow \frac{kx}{A}, V \rightarrow Ax$$

$$= C_V + R \frac{kx}{A} \cdot \frac{A dx}{k 2x dx}$$

$$= C_V + \frac{R}{2}$$

$$= 2R //$$



36. One mole of an ideal mono-atomic gas undergoes two quasi-static processes $A \rightarrow B$ and $B \rightarrow C$ in sequence as shown in the following figure. If in the first process, the pressure p is proportional to the temperature T and in the second process, the pressure p is proportional to $\sqrt{T}$, find the total heat supplied to the gas in both the processes. Universal gas constant is R . Answer in terms of T_A , T_B , T_C only.

$$Q = \Delta U + W$$

$\swarrow \quad \searrow$
 $W_1 \quad W_2$
 $\rightarrow 0$

$$\Rightarrow Q = nC_V(T_C - T_A) + \frac{nR}{2}(T_C - T_B)$$

$$= \frac{nR}{2}(3T_C - 3T_A + T_C - T_B)$$

$$= \frac{nR}{2}(4T_C - T_B - 3T_A)$$

$$W_1 = \int P dV = 0$$

$$W_2 = \int P dV$$

$$= \int_{P_B}^{P_C} P \cdot \frac{nR dP}{k}$$

$$= \frac{nR}{2} \frac{P_C^2 - P_B^2}{k}$$

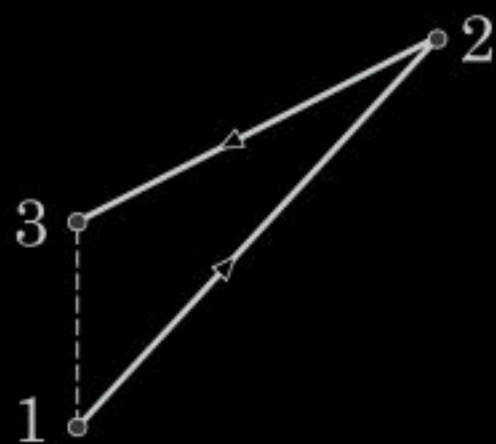
$$= \frac{nR}{2}(T_C - T_B)$$

$$BC: P \propto \sqrt{T}$$

$$\Rightarrow P^2 = kT$$

$$\Rightarrow P^2 = k \left(\frac{PV}{nR} \right)$$

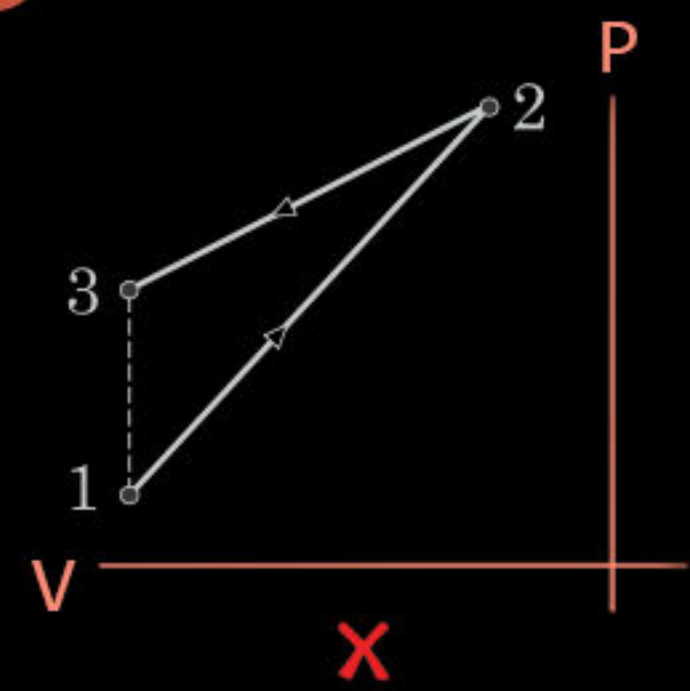
$$\Rightarrow dP = k \frac{dV}{nR}$$



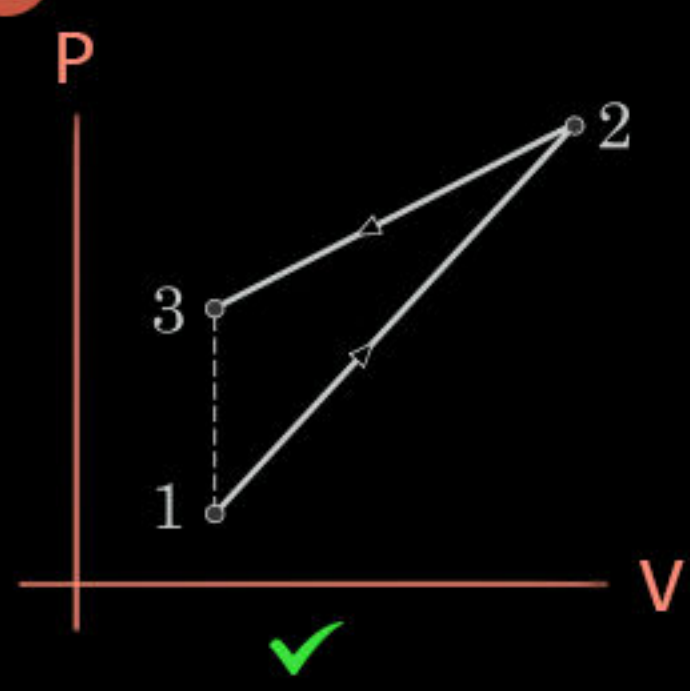
Four cases are possible:
(showing +ve P and V axes)

37. In an old record found in a laboratory, a process $1 \rightarrow 2 \rightarrow 3$ was shown. Over the time, ink faded and it became impossible to see the pressure and volume axes. However, descriptions given there reveal that the states 1 and 3 lie on an isochore corresponding to a volume V , amount of heat supplied during the whole process $1 \rightarrow 2 \rightarrow 3$ is zero and the gas involved is one mole of helium. Find volume occupied by the gas in the state 2.

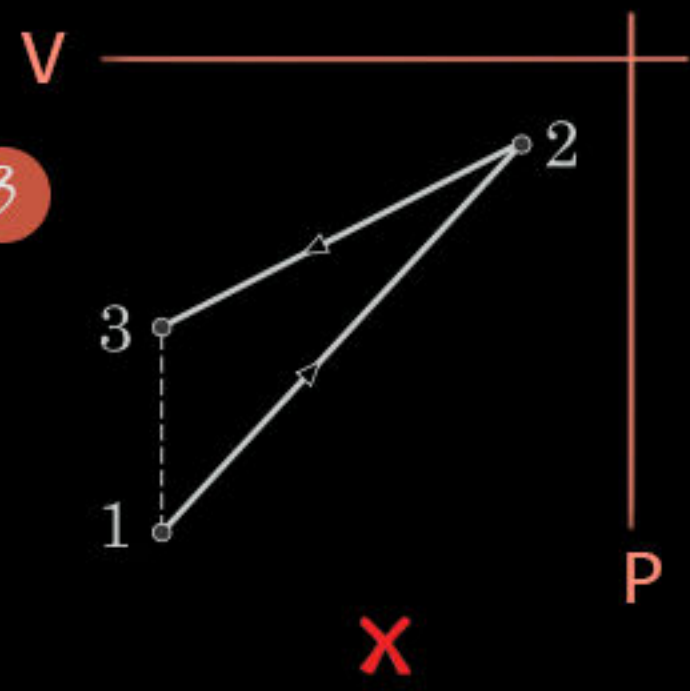
2



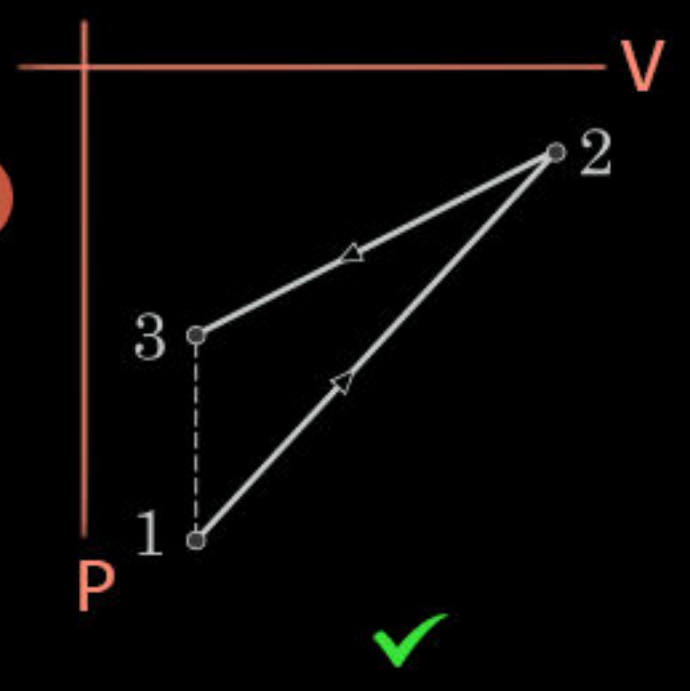
1



3



4



$$Q = \Delta U + W$$

✓ 1	0	=	+	-
✗ 2	0	=	+	+
✗ 3	0	=	-	-
✓ 4	0	=	-	+

1 $n C_V \Delta T = |\text{Area } \Delta|$

$\Rightarrow \frac{3}{2} \Delta(PV) = \frac{1}{2} (P_3 - P_1)(V_2 - V)$

$\Rightarrow \frac{3}{2} V (P_3 - P_1) = \frac{1}{2} (P_3 - P_1)(V_2 - V)$

$\Rightarrow 3V = V_2 - V$

$\Rightarrow V_2 = 4V //$

4 $|n C_V \Delta T| = \text{Area } \Delta$

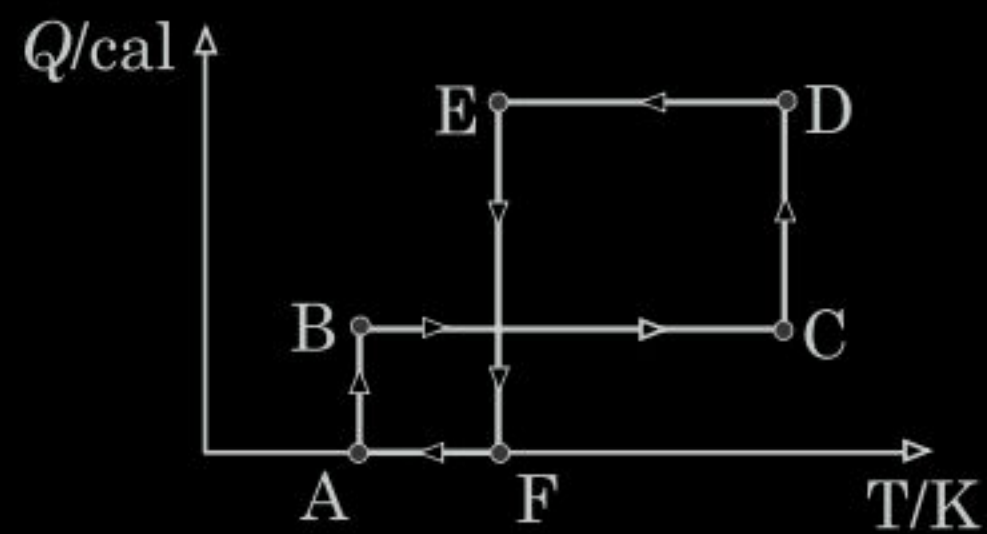
$\Rightarrow \left| \frac{3}{2} \Delta(PV) \right| = \frac{1}{2} (P_1 - P_3)(V_2 - V)$

$\Rightarrow \frac{3}{2} V (P_1 - P_3) = \frac{1}{2} (P_1 - P_3)(V_2 - V) \Rightarrow 3V = V_2 - V$

$\Rightarrow V_2 = 4V //$

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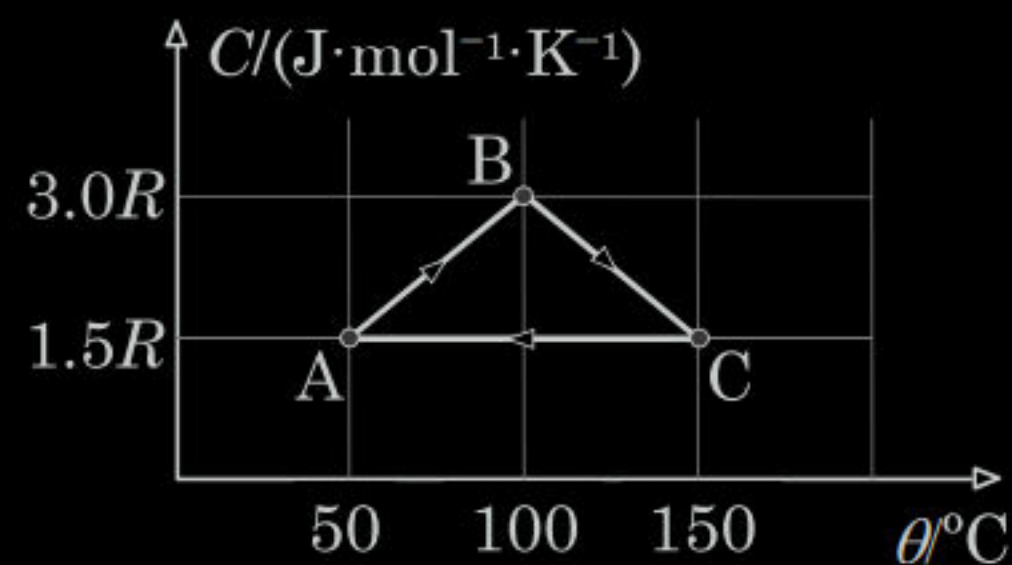
by Ankit Singhvi



38. As shown in the given figure, an ideal **mono-atomic gas** undergoes a quasi-static process $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow A$. Here it is shown that how the amount of net heat Q given to the gas varies with temperature T of the gas. **Identify the processes in which volume of the gas increases, decreases or remains unchanged** respectively.

$$\Delta Q = \underbrace{\Delta U}_{\substack{\downarrow \\ \text{+ve if } T \uparrow}} + \underbrace{\int P dV}_{\substack{\downarrow \\ \text{+ve if } V \uparrow}}$$

	ΔQ	ΔU	$+W$	
AB:	+	0	+	Volume increases
BC:	0	+	-	
CD:	+	0	+	
DE:	0	-	+	
EF:	-	0	-	Volume decreases
FA:	0	-	+	



39. One mole of an ideal mono-atomic gas undergoes quasi-static process $A \rightarrow B \rightarrow C \rightarrow A$ that is shown in the figure, where C is molar heat capacity of the gas and θ is temperature of the gas. Find amount of heat Q received by the gas from the heater heating the gas, work done W by the gas in the entire cycle and efficiency η of the cycle? Molar gas constant is $R = 8.31 \text{ J/(mol}\cdot\text{K)}$.

Concepts: $dQ = C dT$

∴ Q is area under C - T curve

Also, as C is always positive,

∴ Q is received when T increases.

And Q is lost when T decreases.

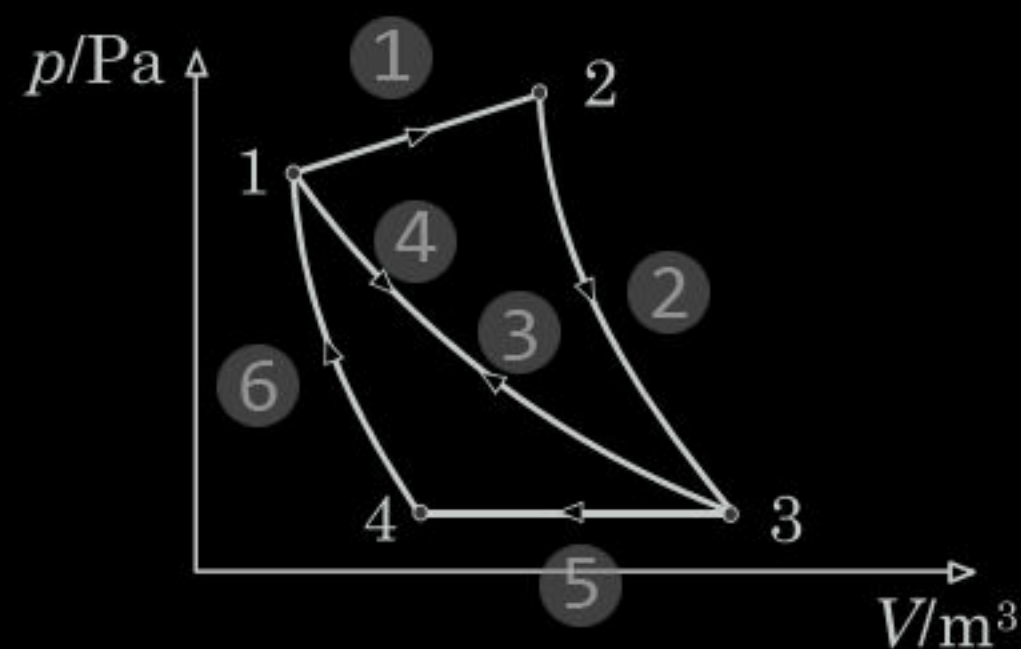
$$Q_{\text{received}} = \text{Area under } A \rightarrow B + \text{Area under } B \rightarrow C = 3(50 \times 1.5R) = 225R //$$

$$\begin{aligned} \text{Work done by gas, } W &= \Delta Q - \cancel{\Delta U} \\ &= \text{Area under } B \rightarrow C = \frac{1}{2} \times 100 \times 1.5R = 75R // \end{aligned}$$

$$\text{Efficiency, } \eta = \frac{W}{Q_{\text{received}}} = \frac{75}{225} = \frac{1}{3} //$$

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40. Pressure (p)-volume (V) indicator diagram of a cyclic process $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$ consists of a straight line $1 \rightarrow 2$, an adiabatic $2 \rightarrow 3$ and an isotherm $3 \rightarrow 1$. Pressure (p)-volume (V) indicator diagram of another cyclic processes $1 \rightarrow 3 \rightarrow 4 \rightarrow 1$ consists of an isotherm $1 \rightarrow 3$, an isobar $3 \rightarrow 4$ and an adiabatic $4 \rightarrow 1$. If efficiencies of these two processes are η_1 and η_2 respectively, determine the efficiency of the cycle $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$.

We know, $\eta = 1 - \frac{|Q_{out}|}{Q_{in}}$

	Q
1	$+Q_1$
2	0
3	$-x$

$$\eta_1 = 1 - \frac{x}{Q_1}$$

4	$+x$
5	$- Q_5 $
6	0

$$\eta_2 = 1 - \frac{|Q_5|}{x}$$

x is a positive number.

For $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$

$$\eta = \frac{W}{Q_1} = \frac{Q_1 + Q_2 + Q_5 + Q_6}{Q_1} = 1 - \frac{|Q_5|}{Q_1} = 1 - (1 - \eta_1)(1 - \eta_2)$$

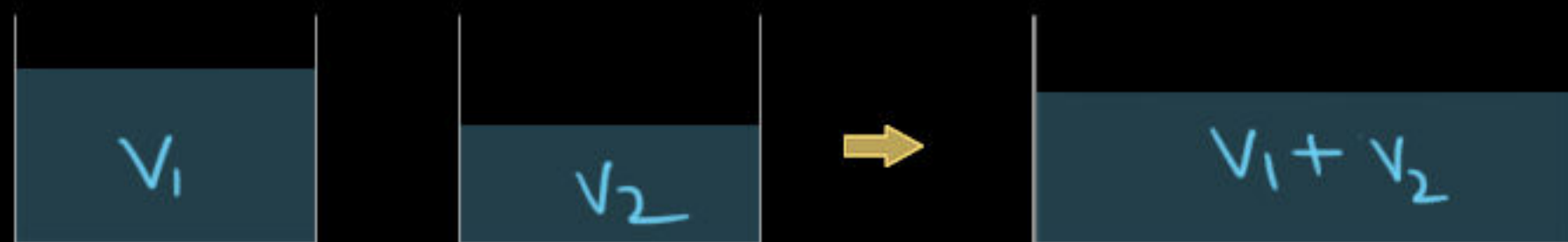
$$= \eta_1 + \eta_2 - \eta_1 \eta_2$$

Whenever we deal with coefficient of expansions, assumptions are:

$$\frac{\Delta \theta}{\theta}, \frac{\Delta l}{l}, \frac{\Delta A}{A}, \frac{\Delta V}{V}, \frac{\Delta \rho}{\rho}$$

etc. are all very small.

Sol: Let final temperature be T .



$$T_1 < T_2$$

$$T_1 < T < T_2$$

Heat gained = Heat lost

$$\Rightarrow m_1 \cancel{\rho} (T - T_1) = m_2 \cancel{\rho} (T_2 - T)$$

$$\Rightarrow \rho_1 V_1 (T - T_1) = \rho_2 V_2 (T_2 - T) \quad (1)$$

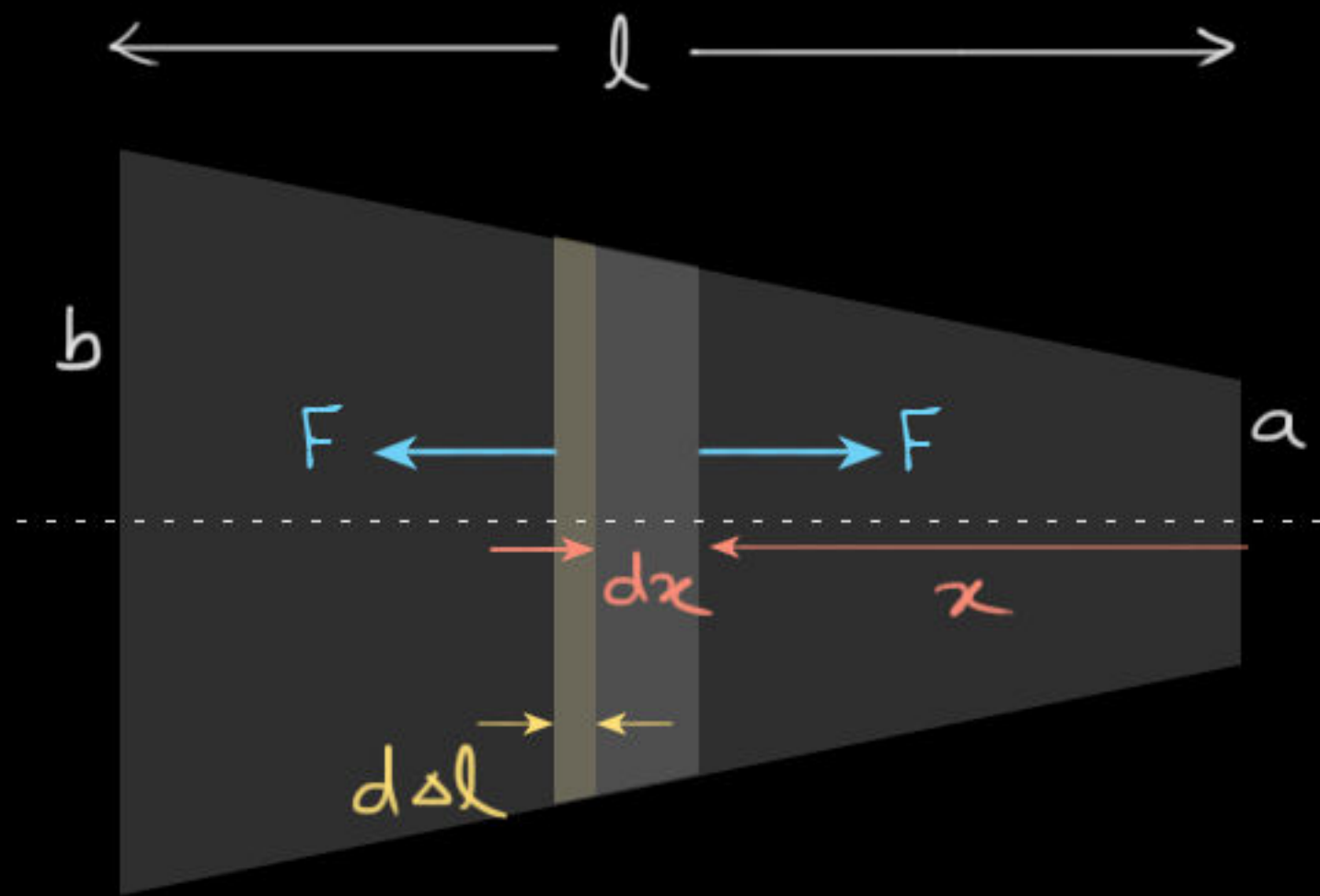
- Volumes V_1 and V_2 of a liquid are maintained in two calorimeters at temperatures θ_1 and θ_2 ($\theta_2 > \theta_1$). Coefficient of volume expansion of the liquid is independent of temperature. Now liquids from both the calorimeter are poured into another calorimeter of negligible heat capacity. Heat loss to the surroundings is strictly restricted. **Final volume V of the mixture** after they are well mixed is **best represented** by the equation

$$\checkmark (a) V = V_1 + V_2$$

After mixing:

$$\begin{aligned} V_f &= V_1 \{1 + \gamma (T - T_1)\} + V_2 \{1 + \gamma (T - T_2)\} \\ &= (V_1 + V_2) + \gamma V_1 (T - T_1) \left[1 + \frac{V_2}{V_1} \left(\frac{T - T_2}{T - T_1}\right)\right] \\ &= (V_1 + V_2) + \gamma V_1 (T - T_1) \left[1 - \frac{\rho_1}{\rho_2}\right] \\ &= (V_1 + V_2) + \gamma V_1 \frac{\Delta \theta \Delta \rho}{\rho_2} \rightarrow \approx 0 \\ &\approx V_1 + V_2 // \end{aligned}$$

2. Radii of a slightly tapered cylindrical wire of length L at its ends are a and b . It is stretched by two forces each of magnitude F applied at its ends. The forces are uniformly distributed over the end faces. If Young's modulus of material of the wire is denoted by Y , extension produced is



$$\text{Stress} = Y \text{ Strain} \quad \checkmark (d) \frac{FL}{\pi abY}$$

$$\Rightarrow \frac{F}{\pi \left[a + \left(\frac{b-a}{l} \right) x \right]^2} = Y \frac{d\Delta l}{dx}$$

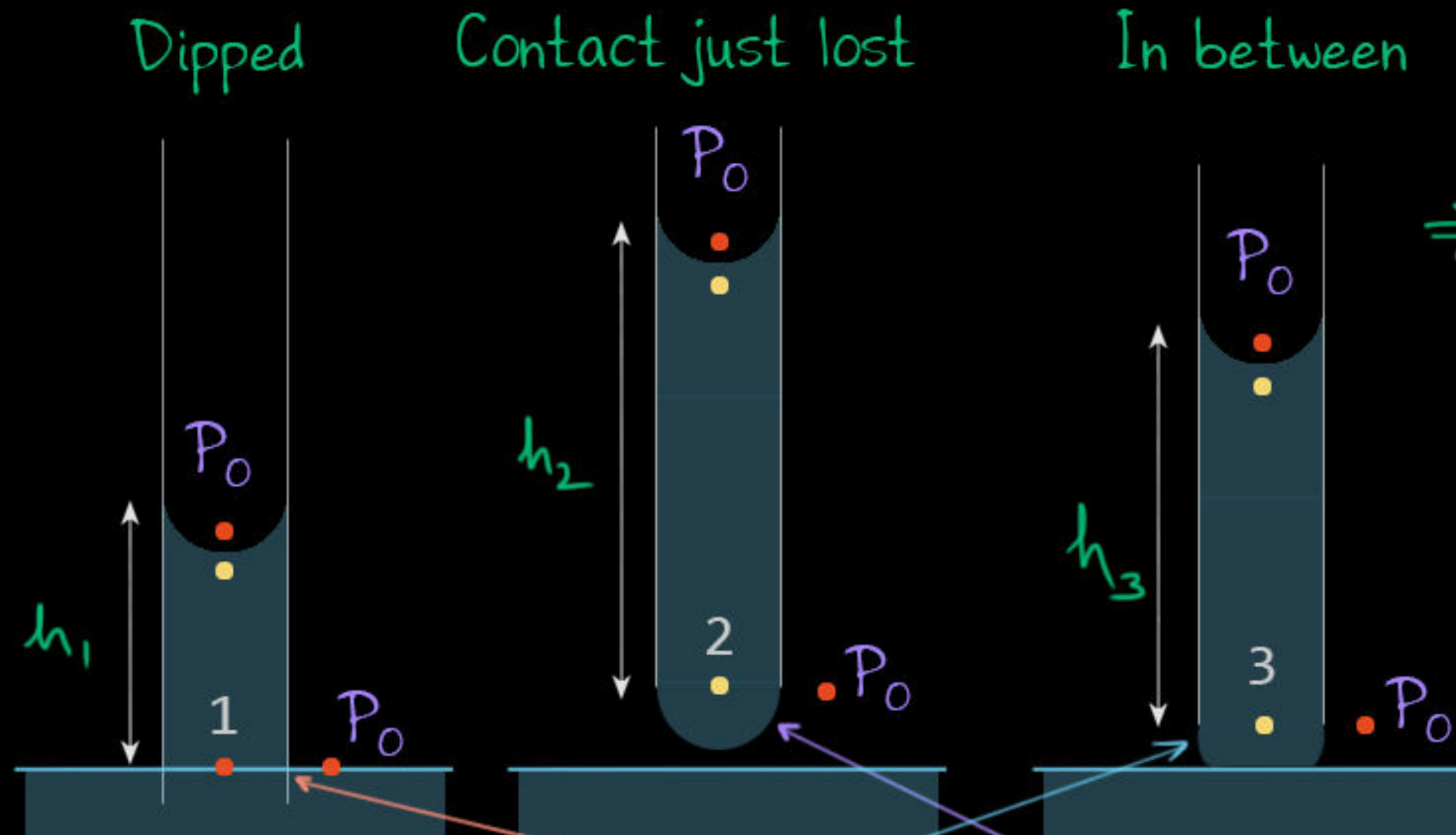
$$\Rightarrow F \int_0^l \frac{dx}{\left(a + \left(\frac{b-a}{l} \right) x \right)^2} = \pi Y \int_0^{\Delta l} d\Delta l$$

$$\Rightarrow \frac{Fl}{b-a} \left[\frac{-1}{a + \left(\frac{b-a}{l} \right) x} \right]_0^l = \pi Y \Delta l$$

$$\Rightarrow \frac{Fl}{b-a} \frac{b-a}{ab} = \pi Y \Delta l \Rightarrow \Delta l = \frac{Fl}{\pi Y ab} //$$

Lets see all three possible cases of a capillary holding maximum liquid.

3. A long capillary tube of radius r is put in contact with surface of a perfectly wetting liquid of density ρ and very low viscosity. What maximum height h the liquid can rise inside the capillary? Here height is measured above the level of the liquid outside the capillary. Surface tension of the liquid and acceleration due to gravity are denoted by σ and g respectively.



Concept: Curve here is inbetween flat and spherical. So pressure at 3 is inbetween pressure at 1 and 2.

$$P_1 < P_3 < P_2$$

$$\checkmark (c) \frac{2\sigma}{\rho g r} < h < \frac{4\sigma}{\rho g r}$$

$$\Rightarrow P_0 - \frac{2\sigma}{R} + \rho g h_1 < P_0 - \frac{2\sigma}{R} + \rho g h_3 < P_0 - \frac{2\sigma}{R} + \rho g h_2$$

$$\Rightarrow h_1 < h_3 < h_2$$

$$\Rightarrow \frac{2\sigma}{\rho g R} < h_3 < \frac{4\sigma}{\rho g R} //$$

$$P_0 - \frac{2\sigma}{R} + \rho g h_1 = P_0 \Rightarrow h_1 = \frac{2\sigma}{\rho g R}$$

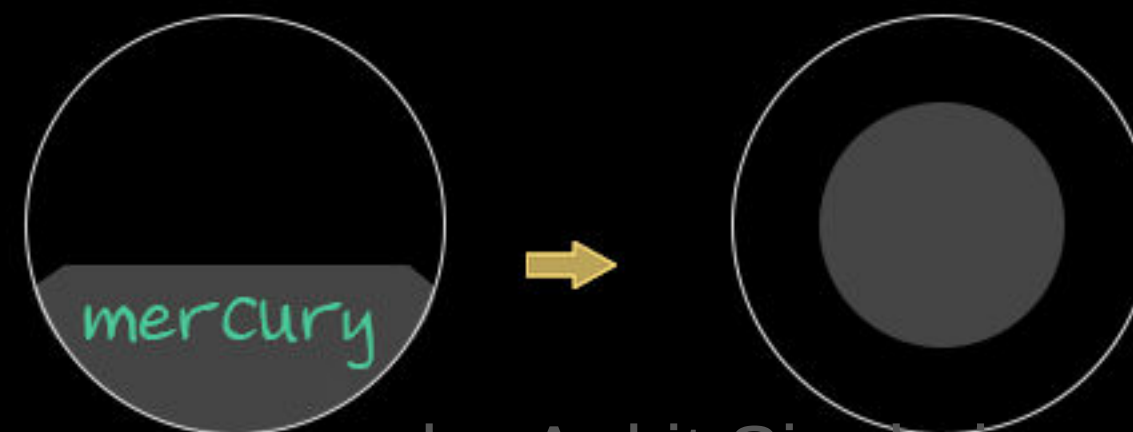
$$P_0 - \frac{2\sigma}{R} + \rho g h_2 = P_0 + \frac{2\sigma}{R} \Rightarrow h_2 = \frac{4\sigma}{\rho g R}$$

4. Consider two hollow glass spheres; one of them containing water in approximately 10% of its volume, and the other containing a similar volume of mercury. If the spheres are brought in a zero gravity environment of a space-shuttle, what will you observe?
- (a) Water and mercury both float in their spheres as spherical drops.
 - ✓ (b) Water forms a layer on inner surface of the sphere while mercury floats as a spherical drop.
 - (c) Mercury forms a layer on inner surface of the sphere while water floats as a spherical drop.
 - (d) In each case, some amounts of the liquids form layers on inner surfaces of the spheres and remaining amounts float as spherical drops.

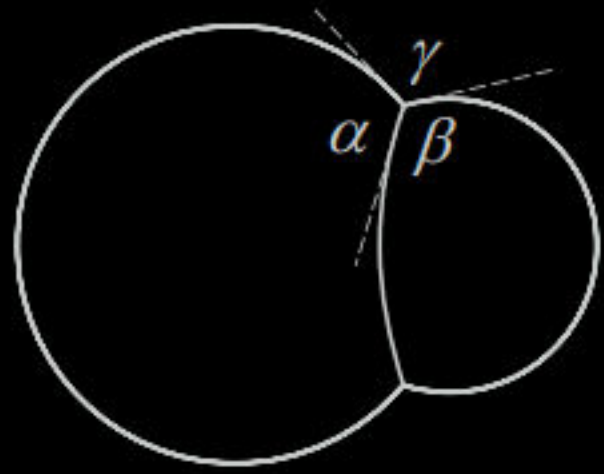
Concept:
In absence of gravitational force, other two forces (cohesive and adhesive) will determine the outcome.



As adhesive forces (between glass and water) are stronger than cohesive forces (between liquid molecules).
Maximum wetting will happen.



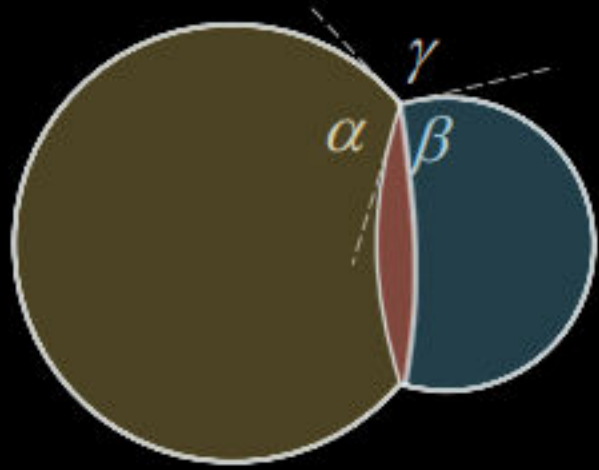
As cohesive forces are stronger than adhesive forces.
Minimum wetting will happen.



5. When two soap bubbles of different radii coalesce, some portions of their surfaces make a common surface. At any point on the circumference of the common surface, the three surfaces meet at angles α , β and γ . What relation should these angles bear?

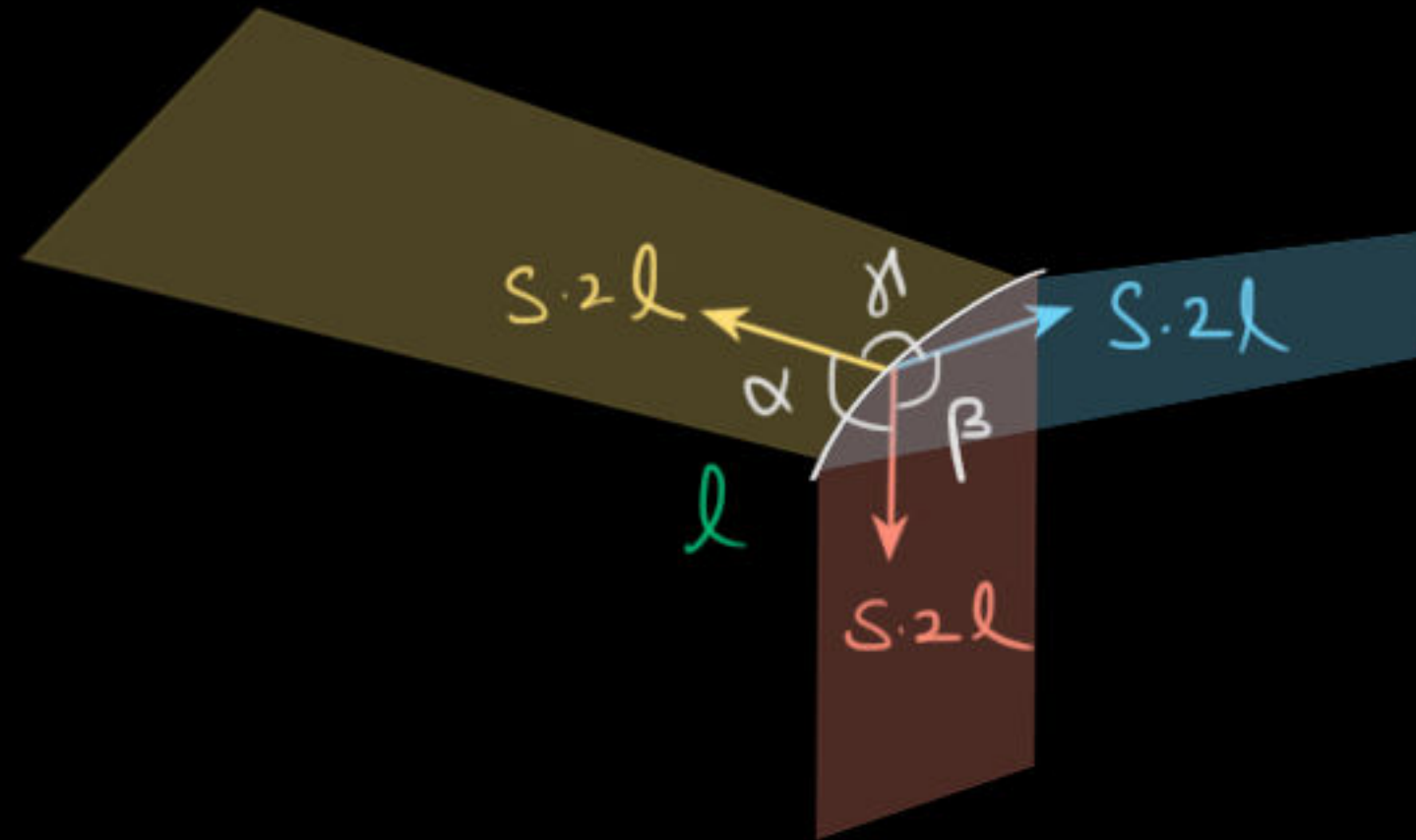
(a) $\alpha > \beta$
(c) $\alpha = \beta < \gamma$

(b) $\alpha > \beta > \gamma$
✓ (d) $\alpha = \beta = \gamma$



≡

Lets take l length of edge common to all three surfaces.



Edge l is at rest, and all three forces are equal in magnitude, i.e $S \cdot 2l$.

$$\therefore \alpha = \beta = \gamma$$

by Ankit Singhvi

Youtube / Arihant Senior

Any system is in stable equilibrium if energy of the system is at local minima. In this case, surface area of films should be minimum.

So let's push the film on any one side a bit. System is at:

- 1) Stable equilibrium, if total area of the film increases, then it's in
- 2) Neutral equilibrium, if area doesn't change.
- 3) Unstable equilibrium, if area reduces.

Initially all four are in equilibrium, as curvatures are equal and in same sense.

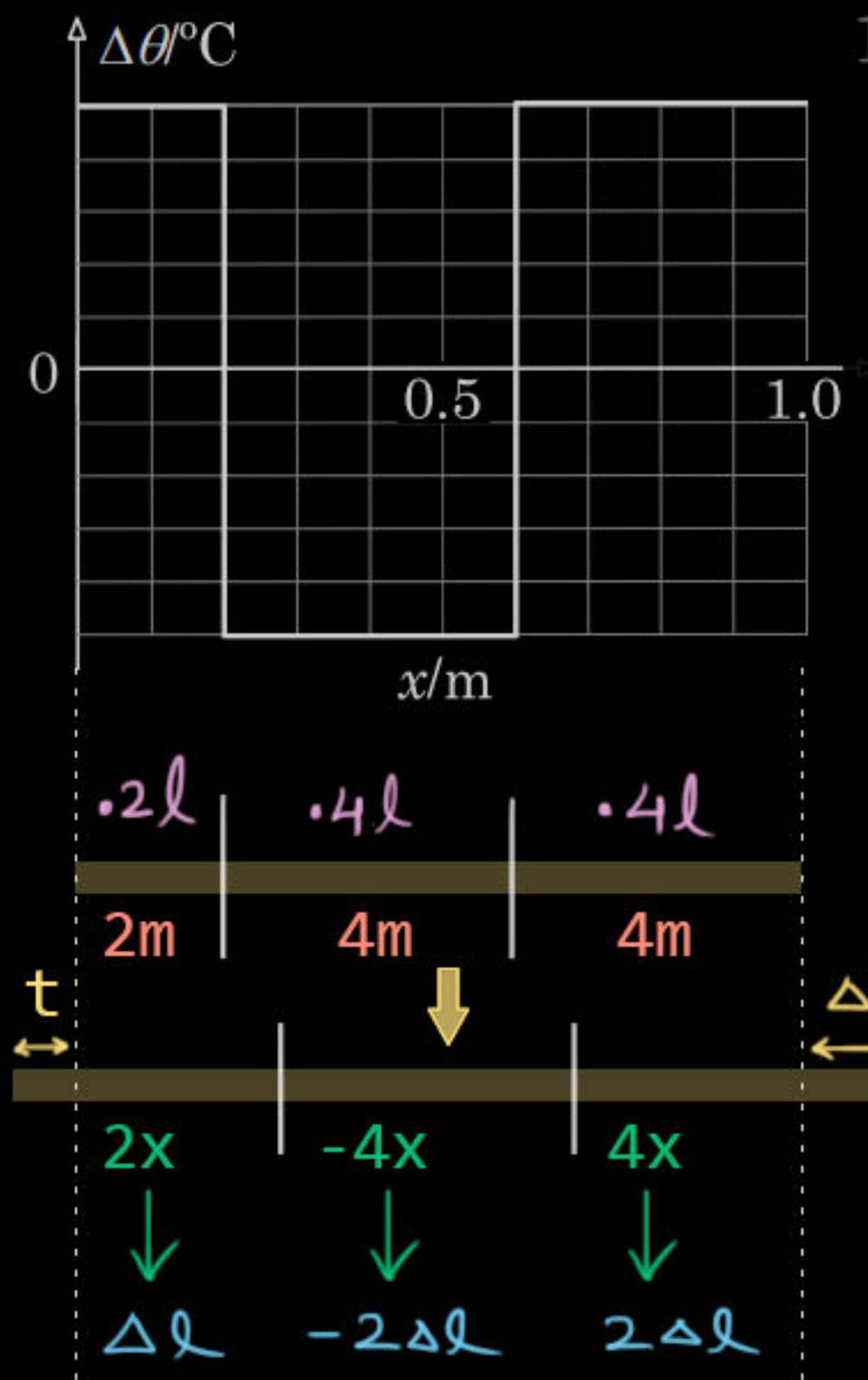
6. Air is confined in a thin-walled glass tube with the help of soap films at the ends of the tube. These films on both the ends have equal radii of curvatures. By some arrangement not shown here, these soap films are given different shapes. In the first and the second columns of the following table, different shapes of the soap films and possible nature of equilibrium of the system are shown respectively. Suggest suitable matches.

You may need the expression $V = \frac{1}{3}\pi h^2(3R - h)$. Here V is volume of a spherical cap of height h cut from a sphere of radius R .

	≡		→		Net area increases ∴ Stable
	≡		→		
	≡		→		
	≡		→		Net area reduces ∴ Unstable

by Ankit Singhvi

Youtube / Arihant Senior



1. With the help of some arrangement, different portions of a uniform metal rod of length 1.0 m placed on a frictionless horizontal tabletop are maintained at different temperatures as shown in the given graph. Here $\Delta\theta$ is deviation in temperature of a point from room temperature and x is distance of the point from the left end of the rod. After a steady state is established, total extension of the rod is found to be Δl .

- (a) Find points on the rod that do not shift. *If expand/compress in opposite dirⁿ.*
 (b) Find displacements of both the ends of the rod. *My answer is different.*

Let the left part of rod shifts left by t .

Let the extensions in three parts be $2x$, $-4x$, $4x$.

Given: $2x - 4x + 4x = \Delta l$
 $\Rightarrow 2x = \Delta l$

∴ CoM is at rest.

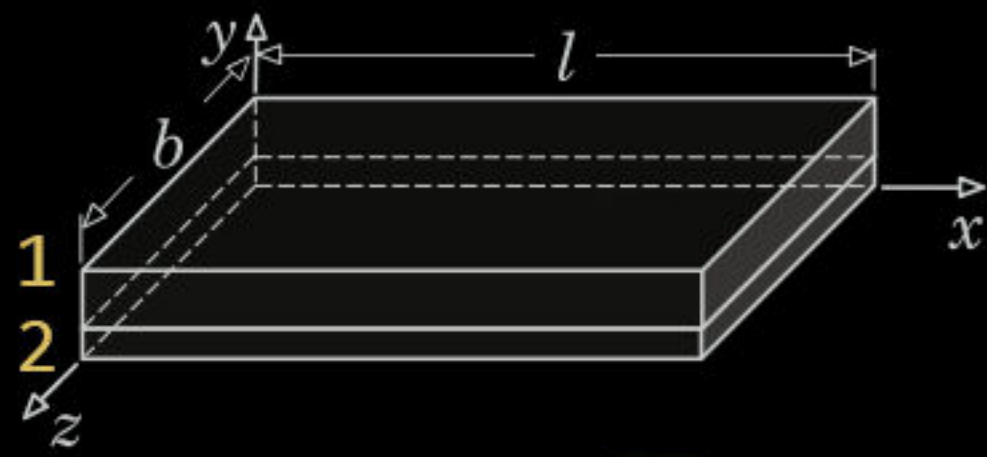
$$Mx_{cm} = \sum_{i=1}^3 m_i x_i$$

$$\frac{5}{10m} \frac{l}{2} = 2m \left[-t + \frac{(0.2l + \Delta l)}{2} \right] + 4m \left[-t + \frac{(0.2l + \Delta l) + (0.4l - 2\Delta l)}{2} \right] + 4m \left[\frac{-t + (0.2l + \Delta l) + (0.4l - 2\Delta l) + (0.4l + 2\Delta l)}{2} \right]$$

mid point coordinates of expanded parts

$$\Rightarrow 5l = -10t + 5l + \Delta l \Rightarrow t = \Delta l / 10 //$$

$l_1 + l_2 = l$ (1)
 $\frac{\Delta x_1}{l_1} = \frac{\Delta x_2}{l_2}$ (2)
 2 eqn, 2 variables



2. A steel plate and an aluminium plate each of length l and width b are welded on each other to make a composite plate. Thickness of the steel plate is t_s and that of the aluminium plate is t_a . Young's moduli of steel and aluminium are Y_s and Y_a respectively. Find force constants of the composite plate along the x , y and z -axes.

We know: $F = \frac{AY}{l} \Delta l$
 $\frac{AY}{l}$ Force constant

When pulled along X and Z axes: $k_{eq} = k_1 + k_2 = \frac{A_1 Y_1}{l_1} + \frac{A_2 Y_2}{l_2}$

$$X: = \frac{(bt_1) Y_1}{l} + \frac{(bt_2) Y_2}{l} = \frac{b}{l} (Y_1 t_1 + Y_2 t_2) //$$

$$Z: = \frac{(lt_1) Y_1}{b} + \frac{(lt_2) Y_2}{b} = \frac{l}{b} (Y_1 t_1 + Y_2 t_2) //$$

When pulled along Y axis: $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$

$$Y: \frac{1}{k_{eq}} = \frac{t_1}{(bl) Y_1} + \frac{t_2}{(bl) Y_2} \Rightarrow k_{eq} = \frac{bl Y_1 Y_2}{Y_2 t_1 + Y_1 t_2} //$$



V	ρ	ρ
$V + dV$	$\rho + d\rho$	$\rho + d\rho$

3. Consider a real fluid; compressibility of which is β and density at atmospheric pressure p_0 is ρ_0 .

(a) Express density ρ of a real fluid as a function of depth h .

(b) Express pressure p as a function of depth h in a real fluid that is open to the atmosphere.

From 1, 2, 3: A $\frac{d\rho}{\rho} = \beta dp = \beta \rho g dh$

$$\Rightarrow \int_{\rho_0}^{\rho} \frac{d\rho}{\rho^2} = \beta g \int_0^h dh$$

$$\Rightarrow \rho = \frac{\rho_0}{1 - \beta g \rho_0 h} \approx \rho_0 (1 + \beta g \rho_0 h) \quad 4$$

$$dp = \rho g dh \quad 1$$

We know: $\beta = \frac{1}{\rho} = -\frac{d\rho}{dV/V} \quad 2$

We know: $\rho V = \text{mass} = \text{constant} \Rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V} \quad 3$

From 1, 4: B $\int_{p_0}^p dP = \int_0^h \rho_0 (1 + \beta g \rho_0 h) g dh$

$$\Rightarrow p = p_0 + \rho_0 g h \left(1 + \frac{\rho_0 \beta g h}{2} \right)$$

Force on liquid inside general capillary is given by :

$$\text{perimeter} \times \sigma = \rho_w A h g$$

$$\Rightarrow h = \frac{\text{perimeter} \times \sigma}{\rho_w A g}$$

We know, minimum perimeter of a fixed area shape is $2\pi r$. i.e, when its circular.

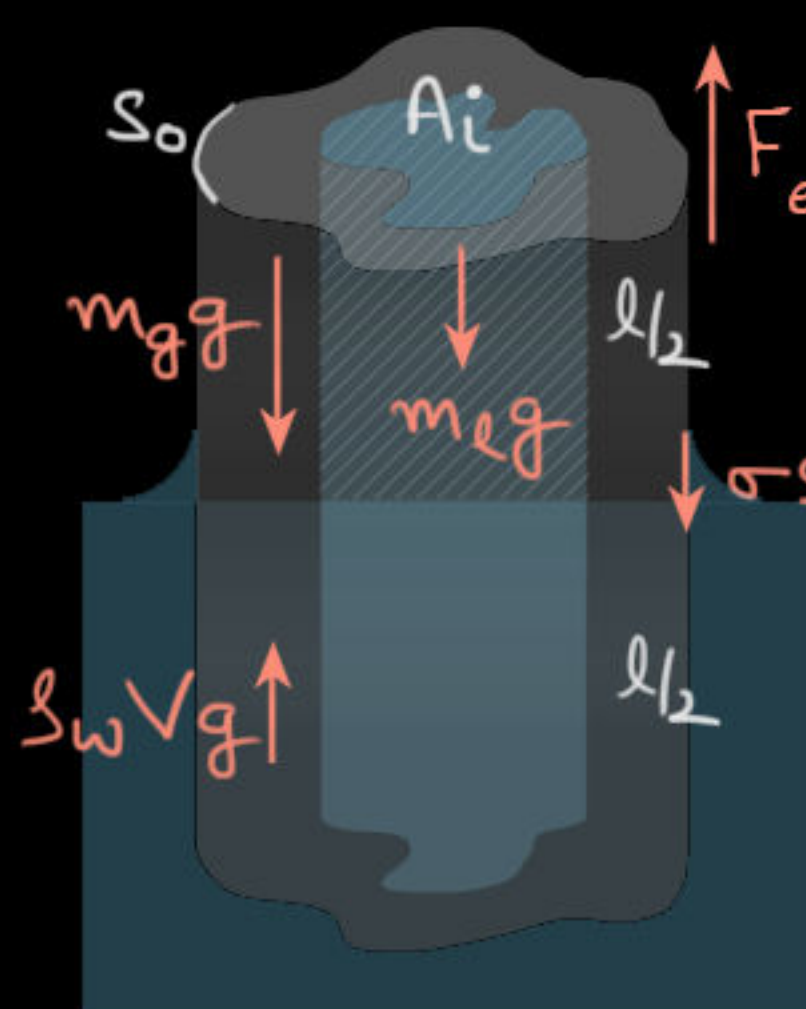
$$\therefore h_{\min} = \frac{2\pi r \sigma}{\rho_w \pi r^2 g} = \frac{2\sigma}{\rho_w r g} = \frac{2\sigma}{\rho_w \sqrt{\frac{A}{\pi}} g}$$

$$= \frac{2(0.07)}{1000 \sqrt{\frac{0.2 \times 10^{-6}}{\pi}} g} = 5.5 \text{ cm}$$

Available length given = 3 cm

Water will fill up to brim.

4. A capillary made of a glass sheet has length $l = 6 \text{ cm}$, mass $m = 12 \text{ g}$, inner sectional area $A_i = 0.2 \text{ mm}^2$ and outer perimeter $s_o = 16 \text{ mm}$. Half-length of this capillary is vertically dipped in water. Surface tension of water is $\sigma = 0.07 \text{ N/m}$, density of water is $\rho_w = 1.00 \text{ g/cm}^3$, density of glass of the capillary is $\rho_g = 2.00 \text{ g/cm}^3$ and acceleration of free fall is $g = 10.00 \text{ m/s}^2$. How much additional force is required to keep the capillary in equilibrium? Assume water a perfect wetting liquid for glass.



Sol

$$F_{\text{ext}} = F_w + F_{gl} + F_{ST}$$

$$= m_l g + m_g g - \rho_w V g + \sigma s_o$$

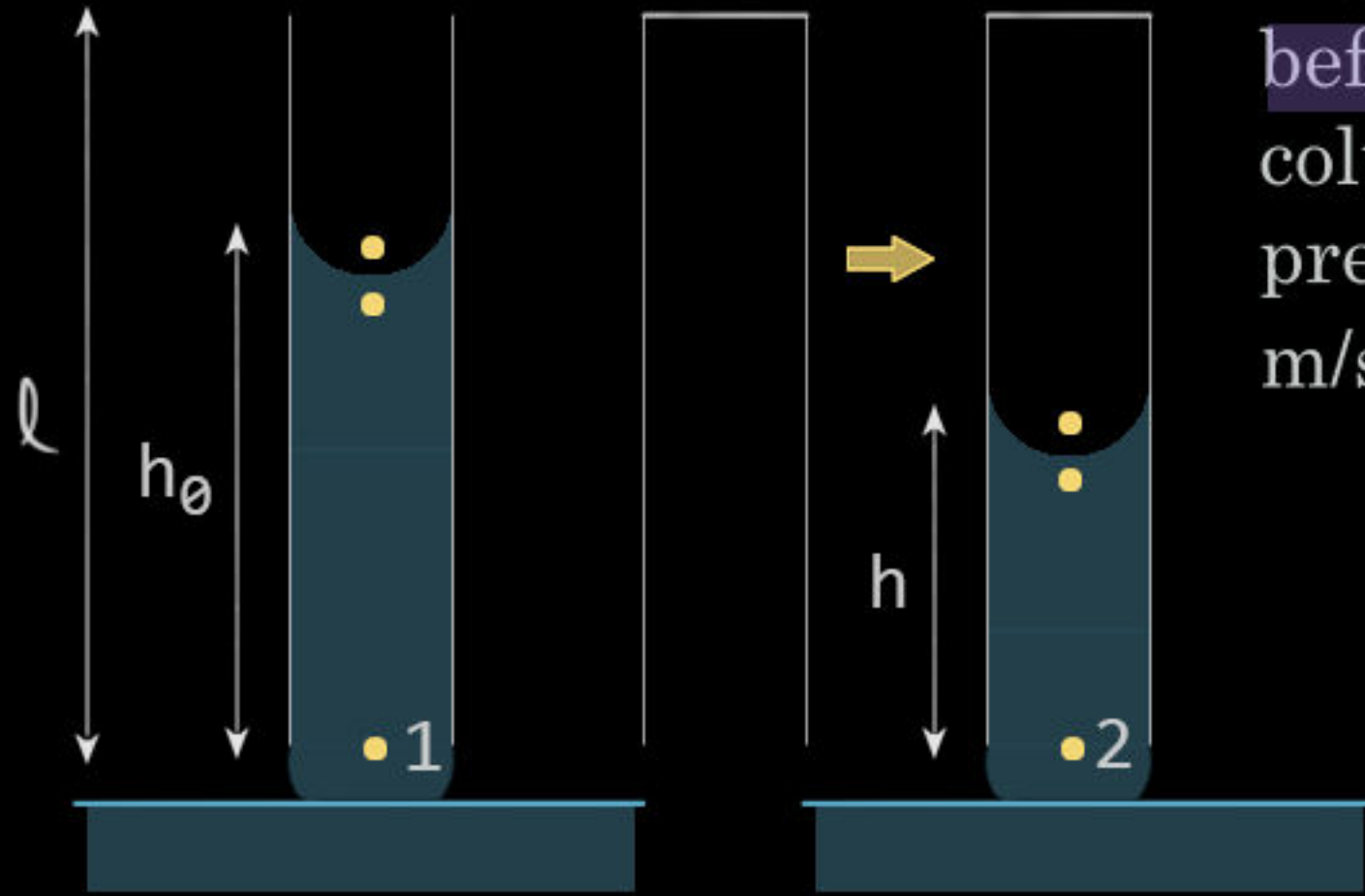
In top half Total Bottom half On outer surface

$$= \rho_w \left(A_i \frac{l}{2} \right) g + \left[m g - \rho_w \left(\frac{1}{2} \frac{m}{\rho_g} \right) g \right] + \sigma s_o$$

$$= \frac{\rho_w A_i l g}{2} + \frac{3mg}{4} + \sigma s_o$$

Similar: PF, MCQ 3

5. When a glass capillary of length $l = 1.01 \text{ m}$ is held vertically, touching water surface of a large reservoir at its lower end, water rises in the capillary to a height $h_0 = 1.10 \text{ cm}$. If top end of the tube were closed before touching the water surface, what would have been height of water column in the capillary? Density of water is $\rho = 1000 \text{ kg/m}^3$, atmospheric pressure is $p_0 = 1.01 \times 10^5 \text{ Pa}$ and acceleration due to gravity is $g = 10 \text{ m/s}^2$. Consider water a perfect wetting liquid for glass.



Glass maintains the temperature.

$$\therefore PV = \text{constant}$$

$$\Rightarrow |dP| = \left| P \frac{dV}{V} \right| (\because \text{given } dV \text{ is small})$$

$$= p_0 \frac{(hA)}{lA} = p_0 \frac{h}{l}$$

$$P_1 = P_2 \neq P_0$$

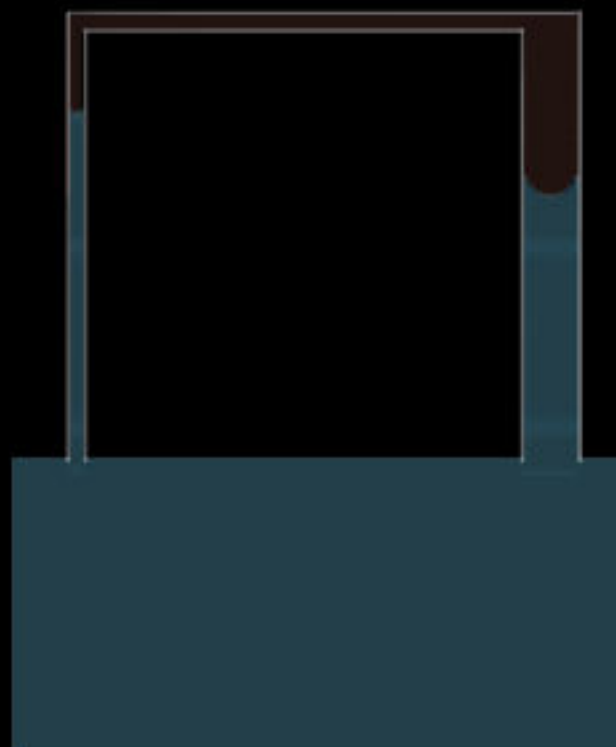
Not dipped!

$$\Rightarrow \cancel{p_0} - \cancel{\frac{2s}{r}} + \rho g h_0 = (\cancel{p_0} + dP) - \cancel{\frac{2s}{r}} + \rho g h$$

$\downarrow$
 $p_0 \frac{h}{l}$

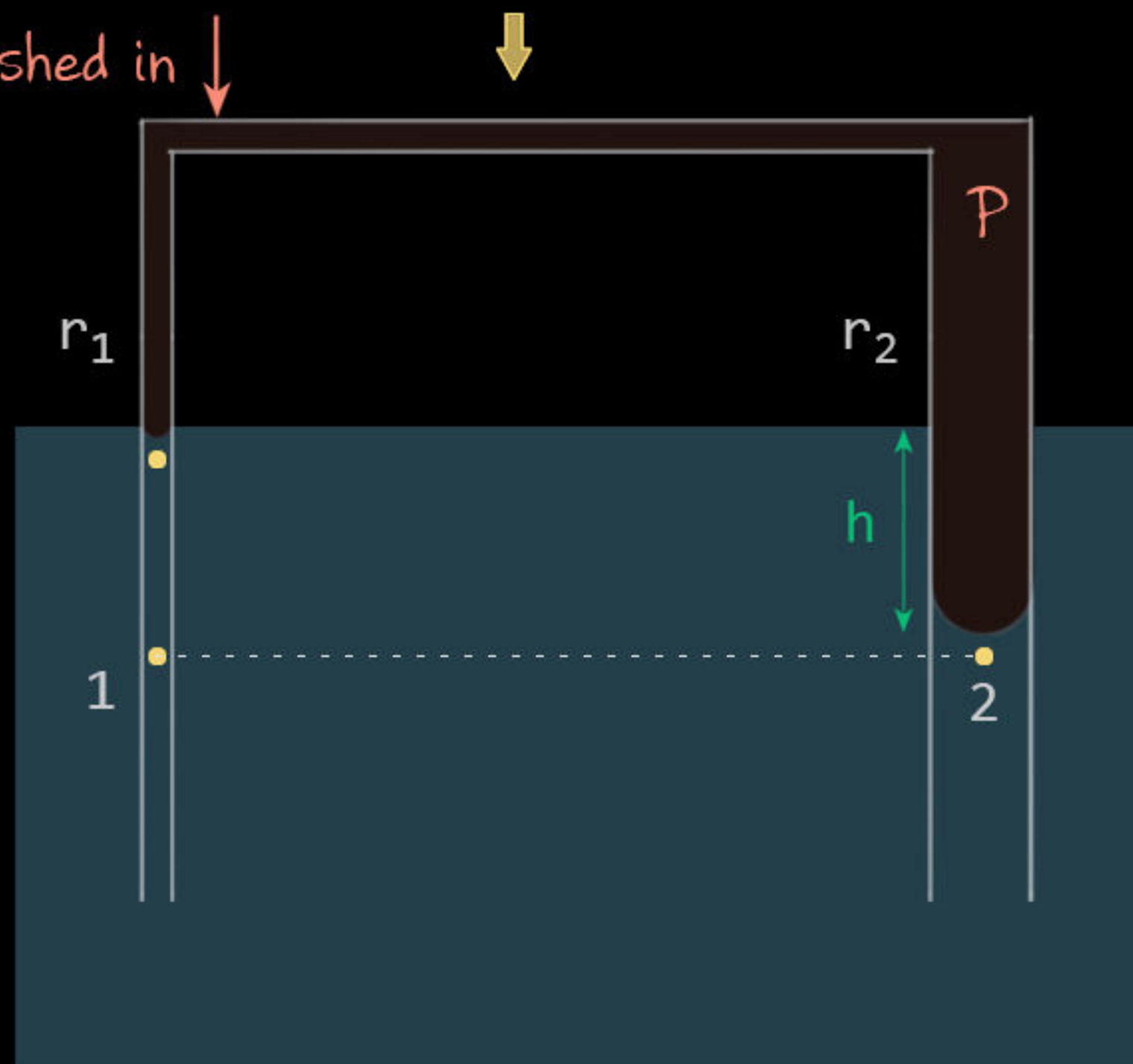
$$\Rightarrow h = \frac{h_0}{\frac{p_0}{\rho g l} + 1} //$$

just dipped



6. A U tube, vertical arms of which have length l and radii r_1 and r_2 ($r_2 > r_1$), is dipped in water with its open ends down to such an extent that the water level in the narrow arm coincides with that of outside the tube. Surface tension and density of water are σ and ρ respectively and atmospheric pressure is p_0 . Consider volume of the horizontal part connecting both the vertical arm to be negligible, find water level in the wider arm relative to the water level outside. Assume water a perfect wetting liquid for glass.

pushed in

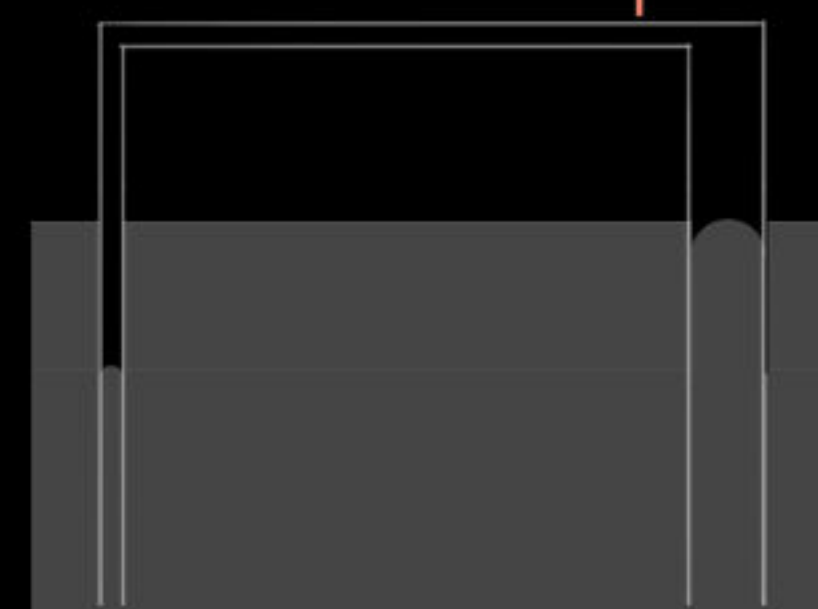


$$P_1 = P_2$$

$$\Rightarrow \rho - \frac{2\sigma}{r_1} + \rho g h = \rho - \frac{2\sigma}{r_2}$$

$$\Rightarrow h = \frac{2\sigma}{\rho g} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

pulled out



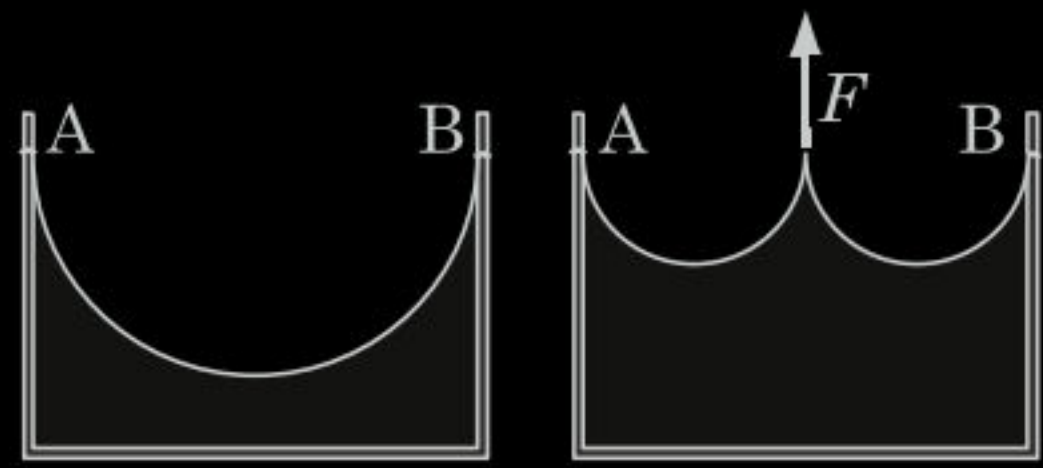
Note: Effect is opposite if it were mercury instead of water.

U tube then has to be pulled up, so mercury column in wider tube will match the surface level.

Level in thinner tube will still be lower than surface then.

by Ankit Singhvi

Youtube / Arihant Senior



7. A film of a soap solution created in a loop formed by a rectangular wire frame and an inextensible light thread AB of length l , pulls the thread in the shape of a semicircle. If a force F applied at midpoint of the thread perpendicular to line segment AB turns the wire into two semicircles as shown in the figure, calculate surface tension of the soap solution.

As the thread is forming perfect circle, whole assembly has to be horizontal,

F_{net} due to Surface tension on semicircular thread.

$$dF = 2S(R d\theta)$$

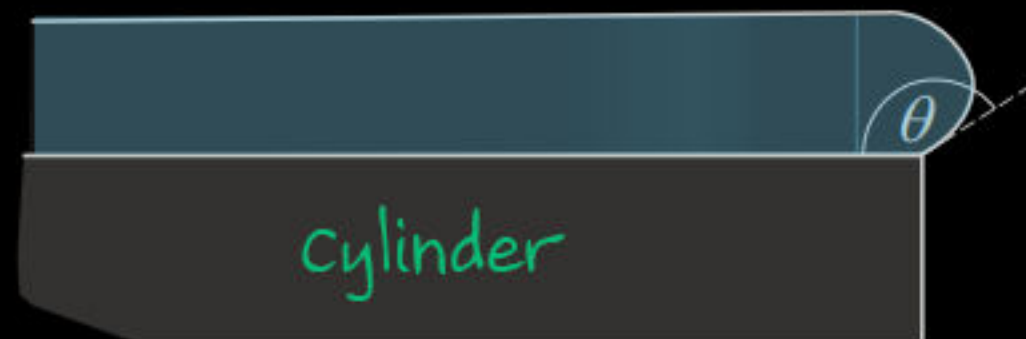
$$\Rightarrow F_y = \int_{-\pi/2}^{\pi/2} 2S(R d\theta) \cos \theta$$

$$\Rightarrow F_y = 4SR \quad F_x = 0$$

Sol: Net force and torque on thread is zero.

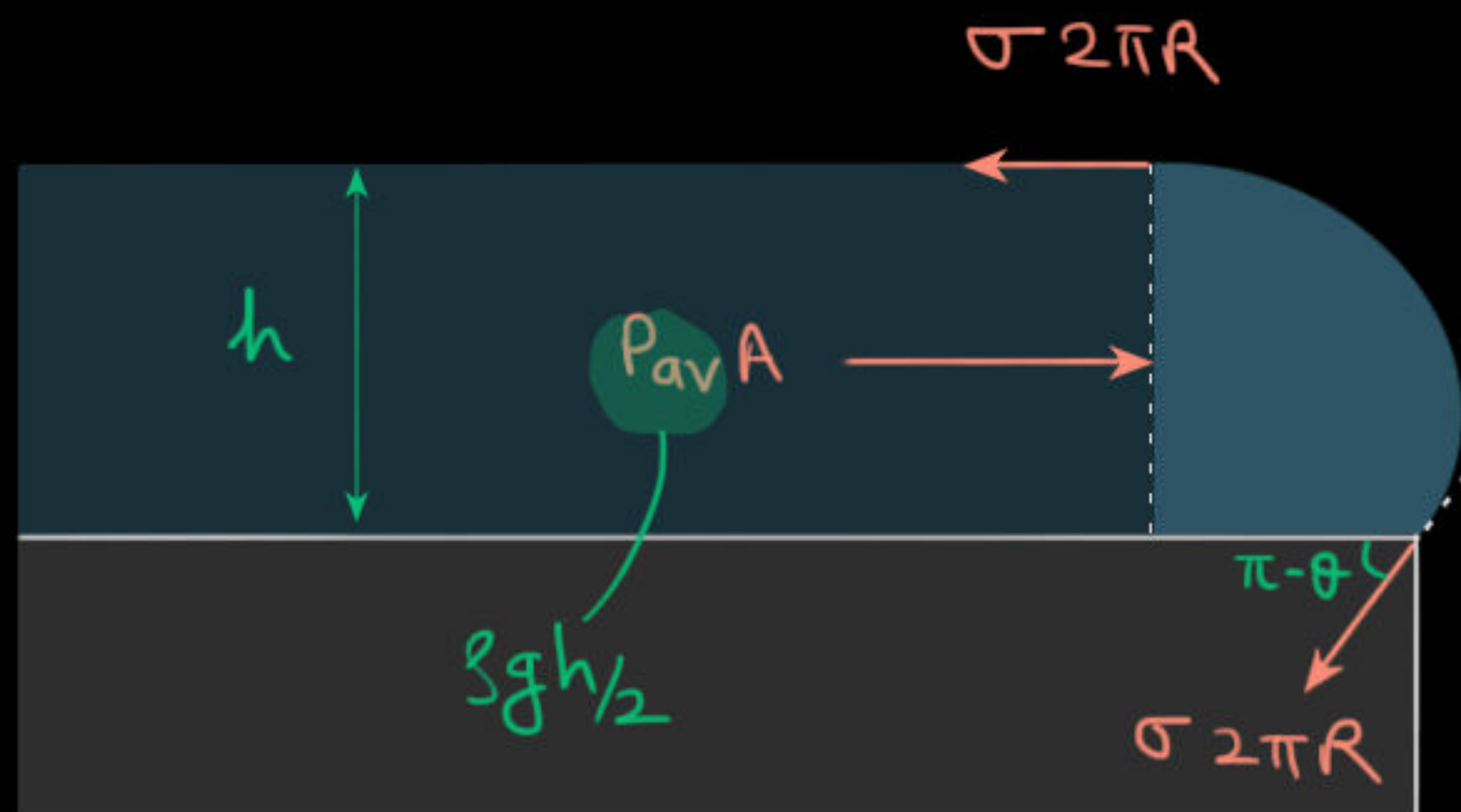
$$\therefore F = 4Sr$$

$$\Rightarrow S = \frac{F}{4r} = \frac{F}{4 \cdot \frac{l}{2\pi}} = \frac{\pi F}{2l}$$

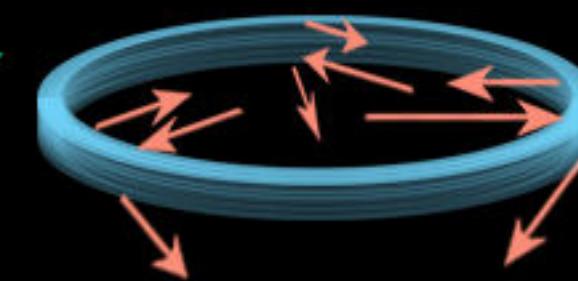


8. An amount of water is poured on the top horizontal circular face of a glass cylinder. Excess amount of water spills over the edges and a little amount of water remains there in a thin uniform layer, a portion of which is shown in the figure. If contact angle is θ and surface tension of water is σ , determine thickness of the water layer left.

Showing radial forces on outer ring.



Balancing radial forces on outer ring of water.

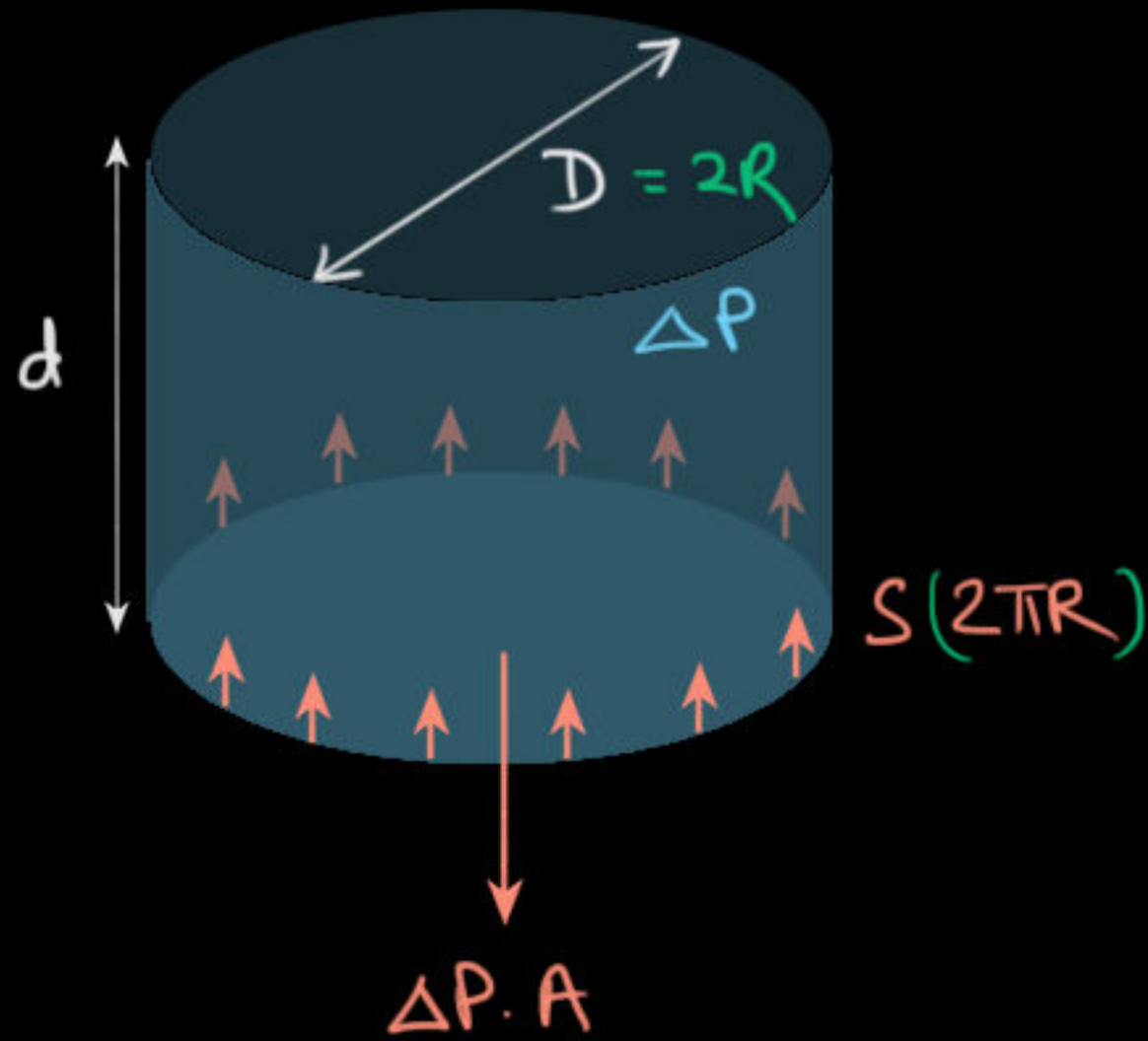


$$\cancel{\sigma 2\pi R} + \cancel{\sigma 2\pi R} \cos(\pi - \theta) = \rho g \frac{h}{2} \cdot \cancel{2\pi R h}$$

$$\Rightarrow h^2 = \frac{2}{\rho g} \sigma (1 - \cos \theta)$$



9. In a zero gravity region, a drop of a liquid of surface tension σ assumes a cylindrical shape of diameter D between two parallel glass plates that are a distance d apart. If curved surface of the drop is at right angles to the plates as shown in the figure, find force exerted by the drop on a plate.



$$\Delta P = \underbrace{\sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)}_{\substack{R \\ \infty}} = \frac{\sigma}{R}$$

Drop is exerting two forces on lower plate.

1	2
Due to S.T	Due to ΔP
↓	↓

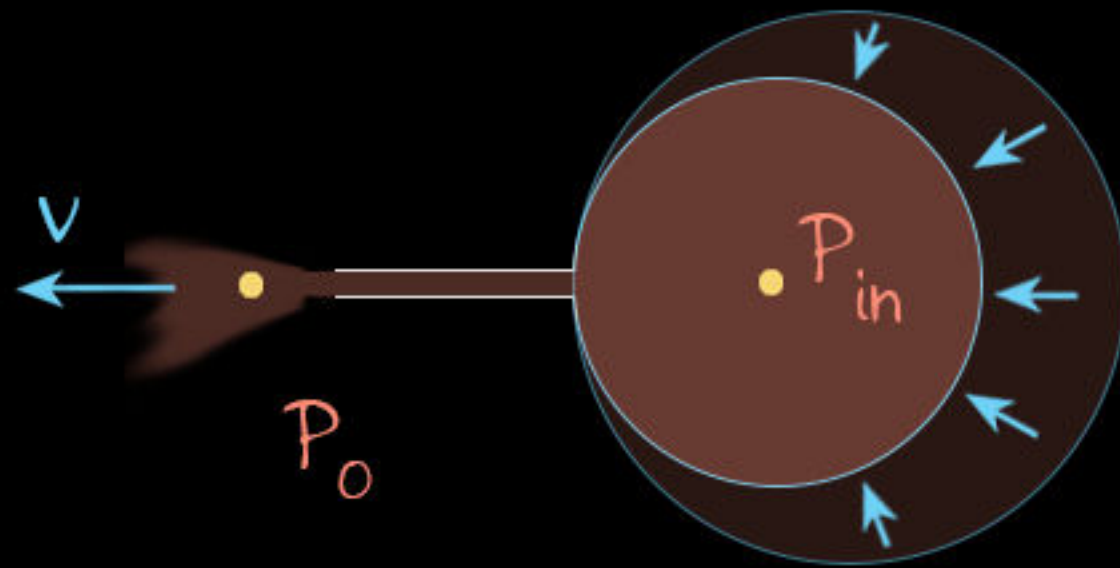
$$\therefore \uparrow F_{\text{net}} = S \cdot 2\pi R - \Delta P A$$

$$= S \cdot 2\pi R - \frac{S}{R} A \quad \text{--- } \pi R^2$$

$$= S \pi R$$

$$= \frac{\pi \sigma D}{2} //$$

+ve, $\therefore$ Plates will attract each other.



10. A bubble of radius r is made of a soap solution at one end of a long capillary tube, other end of which is closed. If the closed end is opened, the bubble collapses in a time interval Δt_0 . Making the following assumptions, estimate time interval taken by another bubble of radius ηr made from the same soap solution at the end of the same tube to collapse.

- Air is non-viscous and its flow in the capillary is laminar.
- The process is slow enough to regard it a quasi-static one and disregard effects of viscosity and time required for liquid in the wall of the bubble to readjust its shape.

With given assumptions, we can use bernoulli's equation.

$$P_o + \frac{1}{2} \rho v^2 = P_{in}$$

$$\Rightarrow \frac{1}{2} \rho v^2 = \frac{4S}{R}$$

$$\Rightarrow v \propto 1/\sqrt{R} \quad (1)$$

Volume flow rate:

$$-\frac{dV}{dt} = vA$$

$\frac{4}{3}\pi R^3$

$$\Rightarrow -4\pi R^2 \frac{dR}{dt} = vA$$

$$\Rightarrow v \propto R^2 dR/dt \quad (2)$$

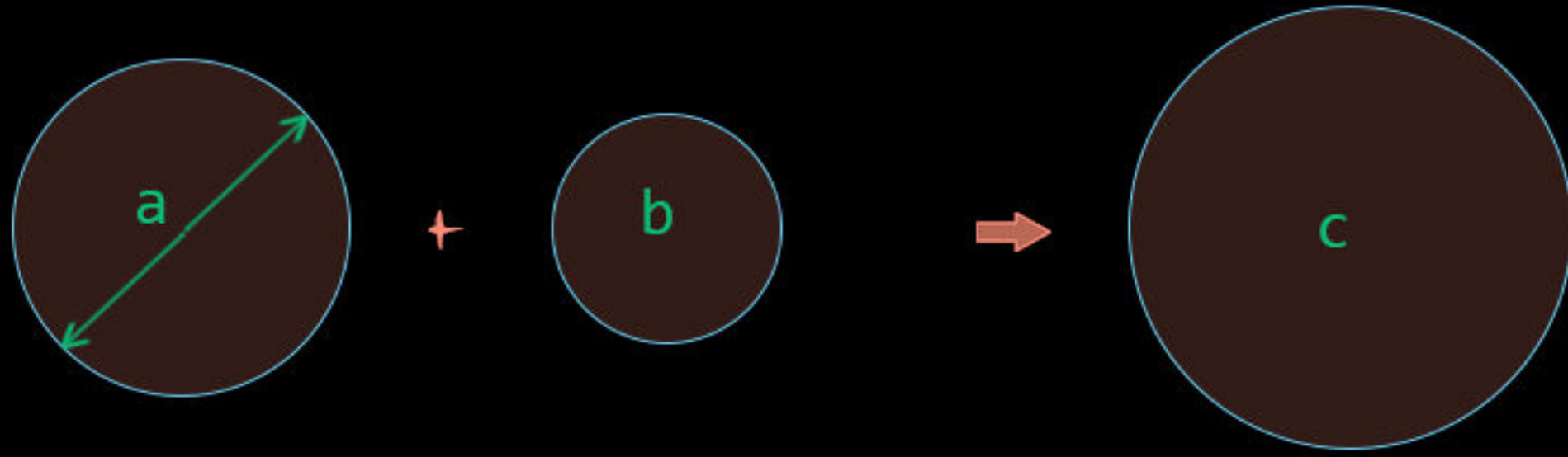
From 1,2: $\frac{1}{\sqrt{R}} \propto R^2 \frac{dR}{dt}$

$$\Rightarrow \int R^{5/2} dR \propto \int dt$$

$$\Rightarrow R^{7/2} \propto t$$

$$\therefore \frac{\Delta t_1}{\Delta t_2} = \frac{R^{7/2}}{(\eta R)^{7/2}} \Rightarrow \Delta t_2 = \eta^{7/2} \Delta t_1$$

11. Two soap bubbles of diameters a and b coalesce to form a larger bubble of diameter c . If surface tension of the soap solution used is σ , find a suitable expression for the atmospheric pressure.



$$n_1 + n_2 = n$$

$$\Rightarrow P_1 V_1 + P_2 V_2 = P V$$

$$\Rightarrow \left(P_0 + \frac{4s}{a/2} \right) \frac{4}{3} \pi \left(\frac{a}{2} \right)^3 + \left(P_0 + \frac{4s}{b/2} \right) \frac{4}{3} \pi \left(\frac{b}{2} \right)^3 = \left(P_0 + \frac{4s}{c/2} \right) \frac{4}{3} \pi \left(\frac{c}{2} \right)^3$$

$$\Rightarrow \left(P_0 + \frac{8s}{a} \right) a^3 + \left(P_0 + \frac{8s}{b} \right) b^3 = \left(P_0 + \frac{8s}{c} \right) c^3$$

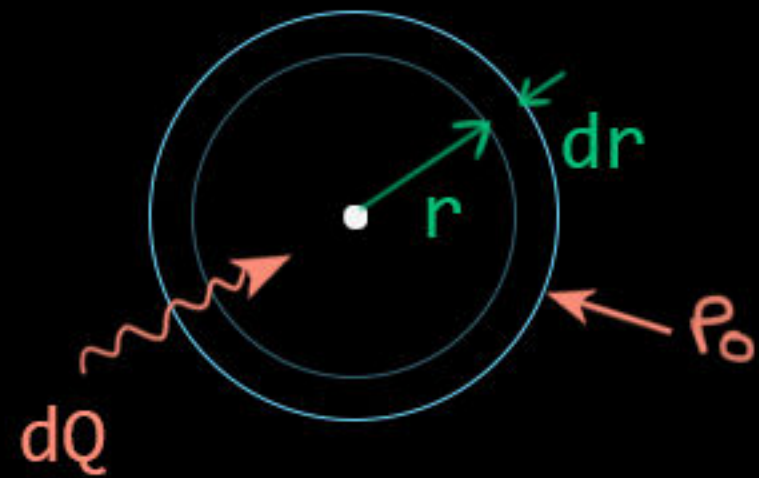
$$\Rightarrow P_0 = \frac{8s(a^2 + b^2 - c^2)}{(c^3 - a^3 - b^3)} //$$

Method 1 Irodov 2.182

Considering only gas.

Method 2

Considering gas and bubble as system.



12. An amount of heat is slowly supplied to a bubble of radius r made of a soap solution of surface tension σ . If pressure p_0 outside the bubble remains constant, find an expression for the molar specific heat of the heating process. Molar specific heat of the gas inside the bubble at constant pressure is C_P and universal gas constant is R .

$$PV = nRT$$

$$\Rightarrow \left(P_0 + \frac{4S}{r} \right) \frac{4}{3} \pi r^3 = RT$$

↓ diff.

$$\frac{dr}{dT} = \frac{R}{4\pi r^2} \left(\frac{1}{P_0 + \frac{8S}{3r}} \right)$$

$$\begin{aligned} Q + W_{\text{ext}} &= \Delta U \\ \begin{matrix} \swarrow & \searrow \\ nC dT & P_0 dV \end{matrix} & \quad \begin{matrix} \swarrow & \searrow \\ \text{gas} & \text{bubble} \end{matrix} \\ \Rightarrow nC dT - P_0 4\pi r^2 dr &= nC_v dT + 16\pi S r dr \end{aligned}$$

$$\Rightarrow C = C_v + \left(P_0 + \frac{4S}{r} \right) 4\pi r^2 \frac{dr}{dT} = C_v + \left(\frac{P_0 + 4S/r}{P_0 + 8S/3r} \right) R = C_P + \frac{4SR}{3P_0 r + 8S} //$$

13. A mercury drop on a clean horizontal glass surface and a water drop on a horizontal glass plate coated with a thin layer of talcum powder are **Not same** ← similar in shape. If it is known that for a particular shape of a drop, surface energy of surface tension and the gravitational potential energy bear a definite ratio that is independent of the size of the drop, estimate the ratio masses of these drops.

Density of mercury is $\rho_H = 13.60 \text{ g/cm}^3$, density of water is $\rho_W = 1.00 \text{ g/cm}^3$, surface tension of mercury is $\sigma_H = 0.46 \text{ N/m}$ and surface tension of water is $\sigma_W = 0.07 \text{ N/m}$.



Let CoM of a sample drop be at x

Then, $A \propto x^2$ ①

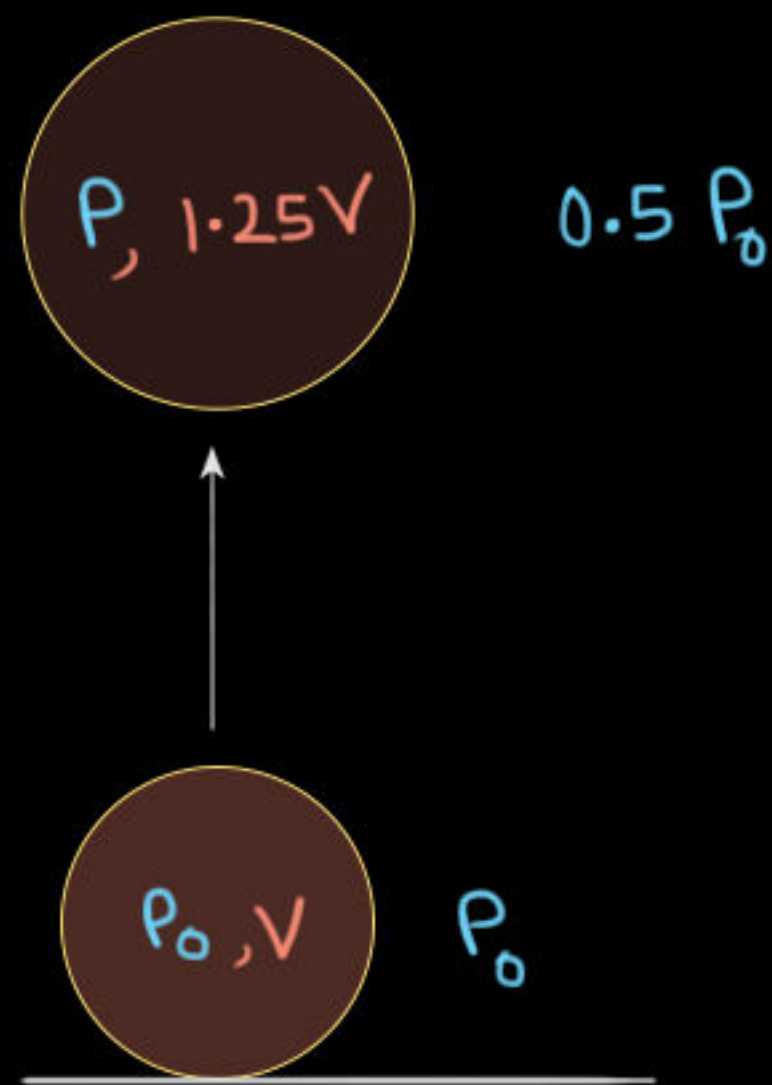
and, $m \propto x^3$ ②

Given: $\sigma A \propto mgx \Rightarrow \sigma A \propto mx$ ③

Eliminating A and x from 1,2,3: $m \propto \left(\frac{\sigma}{g}\right)^3$

$\Rightarrow m \propto \sigma^{3/2} / g^{1/2}$

$\therefore \frac{m_1}{m_2} = \left(\frac{\sigma_1}{\sigma_2}\right)^{3/2} \left(\frac{g_2}{g_1}\right)^{1/2}$



Let final radius be R .

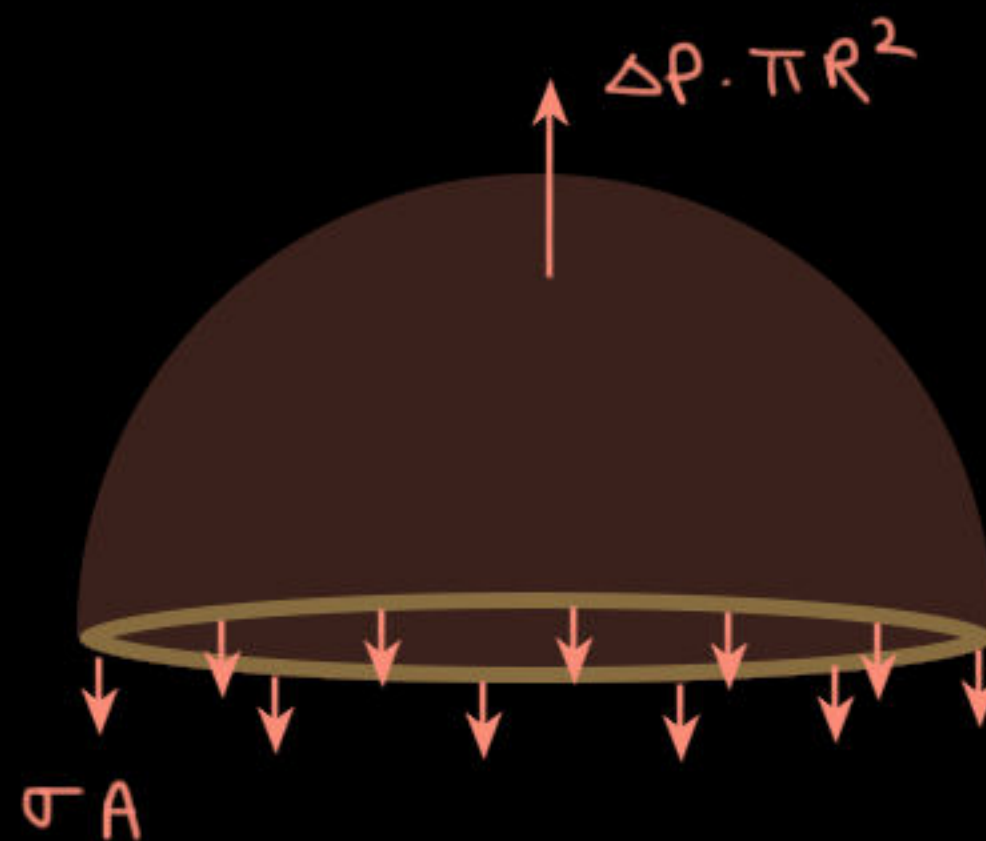
$$\frac{4}{3} \pi R^3 = 1.25V \quad (1)$$

Isothermal

$$\Rightarrow P(1.25)V = P_0 V$$

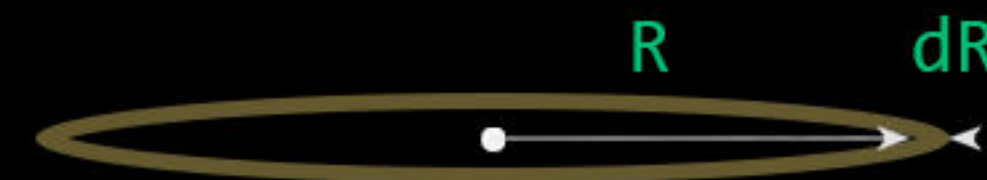
$$\Rightarrow P = 0.8 P_0 \quad (2)$$

15. A balloon filled with helium at the atmospheric pressure p_0 occupying volume V , when released rises up and bursts at an altitude where atmospheric pressure is $0.5p_0$. Immediately before bursting, the balloon has a volume of $1.25V$. If rubber used in making the balloon has mass m and density ρ , find breaking stress of the rubber. Assume the balloon always spherical in shape, the density of the rubber constant and temperature outside uniform and constant.



$$\Delta P \pi R^2 = \sigma A$$

$$\Rightarrow (P - 0.5 P_0) \pi R^2 = \sigma A \quad (3)$$



$$A = 2\pi R dR \quad (4)$$

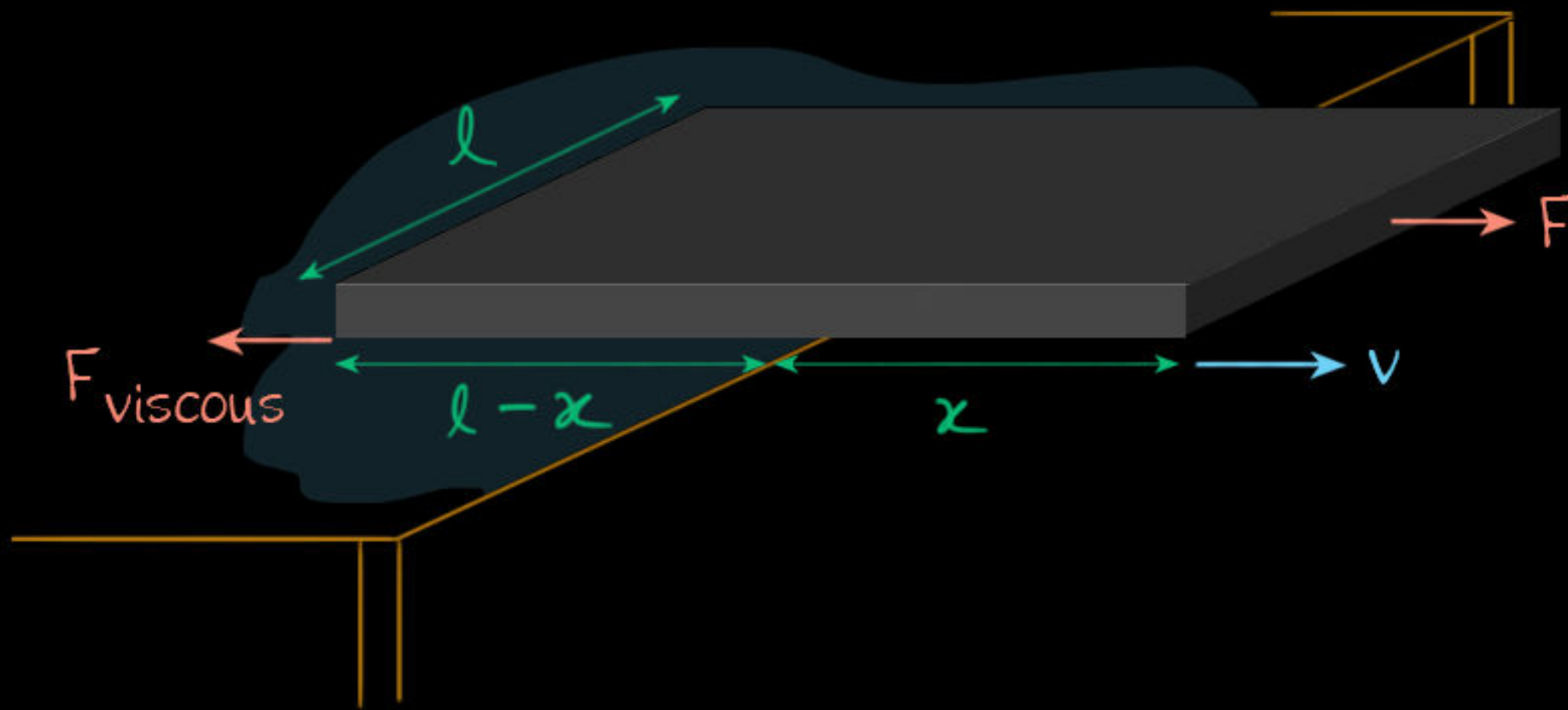
$$m = \int 4\pi R^2 dR \quad (5)$$

From 1,2,3,4,5:

$$\sigma = \frac{g}{16} \frac{P_0 8V}{m} //$$

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16. An almost inertia-less metal sheet is placed on a horizontal tabletop lubricated with oil. The sheet is a square of side length $l = 1.0$ m and the oil layer has thickness $h = 1.0$ mm. Initially one edge of the sheet coincides with one edge of the table. The sheet is pulled outwards without rotation with a constant force $F = 15$ N. If coefficient of viscosity of the oil is $\eta = 0.2$ N·s/m², how long will it take to pull half of the sheet out of the table?

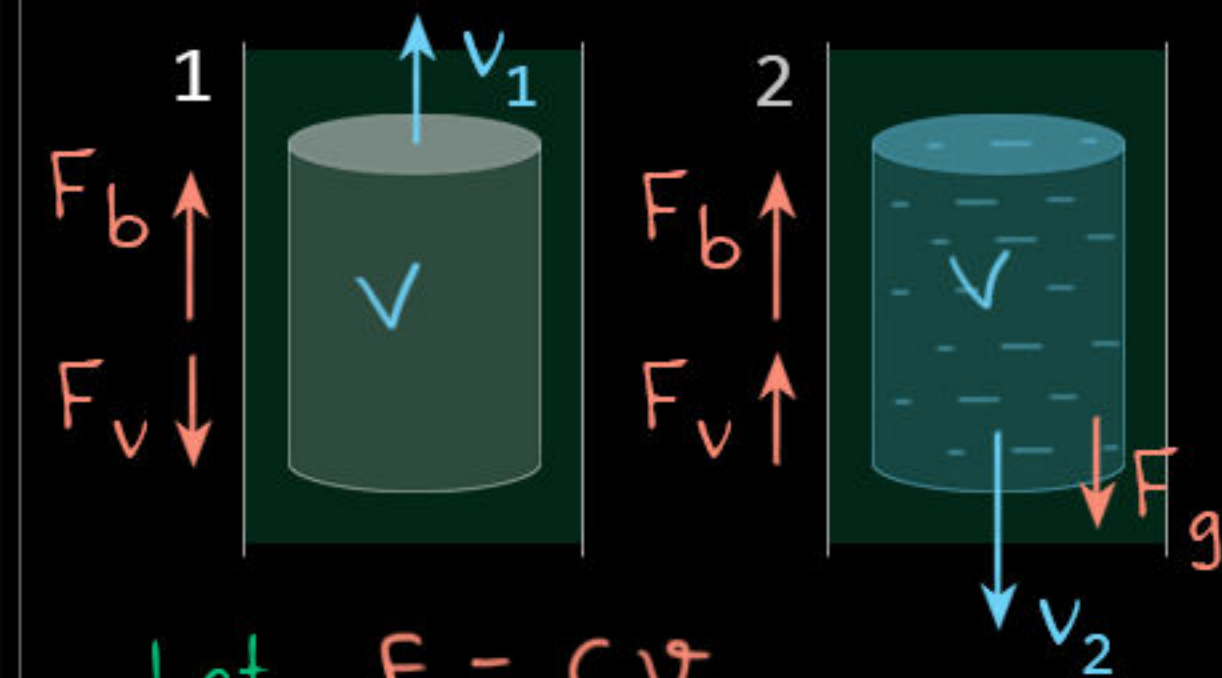


∴ massless, $F_{\text{net}} = 0$

$$\Rightarrow F = \eta A \frac{v}{h} = \eta l \frac{(l-x)}{h} \frac{dx}{dt}$$

$$\Rightarrow F \int_0^t dt = \frac{\eta l}{h} \int_0^{l/2} (l-x) dx$$

$$\Rightarrow t = \frac{3\eta l^3}{8Fh}$$



17. A cylindrical capsule can move inside a long vertical cylindrical tube filled with a viscous Newtonian liquid. Diameter of the capsule is slightly smaller than the inner diameter of the tube and walls of the capsule have negligible thickness. Drag force on the capsule is proportional to speed of the capsule. If the capsule is empty, it rises up with a uniform velocity v_1 and if the capsule is completely filled with the liquid, it moves down with a uniform velocity v_2 .

Let, $F_v = C v$

Cylinder is given as thin.

So massless. So, $F_{net} = 0$

Let liquid density be ρ_l

$\therefore 1: \rho_l V g = C v_1$

And, $2: \rho_l V g - \rho_l V g = C v_2$

$3: \rho_l (\eta V) g - \rho_l V g = C v_3$ it will become

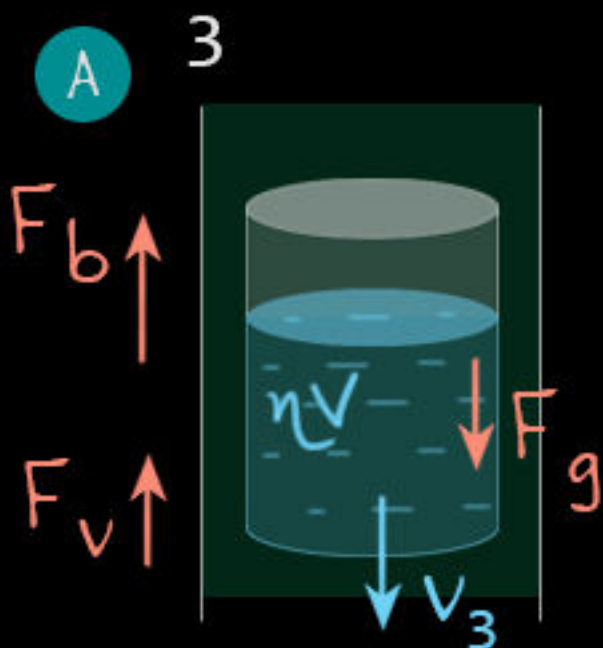
$B \quad k \rho_l (\eta V) g - \rho_l V g = C v_4$

$\Rightarrow \eta (C v_2 + \rho_l V g) - \rho_l V g = C v_3$

$\Rightarrow v_4 = \eta k (v_1 + v_2) - v_1 \downarrow$

$\Rightarrow \eta (C v_2 + C v_1) - C v_1 = C v_3$

$\Rightarrow v_3 = \eta (v_2 + v_1) - v_1 \downarrow$



1. A particle of mass m can move in a force field. Potential energy U varies on location of the particle according to the following equation.

$$U = -k(x-a)^2(x-b)^2$$

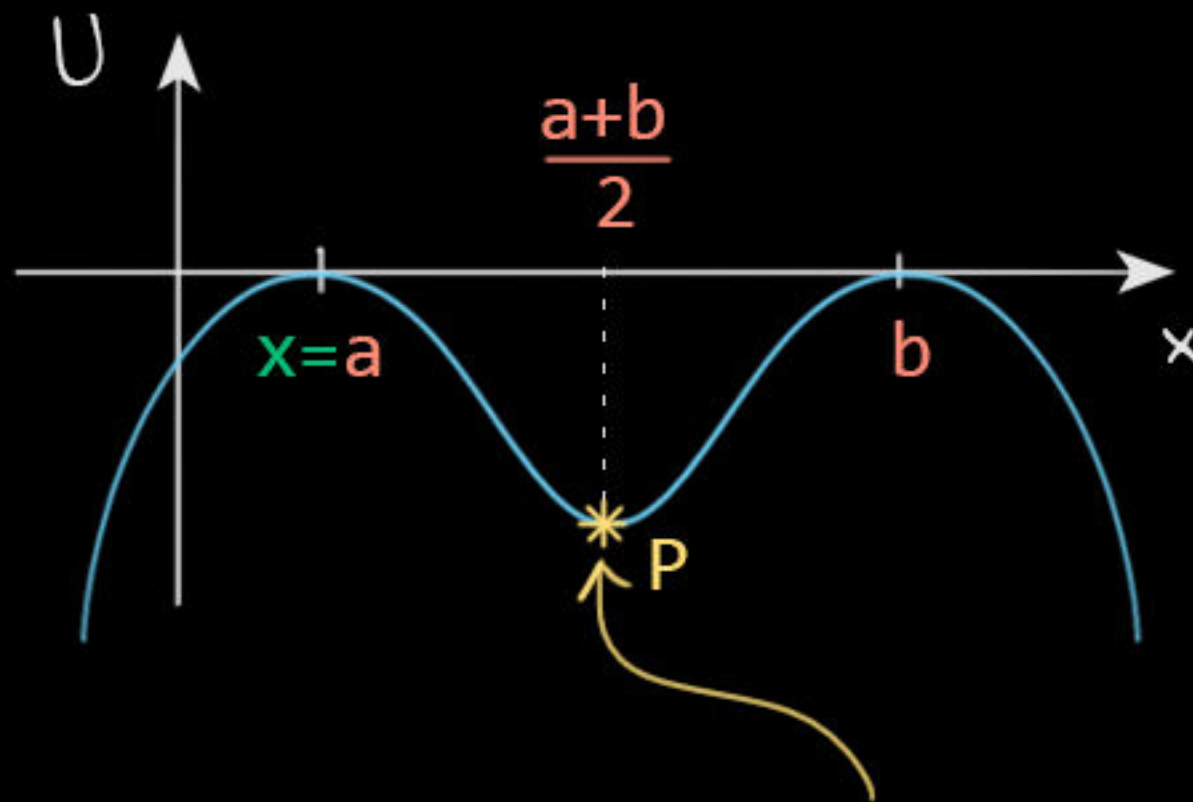
Here k , a and b are positive constants.

What can you conclude about small amplitude oscillations of the particle?

$$F = -\frac{dU}{dx} = 2k(x-a)(x-b)[2x-(a+b)]$$

- ✓ (c) It can oscillate about a specific location with angular frequency $(b-a)\sqrt{k/m}$.

Lets plot $U-x$, assuming $b > a$.



It can oscillate only here.
(∵ its the only minima)

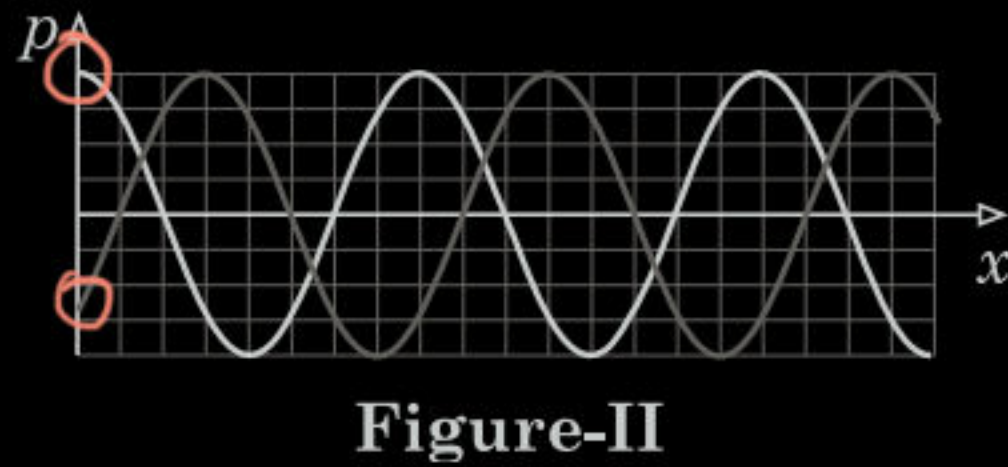
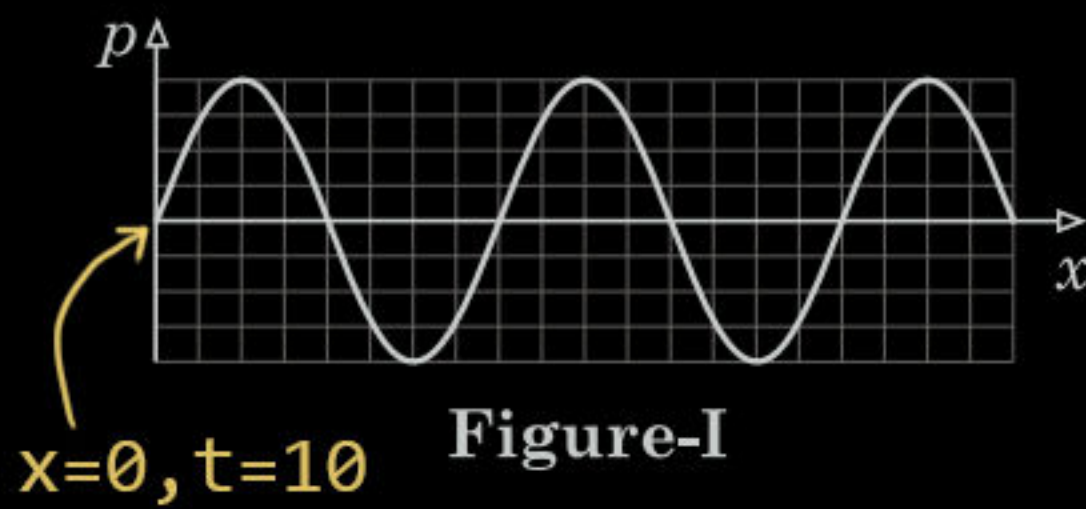
Lets displace it a bit from P. i.e, $x \rightarrow \frac{a+b}{2} + dx$

$$\begin{aligned} \therefore F_{\text{restoring}} &= |2k(x-a)(x-b)[2x-(a+b)]| \\ &= \underbrace{2k(b-a)^2}_{k_{eq}} (dx) \end{aligned}$$

$$\begin{aligned} \therefore T &= \sqrt{\frac{m}{k_{eq}}} \\ &= \sqrt{\frac{m}{k(b-a)^2}} \end{aligned}$$

by Ankit Singhvi

Youtube / Arihant Senior



2. In figure-I, is shown acoustic pressure variation with position in a harmonic sound wave travelling in the positive x -direction. Period of the sound wave is 8 s. Figure-I was recorded at $t = 10$ s. In figure-II are shown variation in acoustic pressure by two curves recorded at two successive instants of time $t = t_1$ and then $t = t_2$. Both the figures are drawn to the same scale. Its not given which is which in graph
- Which of the following sets correctly express possible values of the instants t_1 and t_2 ?

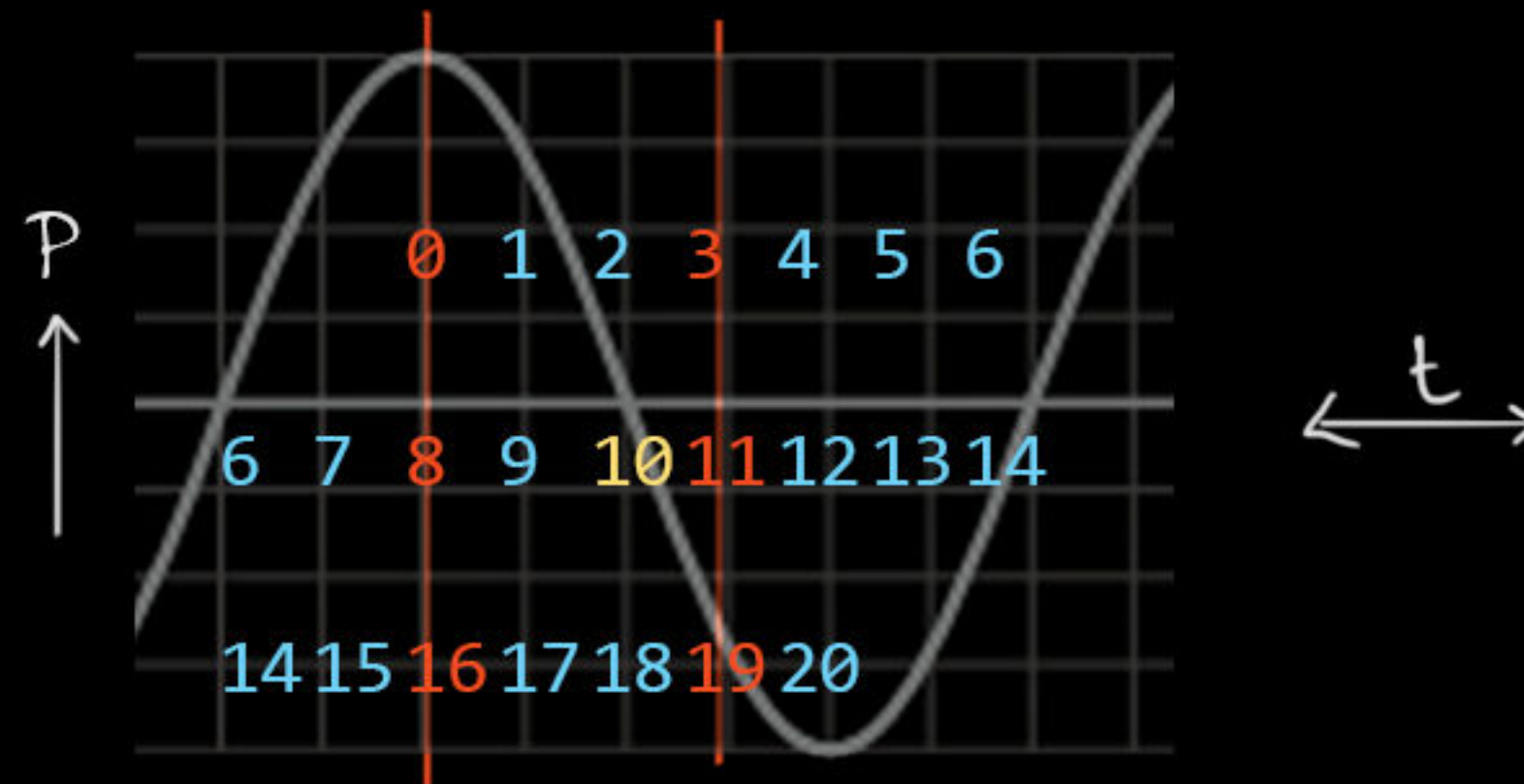
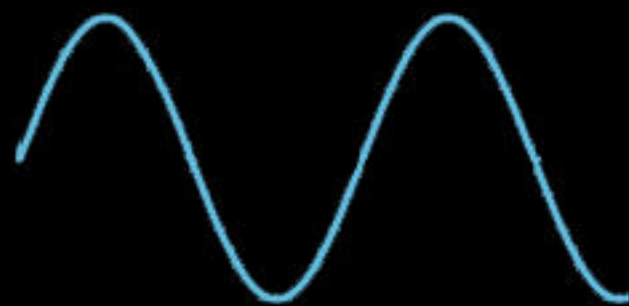
✓(a) $t_1 = 0$ s and $t_2 = 3$ s

✓(b) $t_1 = 11$ s and $t_2 = 16$ s

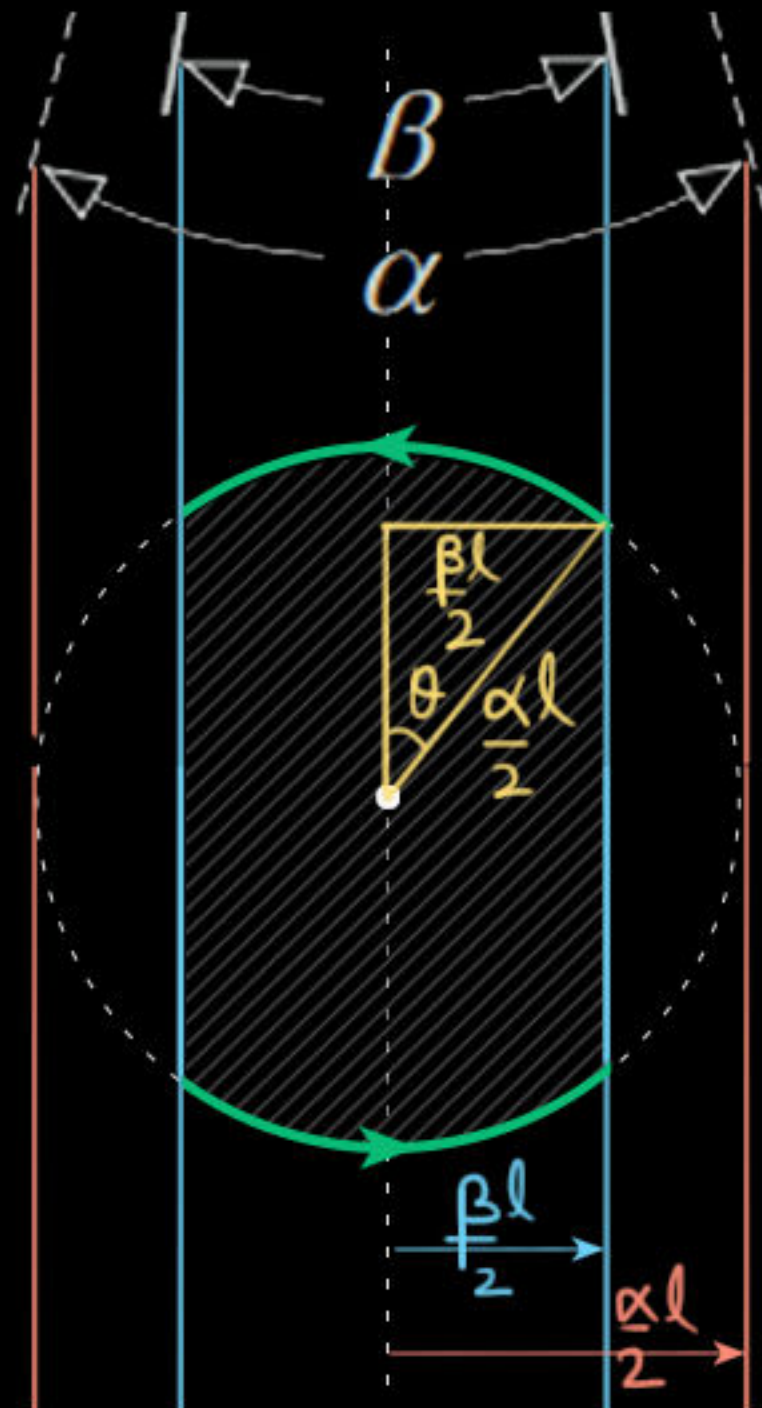
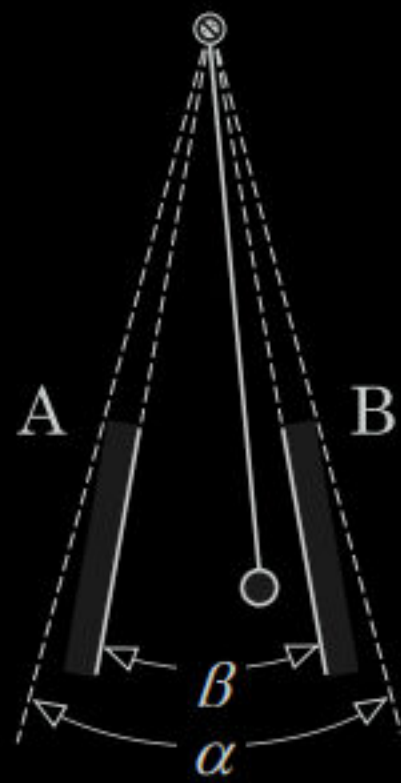
✓(c) $t_1 = 8$ s and $t_2 = 11$ s

✓(d) $t_1 = 16$ s and $t_2 = 19$ s

Lets show P at Origin at different times



Similar:
Irodov 4.20



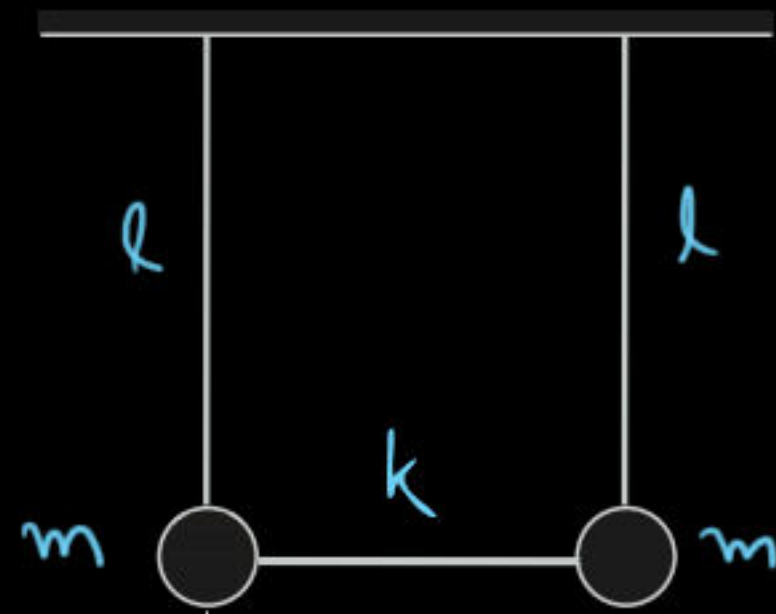
3. A simple pendulum initially oscillating simple harmonically with angular amplitude α and period T_0 is symmetrically confined between two rigid fixed planes A and B making angle $\beta < \alpha$ with each other as shown in the figure.

- (a) If collisions at both the walls are elastic, period is $T_0(1 - \beta/\alpha)$.
- ✓ (b) If collisions at both the walls are inelastic, period is T_0 .
- ✓ (c) If collision at one wall is elastic and at the other is inelastic, the period is T_0 .
- (d) If collision at one wall is elastic and at the other is inelastic, the period is less than T_0 .

$$T = \frac{2\theta}{2\pi} \cdot T_0$$

$$\sin^{-1}\left(\frac{\beta l/2}{\alpha l/2}\right) = \sin^{-1}\left(\frac{\beta}{\alpha}\right)$$

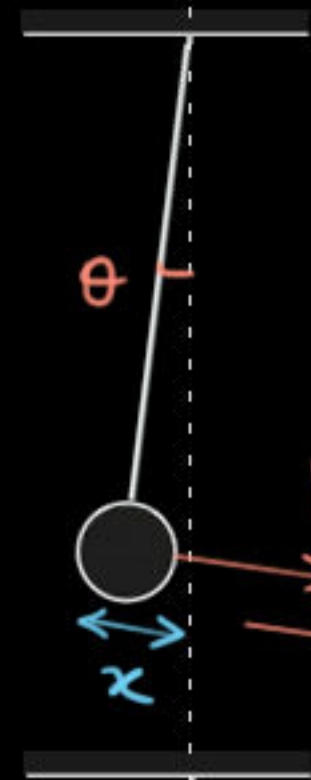
$$= \frac{2T_0}{\pi} \sin^{-1}(\beta/\alpha)$$



4. Two identical simple pendulums each of length l are suspended from the ceiling. Their bobs each of mass m are connected by a very light relaxed elastic cord of force constant k as shown in the figure. If the bobs are symmetrically pulled apart slightly and released, what is period of the ensuing oscillations?

Lets say, bobs are moved by x each.
 $x = l\theta$

$$\checkmark (d) \pi \left\{ \left(\frac{2k}{m} + \frac{g}{l} \right)^{-\frac{1}{2}} + \left(\frac{g}{l} \right)^{-\frac{1}{2}} \right\}$$



$mg \sin \theta$
 $\approx \left(\frac{x}{l} \right)$
 $k(2x)$

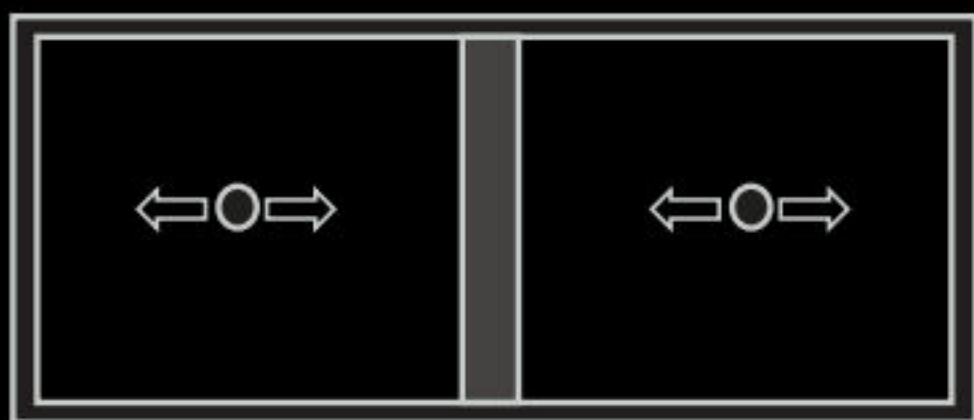
net $= \left(\frac{mg}{l} + 2k \right) x$

$$\Rightarrow T_{\text{left}} = \pi \sqrt{\frac{m}{\frac{mg}{l} + 2k}}$$

$\Rightarrow T_{\text{right}} = \pi \sqrt{\frac{l}{g}}$ Like a normal pendulum

$$\therefore T_{\text{total}} = T_{\text{left}} + T_{\text{right}}$$

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Concept:

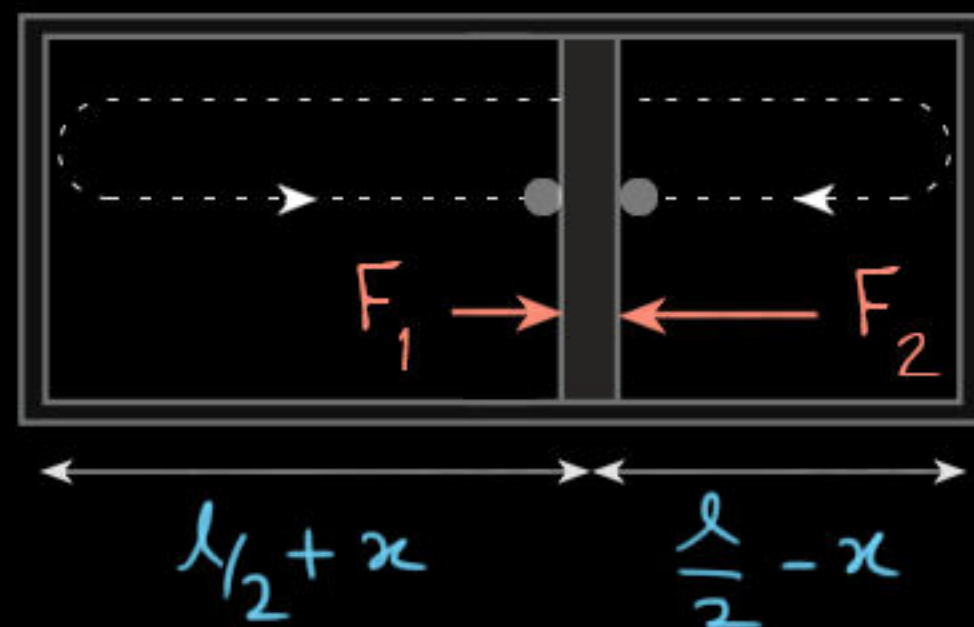
Initially, piston in middle is at equilibrium, as both balls are striking it at the same frequency.

But, when the piston is shifted to right (lets say), ball in right chamber is going to strike the piston more frequently than other. So there will be a 'restoring force' on piston towards left.

After every strike, change in momentum of a ball = $2mv$

5. Consider a fixed highly evacuated cylinder in a gravity free region. Inside the cylinder, a piston of mass M can move without friction. In each part of the cylinder, a ball of mass m ($m \ll M$) is moving back and forth with very high frequency and in doing so they are colliding elastically with the ends of the cylinder and with the piston. When the piston is at the centre, it is in equilibrium as shown in the figure. Now the piston is shifted slightly towards one of the ends of cylinder with a negligible speed and then released. What will happen to the piston?

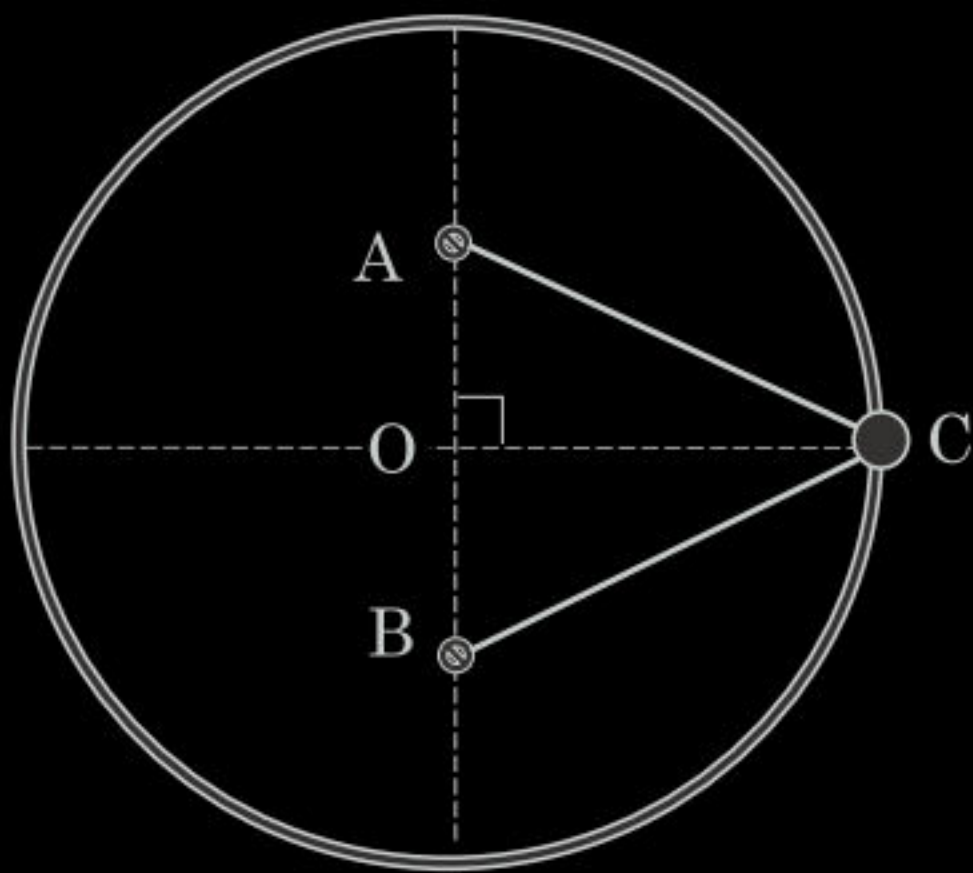
- (a) It will stay where it is released.
- (b) It will ultimately come at the centre and stop.
- ✓ (c) It will start oscillating back and forth simple harmonically.
- (d) It will start oscillating back and forth but not simple harmonically.



$$F_1 = \frac{\Delta p}{\Delta t_1} = \frac{2mv}{2(l/2 + x)/v}$$

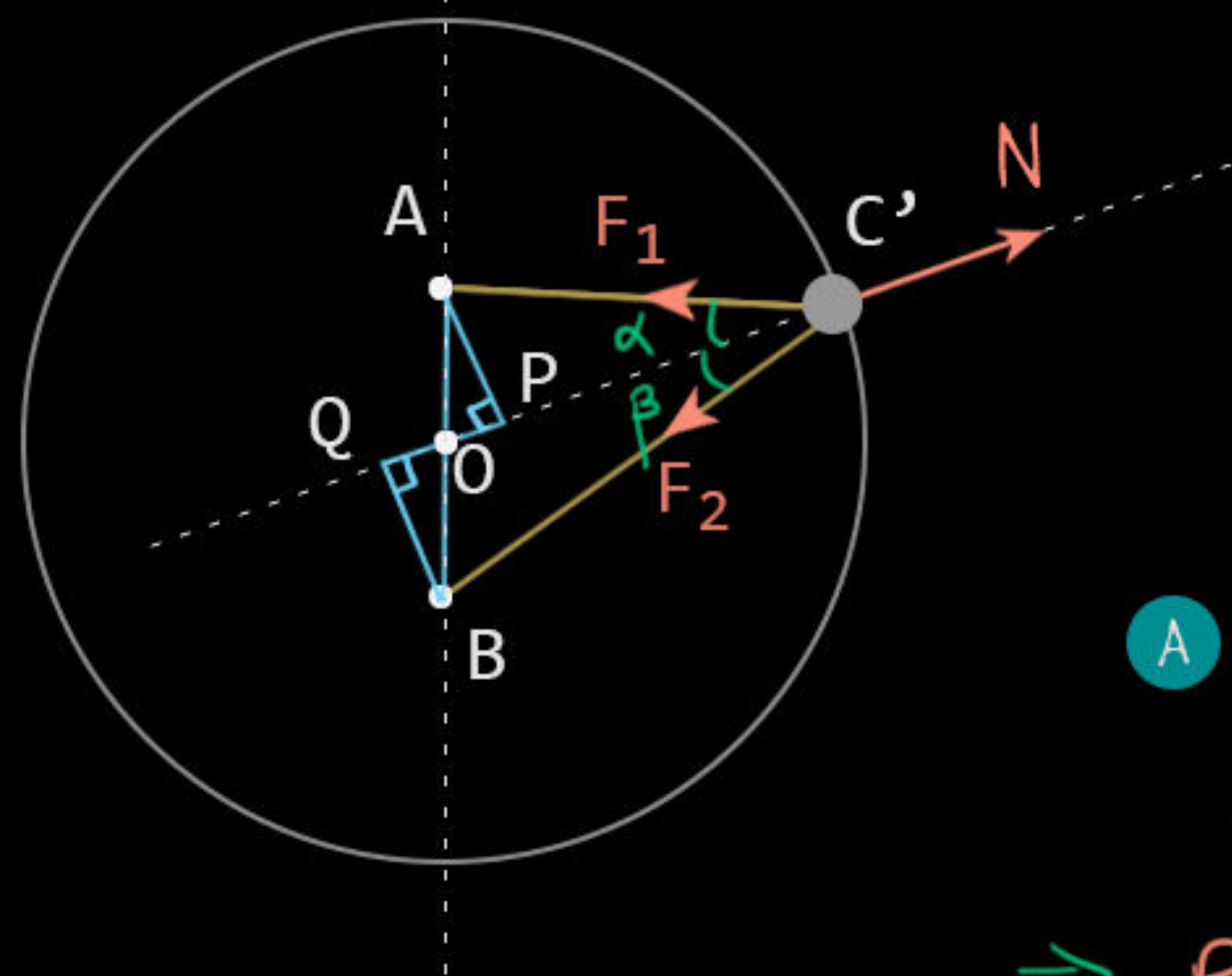
$$F_2 = \frac{\Delta p}{\Delta t_2} = \frac{2mv}{2(l/2 - x)/v}$$

$$\therefore F_{\text{restoring}} = F_2 - F_1 = 2mv^2 \left(\frac{1}{l - 2x} - \frac{1}{l + 2x} \right) = \frac{2mv^2 \cdot 4x}{l^2 - 4x^2} \approx \frac{8mv^2}{l^2} x$$



6. A small bead of mass m can slide on a frictionless fixed ring of radius r . With the help of two identical strings of force constant k , the bead is connected to two nails A and B each on a diameter at a distance $0.5r$ from centre O of the ring. Relaxed length of each string is negligible as compared to the radius of the ring. The bead is given a small velocity at point C. What can you predict about subsequent motion of the bead before any of the string strikes a nail?

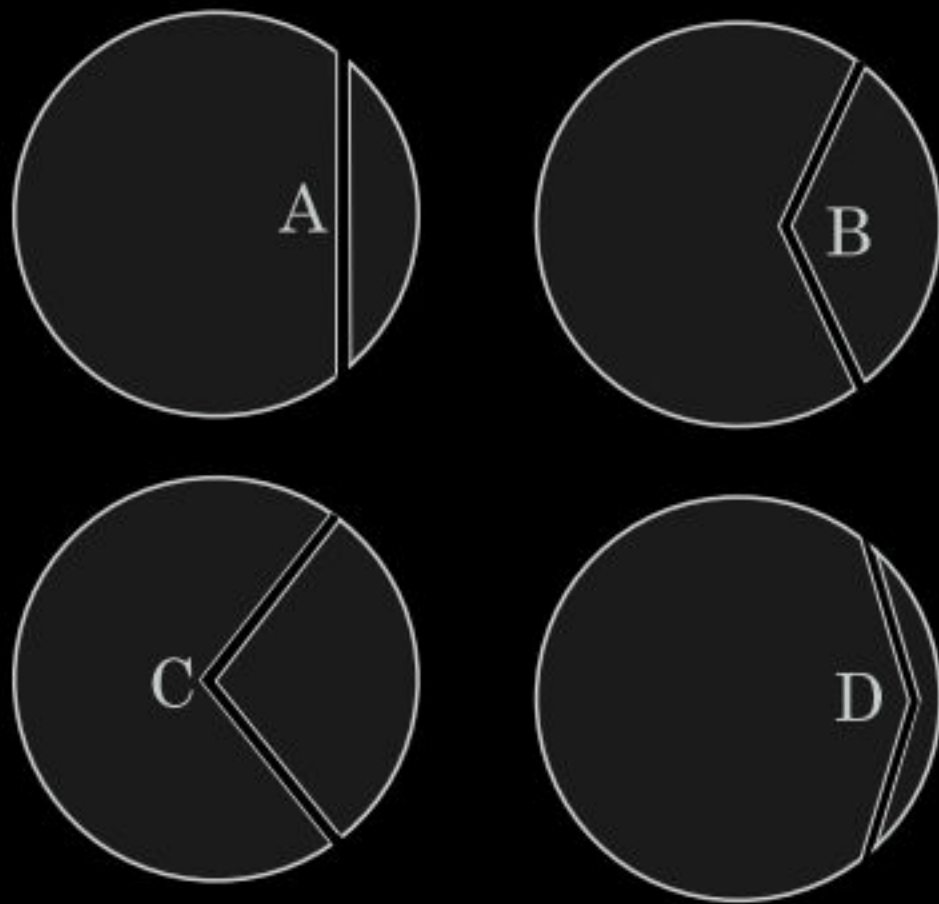
- ✓(a) It will keep moving with its initial speed.
- (b) It will oscillate simple harmonically about the point C.
- ✓(c) Its angular momentum about centre O of the ring is conserved.
- ✓(d) Total elastic potential energy stored in both the strings is $1.25kr^2$.



$$\begin{aligned}
 \text{Tangential } F_{\text{net}} &= F_1 \sin \alpha - F_2 \sin \beta \\
 &= k AC' \sin \alpha - k BC' \sin \beta \\
 &= k AP - k BQ \quad \{AP=BQ, \therefore\} \\
 &= 0
 \end{aligned}$$

- Ⓐ Ⓒ $\therefore$ It will continue to move with same speed.
- Ⓓ As speed is constant, PE is unchanged too.

$$\Rightarrow PE = 2 \times \frac{1}{2} k (AC)^2 = k (AC)^2 = k [R^2 + (0.5R)^2] = 1.25 k R^2$$



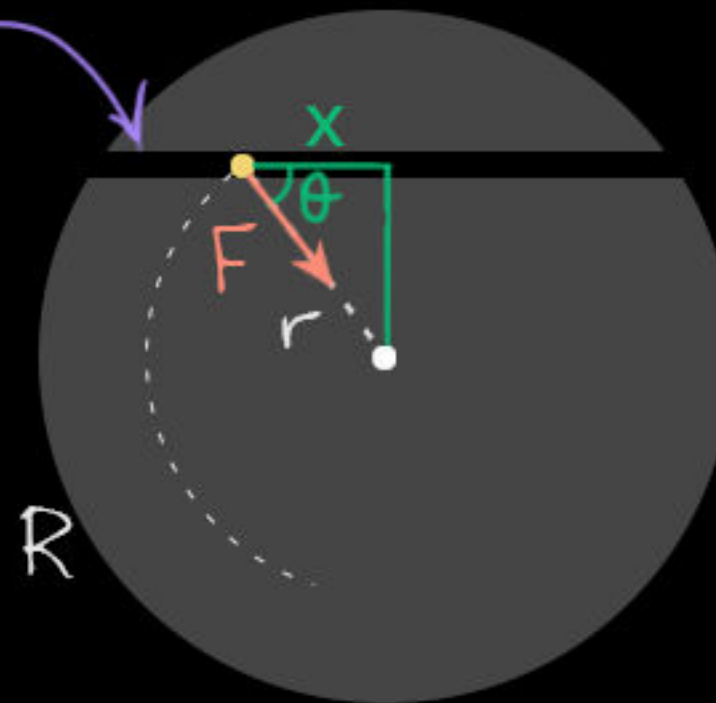
7. **Four tunnels** are supposed to be bored through the earth between two places on a longitude as shown in the figure. One of them with midpoint at A is straight, whereas other three have bent at their midpoints B, C and D. Both the bent sections are coplanar. A particle released from one end of the tunnels, spends time intervals T_A , T_B , T_C and T_D respectively to reach the other end. If the particle passes the bent smoothly without a change in speed there, which of the **following relations** will these time intervals bear?

✓ (d) $T_B < T_A = T_C < T_D$

Concept: A particle will perform SHM in any straight tunnel from surface to surface with same time period.

$$\therefore T_B < T_A = T_C < T_D$$

example



We know, $f = \frac{GmMx}{R^3}$

∴ Along the tunnel:

$$F_{\text{restoring}} = F \cos \theta$$

$$= \frac{GmM}{R^3} x \cos \theta = \frac{GmM}{R^3} x$$

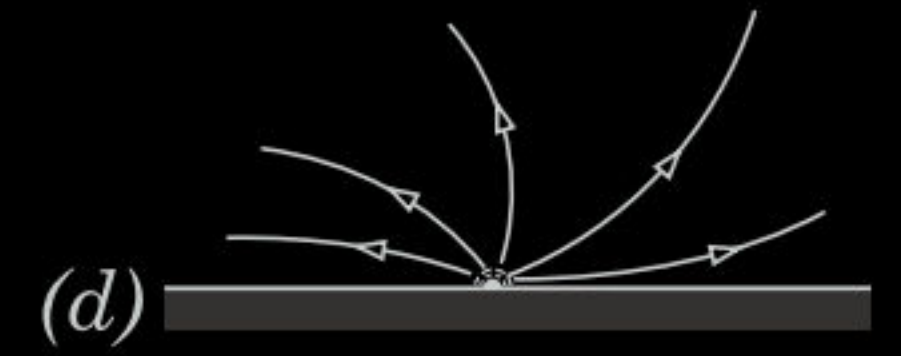
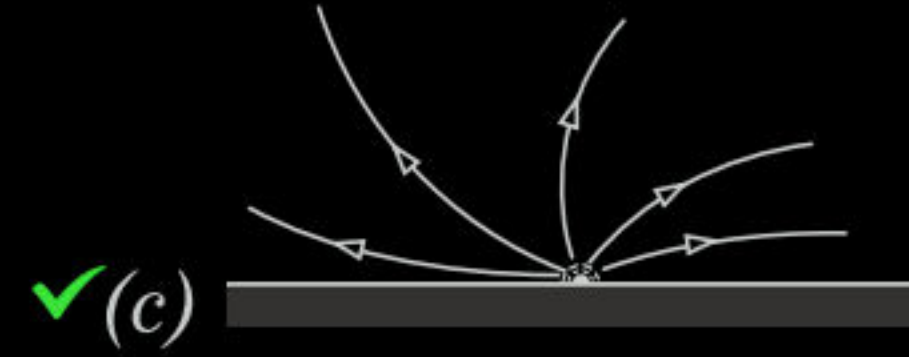
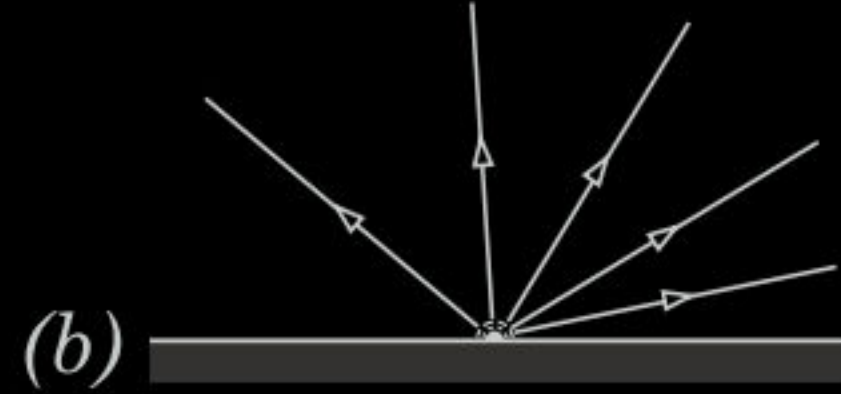
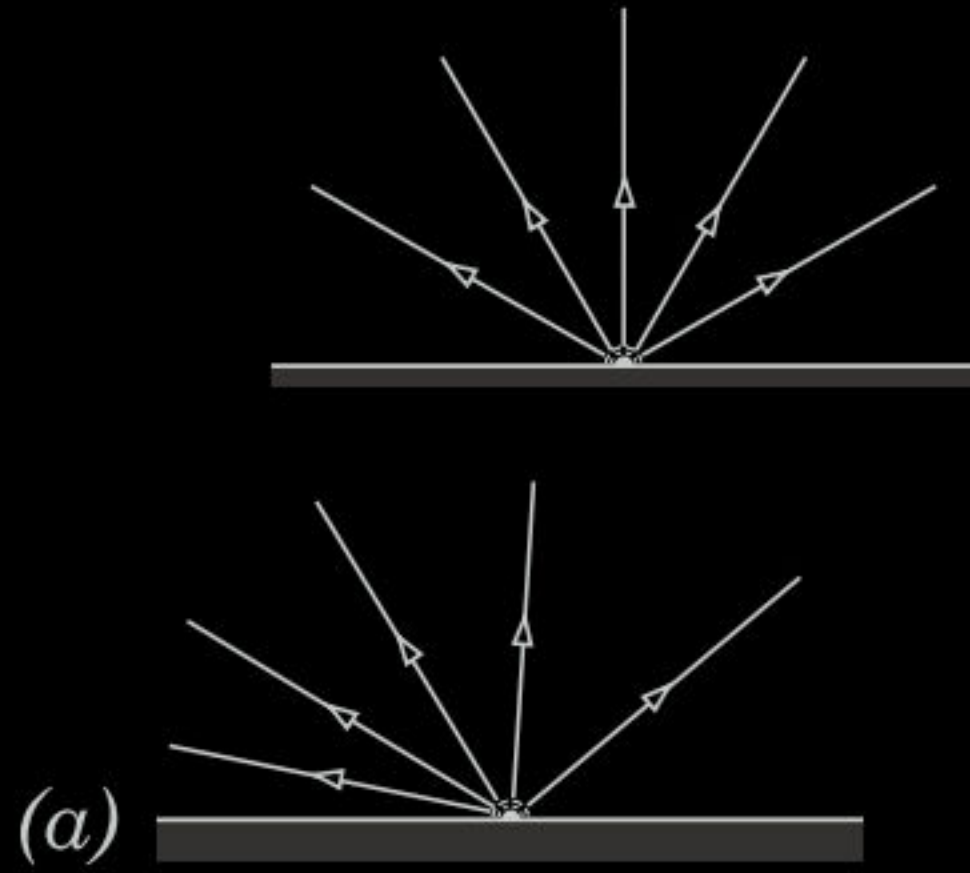
∴ SHM: k

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{R^3}{Gm}} = 2\pi \sqrt{\frac{R}{g}}$$

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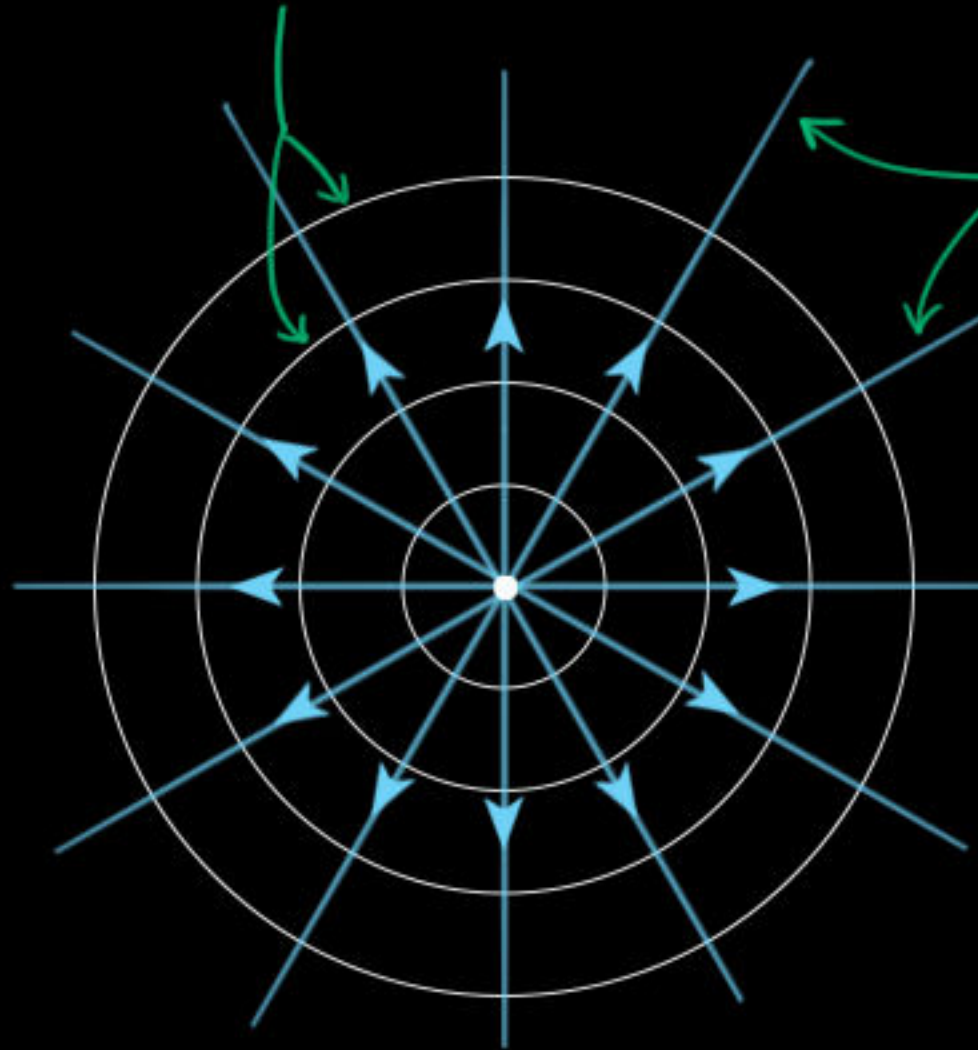
by Ankit Singhvi

8. On a windless day, rays of the sound waves emanating from an isotropic point source placed close to the ground are shown in the figure. If a horizontal wind starts blowing towards the left with a constant and uniform velocity, which of the following will best represent pattern of the sound rays?



Centres of these spherical wavefronts keep shifting towards left with wind velocity. (while expanding at the same time)

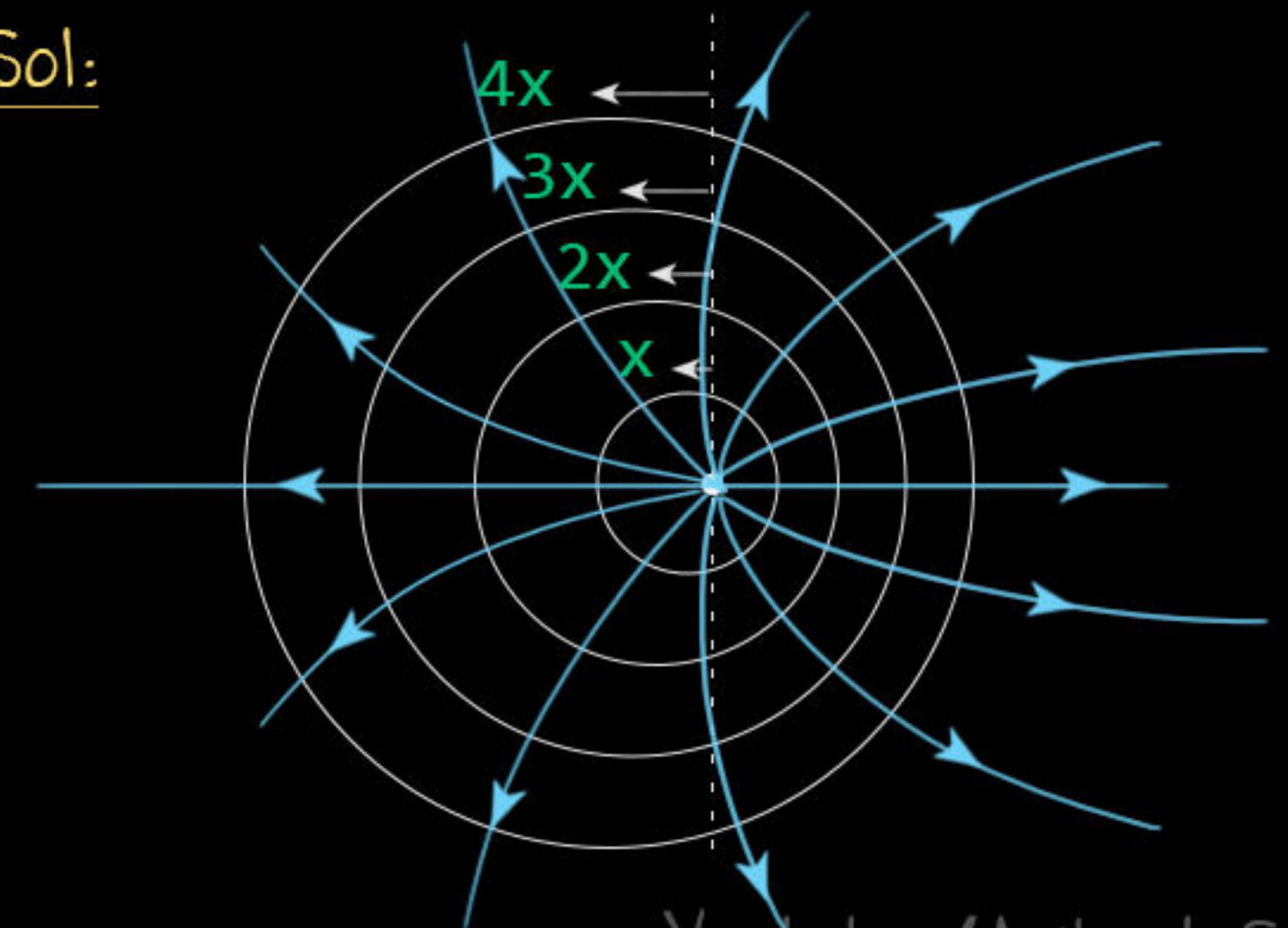
Wavefronts (they keep expanding)



Direction of propagation of energy. Also called as rays

Wavefronts and rays (be it light or sound or water surface waves) are perpendicular to each other.

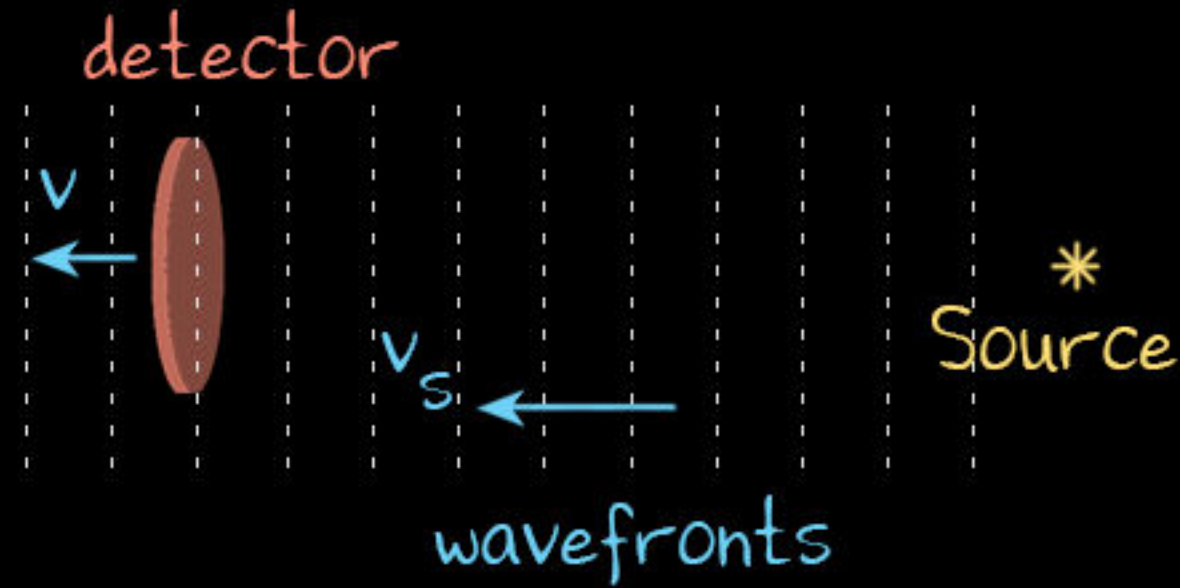
Sol:



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Let v be velocity of detector.

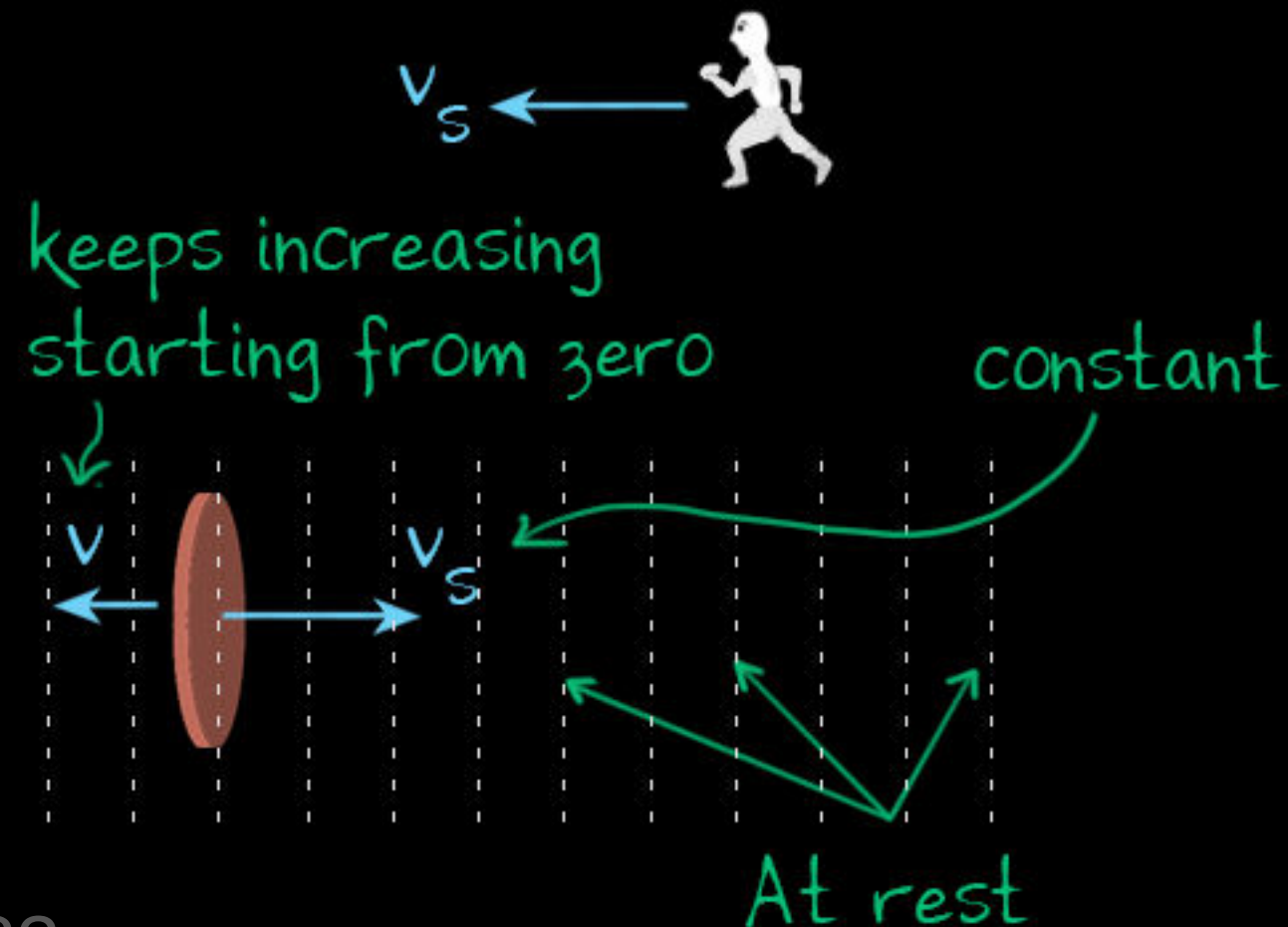
w.r.t ground



9. A detector receding from a stationary sound source with a speed that increases continuously without a limit. Which of the following statements correctly describes frequency of the sound detected by the detector? Assume that sound waves propagate indefinitely without attenuation and the detector does not create any air drag.

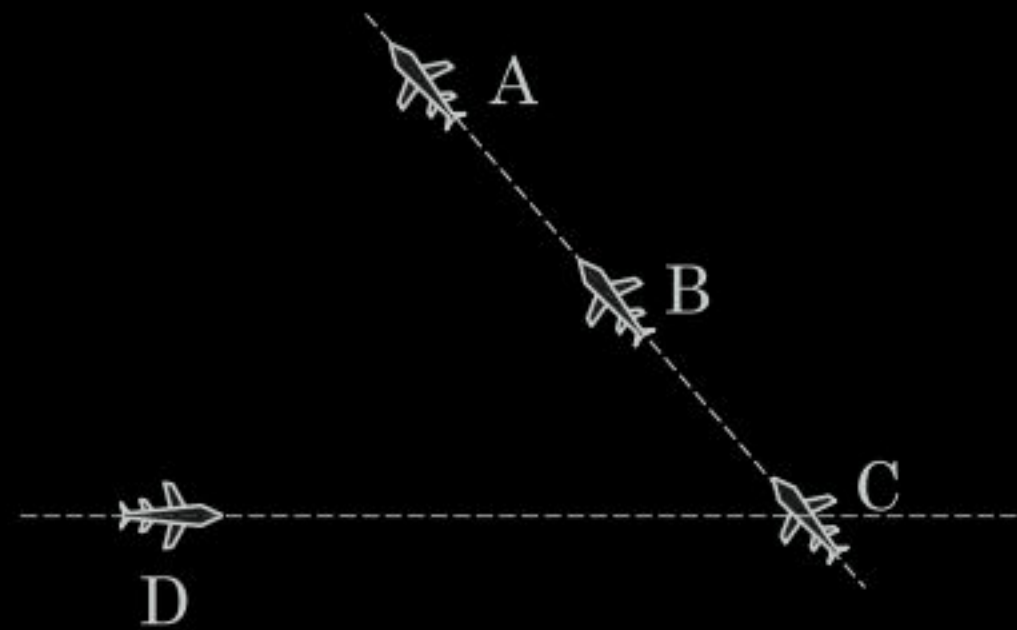
- (a) It first decreases, becomes zero and then increases.
- (b) It continuously decreases and eventually no sound is detected.
- (c) It continuously decreases and eventually acquires a small value.
- ✓(d) It first decreases, becomes zero and then increases and eventually no sound is detected.

w.r.t reference frame (v_s)



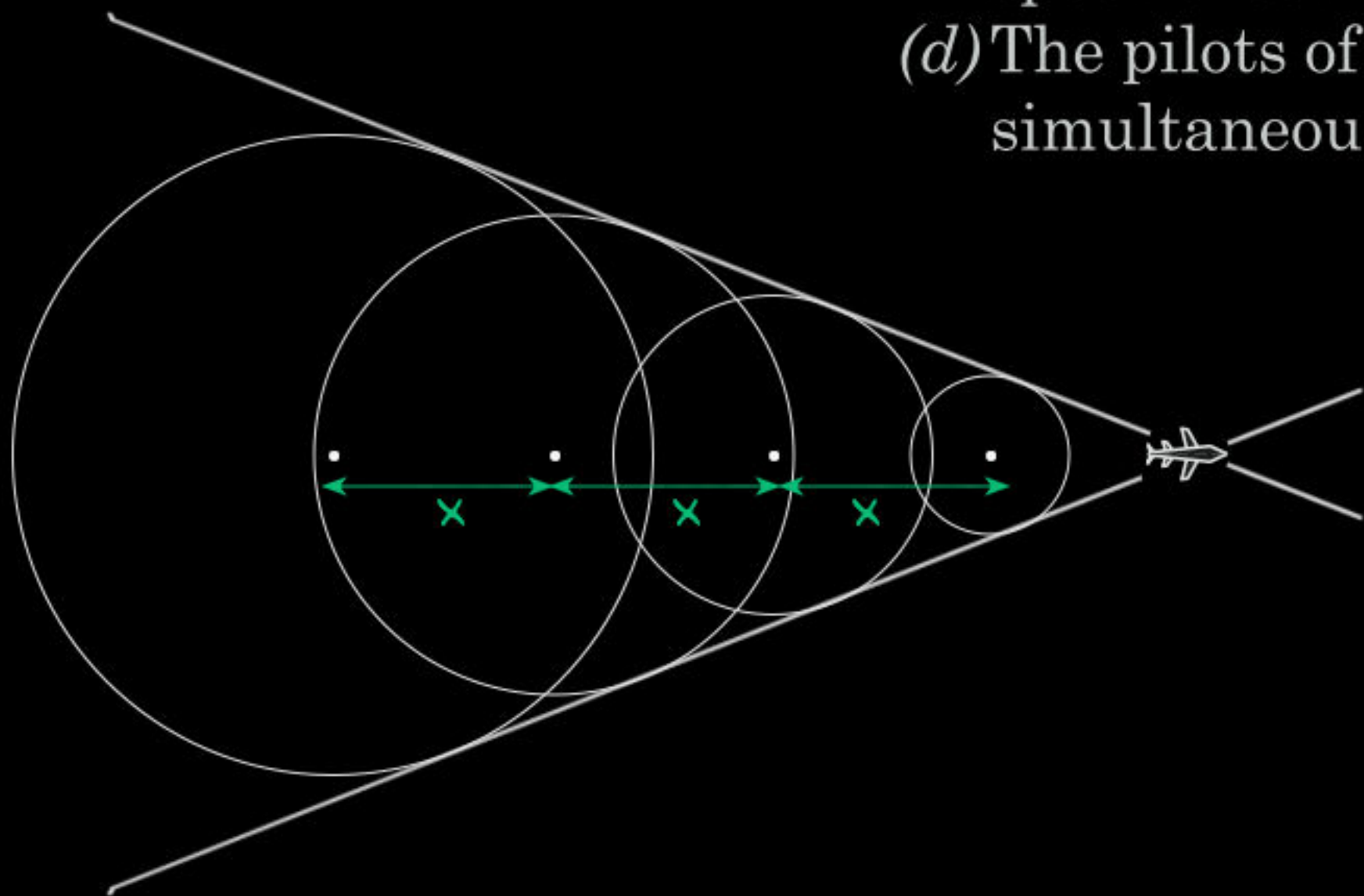
As v goes from 0 to ∞ , wavefronts it will cross per second (i.e, frequency) will:

- Decrease (for $v < v_s$)
- Become zero (at $v = v_s$)
- Then increase again (for $v > v_s$)
- Then abruptly become zero, when first ever wave front is crossed.



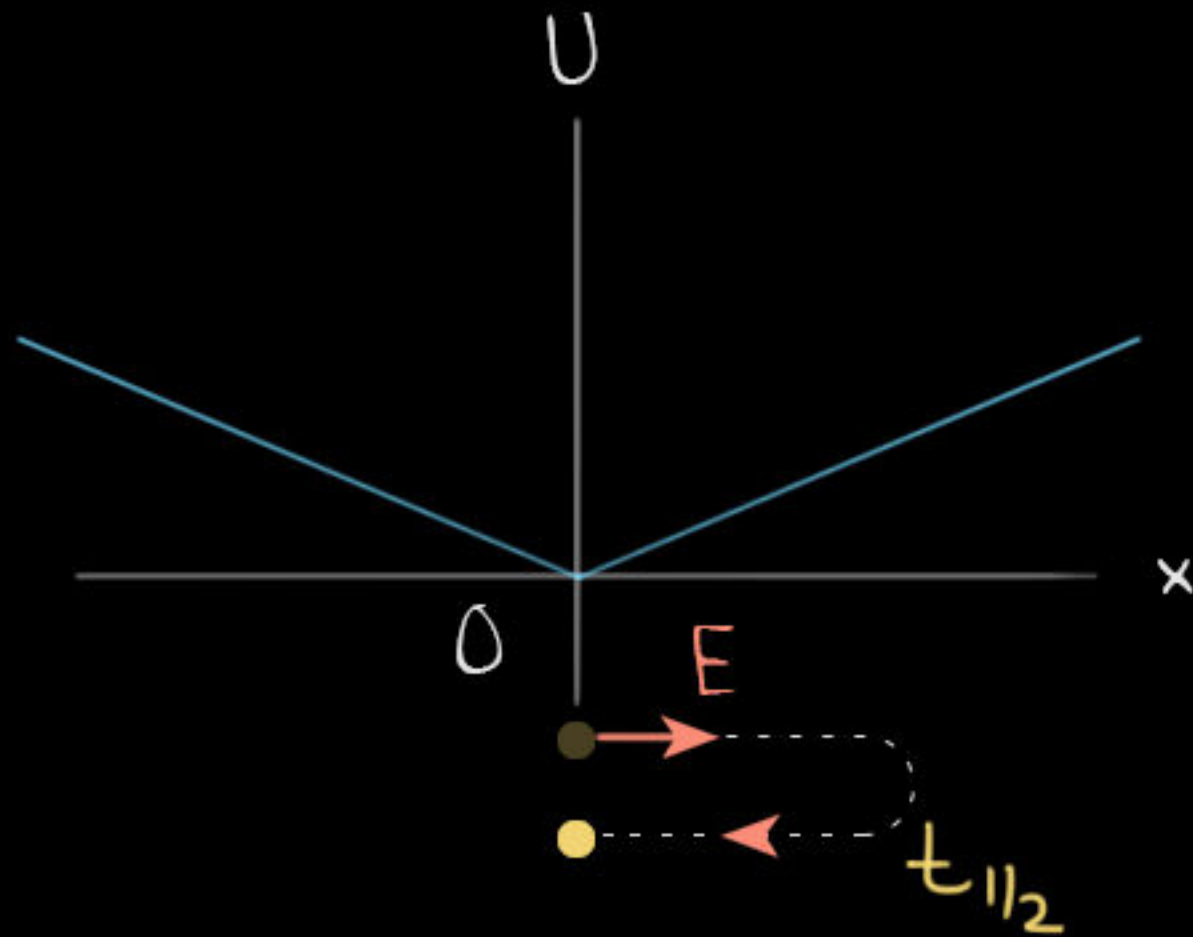
10. Three aircraft A, B and C are flying in a line with the same speed. Another aircraft D is flying on another straight line making an acute angle with the line of motion of the aircrafts A, B and C as shown. If pilots of these aircrafts hear sound of the aircraft D simultaneously, what can you conclude about speed of the aircraft D?

- (a) It is possible only when the aircraft D is moving at speed of sound.
- (b) It is possible only when the aircraft D is moving at speed lower than speed of sound.
- ✓ (c) It is possible only when the aircraft D is moving at speed higher than speed of sound.
- (d) The pilots of aircrafts A, B and C cannot hear sound of the aircraft D simultaneously.



A, B, C will all hear the sound for the first time when the 'bow wave' will reach them. A straight line 'bow wave' is only possible if plane is moving at speed higher than speed of sound.

1. A particle of mass m can move along x -axis of a coordinate frame in a force field of stationary sources. Potential energy U of system varies with position x the particle according to equation $U = k|x|$, where k is a positive constant. If the particle is projected from the origin with a kinetic energy K , find period of its bound motion.



0 is the minima.
 $\therefore$ Force is restoring in nature at 0 .

$$F = \frac{dU}{dx} \quad \text{where } U = kx$$

$$\Rightarrow ma = k$$

$$\Rightarrow a = \frac{k}{m} \text{ (constant)}$$

Half time taken be $t_{1/2}$

$$s = ut + \frac{1}{2}at^2$$

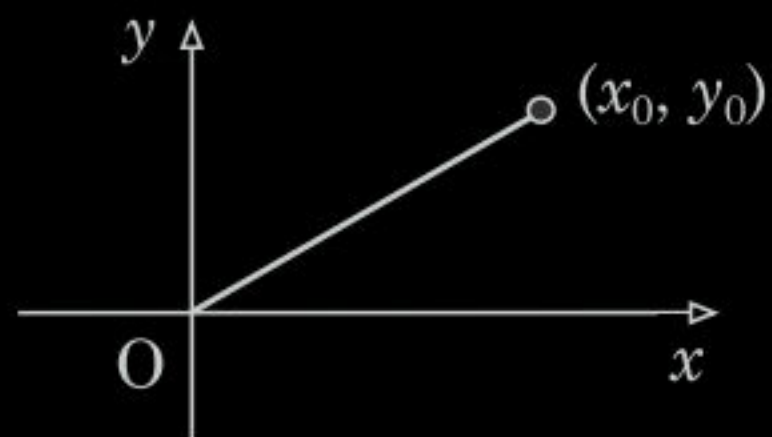
$$\Rightarrow 0 = \sqrt{\frac{2E}{m}} t_{1/2} - \frac{1}{2} \frac{k}{m} t_{1/2}^2$$

$$\Rightarrow t_{1/2} = \frac{2}{k} \sqrt{2Em}$$

$$= \frac{2}{k} \sqrt{2 \cdot m \cdot E}$$

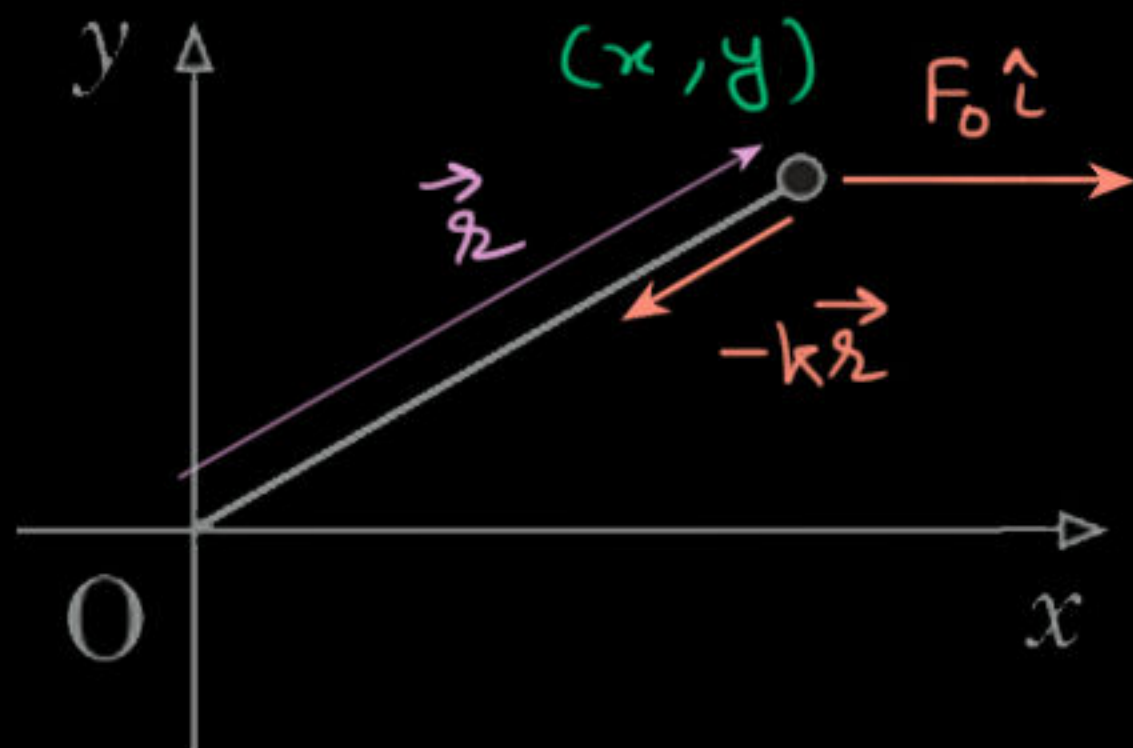
$$\therefore T = 2t_{1/2} = \frac{4}{k} \sqrt{2mE}$$

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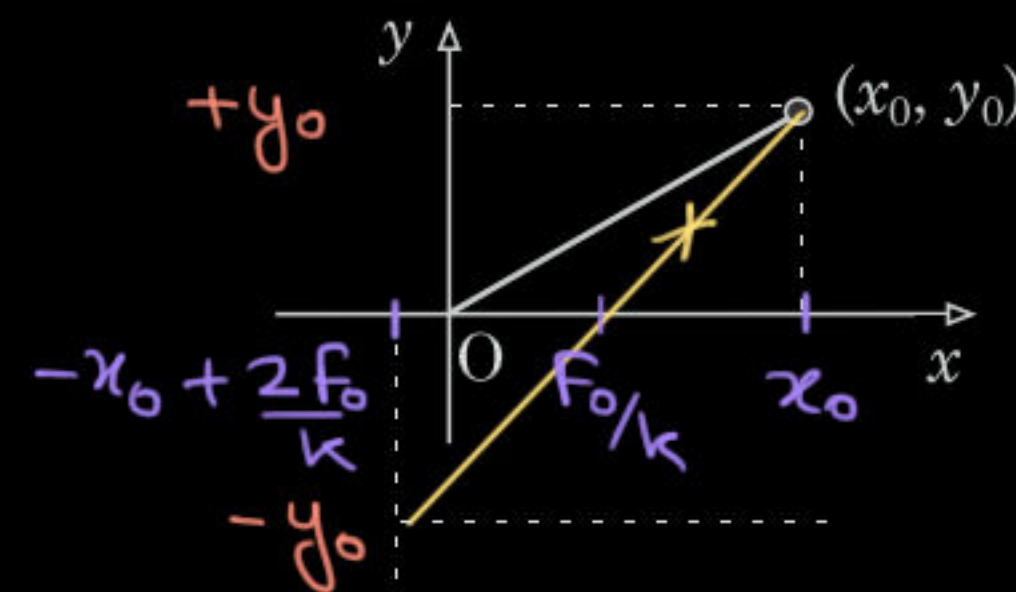


2. A particle of mass m is connected from the origin O of an inertial frame with the help of an elastic cord of force constant k and negligible relaxed length. In addition to the elastic force of the cord, a constant force $\vec{F} = F_0 \hat{i}$ also acts on the particle. The particle is pulled to an arbitrary position (x_0, y_0) as shown in the figure and released. Find the positions between which the particle oscillates and period of the oscillations.

At a general position, $\vec{r}$



$$\begin{aligned} F_{\text{net}} &= -k\vec{r} + F_0 \hat{i} \\ &= -k(x\hat{i} + y\hat{j}) + F_0 \hat{i} \\ &= (-kx + F_0)\hat{i} + (-ky)\hat{j} \\ &\quad \quad \quad F_x \quad \quad \quad F_y \end{aligned}$$



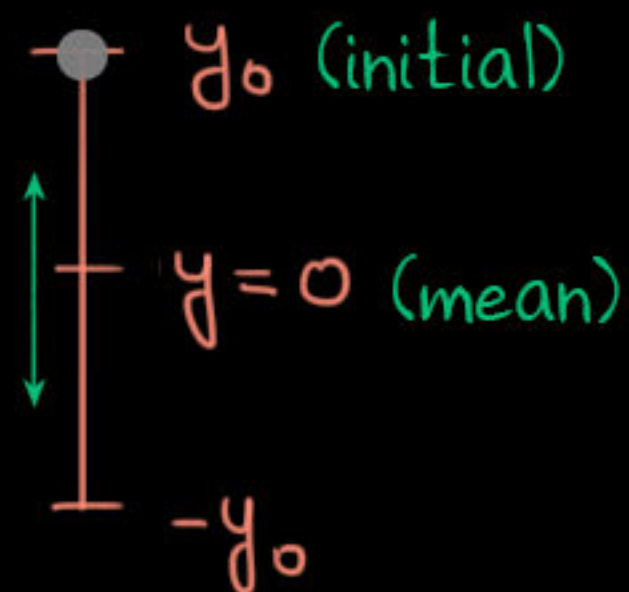
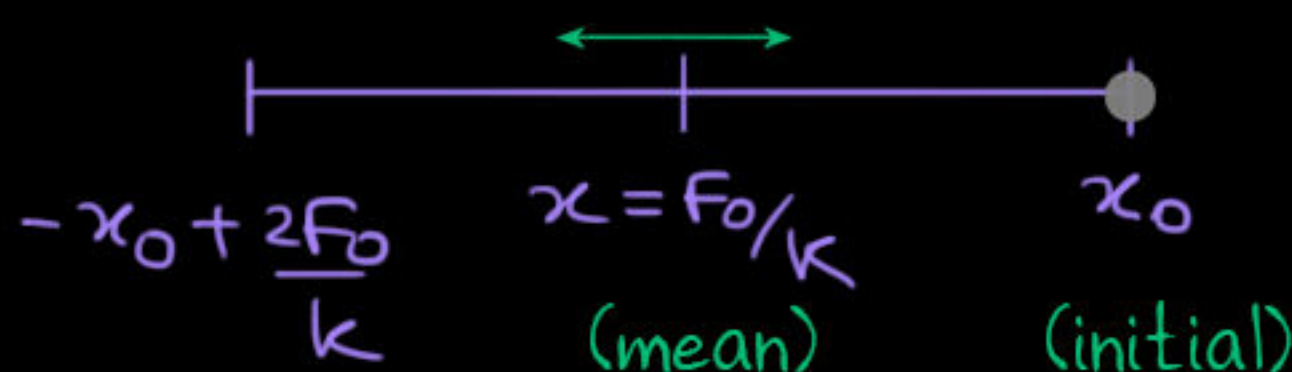
Equations:

$$\left(x - \frac{F_0}{k}\right) = \left(x_0 - \frac{F_0}{k}\right) \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$y = y_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

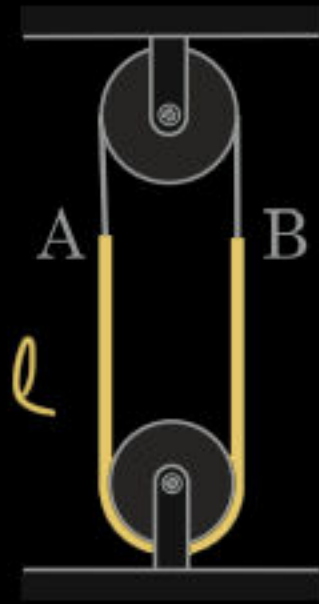
Trajectory: Straight line

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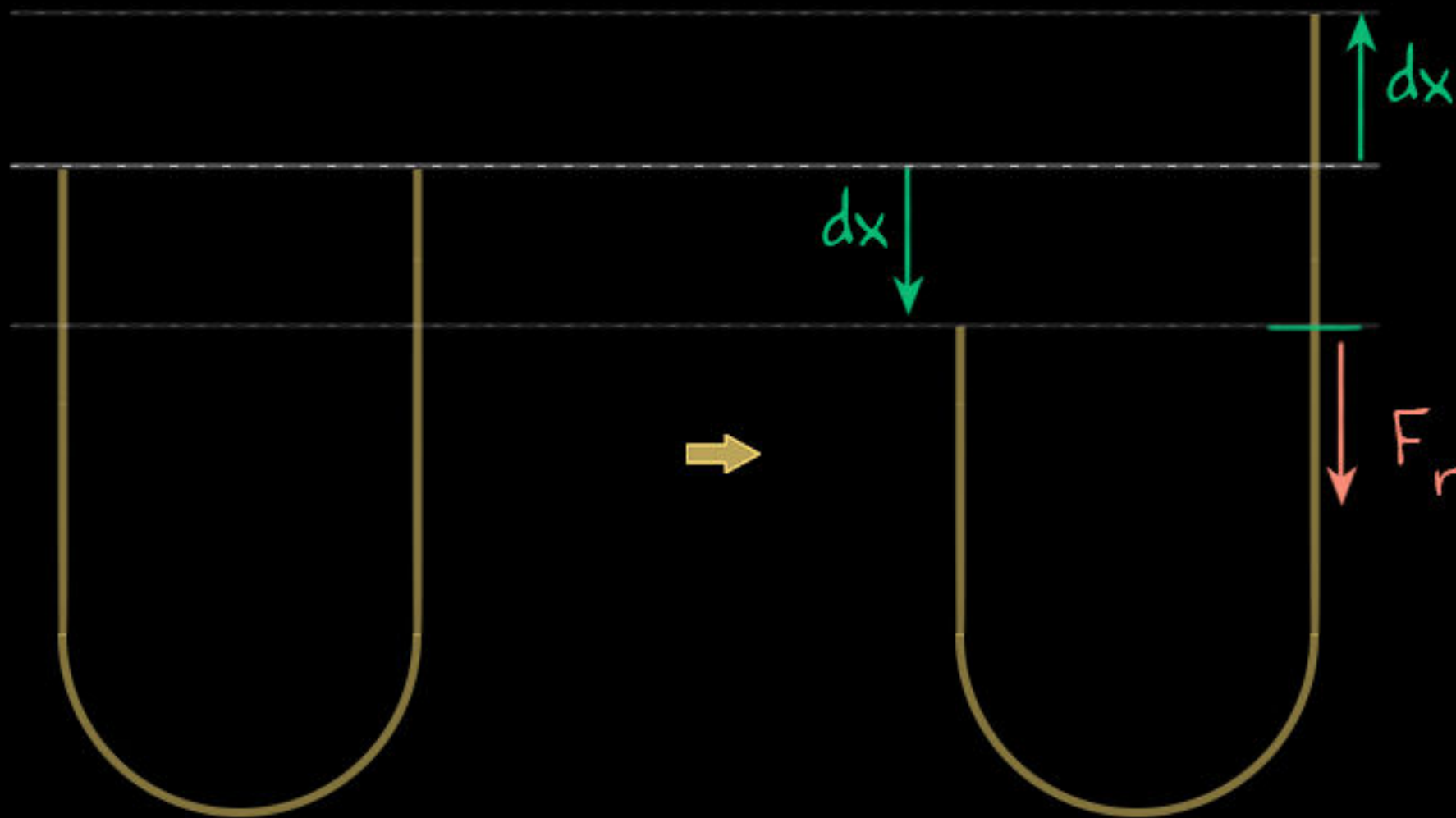
$$T = 2\pi \sqrt{\frac{m}{k}}$$

by Ankit Singhvi



3. A uniform inextensible rope of length l and a light inextensible thread are connected at their ends A and B to make a loop. The thread passes over a fixed ideal pulley and the rope passes under another fixed ideal pulley. The junction B is pulled slightly downwards and then released. Find period of the ensuing oscillations. Acceleration of free fall is g .

Lets say B is moved up by dx and released.

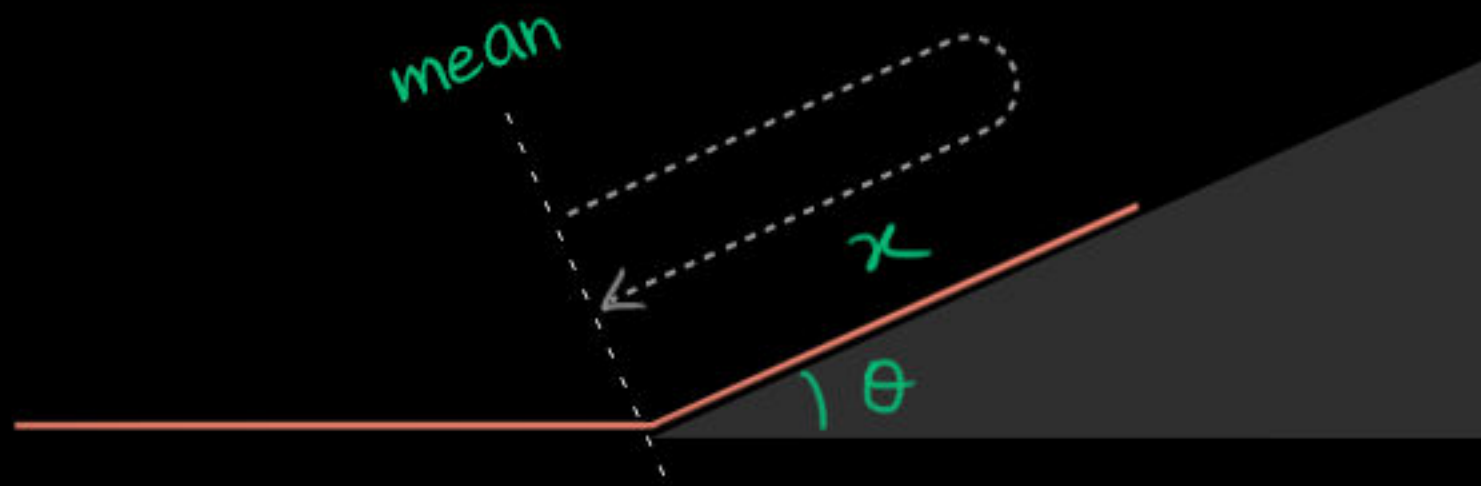


$$\begin{aligned}
 F_{\text{restoring}} &= dm g \\
 &= (\lambda \cdot 2dx) g \\
 &= \underbrace{2\lambda g}_{k} dx
 \end{aligned}$$

$$\begin{aligned}
 T &= 2\pi \sqrt{\frac{m}{k}} \\
 &= 2\pi \sqrt{\frac{\cancel{\lambda} l}{2\cancel{\lambda} g}} \\
 &= 2\pi \sqrt{\frac{l}{2g}}
 \end{aligned}$$

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4. A train of length $l = 900 \text{ m}$ running on a horizontal track with its engine off starts moving up a hill of uniform inclination $\theta = 9.2^\circ$ to the horizontal. If $\eta = 0.6$ fraction of length of the train rises the hill before it starts moving back due to gravity, how much time the train spends on the hill? Assume all the resistive forces negligible, distribution of mass uniform in the train and no impact at the beginning of the hill. Acceleration due to gravity is g . Use $\sin(9.2^\circ) = 0.16$ and $\sqrt{g} = \pi$.

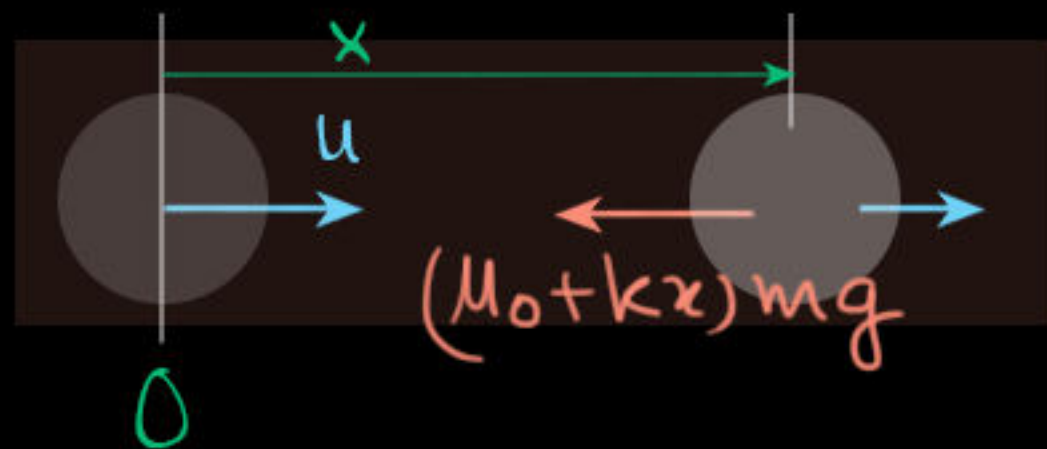


For x length of train on the hill.

$$\begin{aligned}
 F_{\text{restoring}} &= mg \sin \theta \\
 &= (\lambda x) g \sin \theta \\
 &= \underbrace{\lambda g \sin \theta}_k x
 \end{aligned}$$

$$\begin{aligned}
 \therefore T_{1/2} &= \pi \sqrt{\frac{m}{k}} \\
 &= \pi \sqrt{\frac{\cancel{\lambda} l}{\cancel{\lambda} g \sin \theta}} \\
 &= \pi \sqrt{\frac{l}{g \sin \theta}} //
 \end{aligned}$$

Top view



5. A small disc is projected on a horizontal floor with a speed u . Coefficient of friction between the disc and the floor varies according to equation $\mu = \mu_0 + kx$, where μ_0 and k are positive constants and x is distance travelled by the disc. Find distance slid by the disc on the floor. Acceleration of free fall is g .

Method 1 Energy

$$\int F \cdot dx = \Delta KE$$

$$\Rightarrow \int_0^{x_0} -(\mu_0 + kx)mg \, dx = 0 - \frac{1}{2}mu^2$$

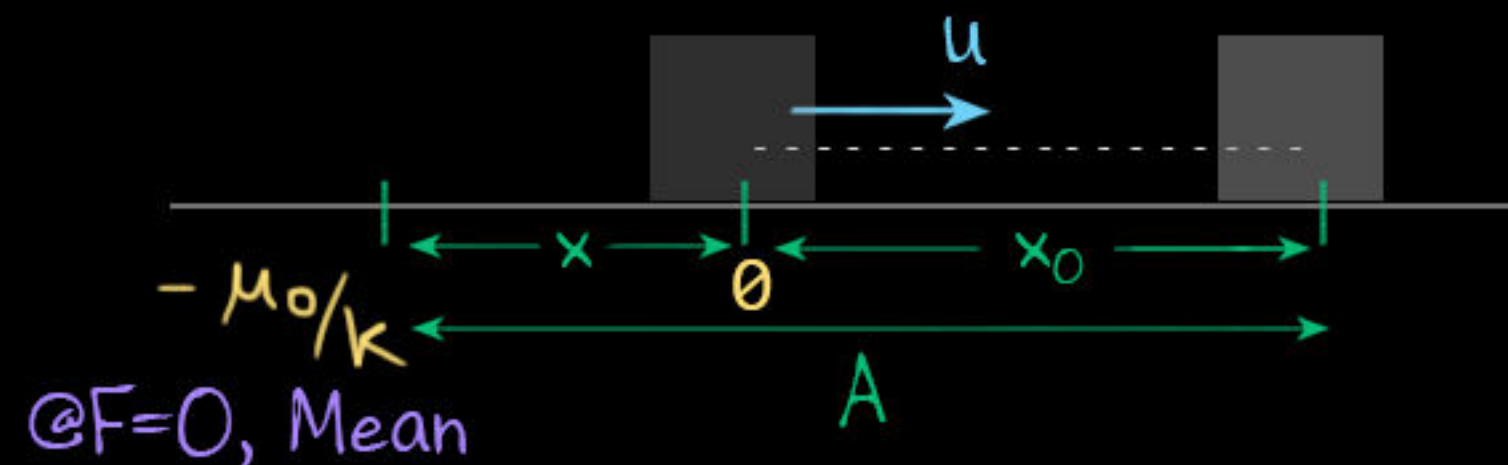
$$\Rightarrow \cancel{mg} \left(\mu_0 x_0 + \frac{kx_0^2}{2} \right) = \frac{1}{2}\cancel{m}u^2$$

Solving the quadratic eqn: $x_0 = \frac{-\mu_0 + \sqrt{\mu_0^2 + \frac{u^2 k}{g}}}{k}$

Method 2 SHM

Force constant k'

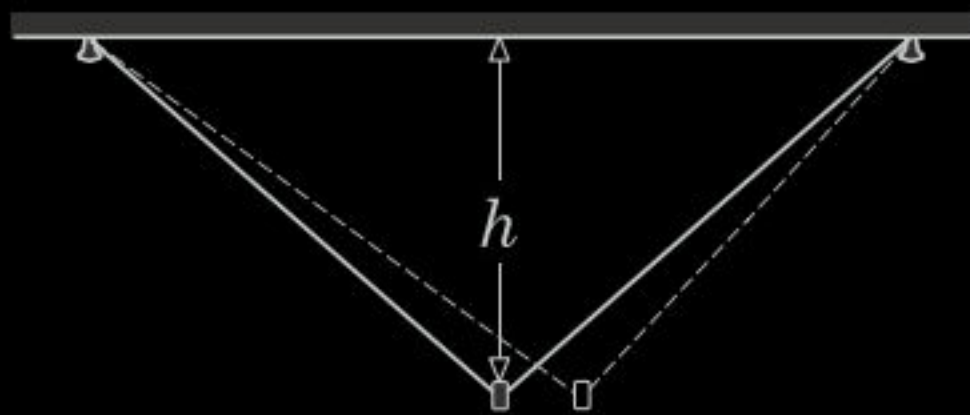
$$F = -(\mu_0 + kx)mg$$



$$v = \omega \sqrt{A^2 - x^2}$$

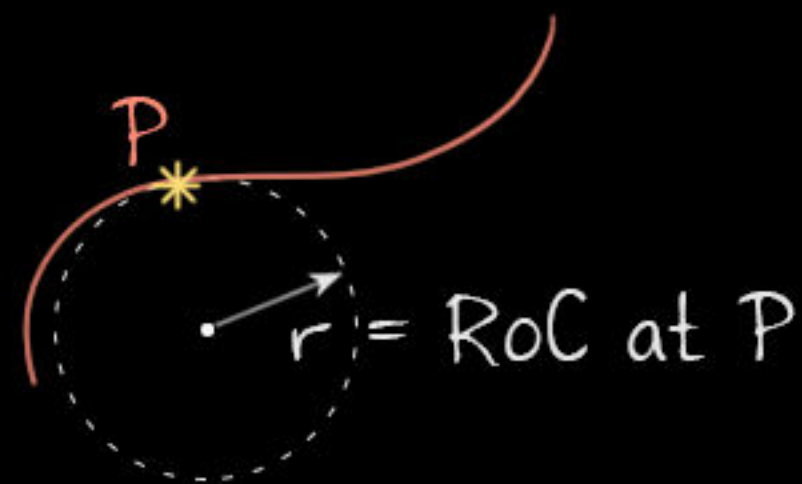
$u = \sqrt{\frac{k'}{m}} = \sqrt{k'g}$ $\frac{\mu_0}{k}$

$$\Rightarrow A = \sqrt{\frac{u^2}{kg} + \frac{\mu_0^2}{k^2}} \quad x_0 = A - \frac{\mu_0}{k}$$

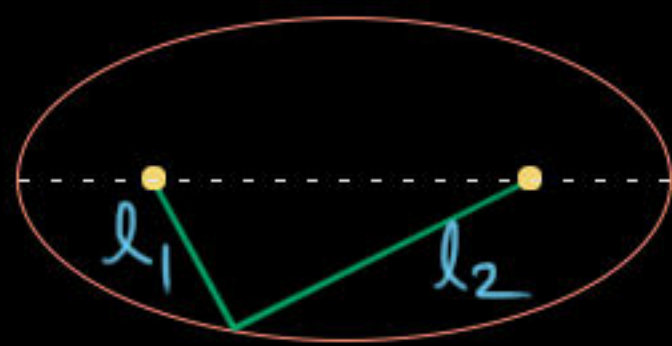


6. A small ring is threaded on an inextensible frictionless cord of length $2l$. The ends of the cord are fixed to a horizontal ceiling. In equilibrium, the ring is at a depth h below the ceiling. Now the ring is pulled aside by a small distance in the vertical plane containing the cord and released. Find period of small oscillations of the ring. Acceleration of free fall is g .

Concept 1: A small part of any smooth curve can be approximated as circle of given RoC (Radius of Curvature).

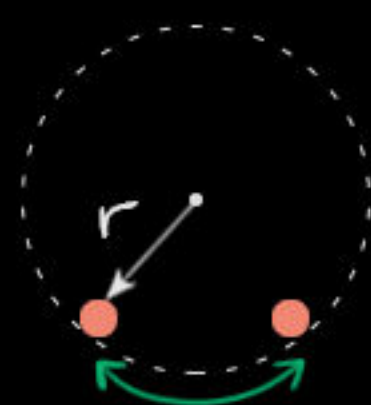
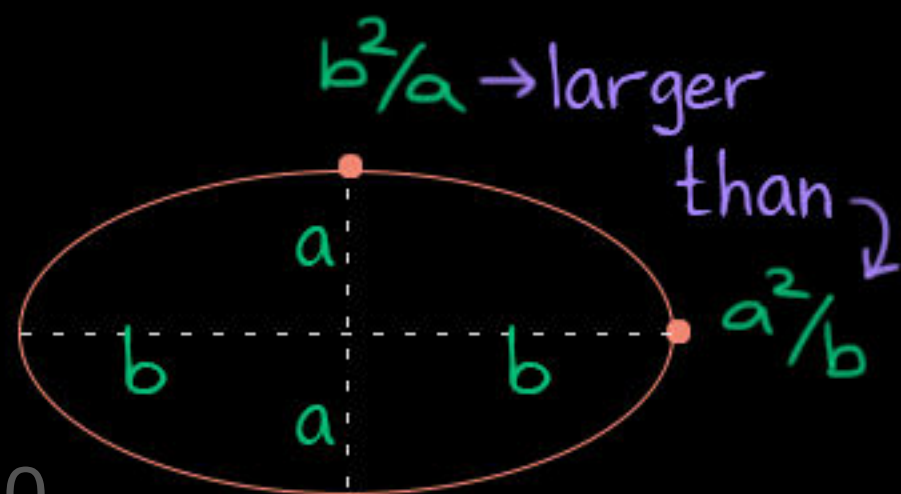


Concept 2: On ellipse, the sum of the two distances to the focal points is constant



$$l_1 + l_2 = \text{constant}$$

Concept 3: RoC of ellipse at vertices:

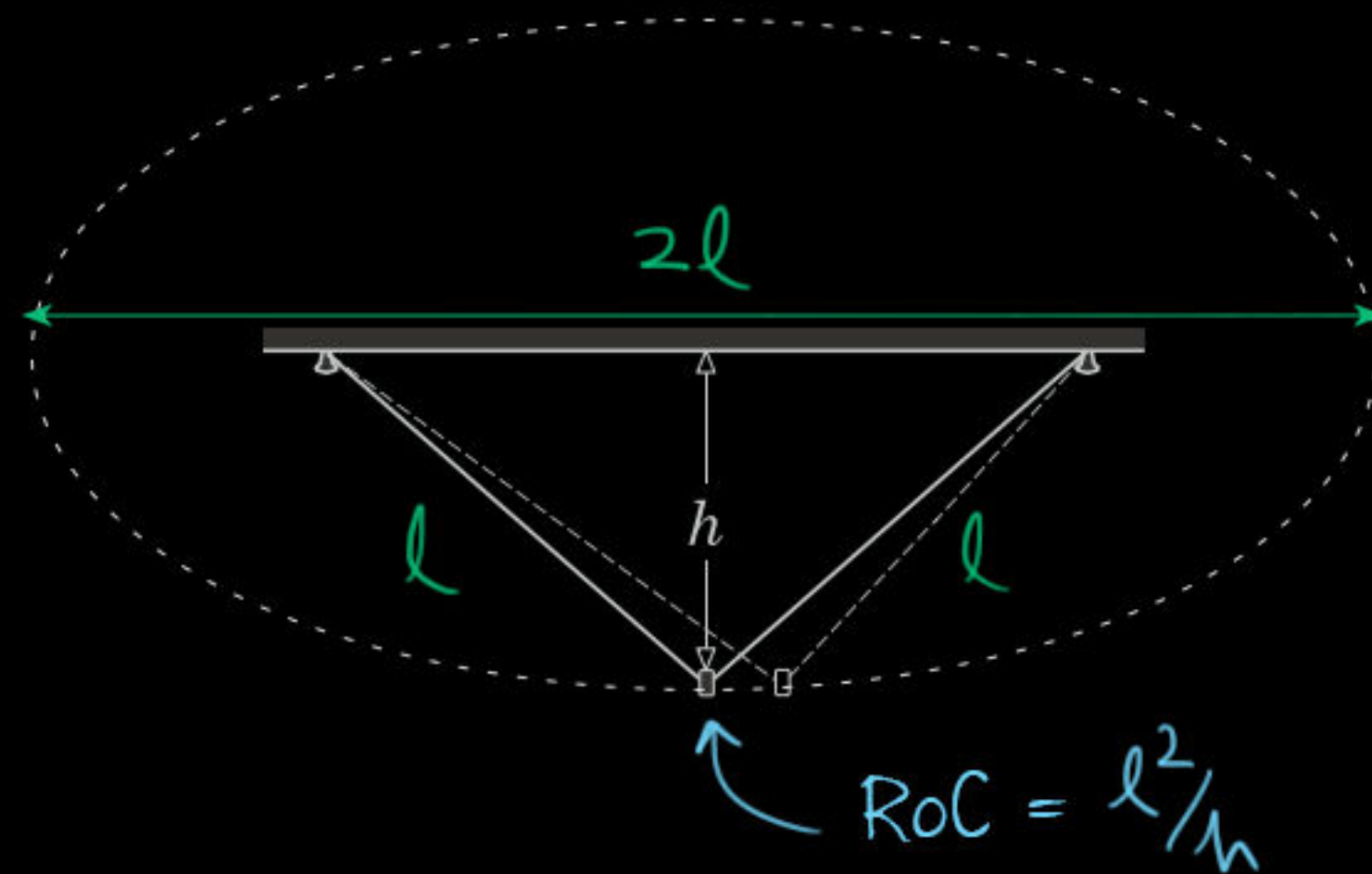


Concept 4:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

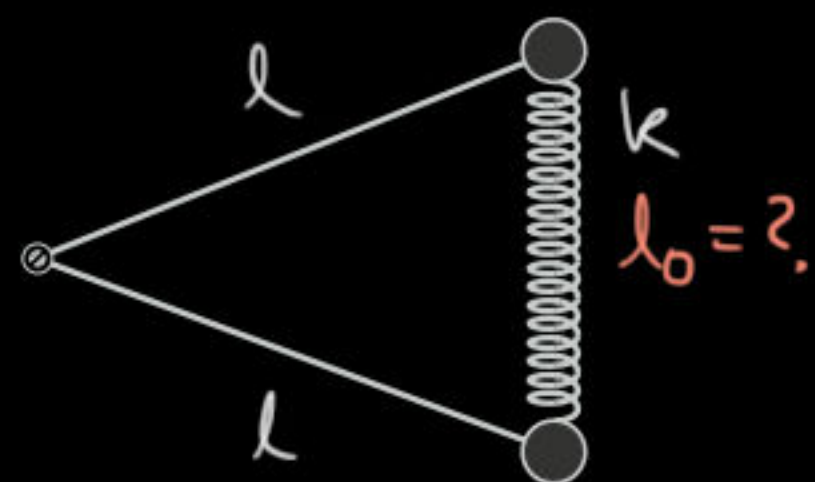
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Sol:

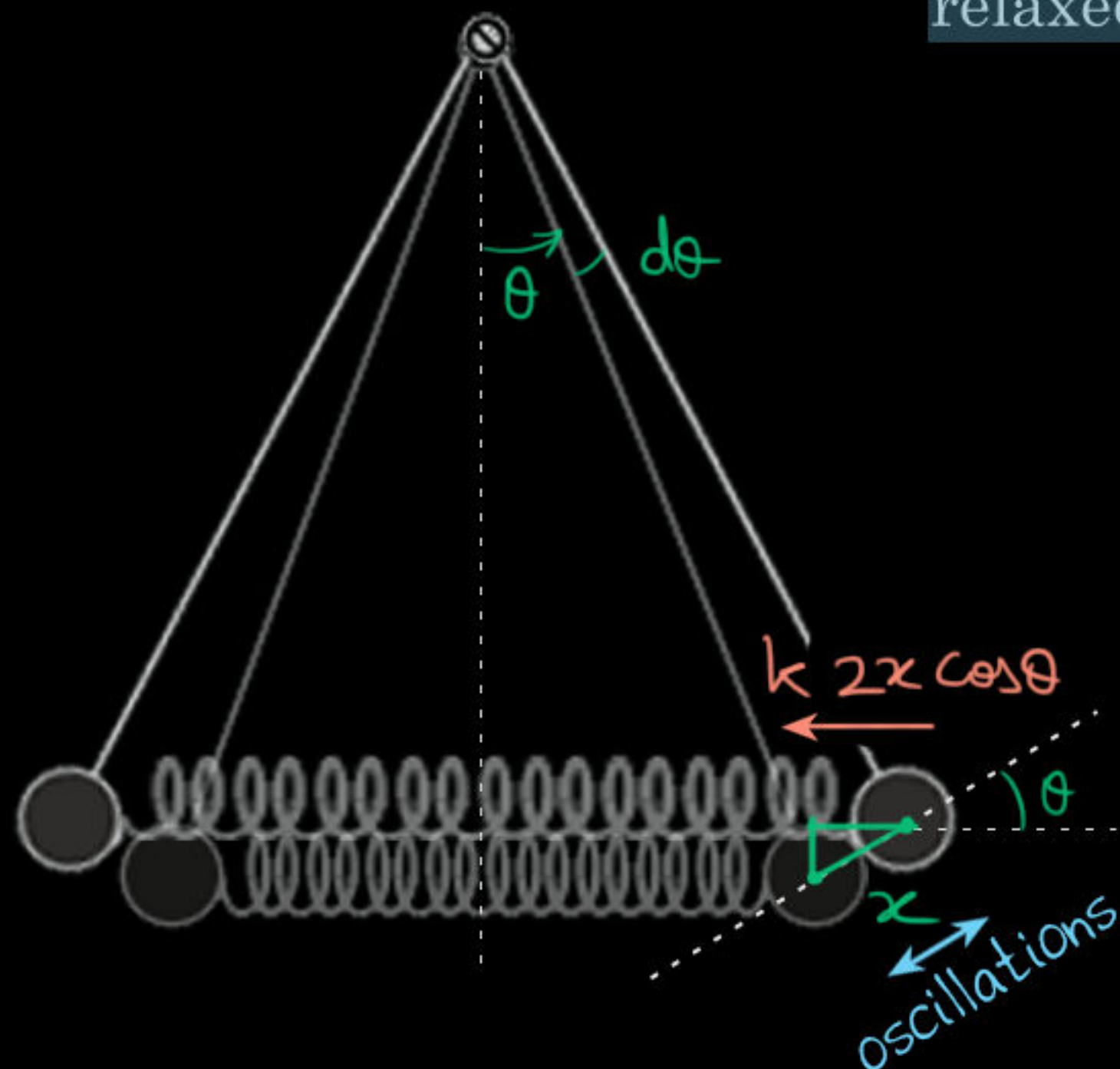


$$\therefore T = 2\pi \sqrt{\frac{\text{RoC}}{g}} = 2\pi \sqrt{\frac{l^2}{hg}}$$

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7. Two identical small discs each of mass m placed on a frictionless horizontal floor are connected with the help of a spring of force constant k . The discs are also connected with two light rods each of length l that are pivoted to a nail driven into the floor as shown in the figure by a top view. If period of small oscillations of the system is $2\pi\sqrt{m/k}$, find relaxed length of the spring.



Lets displace the balls by x each.

$\therefore$ Total elongation in spring = $2x \cos \theta$

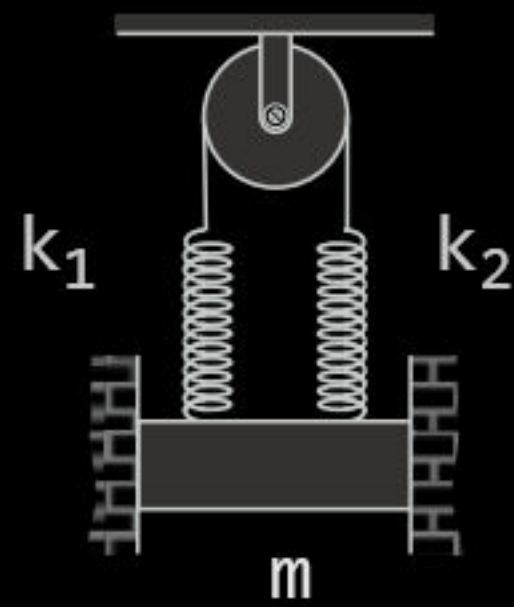
$$F_{\text{restoring}} = [k(2x \cos \theta)] \cos \theta$$

($\because$ Restoring force needs to be written opposite to direction of disturbance x)

$$= 2k \cos^2 \theta x$$

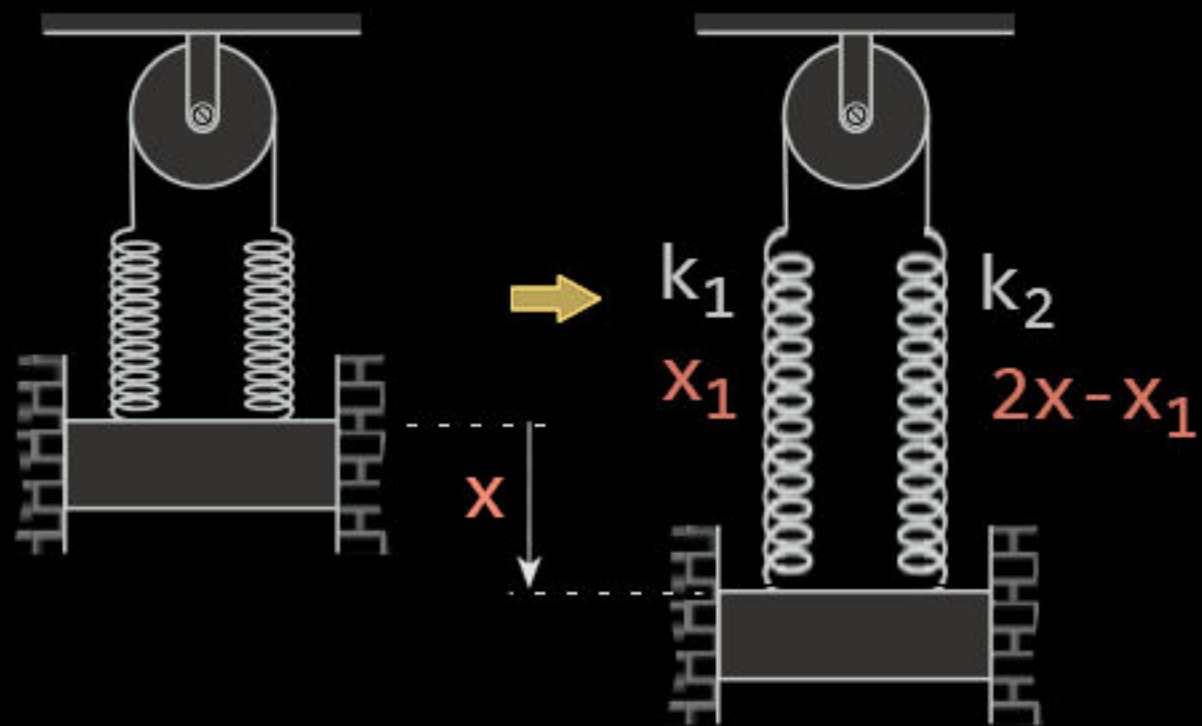
From 1,2: $2k \cos^2 \theta = k$
 $\Rightarrow \theta = 45^\circ$

$$\therefore l_0 = 2l \sin \theta = l\sqrt{2} //$$



8. A block of mass m , which can slide up and down without rotation between two frictionless walls, is suspended with the help of two light springs and a light inextensible cord that passes over an ideal pulley as shown in the figure. Force constants of the springs are k_1 and k_2 . If the bar is pulled down slightly and released, it will oscillate up and down. Find period of these oscillations.

Let extra elongation in springs be x_1 and $2x - x_1$



$$k_1 x_1 = k_2 (2x - x_1) \Rightarrow x_1 = \frac{2k_2 x}{k_1 + k_2}$$

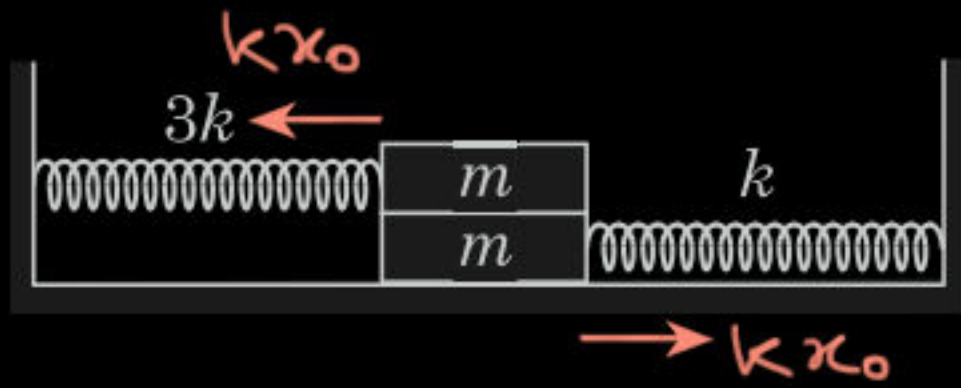
$$F_{\text{restoring}} = 2T_{\text{extra}}$$

$$= 2k_1 x_1$$

$$= \frac{4k_1 k_2 x}{k_1 + k_2}$$

$$\therefore T = 2\pi \sqrt{\frac{m}{K}} = 2\pi \sqrt{\frac{m(k_1 + k_2)}{4k_1 k_2}}$$

Initial forces:



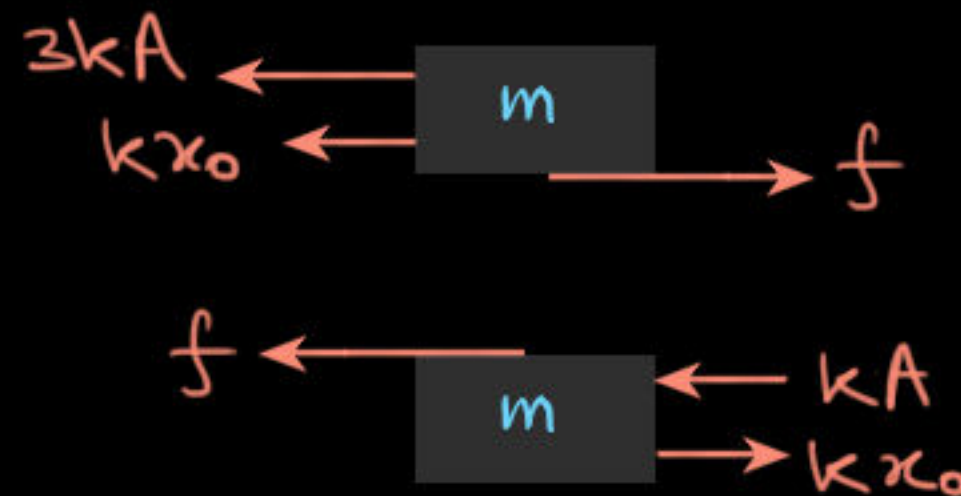
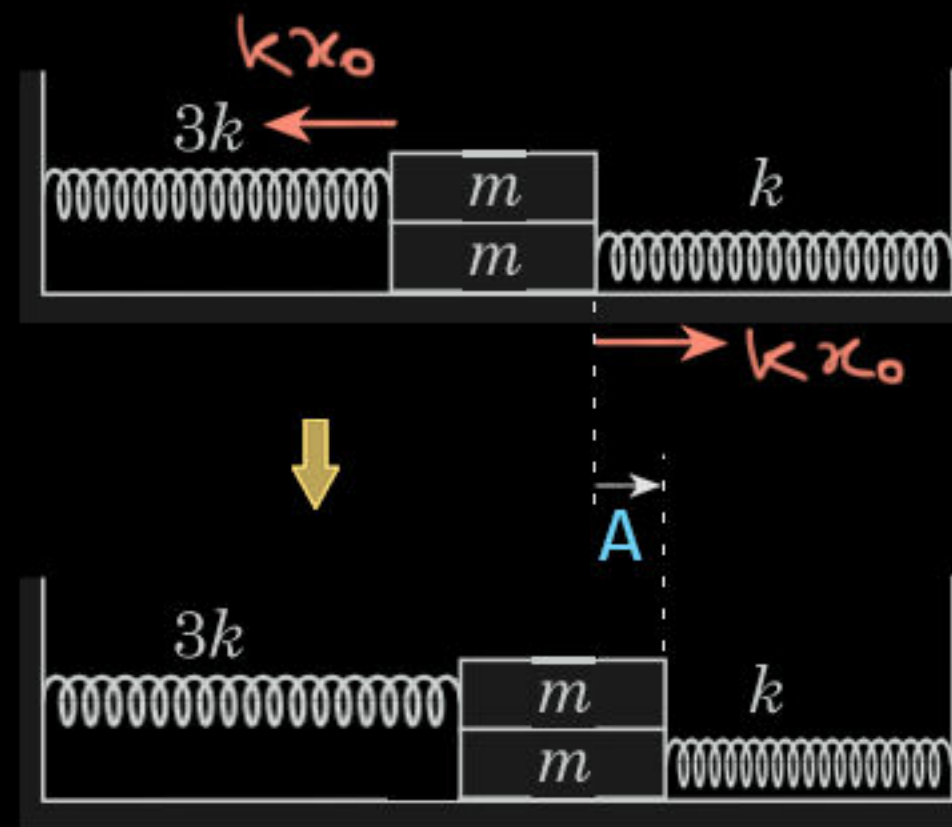
If blocks are pulled apart by greater force, chances of slipping are higher.

If blocks are shifted left, force pulling them apart reduces.
however

If blocks are shifted right, force pulling them apart increases.

So we will get the required maximum amplitude by shifting the blocks right. Let's say it is A .

9. When the arrangement shown is in equilibrium, the spring on the right is stretched by an amount x_0 . Coefficient of static friction between the blocks is μ and the horizontal floor is frictionless. Both the blocks have equal mass m and force constants of the spring are $3k$ and k as shown in the figure. Find the maximum amplitude of oscillations of the blocks along the springs that does not allow them to slide on each other. Acceleration of free fall is g .



by Ankit Singhvi

∴ At A, acceleration of both blocks is same AND friction is limiting.

$$\frac{3kA + kx_0 - f}{m} = \frac{f + kA - kx_0}{m}$$

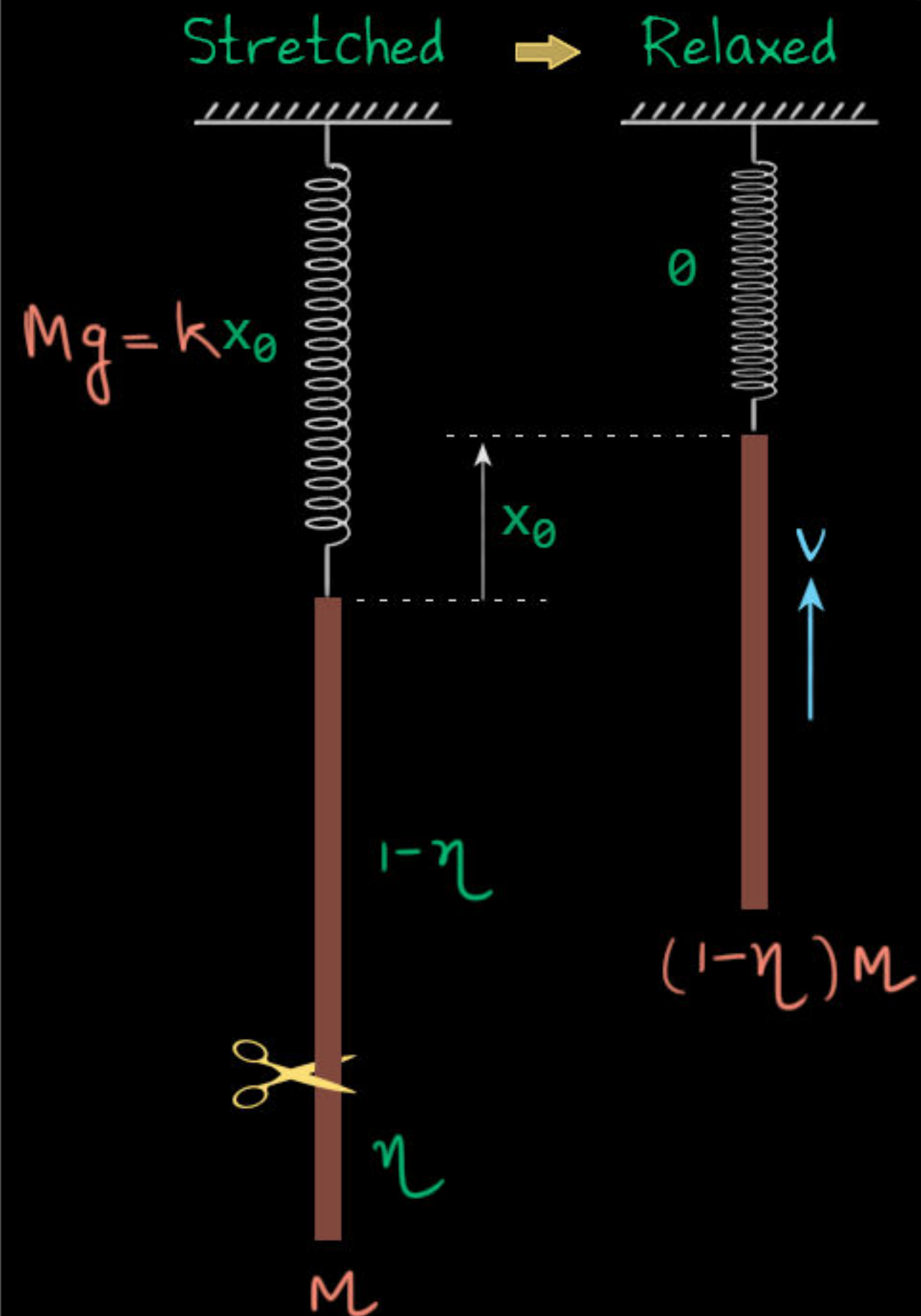
$$\Rightarrow f = kA + kx_0 \leq \mu mg$$

$$\Rightarrow A \leq \frac{\mu mg}{k} - x_0$$

Note: No matter you shift left or right, mean position of SHM is still middle, and

Time period is still $2\pi\sqrt{\frac{2m}{4k}}$

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10. A uniform rope is suspended from the ceiling with the help of a spring. The lower end of the rope is in the air. At what distance from the lower end (fraction of the total length of the rope), the rope can be cut so that the portion hanging from the spring oscillates remaining straight?

Let v be rope's velocity when spring is relaxed.

$$W_{sp} + W_g = \Delta KE$$

$$\therefore \text{Energy: } \frac{1}{2} k x_0^2 - \underset{\substack{\uparrow \\ (1-\eta)M}}{m g x_0} = \frac{1}{2} m v^2 \quad (1)$$

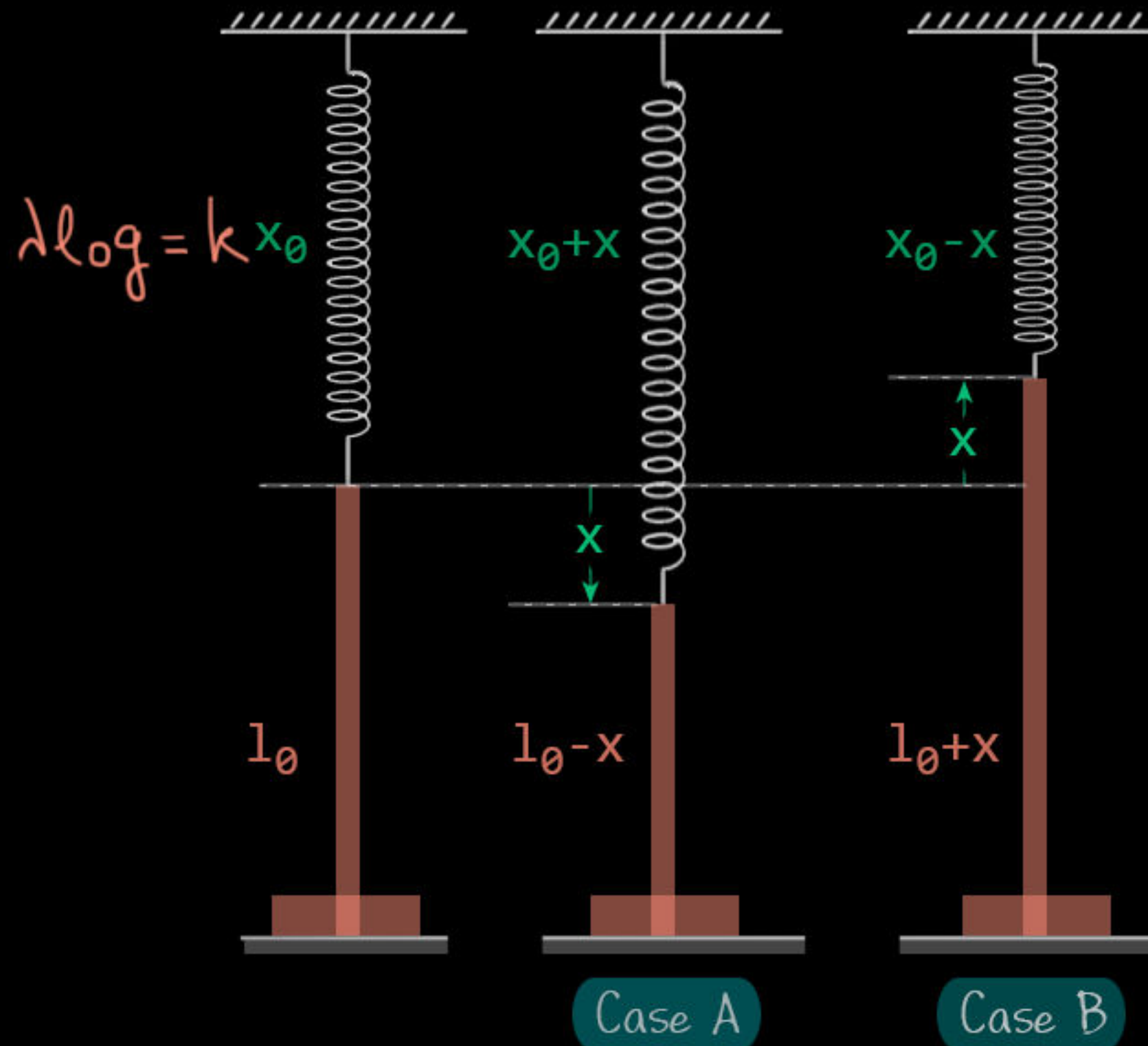
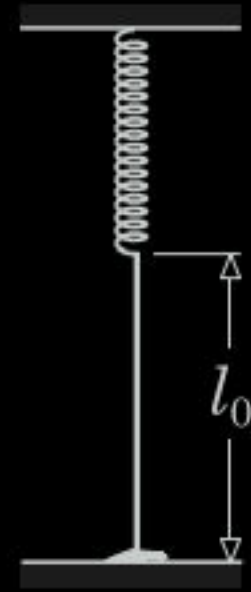
If rope still has velocity once spring is relaxed, then spring will compress and push rope down. It will cause the rope to slack.

So to remain straight $v \leq 0 \quad (2)$

$$\text{From 1,2: } \frac{1}{2} k x_0^2 - (1-\eta) M g x_0 \leq 0$$

$$\Rightarrow \frac{Mg}{2} - (1-\eta) Mg < 0 \Rightarrow \eta \leq \frac{1}{2} //$$

11. A uniform rope of linear mass density λ is suspended from the ceiling with the help of a light spring of force constant k . In equilibrium, a length l_0 of the rope is in air and rest in a heap on the floor as shown in the figure. Height of the heap is negligible as compared to the length of the rope in air. If the rope is pulled down slightly and released, find angular frequency of ensuing oscillations of the rope. Acceleration due to gravity is g .



Case A $k(x_0 + x) - \lambda(l_0 - x)g = \lambda(l_0 - x)a$

$$\Rightarrow a = \frac{(k + \lambda g)x}{\lambda(l_0 - x)} \quad (1)$$

$\downarrow \approx 0$

Case B $\lambda(l_0 + x)g - k(x_0 - x) = \lambda(l_0 + x)a$

$$\Rightarrow a = \frac{(k + \lambda g)x}{\lambda(l_0 + x)} \quad (2)$$

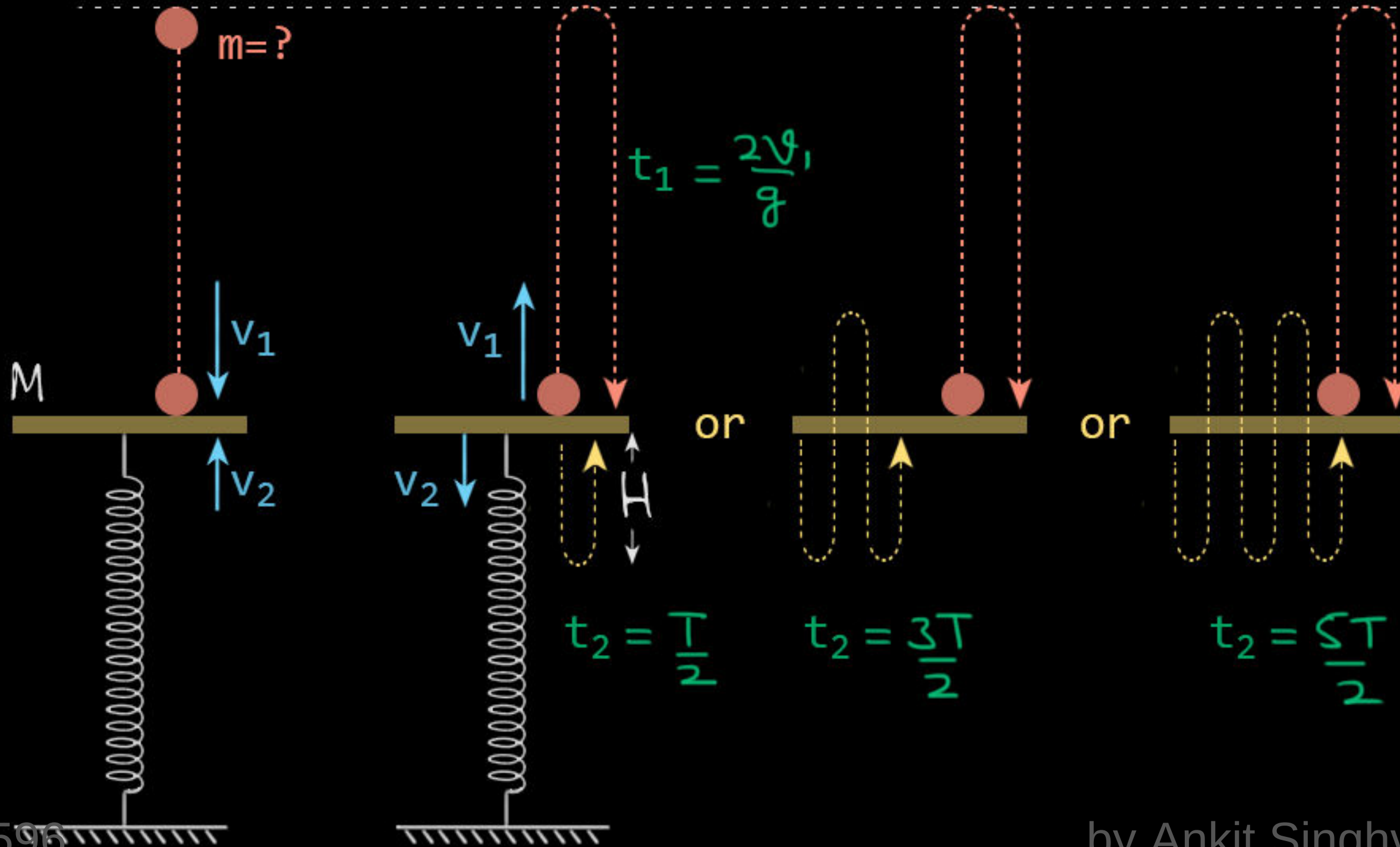
$\downarrow \approx 0$

$\therefore$ From 1, 2:

Rope performs SHM with same $\omega = \sqrt{\frac{(k + \lambda g)}{\lambda l_0}}$

(Regardless of upward or downward motion)

12. A platform of mass M mounted on a vertical spring is made to oscillate up and down with a period T . The very moment, the platform passes its equilibrium position; it collides elastically with a small ball falling from a height. Due to the collision, speeds of the platform and the ball remain unchanged but their directions are reversed. After the collision, the platform moves downwards a distance H then starts moving up and again collides with the ball at its equilibrium position. If this process repeats indefinitely, find mass of the ball. Acceleration of free fall is g .



$$p: mv_1 - Mv_2 = Mv_2 - mv_1 \quad (1)$$

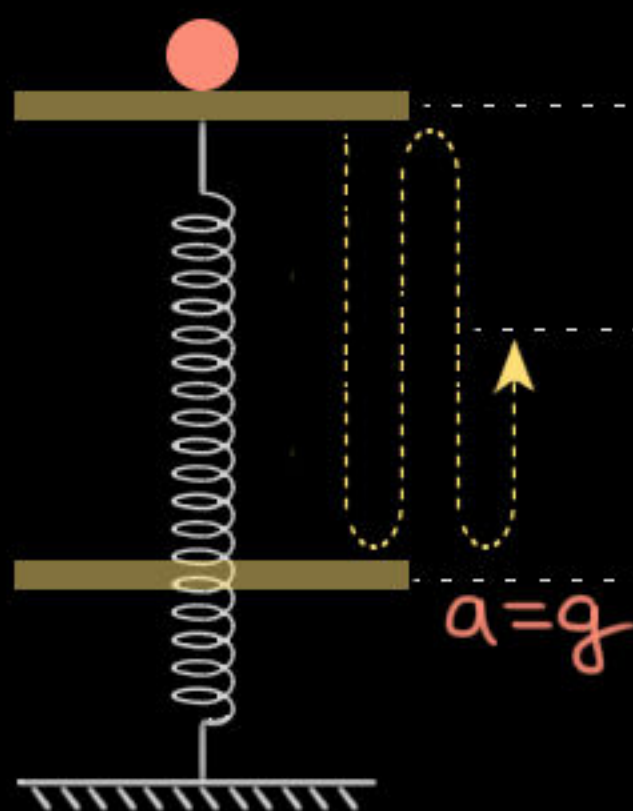
$$t_1 = t_2: \quad \frac{2v_1}{g} = \frac{T}{2}(2n+1) \quad (2)$$

$$v = \omega A: \quad v_2 = \frac{2\pi}{T} H \quad (3)$$

From 1,2,3:

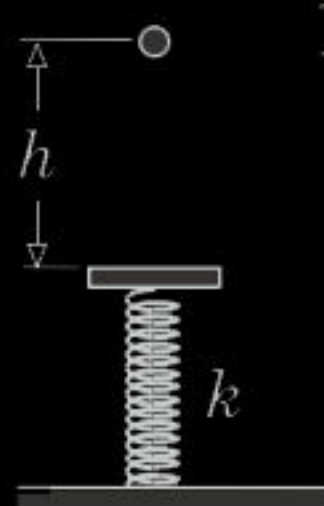
$$m = \frac{8\pi MH}{(2n+1)gT^2} \quad n=1,2,3,\dots$$

Lets stick the ball to the platform (as platform is light and hence no rebound is possible)

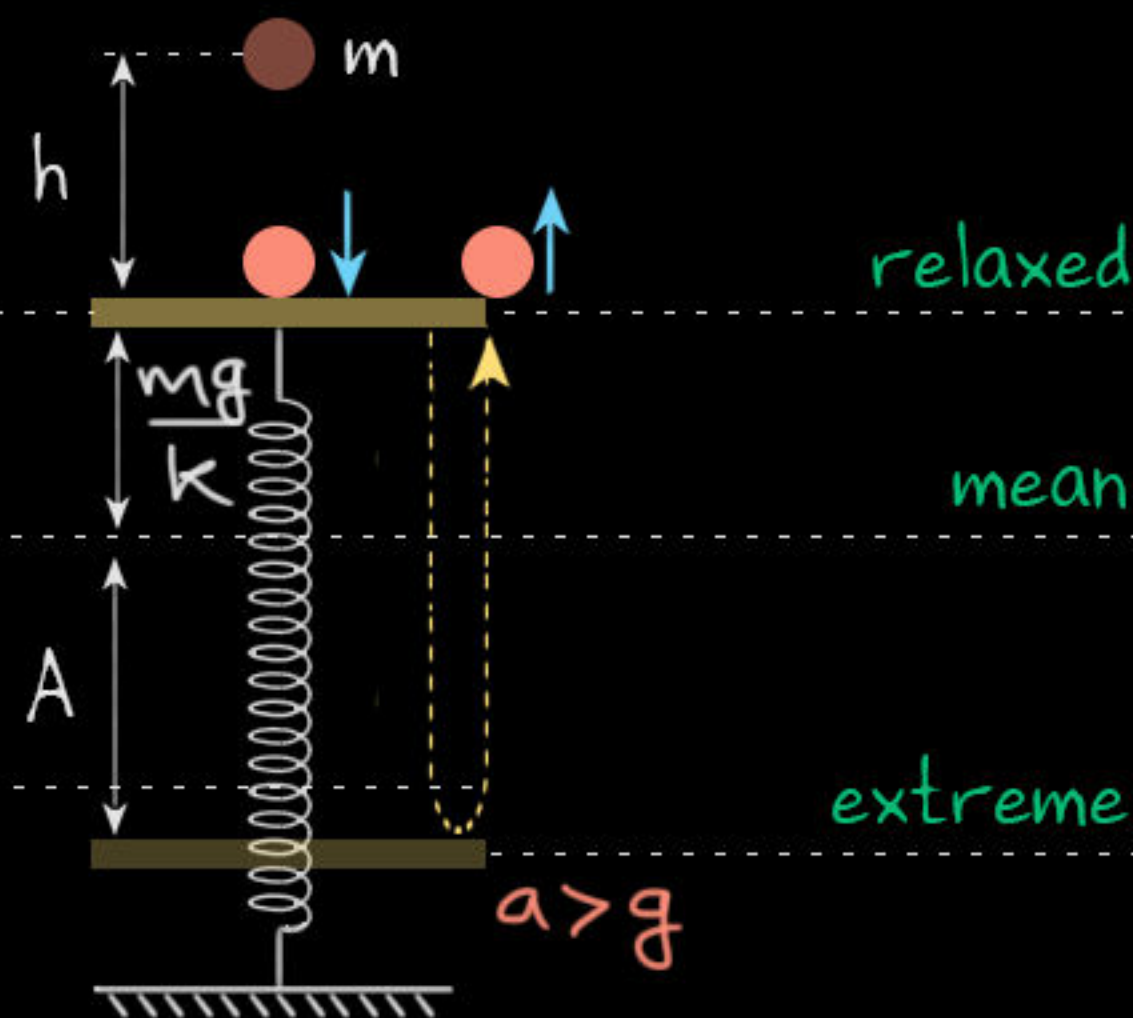


If the ball is gently placed
Then in the resulting SHM,

$$a_{\text{top}} = a_{\text{bottom}} = g.$$



13. A light platform is affixed on the top of a vertical spring of force constant k , lower end of which is affixed on the ground. A small ball of mass m falling from a height h above the platform collides with it elastically. Find the maximum velocity and maximum acceleration of the ball in ensuing oscillatory motion. Acceleration of free fall is g .



So, if ball hits the platform
with any velocity,

$a_{\text{bottom}} > g$
and its maximum there.

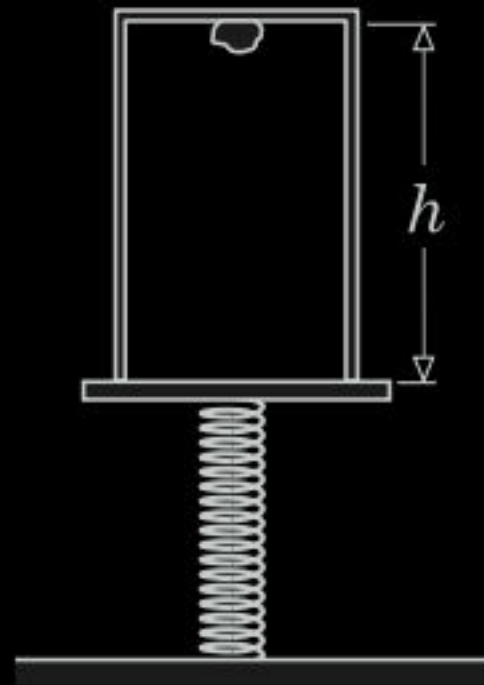
Sol:

$$E: mg \left(h + \frac{mg}{k} + A \right) = \frac{1}{2} k \left(\frac{mg}{k} + A \right)^2$$

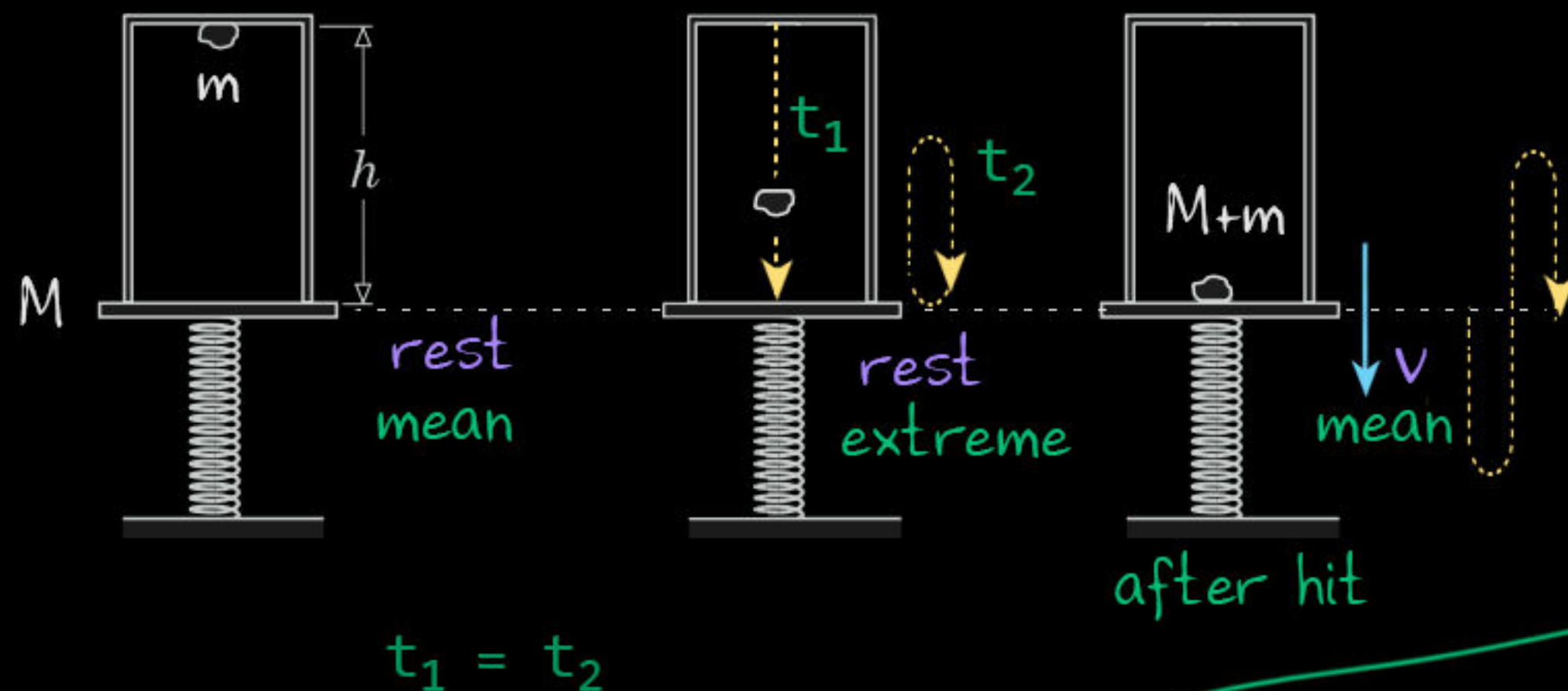
$$\Rightarrow A = \sqrt{\frac{m^2 g^2}{k^2} + \frac{2mgh}{k}}$$

$$v_{\text{max}} = \omega A = \sqrt{\frac{k}{m}} A //$$

$$a_{\text{max}} = \omega^2 A = \frac{k}{m} A //$$



14. A thin light plate is affixed on the upper end of a spring of force constant k , lower end of which is affixed on the ground. An inverted beaker of mass M is placed on the plate. A putty blob of mass m is stuck inside the beaker at its bottom as shown in the figure. After some time, when the adhesion of the putty become too weak to support it, the putty blob falls. The putty hits the plate after the plate has completed exactly one oscillation and sticks there. If the beaker does not lose contact with the plate, find height h of beaker and amplitude A of oscillations of the system after the putty sticks to the plate. Acceleration of free fall is g .



$$p: mv_1 = (m+M)v$$

$$\Rightarrow m\sqrt{2gh} = (m+M)v$$

ωA

$\sqrt{\frac{k}{m+M}}$

$$\Rightarrow m\sqrt{\frac{2g \cdot 2\pi^2 Mg}{k}} = (m+M)\sqrt{\frac{k}{m+M}} A$$

$$\Rightarrow \sqrt{\frac{2h}{g}} = 2\pi\sqrt{\frac{M}{k}} \Rightarrow$$

$$h = \frac{2\pi^2 Mg}{k}$$

$$\Rightarrow A = \frac{2\pi mg}{k} \sqrt{\frac{M}{m+M}}$$

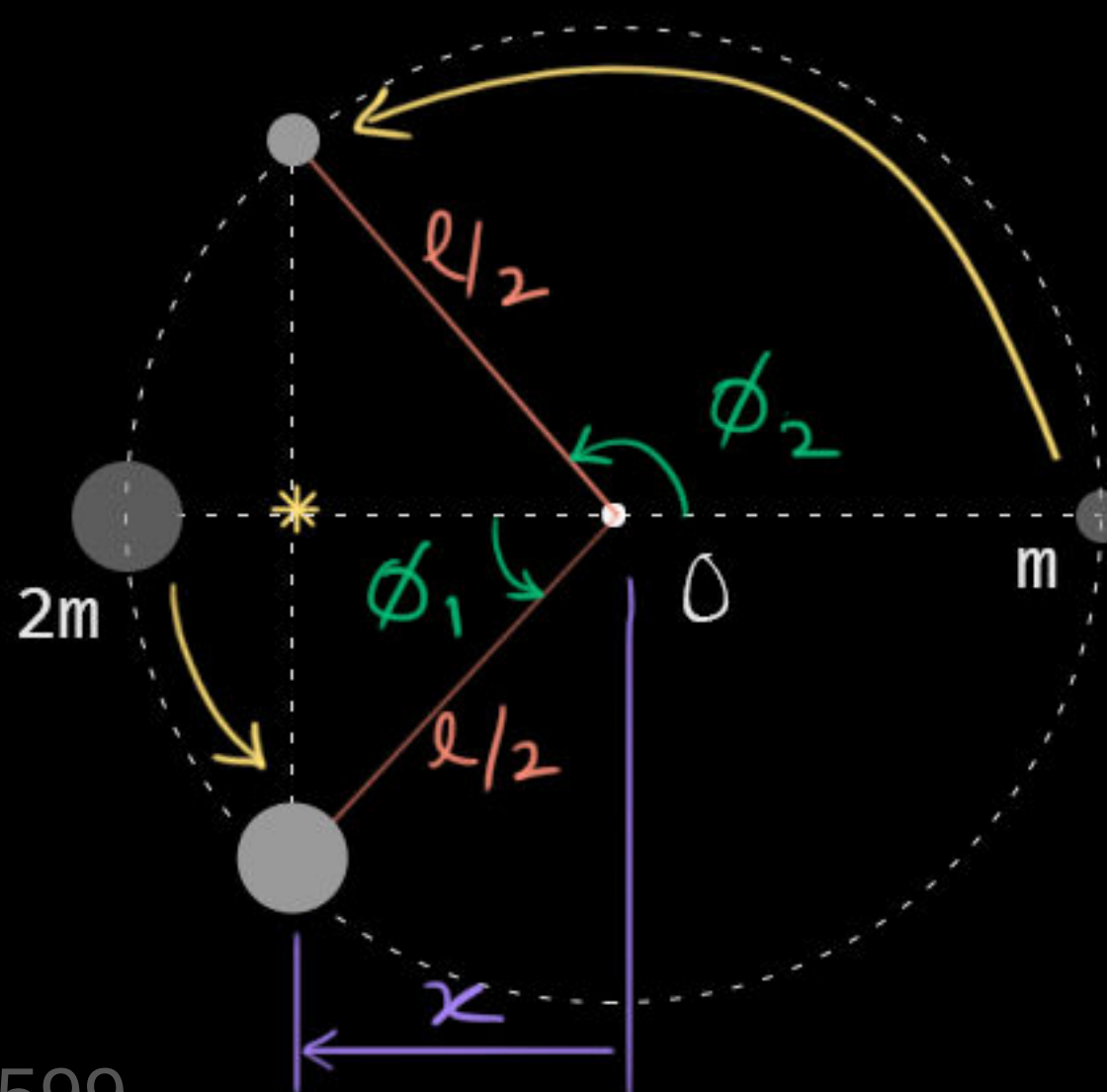
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$$\leftarrow \frac{l}{2} \rightarrow \leftarrow \frac{l}{2} \rightarrow$$

Amplitudes of both SHM's is same ($\frac{l}{2}$). $\sqrt{\frac{k}{2m}}$ $\sqrt{\frac{2k}{m}}$
Also, clearly $\omega_1 < \omega_2$

So m will cross mean first and hit $2m$ at $\times$ let's say.



15. In the arrangement shown, two blocks of masses m and $2m$ affixed on the ends of springs of force constant $2k$ and k are held at rest. The other ends of the springs are affixed on fixed supports. Relaxed length of each spring is l , distance between the supports is $2l$, size of the blocks is negligible and the floor is frictionless. Initially the blocks are held a distance l apart compressing the spring identically. After the blocks are released, they rush towards each other, collide head on and stick to each other. Find maximum speed of the blocks in the ensuing oscillations.

$$\phi_1 + \phi_2 = \pi \Rightarrow \omega_1 t + \omega_2 t = \pi$$

$$\Rightarrow t = \frac{\pi}{\sqrt{\frac{k}{2m}} + \sqrt{\frac{2k}{m}}} = \frac{\sqrt{2}\pi}{3} \sqrt{\frac{m}{k}}$$

Finding x (collision location) w.r.t O .

$$x = \frac{l}{2} \cos(\omega_2 t)$$

$$= \frac{l}{2} \cos\left(\sqrt{\frac{2k}{m}} \cdot \frac{\sqrt{2}\pi}{3} \sqrt{\frac{m}{k}}\right)$$

$$= \frac{l}{2} \cos \frac{2\pi}{3} = -\frac{l}{4}$$

by Ankit Singhvi

$$v_1 = \omega_1 \sqrt{\left(\frac{l}{2}\right)^2 - \left(\frac{l}{4}\right)^2} = \sqrt{\frac{k}{2m}} \frac{l\sqrt{3}}{4}$$

$$v_2 = \omega_2 \sqrt{\left(\frac{l}{2}\right)^2 - \left(\frac{l}{4}\right)^2} = \sqrt{\frac{2k}{m}} \frac{l\sqrt{3}}{4}$$

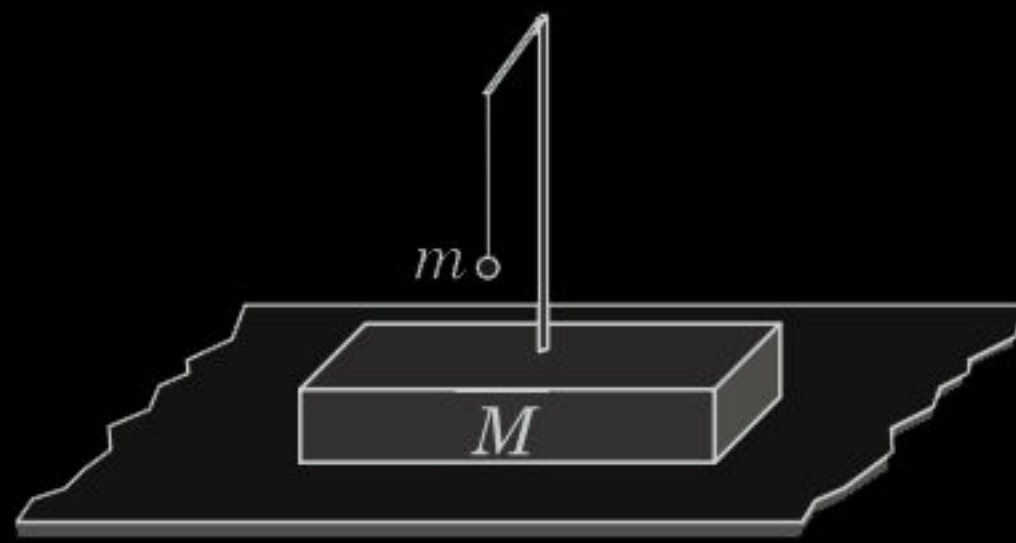
$$p: 2mv_1 - mv_2 = 3mv$$

$$\Rightarrow v = 0$$

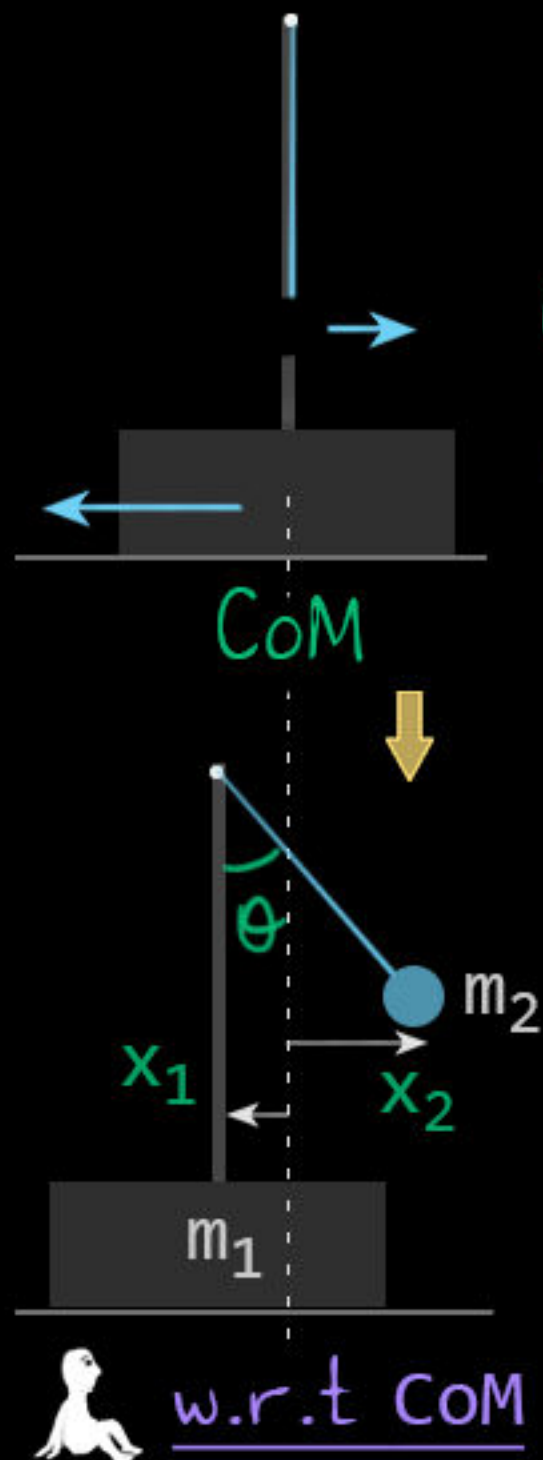
So they come to rest immediately after collision. Then perform SHM with amplitude $\frac{l}{4}$

$$\therefore v_{\max} = \omega A = \sqrt{\frac{3k}{3m}} \frac{l}{4} = \frac{l}{4} \sqrt{\frac{k}{m}}$$

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CoM



16. A block of mass M can slide without friction on a horizontal frictionless floor. A small ball of mass m is suspended by a light inextensible cord of length l from a hanger fixed on the top of a vertical pole rigidly mounted on the block so that the ball can swing freely in a vertical plane. When the block is held fixed, the period of small amplitude oscillations of the simple pendulum consisting of the ball and the thread is T_0 . What would be period of small amplitude oscillations of the simple pendulum, if the block were not held fixed?

$$m_1 x_1 = m_2 x_2$$

Observing m_2 in any frame:

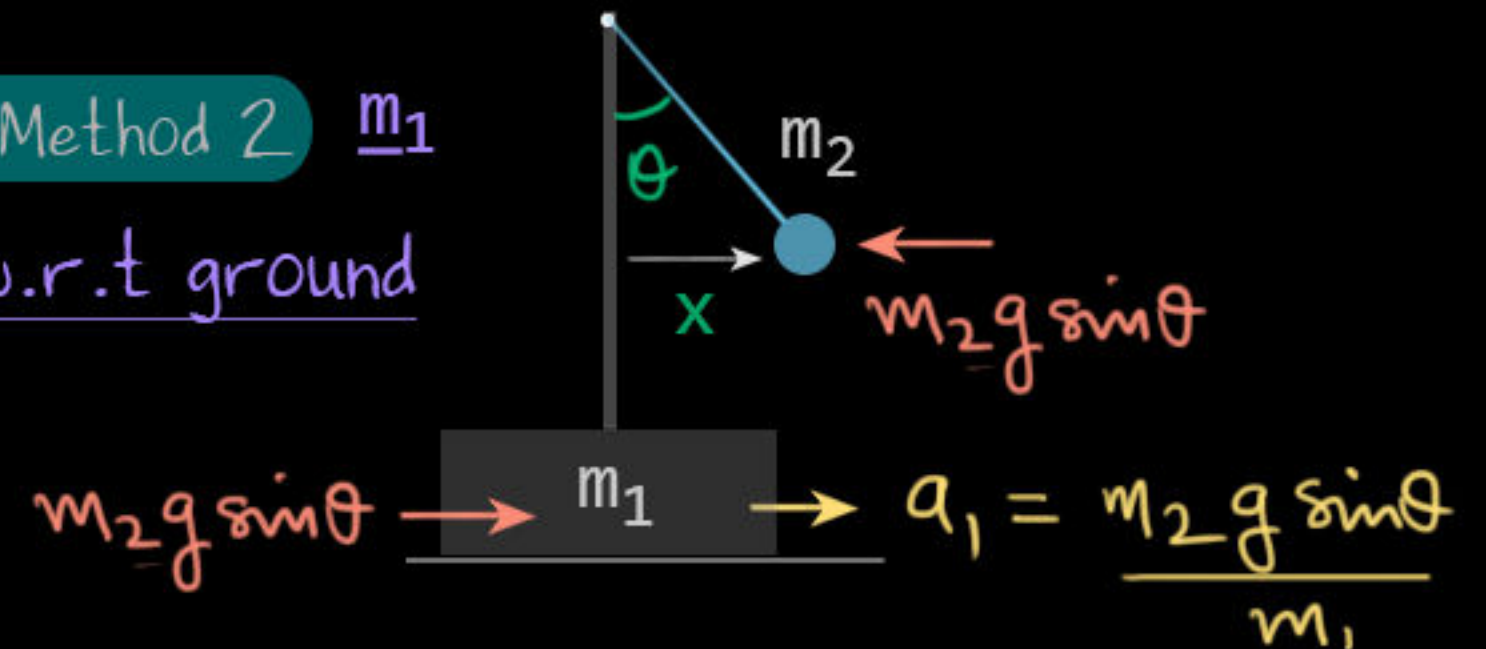
$$F_{\text{restoring}} = m_2 g \sin \theta$$

$$= m_2 g \left[\frac{m_2 x_2}{m_1} + x_2 \right]$$

$$= \frac{m_2 (m_1 + m_2) g}{m_1 l} x_2$$

Method 2 m_1

w.r.t ground



w.r.t m_1



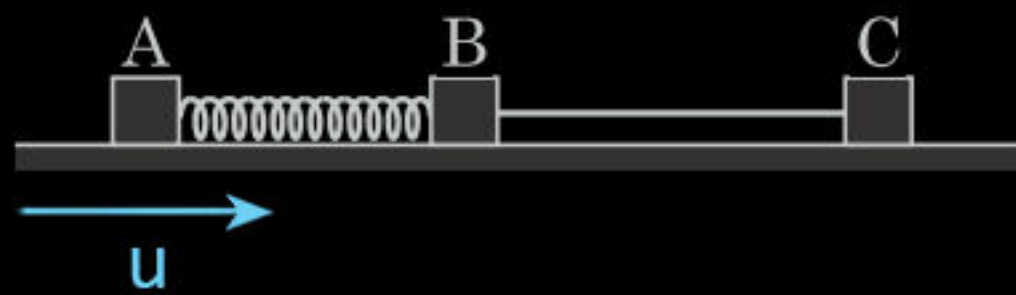
$$F_{\text{restoring}} = m_2 \left[\frac{m_2 g \sin \theta}{m_1} \right] + m_2 g \sin \theta$$

$$T = 2\pi \sqrt{\frac{m_2}{k}}$$

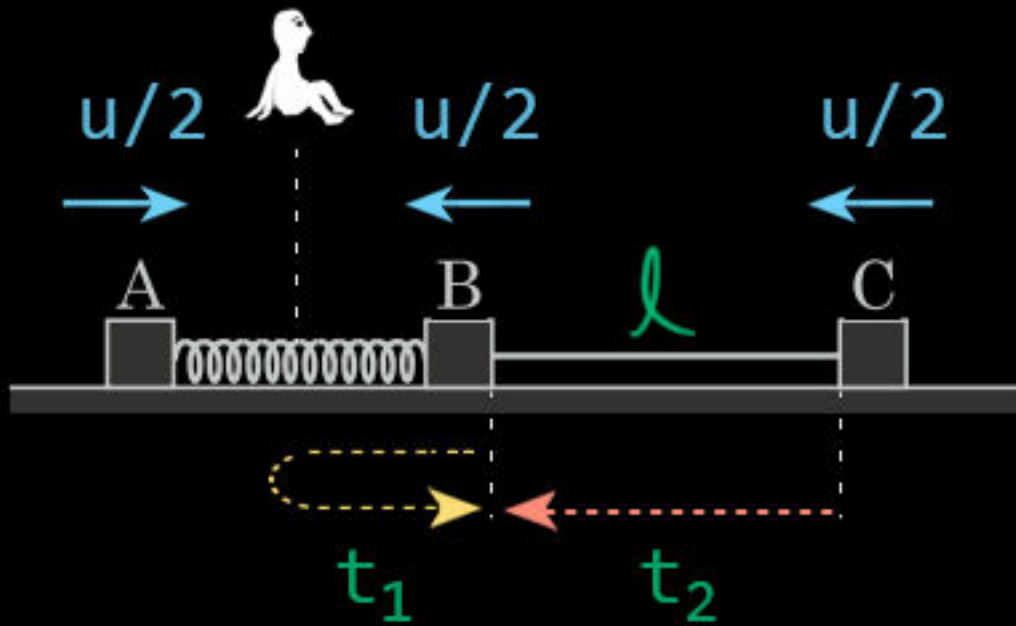
$$= \frac{m_2 (m_1 + m_2) g}{m_1 l} x$$

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w.r.t CoM of A and B



17. Three identical blocks A, B and C are placed in a line on a frictionless horizontal floor. The blocks A and B are connected with a light spring of force constant k and the blocks B and C with a light inextensible cord. Initially the system is motionless. The block A is given a velocity u by a sharp hit towards the block B. When the blocks B and C collide, a sound bang is created. What should the minimum length of the cord be in order to create a sound bang of maximum loudness?

For a two block spring SHM, we know:

$$T = 2\pi \sqrt{\frac{\mu}{k}} \quad \text{where } \mu = \frac{m \cdot m}{m + m} = \frac{m}{2}$$

$$= 2\pi \sqrt{\frac{m}{2k}}$$

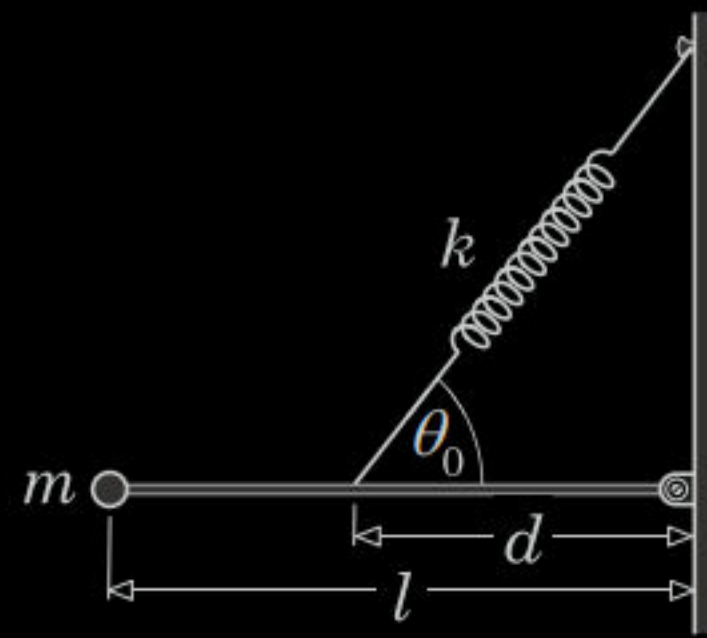
where, μ = reduced mass.

$$t_1 = t_2$$

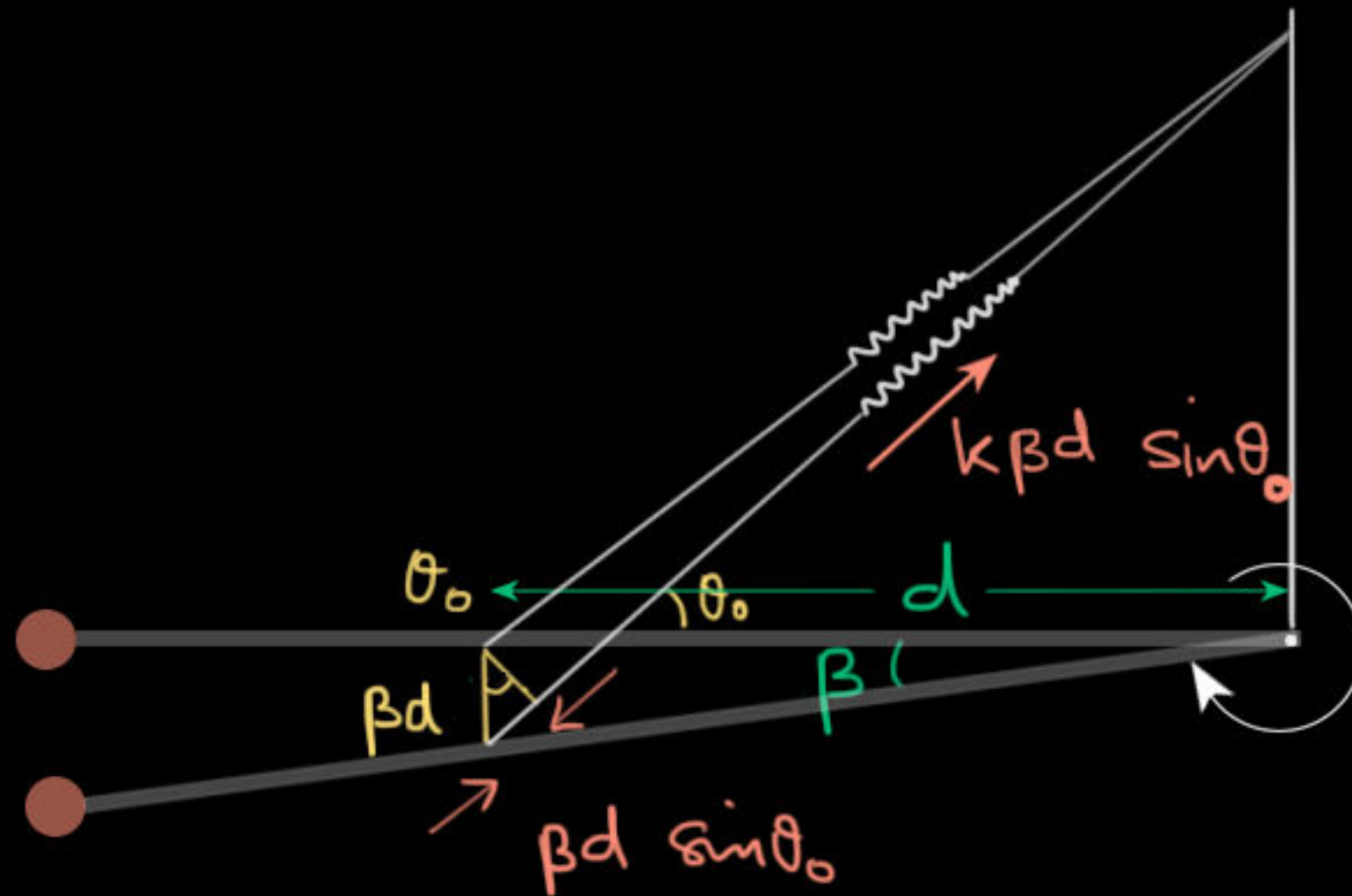
$$\Rightarrow T_{1/2} = \frac{l}{u/2}$$

$$\Rightarrow \frac{1}{2} \left(2\pi \sqrt{\frac{m}{2k}} \right) = \frac{2l}{u}$$

$$\Rightarrow l = \frac{\pi u}{2} \sqrt{\frac{m}{2k}}$$



18. A small ball of mass m is affixed at one end of a light rod of length l , the other end of which is hinged to a fixed pivot on a wall. One end of a spring of force constant k is attached on the rod at a distance d from the hinge and the other end of the spring is attached to a nail on the wall. In equilibrium, the rod stays horizontal and angle between the spring and the rod is θ_0 . If the ball is pulled slightly downwards and released, the system begins to oscillate. Find period of these small amplitude oscillations.



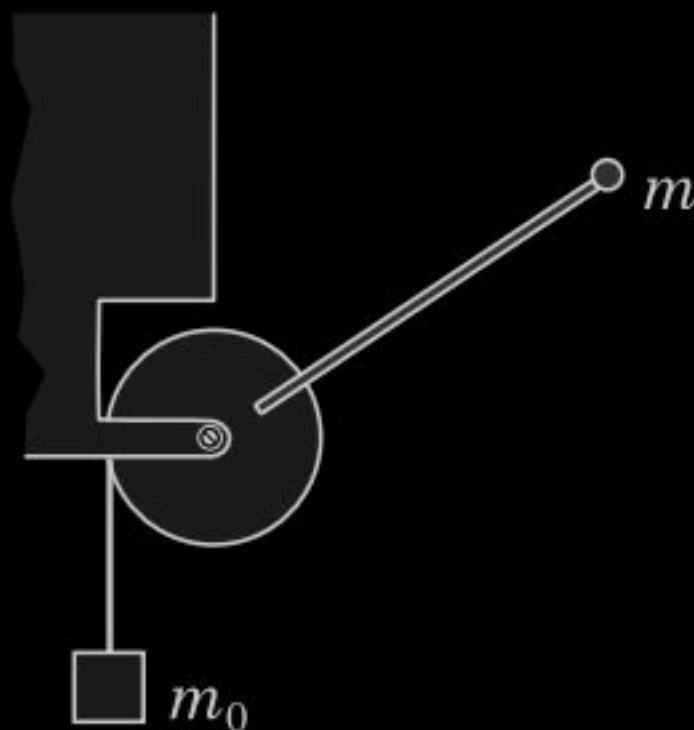
Writing only extra torque produced

$$\text{Restoring Torque} = ml^2 \alpha = (k \beta d \sin \theta_0) \sin \theta_0 \cdot d$$

$$\Rightarrow \alpha = \frac{k d^2 \sin^2 \theta_0}{m l^2} \beta$$

ω^2

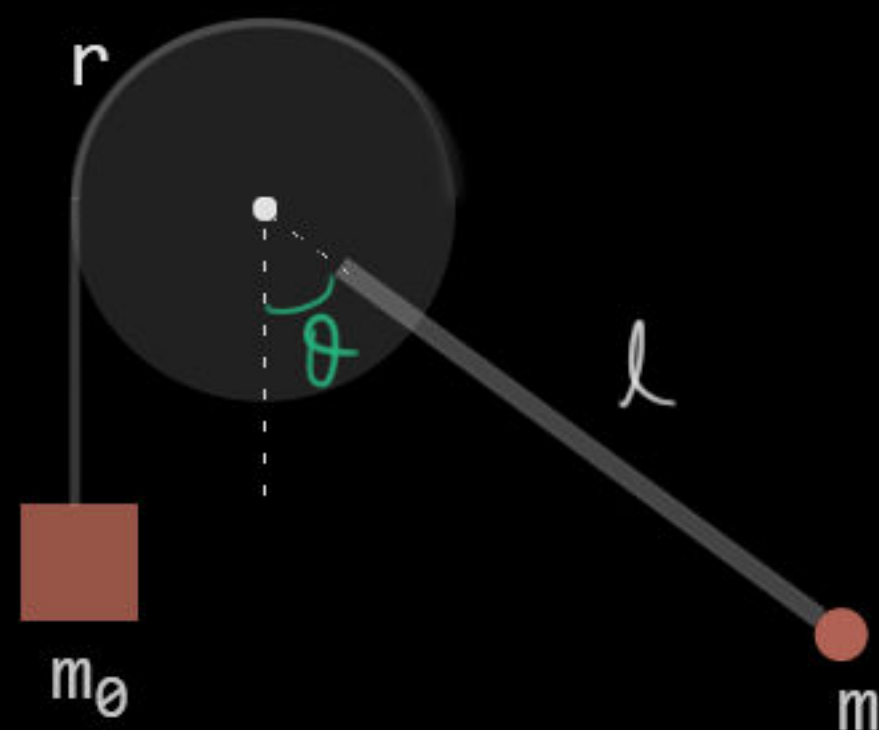
$$\therefore T = \frac{2\pi}{d \sin \theta_0} \sqrt{\frac{m}{k}}$$



19. In the arrangement shown, a block of mass m_0 is suspended at one end of a light inextensible cord that is wrapped on a light drum of radius r . The drum can rotate without friction about a fixed horizontal axle that coincides with axis of the drum. A light rod is attached radially with the drum and a particle of mass m is affixed at the free end of the rod. Distance between the particle and the axle is l .

- (a) Find orientation of the rod when the system is in stable equilibrium.
 (b) Find period of small oscillations about this stable equilibrium.

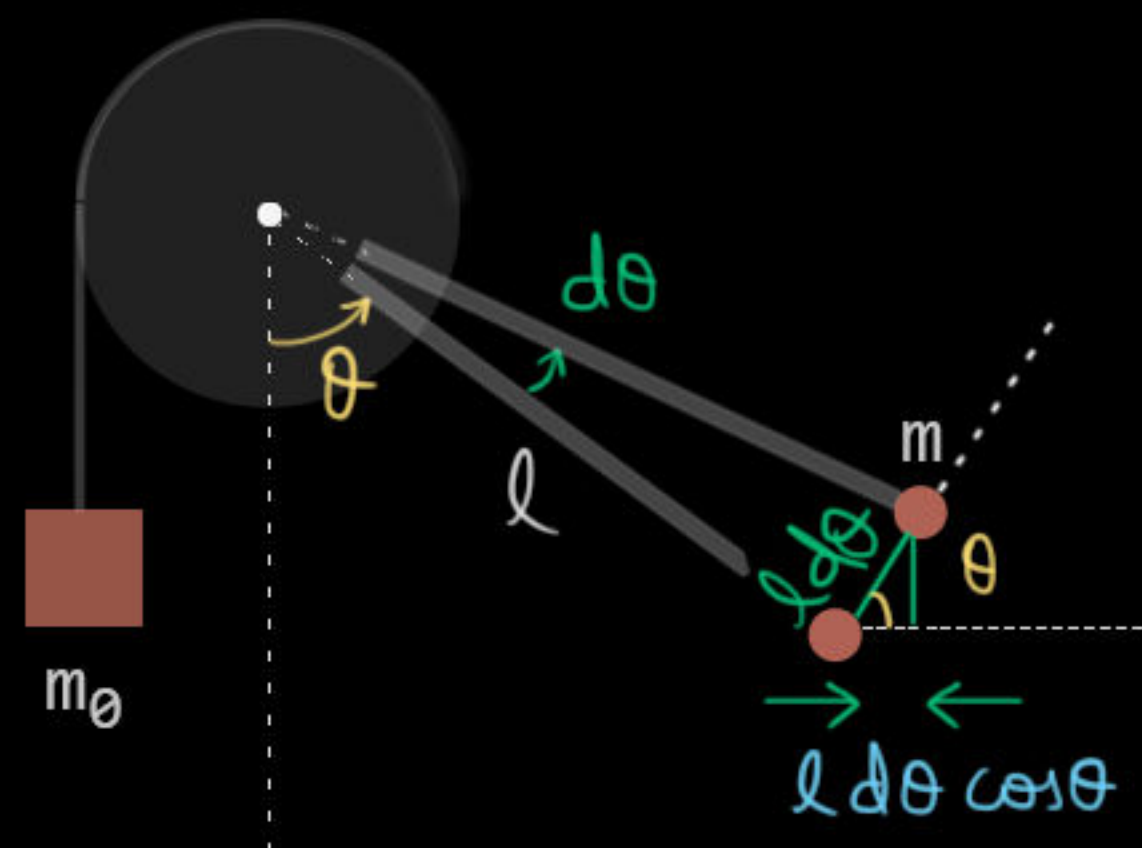
Acceleration of free fall is g .



At equilibrium:

$$m_0 g r = m g l \sin \theta$$

$$\Rightarrow \theta = \sin^{-1} \frac{m_0 r}{m l}$$



$$\frac{m l}{\sqrt{m^2 l^2 - m_0^2 r^2}} \sin \theta = \frac{m_0 r}{\sqrt{m^2 l^2 - m_0^2 r^2}}$$

$$\text{Restoring Torque} = I \alpha = m g l d \theta \cos \theta$$

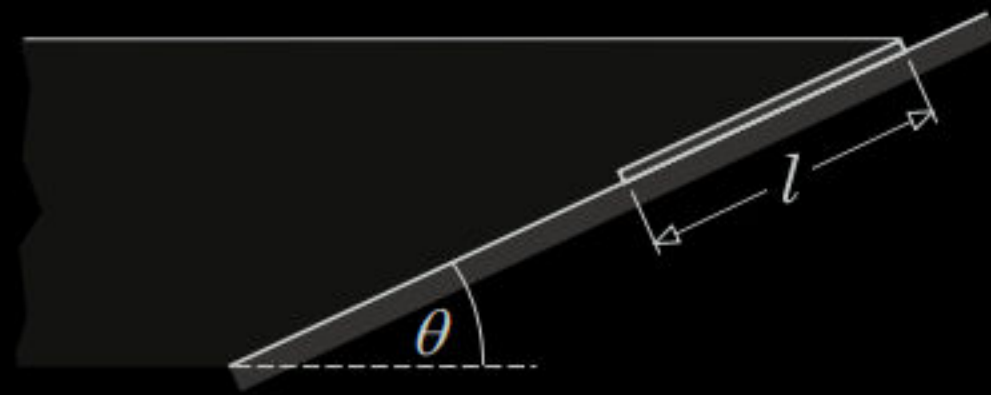
$$\Rightarrow (m_0 r^2 + m l^2) \alpha = m g l \sqrt{m^2 l^2 - m_0^2 r^2} d \theta$$

$$\Rightarrow \alpha = \frac{g \sqrt{m^2 l^2 - m_0^2 r^2}}{m_0 r^2 + m l^2} d \theta$$

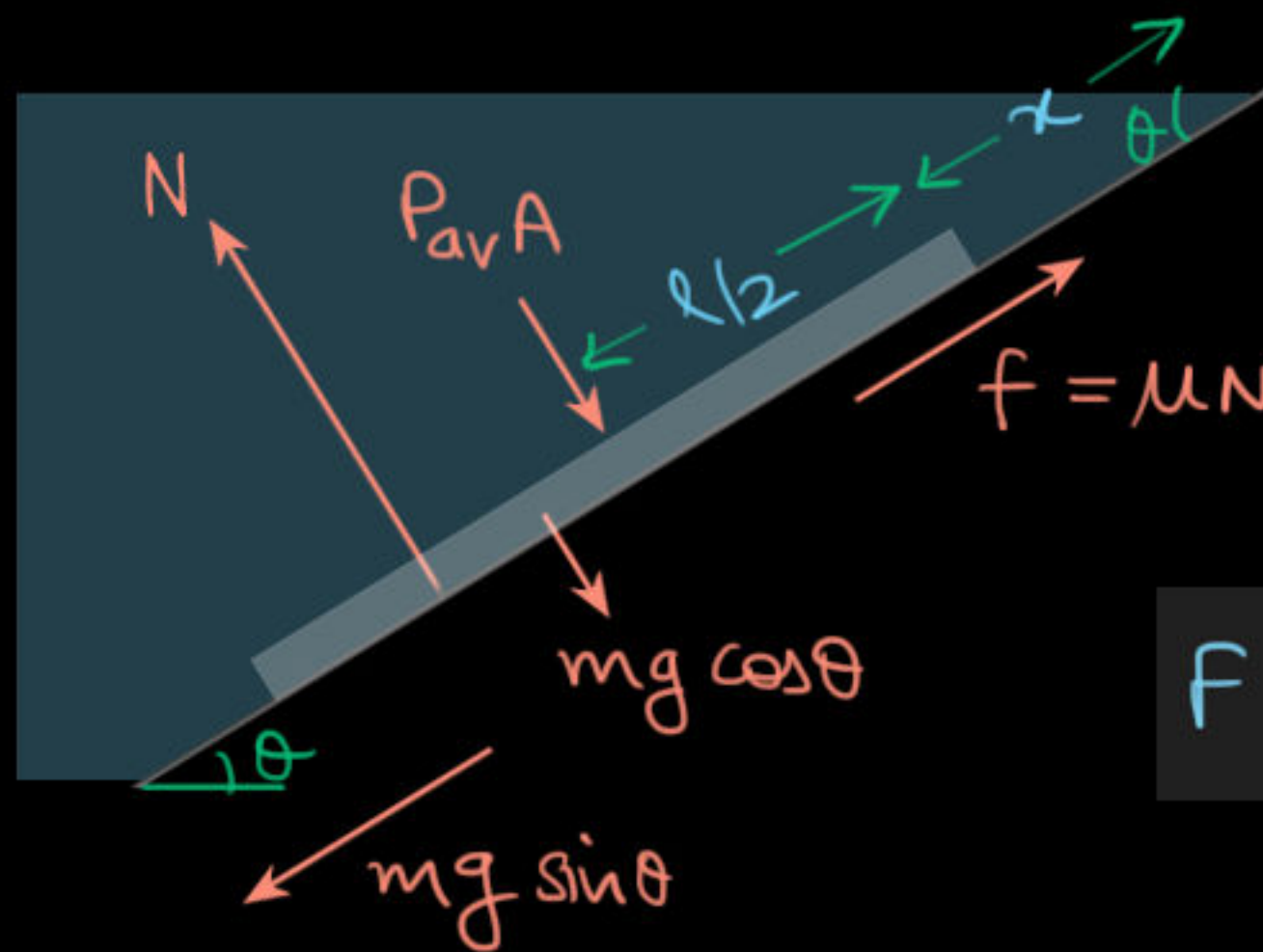
$$\therefore T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m_0 r^2 + m l^2}{g \sqrt{m^2 l^2 - m_0^2 r^2}}}$$

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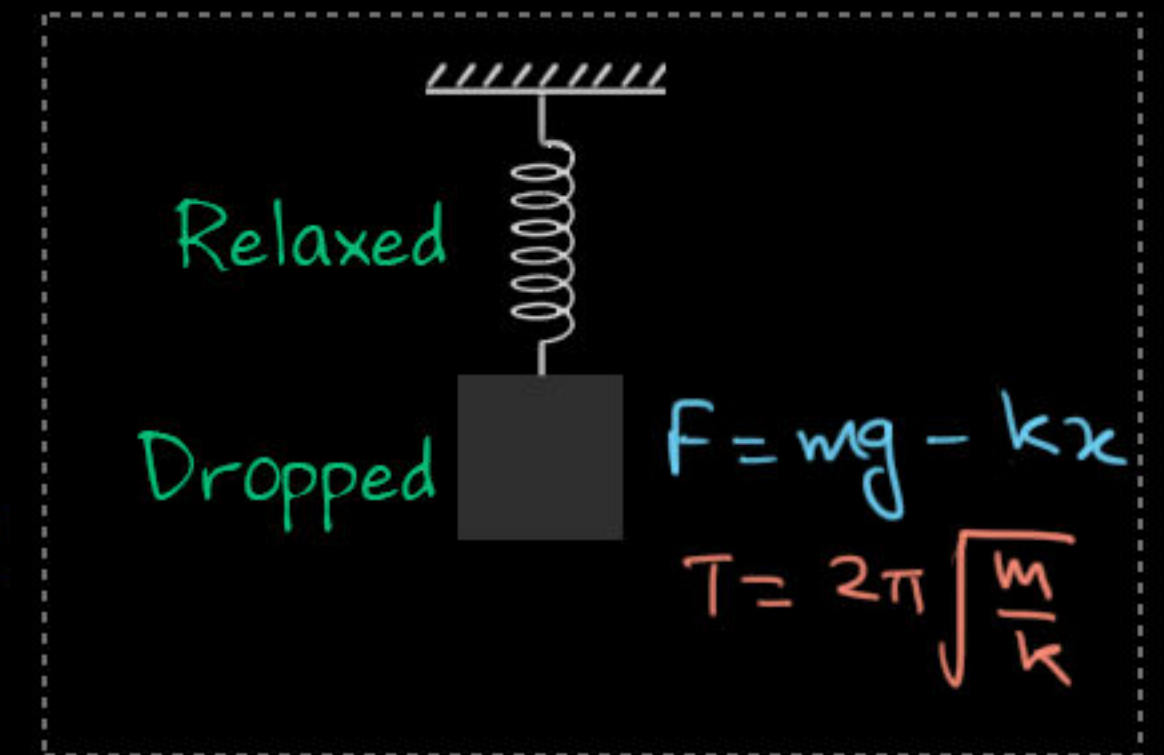
20. One wall of a water tank is inclined at angle θ to the horizontal. On this wall, a thin square plate of mass m and side l is held with its upper edge coinciding with the water level. Coefficient of friction between the plate and the wall is μ . When the plate is released, it slides down the wall and water does not enter between the plate and the wall. How long will the plate keep sliding? Neglect viscosity and turbulence in the water. Density of water is ρ_0 and acceleration of free fall is g



$$f = \mu N = \mu (P_{av} A + mg \cos \theta)$$

$$= \mu l^2 \rho_0 g \left(\frac{l}{2} + x \right) \sin \theta$$

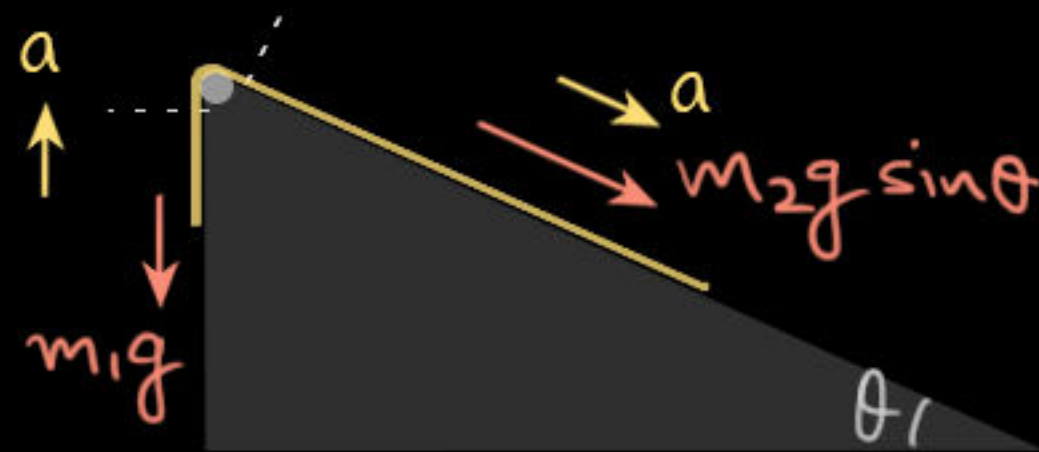
$$F = \text{constant} - \mu l^2 \rho_0 g \sin \theta x$$



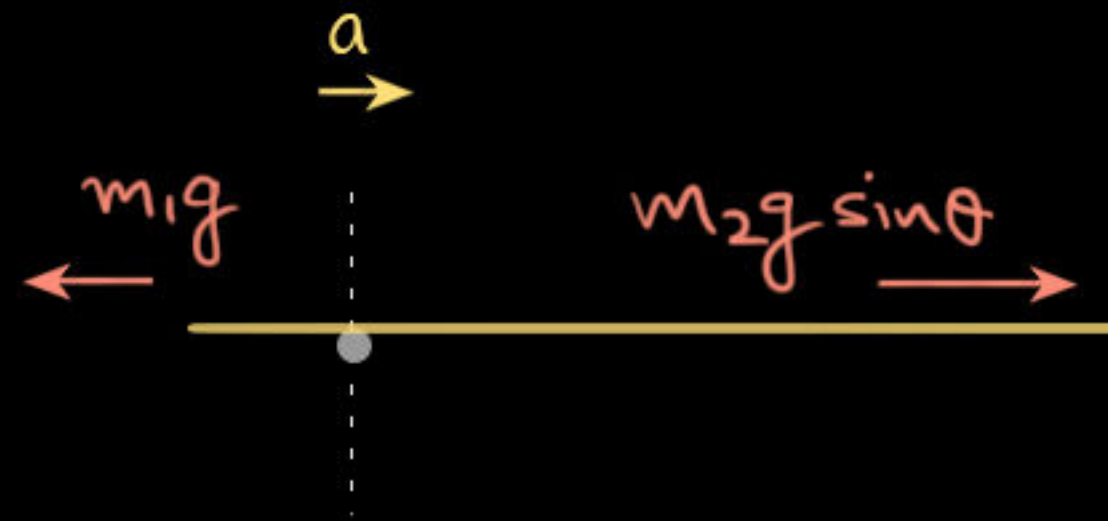
$$t = T_{1/2} = \frac{1}{2} \cdot 2\pi \sqrt{\frac{m}{\mu l^2 \rho_0 g \sin \theta}}$$

Acceleration of liquid in a U tube By writing forces "along" the water column.

Rope on inclined plane



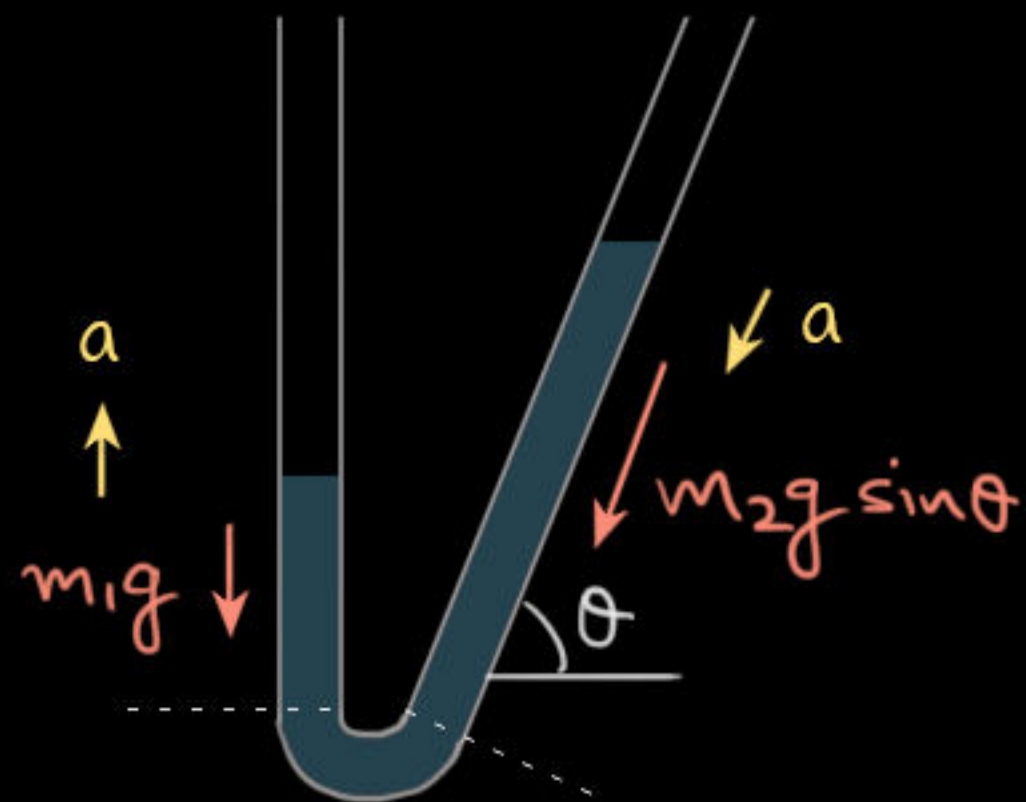
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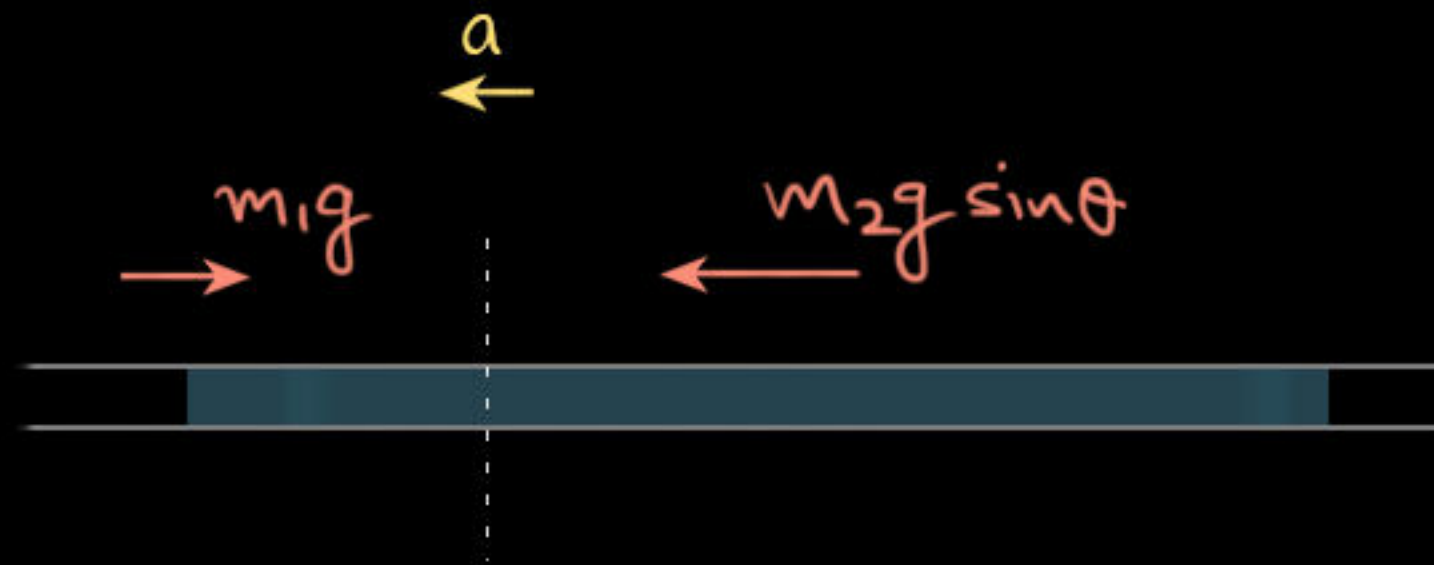
⇒

$$\therefore a = \frac{m_2 g \sin \theta - m_1 g}{m_1 + m_2}$$

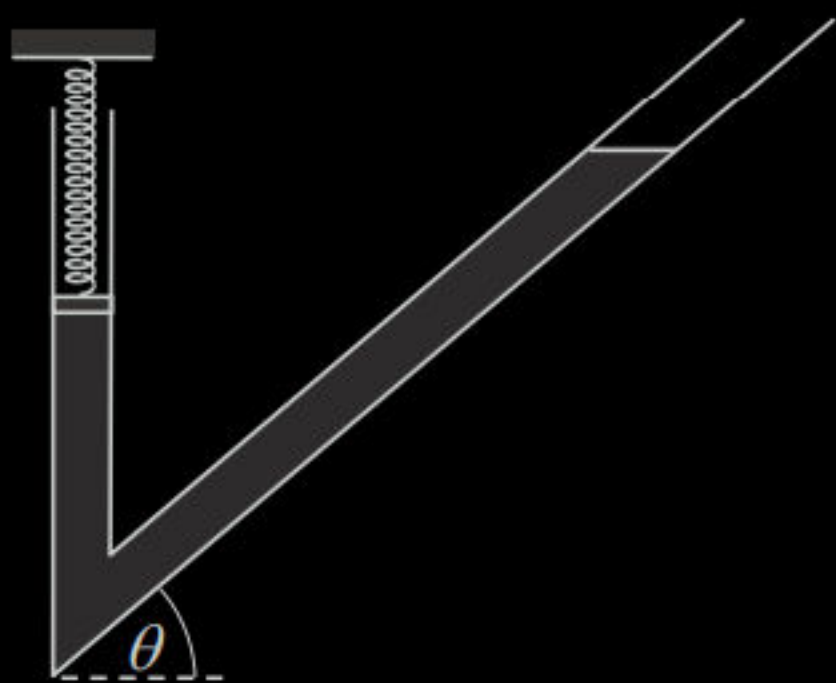
Water in a U tube



≡



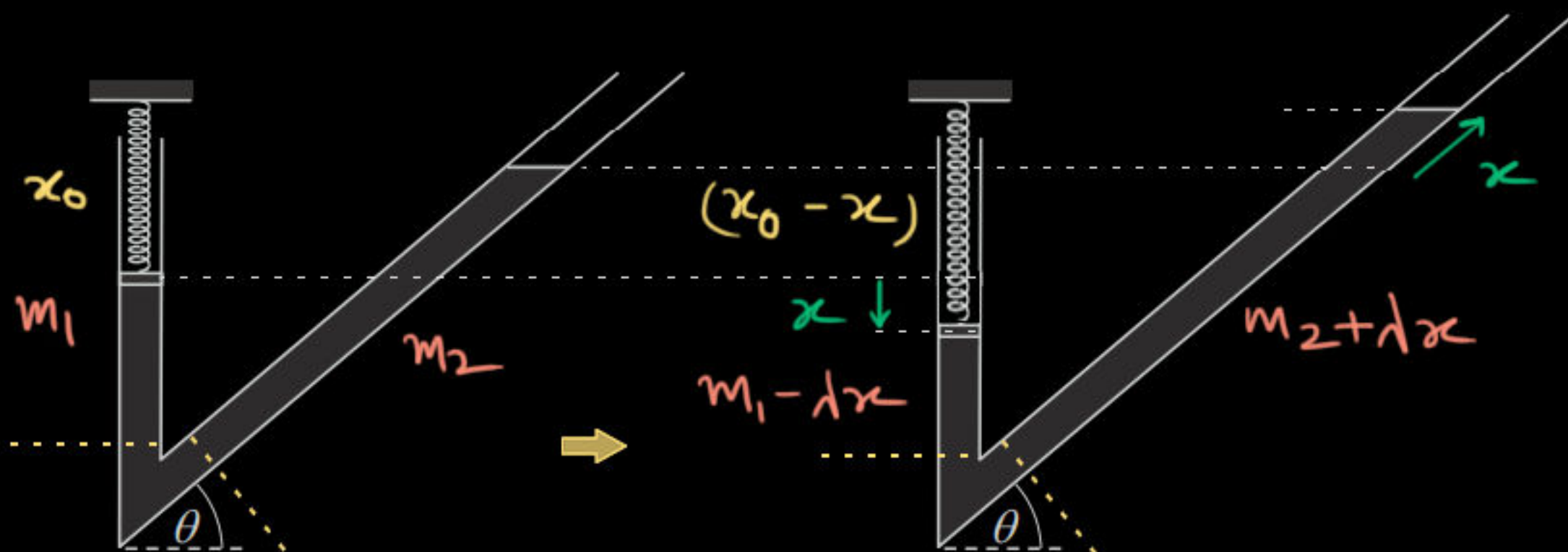
⇒



21. A glass tube of circular section of area S bent in a shape shown, has one arm vertical and the other inclined at an angle θ . A light piston that can slide without friction in the vertical arm is connected at the lower end of a vertical spring of force constant k , the upper end of which is connected to a fixed support. When a mass m of a non-viscous liquid of density ρ is poured in the tube, level of liquid in the inclined arm stays higher than the piston as shown in the figure. Find period of small oscillations of the liquid. There is no friction between the piston and the tube and acceleration due to gravity is g .

Lets push down the spring by x .

∴ If initial compression is x_0 , it becomes $x_0 - x$.



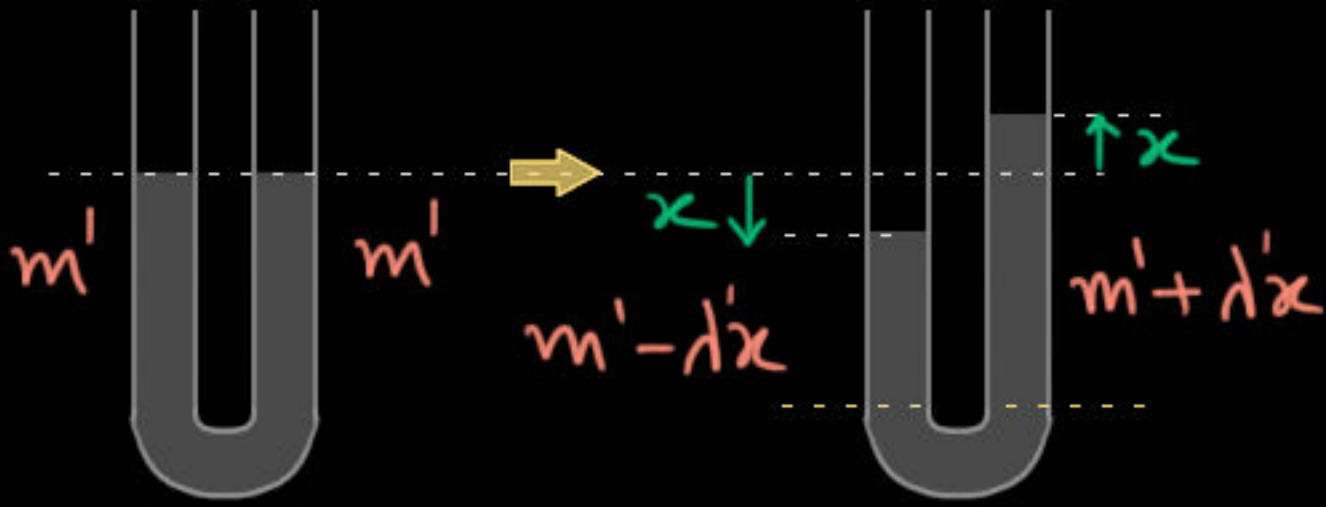
$$kx_0 + m_1g = m_2g \sin \theta$$

$$F_{\text{restoring}} = (m_2 + \rho S x)g \sin \theta - \{ k(x_0 - x) + (m_1 - \rho S x)g \}$$

$$F_{\text{restoring}} = \underbrace{\left[k + \rho S g (1 + \sin \theta) \right]}_{k'} x$$

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{k + \rho S g (1 + \sin \theta)}}$$

Let mercury be $2m'$.



$$F_{\text{restoring}} = (\cancel{m'} + \Delta x)g - (\cancel{m'} - \Delta x)g$$

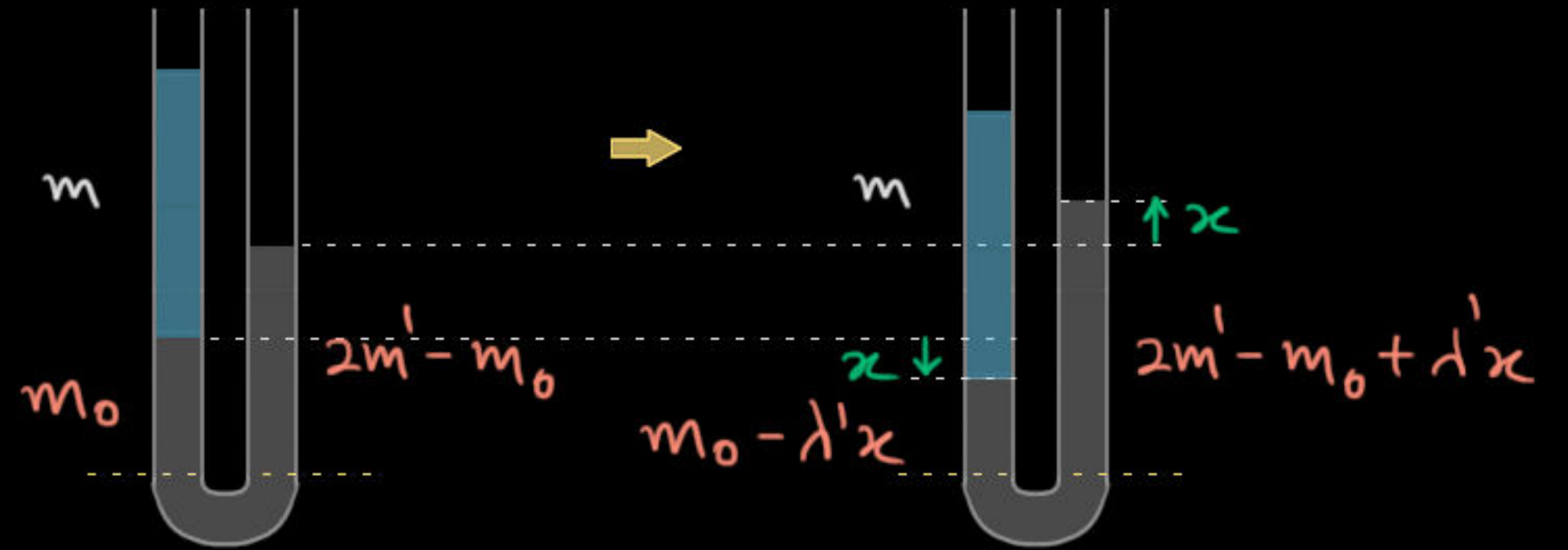
$$= \underbrace{2\Delta x g}_{k}$$

$$\Rightarrow T_1 = 2\pi \sqrt{\frac{2m'}{2\Delta x g}} = 2\pi \sqrt{\frac{m'}{\Delta x g}} \quad (1)$$

Dividing 1 and 2:

$$2m' = \frac{m T_1^2}{T_2^2 - T_1^2}$$

22. A thin vertical U-tube of very long arms contains some amount of mercury. Period of small amplitude oscillations of mercury is $T_1 = 2.0$ s. If in one arm, a mass $m = 100$ g of water is added, period of small oscillation becomes $T_2 = 3.0$ s. Find mass of mercury in the tube. Neglect effects of surface tension.



$$mg + m_0 g = (2m' - m_0)g$$

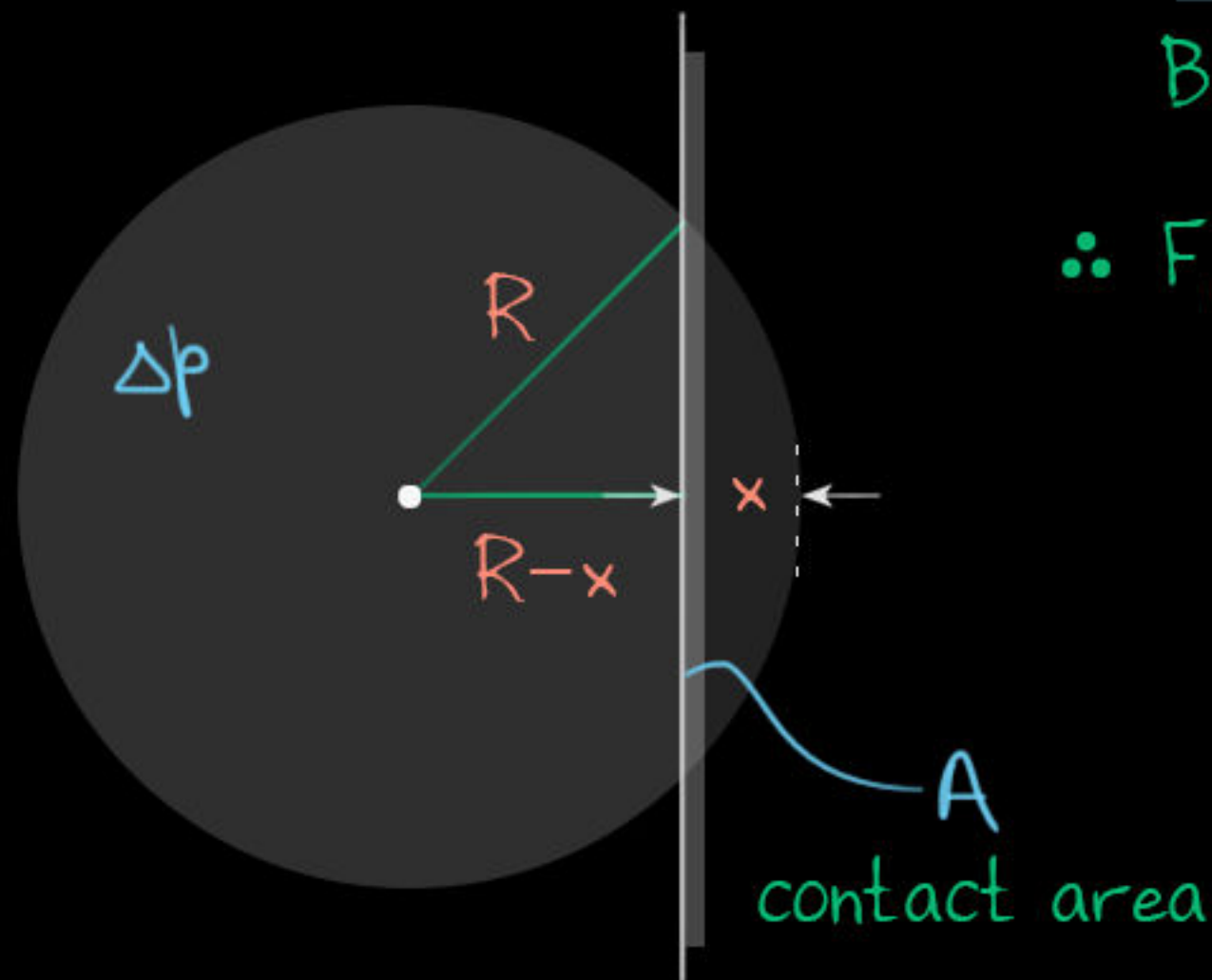
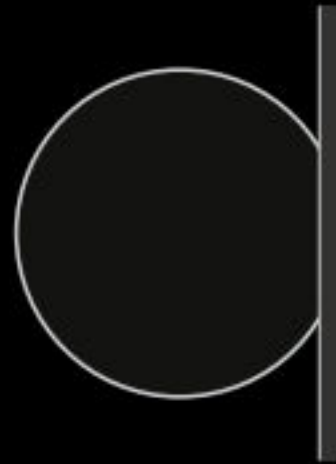
$$F_{\text{restoring}} = (\cancel{2m'} - \cancel{m_0} + \Delta x)g - (\cancel{m} + \cancel{m_0} - \Delta x)g$$

$$= 2\Delta x g$$

$$\Rightarrow T_2 = 2\pi \sqrt{\frac{m + 2m'}{2\Delta x g}} \quad (2)$$

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23. A volleyball can be modelled as a non-stretchable but flexible spherical envelope of mass m and radius R filled with air at excess pressure Δp . The excess pressure remains unchanged with small deformation in the volleyball when it is hit or when it strikes some rigid surface. Mass of the air inside the volleyball can be neglected. If such a volleyball strikes a rigid wall and bounces back without losing speed, how long will the ball remain in contact with the wall?

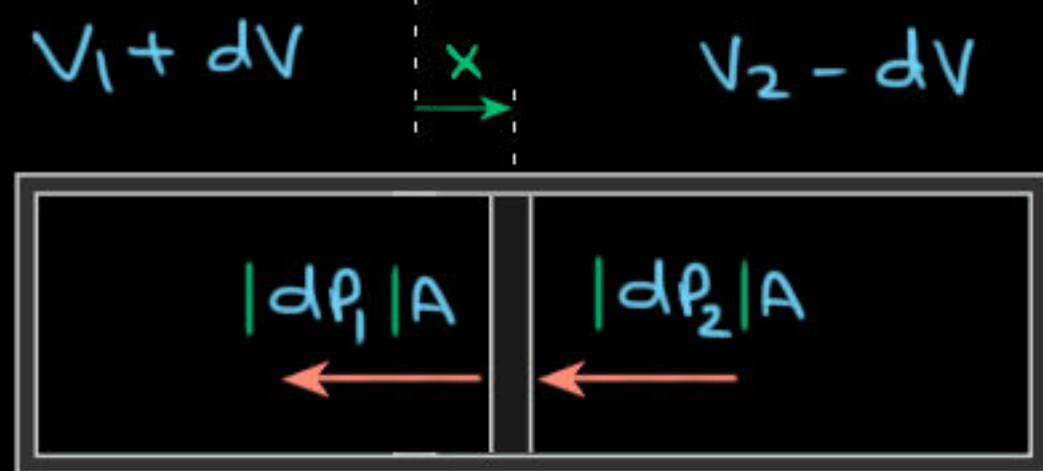
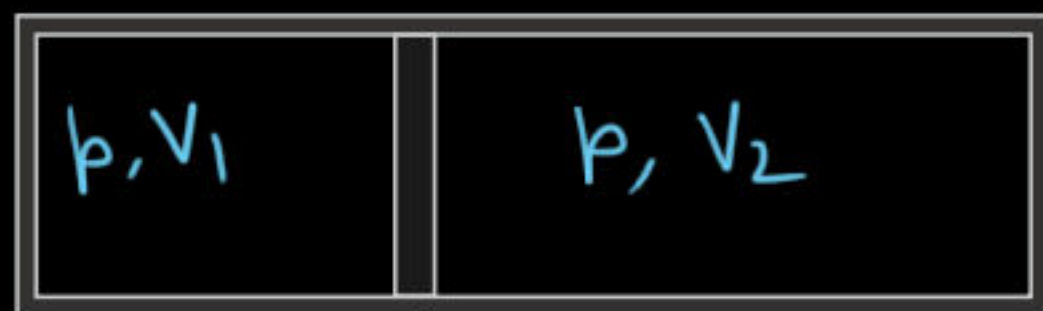
By action reaction, ball and wall exert $\Delta p \cdot A$ on each other.

$$\begin{aligned}
 \therefore F_{\text{restoring}} &= \Delta p \cdot A \\
 &= \Delta p \pi [R^2 - (R-x)^2] \\
 &= \Delta p \pi R^2 \left[1 - \left(1 - \frac{x}{R} \right)^2 \right] \\
 &\approx \Delta p \pi R^2 \cdot \frac{2x}{R} \\
 &= \underbrace{2\pi \Delta p \cdot R}_k x
 \end{aligned}$$

by Ankit Singhvi

$$\begin{aligned}
 \text{Contact time} &= T_{1/2} \\
 &= \pi \sqrt{\frac{m}{k}} \\
 &= \pi \sqrt{\frac{m}{2\pi \Delta p R}}
 \end{aligned}$$

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∴ Isothermal, $pV = \text{constant}$

$$\Rightarrow p dV + V dp = 0$$

$$\Rightarrow |dp| = p \frac{dV}{V}$$

24. A heat-insulating piston of mass m divides a horizontal cylinder into two chambers. Area of a circular section of the cylinder is A . Initially when the piston is in equilibrium, pressure in both the chambers is p and volumes of the chambers are V_1 and V_2 . The chambers are maintained at temperatures T_1 and T_2 . The piston is given a small horizontal displacement and then released. Find period of its oscillations.

On piston:

$$F_{\text{restoring}} = (|dp_1| + |dp_2|) A$$

$$= \left(p \frac{dV}{V_1} + p \frac{dV}{V_2} \right) A$$

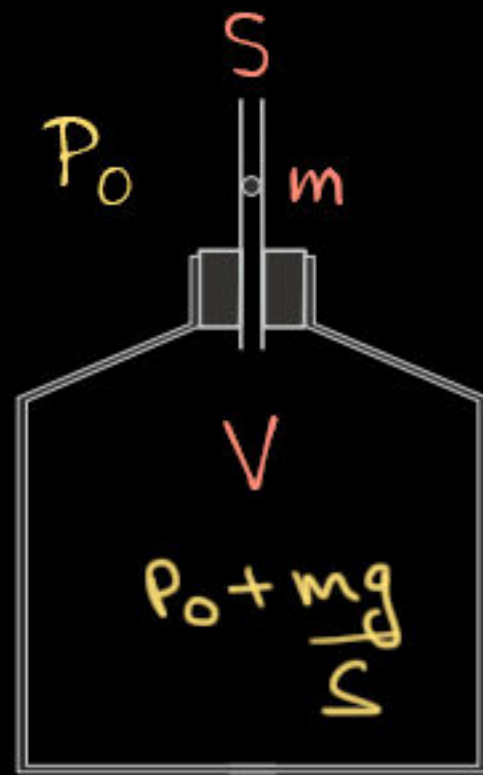
$$= \underbrace{\frac{pA^2(V_1 + V_2)}{V_1 V_2}}_k dx$$

$$T = 2\pi \sqrt{\frac{m V_1 V_2}{pA^2(V_1 + V_2)}}$$

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At equilibrium:



25. Opening of a glass bottle containing an ideal monoatomic gas is tightly closed with a cork fitted with a thin frictionless glass tube of cross-section area S . Inner volume of the tube is negligibly small as compared to volume V of the bottle. A small glass marble of mass m , when dropped in the tube, it starts oscillating up and down in the tube without friction. The marble exactly fits inside the tube so that there is no leakage of air. Denoting atmospheric pressure by p_0 , find expression for period of oscillations of the marble. Treat the glass, the cork and the material of the marble as perfect insulator of heat.

• Adiabatic, $PV^\gamma = \text{constant}$

$$\Rightarrow dP V^\gamma + P \gamma V^{\gamma-1} dV = 0$$

$$\Rightarrow |dP| = \frac{\gamma P dV}{V}$$

On marble:

$$F_{\text{restoring}} = dP \cdot S$$

$$= \frac{\gamma}{3} P \frac{dV}{V} \cdot S$$

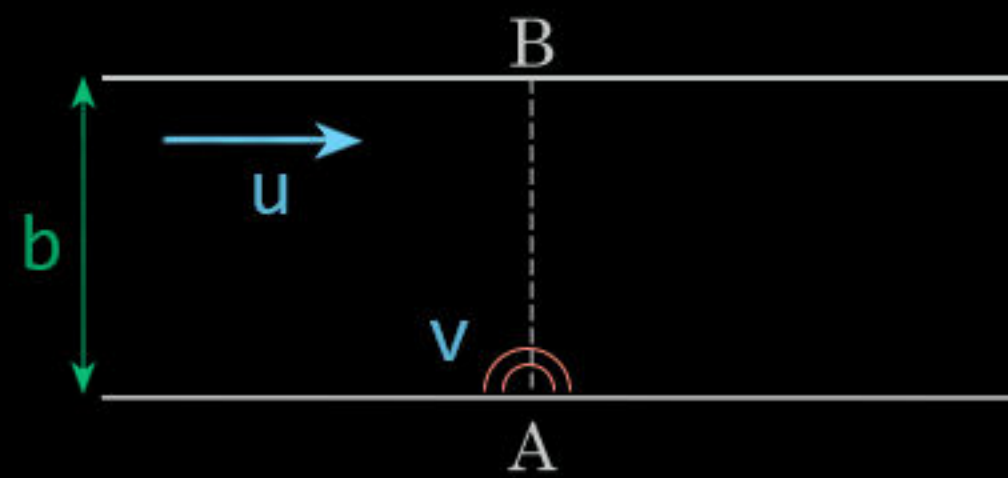
$$= \frac{\gamma}{3} \left(P_0 + \frac{mg}{S} \right) \frac{S dx}{V} \cdot S$$

$$= \frac{\gamma}{3V} (P_0 S + mg) S dx$$

$$T = 2\pi \sqrt{\frac{m 3V}{5S(P_0 S + mg)}}$$

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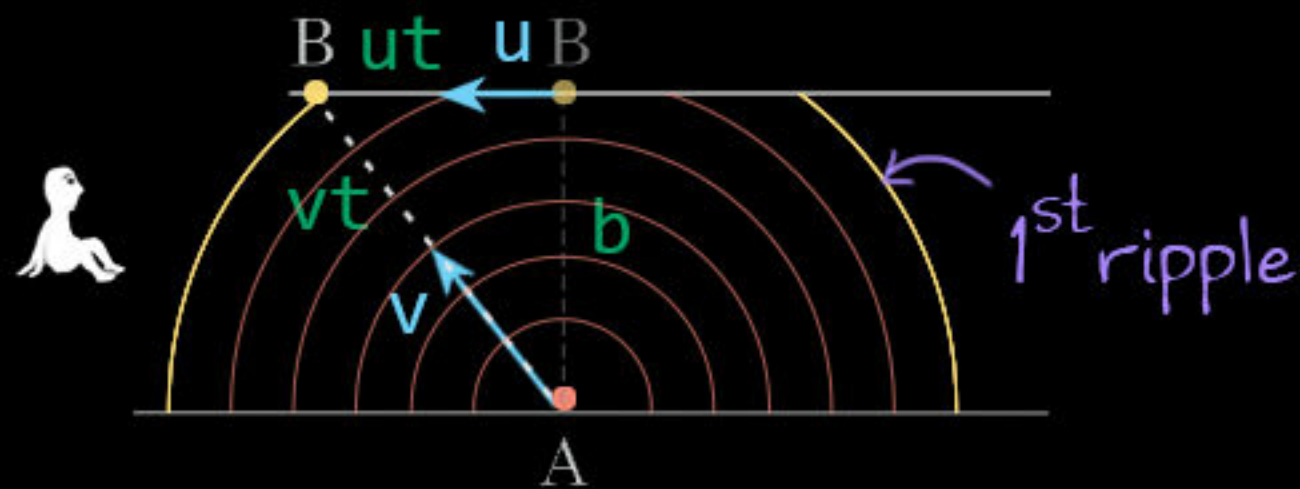
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27. When a stone is dropped at point A in the water near a bank of a river, ripples produced spread in the water. Width of the river is b , water in it flows everywhere at a velocity u and velocity of propagation of surface wave relative to water is v . How long after the stone was dropped, will the ripples reach the point B on the opposite bank in front of A? Consider the conditions $v > u$, $v = u$ and $v < u$.

Concept: Stone is dropped 'inside' the river. So, concentric ripples will travel along the river and their center will be at rest w.r.t river. However, B is a point on bank, so w.r.t river it will travel left.

w.r.t river



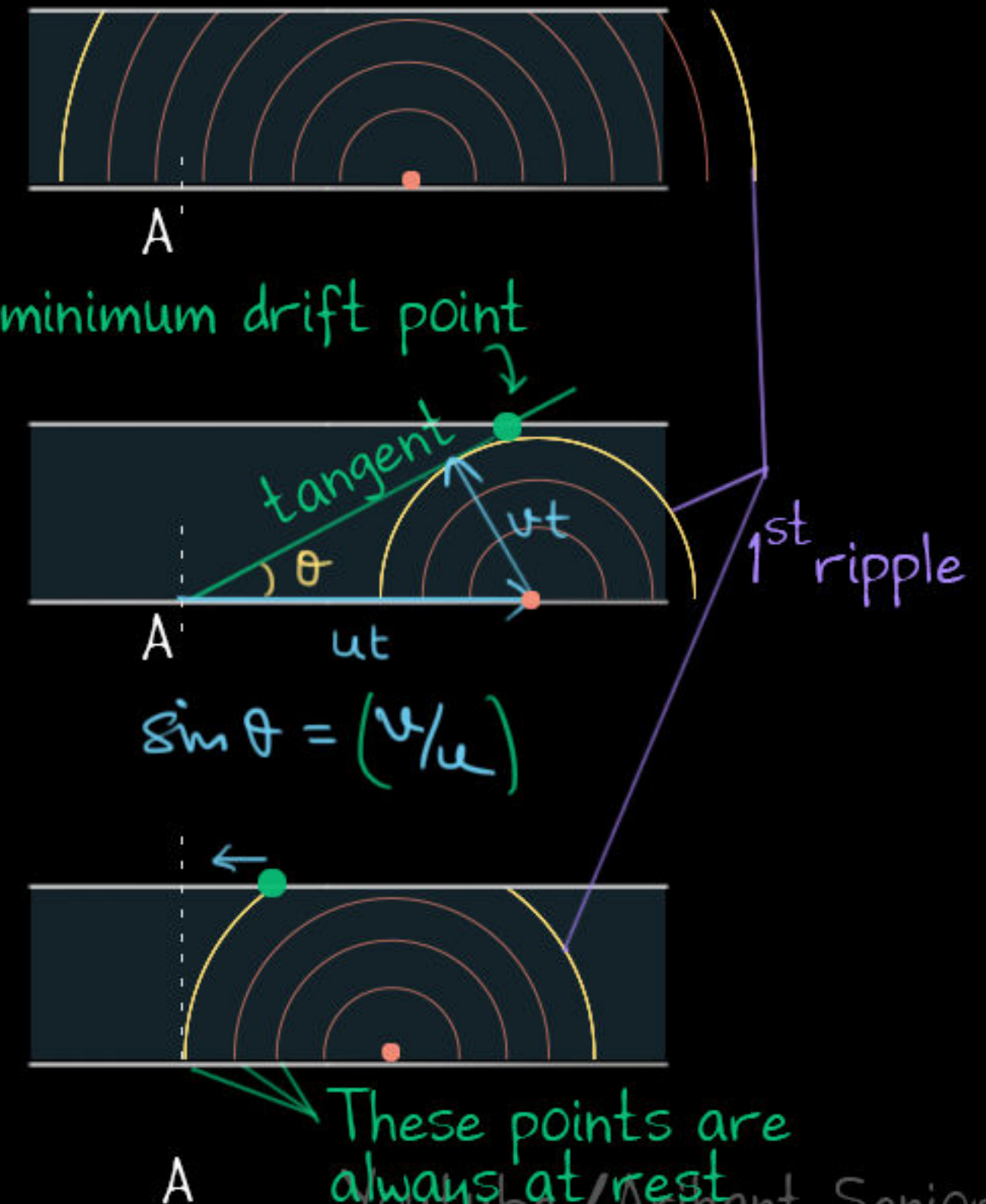
$$v^2 t^2 - u^2 t^2 = b^2$$

$$\Rightarrow t = \frac{b}{\sqrt{v^2 - u^2}} \quad \text{If } v > u$$

Never If $v < u$

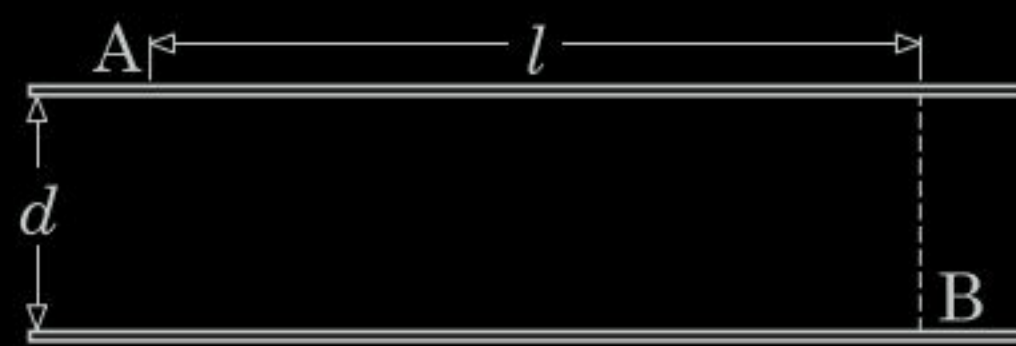
Never If $v = u$

w.r.t ground



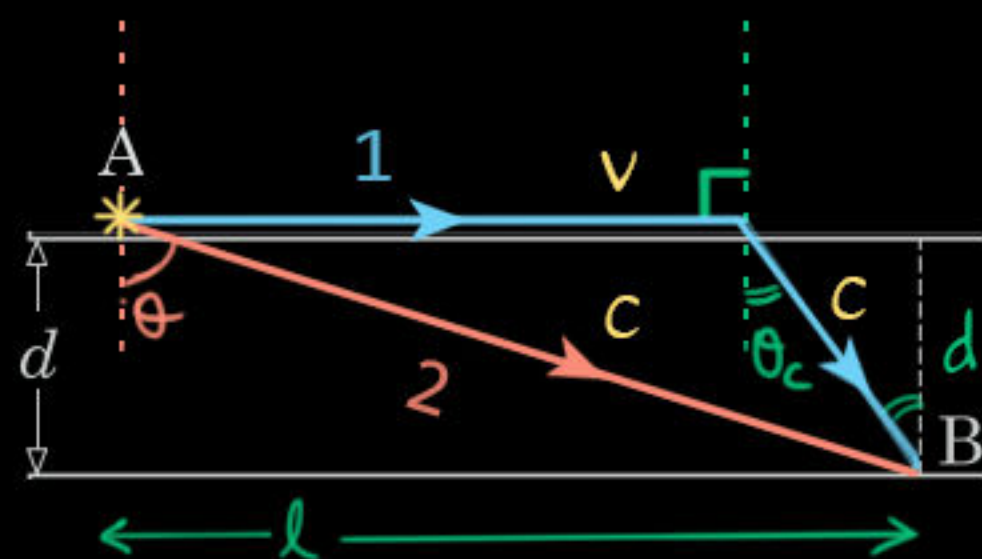
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Let A be a light source. And c and v be light velocities in two different media.

Then, applying the Fermat's theorem, ray that reaches B will take route of shortest time.



$$\frac{v}{\sin \theta} = \frac{c}{\sin \theta_c} \Rightarrow \text{Right Triangle with hypotenuse } v, \text{ opposite side } c, \text{ and angle } \theta_c$$

28. Two thin metal rods are arranged in water parallel to each other a distance d apart. Speeds c and v of sound in the water and in the rod respectively are not equal. If one of the rods is tapped at point A, find the shortest time the sound takes to travel from the point A to a point B on the other rod.

$$1: \theta_c < \theta \Rightarrow \sin \theta_c < \sin \theta \Rightarrow \frac{c}{v} < \frac{l}{\sqrt{l^2 + d^2}}$$

$$t = \frac{l - d \tan \theta_c}{v} + \frac{d}{\cos \theta_c \cdot c}$$

$$= \frac{l \sqrt{v^2 - c^2} - d c}{v \sqrt{v^2 - c^2}} + \frac{d v}{c \sqrt{v^2 - c^2}}$$

$$= \frac{l}{v} + \frac{d(v^2 - c^2)}{c v \sqrt{v^2 - c^2}} = \frac{l}{v} + \frac{d \sqrt{v^2 - c^2}}{c v} //$$

$$2: \theta_c \geq \theta \Rightarrow \frac{c}{v} \geq \frac{l}{\sqrt{l^2 + d^2}}$$

$\therefore$ As the crow flies!

$$\text{i.e., } t = \frac{\sqrt{l^2 + d^2}}{c}$$

We know:

1 Intensity after interference is:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

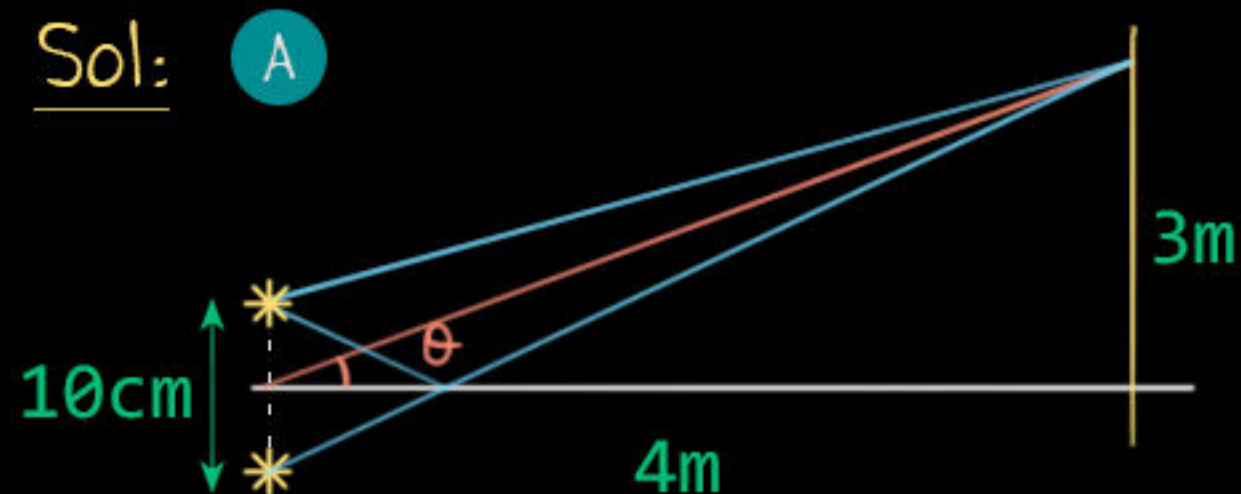
$$= 4I_0 \cos^2 \phi/2 \quad (\text{if } I_1 = I_2)$$

2 On reflection there is NO phase change as sound wave is longitudinal.

3 $I \propto A^2$

Sol:

A



$$\phi = \frac{d \sin \theta}{\lambda} \cdot 2\pi = \frac{(0.1) \frac{3}{5}}{(340/2000)} \cdot 2\pi = \frac{12\pi}{17}$$

29. Walls, floor and the ceiling of a large hall are covered with a perfect sound absorbing coating. A powerful point source that can emit sound waves of frequency 2.0 kHz isotropically in all directions is installed at a height 5.0 cm above the floor. Speed of sound waves in the air is 340 m/s. A small but very sensitive microphone is installed at a horizontal distance 4.0 m from the source and a height 3 m above the floor. The microphone is connected to a very sensitive voltmeter, reading of which is proportional to amplitude of sound waves received by the microphone. When the sound source is switched on, the voltmeter reads 0.10 V.

(a) If sound absorbing coating on the floor is completely removed, how much will the voltmeter read?

(b) Now the floor is again covered with some inferior quality of sound absorbing material that absorbs 50% of incident sound energy, how much will the voltmeter read?

$$\text{Reading } R \propto A \propto \sqrt{I}$$

$$\therefore \frac{R_1}{R_2} = \frac{\sqrt{I_0}}{\sqrt{4I_0 \cos^2 \phi/2}}$$

$$\Rightarrow R_2 = 2 \cos \phi/2 R_1$$

$$= 2 \cos \left(\frac{6\pi}{17} \right) (0.1) = 0.09 \text{ V}$$

by Ankit Singhvi

$$\text{B } I = I_0 + \frac{I_0}{2} + 2\sqrt{I_0 \frac{I_0}{2}} \cos \frac{12\pi}{17}$$

$$= 0.65 I_0$$

$$\therefore \frac{R_1}{R_2} = \frac{\sqrt{I_0}}{\sqrt{0.65 I_0}}$$

$$R_2 \approx 0.08 \text{ V}$$

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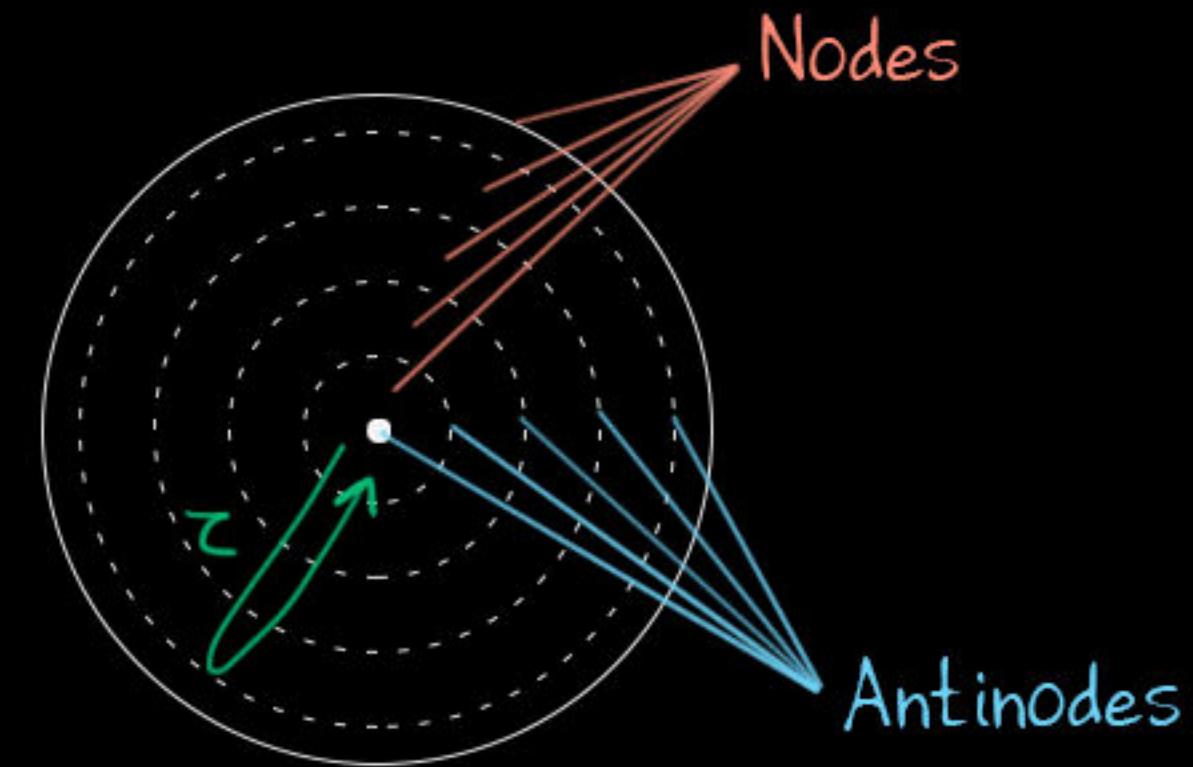
30. In a cylindrical vessel of radius r water is filled up to some height. A small iron ball is dropped at the centre of water surface. A time τ after the ball is dropped, water surface starts moving up and down with a portion having amplitude more than any other place on the surface. Explain the reason for this occurrence and find an expression to obtain a best approximate value of the speed of the surface waves.

When we drop a stone in water, ripples/surface waves/waveforms generate and travel outwards from the impact location.

As a wavelet moves outward, same energy gets distributed in ever increasing circumference. Therefore its amplitude decreases.

These waveforms reflect back from cylindrical walls and interfere with incoming waveforms to form standing waves.

The amplitude of these standing waves will therefore again increase towards the center, with maximum at the center.



$$\tau = \frac{2r}{v}$$

$$\Rightarrow v = \frac{2r}{\tau}$$

Velocity of detector be v_d

By dopplers effect:

$$\left(\frac{c + v_d}{c} \right) \nu_0 = \nu$$

$$\Rightarrow v_d = \left(\frac{\nu - \nu_0}{\nu_0} \right) c$$

∴ Duration of beep as

heard by detector, $t = \frac{\Delta x}{v_d}$

$$= \frac{\nu_0 \Delta x}{(\nu - \nu_0) c}$$

31. A sound detector is moving with a constant velocity towards a stationary sound source. The source emits a beep of sound of frequency $\nu_0 = 262 \text{ Hz}$ that propagates with speed $c = 330 \text{ m/s}$. The detector detects the beep of sound over a distance $\Delta x = 78 \text{ m}$ and registers a frequency $\nu = 275 \text{ Hz}$. Find duration of the beep emitted by the source.



Cycles released = cycles detected.

$$\therefore \nu_0 t_0 = \nu t$$

$$\Rightarrow t_0 = \frac{\nu}{\nu_0} t = \frac{\nu \Delta x}{(\nu - \nu_0) c} //$$